Examination of the first excited state of $^{4}\text{He}$ as a potential breathing mode

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The isoscalar monopole excitation of $^{4}\text{He}$ is studied within a few-body $ab\ initio$ approach. We consider the transition density to the low-lying and narrow $0^+$ resonance, as well as various sum rules and the strength energy distribution itself at different momentum transfer $q$. Realistic nuclear forces of chiral and phenomenological nature are employed. Various indications for a collective breathing mode are found: (i) the specific shape of the transition density, (ii) the high degree of exhaustion of the non-energy-weighted sum rule at low $q$, and (iii) the complete dominance of the resonance peak in the excitation spectrum. For the incompressibility $K$ of the $\alpha$ particle, two different definitions give two rather small values (22 and 36 MeV).

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I. INTRODUCTION

The quantum breathing mode (monopole oscillation) is the object of continuous theoretical and experimental investigations in a large variety of systems as nuclei and trapped nanoplasm or cold atoms (see, e.g., Refs. [1,2] and references therein). In fact it appears as one of the most important properties that allows diagnosis of the underlying force. The isoscalar giant monopole resonance (ISGMR), also known as nuclear breathing mode, is one of the collective nuclear excitations that are well established and much discussed in heavier nuclei, also because of possible interesting relations to the nuclear matter incompressibility (see, e.g., Refs. [3,4] and references therein).

The lightest nucleus where the breathing mode has been discussed is the $\alpha$ particle. The discussion has a rather long history. It was triggered by the results of an inclusive electron scattering experiment on $^{4}\text{He}$ in 1965 [5], where the transition form factor $|F_{\mu}(q)|^2$ to the $0^+$ resonance ($0^+_2$) had been measured for various momentum transfer $q$. The interpretation of the resonance as a collective breathing mode was suggested a year later [6]. A fair agreement both for the excitation energy $E_R$ and $|F_{\mu}(q)|^2$ was obtained. In the following years the collectivity of the resonance was an object of further discussion.

The intent of the present work is to analyze the collectivity issue of the $0^+_2$ from a modern few-body $ab\ initio$ point of view [7], as an interesting bridge between few- and many-body physics. Further below it will become evident that various results of our $ab\ initio$ few-body calculation point to a breathing-mode interpretation of the resonance.

As an introduction, we give a summary of the discussion on the $0^+_2$, leaving out other theoretical work where the issue of collectivity is not addressed explicitly. After the work of Refs. [5,6] mentioned above, in 1970 the $0^+_2$ was the object of an electron scattering experiment at lower $q$ [8], and in the following decade the generator coordinate method [9,10] was applied to investigate the resonance [11,12]. Within this method collective motions are studied microscopically, introducing a collective path. Again a fair agreement with experiment was found. The results gave “further evidence that the first excited state in $^{4}\text{He}$ can be interpreted as a compressionional monopole state” [12]. A translation invariant shell-model calculation, which included two-particle two-hole (2p-2h) configurations, was carried out in 1981 [13] and there it was concluded that the comparison of the obtained results with the experimental $|F_{\mu}(q)|^2$ “casts doubts on the usual breathing mode interpretation.” A similar conclusion was drawn in 1986 [14] by comparing the results of a two- and four-particle excitation models to data that included new results at higher $q$ [15]. There it was stated that the resonance “has very little of the breathing mode in it and consists basically of more complex excitations.” In 1987 two microscopic models were applied [16]: one in terms of a collective variable, the other in terms of a cluster variable (similar to the resonating group approach). It was concluded that the $0^+_2$ “has a cluster character” (3 + 1). A year later it was stated [17] that “these two hypotheses are not mutually exclusive.” In fact it was already shown in Ref. [6] that in a translation-invariant harmonic-oscillator model, where one nucleon is excited from the 0r to the 1s shell, the necessary change of sign of the transition density (its integral in space vanishes) happens just at the $^{4}\text{He}$ radius. This is precisely the form of the excess density of a breathing mode, though the model may appear as noncollective because of its interpretation as mean field or (3 + 1) cluster. This common feature is not very surprising in a s-shell scenario, since the transition density for $q \rightarrow 0$ can be written as a function of just the hyper-radius $R^2 = \sum_\tau \tau^2$, one of the six collective coordinates defined by the group $GL(3,R)$ [18,19]. At the end of the 1980s the $0^+_2$ was understood as “a superposition of simple 1p-1h excitations and not as a collective state” [20].

We observe that in these early works the criterion for the collectivity of the $0^+_2$ has been mainly the agreement with experimental data: If in a collective (noncollective, i.e., purely mean field based) model the data could be described
sufficiently well, it was simply concluded that the $0^+_k$ has a collective (noncollective) character.

No further insight in the issue of collectivity came up until 2004, when it was reconsidered within a few-body $ab$\ textit{initio} approach [21]. Using a truncated version of a realistic nucleon-nucleon (NN) potential, Argonne V8' (AV8') [22], and a phenomenological three-nucleon force (N3F), it was concluded with a sum rule argument that the $0^+_k$ is not a breathing mode. Another motivation against the collectivity was the large overlap of the $0^+_k$ wave function with the trimucleon ground state. However, the density is an integral property; therefore it is perfectly possible, as already indicated in Refs. [6,17], that a large overlap of the $0^+_k$ state with a $(3 + 1)$ cluster gives rise to the density of a breathing mode. The sum rule arguments are questioned below.

For many-body systems there is a general consensus that the breathing mode collectivity is signaled by two typical features: (i) the above mentioned peculiar form of the transition density and (ii) the degree of exhaustion of the energy-weighted sum rule [23] by the resonance strength. For $^4$He the latter was often examined, while the former was considered (besides in Ref. [6]) only in Ref. [21].

As already pointed out, the aim of the present work is the study of collectivity from a modern $ab$\ textit{initio} few-body point of view. We investigate whether an $ab$\ textit{initio} calculation of the $^4$He inelastic isoscalar monopole (InlSM) strength exhibits features that are believed to characterize a collective behavior in many-body systems and how they depend on different nuclear forces. We use two realistic potential models including the Coulomb force: the chiral effective field theory NN potential at next-to-next-to-leading order (N$^3$LO) [24] plus the N$^3$LO N3F [25] and the AV18 NN potential [26] plus the UX3 N3F [27] (both NN potentials lead to excellent fits to NN scattering data). In this work we consider both the form of the transition density to the $0^+_k$ and the degree of exhaustion by the resonance strength of various sum rules. We investigate to what extent these features appear and conclude accordingly whether realistic nuclear forces, the only input of a four-body $ab$\ textit{initio} calculation, allow us to picture the excitation as collective. In addition we show that the strength distribution itself leads to much better insights.

II. FORMALISM

The InlSM strength distribution is given by

$$ S_M(q,\omega) = \sum_{n=0}^{\infty} \langle n | M(q) | 0 \rangle^2 \delta(\omega - E_n + E_0), \tag{1} $$

where $\omega$ is the excitation energy, and $|0\rangle$, $|n\rangle$ and $E_0$, $E_n$ are eigenfunctions and eigenvalues of the nuclear Hamiltonian $H$, respectively (for continuum energies the sum is replaced by an integral). The InlSM operator reads ($h = c = 1$)

$$ M(q) = \frac{1}{2} \left( \sum_{i<j} j_0(q r_{ij}) - (0, \sum_{i=1}^{A} j_0(q r_i)) |0\rangle \right), \tag{2} $$

where $j_0$ is the zeroth order spherical Bessel function (the dependence on the nucleon form factor is neglected).

In the low-$q$, long-wavelength (LW) limit, the InlSM operator is

$$ M_{\text{LW}} = \lim_{q^2 \to 0} \frac{6 M(q)}{q^2} = \frac{1}{2} \left( \sum_{i<j} r_{ij}^2 - A r^2 \right), \tag{3} $$

where $r^2$ is the mean square radius of the system. It is important to notice that $M_{\text{LW}}$ depends only on the collective variable $r^2$, unlike $M(q)$.

As well explained in Ref. [28], sum rules "provide useful yardsticks for measuring quantitatively the degree of collectivity of a given excited state." They are particular expressions for the moments [29–31] defined as

$$ m_n(q) = \int d\omega \omega^n S_M(q,\omega). \tag{4} $$

The moment $m_1$ for the operator $M_{\text{LW}}$ is an interesting sum rule. Using the completeness property of the eigenstates of the Hamiltonian $H = T + V$, where $T$ and $V$ are the kinetic and potential energy operators, one finds

$$ m_1(q) = \frac{1}{2} \langle 0 | [M(q), [T + V, M(q)]] | 0 \rangle = m_1^T + m_1^V. \tag{5} $$

For local potentials $m_1$ coincides with $m_1^T$ and in the LW limit one obtains

$$ m_1^\text{LW} = \frac{1}{2} \langle 0 | [M_{\text{LW}}, [T, M_{\text{LW}}]] | 0 \rangle = \frac{A}{2 M_N} \bar{r}^2, \tag{6} $$

where $M_N$ is the nucleon mass and $A$ the number of nucleons. Equation (6) is known as the Ferrell energy-weighted sum rule (FEWSR) [23].

For $m_0$ completeness leads to

$$ m_0^\text{LW} = \langle 0 | M_{\text{LW}} M_{\text{LW}} | 0 \rangle. \tag{7} $$

While the FEWSR is considered "model independent," $m_0^\text{LW}$ is not. In fact the experimental value of $\bar{r}^2$ (corrected for the nucleon finite size) can lead to a good estimate of the FEWSR. In contrast, for the evaluation of $m_0^\text{LW}$ one would need one- and two-body ground-state densities. Particularly the latter is largely model dependent.

For the special case where all the transition strength is concentrated into one specific excited state, the sum rules become very simple. Assuming that the breathing mode $|\Phi_{BM}\rangle$ is such a state, it is evident that it exhausts 100% of the FEWSR [6]. In general, this assumption implies that for any $n$ one has $m_n = (E_{BM} - E_0)^n |\langle \Phi_{BM} | M_{\text{LW}} | 0 \rangle|^2$ and therefore this state exhausts all sum rules completely:

$$ r_n^\text{LW} = \frac{(E_{BM} - E_0)^n |\langle \Phi_{BM} | M_{\text{LW}} | 0 \rangle|^2}{m_n^\text{LW}} = 1. \tag{8} $$

We note in passing that in Ref. [6] also a simple relation between the $|F_M(q)|^2$ and the derivative of the elastic form factor was derived and that the same relation is found in Ref. [32] via the so-called "progenitor sum rule" [33], under the hypothesis of a unique collective state.

It is generally believed that the ratios $r_n$, and in particular $r_1$, are good quantities to infer the degree of collectivity of a state. However, it is usually neglected that for $n = 1$ (and
higher) one emphasizes the high-energy strength contribution. In fact, even in the presence of a pronounced collective state, a negligible higher energy strength could lead to a rather small \( r_1 \) and thus to the wrong conclusion. In Ref. [28] it is made clear that the proper quantity to check is rather \( r_0 \). It is the “model independence” of the FEWSR and the difficulty in calculating \( m_0 \), especially for the heavier nuclei, that has led to the general attitude to consider \( r_1 \). For \(^4\)He, however, we are able to evaluate \( m_0^{SW} \) accurately. The results for the moments \( m_0 \) and \( m_1 \) of \( S_{\omega}(q, \omega) \), reported in the following, are obtained via the Lanczos algorithm, analogously to what was done in Ref. [34] for the dipole operator. We perform our calculations by diagonalizing \( H \) on the hyperspherical harmonics (HH) basis up to sufficient convergence, using the effective interaction HH method [35,36].

We test the accuracy of our results using the ANW potential [22]. Unlike the other models, this potential is almost local (the only nonlocal term, the spin-orbit term, does not contribute to \( m_1^{SW} \)). Therefore we get an estimate of our numerical error via independent calculations of \( m_1^{SW} \) and \( \langle r^2 \rangle \) in Eq. (6). We find \( m_1^{SW} = 183.43 \text{ fm}^3 \text{ MeV} \) to be compared to \( 183.62 \text{ fm}^3 \text{ MeV} \) from the FEWSR.

III. RESULTS AND DISCUSSION

We discuss our first criterion for collectivity, namely the specific shape of the transition density

\[
\rho_{\gamma}(r) = \rho_0^{0^+_K}(r)/(0^+_K).
\]

In Fig. 1 we show \( \rho_{\gamma}(r) \). One sees that the first criterion is met quite well since \( \rho_{\gamma}(r) \) changes sign at a distance approximately equal to the root-mean-square radius of about 1.46 fm, for both our potential models. A similar behavior is found in Ref. [21] (see Fig. 2 there). Therefore we may conclude that this feature is rather independent on the force.

Next we study our second criterion for collectivity, namely sum rules. First we analyze \( r_0^{SW} \), replacing in Eq. (8) \( |\phi_{\omega\omega}\rangle \) by \( |0^+_K \rangle \). We calculate the transition strength in the same way as we did in Ref. [37], but for \( M^{SW} \). Here we recall that it is the Lorentz integral transform method [38,39] and a proper inversion algorithm that allows us to separate the resonant contribution from the background contribution. For the chiral (phenomenological) potential we find for the transition strength the value of 3.53 (2.25) \text{ fm}^3 and \( m_0 = 6.80 (5.99) \text{ fm}^3 \), leading to \( r_0^{SW} = 52\% (38\%) \). Considering the observation by Rowe [28], that “a typical \( \gamma = 0 \) collective state exhausts something like 50 percent” (of \( m_0 \), we are led to conclude that the chiral force generates a \( 0^+_K \) of collective character.

Since in experiments the \( |F_M(q)|^2 \) is measured at finite momentum transfer, it is worth studying \( r_0(q) \) at various \( q \) (in Eq. (8): \( r_0^{SW} \to r_0(q) \), \( M^{SW} \to M(q) \)). Results are given in Table 1. For both potential models \( r_0(q) \) and \( r_1(q) \) decrease with growing \( q \), but differ somewhat. The low-\( q \) results for \( r_0 \) are in line with those in the \( q \to 0 \) limit. For the chiral interaction \( r_0 \) reaches remarkable values of more than 50%. In the case of a collective mode such a \( q \) behavior of \( r_0 \) can be expected, since low-\( q \) correspond to large wavelengths, where the virtual photon “sees” the nucleus as a whole. The table also shows that \( r_1 \) is always smaller than \( r_0 \). This indicates a non-negligible high-energy strength and also shows that the argumentation against the collectivity which relies on \( r_1 \) is based on shaky ground.

Rather than considering its integral properties the study of the strength distribution itself is much more informative. As already mentioned above, in Ref. [37] we had computed both the IslSM background distribution and the integrated \( 0^+_K \) strength (though not the strength distribution of the resonance

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**FIG. 1.** (Color online) Transition density between the ground state and the \( 0^+_K \) for the chiral (black full line) and the phenomenological (red [gray] dashed line) forces. In the inset the densities of the ground state and of the \( 0^+_K \) for the chiral force are also shown.

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**FIG. 2.** (Color online) \( S_{\omega}(q, \omega)/m_0(q) \) for various fixed \( q \). In the insets the strength in the resonance region and the background contribution [40].
TABLE I. Transition form factor $|F \mu(q)|^2$ and the zeroth and first moment of the strength distribution for the InNpSM operator $M(q)$. Also listed are the corresponding ratios $r_0$ and $r_1$. For each $q$ the upper and lower lines refer to $N^3LO+N^3LO$ and AV18+UIX forces, respectively.

| $q$ (MeV$^{-1}$) | $|F \mu(q)|^2$ | $m_0$ (MeV) | $m_1$ (MeV) | $r_0$ (%) | $r_1$ (%) |
|-----------------|----------------|-------------|-------------|-----------|-----------|
| 50              | 0.00034        | 0.00063     | 0.021       | 53        | 34        |
|                 | 0.00024        | 0.00064     | 0.018       | 38        | 28        |
| 100             | 0.0042         | 0.0085      | 0.262       | 50        | 34        |
|                 | 0.0025         | 0.0086      | 0.258       | 37        | 25        |
| 200             | 0.0248         | 0.0683      | 2.42        | 36        | 22        |
|                 | 0.0190         | 0.0710      | 2.48        | 27        | 16        |
| 300             | 0.0297         | 0.129       | 5.89        | 23        | 11        |
|                 | 0.0242         | 0.139       | 6.33        | 17        | 8         |
| 400             | 0.0154         | 0.126       | 8.43        | 12        | 4         |
|                 | 0.0141         | 0.143       | 9.39        | 10        | 3         |

IV. SUMMARY

We have investigated the InNpSM strength of $^4$He and the corresponding sum rules within a few-body $ab$ initio approach, employing realistic nuclear forces (chiral and phenomenological ones). For the $0^+_2$ we find properties that can also be attributed to a collective breathing mode: (i) the transition density changes sign at about the $^4$He radius; (ii) for low momentum transfer $q$, where the excitation operator can be expressed in terms of the collective hyper-radius, the transition strength to $0^+_2$ is large and exhausts between about 40% (phenomenological force) and 50% (chiral force) of the non-energy-weighted sum rule and (iii) the resonance dominates the continuum spectrum completely over a very low and extended background. Moreover, we observe a very interesting similarity in the evolution with $q$ of the spectrum, between our results and the plasmon collective excitation spectrum of an alkali metal. Finally, the inverse-energy-weighted sum rule allows us to give the incompressibility $K_A$ of $^4$He predicted by the modern realistic potentials: $K_A$ results to be rather small and to vary little (<6%) with the potential model. More noticeable is the variation with the definition that one uses to define it. Two different definitions [42] give values around 22 or 36 MeV.

A final clarification of the collectivity issue can only come from experiment. Since the present criterion based on the FEWSR is not really appropriate, it would be necessary to determine the strength distribution at low $q$, where existing electron scattering data are scarce and limited to the resonance strength. Interesting complementary information could come from $\alpha$ scattering.

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[40] The $q = 300$ MeV curve in the inset is the same as in Fig. 3(b) of Ref. [37]. It should be noticed, however, that there the zero of the $\omega$ axis has been erroneously shifted by 11 MeV.