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QFT holography near the horizon of Schwarzschild-like spacetimes.

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Abstract: It is argued that free QFT can be defined on the event horizon of a Schwarzschild-like spacetime and that that theory is unitarily and algebraically equivalent to QFT in the bulk (near the horizon). Under that unitary equivalence the bulk hidden $SL(2,\mathbb{R})$ symmetry found in a previous work becomes manifest on the event horizon, it being induced by a group of horizon diffeomorphisms. The class of generators of that group can be enlarged to include a full Virasoro algebra of fields defined on the event horizon. These generators have a quantum representation in QFT on the event horizon and thus in the bulk.

1. *Introduction.* A number of papers has been concerned with the issue of the statistical origin of black-hole entropy. Holographic principle [1, 2, 3] arose by the idea that gravity near the horizon should be described by a low dimensional theory with a higher dimensional group of symmetry. The correspondence between quantum field theories of different dimensions was conjectured by Maldacena [4] using the machinery of string theory: There is a correspondence between quantum field theory in a, asymptotically AdS, $d+1$ dimensional spacetime (the "bulk") and a conformal theory in a d dimensional manifold (the (conformal) "boundary" at spacelike infinity). Witten [5] described the above correspondence in terms of relations of observables of the two theories. Rehren proved rigorously some holographic results for free quantum fields in a AdS background, establishing a correspondence between bulk observables and boundary observables without employing string machinery [6, 7]. To explain the correspondence one should notice that the conformal group which acts in the d-dimensional AdS_{d+1} boundary can be realized as the group of the isometries of the AdS_{d+1} bulk. In this way, the bulk-boundary correspondence has a geometric nature. The boundary of a Schwarzschild spacetime (dropping the boundary at infinity) is the event horizon of the black hole. The AdS correspondence has been used directly in Minkowski spacetime for massless particle in [8] with the help of the optical metric. Is there any bulk-boundary correspondence in a manifold containing a Schwarzschild-like black hole? Two-dimensional Rindler spacetime embedded in Minkowski spacetime approximates the

nontrivial part of the spacetime structure near a bifurcate horizon as that of a Schwarzschild black hole embedded in Kruskal spacetime. In that context, we have argued in a recent work [9] that free quantum field theory in two-dimensional Rindler space presents a "hidden" $SL(2,\mathbb{R})$ symmetry: The theory turns out to be invariant under a unitary representation of $SL(2,\mathbb{R})$ but such a quantum symmetry cannot be induced by the geometric background. $SL(2,\mathbb{R})$ is the group of symmetry of the zero-dimensional conformal field theory in the sense of [10], so, as for the case of AdS spacetime, it suggests the existence of a possible correspondence between quantum field theory in Rindler space and a conformal field theory defined on its event horizon. In this letter we illustrate the basic results that can be found in a forthcoming technical paper [11] where we have shown that it is possible to build up the wanted correspondence of a free quantum theory defined in the bulk and a quantum field theory defined on the event horizon of a two dimensional Rindler space. Other involved results are that the $SL(2,\mathbb{R})$ symmetry reveals a clear geometric meaning if it is examined on the horizon and, in that context, a whole Virasoro algebra of symmetries arises.
Some overlap with our results is present in the literature. Guido, Longo, Roberts and Verch [12] discussed in some detail the extent to which an algebraic QFT on a spacetime with a bifurcate Killing horizon induces a conformal QFT on that bifurcate Killing horizon. Along a similar theme, Schroer and Wiesbrock [13] have studied the relationship between QFTs on horizons and QFTs on the ambient spacetime. They even use the term "hidden symmetry" a sense similar as we do here and we done in [9]. In related follow-up works by Schroer [14] and by Schroer and Fassarella [15] the relation to holography and diffeomorphism covariance is also discussed.

2. *Hidden $SL(2,\mathbb{R})$ symmetry.* Consider a general Schwarzschild-like metric (namely a static black hole metric with bifurcate event horizon), $ds^2_{\mathbf{S}} = -A(r)dt^2 + A^{-1}(r)dr^2 + r^2 d\Omega^2$, Ω denoting angular coordinates. Near the horizon ($r = r_h$), the nonangular part of the metric reduces to the metric of a two-dimensional Rindler wedge $\mathbf{R}$, $ds^2_{\mathbf{R}} = -\kappa^2 y^2 dt^2 + dy^2$ with $A'(r_h) = 2\kappa$, and $\kappa y^2 = 4(r - r_h)$. Also dropping the angular coordinates, let us consider a free Klein-Gordon scalar field ϕ with motion equation $-\partial_t^2 \phi + \kappa^2 \left(y\partial_y y \partial_y - y^2 m^2 \right) \phi = 0$. To built up the one-particle Hilbert space referred to the quantization with respect to the Rindler Killing time t, any real solution ψ of the K-G equation must be decomposed in ∂_t-stationary modes as follows

$$\psi(t,y) = \int_0^{+\infty} \sum_{\alpha} \Phi_E^{(\alpha)}(t,y) \tilde{\psi}_+^{(\alpha)}(E)\, dE + c.c. \tag{1}$$

$E \in [0, +\infty) = \mathbb{R}^+$ is an element of the spectrum of the Rindler Hamiltonian H associated with ∂_t evolution. Concerning the index α we distinguish between two cases: if $m > 0$ there is a unique mode $\Phi_E^{(\alpha)} = \Phi_E$ whose expression is $\sqrt{2E \sinh(\pi E/\kappa)}/\sqrt{2\pi^2 \kappa\, E}\, e^{-iEt}\, K_{iE/\kappa}(my)$. If $m = 0$ there are two values of α, corresponding to *in*going and *out*going modes, $\Phi_E^{(in)/(out)}$ whose expression are $e^{-iE(t \pm \ln(\kappa y)/\kappa)}/\sqrt{4\pi E}$. If $m > 0$ there is no energy degeneration and the one-particle Hilbert space $\mathcal{H}$ generated by the positive frequency part of the decomposition above is isomorphic to $L^2(\mathbb{R}^+, dE)$. In the other case ($m = 0$), twofold degeneracy implies that $\mathcal{H} \cong L^2(\mathbb{R}^+, dE) \oplus L^2(\mathbb{R}^+, dE)$. Quantum field operators, acting in the symmetrized Fock space

$\mathcal{F}(\mathcal{H})$ and referred to the Rindler vacuum $|0\rangle$ – that is $|0\rangle_{in} \otimes |0\rangle_{out}$ if $m = 0$ – read

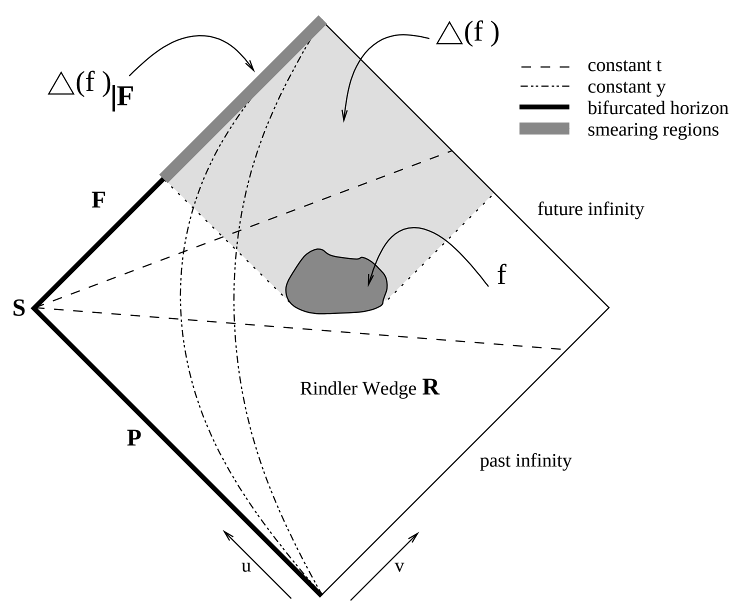
$$\hat{\phi}(t, y) = \int_0^\infty \sum_\alpha \Phi_E^{(\alpha)}(t, y) a_{E\alpha} + \overline{\Phi_E^{(\alpha)}(t, y)} a^\dagger_{E\alpha} dE. \tag{2}$$

As usual, the causal propagator Δ satisfies $[\hat{\phi}(x), \hat{\phi}(x')] = -i\Delta(x, x')$.
In [9] we have found that, if $m > 0$, $\mathcal{H}$ is irreducible under a unitary representation of $SL(2, \mathbb{R})$ generated by (self-adjoint extensions) of the operators iH, iD, iC (which enjoy the commutation relations of the Lie algebra of $SL(2, \mathbb{R})$), with

$$H := E\,, \quad D := -i\left(\frac{1}{2} + E\frac{d}{dE}\right)\,, \quad C := -\frac{d}{dE}E\frac{d}{dE} + \frac{(k - \frac{1}{2})^2}{E}\,. \tag{3}$$

k can arbitrarily be fixed in $\{1/2, 1, 3/2, \ldots\}$. See [11] for details on domains an all that. If $m = 0$ and so $\mathcal{H} \cong L^2(\mathbb{R}^+, dE) \oplus L^2(\mathbb{R}^+, dE)$, an analogue representation exists in each space $L^2(\mathbb{R}^+, dE)$. Making use of Heisenberg representation it is simply proven that the algebra generated by H, D, C, with depending-on-time coefficients, is made of constant of motions [9, 11, 10]. Thus $SL(2, \mathbb{R})$ is a symmetry of the one-particle system (that can straightforwardly be extended to the free quantum field in Fock space). The crucial point is that the found symmetry is *hidden*: it cannot be induced by the background geometry since the Killing fields of Rindler spacetime enjoy a different Lie algebra from that of H, D, C.

3. *Fields on the horizon.* Let us to investigate the nature of the found symmetry exactly on the event horizon assuming **R** to be naturally embedded in a Minkowski spacetime. In particular we want to investigate its geometrical nature, if any, on the event horizon.



(Rindler) *light coordinates* $u = t - \log(\kappa y)/\kappa$, $v = t + \log(\kappa y)/\kappa$ (where $u, v \in \mathbb{R}$) cover the (open) Rindler space **R**. Separately, v is well defined on the future horizon **F**, $u \to +\infty$, and u is well defined on the past horizon **P**, $v \to -\infty$ (see figure). Take the wavefunction in (1) and consider the limit on the future horizon $u \to +\infty$. That is equivalent to restrict the wavefunction on the event horizon when it is considered as a wavefunction in Minkowski spacetime, obtaining

$$\psi(v) = \int \frac{e^{-iEv}}{\sqrt{4\pi E}} e^{i\rho_{m,\kappa}(E)} \tilde{\psi}_+(E)\, dE + \text{c.c.} \tag{4}$$

$e^{i\rho_{m,\kappa}(E)}$ is a pure phase (see [11] for details). In coordinate $u \in \mathbb{R}$, the restriction of ψ to **P** is similar with the v replaced for u and $\rho_{m,\kappa}(E)$ replaced by $-\rho_{m,\kappa}(E)$. If $m = 0$ the restrictions to **F** and **P** read respectively

$$\psi(v) = \int \frac{e^{-iEv}}{\sqrt{4\pi E}} \tilde{\psi}_+^{(in)}(E)\, dE + \text{c.c.}\ , \qquad \psi(u) = \int \frac{e^{-iEu}}{\sqrt{4\pi E}} \tilde{\psi}_+^{(out)}(E)\, dE + \text{c.c.} \tag{5}$$

Discarding the phase it is possible to consider the following real "field on the future Horizon":

$$\varphi(v) = \int_{\mathbb{R}^+} \frac{e^{-iEv}}{\sqrt{4\pi E}} \tilde{\varphi}_+(E)\, dE + \int_{\mathbb{R}^+} \frac{e^{+iEv}}{\sqrt{4\pi E}} \overline{\tilde{\psi}_+(E)}\, dE \tag{6}$$

as the basic object in defining a quantum field theory on the future event horizon. The same can be done for the past event horizon. The one-particle Hilbert space $\mathcal{H}_{\mathbf{F}}$ is defined as the space generated by positive frequency parts $\tilde{\psi}_+(E)$ and turns out to be isomorphic to $L^2(\mathbb{R}^+, dE)$ once again. The field operator reads, on the symmetrized Fock space $\mathfrak{F}(\mathcal{H}_{\mathbf{F}})$ with vacuum $|0\rangle_{\mathbf{F}}$,

$$\hat{\phi}_{\mathbf{F}}(v) = \int_0^\infty \frac{e^{-iEv}}{\sqrt{4\pi E}} a_E + \frac{e^{iEv}}{\sqrt{4\pi E}} a_E^\dagger dE\ . \tag{7}$$

The causal propagator $\Delta_{\mathbf{F}}$ is defined by imposing $[\hat{\phi}(v), \hat{\phi}(v')] = -i\Delta_{\mathbf{F}}(v, v')$ and it takes the form $(1/4)sign(v - v')$. In spite of the absence of any motion equation the essential features of free quantum field theory are preserved by that definition as proven in [11]. Degeneracy of the metric on the horizon prevents from smearing field operators by functions due to the ill-definiteness of the induced volume measure. However, employing the symplectic approach [16], a well-defined smearing-procedure is that of field operators and exact 1-forms $\eta = df$ where $f = f(v)$ vanishes fast as $v \mapsto \pm\infty$. The integration of forms does not need any measure. In other words for a real exact 1-form η as said above

$$\hat{\phi}_{\mathbf{F}}(\eta) = \int_0^\infty \frac{dE}{\sqrt{4\pi E}} \left(\int_{\mathbb{R}} e^{-iEv}\eta(v) \right) a_E + \left(\int_{\mathbb{R}} e^{iEv}\eta(v) \right) a_E^\dagger \tag{8}$$

is well defined and diffeomorphism invariant. In a suitable domain the map $\eta(v) \mapsto \Delta_{\mathbf{F}}(\eta) = \frac{1}{4}\int_{\mathbb{R}} sign(v - v')\eta(v') = \psi_\eta(v)$ defines a one-to-one correspondence between exact one-forms and horizon wavefunctions of the form (1) and $\eta = 2d\psi_\eta$. Finally, similarly to usual quantum field theory [16], it holds

$$[\hat{\phi}_{\mathbf{F}}(\eta), \hat{\phi}_{\mathbf{F}}(\eta')] = -i\Delta_{\mathbf{F}}(\eta, \eta') = \int_{\mathbf{F}} \psi_{\eta'} d\psi_\eta - \psi_\eta d\psi_{\eta'} .$$

The last term define a diffeomorphism-invariant symplectic form on horizon vavefunctions.

4. *Unitary and algebraic holographic theorems.* It is possible to prove the existence of a unitary equivalence between the theory in the bulk and that on the horizon in the sense we are going to describe. Consider the case $m > 0$ and the future horizon $\mathbf{F}$.

Theorem 1. *There is a unitary map* $U_{\mathbf{F}} : \mathcal{F}(\mathcal{H}) \to \mathcal{F}(\mathcal{H}_{\mathbf{F}})$ *such that* $U_{\mathbf{F}}|0\rangle = |0\rangle_{\mathbf{F}}$ *and* $U_{\mathbf{F}}^{-1}\hat{\phi}_{\mathbf{F}}(\eta)U_{\mathbf{F}} = \hat{\phi}(f)$ *for any smooth compactly supported function* f *used to smear the bulk field,* $\eta = 2d(\Delta(f)\restriction_{\mathbf{F}})$*.* (See figure.)

Details on the construction of $U_{\mathbf{F}}$ are supplied in [11], here we give only the main idea. Take f as said and consider the associated bulk wavefunction $\psi_f = \Delta(f)$, restrict ψ_f to $\mathbf{F}$ obtaining a horizon vavefunction as in (4) with positive frequency part $e^{i\rho_{m,\kappa}}(E)\tilde{\psi}_{f+}(E)$. Then define a horizon vavefunction φ_f as in (6) with $\tilde{\varphi}_+$ replaced by $\tilde{\psi}_{f+}$. It is clear that the map $\psi_f \mapsto \varphi_f$ corresponds to a unitary operator from $\mathcal{H}$ to $\mathcal{H}_{\mathbf{F}}$. That is, by definition $U_{\mathbf{F}}\restriction_{\mathcal{H}}$. Imposing $U_{\mathbf{F}}|0\rangle = |0\rangle_{\mathbf{F}}$, by taking tensor products of $U_{\mathbf{F}}\restriction_{\mathcal{H}}$, this map extends to a unitary map $U_{\mathbf{F}} : \mathcal{F}(\mathcal{H}) \to \mathcal{F}(\mathcal{H}_{\mathbf{F}})$. Finally, by direct inspection one finds that, if $\eta = 2d\varphi_f$, one also has $U_{\mathbf{F}}^{-1}\hat{\phi}_{\mathbf{F}}(\eta)U_{\mathbf{F}} = \hat{\phi}(f)$.

The same procedure can be used to define an analogous unitary operator referred to $\mathbf{P}$. If $m = 0$ two unitary operators arises. One is $V_{\mathbf{F}} : \mathcal{F}(\mathcal{H}_{in}) \to \mathcal{F}(\mathcal{H}_{\mathbf{F}})$ such that $V_{\mathbf{F}}|0\rangle_{in} = |0\rangle_{\mathbf{F}}$ and $V_{\mathbf{F}}^{-1}\hat{\phi}_{\mathbf{F}}(\eta_f)V_{\mathbf{F}} = \hat{\phi}_{in}(f)$. $\mathcal{H}_{in}$ is the bulk Hilbert space associated with the ingoing modes ans $\hat{\phi}_{in}(f)$ is the part of bulk field operator built up using only ingoing modes. The other unitary operator $V_{\mathbf{P}} : \mathcal{F}(\mathcal{H}_{out}) \to \mathcal{F}(\mathcal{H}_{\mathbf{P}})$ plays an analogous rôle with *in* replaced for *out* and $\mathbf{F}$ replaced for $\mathbf{F}$ everywhere. (More generally $V_{\mathbf{P}} \otimes V_{\mathbf{F}} : \mathcal{F}(\mathcal{H}) \to \mathcal{F}(\mathcal{H}_{\mathbf{P}}) \otimes \mathcal{F}(\mathcal{H}_{\mathbf{F}})$ define a unitary operator which transforms the vacuum states into vacuum states and field operators into field operators.) As a consequence of the cited theorem, e.g. if $m > 0$, one has the invariance of vacuum expectation values:

$${}_{\mathbf{F}}\langle 0|\hat{\phi}_{\mathbf{F}}(\eta_1)\cdots\hat{\phi}_{\mathbf{F}}(\eta_n)|0\rangle_{\mathbf{F}} = \langle 0|\hat{\phi}(f_1)\cdots\hat{\phi}(f_n)|0\rangle .$$

Similarly to the extent in the bulk case, one focuses on the algebra $\mathcal{A}_{\mathbf{F}}$ of linear combinations of product of field operators $\hat{\phi}_{\mathbf{F}}(\omega)$ varying ω in the space of allowed complex 1-forms. We assume that $\mathcal{A}_{\mathbf{F}}$ also contain the unit operator I. The Hermitean elements of $\mathcal{A}_{\mathbf{F}}$ are the natural observables associated with the horizon field. From an abstract point of view the found algebra is a unital $*$-algebra of formal operators $\phi_{\mathbf{F}}(\eta)$ with the additional properties $[\phi_{\mathbf{F}}(\eta), \phi_{\mathbf{F}}(\eta)] = -i\Delta_{\mathbf{F}}(\eta, \eta')$, $\phi_{\mathbf{F}}(\eta)^* = \phi_{\mathbf{F}}(\overline{\eta})$ and linearity in the form η[1]. $\mathcal{A}_{\mathbf{F}}$ can be studied no matter any operator representation in any Fock space. Operator representations are obtained via GNS

[1] The analogous algebra of operators in the bulk fulfill the further requirement $\phi(f) = 0$ if (and only if) $f = Kg$, K being the Klein-Gordon operator. No analogous requirement makes sense for $\mathcal{A}_{\mathbf{F}}$ since there is no equation of motion on the horizon.

theorem once an algebraic state has been fixed [16]. In the case $m > 0$ we get the following result which is independent from any choice of vacuum state and Fock representation. The proof can be found in [11]. $\mathcal{A}_{\mathbf{R}}$ denotes the unital $*$-algebra of associated with the bulk field operator.

Theorem 2. *There is a unique injective unital $*$-algebras homomorphism $\chi_{\mathbf{F}} : \mathcal{A}_{\mathbf{R}} \to \mathcal{A}_{\mathbf{F}}$ such that $\chi_{\mathbf{F}}(\phi(f)) = \phi_{\mathbf{F}}(\eta_f)$, where $\eta = 2d(\Delta(f)\restriction_{\mathbf{F}})$. Moreover in GNS representations in the respectively associated Fock spaces $\mathfrak{F}(\mathcal{H})$, $\mathfrak{F}(\mathcal{H}_{\mathbf{F}})$ built up over $|0\rangle$ and $|0\rangle_{\mathbf{F}}$ respectively, $\chi_{\mathbf{F}}$ has a unitary implementation and reduces to $U_{\mathbf{F}}$.*

Notice that, in particular $\chi_{\mathbf{F}}$ preserves the causal propagator, in the sense that it must be $-i\Delta(f,g) = [\phi(f),\phi(g)]I = [\phi(f),\phi(g)]\chi_{\mathbf{F}}(I) = \chi_{\mathbf{F}}([\phi(f),\phi(g)]I) = [\chi(\phi(f)),\chi(\phi(g))]$ $= [\phi_{\mathbf{F}}(f),\phi_{\mathbf{F}}(g)] = -i\Delta_{\mathbf{F}}(\eta_f,\eta_g)$.

Analogous algebraic homomorphism theorems can be given for **P** and the massless case [11].

5. *The $SL(2,\mathbb{R})$ symmetry becomes manifest on the horizon.* Consider quantum field theory on **F**, but the same result holds concerning **P**. In $\mathcal{H}_{\mathbf{F}} \cong L^2(\mathbb{R}^+, dE)$ define operators $H_{\mathbf{F}}, D_{\mathbf{F}}, C_{\mathbf{F}}$ as the right-hand side of the equation that respectively defines H, D, C in (3). Exactly as in the bulk case, operators $iH_{\mathbf{F}}, iD_{\mathbf{F}}, iC_{\mathbf{F}}$ generate a unitary $SL(2,\mathbb{R})$ representation $\{\mathfrak{U}_g\}_{g\in SL(2,\mathbb{R})}$. Hence, varying $g \in SL(2,\mathbb{R})$, $U_g = (U_{\mathbf{F}}\restriction_{\mathcal{H}})^{-1}\mathfrak{U}_g U_{\mathbf{F}}\restriction_{\mathcal{H}}$ define a representation of $SL(2,\mathbb{R})$ for the system in the bulk. By construction $(U_{\mathbf{F}}\restriction_{\mathcal{H}})^{-1} H_{\mathbf{F}}\, U_{\mathbf{F}}\restriction_{\mathcal{H}} = H$. As a consequence every U_g turns out to be a $SL(2,\mathbb{R})$ symmetry of the bulk system and the group of these symmetries is unitary equivalent to that generated by iH, iD, iC. In particular the one-parameter group associated with $H_{\mathbf{F}}$ generates v-displacements of horizon wavefunctions which are equivalent, under the action of $U_{\mathbf{F}}$, to t-displacements of bulk wavefunctions. Now, it make sense to investigate the *geometrical nature* of the $SL(2,\mathbb{R})$ representation $\{\mathfrak{U}_g\}$ that, as we said, induces, up to unitary equivalences, the original $SL(2,\mathbb{R})$ symmetry in the bulk. In fact it is possible to prove that:

Theorem 3. *If $k = 1$ in (3), the action of every $\mathfrak{U}_g$ on a state $\tilde{\varphi}_+ = \tilde{\varphi}_+(E)$ is essentially equivalent to the action of a corresponding **F**-diffeomorphism on the associated (by (6)) horizon wavefunction φ. More precisely, take a matrix $g \in SL(2,\mathbb{R})$ and $\varphi = \varphi(v)$ in a suitable space of horizon wavefunction (see [11]). Let $\tilde{\varphi}_+ = \tilde{\varphi}_+(E)$ be the positive frequency part of φ. The wavefunction φ_g associated with $\mathfrak{U}_g\tilde{\varphi}_+$ reads*

$$\varphi_g(v) = \varphi\left(\frac{av+b}{cv+d}\right) - \varphi\left(\frac{b}{d}\right), \qquad g^{-1} = \begin{pmatrix} a & b \\ c & d \end{pmatrix}. \tag{9}$$

The term $-\varphi(b/d)$ assures that φ_g vanishes as $v \to \pm\infty$. Notice that the added term disappears when referring to $d\varphi$ rather than φ. The group of diffeomorphisms of **F**, i.e. the real line[2],

$$v \mapsto \frac{av+b}{cv+d}, \qquad g = \begin{pmatrix} a & b \\ c & d \end{pmatrix} \in SL(2,\mathbb{R}) \tag{10}$$

can be obtained by composition of one-parameter subgroups associated with the following three vector fields on **F**: $\partial_v, v\partial_v, v^2\partial_v$. It is simply proven that the Lie brackets of those fields is a realization of the Lie algebra of $SL(2,\mathbb{R})$. Moreover, it turns out that [11]:

[2]Actually one has to consider the projective line $\mathbf{F} \cup \{\infty\}$.

Theorem 4 *(a) If $k = 1$ in (3), the unitary one-parameter group generated by $iH_{\mathbf{F}}$ is associated, through Theorem 3, to the one-group of $\mathbf{F}$-diffeomorphisms generated by ∂_v, (b) the unitary one-parameter group generated by $iD_{\mathbf{F}}$ is associated to the one-parameter group of $\mathbf{F}$-diffeomorphisms generated by $v\partial_v$ and (c) the unitary one-parameter group generated by $iC_{\mathbf{F}}$ is associated to the one-group of $\mathbf{F}$-diffeomorphisms generated by $v^2\partial_v$.*

6. *Appearance of Virasoro algebra.* The bulk $SL(2,\mathbb{R})$-symmetry is manifest when examined on the event horizon, in the sense that it is induced by the geometry. The Lie algebra generated by vector fields $\partial_v, v\partial_v, v^2\partial_v$ play a crucial rôle in proving this fact. That algebra can be extended to include all the class of fields defined on the event horizon $\{\mathcal{L}_n\}_{n\in\mathbb{Z}}$ with $\mathcal{L}_n = v^{n+1}\partial_v$. It is interesting to notice that these fields enjoy Virasoro commutation relations without central charge, $\{\mathcal{L}_n, \mathcal{L}_m\} = (n-m)\mathcal{L}_{n+m}$. A natural question arises:
Is it possible to give a quantum representation of these generators in the sense of Theorem 4?
At least formally, the answer is positive. Indeed, by employing Theorem 4 one finds out that the infinitesimal action of the one parameter group of diffeomorphisms generated by $\mathcal{L}_n$ on a horizon wavefunction $\varphi = \varphi(v)$ is equivalent to the action of an anti-Hermitean operator L_n on the positive frequency part $\tilde{\varphi}_+ = \tilde{\varphi}_+(E)$. L_n is defined as, respectively for $n \geq -1$ and $n < -1$,

$$(L_n\tilde{\varphi}_+)(E) \;:=\; i^{n+2}\sqrt{E}\frac{d^{n+1}}{dE^{n+1}}\sqrt{E}\tilde{\varphi}_+(E)\,, \tag{11}$$

$$(L_n\tilde{\varphi}_+)(E) \;:=\; -i^{-(n+2)}\sqrt{E}\int_0^E dE_1\int_0^{E_1} dE_2\cdots\int_0^{E_{-(n+2)}} dE_{-(n+1)}\sqrt{E_{-(n+1)}}\tilde{\varphi}_+(E_{-(n+1)})\,. \tag{12}$$

Those operators are at least anti-Hermitean on suitable domains and enjoy Virasoro commutation rules $[L_n, L_m] = (n-m)\,L_{n+m}$.

7. *Four dimensional case.* Up to now we have investigated only the two dimensional spacetimes, but it is possible to extend our results to a four dimensional case which better approximates the Schwarzschild extent. For this purpose consider the near-horizon approximation of a Schwarzschild-like spacetime without discarding the angular variables θ, ϕ, so that $ds^2 = -\kappa^2 y^2 dt^2 + dy^2 + r_h^2 d\Omega^2$. Every field takes an angular part described by the usual spherical harmonics $Y_m^l(\theta,\phi)$. QFT in the bulk involves the one-particle Hilbert space $\oplus_{l=0}^{\infty}\mathbb{C}^{2l+1}\otimes\mathcal{H}_l$ with $\mathcal{H}_l \cong L^2(\mathbb{R}^+, dE)$ if $l > 0$, $\mathbb{C}^{2l+1}$ being the space at fixed total angular momentum l and $\mathcal{H}_0 \cong L^2(\mathbb{R}^+, dE)$ in the massive case but $\mathcal{H}_0 \cong L^2(\mathbb{R}^+, dE) \oplus L^2(\mathbb{R}^+, dE)$ in the massless case. For wavefunctions with components in a fixed space $\mathbb{C}^{2l+1}\otimes L^2(\mathbb{R}^+, dE)$ Klein-Gordon equation reduces to the two-dimensional one with a positive contribution to the mass depending on l. Quantum field theory can be constructed on the future horizon $\mathbf{F} \cong \mathbb{S}^2 \times \mathbb{R}$. The appropriate causal propagator reads

$$\Delta_{\mathbf{F}}(x,x') = (1/4)sign(v-v')\delta(\theta-\theta')\delta(\phi-\phi')\sqrt{g_{\mathbb{S}^2}(\theta,\phi)}\,.$$

The horizon field operator $\hat{\phi}_{\mathbf{F}}$ has to be smeared with 3-forms as $df(v,\theta,\phi)\wedge d\theta\wedge d\phi$ and Theorems 1 and 2, at least in the massive case, can be restated as they stand for the two-dimensional case. Theorems 3 and 4 hold true at fixed angular variables.

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