Investigating the role of sliding friction in rolling motion: a teaching sequence based on experiments and simulations

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ABSTRACT

We designed a teaching-learning sequence on rolling motion, rooted in previous research about student conceptions, and proposing an educational reconstruction strongly centred on the role of friction in different cases of rolling. A series of experiments based on video analysis is used to highlight selected key concepts and to motivate students in their exploration of the topic; and interactive simulations, which can be modified on the fly by students to model different physical situations, are used to stimulate autonomous investigation in inquiry activities.
The activity sequence was designed for students on introductory physics courses and was tested with a group of student teachers. Comparisons between pre- and post-tests, and between our results and those reported in the literature, indicate that students' understanding of rolling motion improved markedly and some typical difficulties were overcome.

1. INTRODUCTION

Although rolling motion is a basic physics topic, included in all introductory courses, the understanding students achieve of it is often quite limited and unsatisfactory, as several studies have shown. Many research works have examined common student difficulties in approaching rotational and rolling motion [1-3], demonstrating that these difficulties are independent of students' background and their level of mathematical preparation.

Some articles were aimed to elucidate the main characteristics and crucial details of rolling motion. As an example, the rolling motion of cylinders has been investigated both on a horizontal plane [4-6] and down an incline at different angles [7]. In both cases the authors suggest that instructional strategies should focus on friction and relative velocity concepts, while paying special attention to the direction and magnitude of friction forces, and to the work done by them. In this paper we present an activity sequence designed to address students' difficulties as well as to help students acquire the elements of an explanatory model for the complex phenomena involved in rolling motion. The sequence proceeds through a combination of real experiments and interactive computer simulations, designed to sustain students' understanding, whose common theme is to explore in detail the role of friction in rolling motion.

The sequence design was rooted on a careful textbook analysis and on research findings on students' difficulties, and was also guided by the results obtained from the initial questionnaire we proposed to the group of student teachers involved in the first trial of the sequence. Twenty student teachers (ST) participated in the study. They performed the experimental and simulation activities in groups of three and completed the sequence in three sessions of 2 hours each. Assistants worked as facilitators, giving support where necessary. Our sources of data on ST's progress and ideas included two questionnaires, worksheets filled in during the experimental activities, discussions during and after the experiments, answers to written questions, and a final report in which they elaborated on what elements of the proposed sequence they considered essential.

One of the questionnaires was given before the activities (pre-test) and the other approximately two months after the end of the teaching sequence (post-test). Some questions were drawn from the literature, to make a comparison possible. The main purposes of testing the sequence were:

(i) to evaluate the effectiveness of the activity sequence in promoting student teachers' reflection on basic physics contents by linking experimental activity, theoretical analysis, and work with simulations;

(ii) to refine the sequence design, taking into consideration also student teachers' comments, attitudes and needs, to produce a version which is suitable for undergraduate students.

2. METHODOLOGY

We made a few fundamental decisions regarding the design of the teaching sequence which can be summarized as follows:

a) Propose activities based on a combination of real experiments and interactive simulations. Measurements are performed through the Tracker Video Analysis open source tool; while interactive simulations are designed and run within the freeware 2D simulation environment Algodoo. These software tools were chosen among others with similar features because, while being high quality products, they are free and easy to use, so that we hope student teachers will continue to use them in their future teaching in high school.

b) Let ST perform the experimental and modelling activities in small groups. They are guided through carefully sequenced activities to make observations that they can use as the basis for their models.

c) Engage ST in the step-by-step process of constructing a qualitative model that they can use to predict and explain the behaviour of rolling bodies. When some degree of formalization becomes necessary, only basic mathematics is used, always in tight connection with qualitative reasoning.

d) Encourage autonomous exploration of a complex problem starting from an initial motivating question (specifically, in this case, the question concerns collision between two rolling spheres). ST analyze the problem de-structuring it into sub-problems that they know how to solve, by designing Algodoo simulations. Such approach requires students to plan a solution through a sequence of steps while keeping in mind the global issue, and leads them to a thorough exploration of the relationship between friction and rolling motion. Moreover, observing ST work and discuss in groups during this activity provides us insight on the role that modelling activity has in scaffolding students' knowledge.

3. DESCRIPTION OF THE TEACHING SEQUENCE

We identified some central themes, focusing on the ubiquitous role that the sliding friction force takes in different cases of rolling motion. Schematically, the main aspects we highlighted with students were:

- In accelerated pure rolling of a body subject to an external force, static friction force appears; however, such force plays no role in pure rolling motion at constant speed.
- Kinetic friction force appears in the case of rolling with slipping, for example when, in accelerated rolling under an external force, static friction is insufficient. In the cases of a sliding ball on a frictionless plane which enters a plane with friction, or immediately after a collision between two spheres, kinetic friction has the crucial role of changing the relative values of linear and angular velocities, leading the body to the condition of pure rolling.
- When a body undergoes rolling motion under the action of a horizontal force applied in an arbitrary point along the radius, the sign of the friction force depends on the point where the force is applied (and for a particular choice of the application point, the friction force is zero).

The sequence of activities is organized into six parts: (A) introductory examples and experiments; (B) kinematics of rolling motion; (C) rolling motion on horizontal plane; (D) rolling motion on an inclined plane; (E) rolling motion on horizontal plane under an external force; (F) head-on collisions between a rolling cue ball and a stationary ball.

In the following we describe the main features of the sequence, paying special attention to experiments and simulations involved. We also focus on students' difficulties, and in discussing them we compare known results from the literature to our findings from student teachers' pre-activity tests.

3.1. Introductory examples and experiments: sliding friction forces

It is well known that students usually neglect the fact that static friction force does not have a unique value, but increases, to prevent relative motion, up to some limit, beyond which motion occurs. They often do not realize that it is just the threshold value at breakaway which is related to the normal force through the coefficient of static friction. Student teachers' answers to question 2 in the pre-test show that only a small fraction (21%) of them recognizes that the value of the friction force has to be less or equal than the product between the static friction coefficient and the normal force, while most of them assign to the friction force its maximum value only.
A further issue related to the nature of friction, which hinders a full understanding of rolling motion, is the fact that students often ignore that action-reaction friction forces are applied on both the surfaces in contact, as specified by Newton's third law. [8,9]

With the aim of overcoming these difficulties we propose some experimental activities. In particular:

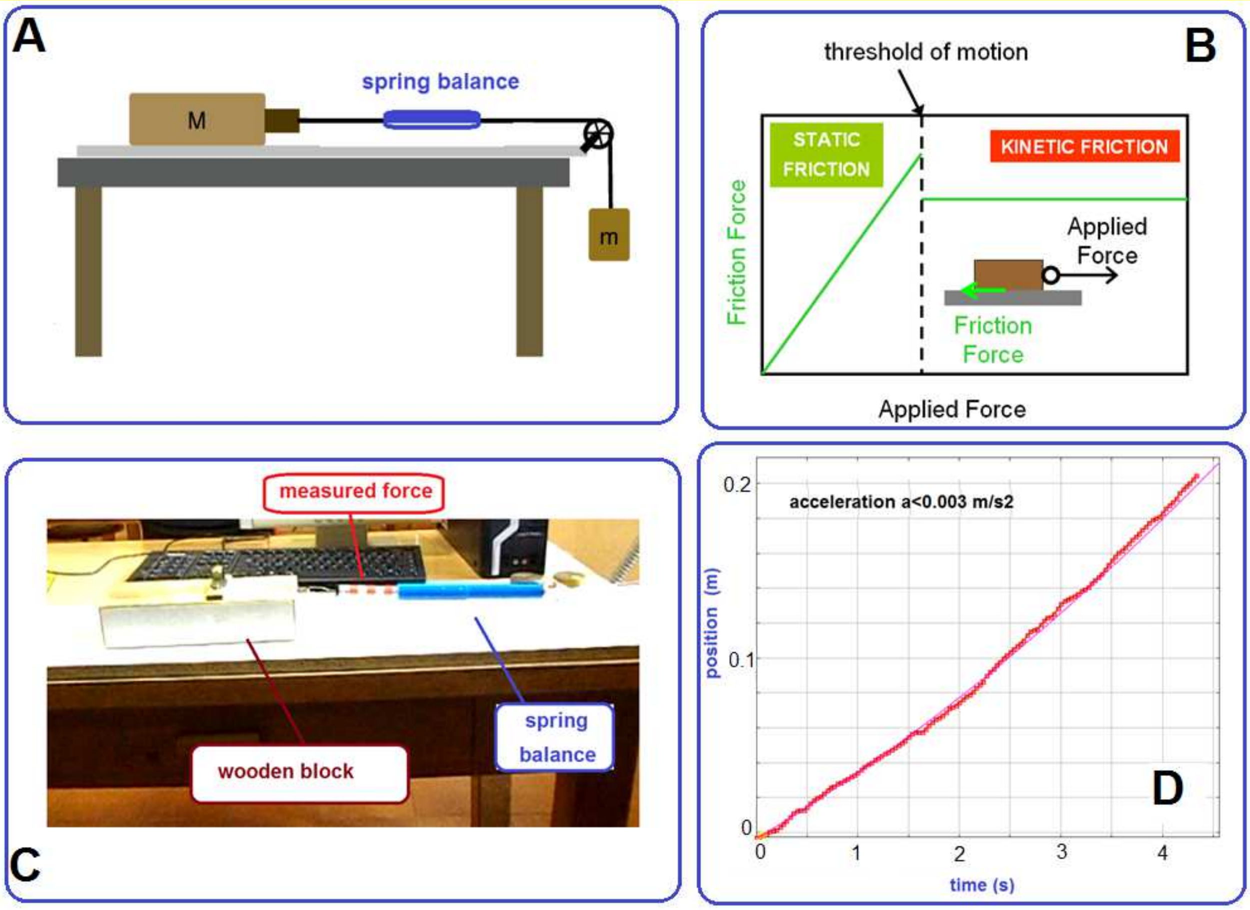


Figure 1: (A) Schematic representation of the apparatus we used to measure sliding friction force. (B) The characteristic graph of the magnitude of friction force as a function of the applied force, before and after the threshold of motion. (C) A photo of the apparatus used. (D) Position-time graph obtained for the value of the suspended mass *m* which almost exactly balances the kinetic friction force.

1. A demonstration to show that action-reaction friction forces occur on the two surfaces in contact: a block is pulled while placed on different materials, for example a strip of paper, a woollen scarf, a small cart. ST observe that the paper, the scarf, and the cart are dragged by the block, due to the friction force exerted by it.
2. A simple experiment for investigating Coulomb's law of sliding friction. A wooden block with mass M is dragged across a wooden plane by weights suspended over a pulley. A spring balance is attached to the front of the block (Fig. 1 A and C). Masses are added slowly until the breakaway

value F_t is reached and measured also by the spring balance. The static friction coefficient is obtained as $\mu_S = \frac{F_t}{Mg}$.

To measure the kinetic friction coefficient the suspended mass is reduced until the block, manually set in motion, moves at constant speed (as seen from the Tracker x vs. time plot, Fig. 1D). In this condition the spring balance measures the kinetic friction force, F_k.

3. 2. Kinematic of the rolling body and relative motion

Students in introductory physics courses have great difficulty distinguishing between the velocity of a point on a rigid wheel, ball or cylinder as with respect the centre of mass, or the ground.[1] The results obtained with the group of 20 student teachers involved in our study show similar difficulties (see Fig.2)

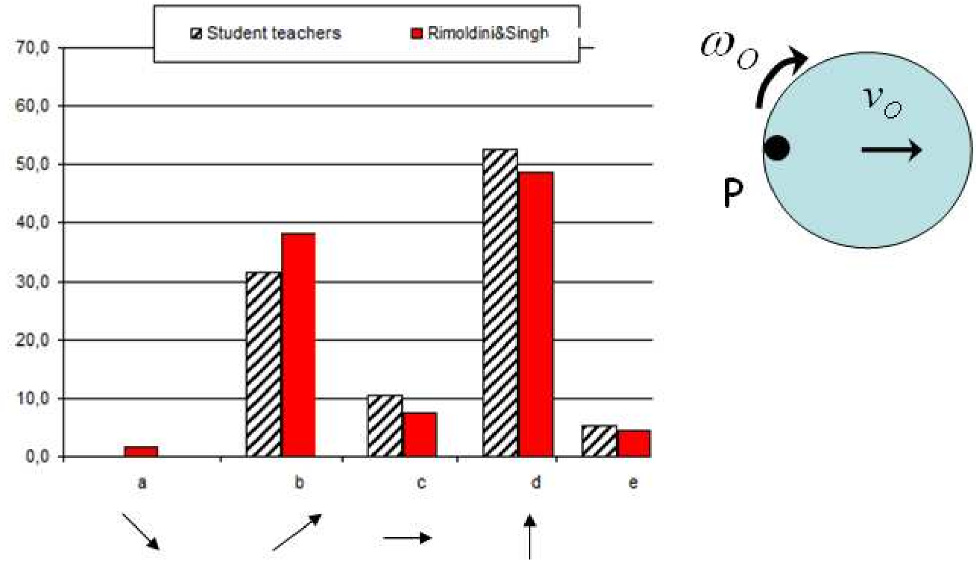


Figure 2
Answers to Q1 of the pre-test. Only 32% of the ST identifies the exact direction of the speed of a point on the edge of the wheel in rolling motion. This result can be compared with the one of the students involved in the study of ref [1] (38%) (a sample of calculus- and algebra based introductory physics students and physics juniors who had learned rotational and rolling motion concepts in an intermediate level mechanics course.)

To help ST grasp the differences between the trajectory shapes and between the velocity vectors computed in the two different reference frames, we designed the following activity: ST record the motion of a rolling disk through a digital video camera, and then analyse the video with Tracker. They compare their predictions about the velocity vectors in the two reference frames with experimental results. Figure 3 shows, as an example, the velocity of a point on the edge of the disk tracked in both the centre of mass and lab reference frames.

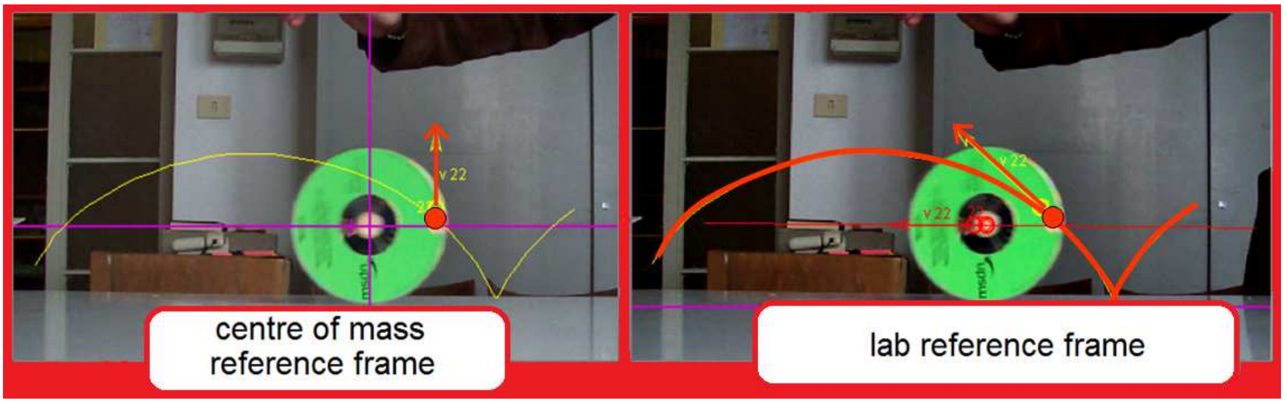


Figure 3. Snapshot showing the Tracker video analysis of the motion for a rolling body. The velocity vectors in the centre of mass and lab reference frames are shown at the same instant.

3.3. Rolling on horizontal plane and the role of friction

Several studies show that the role played by sliding friction forces in shaping the motion of rolling bodies is in some cases underestimated and in others overestimated by students.
For example, in our study, more than 40% of student teachers did not recognize that kinetic friction force produces the transition from sliding to rotational motion of a sphere moving on a rough horizontal plane.
In other cases the role of friction is overestimated. For example, only about 30% of our ST recognized that a sphere rolling without slipping across a rigid and rough horizontal plane is not slowed by friction (32% of ST, versus 25% of the sample tested in ref. [1]). Moreover, from ref. [10] we know that for many students a body cannot rotate or roll in absence of friction because they think that a torque is necessary to maintain rotation.
To explore in detail some cases of transition from sliding to pure rotational motion due to friction force, ST are engaged in simple computer aided activities. By using interactive Algodoo simulations they analyse the role of friction in the dynamics of the rolling disk, when no other accelerating force or momentum is applied.

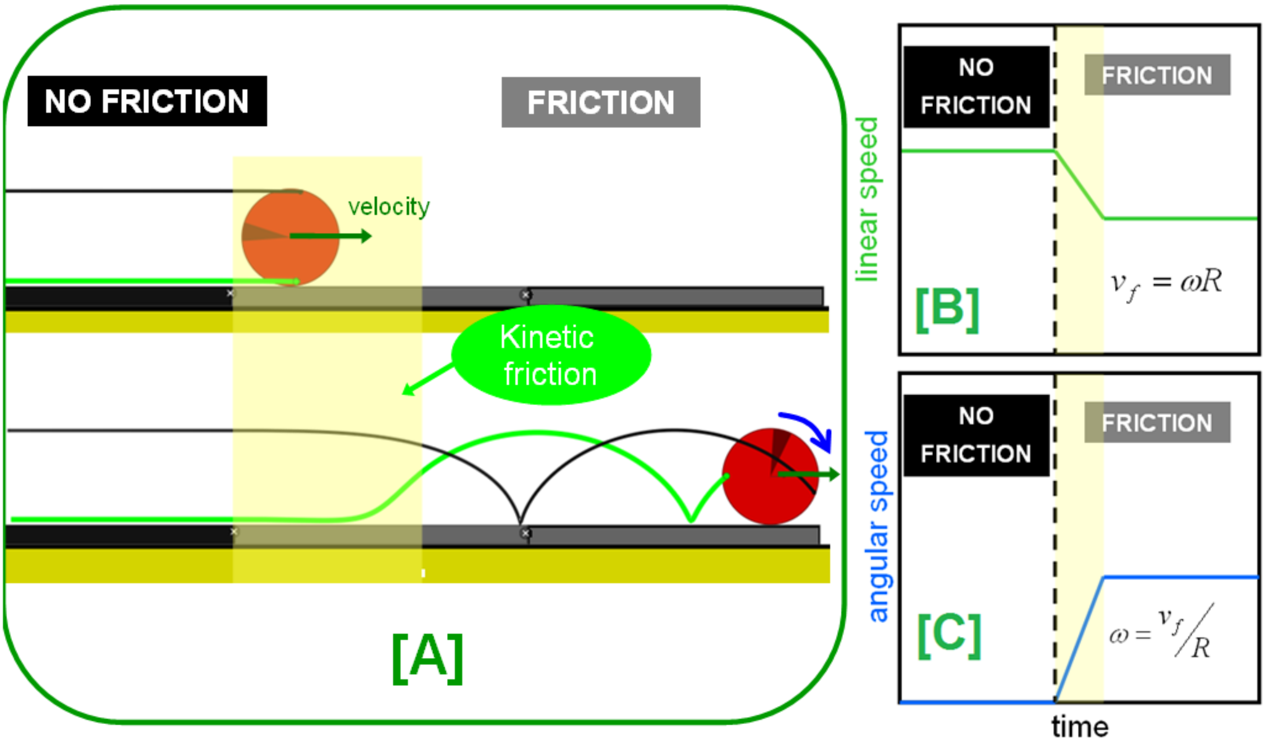


Figure 4. (A) Trajectory of two point on the edge of a disk during the transition from a pure translational motion to pure rolling. Graphs of the linear (B) and angular (C) speeds of the disk as functions of time before, during and after the transition.

In particular, as shown in Figure 4, they study the motion of a disk, which is initially sliding on a rigid horizontal frictionless surface (black in Fig.4) and only has a translational velocity. The disk then encounters a second rough surface (grey plane). ST, working in groups with the simulation, realize that kinetic friction force due to the disk sliding on the plane produces a decrease in the linear velocity of the disk, and an increase in its angular velocity, until finally slipping stops, and pure rolling begins. We focus ST's attention on the fact that, in the first time instants after the disk enters the rough plane, the linear and angular velocities are not yet related by the relation $v = \omega R$, since the disk rolls and slips at the same time. In Figure 4 v and ωR are plotted as functions of time. In this figure two phases of the motion on the rough plane are clearly shown, the first one in which friction force is reducing the translational velocity and increasing the rotational one, and the second one in which the rolling condition has been reached, and friction force vanishes.

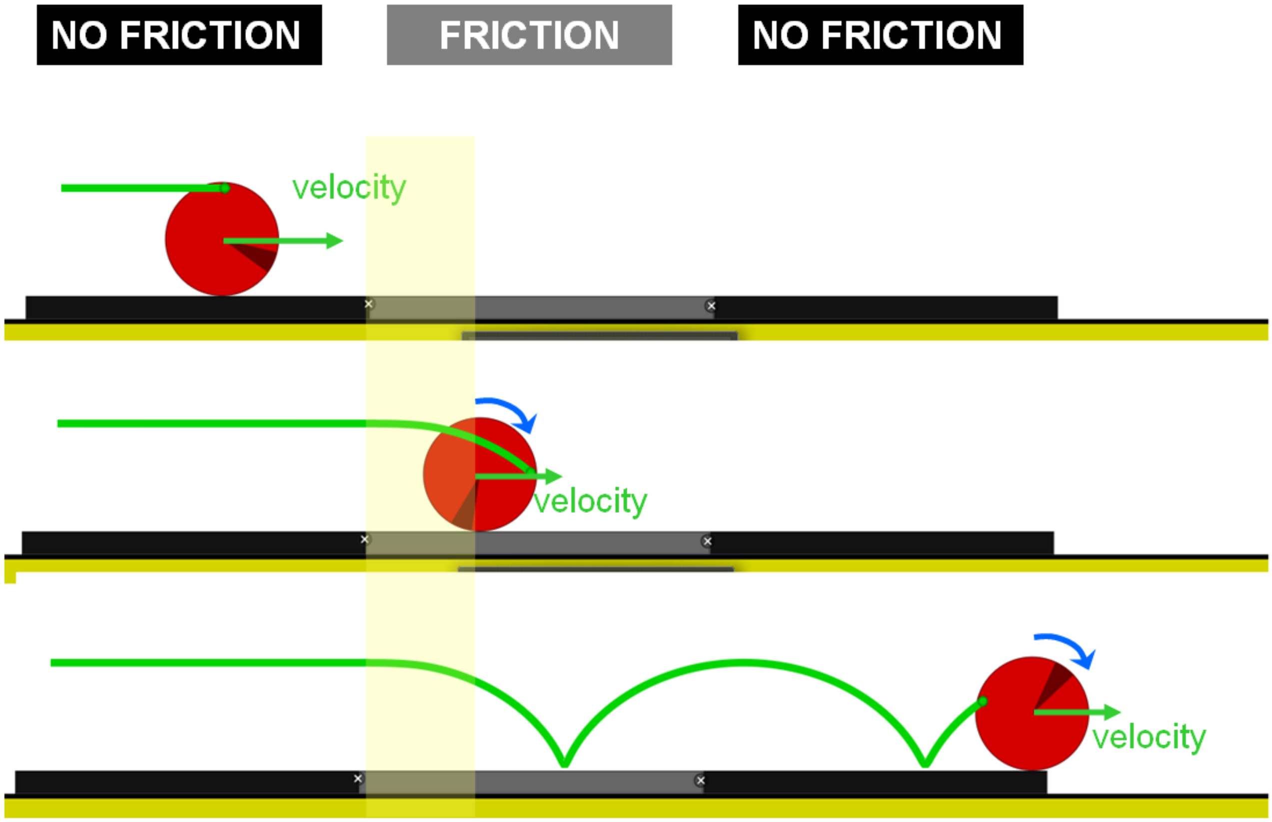


Figure 5 Trajectory of a point on the edge of a disk when it passes from a pure translational motion to a pure rolling motion on a rough plane and then moves in a pure rolling motion on a third frictionless plane

ST can observe these features of the disk motion directly from the simulation. Furthermore they can verify that if a third, frictionless plane is inserted in succession (the black one on the right in Fig.5), the disk continues to roll without slipping although no friction force acts on it, and no change in the trajectories and velocities can be observed.
Thus they realize that static friction plays no role in the pure rolling of a disk at constant speed. While working with the simulation ST usually raise two questions:

- how can the sliding friction force disappear?
- if the sliding friction force disappears, what causes the torque providing the rotation?

The first question reveals a limited understanding of friction as a force that adjusts in magnitude to exactly balance the applied force[11]; the second one shows that ST hold a naïve idea of the relation between rotation and torque, similar to the ingenuous idea of force as necessary for movement (see for example ref.[12]). In Ref. [13] students' wrong conceptions on the relation between angular velocity and torque were highlighted by considering the case of a particle in rectilinear motion.
We point out with students that a friction force appears (a) when two surfaces in contact are in relative motion with respect to one another, or (b) when a force attempts to produce relative motion between two surfaces in contact. Neither of the two conditions occurs when the disk is rolling without slipping and no friction force acts on it. It is of course helpful here to remind ST that, when a body rolls without slipping, the point of contact with the surface is always instantaneously at rest with respect to the surface itself.

3.4. The role of friction in rolling on an inclined plane

As highlighted also by research [1,10] we found that our students teachers had difficulties in explaining rolling motion along an incline. For example only 42% of them were convinced that pure rolling motion along an incline is governed by static friction only. Moreover, 42% answered that a sphere cannot simply slide along a frictionless incline, while for the case in which friction is present, 32% believed that the sphere would remain at rest for small inclination angle, and 26% expected the sphere to roll without slipping for all angles.

To address this problem, ST carry out an experimental activity integrated by the use of computer simulations.

They capture on video the motion of a disk along an inclined plane (as is done in ref. [7]) varying the inclination angle to investigate the differences between the cases of pure rolling, and rolling with slipping. They identify the pure rolling condition using the trajectories of a point on the disk edge, or comparing the angular velocity of the disk with the linear velocity of the geometrical centre. In Figure 6 (left) the different measured trajectories of a point on the edge for pure rolling (small tilting angle, distance covered equal to $2\pi R$) and for rolling with slipping (large tilting angle, distance covered greater than $2\pi R$) are reported.

Experimental results are compared with simulations, in which ST can modify both the slope and the friction coefficient (Figure 6, right).

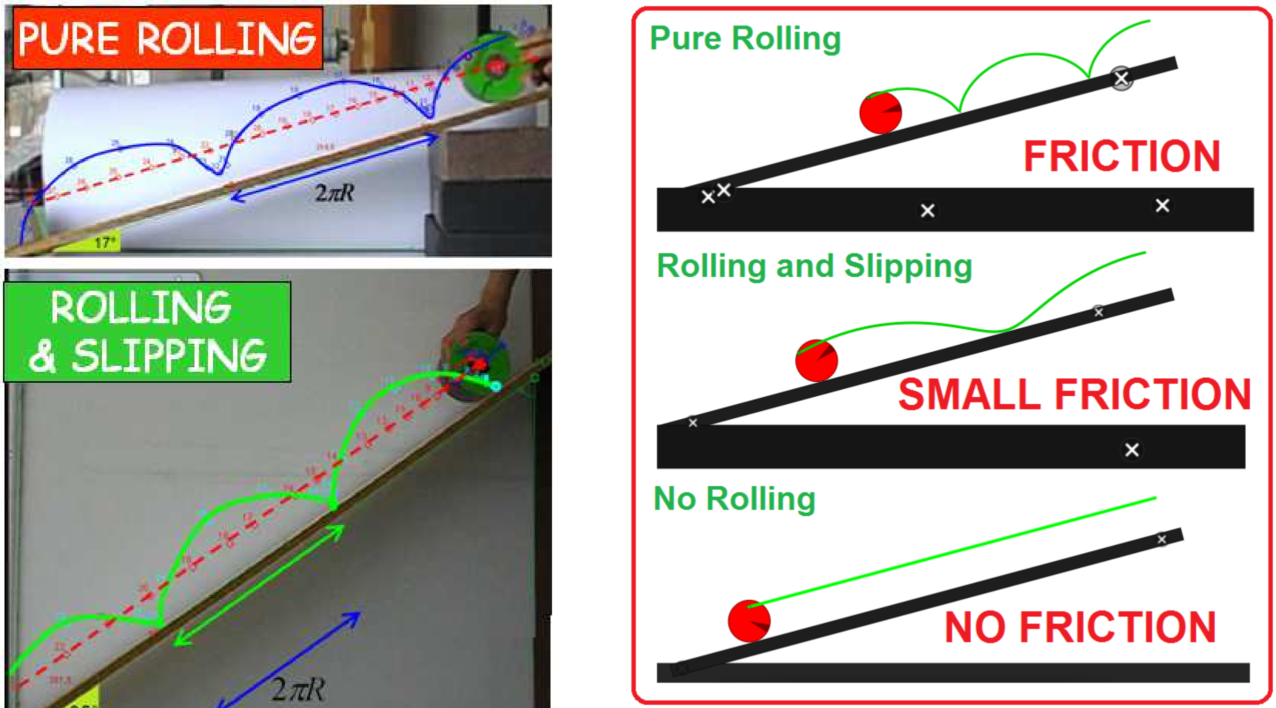
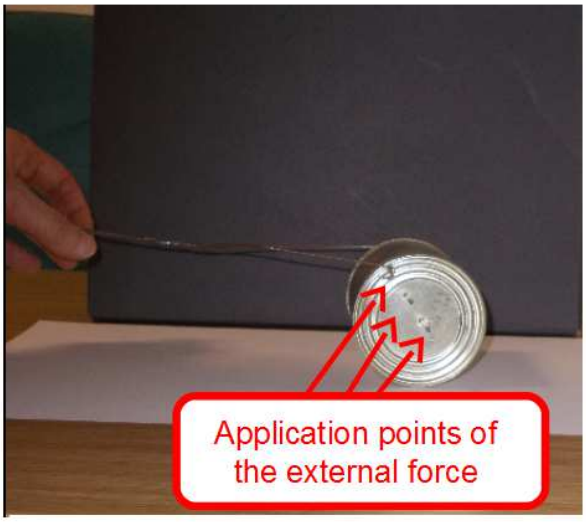
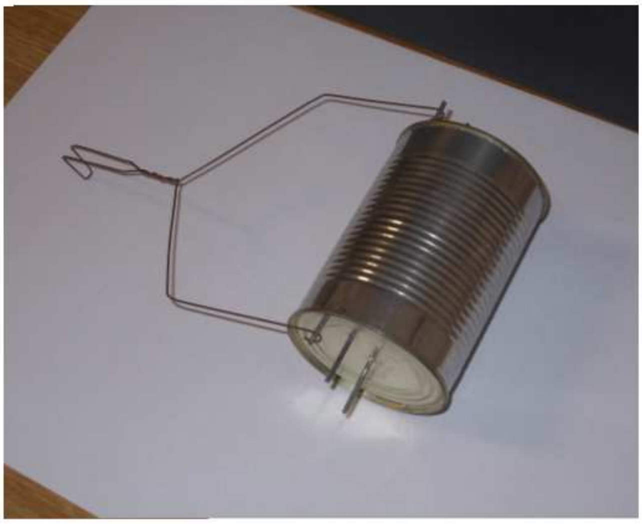


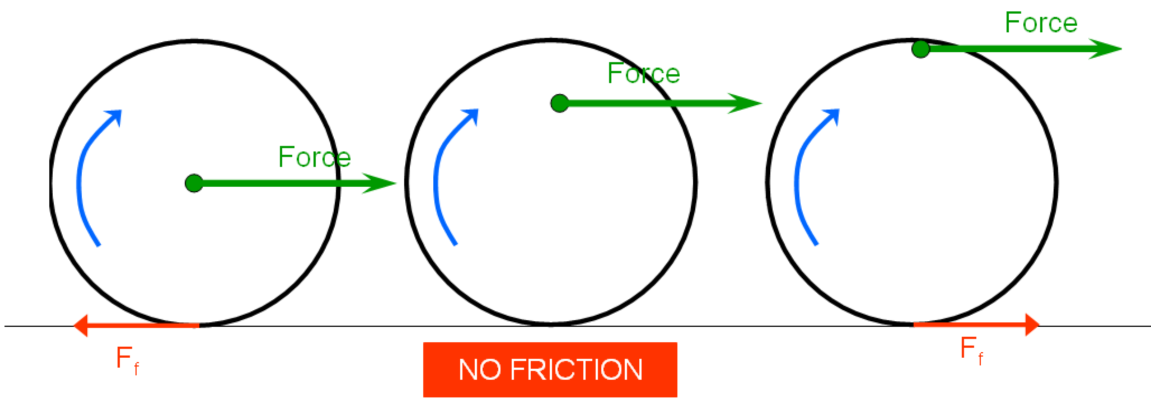
Figure 6 Data from the simulated motion were compared with theory. ST identified the pure rolling condition using the trajectories of a point on the disk edge or comparing the angular velocity and the linear velocity of the geometrical centre; ST computed the total mechanical energy of the disk when the pure rolling condition is fulfilled, demonstrating its conservation and thus confirming that static friction force does no work

3.5. Rolling on a horizontal plane under an external force: the role of friction force and its direction

Rolling on an inclined plane is a special case of rolling motion under the action of an external force; a case in which, in particular, the force is applied on the centre of mass. The general situation in which the force is applied at an arbitrary point is discussed in several papers [14]. In our activity we concentrated on the case of rolling motion on a horizontal plane, under the action of an external force whose application point can be varied. Here the difficulty for the ST is to recognize that the direction of the friction force changes with the application point of the external force, and that friction force can play a motive role (as in the case C of figure 7). Students generally think of friction force as opposed to “actual” motion, and not to the relative motion of the two surfaces in contact.







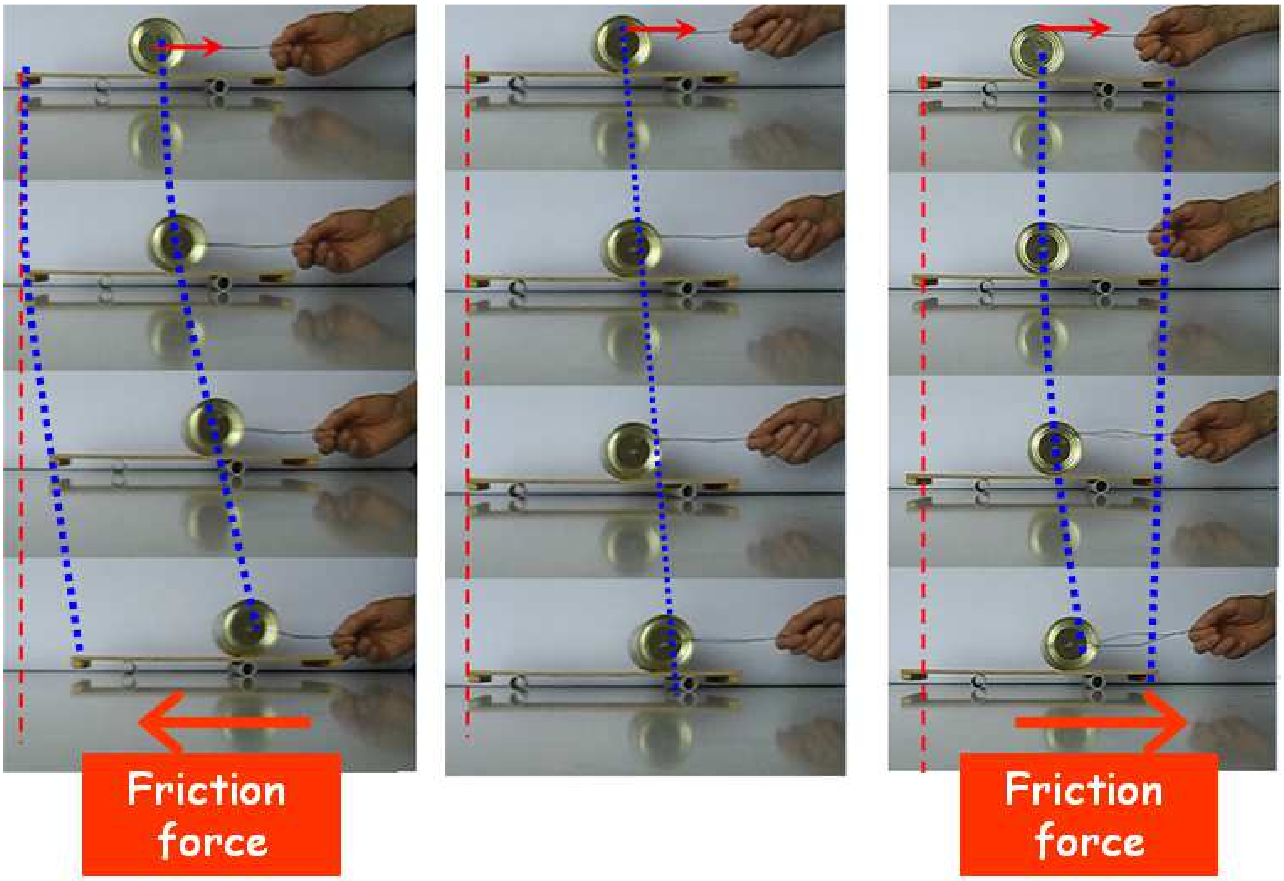


Figure 7 A cylinder rolling without slipping is considered for discussing the 'apparent paradox' of the friction force pointing towards the same direction as the centre of mass velocity. [14] The cylinder can roll on a mobile plane, in such a way that students can observe whether the mobile plane is dragged or pushed by the friction force exerted on it. (top) photo of the simple apparatus used, a cylinder with three possible attachment points for a pulling handle (middle) Schematic representation of the forces acting on the cylinder in the three different cases. (below) Photos and video analysis of the actual experiment. The force acting on the plane is equal in magnitude and opposite in direction to the friction force acting on the cylinder, which is represented by a red arrow.

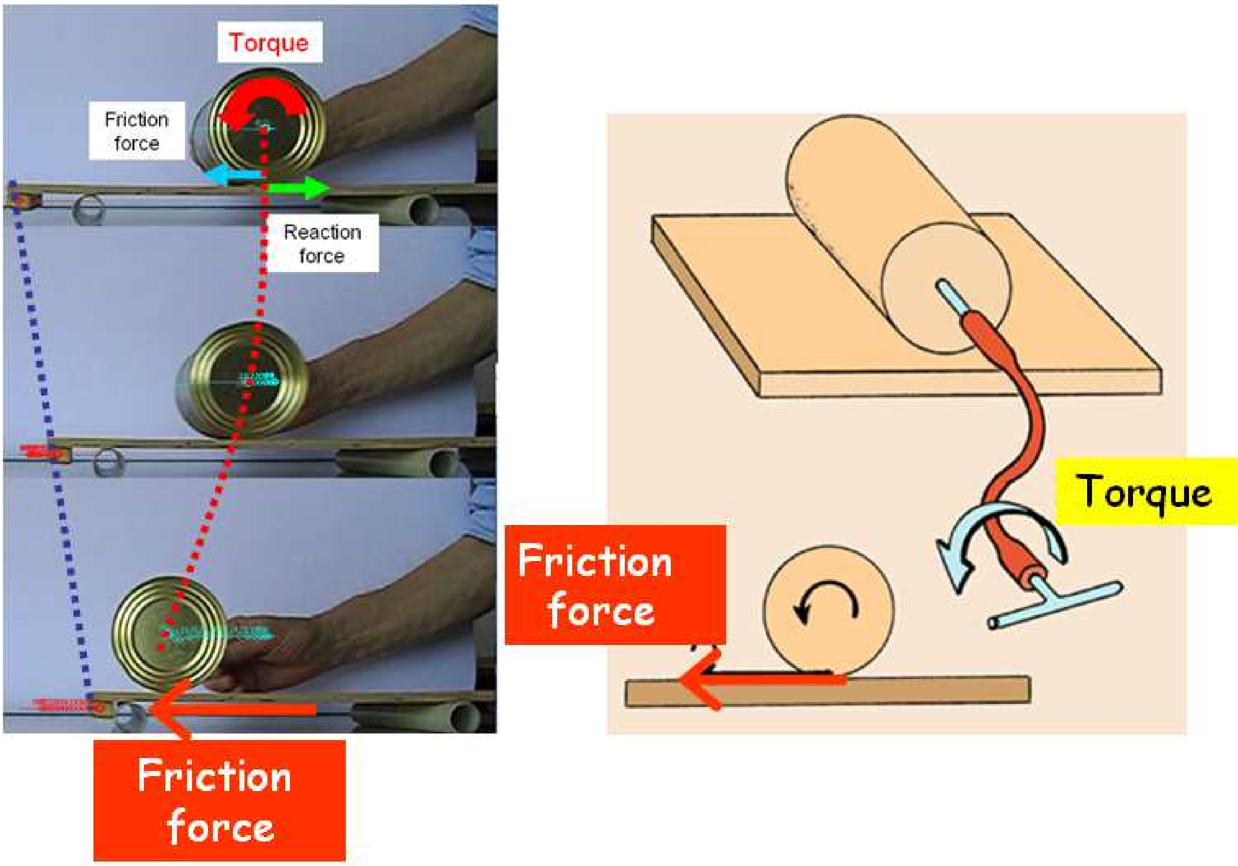


Figure 8 A torque is applied to a cylinder by twisting clockwise a flexible tube. In this case static friction force plays a motive role.

Simple experimental demonstrations, analyzed by means of Tracker, are carried out to address these difficulties.

As shown in Figure 7, a cylinder is initially at rest on a plane which can move on bearings, and it is solicited by a force, parallel to the plane, applied at different distances from the cylinder axis. ST can deduce the direction of the friction force on the cylinder from the acceleration of the plane, which is directed the opposite way. So they can conclude that the friction force switches sign for a particular value of the distance between the application point and the centre of the cylinder.

In Figure 8 a cylinder whose rolling is initiated by a torque is represented. In this case, corresponding to the usual example of the driving wheel of a car or bike [15], the static friction force has always a motive role.

3.6. Inquiry activity: collisions between two balls

As a motivating problem to engage ST in inquiry activity, the collision between two rolling balls is studied. An ingenuous approach to the problem of colliding spheres assumes that no rolling occurs and disregards the effects of friction forces immediately after the collision. However, a meaningful description of "real" collisions between two balls rolling on a surface requires that the role of friction in converting linear motion to rotational and vice-versa is taken into account. [16, 17, 18]

We start from an experimental activity in which we ask ST to observe and compare the elastic collision between two identical carts on a guide, with the one between two identical rolling spheres. ST first examine the elastic collision between the carts, one of which is initially at rest. A quantitative analysis of the collision is carried out, as in the previous activities, by recording the carts' motion and analysing the videos. Using Tracker, ST verify that the results of the experiment are in agreement with the laws of conservation of momentum and energy.

Next we show ST a video of the collision between two identical steel spheres, one of which is initially at rest. We stop the video one instant before the collision, asking ST to make predictions about the following evolution. Contradicting most ST's predictions, the projectile ball does not stop after the collision. In order to explain this unexpected result ST explore several variants of the experiment by designing and manipulating Algodoo simulations. Working in groups with the modelling software, they decompose the initial complex problem into sub-problems to analyze the role of different factors and then construct correlations between them. The main steps of this exploration are summarized in the Table 1, where we report the different cases which ST modelled during the activity. The strategy followed by each group was different, but the steps reported in Table 1 were common to all groups.

Projectile motion	μ_A	μ_B	v_P before	ω_P before	v_P after	ω_P after	v_T after	ω_T after
(I)Translating without friction	=0	=0	≠0	=0	=0	=0	v_P before	=0
(II)Translating and rotating without friction	=0	=0	≠0	≠0	=0	ω_P before	v_P before	=0
(III)Translating and rotating with friction on the plane B	=0	≠0	≠0	≠0	=0	ω_P before	$v_T < v_P$	$\omega_T = v_T / R$
(IV)Translating and rotating with friction on the plane A	≠0	=0	≠0	≠0	≠0	≠0	v_P before	=0
(V)Translating and rotating with friction on both planes	≠0	≠0	≠0	≠0	≠0	≠0	≠0	≠0

Table 1 Collisions simulated by ST during the activity. Plane A is the plane where the projectile moves before the collision; plane B is the plane where projectile and target move after the collision.

The first simulation (the spheres lie on a frictionless plane, and the projectile ball slides without rolling) reproduces the same condition as the cart collision; in fact in this case the projectile ball stops after the collision.

In the second simulation the underlying plane is still frictionless, but the projectile ball approaches with a rolling motion. This helps ST recognize that in the case of head-on collision the target ball only acquires the translational momentum of the projectile ball, while angular momentum is not transferred. (In the following activities they can also verify, by changing the friction coefficient between the two spheres, that angular momentum can only be transferred if the spheres exert a friction force one onto another during the collision).

In the third case the target ball is placed on a plane with friction (plane B in Fig. 9 B). The target ball departs from the collision sliding on the plane, but along the motion kinetic friction produces a decrease in linear velocity and an increase in rotational velocity, until the pure rolling condition is reached.

In the fourth case plane A has a low friction coefficient, while plane B is frictionless. Immediately after the collision, the projectile ball has no longer any translational velocity, but still rotates at its pre-collision angular velocity, because of conservation of angular momentum. Therefore, friction with the underlying plane causes an *increase* in translational velocity and a *decrease* in rotational velocity and the projectile ball rolls forward at low speed.

The fifth case models the real situation initially observed, which now ST are able to reconstruct and explain, based on previous analysis.

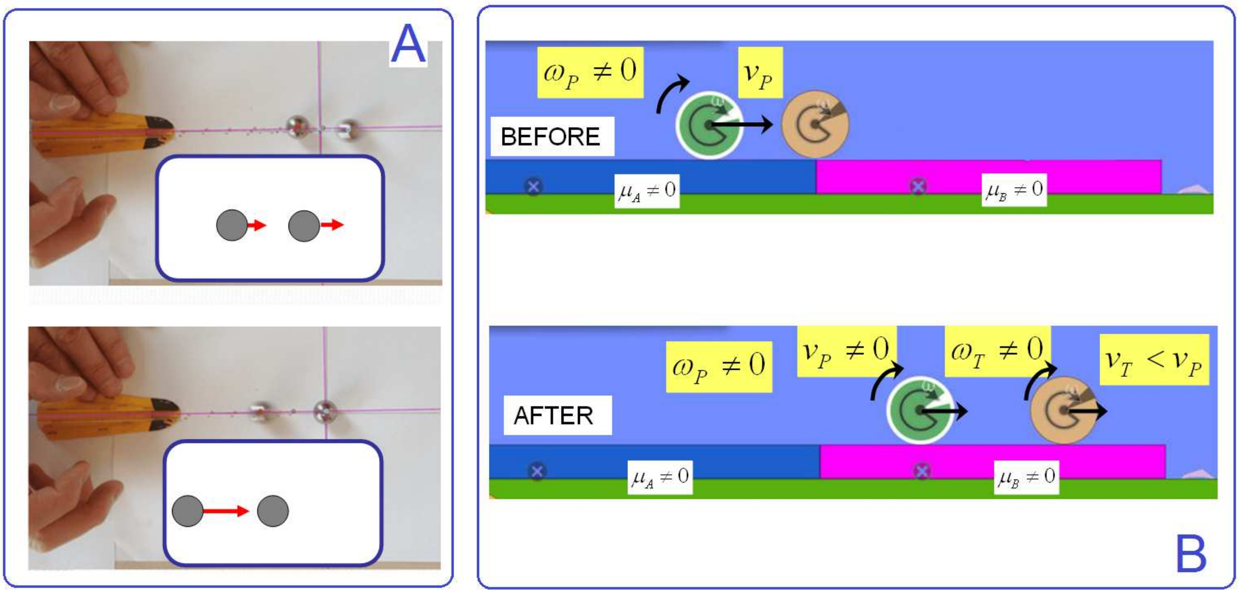


Figure 9 (A) Two frames from the video of the collision between the spheres. (B) Some examples of Algodoo simulations created by students, highlighting the role that modelling activity had in scaffolding students' knowledge.

4. RESULTS

As already mentioned, the sequence was tested with a group of 20 graduate students attending a course for qualification as mathematics and physics teachers (ST). They were graduated in mathematics (9), in physics (8), engineering (3). During their previous studies they attended at least two courses on mechanics, a first introductory course on Newtonian mechanics, and a second one on Lagrangian and Hamiltonian mechanics. The programs of these courses include static and dynamic friction forces and their role in rolling motion.
We analyzed in particular data collected from:
- the two questionnaires given, one before the activities (pre-test) and the other after the end of the sequence (post-test)
- the final report, where students described which elements of the proposed sequence they considered essential.

In the following we briefly summarize both quantitative and qualitative results.

4.1. Quantitative results

The questionnaires, reported in Appendix, were not meant to be comprehensive and cover all topics involved with rolling and rotational motion, but to focus on basic concepts underlying our teaching sequence (see Table I).

TABLE 2. The different topics covered in the items of the two questionnaires. Some questions were drawn from the literature [1] so as to make possible a comparison of our students' results with those obtained in a different context.

Concepts	**Multiple-choice questions**	
	Pre	Post
Rolling and relative motion_	1	1
Sliding friction force	2	2
Rolling on the horizontal plane role of friction and other parameters	3	3,4
Rolling on an incline	4,5	5
Passage from slipping to pure rolling motion, role of the friction force	6	6

Questions drawn from the literature, [1] allow us to compare results from our ST with the ones obtained by a group of calculus-and algebra based introductory physics students and physics juniors who had learned rotational and rolling motion concepts in an intermediate level mechanics course.
We previously mentioned those results from the initial questionnaire that were essential for the sequence design. In the following, we summarize the most relevant conclusions we drew from the pre-test to present a global picture of ST's ideas before the activity sequence.

From the first questionnaire:

(i) ST had great difficulty in distinguishing between the speeds of different points on a rigid wheel with respect to the centre of the wheel or ground. Only 42% of the ST identified the exact direction of the speed of a point on the edge of the wheel in rolling motion. The analogous result in [1] was 32%.

(ii) Only a small fraction of ST recognized that a marble rolling without slipping across a rigid horizontal floor is not slowed by friction (32% versus 25% of [1]).

(iii) More than 40% of the ST were not convinced that a sphere on a frictionless inclined plane slides without rolling (42% of ST vs. 44% in [1]) while, in the case with friction, 32% believed that the sphere remains at rest for a small inclination angle, and 26% that the sphere rolls without slipping for all angles.

(iv) Only 42% of ST were convinced that pure rolling motion along an incline is governed by static friction. Moreover only a small fraction (21%) recognized that the value of the friction force has to be less or equal to the product between the static friction coefficient and the normal force.

More than 40% of ST did not recognize that the kinetic friction force on a sphere which is initially sliding on a rough horizontal plane causes the transition to pure rolling motion.

From the final questionnaire:
In Figure 10 (Top) we compare pre- and post-test results for items related to the same concepts (see Table 2) On the whole, in the post-test the percentage of incorrect answers was, for our ST, below 25%. This result alone is an indication that that the sequence created a fruitful environment for the ST's learning, enabling them to address their initial difficulties.

More in detail

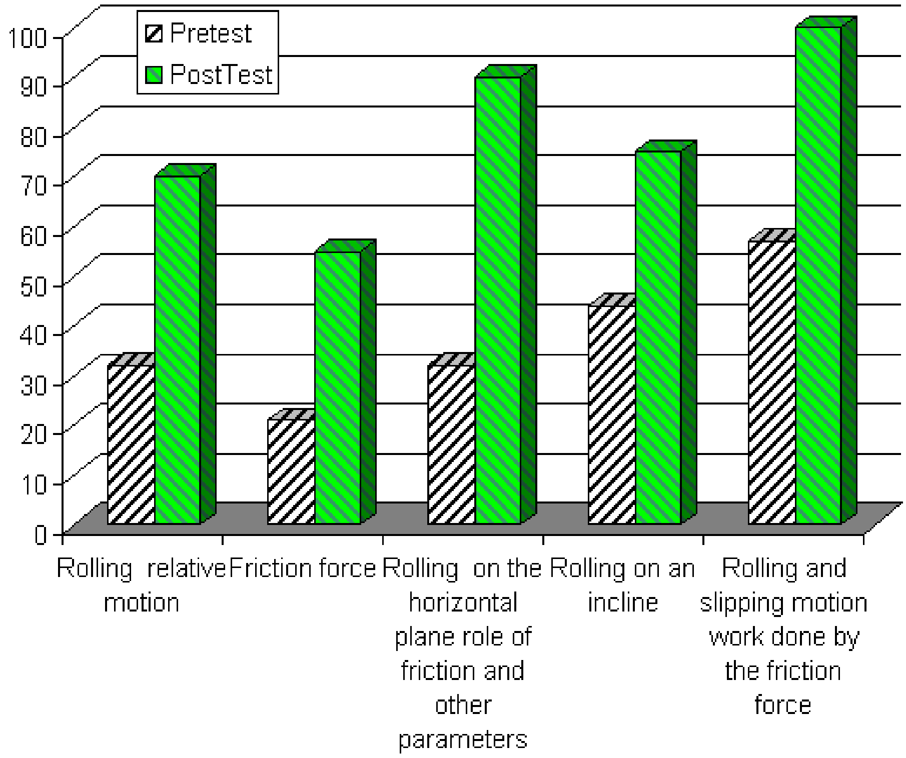
(i) Answers to the post-test confirmed an improvement of ST's understanding of the kinematic of rolling motion and in their capability to distinguish between the speeds of different points with respect to the centre of the wheel or the ground. 70% of the ST correctly answered a question about the velocity of three different points on a rolling wheel with respect to the road, compared to 57% in [1].

(ii) After the sequence a large percentage of ST (89%) was able to recognize that a marble rolling without slipping across a rigid horizontal floor is not slowed down by friction (see Fig.10 (Bottom)). Most ST (79%) also understood that no friction force is acting when a body rolls without slipping along an horizontal plane.

(iii) 70% of ST recognized that whether a sphere moving down along an incline undergoes pure rolling, or rolling with slipping, depends both on the static friction coefficient and on the inclination angle.

(iv) 58% of ST answered correctly that the magnitude of the friction force is not necessarily equal to, but lesser or equal than, the product between the static friction coefficient and the normal force.

(v) 75% of ST recognized the role played by kinetic friction force on a sliding sphere in making it reach the pure rolling condition on a rough horizontal plane.



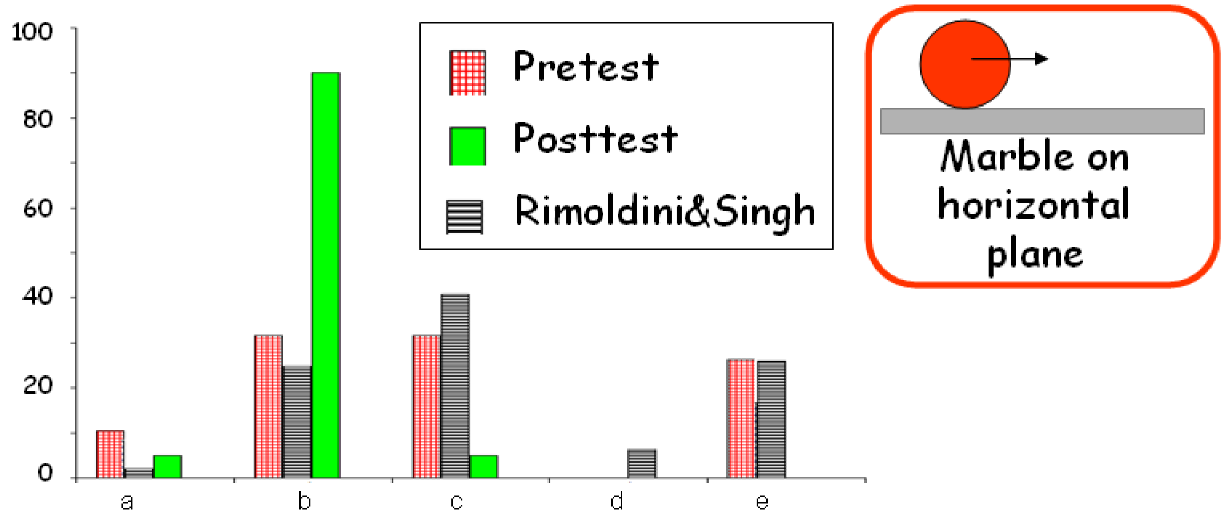


Figure 10 Comparison between pre and post test results analyzing the questions of multi-choice test by concepts. ST's answers to question 3 of the pre and post test.

4.2. Qualitative results

Here we present some results inferred from the final reports written by ST at the end of their activities. In these reports they mainly focus on the autonomous investigation with Algodoo simulations about the collisions between rolling spheres.

ST understood well that rolling motion results from the composition of translational and rotational motion, and that in elastic collisions between two balls only the linear motion of the projectile is transferred to the target, while the angular momentum is not transferred:

One of them writes *"The momentum of the ball projectile is entirely transferred to the target ball, but .... rotational momentum is not transmitted. "*

They are aware of the role of friction (kinetic friction) in the transition from sliding to rotational motion:
"the behavior of the target ball (after the collision) is no doubt due to friction between it and the table; sliding friction force is opposed to velocity, slowing the translational motion and, since it is not applied on the center, also causes an angular acceleration to the sphere. ".

They acknowledge that friction can play a motive role:
"the ball does not have a linear velocity immediately after the impact, but is still rotating; then, under the effect of friction force, angular velocity decreases while linear velocity increases"

They were effectively engaged in decomposing the complex problem of the collision between rolling balls
"When a steel ball collides with a second ball in a central bump, how do they behave? This is a seemingly simple question, but the answer is not obvious and especially dense of physical knowledge. To answer correctly you should ask: is the surface on which the cue ball is located frictionless? Is the plane where the target ball is located before the collision frictionless? …… "

Finally, ST highlighted the role played by software in their learning process
"The software plays a significant role in this activity, because it allows us to freely and easily check the parameters in the game, in order to test our predictions."

5. CONCLUSIONS

A sequence of activities on rolling motion and friction force was designed and tested with student teachers preparing for teaching physics in high school. Throughout the sequence, our aim was to assist ST in scaffolding their knowledge of the phenomenon of rolling, using the role of friction as a guiding principle. Analysis of qualitative data on ST's reasoning suggests that this approach allowed them to obtain a richer and more precise understanding of the subject. Comparison of pre and post test results shows that ST obtained sensible performance improvements, and overcame many common difficulties.
Video analysis based activities were used to highlight experimental situations in which the relationship between friction and rolling is especially complex, or leads to counterintuitive results. Interactive simulations were essential for exploring multiple variations of a given physical situation, and provided the ideal environment for a guided inquiry activity.
One limitation of the study is that the sequence has been tested with a relatively small sample of student teachers having an inhomogeneous background. In the future we are planning a wider experimentation with a larger sample of undergraduate students, to evaluate whether the sequence is suitable for introductory physics courses. In this context we will add a delayed post-test, about six months after the sequence has completed, in order to verify the hypothesis that our approach can lead students to long lasting conceptual understanding of rolling motion.

REFERENCES

[1] L. G. Rimoldini and C.Singh (2005) Student understanding of rotational and rolling motion concepts *Phys. Rev. ST Phys. Educ. Res.* **1** 1.

[2] K. K. Mashood and V. A. Singh (2012) An inventory on rotational kinematics of a particle: unravelling misconceptions and pitfalls in reasoning *Eur. J. Phys.* **33** 1301.

[3] M. Lopez (2003) Angular and linear acceleration in a rigid rolling body: students' misconceptions *Eur. J. Phys.* **24** 553–62.

[4] D. E. Shaw (1979) Frictional force on rolling objects *Am. J. Phys.* **47** 887.

[5] A. Pinto and M. Fiolhais (2001) Rolling cylinder on a horizontal plane *Phys. Educ.* **36** 250.

[6] V. Oliveira (2011) Angular and linear accelerations of rolling cylinder acted by an external force *Eur. J. Phys.* **32** 381.

[7] S. Phommarach, P. Wattanakasiwich and I. Johnston (2012) Video analysis of rolling cylinders *Phys. Educ.* **47** 189.

[8] U. Besson, L. Borghi, A. De Ambrosis, and P. Mascheretti (2007) How to teach friction: Experiments and models *Am. J. Phys*. **75** (12) 1106.

[9] U. Besson, L. Borghi, A. De Ambrosis, and P. Mascheretti (2010) Three-Dimensional Approach and Open Source Structure for the Design and Experimentation of Teaching-Learning Sequences: The case of friction *International Journal of Science Education* **32** 1289.

[10] H.G. Close, L.S. Gomez, P.R.L. Heron (2013) Student understanding of the application of Newton's second law to rotating rigid bodies *Am. J. Phys.* **81** (6) 458.

[11] J. Hierrezuelo and C. Carnero (1993) Sliding and rolling: the physics of a rolling ball *Phys. Educ.* **30** 177; C. Carnero, J. Aguiar and J. Hierrezuelo (1995) The work of the frictional force in rolling motion *Phys. Educ.* **28** 225.

[12] A. diSessa (1993) Toward an epistemology of physics. *Cognition and Instruction* **10** 105

[13] K. K. Mashood, and V. A. Singh, (2012). Rotational kinematics of a particle in rectilinear motion: Perceptions and pitfalls. American Journal of Physics, 80(8), 720-723.

[14] A. Salazar, A. Sanchez-Lavega and M. A. Arriandiaga (1990) Is the frictional force always opposed to the motion? *Phys. Educ.* **25** 82.

[15] P. S. Carvalho, and A. S. Sousa (2005) Rotation in secondary school: teaching the effects of frictional force. *Phys. Educ*. **40** 257

[16] A. Domenech and E. Casasús (1991) Frontal impact of rolling spheres. *Phys. Educ*. **26** (3) 186.

[17] S. Mathavan, M.R. Jackson, R.M. Parkin (2009) Application of high-speed imaging to determine the dynamics of billiards *Am. J. Phys.* **77** 788.

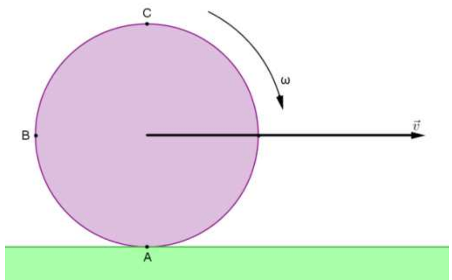
[18] R. E. Wallace and M. C. Schroeder (1988) Analysis of billiard ball collisions in two dimensions *Am. J. Phys.* **56** 815.

Appendix

PRETEST

1 Rolling relative motion

A wheel with radius R rolls without slipping on a horizontal plane. The velocity of the wheel center with respect to the ground is $\vec{v}$ and the angular velocity of the wheel is $\vec{\omega}$. The direction of the instantaneous velocity of point B (see figure) with respect to the ground is approximately:



a. ↘
b. ↗
c. →
d. ↑
e. There is no direction since the instantaneous speed of point B is zero with respect to the ground.

2 Friction force

A cylinder with mass *m* rolls without slipping on an inclined plane. The sliding friction coefficients between the cylinder surface and the plane are μ_S (static) and μ_K (kinetic). What is the absolute value of the friction force?

a. $mg\mu_K \cos\vartheta$.
b. $mg\mu_S \cos\vartheta$.
c. 0.
d. A value lesser than or equal to $mg\mu_K \cos\vartheta$.
e. A value lesser than or equal to $mg\mu_S \cos\vartheta$.

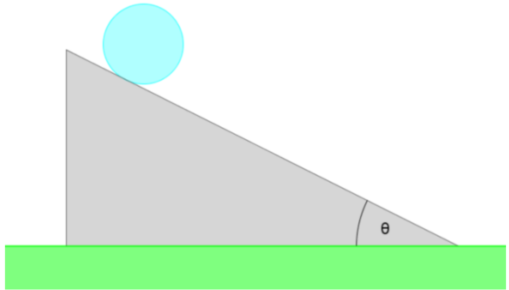
3 Rolling on the horizontal plane role of friction and other parameters

Two identical rigid marbles roll without slipping across rigid horizontal floors. One rolls on a stone floor with coefficient of static friction $\mu_s = 0.80$, and the other rolls on a glass floor with $\mu_s = 0.40$. Which marble is slowed down more by friction, and why? Ignore air-resistance.

a. Both marbles are slowed equally because the marbles themselves are identical.
b. Neither marble is slowed by friction because both roll without slipping.
c. The marble rolling on stone is slowed more, because the greater μ_s makes the force of friction on it greater.
d. The marble rolling on glass is slowed more, because the slippery nature of the glass impedes rolling.
e. It is impossible to answer without knowing the coefficient of kinetic friction μ_k because the marbles are moving.

4 e 5 Rolling on the horizontal plane role of friction and other parameters

A sphere is positioned on an inclined plane with an inclination angle θ, $0° \leq \vartheta \leq 90°$. As in figure. If both the static and kinetic friction coefficients between the sphere and plane are zero, $\mu_s = \mu_k = 0$:



a. The sphere will roll without slipping for small θ and slide down only for θ greater than a certain non-zero value.
b. The sphere will remain at rest for small θ and roll without slipping only for θ greater than a certain non-zero value.
c. The sphere will slide down for all θ > 0◦.
d. The sphere will roll without slipping for all θ > 0◦.
e. None of the above.

Referring to the same system and figure as the previous question, if the static friction coefficient is now $\mu_s \neq 0$,

a. The sphere will roll without slipping for small θ and slide down only for θ greater than a certain non-zero value.
b. The sphere will remain at rest for small θ and roll without slipping only for θ greater than a certain non-zero value.
c. The sphere will slide down for all θ > 0◦.
d. The sphere will roll without slipping for all θ > 0◦.
e. None of the above.

6 rolling and slipping motion: role of the friction force

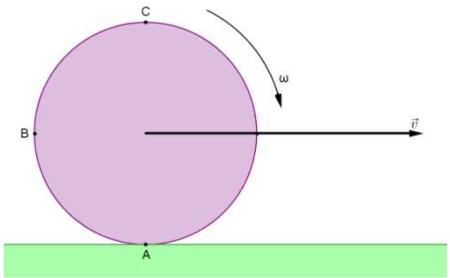
A cylinder lying on horizontal plane has a purely translational motion, with no rotation around it central axis, for a space segment AB in which the plane is frictionless. Then the cylinder reaches a part of the plane with friction, and at point C its motion has become pure rolling, with no sliding. In the space segment BC, the work done on the cylinder is:

a. Zero because the resultant of the forces on the cylinder is zero.
b. Zero because only the static friction force acts.
c. ≠ 0 because the static friction force does work.
d. ≠ 0 because the kinetic friction force does work.
e. The cylinder cannot roll, because there is no resultant torque with respect to the contact point with ground.

POST TEST

1 Rolling relative motion

Rank order the speeds of points A, B, C at the rim of the wheel with respect to the road, largest first (see the figure).



a. $v_A = v_B = v_C$.
b. $v_A > v_B > v_C$.
c. $v_B > v_C > v_A$.
d. $v_C > v_A > v_B$.
e. $v_C > v_B > v_A$.

2 Friction force

A cylinder with mass *m* rolls without slipping on an inclined plane. The sliding friction coefficients between the cylinder surface and the plane are μ_S (static) and μ_K (kinetic). What is the absolute value of the friction force?

a. $mg\mu_K \cos\vartheta$.
b. $mg\mu_S \cos\vartheta$.
c. 0.
d. A value lesser than or equal to $mg\mu_K \cos\vartheta$.
e. A value lesser than or equal to $mg\mu_S \cos\vartheta$.

3 e 4 Rolling on the horizontal plane role of friction and other parameters

Two identical rigid marbles roll without slipping across rigid horizontal floors. One rolls on a stone floor with coefficient of static friction $\mu_s = 0.80$, and the other rolls on a glass floor with $\mu_s = 0.40$. Which marble is slowed down more by friction, and why? Ignore air-resistance.

a. Both marbles are slowed equally because the marbles themselves are identical.
b. Neither marble is slowed by friction because both roll without slipping.
c. The marble rolling on stone is slowed more, because the greater μ_s makes the force of friction on it greater.
d. The marble rolling on glass is slowed more, because the slippery nature of the glass impedes rolling.
e. It is impossible to answer without knowing the coefficient of kinetic friction μ_k because the marbles are moving.

A cylinder rolls without slipping on a horizontal plane, which has a non-zero sliding friction coefficient (both static and kinetic) with respect to the cylinder surface. At a certain time instant, the cylinder encounters a second, frictionless plane (both the static and kinetic siding friction coefficients are zero). What happens, ignoring air friction, is:

a. The cylinder immediately stops rolling and starts sliding with zero angular velocity, conserving the same translational speed it previously had.
b. After some time the cylinder will be sliding with zero angular velocity, with the same translational speed it had previously.
c. Since there are no forces on the cylinder, it will continue its pure rolling motion unperturbed.
d. The motion of the cylinder will be rolling with slipping.
e. After some time the cylinder will be sliding with zero angular velocity, with a greater translational velocity than it previously had during rolling motion.

5 Rolling on the horizontal plane role of friction and other parameter

A sphere placed on an inclined plane, initially at rest, is set free to move. The conditions which allow it to roll without slipping depend on:

a. The static friction coefficient and the plane inclination.
b. The kinetic friction coefficient only .
c. The plane inclination only.
d. The static friction coefficient and the pane inclination.
e. The mass of the sphere and the plane inclination.

6 rolling and slipping motion: role of the friction force

A sphere slides on a frictionless plane. At a certain time instant it encounters a second plane, with a non-zero sliding friction coefficient with the sphere surface. What happens, ignoring air friction, is:

a. The sphere continues sliding but stops after some time because of friction.
b. The motion becomes immediately pure rolling, in a presence even of a very small sliding friction coefficient.
c. While continuing to slide, the sphere gradually acquires an angular velocity and after some time its motion is pure rolling.
d. The motion only becomes pure rolling if the sphere's translational speed is sufficiently high. Otherwise, the sphere just stops.
e. If, as it normally happens, the kinetic friction coefficient is lower than the static friction coefficient, the sphere can never acquire a pure rolling motion.