

Isoscalar Monopole Resonance of the Alpha Particle: A Prism to Nuclear Hamiltonians

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We present an *ab initio* study of the isoscalar monopole excitations of ${}^4\text{He}$ using different realistic nuclear interactions, including modern effective field theory potentials. In particular we concentrate on the transition form factor $F_{\mathcal{M}}$ to the narrow 0^+ resonance close to threshold. $F_{\mathcal{M}}$ exhibits a strong potential model dependence, and can serve as a kind of prism to distinguish among different nuclear force models. Compared to the measurements obtained from inelastic electron scattering off ${}^4\text{He}$, one finds that the state-of-the-art theoretical transition form factors are at variance with experimental data, especially in the case of effective field theory potentials. We discuss some possible reasons for such a discrepancy, which still remains a puzzle.

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The isoscalar monopole strength of large nuclei has been extensively studied since the discovery of a giant monopole resonance in ${}^{144}\text{Sm}$ and ${}^{208}\text{Pb}$ [1]. The reason for the great interest in such excitations originates from their connection to the incompressibility modulus of infinite nuclear matter [2,3]. The alpha particle is a light nucleus that however has a binding energy per particle similar to that of large systems and a high central density. While it possesses no bound excited states, it exhibits a very pronounced narrow resonance (${}^4\text{He}^*$) with the same quantum numbers 0^+ as the ground state, i.e., an isoscalar monopole resonance. Today, the development of few-body theories has reached a point where an *ab initio* calculation of the four-body isoscalar monopole transition strength can be carried out with high precision. As will become evident in the following, the comparison of such four-body results with experimental data can serve as a stringent test for nuclear Hamiltonians that are the sole ingredients of an *ab initio* quantum mechanical approach.

The four-nucleon isoscalar monopole resonance is located at $E_R^{\text{exp}} = -8.20 \pm 0.05$ MeV, with a width of 270 ± 50 keV [4], just above the first two-body breakup threshold $E_{\text{thr}}^p = -8.48$ MeV into a proton and a triton and below the next threshold $E_{\text{thr}}^n = -7.74$ MeV into a neutron and ${}^3\text{He}$. A summary of the experimental studies of the spectrum of ${}^4\text{He}$ can be found in Ref. [5]. Valuable information about the nature of the resonance is given by the transition form factor $F_{\mathcal{M}}(q)$ measured in electron scattering experiments [${}^4\text{He}(e, e'){}^4\text{He}^*$] at various momentum transfers q . Similarly to the case of the elastic form factor, the q dependence of $F_{\mathcal{M}}$ reflects the dynamics at various interaction ranges.

The progress in *ab initio* few-body methods allows us today to obtain accurate results for observables in light nuclear systems using realistic potential models (see the

review in Ref. [6]). In recent years the debate regarding potential models has been boosted, especially after the introduction of the effective field theory (EFT) strategy in nuclear physics [7]. At present, both phenomenological realistic and chiral EFT potentials are used in *ab initio* calculations, but large differences are found only for very few observables, e.g., for the polarization observable A_y of p - ${}^3\text{He}$ scattering [8]. In this Letter, we point out that the calculated ${}^4\text{He}$ isoscalar monopole resonance transition form factor $F_{\mathcal{M}}(q)$ depends dramatically on the nuclear Hamiltonian. Thus, it can serve as a kind of prism to distinguish among nuclear force models.

Main results.—The isoscalar monopole strength $S_{\mathcal{M}}(q, \omega)$ is in general a function of q and the energy transfer ω . It is given by

$$\begin{aligned} S_{\mathcal{M}}(q, \omega) &= \sum_n dn |\langle n | \mathcal{M}(q) | 0 \rangle|^2 \delta(\omega - E_n + E_0) \\ &= -\frac{1}{\pi} \text{Im} \langle 0 | \mathcal{M}^\dagger(q) \\ &\quad \times \frac{1}{\omega - H + E_0 + i\epsilon} \mathcal{M}(q) | 0 \rangle, \end{aligned} \quad (1)$$

where $|0\rangle$, $|n\rangle$ and E_0 , E_n are eigenfunctions and eigenvalues of the nuclear Hamiltonian H , and

$$\mathcal{M}(q) = \frac{G_E^s(q)}{2} \sum_i^A j_0(qr_i) \quad (2)$$

is the isoscalar monopole operator. Here $G_E^s(q) = G_E^p(q) + G_E^n(q)$ is the nucleon electric isoscalar form factor [9], r_i is the nucleon's position, and j_0 is the spherical Bessel function of 0th order. The monopole strength can be written as a sum of a resonance term $S_{\mathcal{M}}^{\text{res}}$ and a nonresonant background contribution $S_{\mathcal{M}}^{\text{bg}}$.

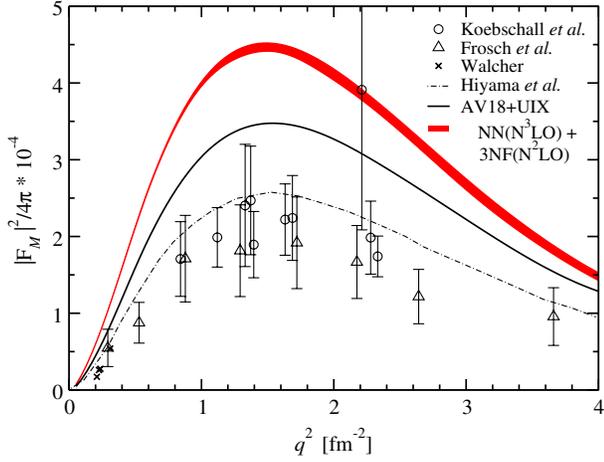


FIG. 1 (color online). Theoretical transition form factor $|F_{\mathcal{M}}(q^2)|^2$ with $G_E^n = 0$ calculated with various force models: AV18 + UIX (full line), $N^3\text{LO} + N^2\text{LO}$ (red band), result from Ref. [17] (dot-dashed line). Data from Frosch *et al.* [10], Walcher [4], and Köbschall *et al.* [11].

$$S_{\mathcal{M}}(q, \omega) = S_{\mathcal{M}}^{\text{res}}(q, \omega) + S_{\mathcal{M}}^{\text{bg}}(q, \omega). \quad (3)$$

For a narrow resonance one defines the resonance transition form factor

$$|F_{\mathcal{M}}(q)|^2 = \frac{1}{Z^2} \int d\omega S_{\mathcal{M}}^{\text{res}}(q, \omega). \quad (4)$$

In Fig. 1, we show results for $F_{\mathcal{M}}(q)$ with two different Hamiltonians including realistic three-nucleon forces (3NFs) in comparison to experimental data from inelastic electron scattering [4,10,11]. As Hamiltonians we use (i) the Argonne V_{18} (AV18) [12] NN potential plus the Urbana IX (UIX) [13] 3NF, (ii) an EFT based potential, where we take the NN potential [14] at fourth order ($N^3\text{LO}$) in the chiral expansion augmented by a 3NF at order $N^2\text{LO}$ [15]. The Coulomb potential is taken into account in all calculations. Both the EFT and the AV18 NN potentials reproduce the NN scattering phase shifts with high precision ($\chi^2/\text{datum} \approx 1$). In the EFT calculations, two different parametrizations of the 3NF have been used, leading to the red band in Fig. 1. The chiral low energy constants c_D and c_E have been determined either by setting c_D to a reasonable value and then fitting c_E to the three-nucleon binding energies [15] ($c_D = 1$ and $c_E = -0.029$) or by fitting to the ${}^3\text{H}$ binding energy and beta decay [16] ($c_D = -0.2$ and $c_E = -0.205$). We also display the result of a previous calculation by Hiyama *et al.* [17], with the AV8' potential, a reduced version of AV18, and a simplified central 3NF, fitted to the binding energy of ${}^3\text{H}$. All three Hamiltonians reproduce the ${}^4\text{He}$ experimental binding energy within one percent. Surprisingly, the results for $F_{\mathcal{M}}(q)$ strongly depend on the Hamiltonian. Furthermore, the realistic Hamiltonians fail to reproduce the experimental data. In particular, this is true for the EFT

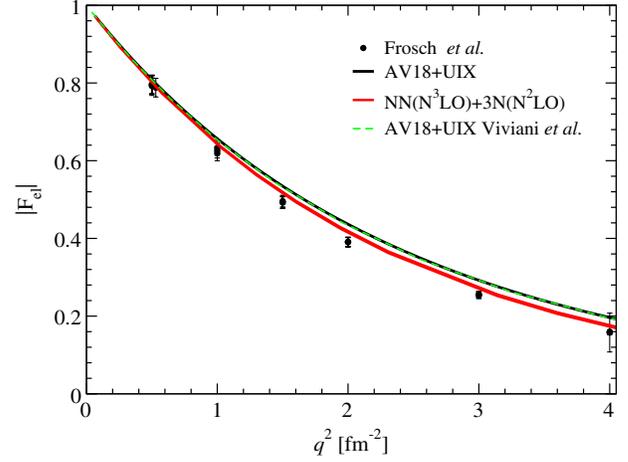


FIG. 2 (color online). Elastic form factor $|F_{\text{el}}(q^2)|$ of ${}^4\text{He}$ calculated with various force models: AV18 + UIX (full line), $N^3\text{LO} + N^2\text{LO}$ (red band), result from Ref. [21] with AV18 + UIX (dot-dashed line). Data from Frosch *et al.* [36].

forces that predict a transition form factor twice as large as the measured one.

In contrast, the realistic Hamiltonians lead to rather similar results for the elastic form factor $F_{\text{el}}(q)$ of ${}^4\text{He}$, defined as

$$F_{\text{el}}(q) = \frac{1}{Z} \langle 0 | \mathcal{M}(q) | 0 \rangle. \quad (5)$$

In Fig. 2, $F_{\text{el}}(q)$ is shown for the AV18 + UIX model and for the chiral EFT potentials. The fact that the results do not differ significantly is not very surprising, since both Hamiltonians give a very similar result for the radius: 1.432(2) fm [18] for AV18 + UIX and 1.464(2) fm for $N^3\text{LO}$ plus the $N^2\text{LO}$ of Ref. [16], which is not far from the experimental value of 1.463(6) fm (obtained from the charge radius of Ref. [19] as explained in Ref. [20]). Also shown in Fig. 2 is the result by Viviani *et al.* [21] with the AV18 + UIX potential, which is indistinguishable from ours, proving the level of accuracy of contemporary four-body calculations.

Computational method.—Our calculations are based on the diagonalization of the Hamiltonian on a square integrable hyperspherical harmonics (HH) basis. The HH convergence is accelerated using the Suzuki-Lee unitary transformation, which then leads to the effective interaction HH (EIH) method [22,23]. The high accuracy of this approach can be inferred from the benchmark results in Ref. [24] and also here from Table I, where we present the binding energies of three- and four-body nuclei obtained from EFT potentials including 3NFs. We agree with other methods at the 10 keV level.

Results for $S_{\mathcal{M}}(q, \omega)$ are often obtained by discretizing the continuum, where the Hamiltonian is represented on a finite basis of square integrable functions and is then diagonalized to obtain the eigenvalues e_ν and eigenfunctions

TABLE I. Ground state energies in MeV for ${}^3\text{H}$, ${}^3\text{He}$, and ${}^4\text{He}$ with N^3LO [14] and N^2LO (parametrizations from Ref. [15] or Ref. [16]). Comparison of present results (EIHH) with no core shell model (NCSM) and hyperspherical harmonics (HH) results.

3NF from Ref. [15]	EIHH	NCSM [15]	HH [25]	Nature
${}^3\text{H}$	-8.474(1)	-8.473(5)	-8.474	-8.48
${}^3\text{He}$	-7.734(1)		-7.733	-7.72
${}^4\text{He}$	-28.357(7)	-28.34(2)	-28.37	-28.30
3NF from Ref. [16]	EIHH	NCSM [16]		Nature
${}^3\text{H}$	-8.472(3)	-8.473(4)		-8.48
${}^3\text{He}$	-7.727(4)	-7.727(4)		-7.72
${}^4\text{He}$	-28.507(7)	-28.50(2)		-28.30

$|\nu\rangle$. In this way one achieves an ill defined discretized representation of $S_{\mathcal{M}}(q, \omega)$. On the contrary in the Lorentz integral transform (LIT) approach [26,27] a continuum discretization can be properly used to reach the correct continuum spectrum (for various benchmark tests of the LIT approach we refer the reader to Ref. [27]).

In the LIT case one has

$$\mathcal{L}_{\mathcal{M}}(q, \sigma, \Gamma) = -\frac{1}{\pi} \text{Im}\langle 0|\mathcal{M}^\dagger(q) \times \frac{1}{\sigma - H + E_0 + i\Gamma} \mathcal{M}(q)|0\rangle, \quad (6)$$

where Γ is finite [compare with Eq. (1)]. It is easy to prove that $\mathcal{L}_{\mathcal{M}}(q, \sigma, \Gamma)$ is connected to $S_{\mathcal{M}}(q, \omega)$ by an integral transform with a Lorentzian kernel $K(\omega, \sigma, \Gamma) = \frac{\Gamma}{\pi} \frac{1}{(\omega + E_0 - \sigma)^2 + \Gamma^2}$,

$$\mathcal{L}_{\mathcal{M}}(q, \sigma, \Gamma) = \int d\omega K(\omega, \sigma, \Gamma) S_{\mathcal{M}}(q, \omega). \quad (7)$$

Since Γ is finite the calculation of $\mathcal{L}_{\mathcal{M}}(q, \sigma, \Gamma)$ is a bound-state-like problem and thus it is legitimate to represent the Hamiltonian on a basis of square integrable functions, which then leads to the following expression:

$$\mathcal{L}_{\mathcal{M}}(q, \sigma, \Gamma) = \frac{\Gamma}{\pi} \sum_{\nu=1}^N \frac{|\langle \nu|\mathcal{M}(q)|0\rangle|^2}{(\sigma - e_\nu + E_0)^2 + \Gamma^2}. \quad (8)$$

The number of basis functions N depends in our EIHH calculation on the maximal value K_{max} of the HH grand angular momentum quantum number K . Note that the set $(e_\nu, |\nu\rangle)$ is Γ independent, but that the convergence of $\mathcal{L}_{\mathcal{M}}$ is strongly correlated with Γ : if Γ is lowered a higher density of states is needed; hence, K_{max} and thus N have to be increased. In our present case we reached convergence of $\mathcal{L}_{\mathcal{M}}$ with Γ as small as 5 MeV employing more than 10^5 states $|\nu\rangle$. Even if this is not sufficient to resolve the ${}^4\text{He}^*$ resonance width of 270 keV, we are nevertheless able to determine the resonance energy E_R . In fact our discrete spectrum shows as the first excitation above the ${}^4\text{He}$ ground state a very pronounced state with strength

$s_1(q) = |\langle 1|\mathcal{M}(q)|0\rangle|^2$; thus, we identify the corresponding energy e_1 with E_R . We find the following results: $E_R = -7.40(20)$ MeV (AV18 + UIX) and $E_R = -7.50(30)$ MeV ($\text{N}^3\text{LO} + \text{N}^2\text{LO}$). Note that error estimates are made by studying the EIHH convergence, i.e., the K_{max} dependence of E_R and that the E_R value for $\text{N}^3\text{LO} + \text{N}^2\text{LO}$ is obtained extrapolating to higher K_{max} with an exponential ansatz $E(K_{\text{max}}) = E^\infty + ae^{-bK_{\text{max}}}$ as in Ref. [28].

In general one obtains the full strength $S(q, \omega)$ from the inversion of a converged LIT, but one has to be aware that structures much smaller than Γ cannot be resolved and thus a regularization procedure has to be used [29,30]. Our standard inversion method consists in an expansion of the response on a set of I continuous functions and in fitting the calculated $\mathcal{L}_{\mathcal{M}}(q, \sigma, \Gamma)$ on the corresponding linear combinations of the transformed basis functions [30]. Note that the regularization consists in the fact that I should not become so large that structures much smaller than Γ appear in the inversion result. We implement many different basis sets and choose the best fit for a given I [for example, we use basis sets of the form $E^\beta \exp(-\alpha E/i)$ with $i = 1, \dots, I$ and different α values; β is known from threshold behavior of the response, e.g., $\beta = 1/2$ for $S_{\mathcal{M}}$]. For example, such a calculation has been made in Refs. [31,32] for the full ${}^4\text{He}$ longitudinal response beyond the ${}^4\text{He}^*$ resonance.

In the presence of a narrow resonance, as in our case, an explicit resonance should be added to the basis, e.g., a Lorentzian with free parameters γ and ω_R : $[(\omega - \omega_R)^2 + (\gamma/2)^2]^{-1}$. If the LIT is determined with a sufficiently small Γ , then the position, width, and strength of the resonance can be determined in the inversion [33]. If we proceed in this way in our present case, imposing $\omega_R = E_R$, we obtain the best fits with $\gamma \rightarrow 0$. This reflects the absence of states $|\nu\rangle$ in the vicinity of E_R . We can nonetheless determine, besides E_R , also the resonance strength $f_R(q)$. For this purpose we note that the above defined strength $s_1(q)$ is equal to the sum of $f_R(q)$ and a background contribution. Thus, formally, we can separate the resonance contribution from $\mathcal{L}_{\mathcal{M}}$:

$$\mathcal{L}_{\mathcal{M}}^{\text{bg}}(q, \sigma, \Gamma) = \mathcal{L}_{\mathcal{M}}(q, \sigma, \Gamma) - \frac{\Gamma}{\pi} \frac{f_R(q)}{(\sigma - E_R + E_0)^2 + \Gamma^2}. \quad (9)$$

Now we proceed as follows. We assume a value for $f_R(q)$ and allow a basis function of Lorentzian shape centered at E_R with $\gamma = 100$ keV in the inversion. If the trial value for $f_R(q)$ is too small (large) one finds an inversion with a positive (negative) resonant structure. The case where this vanishes corresponds to the correct value of the transition form factor $|F_{\mathcal{M}}(q)|^2 = f_R(q)/Z^2$ and the inversion result is just $S_{\mathcal{M}}^{\text{bg}}(q, \omega)$ (see Fig. 3). We would like to emphasize that the results are almost γ independent so long as γ remains small enough ($0 < \gamma \leq 200$ keV) that the Lorentzian approximates sufficiently well a δ function. For the AV18 + UIX potential the

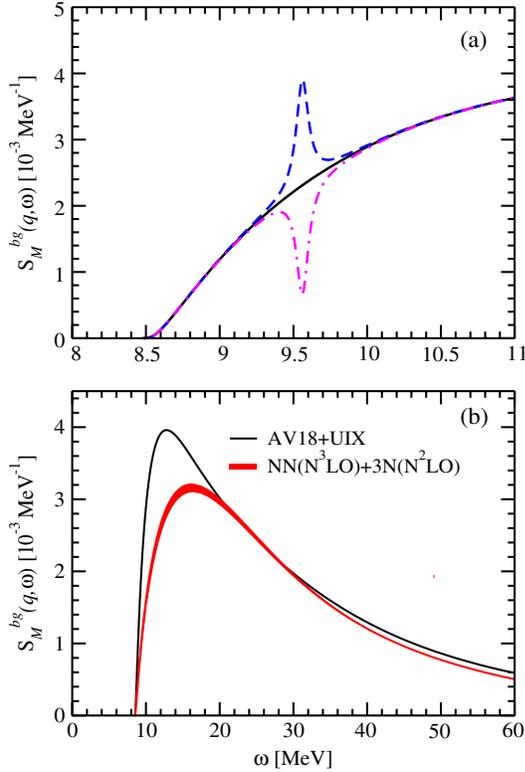


FIG. 3 (color online). (a) $S_M^{\text{bg}}(q, \omega)$ at $q = 1.5 \text{ fm}^{-1}$ for AV18 + UIX obtained with different values of f_R (see text): $f_R = |F_{\mathcal{M}}(q)|^2$ (full line), $f_R = 0.99|F_{\mathcal{M}}(q)|^2$ (dashed line), $f_R = 1.01|F_{\mathcal{M}}(q)|^2$ (dot-dashed line). (b) Nonresonant background contribution $S_M^{\text{bg}}(q, \omega)$: AV18+UIX (full line), $\text{N}^2\text{LO} + \text{N}^3\text{LO}$ (red band).

relative size of the background reduction, about 8%, is roughly q independent. For the $\text{N}^3\text{LO} + \text{N}^2\text{LO}$ interactions the reduction varies between 13% for $q = 0.25 \text{ fm}^{-1}$ and 22% for $q = 2 \text{ fm}^{-1}$.

In Fig. 3(b), the nonresonant monopole strength S_M^{bg} is shown on a larger energy range, into the far continuum region. One sees quite a difference between the results with the EFT and the AV18 + UIX forces. The former leads to a lower low-energy peak and tail than the latter. These results show the power of the LIT approach, which enables one to calculate the strength in the far four-body continuum by reducing a scattering-state problem to a bound-state problem in a rigorous way.

Analysis of the results.—The main findings of this Letter are the dramatic sensitivity of $F_{\mathcal{M}}(q)$ to the nuclear Hamiltonian and the large deviations of realistic calculations from the available experimental data. Even though one can contemplate the possibility of systematic experimental errors, the fact that the experimental results of Fig. 1 correspond to three different sets of data, makes it less likely. Thus we will now list possible sources for theoretical uncertainties. (i) Is our EIH expansion sufficiently convergent? As shown in Table II for a q value of 1.01 fm^{-1} , we find an excellent convergence for both F_{el}

TABLE II. $|F_{\text{el}}|$ and $s_1 = |\langle 1 | \mathcal{M}(q) | 0 \rangle|^2$ for $q = 1.01 \text{ fm}^{-1}$ as a function of the grandangular momentum K_{max} with $\text{N}^3\text{LO} + \text{N}^2\text{LO}$ [16].

K_{max}	12	14	16	18
$ F_{\text{el}} $	0.6248	0.6244	0.6242	0.6241
$10^4 s_1 / 4\pi Z^2$	4.59	4.75	4.85	4.87

and s_1 . (ii) Are there relevant two-body corrections to the one-body operator of Eq. (2)? Such corrections are of relativistic order and appear also in EFT only at fourth order [34] {also for $F_{\text{el}}(q)$ such two-body terms are negligible below $q = 2 \text{ fm}^{-1}$ [21]}. (iii) Can additional 3NF terms change the picture? This is not excluded; however, we notice that the 3NF effect at N^2LO on $F_{\mathcal{M}}(q)$ is rather mild (about 10%). (iv) Does the improper theoretical resonance position E_R affect the $F_{\mathcal{M}}(q)$ result? Both of our potential models (AV18 + UIX, $\text{N}^3\text{LO} + \text{N}^2\text{LO}$) overestimate E_R by almost the same amount (about 700 keV), but still lead to quite different transition form factors. On the other hand, the simplified force model used by Hiyama *et al.* [17] reproduces the correct E_R within 100 keV, and also leads to a much better description of $F_{\mathcal{M}}(q)$. One can envisage a correlation between the ability of a model to reproduce E_R and $F_{\mathcal{M}}$. In fact, if one considers that $F_{\mathcal{M}}$ is the Fourier transform of the transition density from ${}^4\text{He}$ to ${}^4\text{He}^*$, one can imagine that small differences in E_R are reflected in the resonant wave functions and yield larger differences in the transition density. Similar conclusions have been drawn in Ref. [35] in the study of p - ${}^3\text{H}$ scattering. However, the resonant behavior of the nuclear scattering amplitude is barely visible in the data, in contrast to the electromagnetic probe that amplifies the resonance signal considerably (see Fig. 1 of Ref. [11]).

Conclusions.—We have calculated the isoscalar monopole ${}^4\text{He} \rightarrow {}^4\text{He}^*$ transition form factor $F_{\mathcal{M}}(q)$ with realistic nuclear forces ($\text{N}^3\text{LO} + \text{N}^2\text{LO}$, AV18 + UIX). Unexpectedly the results are strongly dependent on the Hamiltonian. Therefore this observable is ideal for testing nuclear Hamiltonians. As surprising as the large potential model dependence is the fact that our $F_{\mathcal{M}}$ results are at variance with the experimental data; particularly large differences are found in the case of the chiral forces. It is very unlikely that corrections to the isoscalar monopole operator can lead to large effects. In order to clarify the situation it is highly desirable to have a further experimental confirmation of the existing data and in particular with increased precision. On the theory side further insight could be gained by an analysis of sum rules, transition densities, effects of D -wave components, and different 3NFs. These issues will be the subject of future studies.

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