UNIVERSITY OF TRENTO

CIFREM

INTERDEPARTMENTAL CENTRE FOR RESEARCH TRAINING
IN ECONOMICS AND MANAGEMENT

DOCTORAL SCHOOL IN ECONOMICS AND MANAGEMENT

MEASURING PRODUCTIVITY AND TECHNOLOGICAL PROGRESS

DEVELOPMENT OF A CONSTRUCTIVE METHOD BASED ON CLASSICAL ECONOMICS AND INPUT-OUTPUT TABLES

A DISSERTATION
SUBMITTED TO THE DOCTORAL SCHOOL OF ECONOMICS AND MANAGEMENT
IN PARTIAL FULFILLMENT OF THE REQUIREMENTS
FOR THE DOCTORAL DEGREE
(PH.D.)
IN ECONOMICS AND MANAGEMENT

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May 2010

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MEASURING PRODUCTIVITY AND TECHNOLOGICAL PROGRESS

Development of a Constructive Method based on Classical Economics and Input-output tables

A Dissertation submitted to the doctoral school of economics and management in partial fulfillment of the requirements for the doctoral degree (Ph.D) in Economics and Management, January 2010

by

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Acknowledgements

Writing a doctoral thesis has been a complex and curious process and the final result is a synthesis of the ideas of many minds. My simple role has been to collect and reorganize the ideas of those who have preceded me in this line of study, those who have worked with me, and those who while not expert in this field have had the patience to listen and discuss with me. Many people deserve gratitude for successful completion of my dissertation.

My first thanks go to Professor Stefano Zambelli for having accepted to be my supervisor and for giving me the passion and support to overcome this challenge.

Another special thanks goes to Thomas Fredholm, with whom I have worked very closely during the writing of my thesis.

I would also like to thank all the members of CIFREM and all the staff at OPES for giving me the opportunity to work in an international and professional environment.

I also wish to express my gratitude to Professor Charlotte Bruun and Prof. Carsten Heyn-Johnsen for making my experience in Denmark a pleasant one.

Finally, I would also like to thank my family for the support and encouragement I received while doing my thesis.

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A.2.Productivity in the Italian Regions: Development of Alternative Indicators Based on Input-Output Tables

INTRODUCTION

The purpose of this introduction is to provide a background to facilitate the understanding of subsequent chapters. The first section describes the traditional approaches to measuring productivity, in order to identify the strengths and weakness of each of them. The second section illustrates the model of Sraffa in the case of single production and deepens some aspects related to the model that will be used in the dissertation. The third section describes the structure of the thesis, while the fourth and fifth sections highlight the advantages and limitations of the new approach proposed.

I.1 Traditional approaches to measuring productivity

The measurement and the examination of productivity has become increasingly important, particularly in those countries where the growth in the number of employees and in the accumulation of capital has peaked, and where, therefore, increases in productivity are the only way to sustain economic growth.

However, productivity analysis is a relatively recent innovation because it requires data, on both output and input, and thus it could not evolve before the emergence of modern national accounts after the end of the Second World War.

The problem of how to measure productivity has been approached from many perspectives[1] and this has led to the development of various methods which sometimes provide dissimilar and even contrasting results. In this respect, there was a debate between Jorgenson and Griliches (1967, 1972) on one hand and Denison (1972) on the other over differences in their estimates of productivity change. According to Denison a substantial part of the post war growth of the US output was due to an increase in productivity, while according to Jorgenson and Griliches, almost all of the increase was due to an increase in factor inputs.

In recent years, the debate shifted from results generated by different methods to the apparent paradox between what is observed and what is measured, which is well summarized by Robert Solow's (Solow, 1987) famous observation "we can see the computer age everywhere but in the productivity statistics."

Following the scheme most usually adopted (see, for example, Hulten, 2000), there are four main approaches to the study of productivity: the growth accounting approach, the index number approach, the distance function approach, and the econometric approach.

The growth accounting approach is also known as Solow's residual (Solow, 1957) because it assumes that the contribution of productivity to economic growth

[1] Some of these perspectives will be discussed later in this introduction.

is a residual factor: what remains after all other factors, such as the growth of labour and capital inputs is deducted[2].

The growth accounting approach is closely linked with the Solow's growth model (Solow, 1956). The model studies the dynamics of a country's economic growth in the long run and it was developed by Solow from the Harrod-Domar model (Harrod 1939; Domar 1946). In particular, in his model Solow relaxes the assumption of constant capital intensity, which characterizes the Harrod-Domar model, and, based on neoclassical assumptions, introduces substitutability between production factors and thus the possibility of adjustments of the capital-labor ratio in the long term.

The introduction of the hypothesis of substitutability between labor and capital has the consequence that in the Solow model, and contrary to what happens in the Harrod-Domar model, the equilibrium growth rate is stable and the growth of output per capita in the long run is determined only by technical progress.

Solow assumes an aggregate production function (I.1), where output Q depends on labor (L) and capital (K).

$$Q(t) = f\,(K(t), L(t)) \qquad \text{(I.1)}$$

The production function exhibits constant return to scale and diminishing marginal productivity to each input.

From the assumption of constant returns to scale it follows that the function is homogeneous of degree one and can be rewritten as follows:

$$q = f\,(k, 1) = F\,(k) \qquad \text{(I.2)}$$

where $q = Q/L$ and $k = K/L$ are respectively the output per capita and capital per capita.

Savings (S) are considered as a constant fraction (s) of income (I.3):

$$S = sQ \qquad \text{(I.3)}$$

where s is precisely the propensity to save.

It is assumed a law of geometric depreciation for capital, a law that ensures that the depreciation in each period is always a constant fraction δ of the capital stock, regardless of the timing of investments that produce it. The law of capital accumulation will be given by

$$\dot{K} = I - \delta K \qquad \text{(I.4)}$$

where $\dot{K}$ is the change in capital stock over time.

[2] In light of this definition, the residual can be interpreted in different ways, but in practice it is simply a measure of our ignorance (Abramovitz, 1956).

Solow analyzes the equilibrium conditions of a closed economy and for this reason he assumes ex-ante equality between investments and savings:

$$S = I \qquad (I.5)$$

Finally, Solow assumes a constant growth rate of the population, which coincides with that of the labor force since it is assumed that in equilibrium there is full employment, equal to n, where:

$$L = L_0 e^{nt} \qquad (I.6)$$

In 1956 article, Solow considers the possibility of including in his model technical progress. In particular, Solow examines the hypothesis of Hicks-neutral technical progress with an aggregate production function of Cobb-Douglas type. Technical progress is modelled as a multiplication factor of the original function, which increases the total output without changing the marginal rate of technical substitution, accordingly

$$Q(t) = A(t) f(K(t), L(t)) \qquad (I.7)$$

An alternative way to introduce technical progress is to assume Harrod-neutral technical progress or labor augmenting technical progress. The production function can be reformulated as follows[3]

$$Q(t) = f(K(t), A(t) L(t)) \qquad (I.8)$$

Solow (1957) used the production function with Hicks neutral technical progress (I.7) to decompose output growth into the different contributions of labor, capital, and technology.

The mathematical presentation of the growth accounting approach follows as:

$$\frac{\dot{A}(t)}{A(t)} = \frac{\dot{Q}(t)}{Q(t)} - (1 - w(t))\frac{\dot{K}(t)}{K(t)} - w(t)\frac{\dot{L}}{L} \qquad (I.9)$$

This expression specifies that the growth rate in total factor productivity is equal to the growth rate in real output minus the growth rate in capital and labour[4] both weighted by their relative income shares. Q_{t}, K_{t}, L_{t}, and the factor

[3] Harrod neutrality implies that relative input shares remain unchanged for a given capital-labor ratio. Hicks neutrality implies that the ratio of the marginal products of capital and labor remains constant. In the Cobb-Douglas production function, Harrod and Hicks neutrality give the same results.

[4] All variables are in logarithmic form.

share can easily be calculated using national account statistics; A_{t} is given by the above formula.

The growth accounting approach is grounded on several restrictive assumptions. First, it requires a constant return to scale production function. This requirement is relevant only when the share of total output that goes to capital is calculated as a residual. In fact, it follows from the Euler's theorem[5] that when a production function exhibits constant return to scale then the total product is equal to the sum of the amounts of the factors multiplied by their marginal products. This requirement together with the assumption that factor are paid according to the marginal products implies that the capital share is equal to the total product minus the wage share.

Second, the growth accounting approach requires the price equal marginal cost conditions because as Hulten (2000, p.12) notes

> the essence of the Solow method is to use prices to estimate the slopes of the production function at the observed input-output configuration, without having to estimate the shape of the function at all other points (i.e., without the need to estimate all the parameters of the technology). The residual is thus a parsimonious method for getting at the shift in the production function, but the price of parsimony is the need to use prices as surrogates for marginal products.

The index number approach is used to classify productivity measures based on the ratio between an output quantity index and an input quantity index. The growth accounting approach can also be included into the index number category, but is treated separately because, unlike the index number approach he develop a precise and elegant link between the production function and the index number.

In the last century, several indexes have been developed, among the most well-known are Laspeyres (1871), Paasche (1874), Fisher (1922) and Tornqvist (1936). As a consequence, there has been a lively debate about which of these indexes is the most appropriate. The two most commonly used approaches are the axiomatic and the economic.

---

[5] The theorem states that where a function is homogeneous of order n in its arguments, so that, for example, if Q = f(L, K), then $f(\lambda L, \lambda K) = \lambda^n f(L, K)$, the sum of the marginal product of each argument times its quantity equals nQ. This implies that if f( ) is a production function with Q as output and L and K the inputs, the amount of factors used times their marginal products equals total output if and only if $n = 1$. Thus if factors are paid their marginal products, only with constant returns to scale does the sum of factor earnings exhaust the total product. With decreasing returns to scale the entrepreneur is left with a profit, and with increasing returns to scale the firm cannot afford to pay its inputs their marginal products.

The axiomatic approach was popularized by Irving Fisher (1922). It lists a series of mathematical properties that an index number has to satisfy, and the preference goes to the one that fulfils the most of these criteria.

In a more recent contribution, Diewert and Nakamura (1993) have listed nine properties that an index number should satisfy[6]. Many of these properties or tests are due to Irving Fisher, but some have been proposed by other authors (Westergaard, 1890; Walsh, 1901; Eichhorn and Voeller, 1976; Vartia, 1985). These properties are

- *Identity*: if prices and quantities are equal, at both time 0 and time 1, the index must be 1
- *Proportionality*: if all prices at time 1 are multiplied by a coefficient a>0, the price index is also multiplied by a
- *Invariance to changes in scale*
- *Invariance to changes in units*
- *Symmetric treatment of countries or time:* the index at time 0 with base 1 must be the reciprocal of the index at time 1 with base 0
- *Symmetric treatment of commodities:* no commodity can be singled out to play an asymmetric role
- *Monotonicity:* if prices in the second period increase in any manner, then the price index cannot decrease
- *Mean value:* the price index should lie between the smallest and largest price ratios over all commodities.
- *Circularity:* if you have two indices, one from time 0 to time 1 and the other from time 1 to time 2, the index from time 0 to time 2 must be equal to their product:

The economic approach instead assumes the production function is a specific functional form, and the index selected is the one that can be derived from that function. The main contributors to this approach are Samuelson (1947), Malmquist (1953) and Pollak (1989).

It is thus evident that the problem of selecting the optimal index is then replaced by the problem of finding the optimal approach of choice. But, many questions remain to be answered, particularly the selection of preferred mathematical properties and appropriate functional form for production function.

In a recent article Van Veelen and Van der Weide (2008, p.1729) discuss the advantages and disadvantages of both approaches and come the following conclusions:

> The difference between the axiomatic and the economic approach in index number theory is that the economic approach

[6] These properties are twenty-one in the latest version of his book.

treats prices and quantities as observations that result from optimizing one single utility function, while the axiomatic approach tries to make meaningful comparisons without the assumption of homogeneity. While the axiomatic approach may include axioms that obstruct finding a good index if this assumption is actually correct, the economic approach is vulnerable to constructing indices of limited use if the assumption turns out to be false...

…A challenge for the future is to explicitly allow for heterogeneity in index number theory.

The distance function approach has been used to disentangle change productivity into two components, which are the movements towards the production frontier and the movement of the frontier. The frontier is usually computed by applying Data Envelopment Analysis (DEA) (Charnes, Cooper, and Rhodes, 1978). DEA develops a function whose form is determined by the most efficient producers using the input and output data of a sample of enterprises.

The measurement of productivity and its decomposition into the two components is done using the Malmquist index. The Malmquist index was introduced by Caves, Christensen, and Diewert (1982a, 1982b) and it was subsequently popularized by Fare, Grosskopf, Norris, and Zhang (1994). This index can be calculated as a ratio of distances from the production function. Essentially, consider a firm at two times periods (0 and 1), the productivity index is defined as

$$\boldsymbol{M}_{\mathrm{i}}\,(\mathbf{y}^1,\mathbf{x}^1,\mathbf{y}^0,\mathbf{x}^0)=[\boldsymbol{D}^0{}_{\mathrm{i}}(\mathbf{y}^1,\mathbf{x}^1)/\boldsymbol{D}^0\mathrm{i}(\mathbf{y}^0,\mathbf{x}^0)]*[\boldsymbol{D}^1{}_{\mathrm{i}}(\mathbf{y}^1,\mathbf{x}^1)/\boldsymbol{D}^1\mathrm{i}(\mathbf{y}^0,\mathbf{x}^0)]^{1/2} \qquad \text{(I.10)}$$

where $\boldsymbol{D}^t{}_i(\mathbf{y},\mathbf{x})$ is the input distance function representing the technology of period t and the vectors $\mathbf{y}^t$, $\mathbf{x}^t$ denote the observed inputs and observed outputs respectively.

The advantage of this method is that it is a non-parametric approach[7], and it allows the estimation of the relative efficiency of production units. Nevertheless, some economists consider DEA unsatisfactory because it treats industries equally and hypothesises that the value added output of each industry can be produced by every other industry. What is more disputable, however, is its assumption of identical aggregate production function for the industries or the countries. Another drawback 'arises from the possibility that measurement errors may lead to data which are located beyond the true best-practice frontier. These outliers will be "enveloped" mistakenly by frontier techniques, resulting in an erroneous best-practice frontier (Hulten, 2000, p.28).'

Finally, the econometric approach is often cited for its flexibility because it allows for the estimation of the parameters of a production function without any a

[7] There are also parametric approaches which are used for the estimation of production frontiers, but they are less common (see Lovell and Schmidt 1988 for a survey).

priori restriction on the production technology. Therefore, this approach is not forced to assume, for example, Hicks-neutral technical change but can accommodate different formulations. Unfortunately, this excess of flexibility sometimes undermines the reliability of the results. The parameters can assume improbable values that require a priori restrictions upon them.

To sum up: each of the most frequently used methodologies of productivity accounting have certain advantages and drawbacks. Notwithstanding marginal improvements in the common approaches for measuring productivity over the last three decades, there is still need for new and enhanced methods.

In particular, the main point of criticism stems from the fact that the traditional approach to measuring productivity often involves the use of an aggregate production function that has theoretical and empirical limitations.

In the subsequent chapters an alternative method for productivity accounting is presented. At this stage, it is not clear to what extent this method is capable of substituting the other approaches. Nevertheless, there are some features that make the proposed approach attractive that will be listed later in this introduction.

I.2 The model of Sraffa

The theoretical model primarily used in this dissertation has been developed by Piero Sraffa over more than three decades and published in his best known work, Production of Commodities by Means of Commodities (1960). In his book Sraffa begins by presenting a simple model of production of subsistence in which the surplus product is just enough to sustain the workers and to be used as inputs in the subsequent period. In such an economy without surplus, there is a unique vector of values of exchange that restores the distribution of goods between sectors, thus ensuring the possibility to continue the production cycle, time after time.

With the extension of this model to the case of production with surplus, the problem of distribution appears on the scene. Firstly, the author assumes that the wage and the profit rate are both uniform. Moreover, given that the surplus should be distributed in proportion to the means of production employed and this can not be done before the heterogeneous means of production are aggregated through prices, and given that prices can not be determined before to know the uniform rate of surplus, it follows that both prices and surplus should be determined simultaneously.

Sraffa, unlike Von Neumann (1945-46) assumes that the salary is variable and not fixed, and then the workers can obtain a share of surplus higher than the subsistence wage. In production of Commodities by Means of Commodities the distribution is not determined endogenously, but the productive relationships, jointly with prices, only determine the net surplus of the system which must be distributed.

To clarify what has been said so far it is introduced now the model of Sraffa by means of the following system of equations (I.11).

$$
\begin{aligned}
&(A_a \cdot P_a + B_a \cdot P_b + \ldots + K_a \cdot P_k) \cdot (1+r) + L_a \cdot w = A \cdot P_a \\
&(A_b \cdot P_a + B_b \cdot P_b + \ldots + K_b \cdot P_k) \cdot (1+r) + L_b \cdot w = B \cdot P_b \\
&\qquad\qquad\qquad . \\
&\qquad\qquad\qquad . \\
&(A_k \cdot P_a + B_k \cdot P_b + \ldots + K_k \cdot P_k) \cdot (1+r) + L_k \cdot w = K \cdot P_k
\end{aligned}
\tag{I.11}
$$

A_a, B_a ... K_a are the quantities of commodities a, b ... k necessary to produce the quantity A of a; A_b, B_b .. K_b are the quantities of commodities a, b ... k required to produce the quantity B of b etc.; L_a, L_b and L_k are the annual amounts of labor employed in sectors a, b ,k respectively. The unknowns of the system are the prices P_a, P_b and P_k of goods, a, b, and k, the unit wage rate w, and the uniform rate of profit r.

At this point is further introduced an equation that defines the national income in terms of which the wage rate and the k prices are expressed. The system now has only one degree of freedom and once the wage rate or the profit rate is exogenously determined the k prices can be determined simultaneously. The system proposed above (I.11) can be rewritten in compact form by introducing a matrix **A** of interindustry coefficients (I.12) and a vector **L** of direct labor coefficients (I.13). Then:

$$
\mathbf{A} = \begin{bmatrix} a_a & b_a & \ldots & k_a \\ a_b & b_b & \ldots & k_b \\ . & . & \ldots & . \\ . & . & \ldots & . \\ a_k & b_k & \ldots & k_k \end{bmatrix} \tag{I.12}
$$

$$
\mathbf{L} = \begin{bmatrix} L_a & L_b & \ldots & L_k \end{bmatrix}, \tag{I.13}
$$

where a_a, b_a and k_a now represent the quantities of goods a,b, and k needed to produce one unit of good a; a_b, b_b and k_b represent the quantity of goods, a,b,k needed to produce one unit of good b, etc.. L_a, L_b and L_k are the coefficients of direct labor used in the sectors a, b, k, measured as the ratio between the total amount of direct labor used in the economic system and the amount of direct labor used in the particular industry.

The representation in compact form thus becomes:

$$\mathbf{P}\cdot\mathbf{A}\cdot(1+r)+\mathbf{L}\cdot w = \mathbf{P} \qquad \text{(I.14)}$$

where P is the column vector of prices.

The system (I.14) has two degrees of freedom because there are k equations and k+2 unknowns represented by the k prices, the wage rate and the rate of profit. By fixing one of the prices as a *numéraire*, the degrees of freedom are reduced to one and to make the system determined is necessary to establish the wage rate or the profit rate.

Now we will consider two cases, one in which all the surplus is attributed entirely to the workers and where therefore the wage rate reaches its maximum value, and another one where the wage rate is equal to zero and the surplus is entirely attribute to the owners of the means of production. In the first case the system in compact form becomes:

$$\mathbf{P}\cdot\mathbf{A}+\mathbf{L}\cdot w = \mathbf{P} \qquad \text{(I.15)}$$

and thus

$$\mathbf{P}\cdot (\mathbf{I}-\mathbf{A}) = \mathbf{L}\cdot w \qquad \text{(I.16)}$$

where **I** is an identity matrix of order k. Since **A** is a non-singular matrix, (**I**-**A**) is also a non-singular matrix and it is therefore invertible. The vector of prices that solve the system is then obtained by dividing both sides of the previous equation by $(\mathbf{I}-\mathbf{A})^{-1}$, consequently:

$$\mathbf{P} = \mathbf{L}\cdot (\mathbf{I}-\mathbf{A})^{-1}\cdot w \qquad \text{(I.17)}$$

In the second case, the one where the salary is equal to zero, the system in compact form is specified as follows

$$\mathbf{P}\cdot\mathbf{A}\cdot(1+R) = \mathbf{P} \qquad \text{(I.18)}$$

where the profit rate is now indicated with a capital R to indicate that it is the highest profit rate attainable by the production system considered. Following Pasinetti (1977), we rewrite the system just proposed (I.18) in the following way

$$\mathbf{P}\cdot [\mathbf{I} - (1+R) \cdot \mathbf{A}] = 0 \qquad (I.19)$$

and introducing for convenience

$$\lambda = 1/(1+R) \qquad (I.20)$$

in this way we obtain

$$\mathbf{P}\cdot [\lambda\, \mathbf{I} - \mathbf{A}] = 0 \qquad (I.21)$$

The system (I.21) is a homogeneous system and it admits non-trivial solutions only if the rank of the matrix (λ **I**-**A**) is less than k[8], then only if the determinant of (λ **I**-**A**) is zero. The values of λ that satisfy the equation **det(λ I-A)=0** are the eigenvalues of matrix A, since it is the characteristic polynomial associated to the matrix A. However, for the theorem of Perron-Frobenius, the only eigenvalue to which corresponds an eigenvector with non-negative prices, in the presence of a non-negative and irreducible matrix, is the maximum eigenvalue. The maximum eigenvalue is therefore capable of ensuring that the solution of the system has economic significance. The maximum rate of profit is then obtained as follows

$$R = (1/ \lambda_m) -1 \qquad (I.22)$$

The formula (I.22) shows quite clearly a further condition that must be satisfied. The maximum eigenvalue must be smaller than one, because otherwise the maximum rate of profit would be less than zero and therefore without economic significance. This condition is commonly defined as a condition of vitality of the system, because when it is not satisfied the system is not viable and therefore unable to generate profits even when the unit wage rate is equal to zero.

In addition to the two extreme cases described up to now, there are an infinite number of intermediate cases in which the profit rate (or alternatively the unit wage rate) is between zero and its maximum value. The system in this case can be expressed using the following equation

[8] Because of the theorem of Rouché-Capelli

$$\mathbf{P} = \mathbf{L} \cdot [\mathbf{I} - (1+r^*) \cdot \mathbf{A}]^{-1} \cdot w \qquad (I.23)$$

where r* is the profit rate selected. The solution of the system is obtained by fixing a price of a commodity (or any combination of commodities) equal to one and therefore obtaining a determined system.

Now it is possible to represent graphically the relationship between the profit rate and the unit wage rate by means of the wage-profit frontier.

Fig.I.1 - Wage-profit frontier in the particular case of standard system

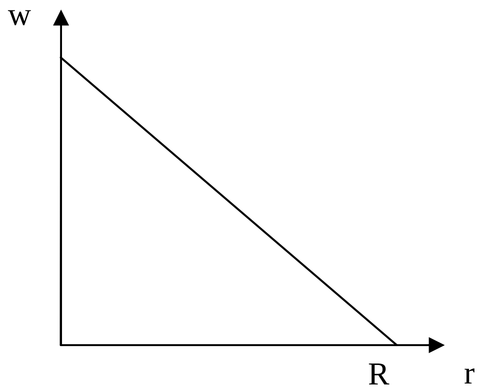

Fig.I.1 shows an example of the wage-profit frontier in a particular case, when the system considered is a standard system. The standard system and the standard commodity are two elements introduced by Sraffa in his search for an invariable measure of value. This search for an invariable measure of value was one of the most important problems posed by David Ricardo, but the English author was not able to find a solution. However, in order not to complicate the discussion, a more thorough description of these elements will be carried out in the next subsection. For the purposes of here is sufficient to recall that regardless of the fact that the production system is a standard system or not, the relationship between the unit wage rate and the profit rate is always strictly monotonic and thus the frontier is always descending and its intercept on the horizontal axis is R.

When a non-standard system is considered, the shape of the wage-profit frontier is no longer a straight line, but a complex path. This is due to the fact that when the system is non-standard prices change with the rate of profit and consequently the value of capital and the surplus also change. A further complication is that the shape of the wage-profit frontier is *numéraire* dependent.

I.2.1 The standard system and the standard commodity

One of the main research objectives of Sraffa was to identify an invariable measure of value that would allow isolating the price changes resulting from the characteristics of the commodities examined from those resulting from the characteristics of the commodity that is used as *numéraire*, by which the other relative prices are measured. In other words, it was shown above that there is a different price vector for each one of the infinite possible combinations of wages and profits. Consequently, it is useful to examine how the price of a particular commodity varies when the profit rate increases from zero to its maximum value.

However, since it is a relative price, its variation is caused by two factors: the first is the characteristics of the commodity itself, and then the intensity of capital used to produce it and the intensity of capital goods used in its production, the second is the characteristics of the commodity used as a *numéraire* that they may influence the relative price.

Therefore, Sraffa proposed the standard commodity as invariable measure of value. In order to uncover this commodity, Sraffa examines the effects of a change in the unit wage rate on the profit rate and the prices of the individual commodities, assuming that the production techniques remain unchanged.

When the entire surplus goes to wages relative prices are determined by the direct and indirect labor required to produce the commodities. This result sustains the labor theory of value supported by classical economists. However, when the rate of profit is positive, the labor theory of value is no longer valid and the key element in determining the movement of relative prices is given by the differences in the proportions of labor and capital that are used in various industries.

Nevertheless, the movement of relative prices depends not only on the proportion of labor and means of production of the commodity in question, but also on the relationship between labor and means of production of each of the other commodities used to produce it. A reduction of the unit wage rate produces a change in relative prices that rebalance the position of the industries in deficit, those with low labor-capital ratio, and the position of the industries in surplus, those with a relatively high labor-capital ratio.

The mathematical formalization of what has been said is now proposed following Pasinetti (1977).

Let us starts from usual price system

$$\mathbf{P}\cdot\mathbf{A}\cdot(1+\mathrm{r})+\mathbf{L}\cdot\mathrm{w} = \mathbf{P} \qquad \text{(I.24)}$$

which is a system with k equations and k +2 unknowns. To solve the system is necessary to fix the value of a distributional variable and select a *numéraire*.

When the *numéraire* is the standard commodity, the system assumes specific characteristics. The standard commodity is given by the combination of commodities that constitutes the standard net product, which is obtained in turn from the standard system, i.e. an economic system in which the proportions in which different commodities are produced are equal to the proportions in which they are used as inputs in the production.

$$\mathbf{P}\cdot(\mathbf{I} - \mathbf{A}) \cdot \mathbf{Q}^* = 1 \qquad \text{(I.25)}$$

where Q* is the column vector containing the total quantity of commodities produced by the standard system. Now it is important to note that the actual net product will be generally different from one, except at the point where the profit rate is zero. At this point, prices are proportional to the quantity of labor incorporated and the wage rate is exactly one[9]. The actual system (I.14) expressed in terms of standard commodity is thus as follows

$$\mathbf{P}\cdot\mathbf{A} + \mathbf{P}\cdot\mathbf{A}\cdot r + \mathbf{L}\cdot w = \mathbf{P} \qquad \text{(I.26)}$$

$$\mathbf{P}\cdot(\mathbf{I} - \mathbf{A}) \cdot \mathbf{Q}^* = 1 \qquad \text{(I.27)}$$

post-multiplying the members of the first equation and rearranging, we obtain

$$\mathbf{P}\cdot\mathbf{A}\cdot\mathbf{Q}^*\cdot r = \mathbf{P}\cdot\mathbf{Q}^* - \mathbf{P}\cdot\mathbf{A}\cdot\mathbf{Q}^* - \mathbf{L}\cdot\mathbf{Q}^*\cdot w = \mathbf{P} \qquad \text{(I.28)}$$

$$\mathbf{P}\cdot\mathbf{A}\cdot\mathbf{Q}^*\cdot r = \mathbf{P}\cdot (\mathbf{I} - \mathbf{A})\cdot\mathbf{Q}^* - \mathbf{L}\cdot\mathbf{Q}^*\cdot w = P \qquad \text{(I.29)}$$

Now, since **P** (**I** - **A**) **Q** * = 1 for (I.27) and **L** **Q** *= 1 by convention, we have then

$$\mathbf{P}\cdot\mathbf{A}\cdot\mathbf{Q}^*\cdot r = 1 - w \qquad \text{(I.30)}$$

or multiplying both terms for the maximum rate of profit

$$\mathbf{P}\cdot\mathbf{A}\cdot\mathbf{Q}^*\cdot r\cdot R = R\cdot (1 - w) \qquad \text{(I.31)}$$

By isolating the term **P** **A** **Q** * R and by considering the equation of the standard system [**I** - (1 + R) **A**] **Q** *= 0, pre-multiplying by the prices and rearranging we have

$$\mathbf{P}\cdot\mathbf{A}\cdot\mathbf{Q}^*\cdot R = \mathbf{P}\cdot \mathbf{Q}^* - \mathbf{P}\cdot\mathbf{A}\cdot\mathbf{Q}^* \qquad \text{(I.32)}$$

$$\mathbf{P}\cdot\mathbf{A}\cdot\mathbf{Q}^*\cdot R = \mathbf{P}\cdot(\mathbf{I} - \mathbf{A})\cdot\mathbf{Q}^* \qquad \text{(I.23)}$$

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[9] This is due to the normalization adopted.

Since **P** (**I** - **A**) **Q** *= 1, then **P A Q** * **R** = 1. Substituting in equation (I.21) yields

$$r = \mathbf{R}\cdot(1 - w) \tag{I.24}$$

which expresses the linear relationship between wages and the rate of profit. In conclusion then we can say that the complicated relationship between wages and the rate of profit, due to changes in the price components of the commodity used as *numéraire*, can be made linear by selecting the standard commodity as *numéraire*.

The linear relationship allows one to examine the income distribution between wages and profits, without its being subjected to distortions caused from price changes of the commodity used as *numéraire*.

I.2.2 The subsystems

Sraffa uses the notion of a subsystem to demonstrate that when the national income is entirely distributed to wages, the relative value of commodities is proportional to their respective labor costs. The description of the subsystems is introduced in Appendix A of Production of Commodities by means of Commodities (Sraffa, 1960, p.89).

The calculation of the subsystems from the original economic system can be made by adopting alternative methods, in what follows we will repropose the method used by Harcourt and Massaro (1964), because it explains in a clear and didactive way the process of decomposition.

Consider an economic system in which three industries produce the commodities a, b and c respectively:

$$\begin{aligned}(x_{aa}\cdot A\cdot P_a+x_{ab}\cdot A\cdot P_b+x_{ac}A\cdot P_c)\cdot(1+r)+L_a\cdot A\cdot w &= A\cdot P_a\\(x_{ba}\cdot B\cdot P_a+x_{bb}\cdot B\cdot P_b+x_{bc}\cdot B\cdot P_c)\cdot(1+r)+L_b\cdot B\cdot w &= B\cdot P_b\\(x_{ca}\cdot C\cdot P_a+x_{cb}\cdot C\cdot P_b+x_{cc}\cdot C\cdot P_c)\cdot(1+r)+L_c\cdot C\cdot w &= C\cdot P_c\end{aligned} \tag{I.25}$$

where r is the uniform rate of profit, w the unit wage rate, P_i the price of commodity i (i = a, b, c), x_{ij} is the input of commodity j required to produce one unit of output of commodity i (i, j = a, b, c)¸ L_i is the labor input per unit of commodity i (i = a, b, c), A, B, C are the total output of commodities a, b and c, respectively.

The production system is shown graphically in Table I.1.

Table.I.1 – A production system with three commodities

Industry	MEANS OF PRODUCTION								TOTAL OUTPUT				
	Commodity						Labor						
a	$x_{aa}A$	(+)	$x_{ab}A$	(+)	$x_{ac}A$	(+)	l_aA	□	$x_{aa}A$	$x_{ba}B$	$x_{ca}C$	S_a	A
b	$x_{ba}B$	(+)	$x_{bb}B$	(+)	$x_{bc}B$	(+)	l_bB	□	$x_{ab}A$	$x_{bb}B$	$x_{cb}C$	S_b	B
c	$x_{ca}C$	(+)	$x_{cb}C$	(+)	$x_{cc}C$	(+)	l_cC	□	$x_{ac}A$	$x_{bc}B$	$x_{cc}C$	S_c	C

The net product components in physical terms are

$$\begin{aligned} S_a &= A - \alpha \\ S_b &= B - \beta \\ S_c &= C - \gamma \end{aligned} \qquad (I.26)$$

where

$$\begin{aligned} \alpha &= x_{aa}\cdot A + x_{ba}\cdot B + x_{ca}\cdot C \\ \beta &= x_{ab}\cdot A + x_{bb}\cdot B + x_{cb}\cdot C \\ \gamma &= x_{ac}\cdot A + x_{bc}\cdot B + x_{cc}\cdot C \end{aligned} \qquad (I.27)$$

The original system can now be divided into as many parts as there are commodities that make up the net product, so that each party is an autonomous self reproducing system with a net product consisting of a single commodity. Each part is called subsystem and in the example described here there are three subsystems.

The net products of each subsystem are equal to the amount of that commodity in net product of the original system. The total sum of each commodity used as means of production in the three subsystems is equal to their use as means of production in the original system. Similarly, the total labor employed in the three subsystems corresponds to that employed in the original system.

In other words, the three subsystems added together represent simply a re-composition of the original system, as shown in the tables proposed below (I.2, I.3).

Tab. I.2 - Decompositions into subsystems of a production system with three commodities

	MEANS OF PRODUCTION					TOTAL OUTPUT			
Industry	Commodity			Labor		Industry	Commodity	Labor	
	a	b	c				A		
	1a	1b	1c	1a		1a	1a	1a	
a	2a	2b	2c	2a	→	2a	2a	2a	S_a
	3a	3b	3c	3a		3a	3a	3a	
	$x_{aa}A$	$x_{ab}A$	$x_{ac}A$			$x_{aa}A$	$x_{ba}B$	$x_{ca}C$	

	MEANS OF PRODUCTION					TOTAL OUTPUT			
Industry	Commodity			Labor		Industry	Commodity	Labor	
	a	b	c				B		
	1a	1b	1c	1b		1b	1b	1b	
b	2a	2b	2c	2b	→	2b	2b	2b	S_b
	3a	3b	3c	3b		3b	3b	3b	
	$x_{ba}B$	$x_{bb}B$	$x_{bc}B$			$x_{ab}A$	$x_{bb}B$	$x_{cb}C$	

	MEANS OF PRODUCTION					TOTAL OUTPUT			
Industry	Commodity			Labor		Industry	Commodity	Labor	
	a	b	c				C		
	1a	1b	1c	1c		1c	1c	1c	
c	2a	2b	2c	2c	→	2c	2c	2c	S_c
	3a	3b	3c	3c		3c	3c	3c	
	$x_{ca}C$	$x_{cb}C$	$x_{cc}C$			$x_{ac}A$	$x_{bc}B$	$x_{cc}C$	

Tab. I.3 - Subsystem 1

a	1a	1b	1c	1a	→	1a	1a	1a	S_a
b	1a	1b	1c	1b	→	1b	1b	1b	
c	1a	1b	1c	1c	→	1c	1c	1c	

I.3 Structure of the thesis

The present work is organized in five chapters and it proposes and applies alternative measures of productivity constructed using input-output tables and based mainly on the Sraffian scheme. The first three chapters are self-contained, so they can be read independently, however they are of course thematically interrelated. The reading of chapter one is necessary for understanding chapters four and five.

The first three chapters of the thesis are devoted to the development and the empirical application of new productivity measures. These chapters form the main part of the work. The last two chapters are devoted to sensitivity analysis.

In the first chapter, entitled 'Productivity accounting based on production prices' an alternative method of productivity accounting is proposed. By using input–output tables from four major OECD countries between 1970 and 2000, we compute the associated wage-profit frontiers and the net national products curves, and from these we derive two measures of productivity growth based on production prices and a chosen *numéraire*. The findings support the general conclusions in the existing literature on the productivity slowdown and later rebound, and supply new important insights to the extent and timing of these events.

The second chapter is entitled 'New measures of sectoral productivity'. The objective of this chapter is to propose alternative methods of sectoral productivity accounting based the theoretical work of Goodwin (1976), Gossling (1972), Pasinetti (1973), and Sraffa (1960). The indexes developed in this study differ from the standard indexes of productivity because they are designed on the basis of some of the following desiderable features: take into account the interconnections among economic sectors, aggregate heterogeneous goods by using production prices, and compute productivity by using quantity of goods instead of their values. These indexes are then be tested empirically by computing productivity of four major OECD countries.

The third chapter is entitled 'Productivity in the Italian regions: development of Alternative Indicators based on input-output tables'. This chapter calculates indices of aggregate productivity, sectoral productivity, and technological progress for a selected sample of Italian regions. Besides these indices, two different versions of the so-called technological frontier were calculated. The contemporary frontiers that are constructed from all the production techniques extracted from the regional input-output tables in a given year and the intertemporal frontier that is computed for the full set of techniques available over time and across regions. The availability of the technological frontiers allows the calculation of the recently developed Velupillai-Fredholm-Zambelli indices of convergence (Fredholm and Zambelli, 2009) that are based on the distance between the region-specific wage-profit frontiers and the technological frontiers. Given the important role played by the production prices, this chapter also examines the price curves for each region and industry and it identifies remarkable regularities.

Not surprisingly, analyses of the findings reveal that there is a productivity gap between the regions of North and South. However, the analysis of sectoral productivity reveals two important facts. The first is that the techniques of some industries are more productive in the South than in the North. The second, who follows from the first, is that all regions could therefore improve productivity through greater integration.

Chapter four is entitled 'An Inquiry into the choice of *Numéraire*'. This chapter has several objectives. The main aim is to examine the robustness of the results obtained by applying the new approach to measuring productivity if we change the *numéraire* chosen. However, it should be mentioned that the problem of the choice of *numéraire* is a general one and for this reason, the chapter also proposes universal guidelines to be followed in choosing the *numéraire* and in testing the robustness of the results to changes in the *numéraire*.

Finally, chapter five is entitled 'An Inquiry into the effect of aggregation of input-output tables'. The aim of this chapter is to test the robustness of the results from a progressive aggregation of the input-output tables.

I.4 Advantages of this approach to measuring productivity

I.4.1 The rejection of the aggregate production function

The production function has been the subject of intense debate between the 50s and 70s during the so-called Cambridge-Cambridge controversy. Almost all the criticisms were directed at the aggregate production function, but also microeconomic production function has been put under scrutiny.

From the theoretical point of view, Felipe and Fisher (2003) showed that the conditions for which an aggregate function can be obtained by individual microeconomic functions are so stringent to be virtually impossible. For this reason, the aggregate production function does not have a sound theoretical foundation.

However, a number of empirical studies conducted up to early 70's showed that a production function of Cobb-Douglass type fit the data well and these results were used to justify the use of an aggregate production function.

In 1974, Shaikh proposed a critique of the neoclassical aggregate production function and its associated marginal-productivity theory of income distribution, by demonstrating that

> when the distribution data (wages and profit) exhibit constant shares, there exist broad classes of production data (output, capital, and labor) which can always be related to each other through a functional form which is mathematically identical to a Cobb-Douglas with constant "return to scale," "neutral technical

> change," and "marginal products equal to factor rewards." Since the above is a mathematical consequence of constant shares, true even for very implausible production data…
> …it is argued that the so-called empirical strength of production function analysis is in reality nothing more that a statistical reflection of the (unexplained) constancy of income shares (Shaikh, 1974 p.119).

The Shaikh's critique of the production function has continued over the years with a series of articles written by the same Shaikh, Felipe, McCombie, and others (see, among many, Shaikh 1980 and 2005, McCombie and Dixon 1991, Felipe and McCombie 2001, Fredholm 2009).

One of the main advantages of the aggregate and sectoral productivity measures proposed in this work is that they do not require any explicit assumption about the production function. In this way, the measures proposed here do not suffer the problems outlined above. Furthermore, the methods presented here do not suffer from the problem of aggregation of capital, which had also been the subject of intense debate during the Cambridge-Cambridge controversy (for an excellent concise survey on this topic see Pasinetti and Scazzieri, 2009).

I.4.2 The use of production prices

Many of aggregate and sectoral productivity measures presented in this work are constructed using prices of production. Thus, the approach followed here is that of the cost-of-production theory of value. This theory argues that the price of a commodity is determined by the cost of all the resources used to produce it. The prices of production are those at which the commodities must be sold in order to guarantee the reproducibility of the economic system. Hence, they differ from the market prices which are obtained by the conditions of supply and demand. The price of production of one commodity can be interpreted as a sign of the relative importance of that commodity for the economy as a whole, and therefore they represent a more appropriate weight for the aggregation of heterogeneous commodities.

I.4.3The scale invariance property of wage-profit frontier

One of the most useful properties of the wage-profit frontier is its invariance to the scale of production. The frontier remains unchanged as a result of changes in the scale of production that are both symmetric and asymmetric between industries. This result is the consequence of a theorem known in literature as the non-substitution theorem (for the original formulation of the theorem, concerning circulating capital only, sees Arrow, 1951; Koopmans, 1951; Samuelson, 1951).

The non-substitution theorem asserts that in a world with only one primary factor (labor) and without joint production, whatever are the possibilities of substitution between production factors, changes in demand imply no change in technical coefficients. The dual interpretation of the non-substitution theorem asserts that 'under certain specified conditions an economy will have one particular price structure for each admissible value of the profit rate, regardless of the pattern of the final demand' (Salvadori, 1987, p.680). It follows that for any kind of change in the scale of production, be it a change in the scale of production for the economy as a whole or a change in the scale of production that is asymmetric across industries, the invariance of the wage-profit frontier is by the non-substitution theorem guarantied in both cases.

Therefore, once a suitable *numéraire* has been selected, it is possible to compare wage-profit frontiers of very different countries and regions, and it is even possible to compare a large state with a small region, because the wage-profit frontier is determined by the technical condition of production and it does not depend on the size of the economy.

I.5 Limitations of this approach to measuring productivity

I.5.1 The use of input-output tables in value-term

This work is mainly based on the application of the Sraffa's model to input-output tables. The Sraffa's scheme of production assumes that the *physical* commodities are produced through the use of *physical* commodities and labor, while the input-output tables currently available are expressed in value terms. Consequently, the application of the Sraffa model to input-output tables would not be legitimate. However, the use of input-output tables in a classical context *à la Sraffa* has some precedents in the literature (see Han and Schefold, 2006).

The hope is that in the near future may be available input-output tables whose values are expressed in physical quantities and with a high level of industry detail. In this way, it would be possible to match the theoretical model with the empirical application.

I.5.2 Fixed capital

Both the theoretical model and the data used do not include fixed capital. Fixed capital is introduced in the Sraffa's model through the notion of joint production (see Sraffa, 1960 Ch.10). Unfortunately, the model of joint production leads to mathematical complications and the results are often of difficult economic interpretation.

However, there is an awareness of the need to improve the indicators proposed in this thesis, so that they can include fixed capital, but there is also awareness that the measurement of fixed capital stock is still problematic.

One of the most widely used methods for measuring the stock of capital is the Perpetual Inventory Method (PIM), but the PIM may frequently give inaccurate results due to inaccurate assumptions. In particular, it is not so easy to obtain precise and current information on the life span of different classes of asset. In an ideal situation of a totally stable economy, and limited technological change, provided the initial estimate of life spans was reasonably accurate, there would be no problem with PIM. But, that type of industrial environment does not exist, and never will. In practice actual asset lives change over time, and sometimes they change very rapidly.

I.5.3 The *numéraire*

The problem of the choice of *numéraire* it is briefly introduced here for completeness, but this will be the subject of a more extended discussion in the next chapters. This is just to recall that the model of Sraffa consists of a system of linear equations with two more unknowns than equations. It is therefore necessary to fix the value of one of the two distributional variables and select a *numéraire* to find the solution. However, changes of the *numéraire* are not without consequences, because all the production prices will vary in a not predictable way.

I.6 Concluding remarks

A first consideration is that this thesis does not have the pretension to be exhaustive in such a large and complex argument. Rather, the aim is to provide "rules of thumb" for measuring productivity and technological progress in a more appropriate way than it is currently done.

I should say that this work is neither purely theoretical nor purely empirical. In this thesis the theory is used to construct indexes of productivity that are then applied to small samples of countries and regions. This work does not make any important theoretical contribution and empirical analysis is not conducted with excessive detail. Yet it is precisely this transition from theory to practice the real value added of the work.

Finally, a broader aim of this research is to contribute to the development of approaches based on the theoretical framework of classical economics. One of the criticisms frequently levelled against classical economics is that it is not very constructive For this reason, it is necessary first to make classical economics applicable and this could help to rehabilitate its theory.

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I

MEASURES OF PRODUCTIVITY WITH A CLASSICAL FLAVOR: THEORY AND APPLICATION

CHAPTER 1

PRODUCTIVITY ACCOUNTING BASED ON PRODUCTION PRICES

Written jointly with Thomas Fredholm

1.1 Introduction

The main aim of this chapter is to introduce an alternative method of productivity analysis using input–output tables and production prices, and to use this method to study productivity growth in four major OECD countries from 1970 to 2000.

This method has several appealing properties, the most important of which is its ability to take into account – for the economy as a whole – the interdependent relationships among industries as a consequence of technological innovations in the single industries.

The method adopted is based on the scheme of production developed by von Neumann (1945–46), Leontief (1941), and Sraffa (1960), while the algorithms employed were first proposed by Velupillai and Zambelli (1993, 2008) and Zambelli (2004).

By doing this we show how productivity accounting can be accomplished without utilising an aggregate production function, which suffers from several serious drawbacks (see Pasinetti, 2000; Cohen and Harcourt, 2003; Felipe and Fisher, 2003; Felipe and McCombie, 2007).

The chapter is structured as follows: Sections two, three, and four present the theory and the algorithms adopted for the productivity accounting. Sections five and six present and analyses the data, section seven concludes the paper.

1.2 The Theoretical Model

Following the tradition of von Neumann, Leontief, and Sraffa, production, growth, and distribution are described in terms of a multi-sector input–output system, where production is described as an interdependent circular process.[1]

The economic system consists of *m* industries producing *n* commodities by means of some combination of the *n* commodities and labour. Let $\mathbf{A}$ be a $(n \times n)$ quadratic non-singular matrix of inter-industry inputs, where the (*i,j*) entry represents the i^{th} industry's use of the j^{th} commodity in the production of the i^{th} commodity. Likewise, $\mathbf{L}$ is a $(n \times 1)$ vector of labour inputs and $\mathbf{B}$ is a $(n \times n)$ positive definite diagonal matrix of outputs, where the i^{th} diagonal entry is the gross output of the i^{th} industry. As usual these elements can be collected in the following long-run equilibrium relationship that captures the distribution of the

[1] This section and the next are inspired by Sraffa (1960), Pasinetti (1977), Zambelli (2004), and Velupillai and Zambelli (1993, 2008).

total production among wages, profits, and means of production, where the wage and profit rates are assumed to be uniform.[2]

$$\mathbf{Ap}(1+r)+\mathbf{L}w=\mathbf{Bp} \tag{1.1}$$

System (1.1) consists of *n* linear independent equations and *n*+2 variables, i.e., the system has initially two degrees of freedom. Choosing a *numéraire* $\boldsymbol{\eta}$, for which it holds that $\boldsymbol{\eta}'\mathbf{p}=1$, the degrees of freedom reduces to one.

Given the profit rate, it is straightforward to calculate the wage rate and the relative prices that solve system (1.1). Isolate $\mathbf{p}$, $\mathbf{p}=(\mathbf{B}-\mathbf{A}(1+r))^{-1}\mathbf{L}w$, premultiply with the *numéraire*, and rearrange to obtain the wage-profit frontier function and the associated prices, *viz.*

$$w=\left(\boldsymbol{\eta}'(\mathbf{B}-\mathbf{A}(1+r))^{-1}\mathbf{L}\right)^{-1} \tag{1.2}$$

$$\mathbf{p}=\frac{(\mathbf{B}-\mathbf{A}(1+r))^{-1}\mathbf{L}}{\boldsymbol{\eta}'(\mathbf{B}-\mathbf{A}(1+r))^{-1}\mathbf{L}} \tag{1.3}$$

We call these prices *production prices*.[3] Using this price vector as a measure of value in terms of a given *numéraire* and a given rate of profit, the value of the NNP is obtained by the following accounting identity, where $\mathbf{e}$ is a $(n\times 1)$ unit vector.

$$\text{NNP}=\mathbf{e}'(\mathbf{B}-\mathbf{A}(1+r))\mathbf{p} \tag{1.4}$$

The following section provides an intuitive description of how this theoretical framework can be employed to study technological change.

[2] The mathematical notation in this paper is kept as parsimonious as possible, e.g., no indexes are used, but everything should be clear from the context.
[3] The production prices give a measure of the cost of production of the *n* commodities.

1.3 Productivity Accounting

This section consists of three parts. The first two parts define and describe what we will call *labour productivity* and *technological progress* based on production prices. The third part describes the major differences between these two interrelated measures and emphasises the main strengths of this method as a whole.

As usual the NNP is the value added in the given accounting period, hence NNP divided by the total use of labour is a measure of labour productivity. Note here that the NNP is a function of the price vector, which again is a function of the rate of profit. As a distribution free measure of labour productivity, we propose to use the area under the NNP per unit of labour curves, i.e., integrate with respect to the rate of profit from zero to maximum rate of profit. Furthermore, to obtain an index, which is comparable across countries and over time, we divide this area with the maximum profit rate.

Given the complicated interdependent structure of the input–output system, changes in labour productivity are not only due to *cet. par.* changes in the quality of labour or innovations that make labour more productive in the single industries. It is also influenced by the effect of a change in the scale of production in the single industries and depends on how the relative prices changes with the profit rate and the relative sizes of the physical net products for the different sectors. A simple example will clarify this point.

Assume that there is an increase in the scale of production in a given sector, without this changing the applied technology. The value of the NNP per unit of labour will change and the change will vary with the profit rate depending on the relative labour intensity in the chosen industry and the industry's weight in the physical NNP. Consequently, if the scale of production is increased in a sector for which the relative price increases with the profit rate, then the difference between the *ex ante* and *ex post* NNP per unit of labour will increase with the rate of profit.

As a supplement to the above measure of labour productivity, we propose to use the area under the wage-profit frontiers as a measure of what we call *technological progress*. Shifts in the wage-profit frontier can as the NNP per unit of labour curves be interpreted as technological change, positive or negative depending on the nature of the shift and the distribution between wages and profits. If the new wage-profit frontier dominates the old frontier and hence we have (production) prices allowing in principle the system to reproduce, we would have a higher wage rate measured in the terms of the same *numéraire* associated with the same profit rate.

The main difference between the two proposed measures is that the one based on the wage-profit frontiers (*technological progress*) will not change as a consequence of simple changes in the scale of production in single industries, but only if real technological innovations are observed in one or more industries. By real technological innovations we mean change in the matrix of technological coefficients and/or in the corresponding (normalised) vector of labour inputs.[4]

One of the main strengths of productivity accounting based on production prices is that it takes into account the effects of technological change in the single industries for the economy as a whole. A way to see this, is to think of the production prices as weights is in the process of aggregation (into for example the NNP) together with the fact that the production prices change with and only with real technological innovations. The fact that the weights/prices only change as a consequence of technological innovations is appealing, because it circumvents the traditional problem of delineating the effects from changes in market prices and that of real technological innovations.

It is important to note that technological changes in the single industries has an effect on all the relative prices (intuitive since this alters the relative scarcity of all commodities in the system), i.e., the total effect on our measures of productivity from technological change in a single industry is not simply the local effect multiplied by some *ex ante* given weight.

1.4 Algorithms and the Choice of *Numéraire*

Given the input–output tables from a given country for a given year and an appropriate *numéraire*, it is straightforward computations to calculate the wage rate, the production prices, and the NNP for any given profit rate using (1.2) – (1.4). After this point it is a simple programming task to compute areas and to collect and organise the results.

The critical step is to choose an appropriate *numéraire*, because all the subsequent results are influenced by this choice. How to construct or select the *numéraire* is a classical problem in economics, because the value of the *numéraire* should be invariant of other economic factors, such as the distribution between wage and profit. This problem, which was posed by Ricardo, was to some extent solved by Sraffa, since his Standard Commodity gives a distribution free measure of value given the set of techniques represented by matrix $\mathbf{A}$, $\mathbf{L}$, and $\mathbf{B}$.

However, since the purpose of our work is to be able to study technological progress over time and across countries, the standard commodity is no longer an invariant measure of value. Instead, we choose the vector of physical sectoral net products (total supply of the i^{th} commodity minus the sum of the i^{th} column in $\mathbf{A}$) in the US in 2000 calculated from a standard system with a zero profit rate and normalised with the hours worked. This is not a perfect *numéraire* – if such a thing exists – but in our opinion the interpretation of this *numéraire* is fairly intuitive and has a number of convenient properties, which will be clear in the following. Still,

[4] The matrix of technical coefficients is a normalised form of **A**, where the (i,j) entry represents the i^{th} industry's use of the j^{th} commodity in the production of one unit of the i^{th} commodity, see Appendix B for further details.

the consequences of the choice among many possible *numéraire* call for further research.

The standard system can be constructed from any viable system,[5] by reproportioning the system, such that the ratios between the final demand and the sum of intermediate goods are the same for all commodities. The multiplier $\mathbf{q}$ used to reproportion the system into a standard system is the (unique) non-trivial solution of the following homogeneous system of equations.

$$\left(\mathbf{B} - \mathbf{A}'(1+R)\right)\mathbf{q} = \mathbf{0} \qquad (1.5)$$

hence the *numéraire* is given by:[6]

$$\mathbf{q}' = \frac{\mathbf{e}'\left((\mathbf{B} - \mathbf{A}) \otimes \mathbf{q}\mathbf{e}'\right)}{\mathbf{L}'\mathbf{q}} \qquad (1.6)$$

This has the appealing property to normalise the maximum wage rate in 2000 to one, i.e., the wage rate by which the workers can buy all the NNP in 2000 given a zero profit rate. Furthermore, the use of the standard system guarantees a strictly non-negative *numéraire*, which is not *a priori* given.[7]

1.5 Data

We use OECD input–output tables that belong to three different data sets for the US, the UK, Germany, and France. All containing matrices in current prices and domestic currency. The first covers roughly five year intervals from around 1970 to 1990 and follows the system of industrial classification 'ISIC Revision 2' (35 sectors) and the System of national accounts 'SNA 68'.[8]

The second data set includes 42-by-42 sector matrices covering one year in the mid-1990s. The matrices follow the new system of industrial classification 'ISIC Revision 3'.

The third data set has been recently published by the OECD in 2006. What is new with respect to the older editions is the high degree of comparability among countries, because the tables are constructed according to the standard industry list based on ISIC Revision 3. The 2006 edition consists of matrices for 28 member countries and 9 non member countries covering 1995 and 2000. Each matrix

[5] The system is said to be viable, if and only if $\lambda \leq 1$, such that the maximum rate of profit will be positive, for further details see Pasinetti (1977, p. 78) and Appendix 1.B.
[6] See Appendix 1.B for details and a numerical example.
[7] The psychical net product can be negative, because imports allow the system to reproduce itself. This is not an uncommon observation in the actual OECD tables.
[8] See Appendix 1.A for details.

describes the inter-industrial relationships for 48 sectors that cover both the industrial part of the economy and services.[9]

The data have been adjusted in order to have matrices that can be adopted within a Sraffian model. In fact, in order to find an inverse matrix, the original matrix must be non-singular. That is, no linear combination of rows and columns and no zero rows and columns. Consequently, the original tables have been modified to satisfy these requirements. The aggregation cancels out the rows and columns with all zero values minimizing the loss of information due to the merge.

As a consequence of the need to both aggregate some sectors in order to clear the null vectors and preserve comparability, the number of sectors is reduced to 23.[10] Each column of the table describes the nominal value of an industry's inputs and each row reports the nominal value of an the industry's output;[11] therefore, we take the transpose matrix.

As previously said, the tables are in current prices and domestic currency. Although previous studies treat the nominal coefficients as physical (see for instance Petrovic, 1991 and Han and Schefold, 2006), we decide to follow an alternative procedure for two reasons. First, from that time on, the OECD website improves the availability of data, and second because, although experimental, an empirical work on productivity growth cannot treat nominal values as physical quantities. The best way is to use the respective deflator for each sector. Unfortunately, the OECD statistics on national accounts are highly aggregated and captures only six macro sectors. Consequently, the ratio previously reported has been calculated for the six sectors available and it has been used on the corresponding micro branches. At the end, we have a set of tables that report the quantity of commodities used and produced with respect to the reference year 2000, the coefficients are expressed in constant Purchasing Power Parities, and the change in relative prices is preserved, although roughly, by using different PPPs.

Finally, the physical quantity of labour is given by the number of hours worked. In default of detailed information for the number of hours worked in each sector we decide to attribute in proportion to the compensation of employees. In any case, further improvements would be achievable when data on labour quality will be available.

1.6 Analysing the data

This section evaluates the areas under the wage-profit frontiers and then compares the areas under the NNP per worker curves for the four countries. The aim is to describe the rate of change in productivity over time in the US, the UK,

[9] For further information see: http://www.oecd.org/ and OECD (2001a, 2001b).

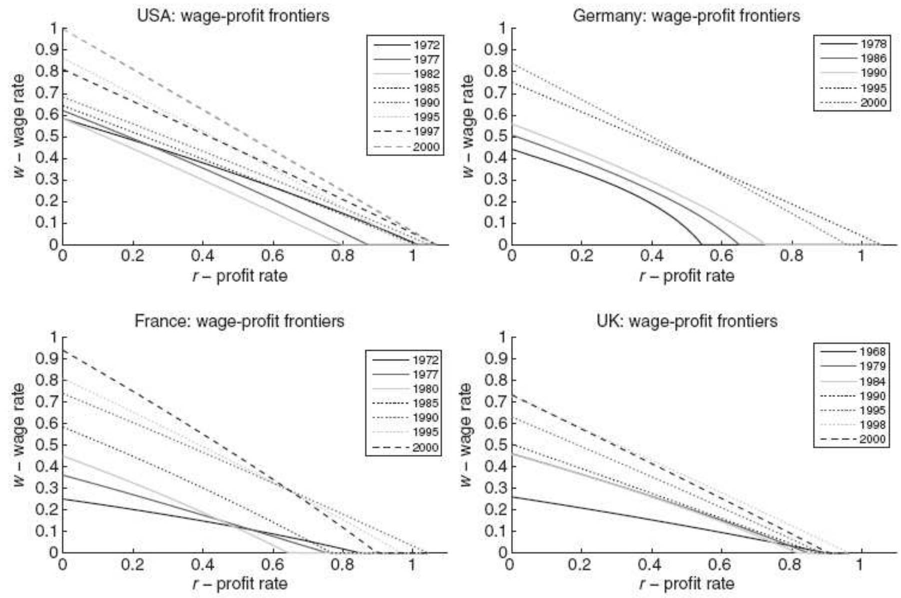
[10] A detailed description of the database and the method of harmonization used is found in Appendix 1.A.

[11] Each country has three different tables: domestic, import and total and the 'total' table sums the coefficients of the others.

Germany, and France. However, it should be noted that the data are not perfectly commensurable across countries and over time. For instance, we dispose of eight input–output matrices for the US, seven for France and the UK, and only six for Germany. Furthermore, the years do not always coincide, for example, we have the 1968 table for the UK, the 1970 table for Germany, and the 1972 tables for France and the US. Nevertheless, in the comparative analysis we use fixed five year intervals from 1970–2000.

Figure 1.1 shows the wage-profit frontiers for the four countries in the period considered. The movement of the frontiers over time is country-specific. In particular, the 1977 and 1982 frontiers for US move back to the origin as well as the 1984 and 2000 frontiers for the UK. On the contrary, we do not observe such behaviour in France and Germany. In looking for an explanation for this behaviour, one should consider the already established literature on the so-called *productivity slowdown* during the 1970s, cf. Nordhaus (2004).

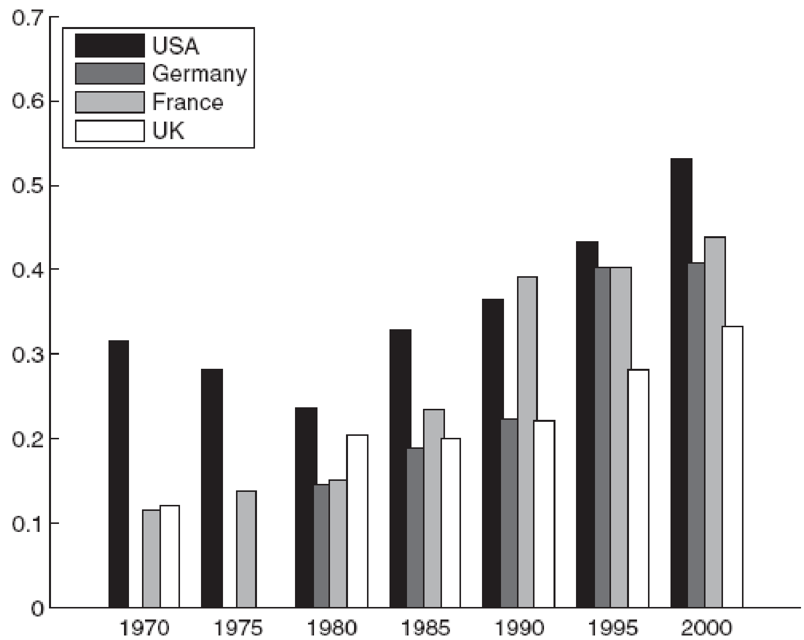
Fig. 1.1 The wage-profit frontiers for the US, Germany, France, and the UK



The following histogram (Figure 1.2) reports the areas under the wage-profit frontiers, thus it can clearly be seen that the slowdown hit mainly the US economy, but also to some extent the UK economy. Conversely, France and Germany remain unaffected and hence they were characterized by a more steady technological progress (catching up).

The 1990s is another decade that deserves special attention. During this period, commonly known as the *new technology era*, the UK and especially the US productivity grew faster than in the other two countries. As a consequence, the level of productivity in the US at the end of the 20th century was much higher than elsewhere.

Fig. 1.2 Technological progress

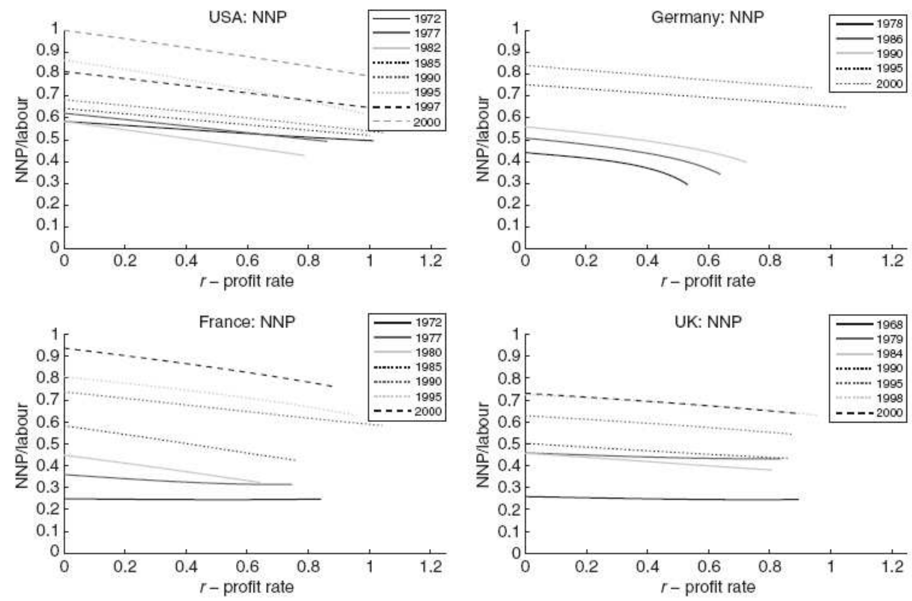


To sum up, the technological development over the thirty years examined in the UK and in particular in the US exhibits a cyclical pattern. At the beginning of the 1970s the UK and especially US were the leading countries, during the economic slowdown from the early 1970s to the mid-1980s, the level of technological progress in Germany and France converged slowly towards the US level and overcame the UK level (or rather the US 'convergence down' toward Germany and France!), and then in the 1990s the US and the UK productivity growth was faster than in the EU's two biggest economies. As a result, the US became again in the 1990s the leading economy.

Thus, our findings support only partially the existing literature and the empirical evidences of the pattern of productivity growth. Notwithstanding, the results are similar to those reported by other studies, see for instance Nordhaus (2004), but the magnitudes are not the same. In particular the US productivity slowdown of the 1970s is more prominent in our case, because not only the rate of growth, but even the level of productivity declines. In addition the US and UK productivity boom begins in the early 1990s, five years before the OECD estimate.[12]

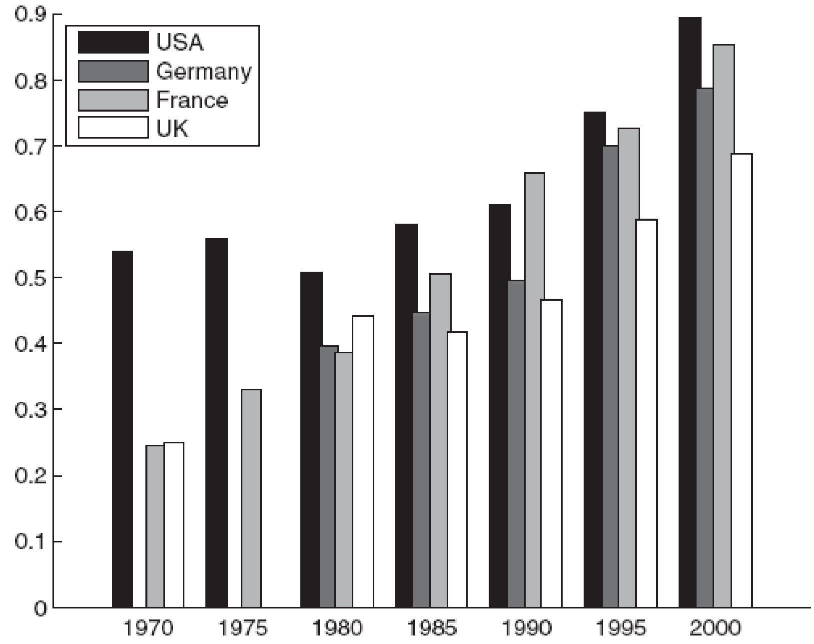
Figure 1.3 and Figure 4 show the NNP curves and the areas under these curves respectively. The histograms in Figure 1.2 and Figure 1.4 are alike, and thus support the story told above. The histograms in Figure 1.4 sometimes differ in term of the magnitude of the change, but with few exceptions the countries' order of rank is the same in the two figures.

Fig. 1.3 The NNP curves for the US, Germany, France, and the UK



[12] See the OECD data on labour and Multi-factor productivity, http://www.oecd.org/

Fig. 1.4 Labor productivity



Before concluding this section, it is worth to mention that the interpretation of the wage-profit frontiers behaviour deserve further investigation. In particular, it is important to identify which sectors are mainly responsible for productivity changes in each period.

It would be also interesting to study in detail the different patterns of technical change among the four countries.

1.7 Conclusion

In this paper we have described an alternative way of productivity accounting based on the work of von Neumann, Leontief, and Sraffa on production systems. We have proposed to compare the areas under the net national product curves and under the wage-profit curves, and we have applied this method to the US, Germany, France, and the UK. The main difference between our method and the orthodox way of measuring productivity consists in the use of industry level input–output data and the associated production prices. We think the use of production prices in the process of aggregation has at least two appealing properties; production prices change only as a consequence of real technological innovations, and take into account the complicated interdependencies among industries in the economic system as a whole.

We have found that the path of technological progress and the growth rates in labour productivity differ substantially between the US and the UK on the one hand and France and Germany on the other. In particular, the US and the UK show a decrease in productivity levels during the 1970s and the early 1980s while France and Germany exhibit more steady technological progress during the same period. Conversely, from 1990 to 2000 the rate of productivity growth was again higher for the US and the UK than for France and Germany. Hence, our findings show both similarities and differences compared to the results based on the traditional ways of productivity accounting. For instance, the well-known literature on the US productivity slowdown identifies a decrease in the rate of growth in productivity, while our results show not only a slowdown, but a clear decline in the level of US productivity in the 1970s.

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APPENDICES

1.A The Datasets

This appendix describes the national input-output tables used and the procedure adopted for making these tables suitable for the computation of wage-profit frontiers and NNP curves. This procedure has two stages: aggregation and statistical error distribution.
The input–output tables are made available by the OECD. They refer to three different time periods and are inconsistent with respect to the number of sectors and the order in which sectors are listed. Therefore, some sectors have been merged and re-ordered in order to harmonize the data.

The first set of tables refers to the period 1970–1990 (ISIC rev.2). The tables are available for the following years:

US:	1972, 1977, 1982, 1985, 1990
Germany:	1978, 1986, 1990
UK:	1968, 1979, 1984, 1990
France:	1972, 1977, 1980, 1985, 1990

The following list describes in detail which sectors were combined

- Chemicals and Pharmaceuticals
- Iron, Steel, and Non-Ferrous Metals
- Electrical machinery and apparatus nec; Radio, Television, and Communication Equipment; Office and Computing Machinery; Professional Goods
- Shipbuilding and Repairing; Other Transport; Motor Vehicles; Aircraft
- Restaurant and Hotels; Transport and Storage
- Finance and Insurance; Real Estate and Business Services
- Community, Social and Personal Services; Producers of Government Services; Other Producers

As a result, the original 35-by-35 sector tables have been reduced to 23-by-23 sector.

The second set of tables (ISIC rev.3) is smaller and refers only to one year: 1997 or 1998. The following matrices are available:

US:	1997
UK:	1998

Unfortunately, this data set does not include data for France and Germany.
The original 41-by-41 sector tables have been reduced to 23-by-23 sector and these sectors coincide with those in the set of tables from 1970 to 1990.
The following sectors were combined

- Chemicals and Pharmaceuticals
- Iron, Steel and Non-Ferrous Metals

- Office Accounting and Computing Machinery, Electrical Machinery and Apparatus nec; Radio, Television and Communication Equipment; Medical Precision and Optical Instruments
- Motor Vehicles, Trailers and Semi-trailers; Building and Repairing of Ship and Boats; Aircraft and Spacecraft; Railroad Equipment and Transport Equipment nec
- Hotels and Restaurant; Transport and Storage
- Financial, Insurance; Real Estate Activities; Renting of Machinery and Equipment; Computer and Related Activities; Research and Development; Other Business Activities
- Public Administration, Defence, Compulsory and Social Security; Education; Health and Social Work; Other Community Social and Personal Services; Private Household with Employed Persons

Finally, the third set of tables have made accessible by the OECD in 2006. It is the most recent available and refers to two years: 1995 and 2000.
The original 48-by-48 sector tables (ISIC rev.3) have been reduced to 23-by-23 sector and these sectors again coincide with those in the set of tables from 1970 to 1990.
Accordingly, the following sectors were combined

- Mining and quarrying (energy); Mining and quarrying (non-energy)
- Chemicals excluding pharmaceuticals; Pharmaceuticals
- Iron and steel; Non-ferrous metals
- Office, accounting and computing machinery; Electrical machinery and apparatus, nec; Radio, television and communication equipment; Medical, precision and optical instruments
- Motor vehicles, trailers and semi-trailers; Building and repairing of ships and boats; Aircraft and spacecraft; Railroad equipment and transport equip nec
- Production, collection and distribution of electricity; Manufacture of gas; distribution of gaseous fuels through mains; Steam and hot water supply; Collection, purification and distribution of water
- Hotels and restaurants; Land transport; transport via pipelines; Water transport; Air transport; Supporting and auxiliary transport activities; activities of travel agencies
- Finance and insurance; Real estate activities; Renting of machinery and equipment; Computer and related activities; Research and development; Other Business Activities
- Public admin. and defence; compulsory social security; Education; Health and social work; Other community, social and personal services; Private households with employed persons and extra-territorial organisations and bodies

In many cases the tables have a residual sector that is the statistical error and/or the non-comparable import. The values included in the residual sector are distributed in proportion to the ratio between the sum of values of intermediate inputs in that sector and the total value of intermediate goods for the entire economy.

1.B A Note on the Numéraire

The following is a numerical example of how the *numéraire* is constructed.

$$\mathbf{A} = \begin{pmatrix} 2 & 1 & 3 \\ 1 & 2 & 1 \\ 3 & 2 & 4 \end{pmatrix} \qquad \mathbf{B} = \begin{pmatrix} 10 & 0 & 0 \\ 0 & 12 & 0 \\ 0 & 0 & 18 \end{pmatrix} \qquad \mathbf{L} = \begin{pmatrix} 2 \\ 4 \\ 4 \end{pmatrix} \tag{1.6}$$

To calculate the maximum rate of profit, we need the matrix of technical coefficients $\mathbf{A}^*$, which is a normalised form of $\mathbf{A}$, where the (*i,j*) entry represents the i^{th} industry's use of the j^{th} commodity in the production of one unit of the i^{th} commodity, *viz.*

$$\mathbf{A}^* = \frac{\mathbf{A}}{diag(\mathbf{B})\mathbf{e}'} = \begin{pmatrix} 0.200 & 0.100 & 0.300 \\ 0.083 & 0.167 & 0.083 \\ 0.167 & 0.111 & 0.222 \end{pmatrix} \tag{1.7}$$

From this it is straightforward to calculate the maximum eigenvalue of $\mathbf{A}^*$ denoted by λ and the maximum rate of profit, *R*. Here $\lambda = 0.4907$ and hence $R = \lambda^{-1} - 1 = 1.04 = 104\%$. Next, we determine the multiplier $\mathbf{q}$ that allows us to construct the standard system, i.e., the non-trivial solution of the following homogeneous system:

$$(\mathbf{B} - \mathbf{A}'(1+R))\mathbf{q} = \mathbf{0} \tag{1.8}$$

The solution of this example is $\mathbf{q} = [0.582 \;\; 0.533 \;\; 0.614]'$, which gives the following standard system.

$$\overline{\mathbf{A}} = \mathbf{A} \otimes \mathbf{qe}' = \begin{pmatrix} 1.73 & 2.30 & 2.30 \\ 1.55 & 0.52 & 1.55 \\ 1.24 & 1.24 & 3.10 \end{pmatrix} \tag{1.9}$$

$$\overline{\mathbf{B}} = \mathbf{B} \otimes \mathbf{qe}' = \begin{pmatrix} 8.06 & 0 & 0 \\ 0 & 7.25 & 0 \\ 0 & 0 & 12.4 \end{pmatrix} \tag{1.10}$$

$$\overline{\mathbf{L}} = \mathbf{L} \otimes \mathbf{q} = \begin{pmatrix} 2.26 \\ 2.07 \\ 1.29 \end{pmatrix} \tag{1.11}$$

Hence the vector of sectoral net products we use as the *numéraire* is given by:

$$\mathbf{q}' = \frac{\mathbf{e}'\left(\overline{\mathbf{B}} - \overline{\mathbf{A}}\right)}{\mathbf{e}'\overline{\mathbf{L}}} = \frac{\mathbf{e}'\left((\mathbf{B} - \mathbf{A}) \otimes \mathbf{qe}'\right)}{\mathbf{L}'\mathbf{q}} = [0.641 \;\; 0.576 \;\; 0.987] \tag{1.12}$$

1.C The Rotation of the Wage-Profit Frontier

The objective of this appendix is to deepen the discussion about the index of technological progress developed in Chapter one. Two issues will be discussed. First, I will reflect on whether it is appropriate to use the area under the wage-profit frontier as the index of technological change when two frontiers cross each other. Second, I will reflect on whether clockwise and counterclockwise rotations of the frontiers can be interpreted according to the standard categories of technological progress. The answer is that it is a serious mistake to associate a clockwise (counterclockwise) rotation to a Harrod-neutral (Solow-neutral) technological progress.

The Area under the Wage-Profit frontier: some remarks

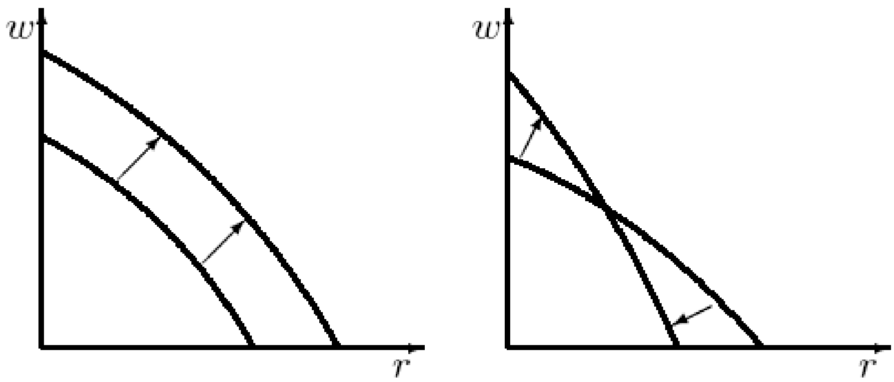
The area under the wage-profit frontier is a synthetic index of technological progress. According to the rule followed in Chapter one there is a one-to-one correspondence between the area and technological progress. Consequently, when the area of one country is larger than the area of another country, this is a sufficient condition to say that technological progress is higher in the former country than in the latter.

This simple rule is meant as a practical guide and is adequate in many circumstances. Nevertheless, we shall distinguish two cases (Fig. 1.5).

1) The first case is when one wage-profit frontier dominates another wage-profit frontier and this is the case in which the rule fits better because technological progress unambiguously occurs. This means that in 'value' terms and for a given rate of profits, a unit of employed labor has a higher purchasing capacity (i.e. can potentially buy more) of the given bundle of goods (the *numéraire*).

2) The second case is when one wage-profit frontier crosses another wage-profit frontier. This is the case in which it shall not be allowed to use the area as the only indicator of technological progress. The area of one country could be larger than that of another country, although for some wage-profit combination, the frontier of the latter dominates the frontier of the former. This could be labelled as an "ambiguous" case of technological progress.

Fig. 1.5 Two different cases of technological progress

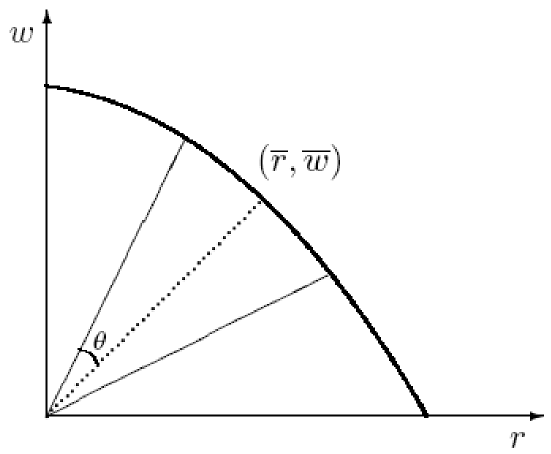


When case two occurs, more appropriate indicators must be used and evaluated. The two presented here represent simple suggestions with pros and cons and further research is needed before a strong recommendation can be made (see Fig. 1.6).

(a) The arc of technology. Instead of measuring the whole area under the wage-profit frontier, it seems reasonable to measure the area encapsulated between two rays starting from the origin and crossing the frontier at those two points, which represent the upper and lower boundaries of realistic distribution of income between wages and profits. This does not ensure that the two frontiers do not intersect, but at least, points corresponding to improbable distributions are not taken into account.

(b) The ray. Alternatively, one could think of measuring the length of the ray starting from the origin and crossing the wage-profit frontier at that point which represents the actual distribution of income between wages and profits. Although this would reduce the usefulness of drawing a wage-profit frontier, it may eliminate the problem that arises when two frontiers intersect each other.

Fig. 1.6 The arc of technology and the ray of technology



<u>*Standard Categories of Technological Progress*</u>

Figure 1.7 can be interpreted in accordance to the taxonomy presented by Hahn and Matthews (1964) and recently used by Foley and Marquetti (1999) and Marquetti (2003).

According to this taxonomy, a clockwise rotation of the frontier around its horizontal intercept corresponds to a labor-saving (Harrod-neutral) technological progress; while a counterclockwise rotation of the frontier around its vertical intercept corresponds to a capital-saving (Solow-neutral) technological progress.

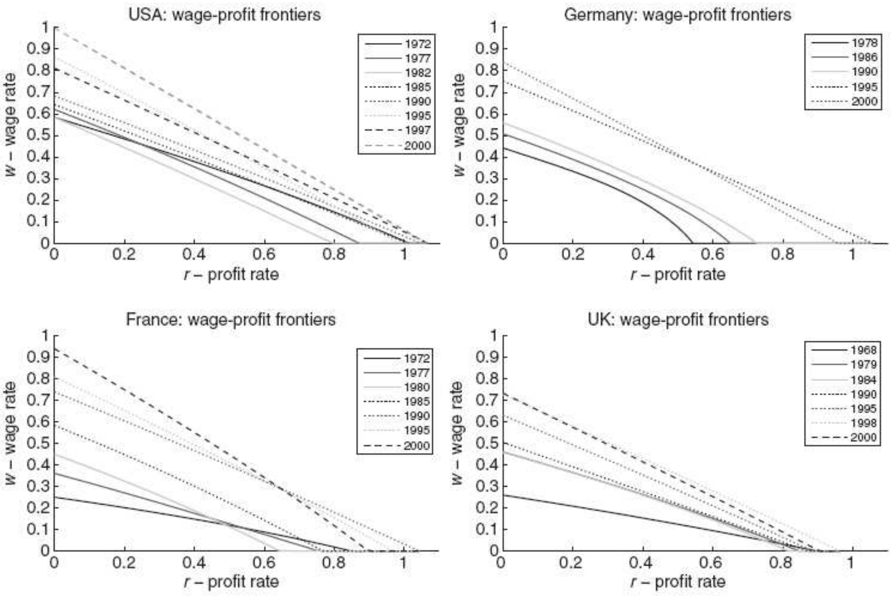
When these two effects occur simultaneously and proportionally (parallel shift) technological progress is usually classified as Hicks-neutral[13.]

Using this classification, an inspection of Figure 1.7 immediately would reveal three interesting facts. First, there is a clear evidence of labor-saving technological progress in all the countries examined. Second, the four countries show different patterns of capital-saving technological progress. The horizontal intercepts are more stable in the UK than in the other countries. From the mid-70s, France, the US, and the UK experienced a capital-using technological change (the horizontal intercepts shift inward). Later, from the early 80s, the horizontal intercepts shift outward, a development that continues to the mid-90s where it

[13] It must be mentioned here that a purely Hicks-neutral technological progress is practically impossible, as shown by Steedman (1985). However, the same author argues that "The reader interested in employing the Hicksian neutrality concept in empirical work-on, say, income distribution and/or explaining economic growth-may nevertheless feel a slight impatience and may argue that, for the purposes of such work, an assumption of 'approximately' Hicksian neutrality may suffice-and this has not been shown above to be ' impossible'. It is not the place of the theorist to deny that, if empirical work is to be done at all, all sorts of aggregations, approximations, and compromises will have to be made. (Steedman 1985, p.757)"

stops or reverses. Third, we observe a few cases of near parallel shifts in the US, France, and Germany and this indicates an approximate Hicks-neutral technological progress.

Fig. 1.7 Technological progress



However, a few remarks must be made. First, this type of classification cannot be directly compared with the classification normally used in the neoclassical theory of production, because the value of capital in our approach is a function of the profit rate and is dependent on the choice of the *numéraire*. In particular, I want to emphasize that the mere observation of the movement of the wage-profit frontier over time cannot, in an unambiguous way, be used to classify technological progress in the above mentioned categories. For instance, a parallel shift of the frontier may not be a sign of Hicks neutrality; instead it may occur as a consequence of a purely capital-saving technological progress, because changes in the matrix of technical coefficients can affect both the vertical and the horizontal intercept.

To conclude, in this framework the very computation of aggregate capital (and hence of an aggregate production function) is problematic and should not be performed, as it is done in standard economics. Therefore, it is not appropriate, especially in this Sraffian-inspired framework, to fake as if there was no problem in the measurement of aggregate capital.

CHAPTER 2

NEW MEASURES OF SECTORAL PRODUCTIVITY*

* I wish to thank Thomas Fredholm, K.Vela Velupillai, and Stefano Zambelli for their useful suggestions and comments. Also the participants at the Second Schumpeter Summer School in Graz 2009 deserve my gratitude for discussing my work, being critical towards it and making valuable suggestions.

2.1 Introduction

The purpose of this chapter is to propose some alternative indices of sectoral productivity accounting based on input-output tables and to contribute to the stream of research on how input-output tables can be used in order to obtain productivity indicators. An input-output table is capable of accounting for the interdependencies among economic sectors and, for this reason, it can be considered a more powerful tool than the aggregate national account statistics for analyzing productivity trends.

In the first chapter, we developed an alternative method of productivity accounting based on production prices that are generated by solving a production system *a la Sraffa*, where the matrix A is the input-output table for a particular country and year. Using this approach, we proposed two measures: labour productivity is calculated integrating the area under the Net National Product curves (NNP) while technological progress is calculated by integrating the area under the wage-profit frontier.

Although this method is appealing in its simplicity and innovation, it can only be used to examine the aggregate technological progress and productivity trend, while it cannot be used to analyze the sectoral productivity trends[1].

As a solution of this problem, this chapter develops some different techniques of sectoral productivity accounting grounded on the works of Goodwin (1976), Gossling (1972), Pasinetti (1973), and Sraffa (1960). These techniques will then be used to identify the pattern of productivity among four major OECD countries. This chapter combines theoretical, empirical, and comparative perspectives and is organized as follows.

Section 2 presents an overview of the productivity measures based on classical tradition. Section 3 describes the theoretical models of Goodwin, Gossling, Pasinetti, and Sraffa while Section 4 shows how each of the new productivity indicators is based on one of each model. Section 5 describes the sources and preparation of the data. Section 6 applies these indicators to assess the sectoral

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[1] The reason for this is that the measure of productivity based on the area under the wage-profit frontier cannot be used as an index of sectoral productivity because of the non-substitution theorem (Arrow, 1951; Koopmans, 1951; Samuelson, 1951). This theorem specify that the relative prices for a given system X={B,A,L} are independent of the actual production or demand vector (see also Zambelli, 2004, p.105). Hence, the wage – profit frontier for the whole economic system is equivalent to the wage-profit frontier for each subsystem.
Conversely, the measure of productivity based on the area under the NNP curves can be decomposed into measures of sectoral productivity (see Fredholm and Zambelli, 2009, for an empirical application).

productivity trends in the US, Germany, France, and the UK and compares the methods from an empirical point of view.

2.2 Productivity measures based on classical tradition

Measuring productivity has attracted considerable attention from the researchers that belong to various schools of thought. Each of the proposed approaches has strengths and limitations and there is no "absolute" optimal measure.

The standard indexes of macroeconomic productivity both at aggregate and sectoral level are not satisfactory because they do not take into account the interdependencies among economic sectors. The use of input-output tables for measuring productivity can overcome this limitation. In order to account for interdependencies, productivity indexes based on input-output tables should be built in such a way that the interrelations among sectors are not lost during the process of aggregation. The underline models that satisfy these conditions are examined in the next section.

The indexes developed in this paper are similar to those used by Rymes (1972), Peterson (1979), Wolff (1985), Panethimitakis (1983), Aulin-Ahmavaara (1999), and De Juan and Febrero (2000). All this literature is rooted in the notion of vertical integration, introduced by Sraffa (1960) and Pasinetti (1973), and the productivity measures used in these papers share some common principles from those realized here.

In particular, as De Juan and Febrero (2000, p.69) emphasize 'the usual productivity indices treat each industry in isolation' thus 'they do not accounts for the transfers of productivity from the innovating sectors to those requiring their inputs, either directly or indirectly'. As a consequence, the standard way of measuring sectoral productivity is imperfect because it isolates sectors of economic activity without taking into account their connections with other sectors and the economic system as a whole.

The following section will present the theoretical approaches, on which some indicators of sectoral productivity are then derived. The approach based on the work of Goodwin is probably the most disconnected from others, while the work of Sraffa, Pasinetti and Gossling are very similar.

In a recent contribution, Fredholm and Zambelli (2009) show with simple examples that the Gossling's (1972) iterative method, the Pasinetti's (1973) concept of vertical integration of and the Sraffa's (1960) reduction to date quantities of labor are different procedures for building subsystems, but all leads to the same results. In the same work Fredholm and Zambelli develop indices of productivity and technical change based on subsystems.

This chapter incorporates part of what has been done by the two authors and offers some original features. A first innovative element is precisely the development of indicators based on the work of Goodwin. A second new element is the calculation of productivity measures through the method named *reduction to dated quantities of a commodity*. It is not by chance that this name is similar to the Sraffa's method of reduction to dated quantities of labor, from which it draws inspiration.

2.3 The theoretical models

2.3.1 Goodwin's Normalized General Coordinates

In the preface to his deceptively titled but highly original textbook, Elementary Economics from the Higher Standpoint, Goodwin (1970) identified the most important problems of economics as global behaviour, interaction of the parts, and dynamics.

The problem of the interaction of the parts is almost as central a motif as nonlinear macro dynamics, in the larger canvas of his economics. He chose to tackle the formal problem of the interaction of the parts – that is, economic interdependence – with the tools and within the framework of, linear mathematics. His formulation of this problem was guided by the wisdom of the classic – Walras, Wicksell and Leontief.

Goodwin (1976, 1983) suggested the use of Normalized General Coordinates. This method transforms the original observed sectors to new sectors that are independent from one another. The method of Normalized Coordinates as formulated by Goodwin (1976) was based on two systems. The original value system is given by the following equation

$$\mathbf{P}_0[\mathbf{I} - (1+\pi)\breve{\mathbf{A}}] = (1+\pi)w\mathbf{L}_0 \qquad 2.1)$$

where $\mathbf{P}_0$ is the price vector, $\breve{\mathbf{A}}$ is the square matrix of interindustry coefficients, w the money wage rate, π the profit rate, and $\mathbf{L}_0$ is the vector of direct labour inputs. The output system is instead given by the following equation

$$[\mathbf{I} - (1+g)\breve{\mathbf{A}}]\mathbf{Q}_0 = (1+g)\gamma\mathbf{C}_0 \qquad (2.2)$$

where $\mathbf{Q}_0$ is the output vector, g is the growth rate, $\mathbf{C}_0$ is a given consumption vector, and Y its scale factor.

Now it is possible to transform the original system to normalized general coordinates in the following way. For each eigenvalue, λ_i, there exists a correspondent value eigenvector $\mathbf{M}$ and a correspondent output eigenvector $\mathbf{M}^{-1}$. Thus, $\mathbf{MAM}^{-1} = \boldsymbol{\lambda}$, where $\boldsymbol{\lambda}$ is the eigenvalues diagonal matrix. Hence, the observed original quantities $\mathbf{P}_0$, $\mathbf{Q}_0$, $\mathbf{L}_0$, and $\mathbf{C}_0$ are transformed to $\mathbf{P}$, $\mathbf{Q}$, $\mathbf{L}$, and $\mathbf{C}$, so that $\mathbf{PM}=\mathbf{P}_0$; $\mathbf{LM}=\mathbf{L}_0$; $\mathbf{M}^{-1}\mathbf{Q}=\mathbf{Q}_0$; $\mathbf{M}^{-1}\mathbf{C}=\mathbf{C}_0$.

The original systems are thus transformed to:

$$\mathbf{P}[\mathbf{I}-(1+\pi)\boldsymbol{\lambda}]=(1+\pi)w\mathbf{L} \quad (2.3)$$

$$[\mathbf{I}-(1+g)\boldsymbol{\lambda}]\mathbf{Q}=(1+g)\gamma\mathbf{C} \quad (2.4)$$

with sectors completely separated.

2.3.2 Pasinetti's vertical integration

Pasinetti (1973)[2] introduces the approach of vertical integration that can be described as follows, in the case of no joint production and no fixed capital.

Let $\breve{\mathbf{A}}$ denote the square $(n\times n)$ matrix of interindustry coefficients and let $\mathbf{L}$ the $(n\times 1)$ labour input vectors. Then, the vector of vertically integrated labor coefficient is given by

$$\mathbf{v}=(\mathbf{I}-\breve{\mathbf{A}})^{-1}\mathbf{L} \quad (2.5)$$

where $\mathbf{I}$ is the $(n\times n)$ identity matrix and $(\mathbf{I}-\mathbf{A})^{-1}$ is the well known Leontief inverse. Finally,

$$\boldsymbol{\tau}=\mathbf{F}(\mathbf{I}-\breve{\mathbf{A}})^{-1}\mathbf{L} \quad (2.6)$$

where $\mathbf{F}$ is a $(n\times n)$ matrix of final output in physical quantities, gives the total quantity of labor directly and indirectly required to produce the different commodities ($\boldsymbol{\tau}$).

2.3.3 Gossling's subsystems

Gossling (1972) proposed an algorithm to compute the gross output sub-system and the final output sub-system[3]. The first step consists in the calculation of

[2] This paper describes a simplified version of the model, without fixed capital. This simplified version is then used for the empirical analysis and it allows a more clear and direct comparison with Goodwin as done by Cozzi (1990)

[3] Gossling's iterative procedure is not the only method for obtaining subsystems (for an overview see Fredholm and Zambelli, 2009).

the respective market share coefficients. Therefore, given an input-output table for a three-sector economy as that one presented in Table 2.1

Table 2.1 Transactions in a three-sector economy

Economic Activities	Inputs to Agriculture	Inputs to Manufacturing	Inputs to Transport	Final Demand	Total Output
Agriculture	x_{11}	x_{12}	x_{13}	c_1	x_1
Manufacturing	x_{21}	x_{22}	x_{23}	c_2	x_2
Transportation	x_{31}	x_{32}	x_{33}	c_3	x_3
Labor	L_1	L_2	L_3		

the market share coefficients are given as follows,

$$u_{ii} = x_{ii} / x_i \tag{2.7}$$

$$u_j = x_{ij} / x_i \tag{2.8}$$

$$t_i = c_i / x_i \tag{2.9}$$

for i, j = 1, 2… n.

The determination of the sub-system for each sector is then obtained by an iterative procedure. For example, the proportion of Manufacturing and Transportation involve in Agriculture's sub-system is given by the following steps:

(a)

$$\begin{Bmatrix} u_{21} + u_{22}u_{21} + u_{23}u_{31} \\ u_{31} + u_{32}u_{21} + u_{33}u_{31} \end{Bmatrix} \tag{2.10}$$

Write $\begin{Bmatrix} g_2 \\ g_3 \end{Bmatrix}$ in place of these

(b)

$$\begin{Bmatrix} u_{21} + u_{22}g_2 + u_{23}g_3 \\ u_{31} + u_{32}g_2 + u_{33}g_3 \end{Bmatrix} \tag{2.11}$$

(c)

Continue this calculation until its reach the limiting proportions $\begin{Bmatrix} p_{21} \\ p_{31} \end{Bmatrix}$

Obviously, this calculation can be repeated for the remaining sub-systems and the result can be summarized in an $[\mathbf{I}+\mathbf{P}]$ matrix, where **I** is the identity matrix, which represent the fact that the whole of principal activity belong to its sub-system, and **P** is the matrix of the proportion of the other sectors activity that enter in the principal activity's sub-system.
Hence,

$$[\mathbf{I}+\mathbf{P}] = \begin{bmatrix} 1 & p_{12} & p_{13} \\ p_{21} & 1 & p_{23} \\ p_{31} & p_{32} & 1 \end{bmatrix} \tag{2.12}$$

Finally, it remains to distinguish between the parts of the principal activity that comprise the internal and external sales of its sub-system. This distinction is needed in order to find the final output sub-systems. The internal output sub-system is formed by the following sub-components:

$$\begin{aligned} (1) &= x_1(u_{11} + u_{12}p_{21} + u_{13}p_{31} + u_{14}p_{41}) \\ (2) &= x_2(u_{21} + u_{22}p_{21} + u_{23}p_{31} + u_{24}p_{41}) \\ (3) &= x_3(u_{41} + u_{42}p_{21} + u_{43}p_{31} + u_{44}p_{41}) \end{aligned} \tag{2.13}$$

while the external output sub-system is given by the following expression:

$$x_1[u_{12}(1-p_{21}) + u_{13}(1-p_{31}) + u_{14}(1-p_{41}) + c_1] \tag{2.14}$$

The final output sub-system for an economic activity coincides with the notion of sub-system stated by Sraffa (1960, p.89).

> 'Consider a system of industries (each producing a different commodity) which is in a self-replacing state. The commodities forming the gross product (i.e. all quantities on the right hand side of the equation in §11) can be unambiguously distinguished as those which go to replace the means of production and those which together form the net product of the system. Such a system can be subdivided into as many parts as there are commodities in its net product, in such a way that each part forms a smaller self-replacing

system the net product of which consists of only one kind of commodity. These parts we shall call "sub-systems".'

The final output of the k_{th} subsystem is given by the following proportion of the external output of the k_{th} gross output subsystem

$$\alpha_k = c_k / (1 - h_{kk} - \theta_{kk}) w_k \tag{2.15}$$

where $(1 - h_{kk} - \theta_{kk}) w_k$ is the external output of activity k_{th}'s subsystem.

The multiplication of the k_{th} column vector of **[I+P]** by α_k specifies the proportions of activities 1,2,..,n involved in the k_{th} final output subsystem. Repeating for all k from 1 to n gives the proportion for the respective subsystem. The resulting set of column vectors is given by **A'=[I+P]** $\hat{\alpha}$, where $\hat{\alpha}$ is the transpose matrix which has α_k's on its main diagonal.

2.3.4 Sraffa's reduction to dated quantities of labor and the reduction to dated quantities of a commodity

Le us start from the usual representation of the Sraffa's scheme of production:

$$\mathbf{p} = w\mathbf{L} + (1+r)\breve{\mathbf{A}}\mathbf{p}\,. \tag{2.16}$$

Substituting **p** on the right-hand side of the equation gives

$$\mathbf{p} = [w\mathbf{L} + (1+r)\breve{\mathbf{A}}][w\mathbf{L} + (1+r)\breve{\mathbf{A}}\mathbf{p}]\,. \tag{2.17}$$

By repeating this substitution recursively, one obtains

$$\mathbf{p} = w[\mathbf{L} + (1+r)\breve{\mathbf{A}}\mathbf{L} + (1+r)^2\breve{\mathbf{A}}^2\mathbf{L} + ... + (1+r)^t\breve{\mathbf{A}}^t\mathbf{L} + ...]\,, \tag{2.18}$$

and this formula is known as the *reduction to dated quantities of labor*. The vector of prices is equal to the wage rate multiplied by the sum of the direct labour inputs (**L**) and a flow of discounted indirect labor components[4].

Alternatively, one can think of reducing the original system to dated quantities of one of the commodities produced. Given the similarity with the method of Sraffa, this procedure can be named *reduction to dated quantities of a commodity*.

[4] It can be shown that when the profit rate is equal to zero and the wage rate is chosen as the *numéraire* the price vector and Pasinetti's vector of vertically integrated labor coefficient coincide.

Let us suppose an economic system in which two commodities are produced by means of commodities and labour. Labour is then replaced by the quantity of the two commodities that are needed in order to sustain the workers.

Hence, the usual matrix of inputs **A** is transformed as follows:

$$\mathbf{A} = \begin{bmatrix} a_{11} + z_{11} & a_{12} + z_{12} \\ a_{21} + z_{21} & a_{22} + z_{22} \end{bmatrix}, \tag{2.19}$$

where the coefficient z_{11} represents the quantity of commodity 1 that is needed in order to sustain the workers employed in the production of one unit of commodity 1 and the coefficient z_{12} represents the quantity of commodity 2 that is needed in order to sustain the workers employed in the production of one unit of commodity 1. All the other z_{ii} are to be interpreted in the same way.

Write b_{ii} in place of $a_{ii}+z_{ii}$, accordingly

$$\mathbf{A} = \begin{bmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{bmatrix}. \tag{2.20}$$

.

Assume one is interested in computing the direct and indirect quantity of good 1 used in the production of both the commodities. Hence, the first step of the iterative procedure is as follows:

$$\breve{\mathbf{A}} = \begin{bmatrix} (b_{11} + b_{21} \cdot b_{12}) & (b_{22} \cdot b_{12}) \\ (b_{21} + b_{21} \cdot b_{22}) & (b_{22} \cdot b_{22}) \end{bmatrix}. \tag{2.21}$$

where **Ă** is now the matrix of interindustry coefficients.

The coefficient b_{11} is now augmented by the quantity of commodity 1 that is needed in order to produce b_{12} and the coefficient b_{12} is reduced to the quantity of commodity 2 that is needed to produce b_{12}. The same method applies to the other coefficients.

By continuing this iteration until the coefficients of commodity 2 in the A matrix are negligible, one obtains the total quantity of commodity 1 incorporated in the production of the two commodities. The same procedure can be applied to the commodity 2.

2.4 New indicators of sectoral productivity

2.4.1 Productivity indicator based on Goodwin's Normalized General coordinates

First of all, it is worth to underline that the indicators based on Goodwin's Normalized General coordinates suffers for a lack of realism because they original goods are transformed into *n* distinct composite commodities called *eigengoods*. As Goodwin emphasized, this transformation has positive and negative aspects. The positive aspect is that the *eigengoods* are produced entirely out of inputs of its own products; wages in each sector are paid out of its own product; any surplus of profit consists of each good itself. The negative aspect of this device is that it will ordinarily involve negative and complex quantities, so that is difficult to give commonsense interpretation to the analysis.

For this reason, the only indicator proposed is the ratio between the identity matrix and the diagonal matrix made up of the *n* distinct eigenvalues (λ)[5]. This indicator stands for a measure of capital productivity in each industry. It is worth saying that the numerator and the denominator now consist of the same kind of *eigengood*. Accordingly,

$$\alpha_{(s,i,t)} = 1 / \lambda_{(s,i,t)} \tag{2.22}$$

where $\alpha_{(s,i,t)}$ is the value of capital productivity of country i and industry s at time t and $\lambda_{(s,i,t)}$ is the eigenvalue which refers to the same country, industry, and time period.

Note that the eigenvalues of the original system are equal to the eigenvalues of the associated subsystems. Consequently, the indicator yields the same results.

2.4.2 Productivity indicator based on Pasinetti's vertical integration

Pasinetti's method of vertical integration allows the calculation of the total quantities of direct and indirect labor used in each industry. In this way, the heterogeneous inputs used in the production are reduced to a uniform measure. Accordingly, a straightforward and intuitive indicator of productivity is given by the following relationship:

[5] This indicator is the only one, among those studied, capable of providing a wide range of values with a clear economic interpretation.

$$\xi_{(s,i,t)} = \frac{f_{(s,i,t)}}{v_{(s,i,t)}} \qquad (2.23)$$

where, $f_{(s,i,t)}$ is the final output of country *i*, industry *s*, at time *t* , and $v_{(s,i,t)}$ is the quantity of direct and indirect labor used in the production of industry *s* at time *t*.

2.4.3 Productivity indicators based on Gossling'subsystems

As it has been shown in section three, Gossling proposed an iterative procedure for computing two kinds of subsystems: the gross output subsystems and the final output subsystems. The final output subsystems have a desirable property of additivity, i.e.,. the sum of all the subsystems gives the original comprehensive system. However, the following three indicators can be calculated from both types of subsystem.

The first indicator suggested is an index of capital productivity. This index is calculated as the ratio between the value of the gross output subsystem and the value of the intermediate inputs. The values are obtained by multiplying the physical quantities of goods by their production prices.

As it is known, prices of production are function of the profit rate, consequently a distribution free index of capital productivity can be obtained by the following definite integral

$$\theta_{(s,i,t)} = \frac{1}{R_{(i,t)}} \int_0^{R(i,t)} \frac{g_{(s,i,t)} p_{(s,i,t)}(r)}{\boldsymbol{e}' \mathbf{A}_{(s,i,t)} \mathbf{p}_{(i,t)}(r)} dr \qquad (2.24)$$

where $g_{(s,i,t)}$ is the gross output in physical quantities of the country *i*, subsystem *s*, and time *t*; $\mathbf{A}_{(s,i,t)}$ is the input matrix in physical quantities; $\mathbf{p}_{(i,t)}$ is a vector of associated production prices; $\boldsymbol{e}$ is the unit vector; and $R_{(i,t)}$ is the maximum profit rate[6].

The second indicator proposed is a measure of labor productivity. This index is calculated as the ratio between the value of the gross output subsystem and the sum of the labor inputs. Thus,

$$\vartheta_{(s,i,t)} = \frac{1}{R_{(i,t)}} \int_0^{R(i,t)} \frac{g_{(s,i,t)} p_{(s,i,t)}(r)}{\boldsymbol{e}' \mathbf{L}_{(s,i,t)}} dr \qquad (2.25)$$

[6] The maximum profit rate is the same in all the subsystems which refer to the same country.

where $g_{(s,i,t)}$ is the gross output in physical quantities; $\mathbf{p}_{(i,t)}$ is a vector of associated production prices; $\mathbf{L}_{(s,i,t)}$ is a vector of labor inputs; and $R_{(i,t)}$ is the maximum profit rate.

Finally, the third indicator proposed is a measure of multi-factor productivity. This index is given by the ratio between the value of the gross output subsystem and the value of all the intermediate inputs utilized by the industry in each time period. Accordingly,

$$\rho_{(s,i,t)} = \frac{1}{R_{(i,t)}} \int_{0}^{R(i,t)} \frac{g_{(s,i,t)} p_{(s,i,t)}(r)}{e' \mathbf{A}_{(s,i,t)} \mathbf{p}_{(i,t)}(r) + e' \mathbf{L}_{(s,i,t)} w_{(i,t)}(r)} dr \tag{2.26}$$

where $g_{(s,i,t)}$ is the gross output in physical quantities; $\mathbf{p}_{(i,t)}$ is a vector of associated production prices; $\mathbf{A}_{(s,i,t)}$ is the input matrix in physical quantities; $\mathbf{L}_{(s,i,t)}$ is a vector of labor inputs; $w_{(i,t)}$ is the wage rate and $R_{(i,t)}$ is the maximum profit rate.

2.4.4 Productivity indicators based on the reduction to dated quantities of a commodity

This subsection proposes a final battery of indicators that are based on the method named *reduction to dated quantities of a commodity*.

As it has shown, this method reduces the original system of heterogeneous commodities to a homogenous measure by means of an iterative procedure. This homogenous measure is the total quantity (direct + indirect) of a single commodity used in production. This procedure can be repeated as many times as the number of commodities produced.

The empirical analysis undertaken in this study will use only the final output subsystem, but this procedure can also be applied to the gross output subsystems and to the original system.

A simple and appealing indicator of productivity based on this method is the following

$$\chi_{(c,s,i,t)} = \frac{1}{b_{(c,s,i,t)}} \tag{2.27}$$

where $\chi_{(c,s,i,t)}$ is the value of productivity for the subsystem s in the country i at time t calculated by reducing the original matrix to dated quantities of the commodity c; and $b_{(c,s,i,t)}$ is the direct and indirect quantity of the commodity c utilized in the production of the subsystem s.

2.5 Data

The data used in this chapter are from the OECD input-output database. The database contains industry by industry input-output tables of the OECD countries from the 1970 to 2005. Generally, the tables are available at intervals of 5 years, although in some periods the intervals are shorter.

Since the objective of this chapter is to develop sectoral productivity measures that can be compared with measures of aggregate productivity developed in the previous chapter, the data correspond with those used in chapter one[7].

Consequently, we use input-output tables that cover roughly five year's intervals from around 1970 to 2000 for the US, Germany, France, and the UK. In order to have comparable input-output tables and non singular matrices, some sectors were aggregated and industries have been reduced to 23.

Because the data are expressed in current prices, macro-industry deflators were used to obtain values in constant prices. In this way, values can be taken as proxies of the quantity of goods traded between industries.

Finally, the physical quantity of labor is given by the number of hours worked. Total hours worked were allocated to various industries in proportion to the share of compensation of employees.

2.6 The empirical investigation and comparison

The indexes previously described are now used to measure the sectoral productivity trends in the US, Germany, France, and the UK. Tables and figures with the empirical findings were collected in the statistical appendix [A.1] to which the reader should refer for a complete view of the results.

In this section we illustrate the main results, dividing the analysis into two parts. In the first part will be summarized the empirical evidence derived from indices based on the theoretical works of Goodwin, Pasinetti and Gossling. The

[7] Please refer to chapter one for a more precise description of the source and data preparation (see section 1.5 and appendix 1.A).

second part will instead examine the indices based on the method of reduction to dated quantities of a commodity. This is necessitated by the fact that the results of indices based on the method of reduction to dated quantities of a commodity are special and deserve separate discussion.

The first issue concerns the distinction between labor productivity, capital productivity, and multi-factor productivity[8]. Two of the indicators proposed measure the productivity of capital (indicator $\alpha_{(s,i,t)}$ and $\theta_{(s,i,t)}$), an indicator measures the productivity of labor (indicator $\vartheta_{(s,i,t)}$), and other two indicators measure the multi-factor productivity (indicator $\xi_{(s,i,t)}$ and $\rho_{(s,i,t)}$).

Regarding *capital productivity*, (see fig.A.1.6, A.1.9, A.1.12, A.1.15 and tab.A.1.1) we find that it is higher in tertiary sector of industry that in other industries. In particular, the productivity is high in the industries of wholesale and retail trade, post and telecommunications, and public administration, in each of the four countries studied. Capital productivity exhibits a quite peculiar trend. In the early seventies the productivity is very high in all countries and almost all industries, and then it decreases until the early eighties before rising again. This trend is noticeable in all countries except Germany, since data for Germany are available only since 1980. In some countries and industries productivity revival since the mid-eighties is very evident, so that the level of productivity in 2000 is greater than the level in 1970. However, there are industries where the productivity rebound is not so pronounced and the level of productivity at the end of the time period considered is lower than the initial level[9]. These points are primarily derived from the analysis of the indicator based on the Gossling's subsystems, given that the Goodwin-based indexes of capital productivity cannot be computed for each sector and year because they sometimes take negative quantities that have no economic meaning (see tab.A.1.1).

Regarding *labor productivity*, (see fig.A.1.5, A.1.8, A.1.11, A.1.14) we find a complementary pattern. Labor productivity is very low in the service sectors, while it tends to be higher in agriculture and industry. In particular, productivity is remarkably high in chemicals, metals, and transport equipments industries, in each of the countries surveyed. A rather high value was also recorded in agriculture in all countries with the sole exception of Germany. Labor productivity grows over time in almost all industries except agriculture and chemical industry[10].

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[8] In this case it is preferable to use the term multi-factor productivity rather than the term total factor productivity because fixed capital has not been considered.

[9] See in particular some service industries in the U.S.

[10] In these two industries, the productivity is not increasing in all countries. However it must be said that there are other industries where productivity is not increasing, but this does not happen in all countries. For a complete view see figure A.1.5, A.1.8, A.1.11, A.1.14 in the Statistical Appendix.

Finally, *multi-factor productivity* analysis is conducted by means of the index based on Pasinetti's vertical integration (see fig.A.1.1, A.1.2, A.1.3, A.1.4) and the index based on Gossling's subsystems (see fig.A.1.7, A.1.10, A.1.13, A.1.16). Pasinetti's index is considered an index of multifactor productivity because the denominator is the total quantity of direct and indirect labor used in production. The indirect labor represents labor, which was previously deposited in capital goods and which gradually becomes re-embodied in the finished commodities. Unfortunately, a simple visual inspection of the two indicators does not allow one to draw unequivocal conclusions[11]. Both indicators suggest a decline in productivity in the seventies and early eighties and a later rebound in all countries surveyed, except for Germany for which data are not available. However, the level and rate of growth of productivity in industries varies from index to index. One can say that in this case multi-factor productivity is not generally higher in services compare to other industries, or vice versa. Some industries tend to have higher productivity than others, particularly the industry of business activities.

Now, we move on the examine the results obtained from the use of the method of reduction to dated quantities of a commodity. The introduction of the quantity of goods needed to sustain the workers is done as follows.

(a) It is assumed that the vector of sectoral net products in physical quantities of the original system corresponds to the consumption bundle of the workers employed.

(b) For each subsystem and for each industry, we multiply each element of the consumption bundle for the ratio between the workers employed in the industry and total workers employed in the original system.

(c) Finally, we add these components to the respective amounts of intermediate goods used in production. In this way, each component of the input-matrix is increased by the amount needed to sustain workers, according to the assumptions made above.

[11] However, the simple visual inspection is not able to determine precisely the degree of association between the two indices. For this reason, some indices of correlation were calculated. One is a index of global correlation, the others are indexes of country specific correlation. The index of global correlation is equal to 0.30, the index of correlation for the US is equal to 0.24, the index of correlation for Germany is equal to 0.53, the index of correlation for France in equal to 0.35, and the index of correlation for the UK is equal to 0.14.Therefore, we find that the correlation is positive but not high.

The following tables (2.2-2.5) show the productivity index calculated for each country and each subsystem[12]. Obviously, for each country and subsystem there is an index for each of the commodities produced.

An important observations concern the values on the main diagonal, which are much higher compared to the other values of the table. Cells in the main diagonal show the productivity of the subsystem using the same commodity for which the subsystem is calculated. Consequently, the interindustry coefficients which relate to the commodity are lower because the input quantities are divided by a rather substantial component of output.

General overviews of the results suggest the following consideration.

The method of reduction to dated quantities of goods clearly highlights the index numbers problem. The productivity measures are influenced by the choice of the commodity. This means that not only the levels of productivity in various industries change when productivity is measured in terms of different commodities, but also that these changes occur in a non-uniform manner. While the calculation of the correlation coefficients[13] indicates that some underlying trend is preserved, on the other hand, the results obtained with this method raise serious doubts about the ability of the index numbers to cope with the problem of heterogeneity.

Table 2.2 Index based on the reduction to dated quantities of a commodity – US 2000

Commodity/ Industry	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23
1	3,77	0,29	0,87	0,66	0,81	0,60	0,34	0,49	0,56	0,36	0,34	0,29	0,30	0,29	0,32	0,47	0,36	0,44	0,38	0,57	0,29	0,32	0,39
2	0,41	4,86	0,32	0,28	0,29	0,28	0,84	0,40	0,36	0,61	0,47	0,29	0,26	0,23	0,25	0,26	0,84	0,37	0,25	0,39	0,21	0,24	0,32
3	0,46	0,07	3,79	0,15	0,16	0,10	0,09	0,13	0,11	0,10	0,08	0,06	0,06	0,05	0,06	0,09	0,11	0,06	0,11	0,47	0,04	0,06	0,13
4	0,21	0,10	0,15	3,37	0,21	0,33	0,09	0,20	0,53	0,19	0,10	0,10	0,16	0,10	0,38	0,64	0,08	0,18	0,15	0,15	0,09	0,10	0,18
5	0,30	0,17	0,22	0,24	4,00	0,41	0,14	0,20	0,22	0,32	0,21	0,20	0,20	0,19	0,20	0,74	0,27	0,71	0,29	0,23	0,27	0,33	0,22
6	0,20	0,18	0,38	0,25	0,19	3,52	0,17	0,33	0,34	0,30	0,16	0,20	0,20	0,31	0,17	0,33	0,16	0,22	0,26	0,23	0,28	0,27	0,27
7	0,44	0,40	0,23	0,22	0,22	0,21	5,42	0,46	0,34	0,23	0,23	0,18	0,15	0,13	0,14	0,17	0,35	0,32	0,18	0,47	0,12	0,13	0,26
8	0,58	0,39	0,36	0,66	0,35	0,43	0,36	22,88	5,35	0,47	0,23	0,34	0,30	0,35	0,32	0,49	0,22	0,30	0,18	0,22	0,17	0,18	0,32
9	0,27	0,25	0,40	0,26	0,26	0,28	0,19	0,45	5,54	0,29	0,17	0,23	0,37	0,31	0,37	0,53	0,22	0,38	0,27	0,29	0,20	0,17	0,24
10	0,25	0,33	0,32	0,21	0,36	0,15	0,27	0,28	0,34	6,84	0,41	0,27	0,28	0,28	0,30	0,30	0,34	0,68	0,17	0,21	0,22	0,27	0,21
11	0,30	0,46	0,25	0,20	0,27	0,24	0,30	0,26	0,34	0,39	2,71	0,78	0,67	0,53	0,56	0,59	0,34	0,40	0,25	0,24	0,28	0,19	0,22
12	0,23	0,35	0,33	0,21	0,34	0,24	0,25	0,31	0,32	0,30	0,38	4,74	0,58	0,38	0,44	0,43	0,27	0,51	0,19	0,23	0,26	0,17	0,19
13	0,29	0,41	0,14	0,13	0,13	0,14	0,22	0,18	0,21	0,13	0,25	0,25	5,45	0,19	0,34	0,13	0,21	0,32	0,11	0,13	0,07	0,08	0,12
14	0,15	0,15	0,15	0,22	0,25	0,27	0,14	0,24	0,28	0,22	0,30	0,26	0,43	2,77	0,39	0,28	0,23	0,31	0,21	0,14	0,39	0,20	0,20
15	0,11	0,13	0,08	0,06	0,11	0,07	0,09	0,07	0,07	0,08	0,09	0,10	0,20	0,08	2,60	0,07	0,09	0,12	0,21	0,23	0,06	0,08	0,20
16	0,08	0,08	0,09	0,15	0,19	0,08	0,07	0,13	0,10	0,12	0,08	0,09	0,12	0,12	0,10	10,26	0,08	0,27	0,11	0,11	0,09	0,13	0,21
17	0,28	0,24	0,23	0,21	0,19	0,20	0,26	0,25	0,25	0,33	0,25	0,20	0,15	0,14	0,13	0,17	12,64	0,14	0,19	0,21	0,14	0,22	0,22
18	0,05	0,03	0,04	0,03	0,03	0,04	0,03	0,05	0,04	0,04	0,03	0,03	0,03	0,03	0,02	0,04	0,13	5,59	0,05	0,05	0,06	0,08	0,09
19	0,21	0,14	0,22	0,20	0,21	0,18	0,18	0,23	0,21	0,17	0,22	0,19	0,20	0,21	0,19	0,22	0,11	0,25	3,31	0,16	0,10	0,09	0,11
20	0,25	0,23	0,26	0,23	0,26	0,23	0,25	0,26	0,29	0,36	0,28	0,20	0,19	0,17	0,18	0,21	0,41	0,18	0,20	3,68	0,13	0,19	0,18
21	0,17	0,17	0,19	0,15	0,14	0,25	0,14	0,19	0,18	0,16	0,13	0,15	0,18	0,19	0,14	0,18	0,15	0,21	0,26	0,26	3,31	0,30	0,26
22	0,26	0,38	0,30	0,23	0,19	0,28	0,26	0,31	0,26	0,24	0,18	0,22	0,23	0,26	0,19	0,24	0,26	0,24	0,34	0,28	0,31	1,82	0,27
23	0,11	0,09	0,11	0,11	0,09	0,14	0,09	0,11	0,11	0,11	0,11	0,09	0,08	0,09	0,09	0,09	0,09	0,10	0,14	0,15	0,19	0,18	1,72

[12] The tables present the results in 2000. The results for other years are available upon request.

[13] The correlation coefficients have been calculated by the author and are available upon request. The values of the correlation coefficients are positive and vary from 0.1 to 0.9..

Table 2.3 Index based on the reduction to dated quantities of a commodity – Germany 2000

Commodity/ Industry	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23
1	7,12	0,27	0,84	0,53	0,75	0,42	0,25	0,33	0,40	0,26	0,23	0,22	0,22	0,21	0,25	0,38	0,21	0,30	0,27	0,48	0,23	0,25	0,34
2	0,50	5,92	0,44	0,39	0,40	0,45	0,79	n.a	0,45	0,66	0,50	0,37	0,36	0,36	0,34	0,39	0,73	0,45	0,42	0,42	0,39	0,40	0,44
3	0,49	0,06	3,44	0,10	0,11	0,06	0,05	n.a	0,08	0,07	0,05	0,05	0,05	0,05	0,05	0,07	0,05	0,05	0,09	0,36	0,06	0,04	0,13
4	0,21	0,10	0,12	2,99	0,13	0,11	0,07	n.a	0,40	0,11	0,10	0,13	0,13	0,12	0,20	0,59	0,07	0,18	0,11	0,11	0,11	0,06	0,14
5	0,20	0,19	0,18	0,18	4,09	0,21	0,12	n.a	0,20	0,19	0,16	0,26	0,22	0,17	0,23	0,74	0,15	0,62	0,12	0,13	0,13	0,20	0,20
6	0,20	0,20	0,34	0,34	0,26	2,84	0,15	n.a	0,29	0,24	0,16	0,19	0,21	0,24	0,18	0,33	0,18	0,18	0,35	0,23	0,39	0,33	0,23
7	0,46	0,26	0,29	0,27	0,30	0,24	3,72	n.a	0,36	0,32	0,38	0,24	0,21	0,20	0,20	0,28	0,39	0,30	0,28	0,48	0,23	0,17	0,21
8	0,58	0,25	0,33	0,64	0,55	0,41	0,26	n.a	5,18	0,42	0,30	0,29	0,27	0,30	0,31	0,38	0,18	0,35	0,15	0,15	0,14	0,16	0,21
9	0,21	0,18	0,26	0,23	0,26	0,22	0,11	n.a	4,26	0,33	0,12	0,24	0,32	0,30	0,37	0,39	0,13	0,45	0,18	0,14	0,12	0,12	0,11
10	0,29	0,24	0,21	0,19	0,34	0,10	0,14	n.a	0,24	5,25	0,23	0,21	0,16	0,23	0,17	0,20	0,17	0,66	0,09	0,11	0,12	0,16	0,11
11	0,23	0,35	0,21	0,19	0,23	0,17	0,23	n.a	0,34	0,25	2,16	0,70	0,54	0,45	0,50	0,35	0,29	0,36	0,18	0,17	0,17	0,13	0,13
12	0,19	0,31	0,20	0,16	0,25	0,10	0,20	n.a	0,21	0,16	0,21	3,39	0,45	0,30	0,36	0,38	0,19	0,41	0,11	0,13	0,10	0,10	0,12
13	0,18	0,34	0,12	0,14	0,11	0,11	0,19	n.a	0,21	0,16	0,17	0,23	2,92	0,13	0,22	0,18	0,17	0,15	0,06	0,08	0,06	0,06	0,08
14	0,14	0,17	0,13	0,13	0,13	0,13	0,11	n.a	0,17	0,15	0,16	0,18	0,41	2,38	0,31	0,15	0,37	0,36	0,15	0,14	0,41	0,16	0,18
15	0,07	0,06	0,05	0,04	0,05	0,03	0,04	n.a	0,07	0,07	0,05	0,05	0,08	0,08	2,02	0,06	0,05	0,05	0,09	0,16	0,04	0,04	0,07
16	0,05	0,05	0,05	0,15	0,10	0,18	0,03	n.a	0,17	0,13	0,27	0,13	0,08	0,06	0,22	6,29	0,04	0,08	0,05	0,05	0,04	0,04	0,08
17	0,30	0,41	0,27	0,28	0,26	0,29	0,25	n.a	0,30	0,36	0,28	0,25	0,19	0,18	0,18	0,21	5,80	0,17	0,20	0,23	0,20	0,16	0,19
18	0,09	0,11	0,08	0,06	0,08	0,08	0,07	n.a	0,07	0,10	0,07	0,07	0,07	0,08	0,06	0,07	0,14	3,27	0,08	0,09	0,12	0,24	0,11
19	0,22	0,13	0,23	0,26	0,23	0,14	0,09	n.a	0,15	0,20	0,15	0,19	0,18	0,16	0,15	0,26	0,14	0,18	3,09	0,17	0,11	0,07	0,10
20	0,18	0,26	0,25	0,17	0,22	0,22	0,20	n.a	0,24	0,32	0,21	0,19	0,22	0,20	0,22	0,24	0,20	0,17	0,45	2,57	0,30	0,15	0,16
21	0,17	0,19	0,18	0,17	0,20	0,31	0,15	n.a	0,23	0,21	0,13	0,21	0,22	0,21	0,16	0,17	0,22	0,19	0,35	0,25	3,94	0,29	0,25
22	0,32	0,27	0,31	0,22	0,25	0,31	0,20	n.a	0,31	0,29	0,19	0,22	0,25	0,28	0,24	0,26	0,29	0,30	0,34	0,29	0,36	2,03	0,26
23	0,11	0,09	0,08	0,05	0,07	0,14	0,06	n.a	0,07	0,07	0,05	0,05	0,04	0,05	0,05	0,05	0,17	0,06	0,07	0,08	0,07	0,13	2,32

Table 2.4 Index based on the reduction to dated quantities of a commodity – France 2000

Commodity/ Industry	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23
1	4,51	0,15	0,83	0,44	0,73	0,29	0,15	0,29	0,34	0,16	0,17	0,11	0,12	0,13	0,14	0,33	0,21	0,30	0,11	0,45	0,07	0,10	0,25
2	0,45	6,58	0,43	0,37	0,39	0,38	0,81	n.a	0,40	0,64	0,47	0,34	0,34	0,34	0,31	0,37	0,68	0,51	0,44	0,43	0,42	0,43	0,45
3	0,48	0,04	3,17	0,13	0,12	0,08	0,04	n.a	0,07	0,05	0,04	0,03	0,02	0,03	0,02	0,05	0,03	0,04	0,05	0,36	0,02	0,03	0,09
4	0,28	0,15	0,17	2,64	0,14	0,15	0,18	n.a	0,37	0,17	0,21	0,19	0,18	0,22	0,26	0,54	0,12	0,24	0,22	0,20	0,29	0,13	0,34
5	0,53	0,38	0,47	0,17	3,66	0,34	0,23	n.a	0,14	0,24	0,30	0,22	0,21	0,19	0,19	0,82	0,20	0,61	0,18	0,17	0,11	0,13	0,23
6	0,26	0,22	0,37	0,30	0,19	2,56	0,19	n.a	0,28	0,39	0,19	0,18	0,22	0,21	0,16	0,33	0,21	0,23	0,34	0,20	0,29	0,42	0,39
7	0,38	0,21	0,23	0,16	0,21	0,18	3,78	n.a	0,22	0,26	0,26	0,14	0,13	0,11	0,11	0,21	0,52	0,22	0,27	0,41	0,11	0,12	0,14
8	0,66	0,40	0,37	0,43	0,45	0,40	0,34	n.a	5,35	0,37	0,30	0,37	0,27	0,32	0,28	0,44	0,18	0,26	0,13	0,15	0,11	0,14	0,24
9	0,29	0,38	0,40	0,39	0,15	0,25	0,20	n.a	3,21	0,30	0,18	0,25	0,29	0,38	0,31	0,52	0,19	0,28	0,17	0,19	0,18	0,13	0,13
10	0,26	0,11	0,24	0,09	0,11	0,10	0,12	n.a	0,27	4,85	0,30	0,14	0,17	0,25	0,21	0,28	0,17	0,65	0,14	0,25	0,09	0,10	0,16
11	0,26	0,35	0,24	0,27	0,31	0,30	0,27	n.a	0,35	0,36	1,96	0,63	0,58	0,45	0,45	0,67	0,23	0,43	0,16	0,20	0,20	0,18	0,18
12	0,25	0,41	0,26	0,27	0,40	0,13	0,30	n.a	0,26	0,27	0,30	2,49	0,52	0,40	0,34	0,33	0,24	0,35	0,15	0,19	0,14	0,14	0,14
13	0,34	0,61	0,20	0,22	0,25	0,11	0,32	n.a	0,25	0,25	0,23	0,26	3,36	0,10	0,20	0,22	0,26	0,34	0,13	0,13	0,05	0,07	0,14
14	0,13	0,18	0,11	0,10	0,10	0,15	0,17	n.a	0,17	0,15	0,21	0,25	0,39	2,03	0,40	0,14	0,24	0,32	0,15	0,12	0,48	0,25	0,19
15	0,08	0,04	0,06	0,05	0,06	0,08	0,04	n.a	0,04	0,18	0,10	0,05	0,05	0,05	1,72	0,18	0,04	0,07	0,12	0,25	0,12	0,11	0,15
16	0,09	0,09	0,09	0,15	0,12	0,21	0,10	n.a	0,10	0,12	0,31	0,20	0,18	0,13	0,19	8,58	0,07	0,13	0,07	0,10	0,13	0,17	0,18
17	0,27	0,32	0,26	0,23	0,26	0,24	0,23	n.a	0,27	0,34	0,29	0,19	0,18	0,15	0,14	0,21	4,56	0,15	0,19	0,21	0,11	0,12	0,21
18	0,09	0,08	0,05	0,04	0,04	0,03	0,14	n.a	0,04	0,06	0,05	0,04	0,04	0,03	0,03	0,03	0,26	3,10	0,05	0,04	0,06	0,07	0,10
19	0,23	0,19	0,20	0,14	0,22	0,19	0,13	n.a	0,16	0,18	0,18	0,16	0,16	0,16	0,15	0,20	0,09	0,16	3,37	0,12	0,09	0,08	0,09
20	0,21	0,18	0,21	0,16	0,22	0,29	0,16	n.a	0,14	0,30	0,19	0,13	0,13	0,14	0,12	0,18	0,12	0,17	0,31	2,76	0,11	0,19	0,16
21	0,15	0,14	0,18	0,19	0,21	0,28	0,13	n.a	0,18	0,14	0,21	0,22	0,18	0,18	0,14	0,25	0,18	0,18	0,30	0,18	3,09	0,35	0,27
22	0,27	0,21	0,33	0,29	0,20	0,27	0,24	n.a	0,33	0,27	0,20	0,21	0,31	0,28	0,25	0,30	0,30	0,35	0,32	0,26	0,28	1,79	0,26
23	0,05	0,04	0,05	0,04	0,05	0,05	0,04	n.a	0,06	0,05	0,04	0,04	0,05	0,05	0,04	0,05	0,07	0,04	0,04	0,05	0,03	0,07	2,13

Table 2.5 Index based on the reduction to dated quantities of a commodity – UK 2000

Commodity/ Industry	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23
1	7,89	0,25	0,83	0,27	0,71	0,28	0,22	0,34	0,46	0,26	0,25	0,21	0,22	0,22	0,23	0,35	0,21	0,32	0,37	0,48	0,22	0,21	0,25
2	0,29	6,75	0,22	0,19	0,21	0,18	0,88	n.a	0,21	0,51	0,38	0,23	0,20	0,17	0,18	0,21	0,58	0,33	0,24	0,27	0,18	0,18	0,17
3	0,43	0,11	3,49	0,15	0,11	0,10	0,10	n.a	0,10	0,12	0,09	0,08	0,08	0,08	0,08	0,09	0,08	0,07	0,22	0,41	0,10	0,09	0,12
4	0,23	0,09	0,12	4,29	0,10	0,12	0,07	n.a	0,22	0,12	0,13	0,08	0,09	0,10	0,19	0,47	0,06	0,13	0,20	0,13	0,10	0,09	0,15
5	0,24	0,27	0,25	0,24	3,77	0,48	0,19	n.a	0,28	0,38	0,35	0,35	0,29	0,20	0,30	0,84	0,17	0,57	0,20	0,20	0,18	0,20	0,20
6	0,27	0,18	0,35	0,25	0,26	2,98	0,16	n.a	0,29	0,26	0,16	0,18	0,20	0,23	0,19	0,28	0,19	0,18	0,24	0,21	0,18	0,30	0,24
7	0,40	0,28	0,21	0,17	0,21	0,16	10,7	n.a	0,19	0,23	0,25	0,18	0,16	0,14	0,14	0,20	0,34	0,18	0,30	0,40	0,19	0,18	0,16
8	0,56	0,23	0,32	0,47	0,33	0,40	0,35	n.a	3,27	0,41	0,28	0,28	0,27	0,26	0,29	0,46	0,15	0,25	0,19	0,18	0,17	0,15	0,34
9	0,27	0,17	0,40	0,20	0,35	0,24	0,13	n.a	4,71	0,22	0,19	0,22	0,29	0,30	0,39	0,45	0,13	0,38	0,24	0,22	0,32	0,15	0,15
10	0,21	0,28	0,24	0,09	0,29	0,09	0,16	n.a	0,32	7,25	0,19	0,17	0,13	0,22	0,23	0,15	0,14	0,55	0,18	0,13	0,15	0,12	0,16
11	0,21	0,33	0,23	0,21	0,26	0,19	0,24	n.a	0,32	0,36	2,94	0,74	0,63	0,45	0,54	0,63	0,22	0,30	0,21	0,18	0,23	0,14	0,19
12	0,17	0,41	0,26	0,17	0,36	0,15	0,27	n.a	0,37	0,31	0,30	4,89	0,50	0,34	0,45	0,34	0,22	0,35	0,15	0,13	0,15	0,11	0,13
13	0,17	0,35	0,22	0,18	0,25	0,19	0,22	n.a	0,23	0,27	0,30	0,28	4,24	0,20	0,33	0,22	0,21	0,22	0,15	0,11	0,12	0,10	0,14
14	0,18	0,22	0,18	0,17	0,20	0,17	0,15	n.a	0,21	0,20	0,24	0,22	0,42	2,27	0,37	0,23	0,29	0,26	0,18	0,18	0,58	0,20	0,32
15	0,18	0,17	0,12	0,11	0,13	0,10	0,10	n.a	0,11	0,13	0,13	0,11	0,18	0,10	2,85	0,17	0,09	0,10	0,26	0,21	0,10	0,12	0,21
16	0,10	0,11	0,10	0,16	0,17	0,09	0,08	n.a	0,14	0,15	0,57	0,24	0,27	0,14	0,20	8,95	0,08	0,16	0,10	0,09	0,09	0,08	0,11
17	0,35	0,36	0,31	0,31	0,31	0,28	0,30	n.a	0,35	0,46	0,42	0,33	0,29	0,23	0,26	0,29	2,36	0,20	0,23	0,21	0,19	0,19	0,22
18	0,14	0,26	0,09	0,07	0,08	0,07	0,18	n.a	0,07	0,09	0,09	0,08	0,07	0,07	0,08	0,08	0,19	2,34	0,12	0,11	0,13	0,23	0,13
19	0,28	0,14	0,25	0,27	0,19	0,21	0,10	n.a	0,22	0,19	0,21	0,18	0,20	0,23	0,20	0,21	0,11	0,16	2,07	0,18	0,16	0,11	0,13
20	0,18	0,28	0,22	0,19	0,19	0,21	0,19	n.a	0,21	0,31	0,20	0,17	0,17	0,17	0,16	0,19	0,14	0,16	0,39	2,32	0,22	0,20	0,15
21	0,21	0,19	0,18	0,15	0,15	0,17	0,15	n.a	0,17	0,18	0,14	0,14	0,14	0,15	0,14	0,15	0,16	0,17	0,26	0,22	4,16	0,37	0,20
22	0,31	0,37	0,28	0,25	0,24	0,25	0,26	n.a	0,26	0,25	0,21	0,23	0,23	0,24	0,23	0,25	0,27	0,30	0,38	0,32	0,28	1,55	0,27
23	0,09	0,05	0,06	0,05	0,04	0,10	0,05	n.a	0,05	0,05	0,04	0,04	0,04	0,05	0,04	0,05	0,05	0,04	0,06	0,07	0,07	0,10	1,63

2.7 Conclusions

Although some standard methods of productivity accounting utilize the input-output tables, they do it partially because they fail to capture the interrelationship among economic sectors. This paper has introduced four related, although conceptually distinct, approaches to measure productivity and mainly sectoral productivity. These four methods are linked because they transform the original input-output tables in such a way as to preserve 'as much as they can' the interconnection among sectors. At the same time, each of these methods are distinct because is based on a different theoretical framework. In the previous sections has been made an attempt to compare these procedures and identify advantages and disadvantages of all of them.

As shown above, there is no such a thing as a best method, given that a theoretically well-grounded approach is not feasible from an empirical point of view. Nevertheless, the main aim of this paper is to push the research on productivity toward a development of alternative methods that are each based at least on one of the following principles.

(a) *The adoption of a non-value based approach*

The ideal productivity index is a ratio where the numerator and the denominator are represented by the same commodity. In this case the surplus is physically comparable to their capital, just as in the corn economy. Unfortunately, in the real world situation with heterogeneous commodities such a index is not feasible and therefore an alternative approach has to be used to by-pass this problem. This paper has presented a technique that has much in common with the method of 'the reduced at dated quantity of labour'. Essentially, the idea is to reduce the whole economic system, as represented in the input–output table, to dated quantity of one commodity and then repeat the procedure for all the commodities.

(b) *The superiority of production prices over the market prices*

The prices of production are those at which the commodities must be sold in order to guarantee the reproducibility of the economic system. Hence, they differ from the market prices which are obtained by the conditions of supply and demand. The price of production of one commodity can be interpreted as a sign of the relative importance of that commodity for the economy as a whole, and therefore they represent a more appropriate weight for the aggregation of heterogeneous commodities.

(c) *The influence of interconnections among economic sectors*

An economic system of production is made up by many industries producing various types of intermediate and final goods. It is thus natural that innovation in one industry spills over to other directly or indirectly connected industries. As a corollary, the measurement of productivity in one sector cannot be implemented successfully while disregarding the change in productivity in related sectors.

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CHAPTER 3

PRODUCTIVITY IN THE ITALIAN REGIONS: DEVELOPMENT OF ALTERNATIVE INDICATORS BASED ON INPUT-OUTPUT TABLES

Written jointly with Stefano Zambelli

3.1 Introduction

The objective of this chapter is to analyse productivity and technological progress in a selected sample of Italian regions in recent years using input-output tables. Among the OECD member countries, Italy deserves a special attention because the country is suffering from a decline in competitiveness and low growth rates of its Gross Domestic Product, which has been emphasized by recent empirical studies (Daveri, 2006; Daveri and Jona-Lasinio, 2005, Faini, 2004).

According to the researchers, the main causes of this decline are the structural weakness due to its geographical conformation, the lack of raw materials and energy resources, the lack of infrastructure development, and the influence of a global competitive environment on small and medium enterprises.

Italy is subdivided into twenty regions, but this administrative division is accompanied by an economic division generally portrayed as a north/south divide although this is a rough distinction as every region has its peculiarities formed by history and geography. For this reason, a rigorous analysis of regional productivity in Italy is much needed than in other developed countries.

However, the study of productivity differential at regional level is becoming a subject of interest not only for Italy but also for Europe as a whole. At EU level, regional analysis of productivity is becoming an important goal as it represents the way to close the productivity gap with the United States and ensuring better social cohesion, especially with the new member states (Gardiner et al, 2004, see also European Commission, 2004a; 2004b).

Among the members of the European Union, United Kingdom is the only country where the government has specifically emphasized the importance of the regional dimension to its national economic objectives (HM Treasury, 2001; HM Treasury, 2004; Department of Trade and Industry, 2004).

The issues of regional competitiveness are also characterized by a growing academic literature. In fact, the Regional Studies journal has devoted an entire issue to this theme (Regional Studies, 2004) and increasing research has been carried out to study the problem of how the differences in regional competitiveness can best be defined and measured (Begg, 1999; Camagni, 2002; Krugman, 1990; Porter, 1992, 1998, 2001a, 2001b). There are also recent studies examining the factors explaining the productivity gap (Criscuolo and Martin, 2003; Cambridge Econometrics, 2003; Rice and Venables, 2004).

The main goal of this paper is to analyse regional productivity differentials through the application of a new set of production-based indicators of technological progress and productivity recently developed by Degasperi and Fredholm (2010) and Fredholm and Zambelli (2009a; 2009b). The study is conducted in three steps. Firstly, we compute aggregate productivity and technological progress for eight Italian regions, secondly, we calculate indexes of sectoral productivity, and finally, we compute two different versions of the so-called technological frontier and we construct indices for regional specific technological progress and convergence by combining the regional specific wage-profit frontiers and the contemporary and intertemporal technological frontiers. Given the extensive role of production prices in this chapter, which are used for the calculation of most of the indicators proposed, we will integrate our analysis with a comparison of the behaviour of production prices in each industry and in each region.

The conclusions will be aimed to provide answers to the below pivotal questions.

- Is there a productivity and technological progress differential among the Italian regions?
- How pronounced is this productivity gap and how it has evolved recently?
- What is the relationship between aggregate productivity and sectoral productivity?
- Finally, what is the pattern of the global productivity from 2001 and 2004?

3.2 Indicators[1]

3.2.1 Indicators of aggregate productivity

The aim of the first set of indicators is to provide estimates of aggregate productivity and technological progress. We then calculate two indexes developed by Degasperi and Fredholm (2010) and based on the area under the region specific wage-profit frontiers and Net National Profit (NNP) curves.

These indexes are based on a theoretical model that follows the tradition of von Neumann, Leontief, and Sraffa, production, growth, and distribution. The economic system consists of *n* industries producing *n* commodities by means of some combination of the *n* commodities and labour.

Let $\mathbf{A}$ be a $(n \times n)$ quadratic non-singular matrix of inter-industry inputs, where the (*i,j*) entry represents the i^{th} industry's use of the j^{th} commodity in the production of the i^{th} commodity. Likewise, $\mathbf{L}$ is a $(n \times 1)$ vector of labour inputs and $\mathbf{B}$ is a $(n \times n)$ positive definite diagonal matrix of outputs, where the i^{th} diagonal entry is the gross output of the i^{th} industry. These elements can be collected in the following long-run equilibrium relationship that captures the distribution of the total production among wages, profits, and means of production, where the wage and profit rates are assumed to be uniform.

$$\mathbf{Ap}(1+r)+\mathbf{L}w=\mathbf{Bp} \tag{3.1}$$

[1] The chapters of the thesis have been prepared as independent papers, so there are certain differences in the notation.

System (3.1) consists of n linear independent equations and n+2 variables, i.e., the system has initially two degrees of freedom. Choosing a *numéraire* $\boldsymbol{\eta}$, for which it holds that $\boldsymbol{\eta}'\mathbf{p} = 1$, the degrees of freedom reduces to one.

Given the profit rate, it is straightforward to calculate the wage rate and the relative prices that solve system (3.1). Isolate $\mathbf{p}$, $\mathbf{p} = (\mathbf{B} - \mathbf{A}(1+r))^{-1}\mathbf{L}w$, premultiply with the *numéraire*, and rearrange to obtain the wage-profit frontier function and the associated prices, *viz.*

$$w = \left(\boldsymbol{\eta}'(\mathbf{B} - \mathbf{A}(1+r))^{-1}\mathbf{L}\right)^{-1} \tag{3.2}$$

$$\mathbf{p} = \frac{(\mathbf{B} - \mathbf{A}(1+r))^{-1}\mathbf{L}}{\boldsymbol{\eta}'(\mathbf{B} - \mathbf{A}(1+r))^{-1}\mathbf{L}} \tag{3.3}$$

Using this price vector as a measure of value in terms of a given *numéraire* and a given rate of profit, the value of the NNP is obtained by the following accounting identity, where $\mathbf{e}$ is a $(n \times 1)$ unit vector.

$$\text{NNP} = \mathbf{e}'(\mathbf{B} - \mathbf{A}(1+r))\mathbf{p} \tag{3.4}$$

The first index uses the area under the wage-profit frontiers as a measure of technological progress. Shifts in the wage-profit frontier can be interpreted as technological change, positive or negative depending on the nature of the shift and the distribution between wages and profits. Then, the procedure proposed by Degasperi and Fredholm (2010) will be used to construct a distribution free measure of technological progress given by the following definite integral:

$$\alpha_t^i = \frac{1}{R_t^i} \int_0^{R} w_t^i(r)\,dr \tag{3.5}$$

where, α_t^i indicates the technological progress of the region i at time t. R_t^i indicates the maximum profit rate of the region i at time t[2], and w_t^i is the wage rate of the region i at time t. In order to normalize the values and enable comparisons between different regions, the area under the wage-profit frontier is then divided by the maximum rate of profit R_t^i.

The area is calculated by means of computational methods. Essentially, first we identify a hundred equally spaced points on the horizontal axis, enclosed between the zero profit rate and the maximum profit rate. For each of these values of the rate of profit, we calculate the value of the wage rate. Then, we interpolate

[2] The maximum rate of profit is computed as $R_t^i = \lambda_t^{i\,-1} - 1$, where λ_t^i is the maximum eigenvalue of the matrix of interindustrial coefficient of the region i at time t.

these points using a polynomial of fifth degree and, finally, we determine the area under the polynomial curve.

The second index is calculated from the area under the NNP per unit of labour curves, i.e., integrate with respect to the rate of profit from zero to maximum rate of profit. Furthermore, to obtain an index, which is comparable across regions and over time, we divide this area with the maximum profit rate. The procedure proposed by Degasperi and Fredholm (2010) is used to construct a distribution free measure of labour productivity, accordingly:

$$\beta_t^i = \frac{1}{\boldsymbol{e}'\mathbf{L}_t^i R_t^i} \int_0^{R} \boldsymbol{e}'(\mathbf{B}_t^i - \mathbf{A}_t^i(1+\boldsymbol{r}))\mathbf{p}_t^i dr \tag{3.6}$$

where, β_t^i indicates the labour productivity of the region i at time t, $\mathbf{B}_t^i$ is a positive definite diagonal matrix of outputs, $\mathbf{A}_t^i$ is a quadratic non-singular matrix of inputs, $\mathbf{p}_t^i$ is a vector of production prices, $\boldsymbol{e}$ is a unit vector, R_t^i is the maximum profit rate, and $\mathbf{L}_t^i$ is a vector of labour inputs.

3.2.2 Indicators of sectoral productivity

A recent contribution by Fredholm and Zambelli (2009a) and the second chapter of this thesis propose a series of indicators of sectoral productivity constructed from input-output tables. These indicators are based on several theoretical works which allow in some way to decompose the original input-output tables without loosing the information about the interdependence among industries.

This is the case of subsystems which consists of the portions in which an entire system can be subdivided, in such way that each portion represents a smaller self-replacing system producing only one final product. This is also the case of Goodwin's Normalized General coordinates (Goodwin, 1976), a method that transforms the original observed sectors to new sectors that are independent from one another.

In this paper we focus on few important indicators which have a clear economic interpretation. For this reason we avoid those based on Goodwin's Normalized General coordinates because they often involve negative and complex quantities.

Subsystems can be calculated using different methods: the direct multiplier method, the Pasinetti's vertically integrated sectors (1973), the Gossling's procedure (Gossling and Dovring, 1976), and the reduction to dated quantity of labour (Sraffa, 1960 pp.113-144). In a recent study, Fredholm and Zambelli (2009a) have shown that all the above methods under certain circumstances give identical results.

In order to compute the first index of sectoral productivity, we use the method of vertical integration proposed by Pasinetti (1973), which is briefly described as follows.

Let $\breve{\mathbf{A}}$ denote a square $(n \times n)$ matrix of inter-industrial coefficient and let $\mathbf{L}$ denote a $(n \times 1)$ vector of direct labour input coefficient. The vector of vertically integrated labour coefficient is given by

$$\mathbf{v} = (\mathbf{I} - \breve{\mathbf{A}})^{-1}\mathbf{L} \tag{3.7}$$

Finally, the total quantity of labour directly and indirectly required to produce the different commodities is given by

$$\boldsymbol{\tau} = \mathbf{F}(\mathbf{I} - \breve{\mathbf{A}})^{-1}\mathbf{L} \tag{3.8}$$

where, $\mathbf{F}$ is a $(n \times n)$ matrix of final output.

Hence, an appealing sectoral productivity index based on *physical quantities* can be derived from the total quantity of direct and indirect labour as follows:

$$\xi^s_{i,t} = f^s_{i,t}/\tau^s_{i,t} \tag{9}$$

where, $\xi^s_{i,t}$ is the value of the productivity in industry s in the region i in year t, $f^s_{i,t}$ is the final output in physical quantities in the industry s in the region i at time t, and $\tau^s_{i,t}$ is the direct and indirect labour used by the industry s in the region i at time t.

As a supplement of this first index of sectoral productivity, we compute another indicator which is based on *production prices*[3]. By applying the direct multiplier method (see Fredholm and Zambelli, 2009a), we compute the final output subsystems for each industry and we derive distribution free measures of sectoral productivity given by the area under the industry specific NNP curves. To construct these measures we calculate the usual following definite integral for each industry

$$\delta^s_{i,t} = \frac{1}{e'\,\mathbf{L}^s_{i,t}R_{i,t}} \int_0^R e'\,(\mathbf{B}^s_{i,t} - \mathbf{A}^s_{i,t}(1+r))\mathbf{p}_{i,t}\,dr \tag{3.10}$$

[3] In this chapter, we follow the classification of Fredholm and Zambelli (2009a). According to this classification the productivity indices are divided into two categories, i.e. indices based on physical quantities and indices based on production prices.

where, $\delta^s_{i,t}$ indicates the labour productivity of the industry *s* in the region *i* at time *t*, $\mathbf{B}^s_{i,t}$ is a positive definite diagonal matrix of outputs of the subsystem *s*[4], $\mathbf{A}^s_{i,t}$ is a quadratic non-singular matrix of inter-industry inputs, of the subsystem *s*, $\mathbf{p}_{i,t}$ is a vector of production prices, e is a unit vector, $R_{i,t}$ is the maximum profit rate, and $\mathbf{L}^s_{i,t}$ is a vector of labour inputs of the subsystem *s* of region *i* at time *t*.

3.2.3 Indicators of convergence

By using input-output tables is also possible to calculate the so-called technological frontier, choosing the dominant technique in each industry. The set of available techniques is given by the techniques used in each region and time periods.

Following Fredholm and Zambelli (2009b) we compute two types of technological frontier. The *contemporary frontier* that is constructed from all the production techniques extracted from the regional input-output table in a given year and the *intertemporal frontier* that is given by the envelope formed by the full set of techniques available over time and across regions.

Accordingly, from each techniques $\begin{bmatrix} A^i_t \\ L^i_t \end{bmatrix}$ i=1,2,...N; t=1,2,…M there is a unique wage-profit frontier and the envelope of these frontiers is given by

$$w^{ENV\_INT}\left(r,\begin{bmatrix} A^i_t \\ L^i_t \end{bmatrix}\right) = \max\left\{w\left(r,\begin{bmatrix} A^i_t \\ L^i_t \end{bmatrix}\right)\right\} \text{ i=1,2,…N; t=1,2,…M,} \quad (3.11)$$

where *i* is the region and *t* is the time period.

By applying the above formula we obtain the intertemporal technological frontier, while the contemporary frontier is given by

$$w_t^{ENV\_CON}\left(r,\begin{bmatrix} A^i_{\bar{T}} \\ L^i_{\bar{T}} \end{bmatrix}\right) = \max\left\{w\left(r,\begin{bmatrix} A^i_{\bar{T}} \\ L^i_{\bar{T}} \end{bmatrix}\right)\right\} \text{ i=1,2,…N;} \quad (3.12)$$

where T now is a fixed time period.

To study technological progress and convergence among Italian regions we apply the Velupillai-Fredholm-Zambelli indexes (Fredholm and Zambelli, 2010b). In particular, we compute a region specific index (VFZ(1)) and a global index (VFZ(2)). Accordingly,

[4] In this context industry and subsystem can be labelled with the same letter.

$$VFZ(1)_{i,\bar{T}} = 1 - \frac{1}{R_{i,\bar{T}}} \sum_{r=0}^{R_{i,\bar{T}}} \left[w^{ENV\_INT}\left(r, \begin{bmatrix} A_{\bar{T}}^{i} \\ L_{\bar{T}}^{i} \end{bmatrix} \right) - w\left(r, \begin{bmatrix} A_{\bar{T}}^{i} \\ L_{\bar{T}}^{i} \end{bmatrix} \right) \right] \tag{3.13}$$

I=1,2,…N, t= $\bar{T}$

$$VFZ(2)_{i,t} = 1 - \frac{1}{R_t^{CON}} \sum_{r=0}^{R_{i,t}} \left[w^{ENV\_INT}\left(r, \begin{bmatrix} A_t^{i} \\ L_t^{i} \end{bmatrix} \right) - w_t^{ENV\_CON}\left(r, \begin{bmatrix} A_t^{i} \\ L_t^{i} \end{bmatrix} \right) \right] \tag{3.14}$$

i=1,2,…,N ; t=1,2,…,T

The VFZ(1) index is computed as one minus the vertical distance between the region specific wage-profit frontiers and the intertemporal technological frontier. The VFZ(2) index is computed as one minus the vertical distance between the contemporary frontiers and the intertemporal frontier. Consequently, the first index measures how far is the individual region form the theoretically maximum potential level of productivity, while the second index provides a measure of technological progress for the group of selected regions as a whole.

3.3 Source and preparation of the data and the choice of *numéraire*

The regions examined in this study are Trentino, Sicily, Piedmont, Tuscany, Campania, Emilia-Romagna, Veneto, and Lombardy. From an administrative viewpoint, Trentino is an autonomous province of Italy and it is one of the two provinces which make up Italy's region of Trentino-Alto Adige. However, the province enjoys a large degree of autonomy is some relevant economic sectors that can be classified as a region on its own. The selected regions can be qualified as a representative sample of the "economic diversity" of Italy.

The input-output tables for the above regions are made available by the Regional Institute Economic Planning of Tuscany (Irpet), apart from the input-output table for Trentino, which is made available by the Statistical Office of the province of Trento. The data were limited only to two years: 2001 and 2004. All the input-output tables are based on the ESA 95 – NACE Rev.1 classification with 30 industries.

Given that all the data reported in the tables are in current basic prices, industry deflators have been computed and used to deflate the table. The deflated table can be regarded as proxies for the physical flows among industries for the selected regions.

Labour data are taken from the Regional accounts available on the website of the National Institute of Statistics. Labour input is measured as number of workers weighted by an index of hours worked in each sector and year. Labour data are not available at the same level of industry detail of the input-output tables; therefore there is a lack of coincidence between the sectoral labour input and the sectoral input-output data. Hence, some labour input data were decomposed into smaller aggregate classes so to fit with the sectoral subdivision of the tables.

The 30 industries must be aggregated down to 27 in order to ensure non singular matrices for all periods. The list of industries and details on the aggregation are found in Appendix 3.A.

As a *numéraire*, we choose the vector of physical sectoral net products (total supply of the i^{th} commodity minus the sum of the i^{th} column in $\mathbf{A}$) in Trentino in 2004 calculated from a standard system with a zero profit rate and normalised with the hours worked.

Changes in the *numéraire* are able to significantly influence many of the indicators presented in the work, although chapter four of the thesis has shown that these changes do not seem to be so relevant for the indicators of aggregate productivity.

The technological frontier has, however, a particularly important property that makes it partially immune from the problem of change of *numéraire*. In fact, the points of change of the frontier are independent of the choice of *numéraire* (for the proof, see Pasinetti (1981, p.204-205). This means that the dominant techniques that make up the various segments of the technological frontier do not vary with the *numéraire*. However, the shape of the frontier does not remain unchanged, as the values of the wage rate between the switch points can change. From this, it follows that the greater is the number of switch points, the more uniform is their distribution on the frontier and the greater is the robustness of the frontier to changes of the *numéraire*.

3.4 Empirical analysis

3.4.1 Aggregate productivity and technological progress

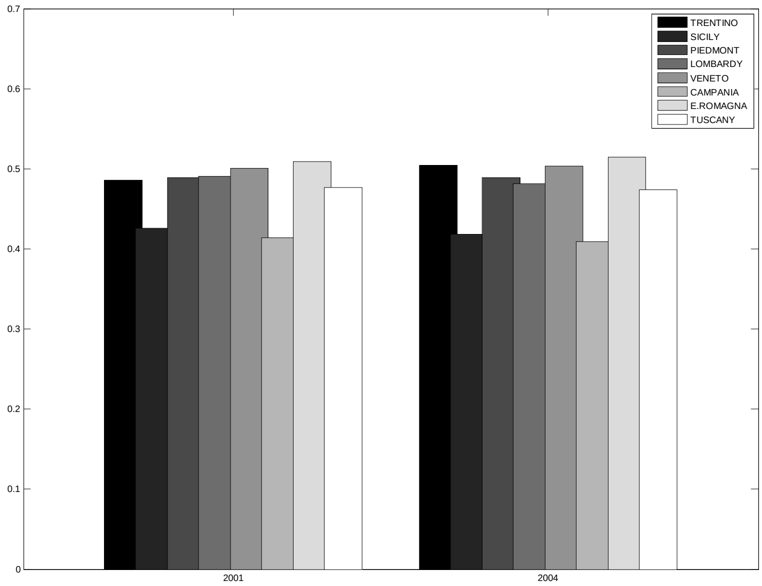
The first step of our investigation is to calculate aggregate measures of technological progress and labour productivity.

The first comment concerns the technological progress in 2001. The region with the highest level of technological progress is Emilia Romagna followed in order by Veneto, Lombardy, Piedmont, Trentino, Tuscany, Campania, and Sicily (Fig.3.1). Hence, the regions in the north show a higher level of technological progress than the regions in the centre-south. Moreover, the regions in the north-east show higher values than the region in the north-west, with the only exception of Trentino.

The situation is generally unchanged in 2004, with the only two exceptions of Trentino and Lombardy. Technological progress increases quite considerably in Trentino and this region moves up from fifth to third place in the regional ranking, while it decreases in Lombardy (Tab.3.1 - 3.2).

Emilia-Romagna can then be used as a point of reference for an estimate of the technological progress differential between regions[5]. The spreads of technological progress compared to Emilia-Romagna are in the order of 1.5-6.5% in the North, 6-8% in Tuscany, and 16-21% in the South. The differential increases in all regions from 2001 to 2004, with the exception of Trentino, where the reduction is more than two percentage points.

Fig. 3.1 Technological progress



Tab. 3.1 –Technological progress - values

Year/Region	Trentino	Sicily	Piedmont	Lombardy	Veneto	Campania	Emilia Romagna	Tuscany
2001	0.486	0.426	0.489	0.491	0.501	0.414	0.509	0.477
2004	0.504	0.419	0.488	0.482	0.504	0.409	0.515	0.474

Tab 3.2 –Ranking of regions by technological progress

Year/Region	Trentino	Sicily	Piedmont	Lombardy	Veneto	Campania	Emilia Romagna	Tuscany
2001	5	7	4	3	2	8	1	6
2004	3	7	4	5	2	8	1	6

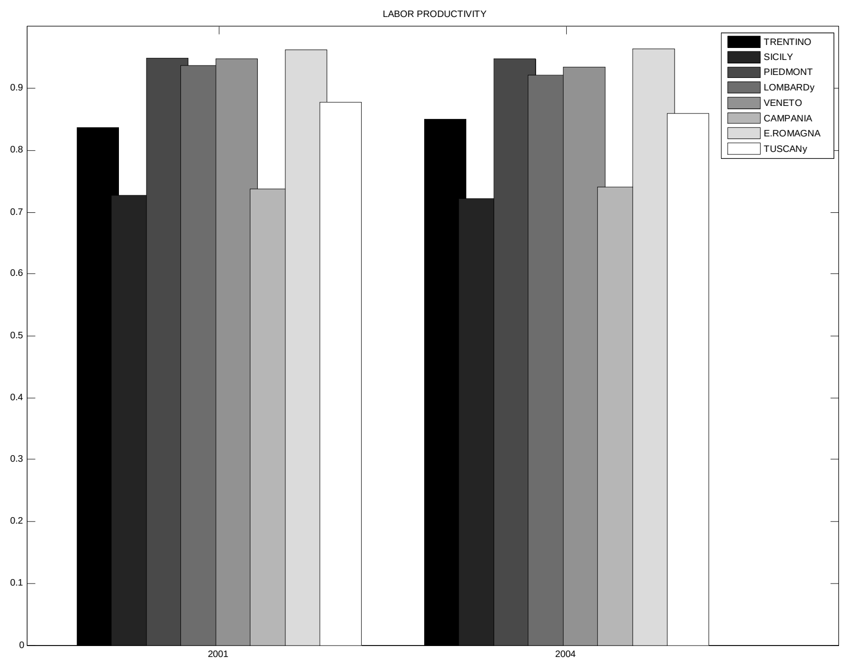
[5] Differentials in technological progress are calculated as follows: [(Value of Technological Progress in the region X - Value of Technological Progress in Emilia-Romagna)/ Value of Technological Progress in Emilia-Romagna]*100.

The indicator of labour productivity depicts a similar situation. Nevertheless, there are some differences which could be partly explained by the fact that the index of technological progress is characterized by a scale invariant property while the index of labour productivity is not.

In particular, it must be made the distinction between changes in the scale of production for the economy as a whole, and changes in the scale of production that is asymmetric across industries. The labour productivity indexes are only invariant in the first case because the numerator and denominator changes proportionally, while the invariance of the wage-profit frontier is by the non-substitution theorem guarantied in both cases (see appendix 3.B for a small example).

The non-scale invariant property of the labour productivity index could penalize the smallest regions with respect to the largest one, and this can explain to some extent the low relative position of Trentino in terms of labour productivity.

Fig.3.2 Labour productivity



Tab. 3.3 – Index of labour productivity - values

Year/Region	Trentino	Sicily	Piedmont	Lombardy	Veneto	Campania	Emilia Romagna	Tuscany
2001	0.837	0.728	0.949	0.937	0.948	0.737	0.962	0.877
2004	0.850	0.722	0.948	0.922	0.935	0.740	0.964	0.859

Tab 3.4 – Ranking of regions by labour productivity

Year/Region	Trentino	Sicily	Piedmont	Lombardy	Veneto	Campania	Emilia Romagna	Tuscany
2001	6	8	2	4	3	7	1	5
2004	6	8	2	4	3	7	1	5

The regional classification in terms of labour productivity remains unchanged from 2001 to 2004. Emilia-Romagna has the highest value followed by Piedmont, Veneto, Lombardy, Tuscany, Trentino, Campania, and Sicily. The productivity gap is 1.5-4.4% in the north, apart from the Trentino for which the differential is 11-13%. Again, the productivity gap is greater in Tuscany (8-11%), and especially in the southern regions (23-25%). The differential increases in all regions from 2001 to 2004, with the exception of Trentino and Campania, where the reduction is 1.2% and 0.2% respectively.

3.4.2 Sectoral Productivity

The first index that we propose is given by the ratio between the final output and the quantity of direct and indirect labour. This indicator is also known as the Gossling I index (Gossling, 1972, p.45).

The histograms below (see Fig 3.3) describe productivity in twenty-seven industries for the eight regions considered in 2001[6]. The values of sectoral productivity represented in the bar graphs are shown in Tables A.2.1 and A.2.2 in the statistical appendix and the following considerations emerge from a combined reading of figures and tables.

The first consideration relates to productivity differentials between sectors. The absolute values of this index are generally higher for manufacturing industries than for agriculture, construction, and services. However, there are some exceptions. Among services, Transport and Telecommunications, Finance and Insurance, and Computer and R&D show a productivity level in line with those of industrial industries.

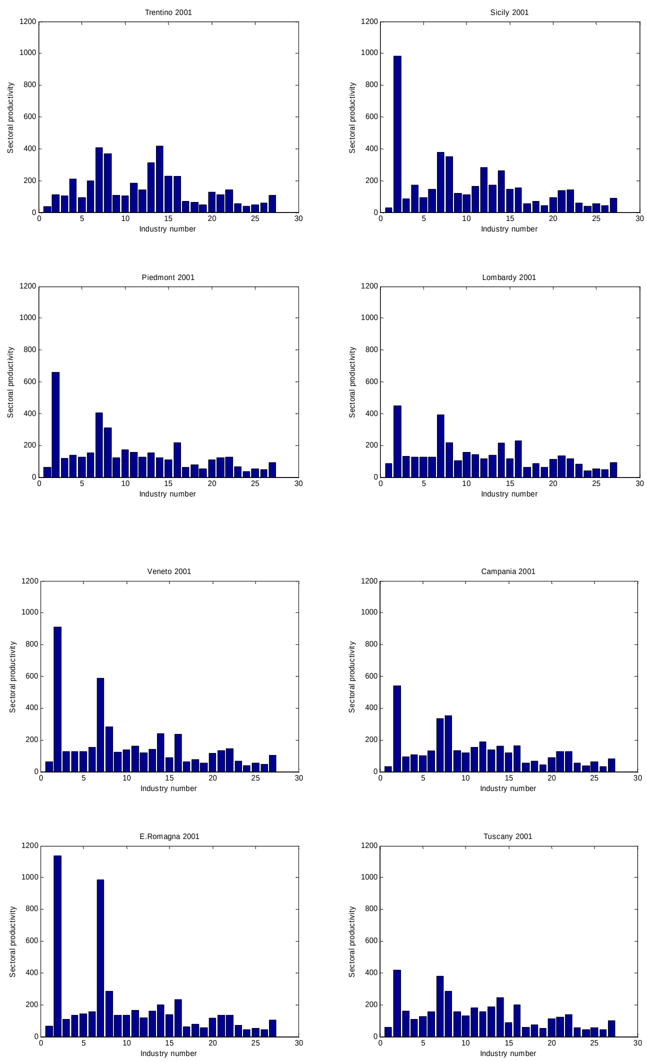
Moreover, some industries exhibit a productivity level much higher than those recorded in the other industries in nearly all the regions. These industries are: Extraction of minerals, Mfr. of Refined Petroleum, and Mfr. of Chemicals and

[6] The histograms for 2004 are not presented, because they are similar to those for 2001.

Man-Made fibres. Overall, we can argue that productivity in energy industries is relatively high as compared to other industries.

Sectors where productivity is relatively low in all regions are Agriculture and Fishing, Hotels and Restaurants, Education, Health Care Activities, and Other Service Activities.

Fig.3.3 Index of sectoral productivity based on physical quantities (ξ)



Industry number - legend

1- Agriculture and Fishing	10 - Mfr. of Other Non Metallic Mineral Products	19 - Hotels and Restaurants
2 - Extraction of minerals	11 - Mfr. And Processing of Basic Metals	20 - Transport, Post and Telecommunications
3 - Mfr. of Food, Beverages and Tobacco	12 - Mfr. of Machinery and Equipment n.e.c.	21 - Financial Intermediation, Insurance
4 - Mfr. of Textiles, Wearing Apparel, Leather	13 - Mfr. of Electrical and Optical Equipment	22 - Computer, Research and Development, Consultancy
5 - Mfr. of Wood and Wood Products	14 - Mfr. of Transport Equipment	23 - Public Administration
6 - Mfr. of Paper Products, Printing and Publishing	15 - Mfr. of Furniture, Mfr. n.e.c	24 - Education
7 - Mfr. of Refined Petroleum	16 - Electricity, Gas and Water Supply	25 - Health Care Activities Etc.
8 - Mfr. of Chemicals and Man-Made Fibers Etc.	17 - Construction	26 - Other Service Activities
9 - Mfr. of Rubber and Plastic Products	18 - Wholesale and Retail Trade	27 - Renting of Machinery and Equipment, Real Estate Activities

Further considerations concern the comparisons of sectoral productivity among regions. Table 3.5 shows the relative position of each region, in each sector, in the years 2001 and 2004 (see also Tables A.2.3 and A.2.4 in the statistical appendix). The relative position in 2001 is indicated by the value enclosed in square brackets, while the relative position in 2004 is indicated by the value enclosed in braces. For example, Lombardy is the region where the index of productivity for agriculture is the highest both in 2001 and 2004.

In order to summarize the results of table 3.5, in the last two rows the mean value and the standard deviation for each region are given. The lower is the mean value and the better is the position of the region within the selected sample. Standard deviation is instead an estimate of heterogeneity in productivity across sectors. It should be further emphasized that we are considering the mean and standard deviation of the relative position of each region, in each sector; this means that we consider neither the absolute value nor the weight of the individual industry on the total.

To some extent, there is a fair correspondence between the position of the regions in terms of aggregate productivity (see section 3.1 and tables 3.3, 3.4) and the position when considering the mean value of the table below. For example, even in this case the regions in the north in general and in the north-east in particular are in a position of advantage over the regions in the centre-south.

More interesting is the measure of standard deviation. In particular, there are three regions Trentino, Sicily and Lombardy where the standard deviation is much greater than in the other regions. This means that in these regions there are industries where productivity is relatively high and industries where productivity is relatively low. Conversely, the values of standard deviation of Piedmont, Veneto and Emilia-Romagna are fairly low, around 1.5, and indicate greater uniformity in the relative sectoral productivity.

Although we are aware of the need for further investigation and more precise measurements, one can say that the regions reviewed suggests two models. The "Trentino model" (present to some extent also in Lombardy and Sicily) is characterized by the joint presence of highly competitive industries and uncompetitive industries. Highly dynamic sectors coexist with backward sectors at least in relative terms. The "Emilia-Romagna model" (present to some extent also in Piedmont and Veneto) where the competitiveness of sectors, in relative terms, is quite similar.

The second index proposed is an indicator based on production prices. Essentially, while the previous index reduced heterogeneous commodities into a commensurable measure given by the quantity of direct and indirect labour, this index aggregates heterogeneous commodities by using prices of production as weights in the process of aggregation.

Table 3.6 is identical to Table 3.5 and shows the relative position of each region in each sector in 2001 (square brackets) and 2004 (braces). We examine again the last two rows of the table, which show the mean value and standard deviation of the relative positions of the regions in each economic sector. Considering the mean value one can say that the ordering of the regions is similar to that found previously. Emilia-Romagna and Veneto are the regions with the best average values, and Campania and Sicily are the regions with the worst average values.

However, the standard deviation of this indicator does not have a direct correspondence with the standard deviation of the previous one. Trentino is once again the area with the greatest variability and this means that it juxtaposes highly competitive sectors with low productive sectors. Nevertheless, some other regions show a level of variability different from that previously observed. For instance, this is the case of Sicily and Lombardy, which have reduced their variability.

Tab. 3.5 - Sectoral productivity index based on physical quantities - region's relative position in each sector in [2001] and {2004}

Industry	Trentino	Sicily	Piedmont	Lombardy	Veneto	Campania	Emilia Romagna	Tuscany
Agriculture and Fishing	[6] {7}	[8] {8}	[3] {4}	[1] {1}	[4] {3}	[7] {6}	[2] {2}	[5] {5}
Extraction of minerals	[8] {8}	[2] {3}	[4] {4}	[6] {6}	[3] {2}	[5] {5}	[1] {1}	[7] {7}
Mfr. of Food, Beverages and Tobacco	[6] {6}	[8] {8}	[4] 4}	[2] {3}	[3] {2}	[7] {7}	[5] {5}	[1] {1}
Mfr. of Textiles, Wearing Apparel, Leather	[1] {1}	[2] {2}	[3] {4}	[6] {6}	[5] {5}	[8] {7}	[4] {3}	[7] {8}
Mfr. of Wood and Wood Products	[7] {6}	[8] {8}	[5] {2}	[3] {3}	[4] {5}	[6] {7}	[1] {1}	[2] {4}
Mfr. of Paper Products, Printing and Publishing	[1] {1}	[6] {6}	[5] {4}	[8] {8}	[4] {3}	[7] {7}	[3] {2}	[2] {5}
Mfr. of Refined Petroleum	[3] {7}	[7] {8}	[4] {3}	[5] {4}	[2] {2}	[8] {6}	[1] {1}	[6] {5}
Mfr. of Chemicals and Man-Made Fibers Etc.	[1] {2}	[3] {6}	[4] {3}	[8] {8}	[7] {5}	[2] {1}	[5] {4}	[6] {7}
Mfr. of Rubber and Plastic Products	[7] {7}	[6] {6}	[4] {4}	[8] {8}	[5] {5}	[2] {3}	[3] {2}	[1] {1}
Mfr. of Other Non Metallic Mineral Products	[8] {8}	[7] {7}	[1] {1}	[2] {2}	[3] {3}	[6] {6}	[4] {4}	[5] {5}
Mfr. And Processing of Basic Metals	[1] {1}	[4] {6}	[6] {4}	[8] {8}	[5] {5}	[7] {7}	[3] {3}	[2] {2}
Mfr. of Machinery and Equipment n.e.c.	[4] {4}	[1] {1}	[5] {5}	[8] {8}	[7] {6}	[2] {2}	[6] {7}	[3] {3}
Mfr. of Electrical and Optical Equipment	[1] {1}	[3] {5}	[5] {4}	[8] {8}	[6] {6}	[7] {7}	[4] {3}	[2] {2}
Mfr. of Transport Equipment	[1] {1}	[2] {2}	[8] {8}	[5] {6}	[4] {4}	[7] {7}	[6] {5}	[3] {3}
Mfr. of Furniture, Mfr. n.e.c	[1] {1}	[2] {2}	[6] {6}	[5] {5}	[8] {8}	[4] {4}	[3] {3}	[7] {7}
Electricity, Gas and Water Supply	[4] {1}	[8] {8}	[5] {4}	[3] {5}	[1] {3}	[7] {7}	[2] {2}	[6] {6}
Construction	[1] {1}	[7] {7}	[4] {3}	[5] {4}	[3] {5}	[8] {8}	[2] {2}	[6] {6}
Wholesale and Retail Trade	[8] {8}	[6] {6}	[4] {4}	[1] {1}	[2] {2}	[7] {7}	[3] {3}	[5] {5}
Hotels and Restaurants	[6] {6}	[7] {7}	[5] {4}	[1] {1}	[3] {3}	[8] {8}	[2] {2}	[4] {5}
Transport, Post and Telecommunications	[1] {1}	[7] {8}	[6] {5}	[4] {4}	[3] {3}	[8] {7}	[2] {2}	[5] {6}
Financial Intermediation, Insurance	[8] {8}	[1] {4}	[7] {7}	[2] {1}	[3] {3}	[5] {6}	[4] {2}	[6] {5}
Computer, Research and Development, Consultancy	[2] {3}	[3] {1}	[6] {6}	[8] {8}	[1] {2}	[7] {7}	[5] {4}	[4] {5}
Public Administration	[7] {6}	[5] {5}	[3] {3}	[1] {1}	[4] {4}	[8] {8}	[2] {2}	[6] {7}
Education	[3] {3}	[6] {6}	[7] {7}	[4] {4}	[5] {5}	[8] {8}	[2] {2}	[1] {1}
Health Care Activities Etc.	[8] {8}	[4] {2}	[6] {7}	[5] {6}	[3] {4}	[1] {1}	[7] {5}	[2] {3}
Other Service Activities	[1] {1}	[7] {7}	[4] {4}	[2] {2}	[3] {3}	[8] {8}	[6] {6}	[5] {5}
Renting of Machinery and Equipment, Real Estate Activities	[1] {3}	[7] {7}	[6] {6}	[5] {5}	[2] {1}	[8] {8}	[3] {4}	[4] {2}
Mean value	**[3.9] {4.1}**	**[5.1] {5.4}**	**[4.8] {4.4}**	**[4.6] {4.7}**	**[3.8] {3.8}**	**[6.2] {6.1}**	**[3.4] {3.1}**	**[4.2] {4.5}**
Standard Deviation	**[2.9] {2.9}**	**[2.4] {2.4}**	**[1.5] {1.6}**	**[2.6] {2.6}**	**[1.8] {1.6}**	**[2.2] {2.1}**	**[1.7] {1.6}**	**[2.0] {2.0}**

Tab. 3.6 - Sectoral productivity index based on production prices - region's relative position in each sector in [2001] and {2004}

Industry	Trentino	Sicily	Piedmont	Lombardy	Veneto	Campania	Emilia Romagna	Tuscany
Agriculture and Fishing	[5] {5}	[7] {7}	[4] {3}	[3] {4}	[2] {2}	[8] {8}	[1] {1}	[6] {6}
Extraction of minerals	[5] {3}	[7] {6}	[4] {2}	[1] {7}	[3] {1}	[8] {8}	[6] {5}	[2] {4}
Mfr. of Food, Beverages and Tobacco	[5] {5}	[7] {6}	[1] {1}	[3] {4}	[4] {3}	[8] {8}	[2] {2}	[6] {7}
Mfr. of Textiles, Wearing Apparel, Leather	[6] {6}	[8] {8}	[3] {4}	[5] {5}	[1] {1}	[7] {7}	[4] {3}	[2] {2}
Mfr. of Wood and Wood Products	[1] {1}	[8] {8}	[4] {3}	[6] {6}	[3] {4}	[7] {7}	[5] {5}	[2] {2}
Mfr. of Paper Products, Printing and Publishing	[1] {1}	[8] {8}	[2] {3}	[6] {6}	[4] {4}	[7] {7}	[5] {5}	[3] {2}
Mfr. of Refined Petroleum	[4] {4}	[5] {2}	[3] {1}	[2] {7}	[6] {6}	[8] {8}	[7] {5}	[1] {3}
Mfr. of Chemicals and Man-Made Fibers Etc.	[7] {7}	[6] {3}	[3] {2}	[5] {6}	[1] {4}	[8] {8}	[4] {1}	[2] {5}
Mfr. of Rubber and Plastic Products	[6] {6}	[8] {4}	[2] {1}	[5] {5}	[1] {3}	[7] {7}	[4] {4}	[3] {2}
Mfr. of Other Non Metallic Mineral Products	[4] {4}	[7] {7}	[3] {2}	[6] {6}	[2] {3}	[8] {8}	[1] {1}	[5] {5}
Mfr. And Processing of Basic Metals	[3] {5}	[8] {8}	[5] {3}	[1] {1}	[2] {2}	[7] {7}	[4] {4}	[6] {6}
Mfr. of Machinery and Equipment n.e.c.	[6] {5}	[7] {8}	[3] {1}	[4] {4}	[2] {3}	[8] {7}	[1] {2}	[5] {6}
Mfr. of Electrical and Optical Equipment	[6] {6}	[7] {8}	[2] {1}	[4] {3}	[3] {4}	[8] {7}	[1] {2}	[5] {5}
Mfr. of Transport Equipment	[8] {8}	[7] {7}	[2] {1}	[5] {4}	[3] {3}	[6] {6}	[1] {2}	[4] {5}
Mfr. of Furniture, Mfr. n.e.c	[3] {3}	[5] {6}	[6] {5}	[2] {2}	[4] {4}	[8] {8}	[1] {1}	[7] {7}
Electricity, Gas and Water Supply	[1] {1}	[5] {6}	[7] {5}	[6] {7}	[3] {3}	[8] {8}	[2] {2}	[4] {4}
Construction	[3] {3}	[7] {8}	[4] {2}	[6] {6}	[2] {4}	[8] {7}	[1] {1}	[5] {5}
Wholesale and Retail Trade	[8] {8}	[7] {7}	[4] {3}	[3] {4}	[2] {2}	[6] {6}	[1] {1}	[5] {5}
Hotels and Restaurants	[5] {4}	[6] {5}	[3] {3}	[4] {6}	[2] {2}	[8] {8}	[1] {1}	[7] {7}
Transport, Post and Telecommunications	[8] {7}	[7] {8}	[2] {1}	[1] {2}	[4] {3}	[6] {5}	[3] {4}	[5] {6}
Financial Intermediation, Insurance	[8] {8}	[6] {6}	[3] {3}	[4] {4}	[1] {2}	[7] {7}	[2] {1}	[5] {5}
Computer, Research and Development, Consultancy	[7] {6}	[6] {5}	[4] {4}	[5] {7}	[2] {2}	[8] {8}	[1] {1}	[3] {3}
Public Administration	[1] {1}	[7] {8}	[4] {3}	[5] {5}	[3] {2}	[8] {7}	[2] {4}	[6] {6}
Education	[1] {2}	[7] {7}	[4] {4}	[5] {6}	[3] {3}	[8] {8}	[2] {1}	[6] {5}
Health Care Activities Etc.	[8] {8}	[7] {4}	[3] {5}	[4] {7}	[1] {3}	[6] {1}	[2] {2}	[5] {6}
Other Service Activities	[1] {1}	[8] {8}	[4] {3}	[2] {2}	[3] {4}	[7] {7}	[5] {5}	[6] {6}
Renting of Machinery and Equipment, Real Estate Activities	[1] {1}	[7] {6}	[5] {5}	[6] {7}	[3] {3}	[8] {8}	[2] {2}	[4] {4}
Mean value	**[4.5] {4.4}**	**[6.9] {6.4}**	**[3.5] {2.7}**	**[4.0] {4.9}**	**[2.6] {3.0}**	**[7.4] {7.1}**	**[2.6] {2.5}**	**[4.4] {4.8}**
Standard Deviation	**[2.6] {2.6}**	**[0.9] {1.7}**	**[1.3] {1.4}**	**[1.7] {1.8}**	**[1.2] {1.1}**	**[0.8] {1.4}**	**[1.8] {1.6}**	**[1.7] {1.6}**

In concluding this section we test the degree of association between the two indices proposed here. Obviously, there are different possibilities to verify the correspondence between the two measures; given the absolute values of productivity we can calculate the overall correlation, the correlation for each region and the correlation for each sector.

In what follows, we are interested only in the first two types of correlation and we use the following Bravais-Pearson linear correlation coefficient to estimate the quantitative differences between the two indexes.

$$Corr(X,Y) = \frac{\sum (x-\bar{x})(y-\bar{y})}{\sqrt{\sum (x-\bar{x})^2 \sum (y-\bar{y})^2}} \tag{3.15}$$

Its values range between minus one and plus one and the following points are the accepted guidelines for interpreting the correlation coefficient:

- 0 indicates no linear relationship.
- +1 indicates a perfect positive linear relationship: as one variable increases in its values, the other variable also increases in its values via an exact linear rule.
- -1 indicates a perfect negative linear relationship: as one variable increases in its values, the other variable decreases in its values via an exact linear rule.
- Values between 0 and 0.3 (0 and -0.3) indicate a weak positive (negative) linear relationship.
- Values between 0.3 and 0.7 (0.3 and -0.7) indicate a moderate positive (negative) linear relationship.
- Values between 0.7 and 1.0 (-0.7 and -1.0) indicate a strong positive (negative) linear relationship.

Tab. 3.7 - Bravais-Pearson linear correlation coefficient between the two indexes of sectoral productivity

	Trentino	Sicily	Piedmont	Lombardy	Veneto	Campania	Emilia Romagna	Tuscany	ALL
Values 2001	0,62	0,46	0,45	0,64	0,39	0,55	0,25	0,74	0,43
Values 2004	0,62	0,45	0,55	0,53	0,46	0,59	0,27	0,72	0,43

The correlations are shown in Table 3.7. The two measures of sectoral productivity are moderately positively correlated when we compare all the regions examined. The range of the region specific correlation coefficients is instead rather wide, ranging form 0.25 for Emilia Romagna in 2001 to 0.74 for Tuscany in 2001. However, the coefficients are always positive and bigger than 0.3, with the only exception of Emilia-Romagna. This indicates that overall the two measures of labour productivity yield similar results.

3.4.3 Convergence

This section examines the indicators for regional specific technological progress and convergence. First, we consider the contemporary technological frontiers for the years 2001 and 2004. The contemporary frontiers indicate a sort of maximum productive efficiency in each of two years. It is recalled that in a given year and for each level of the rate of profit, the frontier is obtained as combination of most productive techniques in each industry.

Thus, in order to describe in detail the frontier, we list the names of regions that have the most productive technique in each industry. When the profit rate varies from zero to its maximum value the so-called switch points can occur. At a switch point, one and only one technique is replaced by another more productive technique, so the change can only occur in one sector.

Tables 3.7-3.8 propose the contemporary frontiers for 2001 and 2004. The first row indicates the value of the rate of profit when a switch occurs, while the other rows indicate the name of the regions that have the dominant technique in each industry. In each column, except the first, the name of the region in industry where switch occurs is in bold type.

There are two items of particular interest in the two technological frontiers. First, the large number of dominant techniques found in Trentino. In 2001, Trentino had 12 dominant techniques out of 27 for profit rate values between 0 and 0.366. This number then gradually decreases for higher values of the rate of profit, but remains noteworthy. Secondly, a considerable number of dominant techniques have been found in Sicily and Campania, though these two regions were the worst in terms of aggregate productivity. Alternatively, one could also stress the low number of dominant techniques found in Emilia-Romagna and Veneto, despite these two regions were the best in terms of aggregate productivity.

A comparison between the two tables shows that there were only subtle changes from 2001 to 2004. Practically, this means that if a region owned the dominant technique in an industry in 2001, it owned the dominant technique in the same industry in 2004.

We now observe the values of the profit rate at every point of change. What emerges is that the switch points occur at very low or very high profit rates. In practice, the contemporary technological frontier remains unchanged in a wide range of profit rate values, while for extreme values of the profit rate some switch points occur.

Table 3.10 describes the intertemporal frontier. The table shows the region and the year to which corresponds the dominant technique in each industry and for each profit rate value. The points made above regarding the contemporary frontiers apply here also, as Table 3.10 emphasizes again the importance of Trentino, Sicily and Campania.

Rather, it is important to note the large number of dominant technique owned in 2001. This means that there was not a rise in productivity in many industries between 2001 and 2004, so that many techniques used in 2001 are even better compared to those used three years later. Finally, note that half of the switch points now take place between the techniques used by the same region, but in different years.

Tab. 3.8 – Contemporary frontier 2001. (The switch points are shown in bold).

Industry / Rate of Profit	0.338	0.644	2.322	2.51	2.574
Agriculture and Fishing	Tuscany	Tuscany	Tuscany	Tuscany	Tuscany
Extraction of minerals	Emilia-Romagna	Emilia-Romagna	Emilia-Romagna	Emilia-Romagna	Emilia-Romagna
Mfr. of Food, Beverages and Tobacco	Tuscany	Tuscany	Tuscany	Tuscany	Tuscany
Mfr. of Textiles, Wearing Apparel, Leather	Trentino	Trentino	**Sicily**	Sicily	Sicily
Mfr. of Wood and Wood Products	Emilia-Romagna	Emilia-Romagna	Emilia-Romagna	Emilia-Romagna	Emilia-Romagna
Mfr. of Paper Products, Printing and Publishing	Sicily	Sicily	Sicily	Sicily	Sicily
Mfr. of Refined Petroleum	Trentino	Trentino	Trentino	Trentino	Trentino
Mfr. of Chemicals and Man-Made Fibers Etc.	Trentino	**Campania**	Campania	Campania	Campania
Mfr. of Rubber and Plastic Products	Campania	Campania	Campania	Campania	Campania
Mfr. of Other Non Metallic Mineral Products	Piedmont	Piedmont	Piedmont	Piedmont	Piedmont
Mfr. And Processing of Basic Metals	Sicily	Sicily	Sicily	Sicily	Sicily
Mfr. of Machinery and Equipment n.e.c.	Sicily	Sicily	Sicily	Sicily	Sicily
Mfr. of Electrical and Optical Equipment	Trentino	Trentino	Trentino	Trentino	Trentino
Mfr. of Transport Equipment	Trentino	Trentino	Trentino	Trentino	Trentino
Mfr. of Furniture, Mfr. n.e.c	Trentino	Trentino	Trentino	Trentino	Trentino
Electricity, Gas and Water Supply	Piedmont	Piedmont	Piedmont	Piedmont	Piedmont
Construction	Trentino	Trentino	Trentino	Trentino	Trentino
Wholesale and Retail Trade	Trentino	Trentino	Trentino	Trentino	Trentino
Hotels and Restaurants	Trentino	Trentino	Trentino	**Lombardy**	Lombardy
Transport, Post and Telecommunications	Trentino	Trentino	Trentino	Trentino	Trentino
Financial Intermediation, Insurance	Trentino	Trentino	Trentino	Trentino	Trentino
Computer, Research and Development, Consultancy	Campania	Campania	Campania	Campania	Campania
Public Administration	Lombardy	Lombardy	Lombardy	Lombardy	Lombardy
Education	Sicily	Sicily	Sicily	Sicily	**Campania**
Health Care Activities Etc.	Trentino	Trentino	Trentino	Trentino	Trentino
Other Service Activities	Sicily	Sicily	Sicily	Sicily	Sicily
Renting of Machinery and Equipment, Real Estate Activities	Piedmont	Piedmont	Piedmont	Piedmont	Piedmont

Tab. 3.9 – Contemporary frontier 2004. (The switch points are shown in bold).

Industry / Rate of Profit	**0.036**	**0.11**	**0.262**	**2.318**	**2.5686**
Agriculture and Fishing	Tuscany	Tuscany	Tuscany	Tuscany	Tuscany
Extraction of minerals	Emilia-Romagna	Emilia-Romagna	Emilia-Romagna	Emilia-Romagna	Emilia-Romagna
Mfr. of Food, Beverages and Tobacco	Tuscany	Tuscany	Tuscany	Tuscany	Tuscany
Mfr. of Textiles, Wearing Apparel, Leather	Trentino	Trentino	**Sicily**	Sicily	Sicily
Mfr. of Wood and Wood Products	Emilia-Romagna	Emilia-Romagna	Emilia-Romagna	Emilia-Romagna	Emilia-Romagna
Mfr. of Paper Products, Printing and Publishing	Trentino	Trentino	Trentino	**Sicily**	Sicily
Mfr. of Refined Petroleum	Trentino	Trentino	Trentino	Trentino	Trentino
Mfr. of Chemicals and Man-Made Fibers Etc.	Campania	Campania	Campania	Campania	Campania
Mfr. of Rubber and Plastic Products	Campania	Campania	Campania	Campania	Campania
Mfr. of Other Non Metallic Mineral Products	Campania	Campania	Campania	Campania	Campania
Mfr. And Processing of Basic Metals	Sicily	Sicily	Sicily	Sicily	Sicily
Mfr. of Machinery and Equipment n.e.c.	Sicily	Sicily	Sicily	Sicily	Sicily
Mfr. of Electrical and Optical Equipment	Trentino	Trentino	Trentino	Trentino	Trentino
Mfr. of Transport Equipment	Trentino	Trentino	Trentino	Trentino	Trentino
Mfr. of Furniture, Mfr. n.e.c	Trentino	Trentino	Trentino	Trentino	Trentino
Electricity, Gas and Water Supply	Trentino	**Campania**	Campania	Campania	Campania
Construction	Trentino	Trentino	Trentino	Trentino	Trentino
Wholesale and Retail Trade	Trentino	Trentino	Trentino	Trentino	Trentino
Hotels and Restaurants	Trentino	Trentino	Trentino	Trentino	**Lombardy**
Transport, Post and Telecommunications	Trentino	Trentino	Trentino	Trentino	Trentino
Financial Intermediation, Insurance	Trentino	Trentino	Trentino	Trentino	Trentino
Computer, Research and Development, Consultancy	Campania	Campania	Campania	Campania	Campania
Public Administration	Lombardy	Lombardy	Lombardy	Lombardy	Lombardy
Education	Campania	Campania	Campania	Campania	Campania
Health Care Activities Etc.	Trentino	Trentino	Trentino	Trentino	Trentino
Other Service Activities	Sicily	Sicily	Sicily	Sicily	Sicily
Renting of Machinery and Equipment, Real Estate Activities	Piedmont	Piedmont	Piedmont	Piedmont	Piedmont

Tab. 3.10 – Intertemporal frontier. (The switch points are shown in bold).

Industry / Rate of Profit	0.026	0.084	0.174	0.316	0.51	1.042	1.406	1.412	1.938	2.118	2.224	2.422	2.6821
Agriculture and Fishing	Tuscany (2004)	Tuscany (2004)	Tuscany (2004)	Tuscany (2004)	Tuscany (2004)	Tuscany (2004)	Tuscany (2004)	Tuscany (2004)	Tuscany (2004)	Tuscany (2004)	Tuscany (2004)	Tuscany (2004)	Tuscany (2004)
Extraction of minerals	E.Romagna (2004)	E.Romagna (2004)	E.Romagna (2004)	E.Romagna (2004)	E.Romagna (2004)	E.Romagna (2004)	E.Romagna (2004)	E.Romagna (2004)	E.Romagna (2004)	E.Romagna (2004)	E.Romagna (2004)	E.Romagna (2004)	E.Romagna (2004)
Mfr. of Food, Beverages and Tobacco	Tuscany (2001)	Tuscany (2001)	Tuscany (2001)	Tuscany (2001)	Tuscany (2001)	Tuscany (2001)	Tuscany (2001)	Tuscany (2001)	Tuscany (2001)	Tuscany (2001)	Tuscany (2001)	Tuscany (2001)	Tuscany (2001)
Mfr. of Textiles, Wearing Apparel, Leather	Trentino (2001)	Trentino (2001)	Trentino (2001)	Trentino (2001)	Trentino (2001)	**Sicily (2004)**	Sicily (2004)	Sicily (2004)	Sicily (2004)	Sicily (2004)	Sicily (2004)	Sicily (2004)	Sicily (2004)
Mfr. of Wood and Wood Products	E.Romagna (2004)	E.Romagna (2004)	E.Romagna (2004)	E.Romagna (2004)	E.Romagna (2004)	E.Romagna (2004)	E.Romagna (2004)	E.Romagna (2004)	E.Romagna (2004)	E.Romagna (2004)	E.Romagna (2004)	E.Romagna (2004)	E.Romagna (2004)
Mfr. of Paper Products, Printing and Publishing	Sicily (2001)	Sicily (2001)	Sicily (2001)	Sicily (2001)	Sicily (2001)	Sicily (2001)	Sicily (2001)	Sicily (2001)	Sicily (2001)	Sicily (2001)	Sicily (2001)	Sicily (2001)	Sicily (2001)
Mfr. of Refined Petroleum	Trentino (2001)	Trentino (2001)	Trentino (2001)	Trentino (2001)	Trentino (2001)	Trentino (2001)	Trentino (2001)	Trentino (2001)	Trentino (2001)	Trentino (2001)	Trentino (2001)	Trentino (2001)	Trentino (2001)
Mfr. of Chemicals and Man-Made Fibers Etc.	Trentino (2001)	Trentino (2001)	Trentino (2001)	Trentino (2001)	**Campania (2001)**	Campania (2001)	Campania (2001)	Campania (2001)	Campania (2001)	Campania (2001)	Campania (2001)	Campania (2001)	Campania (2001)
Mfr. of Rubber and Plastic Products	Campania (2004)	Campania (2004)	Campania (2004)	Campania (2004)	Campania (2004)	Campania (2004)	Campania (2004)	Campania (2004)	Campania (2004)	Campania (2004)	Campania (2004)	Campania (2004)	Campania (2004)
Mfr. of Other Non Metallic Mineral Products	Piedmont (2001)	Piedmont (2001)	Piedmont (2001)	Piedmont (2001)	Piedmont (2001)	Piedmont (2001)	Piedmont (2001)	Piedmont (2001)	Piedmont (2001)	Piedmont (2001)	Piedmont (2001)	**Campania (2004)**	Campania (2004)
Mfr. And Processing of Basic Metals	Sicily (2001)	Sicily (2001)	Sicily (2001)	Sicily (2001)	Sicily (2001)	Sicily (2001)	**Sicily (2004)**	Sicily (2004)	Sicily (2004)	Sicily (2004)	Sicily (2004)	Sicily (2004)	Sicily (2004)
Mfr. of Machinery and Equipment n.e.c.	Sicily (2001)	Sicily (2001)	Sicily(2001)	Sicily (2001)	Sicily (2001)	Sicily (2001)	Sicily (2001)	**Sicily (2004)**	Sicily (2004)	Sicily (2004)	Sicily (2004)	Sicily (2004)	Sicily (2004)
Mfr. of Electrical and Optical Equipment	Trentino (2001)	Trentino (2001)	Trentino (2001)	**Trentino (2004)**	Trentino (2004)	Trentino (2004)	Trentino (2004)	Trentino (2004)	Trentino (2004)	Trentino (2004)	Trentino (2004)	Trentino (2004)	Trentino (2004)
Mfr. of Transport Equipment	Trentino (2001)	Trentino (2001)	**Trentino (2004)**	Trentino (2004)	Trentino (2004)	Trentino (2004)	Trentino (2004)	Trentino (2004)	Trentino (2004)	Trentino (2004)	Trentino (2004)	Trentino (2004)	Trentino (2004)
Mfr. of Furniture, Mfr. n.e.c	Trentino (2004)	Trentino (2004)	Trentino (2004)	Trentino (2004)	Trentino (2004)	Trentino (2004)	Trentino (2004)	Trentino (2004)	Trentino (2004)	Trentino (2004)	Trentino (2004)	Trentino (2004)	Trentino (2004)
Electricity, Gas and Water Supply	Trentino (2004)	**Campania (2004)**	Campania (2004)	Campania (2004)	Campania (2004)	Campania (2004)	Campania (2004)	Campania (2004)	Campania (2004)	Campania (2004)	Campania (2004)	Campania (2004)	Campania (2004)
Construction	Trentino (2004)	Trentino (2004)	Trentino (2004)	Trentino (2004)	Trentino (2004)	Trentino (2004)	Trentino (2004)	Trentino (2004)	**Trentino (2001)**	Trentino (2001)	Trentino (2001)	Trentino (2001)	Trentino (2001)
Wholesale and Retail Trade	Trentino (2004)	Trentino (2004)	Trentino (2004)	Trentino (2004)	Trentino (2004)	Trentino (2004)	Trentino (2004)	Trentino (2004)	Trentino (2004)	Trentino (2004)	Trentino (2004)	Trentino (2004)	Trentino (2004)
Hotels and Restaurants	Trentino (2001)	Trentino (2001)	Trentino (2001)	Trentino (2001)	Trentino (2001)	Trentino (2001)	Trentino (2001)	Trentino (2001)	Trentino (2001)	Trentino (2001)	Trentino (2001)	Trentino (2001)	**Lombardy (2001)**
Transport, Post and Telecommunications	Trentino (2001)	Trentino (2001)	Trentino (2001)	Trentino (2001)	Trentino (2001)	Trentino (2001)	Trentino (2001)	Trentino (2001)	Trentino (2001)	Trentino (2001)	Trentino (2001)	Trentino (2001)	Trentino (2001)
Financial Intermediation, Insurance	Trentino (2001)	Trentino (2001)	Trentino (2001)	Trentino (2001)	Trentino (2001)	Trentino (2001)	Trentino (2001)	Trentino (2001)	Trentino (2001)	Trentino (2001)	Trentino (2001)	Trentino (2001)	Trentino (2001)
Computer, Research and Development, Consultancy	Campania (2001)	Campania (2001)	Campania (2001)	Campania (2001)	Campania (2001)	Campania (2001)	Campania (2001)	Campania (2001)	Campania (2001)	Campania (2001)	**Campania (2004)**	Campania (2004)	Campania (2004)
Public Administration	Lombardy (2004)	Lombardy (2004)	Lombardy (2004)	Lombardy (2004)	Lombardy (2004)	Lombardy (2004)	Lombardy (2004)	Lombardy (2004)	Lombardy (2004)	Lombardy (2004)	Lombardy (2004)	Lombardy (2004)	Lombardy (2004)
Education	Sicily (2001)	Sicily(2001)	Sicily (2001)	Sicily (2001)	Sicily (2001)	Sicily (2001)	Sicily (2001)	Sicily (2001)	Sicily (2001)	**Campania (2004)**	Campania (2004)	Campania (2004)	Campania (2004)
Health Care Activities Etc.	Trentino (2001)	Trentino (2001)	Trentino (2001)	Trentino (2001)	Trentino (2001)	Trentino (2001)	Trentino (2001)	Trentino (2001)	Trentino (2001)	Trentino (2001)	Trentino (2001)	Trentino (2001)	Trentino (2001)
Other Service Activities	Sicily (2004)	Sicily (2004)	Sicily(2004)	Sicily (2004)	Sicily (2004)	Sicily (2004)	Sicily (2004)	Sicily (2004)	Sicily (2004)	Sicily (2004)	Sicily (2004)	Sicily (2004)	Sicily (2004)
Renting of Machinery	Piedmont (2001)	Piedmont (2001)	Piedmont (2001)	Piedmont (2001)	Piedmont (2001)	Piedmont (2001)	Piedmont (2001)	Piedmont (2001)	Piedmont (2001)	Piedmont (2001)	Piedmont (2001)	Piedmont (2001)	Piedmont (2001)

A comprehensive examination of the three frontiers highlights two important aspects.

The first concerns the number of switch points and their distribution on the frontier. This element is closely linked with the problem of the choice of *numéraire* (see Section 3). The switch points are less than those found by Fredholm and Zambelli (2009b), both in the contemporary and intertemporal frontiers. As the two authors note, the number of points increases with the number of available techniques and this explain the low number of switch points in this study. Their distribution on the contemporary frontier is skewed, while the distribution on the intertemporal frontier is much more uniform. This means that the intertemporal frontier is more robust to changes in *numéraire* than the contemporary frontiers.

The second aspect is that no region at a single point in time dominates the entire technological frontier. Hence, all regions could potentially gain through greater integration.

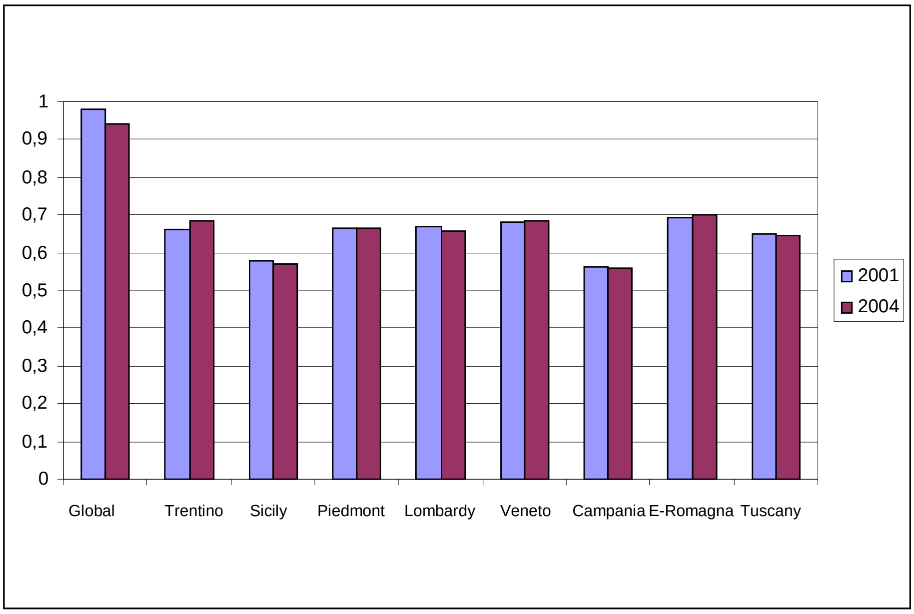
The Velupillai-Fredholm-Zambelli indices computed for the eight regions are collected in the following Figure 3.4, while the numerical values are collected in Table A.2.5 in the statistical appendix. Bars called *Global* measure the distance of the contemporary frontiers from the intertemporal frontier, which represents the maximum attainable value

The value of the index for the intertemporal frontier is equal to one and that is the maximum attainable value. Obviously, the difference between the index value for the specific region and one measures the distance between the region specific technological progress and the intertemporal maximum.

At the global level, we can say that the contemporary frontiers are very close to the maximum attainable value, but this is quite normal since the intertemporal frontier is constructed by taking only two points in time. Rather, it is important to emphasize that there is a slight process of divergence from 2001 to 2004. It therefore appears that there has been a phenomenon of technological regression between 2001 and 2004 and this coincides with the global economic crisis.

The index for the individual regions varies from 0.55 to 0.7 and is fairly stable in the two years examined. This range is rather limited and so, although the productivity gap between regions is not negligible, their position relative to the intertemporal maximum is fairly homogeneous. Note that there is a good correspondence between this index and the index of technological progress set out in paragraph four. This means that the relative position of the regions in the two years is the same as given in Table 3.2.

Fig.3.4 The Velupillai-Fredholm-Zambelli index



3.5 The behaviour of production prices

One of the characteristics of the indicators previously proposed is to be distribution free measures of productivity. It is therefore not fixed *a priori* any specific distribution of the net product between wages and profits, but it is considered all possible distributions in the spectrum where the rate of profit ranges from zero to its maximum value. Then all economically significant combinations.

The variation in prices to changes in the rate of profit is determined by a rather complex relationship (see Pasinetti, 1981 p.105) that can be divided into two main components: the capital intensity effect and the price effect. The capital intensity effect is always positive for the commodities that are produced with technical processes more capital-intensive than the one used by the commodity taken as *numéraire*. Conversely, the capital intensity effect is always negative for the commodities that are produced with technical processes less capital-intensive than the one used by the commodity taken as *numéraire*.

The price effect is rather unpredictable, since it depends on the variation of all prices of the system. The price effect can then accentuate, attenuate, or reverse the effect of capital intensity, and this depends on whether it happens in the same direction or in opposite direction.

In the accompanying statistical appendix we illustrate the production price curve for each region and each sector in 2001 and 2004 (see figA.2.1-A.2.54). A simple visual inspection of the figures suggests that the curves for a specific industry are often similar among regions. For instance, the curves for Agriculture are always nearly linear and they have a negative slope (see fig.A.2.1 and fig.A.2.2), whereas the curves for the Industry of Food Manufacturing (see fig.A.2.5 and fig.A.2.6) are convex and they have a positive slope for low rates of interest and a negative slope for high rate of interest. The number of industries in which these similarities occur is considerable, so it cannot be by chance.

The aim of this investigation is twofold: first to assess whether there is a common pattern of price movement among industries and regions, and second to detect cases on non-monotonic behaviour of prices when the rate of profit varies form zero to its maximum value.

To perform our inspection we calculate prices of production for each region and industry using [3.3]. Obviously, there are infinite numbers rates of profit and infinite numbers of associated vectors of prices of production. Therefore, we identify one hundred equally spaced points within the interval between zero and the maximum profit rate for each region and time period and subsequently, we calculate one hundred vectors of prices of production associated with the respective profit rate. Finally, we analyze these sequences of prices of production and we classify the pattern into the following four categories (see also table A.2.6 and A.2.7 in the statistical annex).

a) Monotonic-decreasing (Price of production decreases when the rate of profit increases).
b) Monotonic-increasing (Price of production decreases when the rate of profit increases).
c) Parabolic (Price of production first decreases and then increases when the rate of profit increases).
d) Reverse parabolic (Price of production first increases and then decreases when the rate of profit increases).

Subsequently, we classify the twenty-seven industries into two categories: uniform and mixed. An industry is said uniform when the category of price of production pattern is the same in at least six regions; an industry is said mixed otherwise[1].

Following this classification, we identify twenty one uniform industries and five mixed industries in 2001 and twenty four uniform industries and three mixed industries in 2004. We suspect that the proportion of mixed industries could diminish using a finer level of aggregation. In fact, three of the five mixed industries in 2001 are obtained by aggregation of smaller industries of very different kind.

This result suggests that the industry specific technology of the Italian regions (at least for those examined here) is very much alike. In fact, the movement of prices of production to changes of the profit rate for a specific industry is determined by the technology of that specific industry, and the technology of the supporting industries. Consequently, similar pattern of prices of production among different regions in some ways indicates that the industries adopt a similar

[1] We are aware that in setting this threshold there is a certain degree of arbitrariness. However, this seems the most appropriate threshold for classifying prices.

technology across the country. Whether this result could be replicated in other countries, needs to be proven by further investigation.

The second objective of this investigation is to assess the proportion of cases where the prices of production have a non-monotonic pattern (see tab.A.2.6 and A.2.7 in the statistical appendix).We record 31 cases out of 316 (9.8%) of not monotonic price behaviour both in 2001 and 2004. In particular, we note 3 cases of parabolic price behaviour and 28 cases of reverse parabolic price behaviour in both years. The region with the largest number of occurrences is Trentino (14 in 2001, 13 in 2004), while other regions do not exceed the 4 cases. The industries with the largest number of occurrences are Food, Beverages and Tobacco (8 in both 2001 and 2004) and Computer, Research and Development, Consultancy (5 in 2001, 7 in 2004).

3.6 Conclusions

This chapter has examined productivity and technological progress in a selected sample of Italian regions in 2001 and 2004 using a set of recently developed indicators based on input-output tables. Some problems of data availability limit the scope of this study, but overall, however, we can say that this work can provide a fairly comprehensive framework, although in a limited time period.

The analyses were divided into four main groups: the study of aggregate productivity and technological progress, the study of sectoral productivity, the study of technological frontiers and indices of convergence, and the study of prices of production.

Before presenting the conclusions, a brief description of the strengths and the weaknesses of this empirical work are necessary. The dataset is indeed the most objectionable element; it is in fact a very limited dataset, which cover less than half of the Italian regions and lacking of the data of Italy as a whole.

Unfortunately, the input-output tables of the Italian regions are not freely available and the availability of a subset of them represents a positive start. Furthermore the Italian table is not comparable with the regional tables and can not be used. The absence of the national figure is perhaps the biggest gap since we lose a necessary point of reference.

However the sample of selected regions is quite representative, since there are regions in all the macro areas of Italy and there are also the most important regions in dimensional terms. This work is also original, not only for the methodology used, but also for the same content: it constitutes one of the first empirical works which measure and compare productivity in the Italian regions.

Another criticism could be that this study uses many different measures of productivity and then the results are not uniquely interpretable. In this regard we consider the differences in results more as a strength that a weakness of this work. It is not necessary to reiterate that productivity is a concept difficult to measure and therefore it is reasonable that different indicators do not lead to the same result but rather they identify common trends.

Moreover, this empirical study was also intended as a test of a set of alternative indicators for measuring productivity that has been recently developed. Therefore, the purpose of measuring productivity in the Italian regions is mixed with the objective of evaluating the practical aspect of this new set of indicators.

We return now to the questions posed in the introduction and we try to give an answer to each of them. The first question was whether in Italy there is productivity differential between regions and the answer to this question is certainly yes. The biggest difference is found between the regions in the centre-north and the regions in the south, but less significant differences are found between regions in the north and regions in the centre and between regions in the north-west and regions in the north-east.

However, even within rather homogeneous geographic areas, there are differences. For example, productivity in Trentino is not in line with that of other North-East regions. Thus, the data at our disposal are sufficient to say that in Italy there is a gap in productivity between regions, although we can not draw a complete map.

The second question was more specific as it requires to measure the productivity gap and to examine its evolution over the past years. First of all it was necessary to identify a region that could act as reference point. In light of the results obtained with both indicators of aggregate productivity and indicators of sectoral productivity, Emilia-Romagna was selected as the benchmark. Without going into too much detail, the productivity gap compared to Emilia-Romagna is equal to about 2-6% for the regions in the north, 6-13% for the region in the Centre and 16-25% for the regions in the South. An examination of productivity in the two years for which the data are available indicates a widening of the gap between Emilia-Romagna and the other regions, with the exception of Trentino. It should be kept in mind however that the time horizon is very limited and, moreover, it coincides with a period of economic stagnation that may have influenced the results.

Regarding the relationship between aggregate productivity and sectoral productivity, we have identified two types of association. The first type, in which the relative levels of sectoral productivities are similar to the relative level of aggregate productivity. Therefore, if a region is in first place for aggregate productivity is in the first place for most of the sectoral productivities too. The second type, in which the relative levels of sectoral productivities are very heterogeneous and the relative level of aggregate productivity is a kind of average of these heterogeneous values. Trentino and, to some extent, Sicily and Lombardy are characterized by the second type of association, while the other regions are characterized by the first type.

Finally, the evolution of the global technological development of the selected region is slightly decreasing between 2001 and 2004. This means that the level of technological progress that would be achieved using the dominant techniques in 2001 is greater than the level of technological progress that would be achieved using the dominant techniques in 2004. We suspect that this trend is related to the global economic crisis that began at the end of 2001, however this issue requires further research.

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APPENDICES

3.A Industry classification and aggregation

This work uses the following industry classification based on Esa 95 – NACE Rev.1.

(1)	A	Agriculture, hunting and forestry
(2)	B	Fishing
(3)	CA	Mining and quarrying of energy producing materials
(4)	CB	Mining and quarrying, non energy producing materials
(5)	DA	Food products, beverages and tobacco
(6)	DB	Textiles and textile products
(7)	DC	Leather and leather products
(8)	DD	Wood and wood products
(9)	DE	Pulp, paper and paper products
(10)	DF	Coke, refined petroleum products and nuclear fuel
(11)	DG	Chemicals, chemical products and man-made fibres
(12)	DH	Rubber and plastic products
(13)	DI	Other non-metallic mineral products
(14)	DJ	Basic metals and fabricated metal products
(15)	DK	Machinery and equipment n.e.c.
(16)	DL	Electrical and optical equipment
(17)	DM	Transport equipment
(18)	DN	Manufacturing n.e.c.
(19)	E	Electricity, gas and water supply
(20)	F	Construction
(21)	G	Wholesale and retail trade
(22)	H	Hotels and restaurants
(23)	I	Transport, storage and communication
(24)	J	Financial intermediation
(25)	72 - 73 - 74	Business activities, R&D and IT
(26)	L	Public administration
(27)	M	Education
(28)	N	Health and social work
(29)	O-P-Q	Other community, social and personal service activities
(30)	70 - 71	Real estate and renting

The following industries have been aggregated: (1)&(2); (3)&(4); (6)&(7).

3.B Scale Invariance Property of the wage-profit frontier – A small example

In this appendix we will show, through a simple example, that a change in the scale of production that is asymmetric across industries leaves unchanged the wage-profit frontier, while changing the NNP curve.

We compare the productivity between two different hypothetical countries.
The first country, Alpha, is characterized by the following input-output matrix (A_{alpha}) and the following vectors of labor inputs (L_{alpha}) and sectoral outputs (B_{alpha}).

$$A_{alpha} = \begin{bmatrix} 4 & 6 & 6 \\ 6 & 4 & 8 \\ 4 & 4 & 10 \end{bmatrix} \quad L_{alpha} = \begin{bmatrix} 8 \\ 6 \\ 4 \end{bmatrix} \quad B_{alpha} = \begin{bmatrix} 20 \\ 30 \\ 35 \end{bmatrix}$$

The second country, Beta, is instead characterized by the following input-output matrix (A_{beta}) and the following vectors of labor inputs (L_{beta}) and sectoral outputs (B_{beta}).

$$A_{beta} = \begin{bmatrix} 4 & 6 & 6 \\ 7{,}8 & 5{,}2 & 10{,}4 \\ 4 & 4 & 10 \end{bmatrix} \quad L_{beta} = \begin{bmatrix} 8 \\ 7{,}8 \\ 4 \end{bmatrix} \quad B_{beta} = \begin{bmatrix} 20 \\ 39 \\ 35 \end{bmatrix}$$

One can easily verify that the values of the second industry in the country Beta have increased by 30 percent compared to the corresponding values of the country Alpha.
This increase has affected both the matrix A and the vectors L and B.

At this point we calculate NNP curves and the wage-profit frontiers using formulas (3.2) and (3.4) of section 3.1 . Figures 3.5 and 3.6 show the NNP curves the wage-profit frontiers respectively.

Fig 3.5 NNP per unit of labor curves

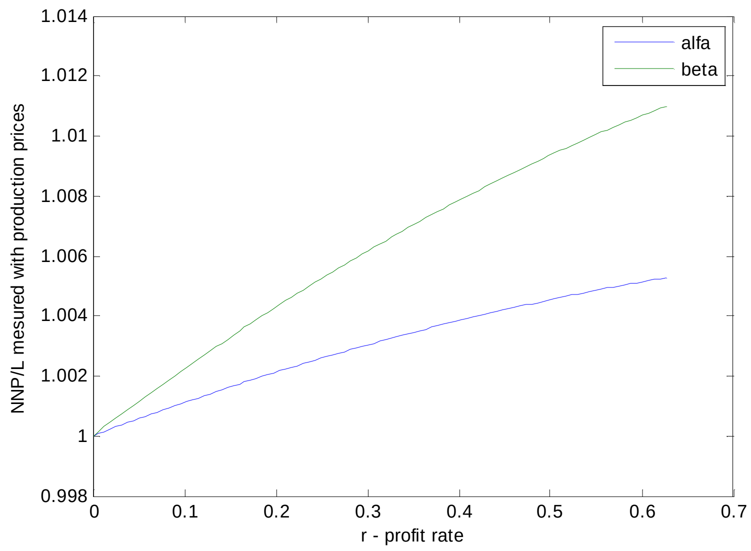
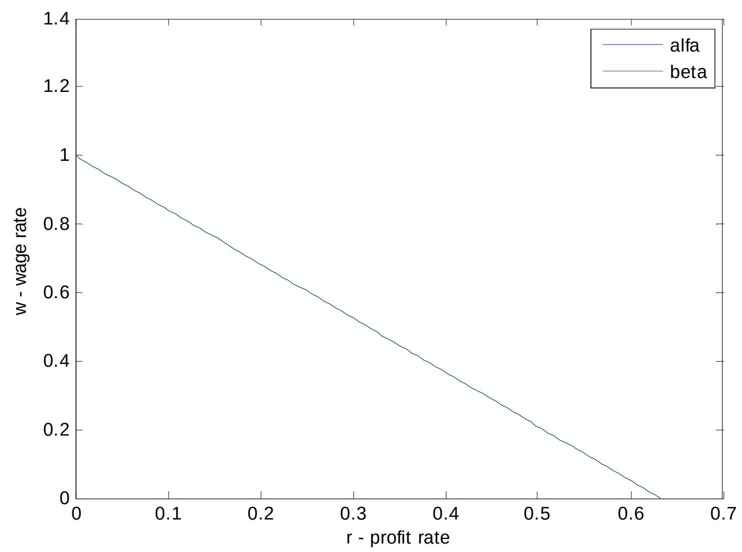


Fig 3.6 Wage-profit frontiers



It can clearly be seen, the NNP curves are different, while the wage-profit frontiers coincide. Therefore the indicators of labor productivity will differ while the indices of technological progress will be equal.

The NNP curve remains the same only when the scaling is uniform in all industries.

II

SENSITIVITY ANALYSES

The aim of the second part of this thesis is to test the robustness of some of the interesting and influential indicators, previously proposed and discussed, as a result of changes in *numéraire* and of a progressive aggregation of input-output tables.

This second part is further divided into two chapters. Chapter four is entitled 'An inquiry into the choice of *numéraire*' and its main objective is to measure the robustness of the indicators of technological progress and labor productivity proposed in the first chapter of the thesis to changes in *numéraire*. Chapter five is entitled 'An inquiry into the effect of aggregation of input-output tables' and its aim is to measure the robustness of the indicators as a result of a gradual aggregation of input-output tables. I should say that the contents of the two contributions are characterized by two main features. First, the aim of the two chapters is not only to test the robustness of some indicators presented in the first part of the thesis, but also to address the problems of the choice of *numéraire* and aggregation of input-output tables more generally. Secondly, a control of robustness is implemented only for the indicators of aggregate productivity and technological progress presented in the first chapter.

There is a reason why only the indicators of the first chapter are analyzed. Many of the indicators of sectoral productivity proposed in the second chapter do not suffer from the problem of the choice of *numéraire*. Many of them are in fact indicators based on physical quantities and the indicators based on production prices is simply a restatement of the indicator of aggregate productivity at the industry level. Furthermore, a robustness check of indicators of sectoral productivity as a result of a gradual aggregation of input-output tables is obviously illogical. Finally, the technological frontiers introduced in the third chapter do not suffer excessively from changes in the *numéraire*, given that the switch points remain unchanged.

It should be said immediately that the following two chapters are not able to give a general answer to the problems of the choice of *numéraire* and aggregation of input-output tables, because there are too many variables involves. Just to mention the most important:

- the set of all possible *numéraires* is infinite, since the *numéraire* can be a single commodity or any combination of several commodities.
- Aggregation can also be done in a large number of possible ways.
- There is an overlap between the two studies of robustness. When the input-output tables are aggregated, this often means that the *numéraire* must also change.

Despite their limitations, the two following contributions are still to be regarded as one of the first attempts to empirically explore these complex issues.

CHAPTER 4

AN INQUIRY INTO THE CHOICE OF NUMERAIRE[*]

[*] I am deeply indebted to Thomas Fredholm for useful discussions and comments.

4.1 Introduction

To our knowledge, very little research has been conducted assessing the effect of the *numéraire* on theoretical and empirical results. Some authors tried to investigate the effect of a change in the *numéraire* on a two-sector linear capital model (Akyuz, 1972; Yi, 1982; Ahmad, 1986). In particular, these three authors were interested in comparing the wage-profit relationships generated using two different *numéraires*: the Hicks and the Sraffa.

However, the problems connected with the selection of a standard of value are not only related to models with 'classical flavor' as those based on the Von Neumann-Leontief-Sraffa schemes of production, although in this context the consequences of a choice of the *numéraire* received much attention especially during the so-called Cambridge-Cambridge Controversy.

As a matter of fact, the literature on this topic, though sparse, is spread across a number of different frameworks. Veendrop (1970) and Mukherji (1973) analyzed the effect of the *numéraire* on the stability of general equilibrium. Papell and Theodoridis (2000) examined the implications of the choice of *numéraire* currency on panel tests of Purchasing Power Parity. Geman, El Karoui, and Rochet (1995) investigated the effect of *numéraire* changes on probability measures used to solve option pricing problems. Brekke (1994) showed the importance of the choice of *numéraire* in cost-benefit analysis.

Nevertheless, the above cited papers, without any pretension of generalization, tend to adopt a common methodology that is to show as a specific result can in some cases be '*numéraire* dependent' and then the conclusions drawn from it lack of general validity.

4.2 Theoretical prologue

In this chapter we propose a general and systematic method to compare and select an appropriate *numéraire*. The fundamental point of this essay is the claim that there is no such thing as a general and perfect *numéraire*. This, however, does not imply that we cannot construct some ranking from which the best possible *numéraire* could be chosen.

In particular, we think that there are three basic steps that should be followed in order to selecting the best possible *numéraire*:

a) Specify what do we want to measure?
b) Select a set of possible *numéraires*?
c) Given (a) and (b), decide what *numéraire* we should choose?

Ad a) At first sight, this question could seem obsolete but it is crucial for the rest of the process, since the purpose of the research – i.e. the process of which a *numéraire* is needed – will define the source for the ranking among the possible *numéraires*. Consequently, it is not *ex ante* given that this ranking will be independent of the measure (typically some kind of index) we want to construct.

Ad b) The set of all possible *numéraires* is infinite. As a consequence, the choice of the best *numéraire* requires necessarily the selection of a finite and a reasonably small subset. Selecting this finite subset must necessarily be a subjective choice based on practical and theoretical reasoning. One criterion could be that the *numéraire* should have a clear economic interpretation

Ad c) Having specified what we want to measure and having selected a finite and sufficiently small subset of possible *numéraires*, the crucial point remains – if and how we can determine the quality of a given *numéraire* and how these can be compared? (objectivity, ordinal ranking, etc).

In general we could say that the best *numéraire* is the one that produce results that are most resilient toward changes in the *numéraire*, meaning first of all, an acute inertia in the fundamental pattern in the produced results.

4.3 Criteria for the choice of the numéraire: an example

In chapter one, we introduced alternative measures of technological progress and labor productivity using input-output data. Essentially, we computed the associated wage-profit frontiers and the net national products curves, and from these we derive two measures of productivity growth based on production prices and a chosen *numéraire*.

The vector of production prices, which is a function of the rate of profit, is obtained by solving a system of n linear equations with n+2 unknowns. Hence, the solution of each system (given a uniform rate of profit) can be found only by fixing the price of one commodity or a bundle of commodities, i.e., choosing a *numéraire*. Consequently, the choice of the *numéraire* is a key step in determining these two indexes.

This paragraph re-examines the chapter one with the only focus on how the *numéraire* affects the final outcome. Following the approach specified above, the first step is to identify "what we want to measure", the answer to which is simply the two indexes: labor productivity and technological progress.

Furthermore, we must specify a subset of possible *numéraires*. In chapter one, we used the vector of Net Sectorial Products (NSP) for the standard system US 2000. This is because it provides a useful and straightforward interpretation of the results. In particular the point of intersection between each frontier and the vertical axis represents the proportion of US 2000 net national product that wages can acquire given a zero rate of profit.

It seems natural to include the NSP for each country and each time period in the subset of possible *numéraires*. It goes without saying that the NSP are only a small set of all possible applicable *numéraires*. For instance, one could fix the price of each one of the 23 industries and define products of the input-output tables. Alternatively, one could take some combination of industries. For simplicity, we limit attention to the NSP.

In chapter one, we were interested in comparing the technological progress among countries and over time. Consequently, our main concern is the pattern of technical progress in each country over time and the different evolution of this index among the four countries examined.

Accordingly, the best *numéraire* would provide a result that on average does not vary much when another *numéraire* is chosen.

4.4 The procedures adopted

In the following paragraphs, a detailed description of the procedures adopted is given.

As mentioned before, the main aim is to find a *numéraire* that produces results that are most resilient towards changes in the *numéraire*. Accordingly, the growth of labour productivity as well as the level of technological progress should neither change nor change proportionally.

In this respect, we envisage two methods. The first method based on the sign direction is rather simple and it provides a first assessment of how resilient the unit of measurement is. The second method, based on standard deviation, is more complex but is capable of calculating more precisely the level of variance of the pattern of labor productivity and technological progress to a change in the *numéraire*.

4.4.1 The sign direction approach

The analysis of change in sign represents a first naive approach of evaluating the resilience of a *numéraire*. Essentially, the procedure is as follows. The starting point is the calculation of technological progress and labor productivity for a given *numéraire* that we call the "base *numéraire*". When the base *numéraire* is replaced by another unit of measurement, the value of labor productivity and technological progress changes for each country and year. The main concern in this evaluation is the proportion of changes that have the same sign. A high proportion means that most of the changes go in the same direction when a new *numéraire* is adopted.

When the comparison is among many *numéraires*, it would be useful to have a single number to summarize their individual "performance". The way in which this number is calculated can be better explained with the help of Table 4.1

The first row reports the value of technological progress (or alternatively the labor productivity) for each country and year for a given "base *numéraire*". From the second row onward, each cell reports the sign of the change of technological progress when an alternative *numéraire* is used and the unit of measurement in use is described in the first column.

For our purposes, the last column is the most relevant. It shows the proportion of changes of the same sign. Obviously, when there are more plus signs than minus signs this fraction will refer to positive changes whereas when there are more minus signs than plus signs, it will refer to the negative changes. The bottom right cell reports the mean of these ratios that can be regarded as a synthetic measure of the base *numéraire* resilience. This value is bounded between 0.5 and 1. It is equal to 1 when for each country and year the sign is the same for whatever *numéraire* is used. It is equal to 0.5 when the number of plus and minus signs is equal for whatever *numéraire* is used.

Table 4.1 –The sign direction approach

	Area under the w-p frontier US 1970	**Area under the w-p frontier Ger1970**	**.....**	**Area under the w-p frontier US 2000**	**Area under the w-p frontier Ger 2000**	**% changes of the same sign**
Base numéraire	Value in absolute term	Value in absolute term	Value in absolute term	Value in absolute term	Value in absolute term	
Alternative numéraire 1	Change of sign (+ or -)	Change of sign (+ or -)	Change of sign (+ or -)	Change of sign (+ or -)	Change of sign (+ or -)	
Alternative numéraire 2	Change of sign (+ or -)	Change of sign (+ or -)	Change of sign (+ or -)	Change of sign (+ or -)	Change of sign (+ or -)	
Alternative numéraire 3	Change of sign (+ or -)	Change of sign (+ or -)	Change of sign (+ or -)	Change of sign (+ or -)	Change of sign (+ or -)	
Alternative numéraire 4	Change of sign (+ or -)	Change of sign (+ or -)	Change of sign (+ or -)	Change of sign (+ or -)	Change of sign (+ or -)	
Alternative numéraire 5	Change of sign (+ or -)	Change of sign (+ or -)	Change of sign (+ or -)	Change of sign (+ or -)	Change of sign (+ or -)	
						Mean value

4.4.2 The standard deviation approach

The main limitation of the previous method is that it takes into account only the direction of change and not its magnitude. Given that our concern is to preserve the pattern of technical progress and labor productivity as well as the relative position of each country with respect to others, the magnitude of change is also important.

As before, the procedure adopted is described with the help of Table 4.2 and, again, the goodness of a *numéraire* is summarized by a number that constitutes the final outcome of the procedure used. Given that this approach is more complicated, it is presented in *an algorithmic way*.

Step 1) Select a *numéraire* as the "base *numéraire*" and compute the NNP curves; then write these numbers in the first row of Table 4.2.

Step 2) Select an alternative *numéraire*, repeat the calculations of the indices, and then compute the percentage relative change of each entry with respect to the value obtained using the base *numéraire*. Write these values in the second row of Table 4.2. Finally, compute the standard deviation of this set of numbers. The lower the standard deviation is the more resilient is the "base *numéraire*".

Step 3) Repeat step 2 for each *numéraire* in the chosen subset. In our example, the set consists of NSP for each country and for each year. At the end, the last column of the Table 2 will give the list of all standard deviations.

Step 4) Finally, take the sum of all the standard deviations. The result is a number that summarizes the inertia of the base *numéraire*. The lower this number is, the more resilient is the outcome obtained using the base *numéraire* toward changes in the *numéraire*.

Step 5) Repeat all steps from step number 1 for each "base *numéraire*". The "best" *numéraire* would be the one that produces the lowest sum of standard deviations.

Table 4.2 –The standard deviation approach

	Area under the w-p frontier US 1970	**Area under the w-p frontier Ger1970**	**.....**	**Area under the w-p frontier US 2000**	**Area under the w-p frontier Ger 2000**	**Std. Dev**
Base numéraire	Value in absolute term	Value in absolute term	Value in absolute term	Value in absolute term	Value in absolute term	
Alternative numéraire 1	% relative change	% relative change	% relative change	% relative change	% relative change	
Alternative numéraire 2	% relative change	% relative change	% relative change	% relative change	% relative change	
Alternative numéraire 3	% relative change	% relative change	% relative change	% relative change	% relative change	
Alternative numéraire 4	% relative change	% relative change	% relative change	% relative change	% relative change	
Alternative numéraire 5	% relative change	% relative change	% relative change	% relative change	% relative change	
						Sum

4.5 The empirical result

Although the change in the unit of measure has an impact on the final outcome, we assess that the general patterns are preserved. In particular, we observed a productivity slowdown in the US and the UK during the seventies and a sharp increase in technological level during the nineties. Also, France and Germany maintained the same pattern of labour productivity and technological progress practically unchanged.

However, this favorable outcome does not imply that we should avoid a careful investigation of the consequences of a change in the *numéraire*. The following histograms (Fig.4.1-4.2) show the final results of the procedure presented above. Figure 4.1 shows the achievements of the change of sign approach. Each bar describes the average proportion of change of sign for each of the base *numéraires*. The top histograms show the results for the indicator of technological progress, while the histograms below refer to the labor productivity. The results of this test are very encouraging since the average of the proportion of sign changes is always greater than 0.9 and in some cases is even equal to one. This means that the values of the areas below the wage-profit frontier and the area under the curves of the NNP tend to change in the same direction.

Figure 4.1 Results of the change of sign approach.

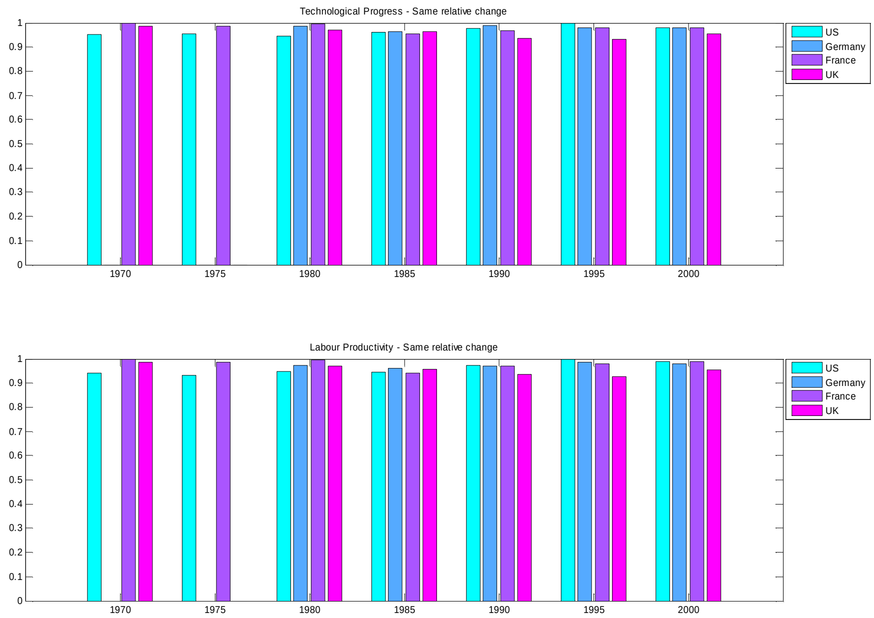
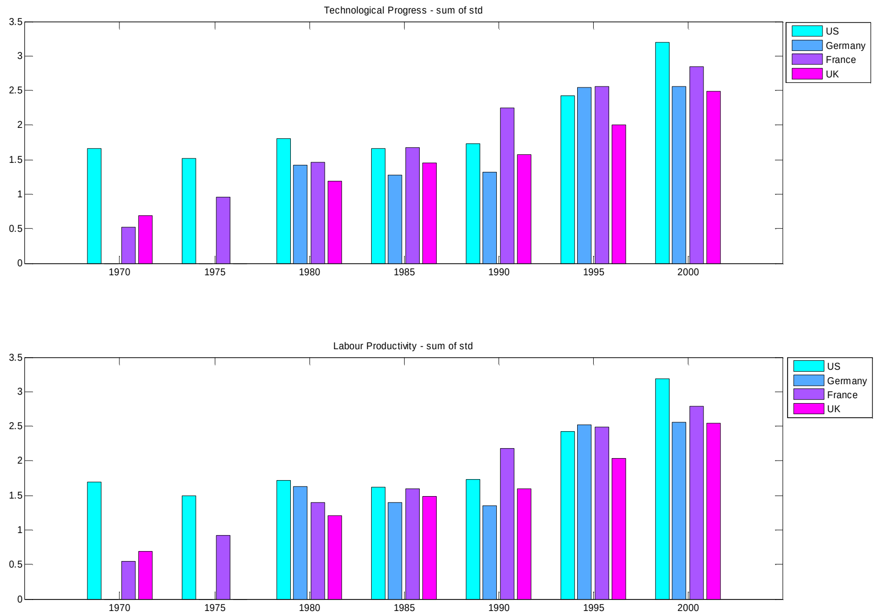


Figure 4.2 shows the achievements of the standard deviation approach. Each bar describes the sum of the standard deviations for each of the base *numéraires*. Again, the top histograms show the results for the indicator of technological progress, while the histograms below refer to the indicator of labor productivity.

Figure 4.2 Results of the standard deviation approach.



The outcome is counterintuitive because it shows a clear inverse relationship between the goodness of the *numéraire* and the level of technical progress (labor productivity) of the country and year for which the *numéraire* is chosen. In other words, the "best" *numéraire* (among the set used herein) is NSP of the country and year with the lowest level of technical progress.

We expected that the best standard of value should be the Net Sectorial Product of the country and year that exhibits a level of technical progress closer to the average of the entire distribution. This intuition is based on the theoretical work of Pasinetti (1993), even if this work is not similar to ours. In his book, he argues

> "In the general case, no physical commodity, if chosen as *numéraire*, will have the property of keeping the general price level perfectly stable over time…..
>
> ….it follows that the degree of maximum instability (within the physical *numéraires*) is associated with the choice of one or the other – commodity 1 or commodity m – that are

> found at the two opposite ends of the ordered scale of the rates of change of productivity. Clearly, the degree of instability decreases as we move towards the central area of such a scale; and ideally reduces to zero (reaching stability), if it were possible to choose that particular commodity which lies exactly half way between the two extremes. More precisely, instability would be eliminated if we were able to choose as *numéraire* that particular physical commodity to which there correspond the average (appropriately weighted) of the rates of growth of productivity of the entire economic system. (Pasinetti 1993, p69)".

More precisely, a robustness check of the indicators presented in the first chapter shows that, as we choose a *numéraire* far away form the year of the base *numéraire* the standard deviation of the relative changes increases sharply.

4.6 Toward a general case

Sections four and five explicitly refer to a specific case study, but as mentioned in the introductory part of this paper, the problem of the selection of the *numéraire* is a general one.

However, as pointed out many times there are no general rules, and the choice of the "best" *numéraire* must be accomplished following different procedures, depending on the specific case under examination.

The best strategy is thus to collect and analyze as many examples as possible in order to set a precedent for all future specific studies. Clearly, it is not feasible in one paper to explore all the possible alternatives; nevertheless in this section we provide another example that is totally different from the previous one in order to broaden our understanding of this topic. This example will show that the change in *numéraire* can have a significant impact.

The solution of the Sraffian system of production and distribution is a vector of relative prices. This vector of prices is not unique but instead changes as the distributional parameters change. As a consequence of the movement of the relative prices, the value of capital (given by the quantity of each commodity multiplied by its relative price) is dependent on the distribution of income. In a "normal" case, the value of capital (or the capital-labor ratio) would decrease as the rate of profit increases because the entrepreneurs will substitute the factor of production that is becoming more expensive with the factor of production that is becoming cheaper. Nevertheless, solving the Sraffian system, it is possible to identify peculiar cases where the capital-labor ratio increases when the rate of profit increases. In the literature, this occurrence is called reverse capital deepening effect because a lower value of capital is associated with a lower rate of profit.

Some empirical studies have tried to assess the likelihood of reverse capital deepening, but these studies do not consider the effect of the *numéraire* on the probability of reverse capital deepening (D'Ippolito, 1987; Mainwaring and Steedman, 1995; Petri, 2000; Han and Schefold, 2006).

In these sections, the same OECD input-output tables used in chapter one are examined. Each of these tables is then used within a Sraffian framework of production and for each country and year the ratio between value of capital and labour as a function of the rate of profit is calculated.

Then, for a small set of *numéraire* consisting of the NSP for each country and each time period, we record the number of times the reverse capital deepening occurs. Finally, we are interested in observing whether changes of the *numéraire* will cause changes in the number of times the reverse capital deepening occurs.

As can clearly be seen in the following table (Tab. 4.3) the number of occurrences of reverse capital deepening varied form zero to five. Hence, it is possible to argue that in this example the *numéraire* has a non-negligible effect on the likelihood of reverse capital deepening.

Table 4.3 –The effect of numéraire on the likelihood of reverse capital deepening

Numéraire	Number of occurrences of reverse capital deepening
NSP US 1972	0
NSP US 1977	1
NSP US 1982	4
NSP US 1985	3
NSP US 1990	2
NSP US 1995	2
NSP US 1997	2
NSP US 2000	4
NSP GER 1970	0
NSP GER 1978	0
NSP GER 1986	0
NSP GER 1990	0
NSP GER 1995	2
NSP GER 2000	2
NSP FRA 1972	0
NSP FRA 1977	0
NSP FRA 1980	2
NSP FRA 1985	3
NSP FRA 1990	3
NSP FRA 1995	1
NSP FRA 2000	1
NSP UK 1968	0
NSP UK 1979	1
NSP UK 1984	1
NSP UK 1990	2
NSP UK 1995	2
NSP UK 1998	0
NSP UK 2000	5

4.7 Conclusions

The main objective of this chapter is to emphasize that the selection of the standard of value is not insignificant. In many cases, the adoption of a specific *numéraire* has important consequences on the final outcome.

Consequently, the selection of the standard of measure has to be driven by some basic rules. In particular, we argue that the most excellent *numéraire* is the one that provides a result more resilient to change in the unit of measure.

The practical implementation of this principle should be evaluated on a case to case basis, because this basic rule can be interpreted differently depending to the area of analysis.

In this chapter we showed a couple of examples in which the choice of a *numéraire* constitutes an essential step. As a matter of fact, the final results were in both cases modified when the standard of values changed.

The very different nature of these two examples confirm our belief that an overall theory on the choice of the *numéraire* is not feasible; nevertheless a collection of case studies and practical illustrations would constitute a desirable background for those who in the future have to cope with the problem of selecting a unit of measure.

The problem is there, whether we like it or not, and no universal solution will probably be found. However, this does not mean that we should ignore the problem.

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CHAPTER 5

AN INQUIRY INTO THE EFFECT OF AGGREGATION

5.1 Introduction

Sraffa (1960) introduced a scheme of production where commodities are produced by means of other commodities and labour. For instance, in chapter one of his work, he assumes a simple economy where there are only three commodities wheat, iron, and pigs and he is interested in finding a set of exchange values relating the various products that would enable production to persist in self-replacing state.

Input-Output tables constitute an empirical counterpart to Sraffa's production scheme. They describe the sales and purchases relationship between producers and consumers within an economy. Clearly, in the advanced economies like those of the OECD countries a great variety of goods are produced and consumed so that input-output tables cannot illustrate all the physical interrelations of an economy. An industry by industry input-output table summarizes and simplifies information of transaction of commodities by aggregating heterogeneous goods into broad classes according to a standard economic classification.

Leontief, for instance, (1986, p.3) describes an input-output table in this following way:

> It is true, of course, that the individual transactions, like individual atoms and molecules, are far too numerous for observation and description in detail. But it is possible, as with physical particles, to reduce them to some kind of order by classifying and aggregating them into groups. This is the procedure employed by input-output analysis in improving the grasp of economic theory upon the facts with which it is concerned in every real situation.

During the last fifty years there has been some empirical works based on Sraffa's framework (see among many Bienefeld,1988; Ochoa, 1984, 1989; Petrovic 1987; Tsoulfidis and Maniatis, 2002; Tsoulfidis, 2008) and some of these works generate wage-profit frontiers from the available input-output tables (Han and Shefold, 2006; Degasperi and Fredholm, 2010; Fredholm and Zambelli, 2009a;, and Fredholm and Zambelli, 2009b). The objective of this chapter is thus to study whether and to what extend the results obtained are affected by a step by step reduction of the level of industry detail of the input-output tables utilized.

It is worth saying from the outset that this study concentrates mainly on the effect of aggregation on the shape of the wage-profit frontier. However, of course, aggregation could exert its effect also on many others elements related to Sraffa's work as well as on many other elements unrelated with Sraffa but linked in some way with input-output analysis.

If this study will show that aggregation of input-output tables does not affect much the form of the wage-profit frontiers, we would claim that input-output tables represent a good proxy of Sraffa's scheme of production. Conversely, we should be aware that the results obtained are not robust to aggregation of input-output tables.

The second section of the chapter briefly describes the steps for the mathematical calculation of wage-profit frontier and NNP curves, the third section presents the data used for the inquiry and focuses in particular on the problem of the choice of *numéraire*, the fourth section describes the procedure of aggregation and presents some measures of robustness applied to the wage-profit frontiers, the fifth section checks the robustness of the indicators proposed in the first chapter of the thesis, and, finally, the sixth section is devoted to the conclusions.

It should be clarified that paragraphs four and five have different objectives. Paragraph four is concerned with an assessment of the robustness of the wage-profit frontier irrespective of the indicators proposed in chapter one, where paragraph five is concerned to test the robustness of the indicators of technological progress and labor productivity.

5.2 The wage-profit frontier

Consider an economic system consists of *n* industries producing *n* commodities by means of some combination of the *n* commodities and labour. Let $\mathbf{A}$ be a $(n \times n)$ quadratic non-singular matrix of inter-industry inputs, where the (*i,j*) entry represents the i^{th} industry's use of the j^{th} commodity in the production of the i^{th} commodity. Likewise, $\mathbf{L}$ is a $(n \times 1)$ vector of labour inputs and $\mathbf{B}$ is a $(n \times n)$ positive definite diagonal matrix of outputs, where the i^{th} diagonal entry is the gross output of the i^{th} industry. As usual these elements can be collected in the following long-run equilibrium relationship that captures the distribution of the total production among wages, profits, and means of production, where the wage and profit rates are assumed to be uniform.

$$\mathbf{Ap}(1+r)+\mathbf{L}w=\mathbf{Bp} \tag{5.1}$$

System (5.1) consists of *n* linear independent equations and *n+2* variables, i.e., the system has initially two degrees of freedom. Choosing a *numéraire* $\boldsymbol{\eta}$, for which it holds that $\boldsymbol{\eta}'\mathbf{p}=1$, the degrees of freedom reduces to one.

Given the profit rate, it is straightforward to calculate the wage rate and the relative prices that solve system (5.1). Isolate $\mathbf{p}$, $\mathbf{p}=(\mathbf{B}-\mathbf{A}(1+r))^{-1}\mathbf{L}w$, premultiply with the *numéraire*, and rearrange to obtain the wage-profit frontier function and the associated prices, *viz.*

$$w=\left(\boldsymbol{\eta}'(\mathbf{B}-\mathbf{A}(1+r))^{-1}\mathbf{L}\right)^{-1} \tag{5.2}$$

$$\mathbf{p} = \frac{\left(\mathbf{B} - \mathbf{A}(1+r)\right)^{-1}\mathbf{L}}{\boldsymbol{\eta}'\left(\mathbf{B} - \mathbf{A}(1+r)\right)^{-1}\mathbf{L}} \qquad (5.3)$$

5.3 Data and the Choice of *Numèraire*

The data used in this chapter are taken from the OECD 1970-2000 input-output database (1995, 2002 and 2006 edition) for the US, Germany, France, and the UK. The industry classification of the database is based on the ISIC Rev. 2 (1995 edition) or ISIC Rev. 3 (2002 and 2006 edition) with 35, 42, and 48 industries respectively (see also appendix 5.A).

In order to allow comparability between countries and across time, the original 48, 42 and 35 industries have been aggregated into 23 industries following standards national account. Moreover, aggregation allows us to ensure non-singular matrices for the whole dataset. As labour inputs we used the number of hours worked. In default of detailed information for the number of hours worked in each sector we decide to attribute in proportion to the compensation of employees. In any case, further improvements would be achievable when data on labour quality will be available.

The original input-output tables are in current prices. Estimates in constant prices are calculated by using macro-industry deflators, these deflators are obtained as a ratio between the macro-industry value added in current prices and the macro industry value added in constant prices. Constant prices input-output tables represent a proxy for input-output tables in physical quantities.

As a *numéraire* we choose to fix the price for agriculture equal to one. We decided to fix the price for agriculture because it is the only sector that has not been aggregated and therefore the *numéraire* does not change.

5.4 The effect of aggregation on the wage-profit frontier

The available OECD input-output tables show the monetary transactions among industries and thus they summarize the large number of transactions of physical goods among producers and consumers within an economic system. Hence, the existing tables have already passed through a process of aggregation given that many heterogeneous goods and services have been aggregated into a unique number, which is the monetary value of these commodities.

The level of industry detail of the input-output tables is arbitrarily chosen by the OECD according to the information available and the uses of these tables. Consequently, the existing tables constitute one of the many possible outcomes obtained by aggregating physical goods and services in different ways. Further aggregations allows to obtain tables which describe the same underline economic system, but with a different level of detail. If the wage-profit frontier will be insensitive to a progressive aggregation of an input-output table, this would enlarge the validity of all the empirical studies based on Sraffa and using input-output tables.

The initial 23 by 23 industries input-output tables for each country and time period has been gradually aggregated and the effect of this aggregation on the shape of the wage-profit frontiers has been studied. Clearly, there are infinitely many paths of aggregation. Therefore, we propose the following pattern of aggregation (Tab.5.1), which reduces step by step the level of industry detail of the input-output tables but it preserves the structure of the national accounts[1].

Aggregation was implemented in six steps and the initial 23 industries[2] have been reduced to 19, 13, 9, 6, 4, and 3 industries respectively. As can be seen from Table 5.1, agriculture is the only sector which remains unchanged from the beginning to step number six.

Table 5.1 – Representation of the pattern of aggregation

	Initial	Step 1	Step 2	Step 3	Step 4	Step 5	Step 6
Industry	1	1	1	1	1	1	1
	2	2	2	2	2	2 - 17	2 - 18
	3	3	3 - 6	3 - 10	3 - 16	18	19 - 23
	4	4 & 5	7	11- 16	17	19 -23	
	5	6	8 - 10	17	18		
	6	7	11 & 12	18	19 - 23		
	7	8 & 9	13 - 16	19			
	8	10	17	20 & 21			
	9	11 & 12	18	22 & 23			
	10	13 & 14	19				
	11	15	20 - 21				
	12	16	22				
	13	17	23				
	14	18					
	15	19					
	16	20					
	17	21					
	18	22					
	19	23					
	20						
	21						
	22						
	23						
N° of industries	23	19	13	9	6	4	3

[1] Step number six, for example, reduces the table to a 3-by-3 industries: agriculture, industry, and services.
[2] The names of the industries are listed in appendix 5.A.

After each step of aggregation the wage-profit frontiers for each country and year has been calculated and compared with the frontiers previously obtained.

The first evaluation is performed by means of visual inspection and it does not reveal significant differences (see Fig.5.2- Fig.5.3 in appendix 5.B). More precise measures of similarity are given by the area under the wage-profit frontier and the correlation coefficients of two series of one hundred equally spaced points of the frontiers for the same country and time period.

The variation of the area under the wage-profit frontier constitutes a first precise indicator of the effect of aggregation on the wage-profit frontiers. In particular, we compute the mean relative change[3] of the areas calculated under different steps of aggregation. The values of the mean relative change are reported in Table 5.2.

As can be seen, the mean relative change increases when the comparison is between tables with large differences in terms of industry details. Look, for instance, at the far right column of Table 5.2, the mean relative change of the area under the wage-profit frontiers is 10.2% when we are confronting 23-by-23 industries tables with 19-by-19 industries tables, but the value increases to 17.8% when we are confronting 23-by-23 industries tables with 3-by-3 industries tables.

Hence, Table 5.2 indicates that the effect of aggregation on the wage-profit frontier is small when few sectors are aggregated, but it becomes more significant as the number of sectors aggregated increases.

Table 5.2. Mean relative change of the area under the wage-profit frontiers (%)

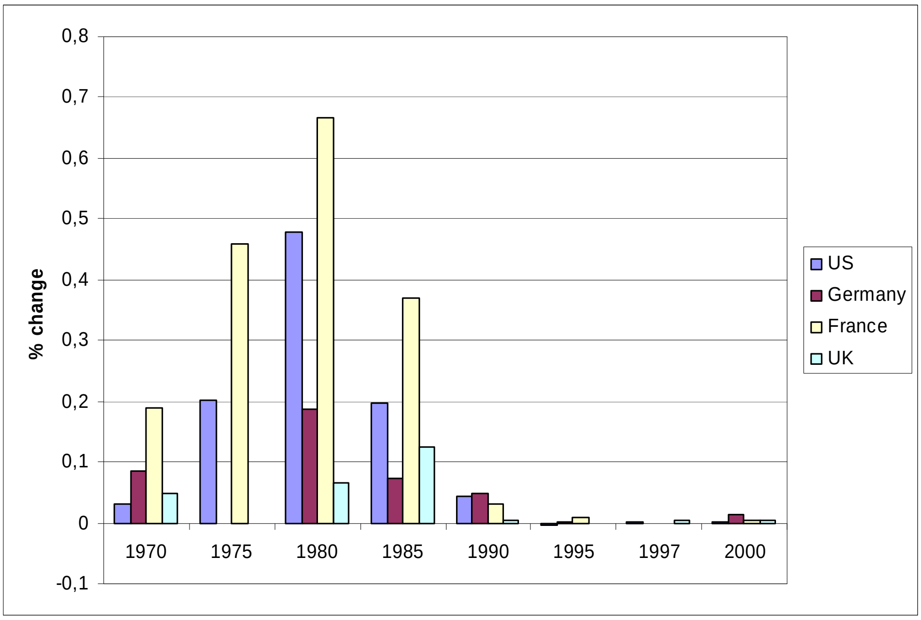
N° of sectors	**3**	**4**	**6**	**9**	**13**	**19**	**23**
3	--	10.1	9.8	8.6	8.8	8.1	17.8
4		--	2.2	2.9	6.8	7.6	11.9
6			--	2.3	6.2	7.2	11.5
9				--	4.9	6.2	10.5
13					--	1.6	9.3
19						--	10.2
23							--

We are not only interested in the variation of the relative change, but also in the sign and the magnitude of it. Taking as base frontiers those obtained using the 23-by-23 industries input-output tables we look at the relative change of the area under the wage-profit frontier for each country and time period. Figure 5.1 below shows, for instance, the relative change of the areas when 19-by-19 industries tables are used. As it can be seen, the change is always positive except in US and in UK for the year 1995.

[3] The relative change of the area under the wage-profit frontier is given by the difference between the area calculated under two different steps of aggregation divided by the mean of the two areas in absolute terms, for a particular country and year. The numbers reported in Table 2 are the mean values of these ratios.

By looking at Figure 5.1, we find one point that is worth considering: the magnitude of the change decreases in recent years. This result is true not only for the example of Figure 5.1, but also in the other cases examined[4]. While we do not have an explanation of this effect, we note that the latest tables seem to be less influenced by the aggregation.

Figure 5.1 Relative differences between the areas obtained by using 23-by-23 industries input-output tables and the areas obtained by using 19-by-19 industries input-output tables.



The final measure examined is the correlation coefficient between two sets of a hundred points[5] uniformly distributed along the wage-profit frontiers. The values of the correlation coefficients are available in Tables 5.6-5.9 in the appendix 5.C. As it can be seen from the tables, the values of the coefficients are all very high and close to one. This means that the aggregation causes a parallel shift of the frontier and it does not significantly alter their shape.

[4] The full list of the values of the areas and their percentage differences is available upon request.
[5] The points correspond to the values of the wage rate for different values of the rate of profit.

5.5 A robustness check of the indicators of technological progress and labor productivity

In the first chapter of this thesis, a method of productivity accounting based on the area under the wage-profit frontier and NNP curves is introduced. By using that method, we compute technological progress and labor productivity in four major OECD countries form 1970 to 2000. Unfortunately, nothing can be said, analytically, about the robustness of this result in relation to the use of alternative *numéraires* and a progressive aggregation of the input-output tables.

In chapter four we checked the robustness of the indicators with respect to the use of a restricted sample of alternative *numéraires* and we proved that the results are quite stable. In an effort to further investigate the robustness of these indicators, we repeat the calculation of the area under the wage-profit frontier using input-output tables with different levels of industry details. This investigation is complicated by the fact that it is not possible to compare the results obtained in the first chapter with those obtained here, because the *numéraire* is now different.

As explained in section three, the *numéraire* is now given by the price of the agricultural goods, because this is the only industry which has not been aggregated. In this way the sensitivity analysis is conducted properly because we isolate the effects of aggregation than those related to the change of *numéraire*. In fact, although the input-output tables are aggregated progressively, the *numéraire* remains always the same.

The examination is then realized in two steps. In the first, measures of technological progress and labor productivity are computed using as a *numèraire* the price of agricultural goods. This, of course, could cause a substantial variation of the results obtained in chapter one, for the reason that, although we proved a substantial invariance of the outcome when different unit of measures are used, this is not universally true. In the second, the measures of technological progress calculated using input-output tables with a different level of industry detail are compared for each country and time period.

In section four, we have already studied the effect of aggregation of input-output tables on the wage-profit frontiers, but that analysis was not specifically designed to asses the validity of the results obtained in the first chapter. Our objective in this section is to make a country-by-country and year-by-year comparison of the area under the wage-profit frontiers and the NNP curves obtained using input-output tables with a different level of industry detail. In particular, we are interested to check whether the relative position of each country in each time period varies or remains constant.

Table 5.3 shows the relative position of each country in each time period in terms of technological progress. The second column shows the level of industry detail of the input-output table utilized, while the last column summarizes the results. We count 37 cases where the relative position of the countries is unchanged with respect to the reference rank order (given by the 23-by-23 industries input-output tables), 7 cases where the relative position changes, and 4 not comparable cases[6]. Then, it is possible to argue that the result are quite robust to a progressive aggregation of input-output tables and this implies that a 3-by-3 industries input-output tables will produce results similar to those obtained using a 23-by-23 industries input-output table.

Table 5.4 shows the relative position of each country in each time periods in terms of labor productivity. The structure of the table is the same as the previous table. In this case, we count 41 cases where the relative position of the countries is unchanged with respect to the reference rank order (given by the 23-by-23 industries input-output tables), 3 cases where the relative position changes, and 4 not comparable cases.

Overall, it can be said that the aggregation of input-output tables alters only minimally the relative position of the four countries examined in terms of technological progress and labor productivity. Clearly, the value of the area below the wage-profit frontier and the NNP curve changes as a result of the aggregation, but this change is not likely to modify the relative positions of the countries considered.

[6] The case is not comparable when some value is missing.

Table 5.3 Technological progress. Relative position of each country in each time period for different levels of aggregation of input-output tables.

		US	Germany	France	UK	
1970	***23 sec***	2	4	1	3	
	19 sec	2	4	1	3	Cons.
	13 sec	2	4	1	3	Cons.
	9 sec	n.a.	3	1	2	N.C.
	6 sec	2	4	1	3	Cons.
	4 sec	2	3	1	n.a.	N.C
	3 sec	2	4	1	3	Cons.
1975	***23 sec***	1	n.a.	2	n.a.	
	19 sec	2	n.a.	1	n.a.	Var.
	13 sec	2	n.a.	1	n.a.	Var.
	9 sec	1	n.a.	n.a.	n.a.	N.C
	6 sec	2	n.a.	1	n.a.	Var.
	4 sec	2	n.a.	1	n.a.	Var.
	3 sec	2	n.a.	1	n.a.	Var.
1980	***23 sec***	2	3	1	4	
	19 sec	2	3	1	4	Cons.
	13 sec	2	3	1	4	Cons.
	9 sec	2	3	1	4	Cons.
	6 sec	2	3	1	4	Cons.
	4 sec	2	3	1	4	Cons.
	3 sec	2	3	1	4	Cons.
1985	***23 sec***	3	2	1	4	
	19 sec	3	2	1	4	Cons.
	13 sec	3	2	1	4	Cons.
	9 sec	2	n.a.	1	3	N.C
	6 sec	3	2	1	4	Cons.
	4 sec	3	2	1	4	Cons.
	3 sec	3	2	1	4	Cons.
1990	***23 sec***	3	2	1	4	
	19 sec	3	2	1	4	Cons.
	13 sec	3	2	1	4	Cons.
	9 sec	3	2	1	4	Cons.
	6 sec	3	2	1	4	Cons.
	4 sec	3	2	1	4	Cons.
	3 sec	3	2	1	4	Cons.
1995	***23 sec***	3	2	1	4	
	19 sec	3	2	1	4	Cons.
	13 sec	3	2	1	4	Cons.
	9 sec	3	2	1	4	Cons.
	6 sec	3	2	1	4	Cons.
	4 sec	3	2	1	4	Cons.
	3 sec	3	2	1	4	Cons.
1997	***23 sec***	1	n.a.	n.a.	2	
	19 sec	1	n.a.	n.a.	2	Cons.
	13 sec	2	n.a.	n.a.	1	Var.
	9 sec	2	n.a.	n.a.	1	Var.
	6 sec	1	n.a.	n.a.	2	Cons.
	4 sec	1	n.a.	n.a.	2	Cons.
	3 sec	1	n.a.	n.a.	2	Cons.
2000	***23 sec***	2	3	1	4	
	19 sec	2	3	1	4	Cons.
	13 sec	2	3	1	4	Cons.
	9 sec	2	3	1	4	Cons.
	6 sec	2	3	1	4	Cons.
	4 sec	2	3	1	4	Cons.
	3 sec	2	3	1	4	Cons.

Table 5.4 Labor productivity. Relative position of each country in each time period with for different levels of aggregation of input-output tables.

		US	Germany	France	UK	
1970	***23 sec***	2	4	1	3	
	19 sec	2	4	1	3	Cons.
	13 sec	2	4	1	3	Cons.
	9 sec	n.a.	n.a.	n.a.	n.a.	N.C.
	6 sec	2	4	1	3	Cons.
	4 sec	2	3	1	n.a	N.C
	3 sec	2	4	1	3	Cons.
1975	***23 sec***	2	n.a.	1	n.a.	
	19 sec	2	n.a.	1	n.a.	Cons.
	13 sec	2	n.a.	1	n.a.	Cons.
	9 sec	1	n.a.	n.a.	n.a.	N.C
	6 sec	2	n.a.	1	n.a.	Cons.
	4 sec	2	n.a.	1	n.a.	Cons.
	3 sec	2	n.a.	1	n.a.	Cons.
1980	***23 sec***	3	2	1	4	
	19 sec	3	2	1	4	Cons.
	13 sec	3	2	1	4	Cons.
	9 sec	3	n.a.	1	2	N.C.
	6 sec	3	2	1	4	Cons.
	4 sec	2	3	1	4	Var.
	3 sec	2	3	1	4	Cons.
1985	***23 sec***	4	1	2	3	
	19 sec	3	2	1	4	Var.
	13 sec	4	2	1	3	Var.
	9 sec	4	2	1	3	Cons.
	6 sec	4	2	1	3	Cons.
	4 sec	4	2	1	3	Cons.
	3 sec	4	2	1	3	Cons.
1990	***23 sec***	4	1	2	3	
	19 sec	4	2	1	3	Cons.
	13 sec	4	2	1	3	Cons.
	9 sec	4	2	1	3	Cons.
	6 sec	4	2	1	3	Cons.
	4 sec	4	2	1	3	Cons.
	3 sec	4	2	1	3	Cons.
1995	***23 sec***	3	2	1	4	
	19 sec	3	2	1	4	Cons.
	13 sec	3	2	1	4	Cons.
	9 sec	3	2	1	4	Cons.
	6 sec	3	2	1	4	Cons.
	4 sec	3	2	1	4	Cons.
	3 sec	3	2	1	4	Cons.
1997	***23 sec***	2	n.a.	n.a.	1	
	19 sec	2	n.a.	n.a.	1	Cons.
	13 sec	2	n.a.	n.a.	1	Cons.
	9 sec	2	n.a.	n.a.	1	Cons.
	6 sec	2	n.a.	n.a.	1	Cons.
	4 sec	2	n.a.	n.a.	1	Cons.
	3 sec	2	n.a.	n.a.	1	Cons.
2000	***23 sec***	3	2	1	4	
	19 sec	3	2	1	4	Cons.
	13 sec	3	2	1	4	Cons.
	9 sec	3	2	1	4	Cons.
	6 sec	3	2	1	4	Cons.
	4 sec	3	2	1	4	Cons.
	3 sec	3	2	1	4	Cons.

5.6 Conclusions

This chapter has shown that aggregation of input-output tables does not affect much the shape of the wage-profit frontier, especially when the level of industry detail is slightly reduced with respect to the initial level. In addition, it has shown that the sign and the magnitude of the variation in the area is not uniform among countries and time periods. In particular, it seems that the areas calculated using the more recent data (from 1995 to 2000) are less sensitive to a progressive aggregation of the input-output tables. We do not have an explanation of this effect, but it is interesting to note that aggregation of input-output tables belonging to the same dataset induces similar changes in the wage-profit frontiers. This chapter has also revealed that all the correlation coefficients between frontiers are very close to one and this indicates that wage-profit frontiers are very often affected by a nearly parallel up-or-down shift. These results are also confirmed by a simple visual inspection of the curves. Overall, this work reveals a robustness of the wage-profit frontier to aggregation of input-output tables and thus it improves the reliability of all the studies of this kind that have been conducted so far. Clearly, this result is valid only for a specific dataset and further research is needed to confirm these results using different input-output tables.

The robustness check on indicators of technological progress and labor productivity presents encouraging results. The relative positions of countries in different years remain largely unchanged when the input-output tables are progressively aggregated. However, this does not mean that the areas below the wage-profit frontiers and below the NNP curves are unchanged, rather the change is not likely to significantly alter the relative position of each country.

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APPENDICES

5.A The Dataset

Table 5.5 shows the available input-output tables from the period 1970-2000. Tables are not necessarily available from the exact five years intervals, e.g., the Germany tables here labelled 1978 and 1985 are actually the 1980 and 1986 tables, respectively.

Table 5.5 Available input-output tables

	1970	1975	1980	1985	1990	1995	1997	2000
US	x	x	x	x	x	x	x	x
Germany	x		x	x	x	x		x
France	x	x	x	x	x	x		x
UK	x		x	x	x	x	x	x

The original 35-by-35, 42-by-42, and 48-by-48 sector tables have been reduced to 23-by-23 sector. These sectors have been further aggregated as explained in Table 5.1. The list below shows the twenty-three sectors with the corresponding number used in Table 5.1.

1. Agriculture, hunting, forestry, and fishing
2. Mining and quarrying
3. Food products, beverages, and tobacco
4. Textiles, textile products, leather, and footwear
5. Wood and products of wood and cork
6. Pulp, paper, paper products, printing, and publishing
7. Coke, refined petroleum products, and nuclear fuel
8. Chemicals
9. Rubber and plastic products
10. Other non-metallic mineral products
11. Metals
12. Fabricated metal products, except machinery and equipment
13. Machinery and equipment, nec
14. Electrical machinery and apparatus
15. Transport equipment
16. Manufacturing nec; recycling (include furniture)
17. Production and distribution of electricity, gas, and water
18. Construction
19. Wholesale and retail trade
20. Service activities (transport, hotels and restaurants)
21. Post and telecommunications
22. Business activities (finance, real estate, and R&D)
23. Public administration, education and health

5.B Wage-profit Frontiers for Different Levels of Aggregation

The following figures show the wage-profit frontiers for different levels of aggregation of input-output tables for 1970 and 2000. The figures for the remaining years are available upon request.

Figure 5.2 Wage-profit frontiers - 1970

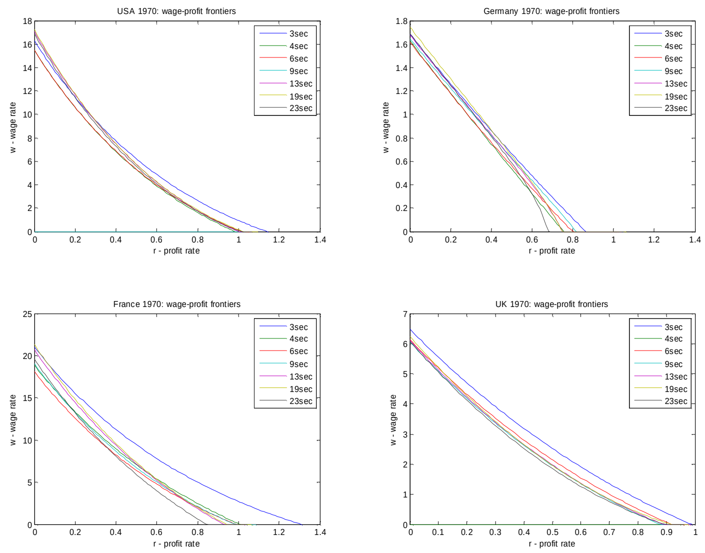
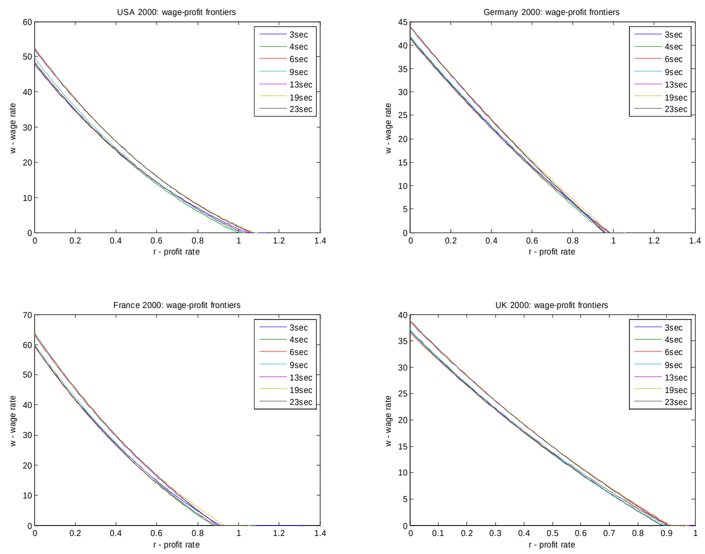


Figure 5.3 Wage-profit frontiers - 2000



5.C Correlation Coefficients

This appendix shows only the correlation coefficients for the year 2000. The values of the correlation coefficients for other years are available upon request.

Table 5.6 Matrix of correlation coefficients – US 2000.

N° of Industries	23	19	13	9	6	4	3
23	1.0000	0.9993	0.9994	0.9992	0.9993	0.9997	0.9995
19	0.9993	1.0000	1.0000	1.0000	1.0000	0.9998	0.9999
13	0.9994	1.0000	1.0000	1.0000	1.0000	0.9999	1.0000
9	0.9992	1.0000	1.0000	1.0000	1.0000	0.9998	0.9999
6	0.9993	1.0000	1.0000	1.0000	1.0000	0.9999	1.0000
4	0.9997	0.9998	0.9999	0.9998	0.9999	1.0000	1.0000
3	0.9995	0.9999	1.0000	0.9999	1.0000	1.0000	1.0000

Table 5.7 Matrix of correlation coefficients - Germany 2000.

N° of Industries	23	19	13	9	6	4	3
23	1.0000	1.0000	1.0000	0.9999	0.9999	1.0000	0.9999
19	1.0000	1.0000	1.0000	1.0000	0.9999	0.9999	0.9998
13	1.0000	1.0000	1.0000	1.0000	0.9999	0.9999	0.9998
9	0.9999	1.0000	1.0000	1.0000	1.0000	0.9999	0.9999
6	0.9999	0.9999	0.9999	1.0000	1.0000	0.9999	0.9999
4	1.0000	0.9999	0.9999	0.9999	0.9999	1.0000	0.9999
3	0.9999	0.9998	0.9998	0.9999	0.9999	0.9999	1.0000

Table 5.8 Matrix of correlation coefficients – France 2000.

N° of Industries	23	19	13	9	6	4	3
23	1.0000	0.9920	0.9927	0.9918	0.9849	0.9872	0.9854
19	0.9920	1.0000	1.0000	1.0000	0.9986	0.9992	0.9989
13	0.9927	1.0000	1.0000	0.9999	0.9983	0.9989	0.9986
9	0.9918	1.0000	0.9999	1.0000	0.9985	0.9990	0.9988
6	0.9849	0.9986	0.9983	0.9985	1.0000	0.9999	1.0000
4	0.9872	0.9992	0.9989	0.9990	0.9999	1.0000	0.9999
3	0.9854	0.9989	0.9986	0.9988	1.0000	0.9999	1.0000

Table 5.9 Matrix of correlation coefficients – UK 2000.

N° of Industries	23	19	13	9	6	4	3
23	1.0000	0.9997	0.9995	0.9996	0.9997	0.9996	0.9994
19	0.9997	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
13	0.9995	1.0000	1.0000	1.0000	0.9999	1.0000	1.0000
9	0.9996	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
6	0.9997	1.0000	0.9999	1.0000	1.0000	1.0000	0.9999
4	0.9996	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
3	0.9994	1.0000	1.0000	1.0000	0.9999	1.0000	1.0000

III

STATISTICAL APPENDIX

A.1 – NEW MEASURES OF SECTORAL PRODUCTIVITY

The following legend is to be used in the statistical appendix A.1

1- Agriculture, hunting, forestry, and fishing	9 – Rubber and plastic products	17 – Production and distribution of electricity, gas, and water
2 – Mining and quarrying	10 – Other non-metallic mineral products	18 - Construction
3 – Food products, beverages, and tobacco	11 -Metals.	19 – Wholesale and retail trade
4 – Textiles, textile products, leather, and footwear	12 – Fabricated metal products, except machinery and equipment	20 – Service activities (transport, hotels, and restaurants)
5 – Wood and products of wood and cork	13 – Machinery and equipment, nec	21 – Post and telecommunications
6 – Pulp, paper, paper products, printing, and publishing	14 – Electrical machinery and apparatus	22 – Business activities (finance, real estate, and R&D)
7 – Coke, refined petroleum products, and nuclear fuel	15 – Transport equipment	23 – Public administration, education, and health
8 -Chemicals.	16 – Manufacturing nec; recycling	

Tab.A.1.1 – Productivity indicator based on Goodwin's normalized general coordinates for the US, Germany, France, and the UK

the U.S.

Industry/Year	1970	1975	1980	1985	1990	1995	1997	2000
1	2,0	1,9	1,8	2,0	2,0	2,0	2,1	2,1
2	2,5	-	-	2,9	2,9	2,8	3,0	3,2
3	2,7	-	-	3,2	3,1	3,2	3,1	3,7
4	-	-	-	-	-	-	3,9	-
5	3,8	3,7	3,9	-	3,9	-	-	-
6	-	4,1	-	4,1	4,4	-	-	4,0
7	-	4,6	-	-	-	-	-	-
8	-	-	5,1	-	-	4,6	-	-
9	-	-	5,2	4,7	-	5,1	5,1	5,5
10	-	-	-	5,9	-	-	-	5,9
11	9,1	5,5	-	-	-	-	-	-
12	9,1	-	6,3	-	-	-	-	-
13	-	-	6,4	-	8,1	-	-	10,0
14	-	-	7,5	-	8,4	-	-	-
15	-	-	-	10,3	11,5	-	-	13,2
16	9,3	-	-	9,5	-	-	9,6	-
17	-	-	-	-	-	-	13,3	-
18	-	-	-	-	-	-	-	-
19	7,2	-	10,0	-	-	-	-	-
20	5,4	19,6	-	-	-	10,0	-	16,7
21	-	6,7	-	-	-	11,1	-	-
22	-	9,0	-	-	-	17,3	-	-
23	6,3	8,4	21,2	28,8	24,2	20,6	-	27,1

France

Industry/Year	1970	1975	1980	1985	1990	1995	1997	2000
1	1,8	1,8	1,6	1,8	2,0	2,0	n.a.	1,9
2	2,5	2,4	2,5	2,6	2,8	2,6	n.a.	2,6
3	3,1	3,0	2,9	-	3,1	3,3	n.a.	
4	3,3	3,3	3,2	-	3,4	-	n.a.	
5	-	3,4	3,3	3,2	-	-	n.a.	
6	-	-	-	-	-	-	n.a.	
7	4,1	-	-	-	-	-	n.a.	
8	-	5,2	-	-	-	4,5	n.a.	
9	-	-	-	-	5,5	-	n.a.	4,2
10	6,2	-	5,2	5,4	6,7	-	n.a.	
11	-	-	6,3	5,7	9,2	5,2	n.a.	
12	-	-	8,1	7,2	-	-	n.a.	
13	-	-	8,9	9,8	-	-	n.a.	
14	-	-	12,6	-	-	-	n.a.	7,8
15	-	13,3	16,2	-	-	-	n.a.	
16	13,0	19,6	-	12,7	-	-	n.a.	
17	-	-	-	17,6	-	24,6	n.a.	
18	-	-	-	-	-	18,5	n.a.	
19	-	-	-	-	-	7,5	n.a.	
20	-	44,8	-	-	-	-	n.a.	
21	-	-	-	-	-	-	n.a.	
22	-	-	-	-	-	-	n.a.	24,5
23	48,9	69,7	60,7	57,6	53,4	-	n.a.	41,6

Germany

Industry/Year	1970	1975	1980	1985	1990	1995	1997	2000
1	1,7	n.a.	1,5	1,6	1,7	2,1	n.a.	2,0
2	2,1	n.a.	2,0	2,1	2,2	-	n.a.	-
3	-	n.a.	2,9	2,9	-	3,2	n.a.	2,7
4	2,6	n.a.	3,2	3,1	-	3,4	n.a.	49,6
5	2,9	n.a.	-	3,7	3,5	-	n.a.	-
6	3,4	n.a.	-	-	-	-	n.a.	-
7	-	n.a.	-	-	-	-	n.a.	-
8	-	n.a.	-	-	4,2	-	n.a.	-
9	4,5	n.a.	-	-	4,5	-	n.a.	13,9
10	5,7	n.a.	-	-	-	-	n.a.	9,4
11	6,1	n.a.	5,4	-	-	-	n.a.	-
12	7,0	n.a.	5,7	6,0	6,3	-	n.a.	-
13	10,1	n.a.	-	-	-	4,7	n.a.	-
14	-	n.a.	-	-	-	4,9	n.a.	-
15	-	n.a.	-	-	8,5	6,4	n.a.	5,8
16	-	n.a.	-	-	8,7	8,0	n.a.	-
17	-	n.a.	-	-	12,1	-	n.a.	-
18	-	n.a.	-	-	14,2	-	n.a.	-
19	-	n.a.	-	22,1	16,0	18,8	n.a.	-
20	-	n.a.	14,6	13,3	83,7	16,5	n.a.	4,3
21	-	n.a.	-	8,3	-	-	n.a.	3,6
22	20,3	n.a.	-	9,3	29,1	-	n.a.	-
23	-	n.a.	10,1	10,3	-	10,8	n.a.	-

the U.K.

Industry/Year	1970	1975	1980	1985	1990	1995	1997	2000
1	1,9	n.a.	1,8	1,8	1,9	1,9	2,0	1,9
2	2,4	n.a.	2,5	2,5	3,0	-	2,9	3,0
3	2,7	n.a.	-	-	-	-	-	-
4	-	n.a.	-	-	-	-	-	-
5	-	n.a.	-	3,1	-	-	-	-
6	4,0	n.a.	-	3,2	-	-	-	-
7	-	n.a.	3,5	-	-	-	-	-
8	-	n.a.	3,7	-	-	-	-	-
9	-	n.a.	4,7	3,7	-	-	-	-
10	-	n.a.	-	3,9	-	-	-	-
11	-	n.a.	-	4,5	16,4	-	-	5,0
12	-	n.a.	-	5,3	13,5	9,8	8,4	-
13	6,6	n.a.	7,1	7,1	3,3	8,9	-	-
14	11,1	n.a.	-	-	-	3,1	-	5,4
15	-	n.a.	-	-	-	-	3,7	5,9
16	-	n.a.	-	-	3,7	-	-	8,0
17	17,6	n.a.	-	-	-	4,6	-	-
18	28,7	n.a.	-	-	-	-	-	-
19	-	n.a.	-	-	-	-	-	16,3
20	-	n.a.	22,3	11,3	-	3,6	6,0	-
21	-	n.a.	-	31,1	4,4	-	4,7	-
22	-	n.a.	-	22,2	4,6	-	5,2	-
23	-	n.a.	-	19,3	5,4	3,8	5,0	-

Fig.A.1.1 – Productivity indicator based on Pasinetti’s vertical integration - US

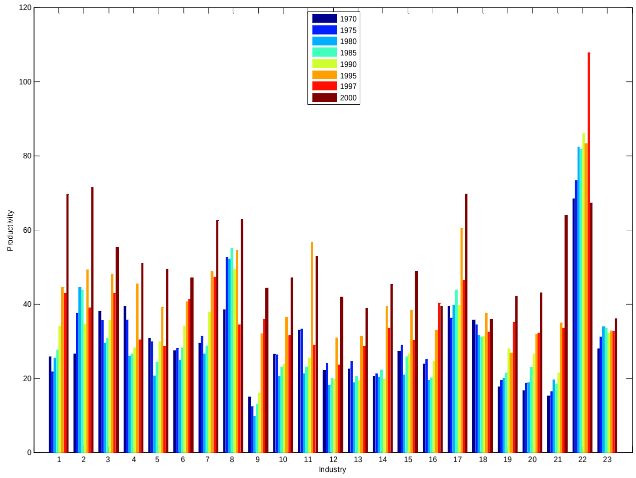


Fig.A.1.2 – Productivity indicator based on Pasinetti’s vertical integration - Germany

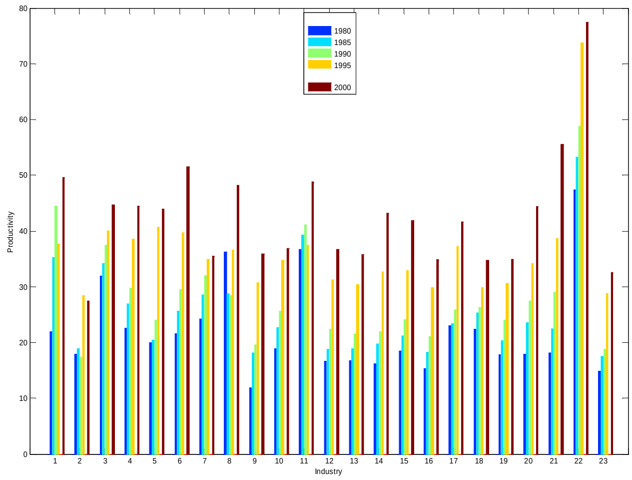


Fig.A.1.3 – Productivity indicator based on Pasinetti's vertical integration - France

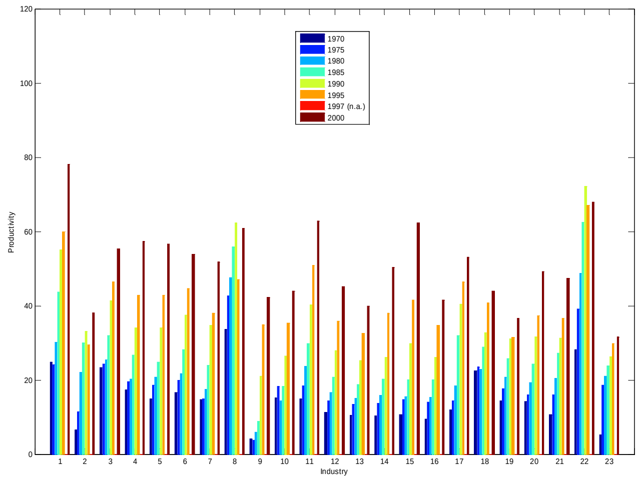


Fig.A.1.4 – Productivity indicator based on Pasinetti's vertical integration - UK

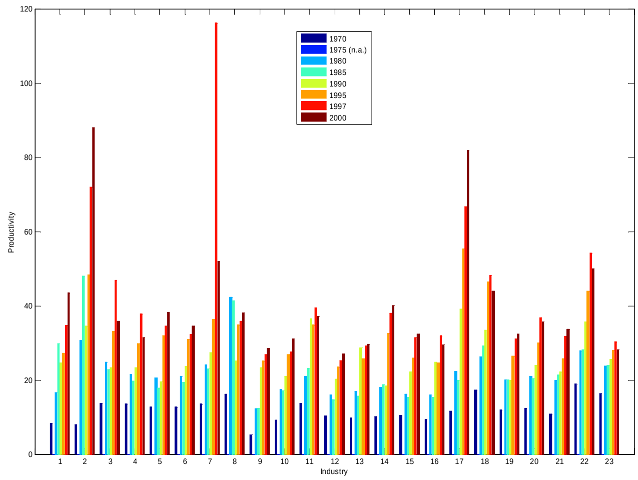


Fig.A.1.5 - Labor productivity indicator based on Gossling's subsystems -US

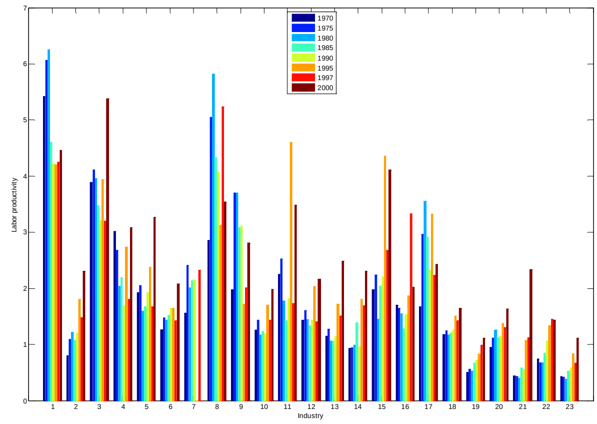


Fig.A.1.6 - Capital productivity indicator based on Gossling's subsystems - US

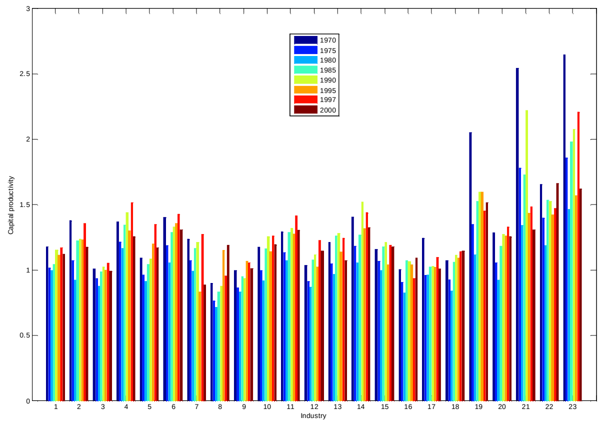


Fig.A.1.7 – Multi-factor productivity indicator based on Gossling's subsystems -US

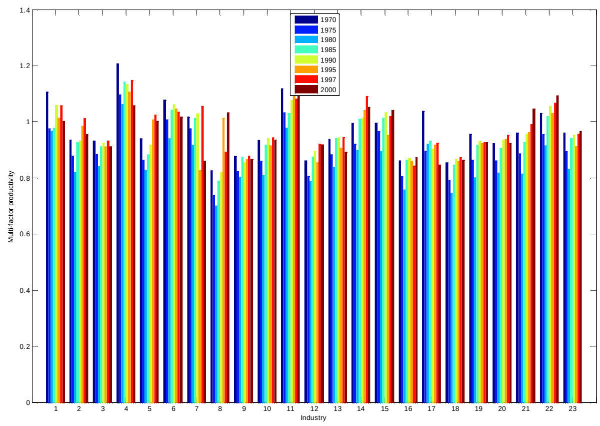


Fig.A.1.8 - Labor productivity indicator based on Gossling's subsystems - GER

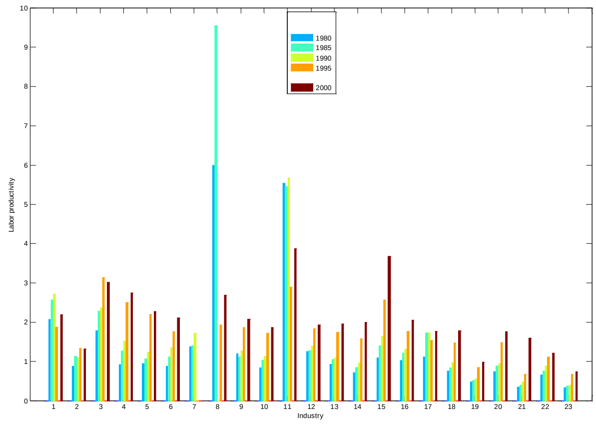


Fig.A.1.9 - Capital productivity indicator based on Gossling's subsystems - GER

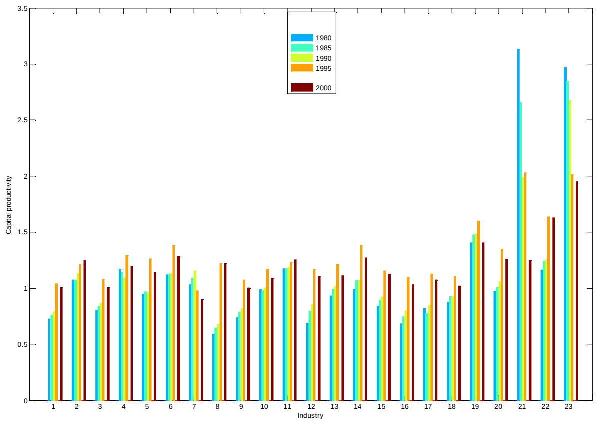


Fig.A.1.10 –Multi-factor productivity indicator based on Gossling's subsystems-GER

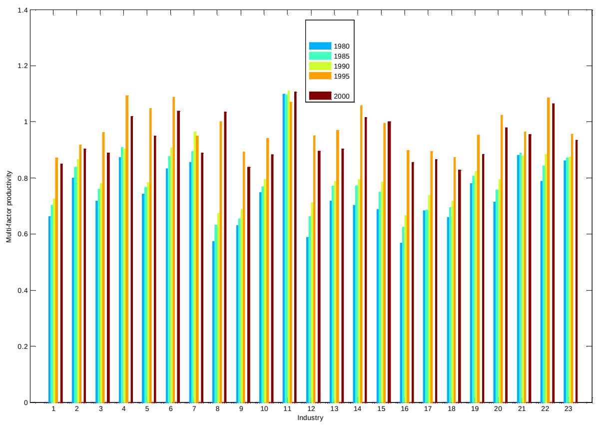


Fig.A.1.11 - Labor productivity indicator based on Gossling's subsystems - FRA

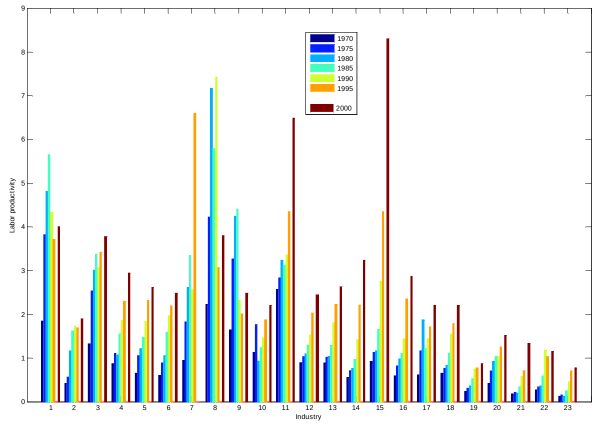


Fig.A.1.12 - Capital productivity indicator based on Gossling's subsystems - FRA

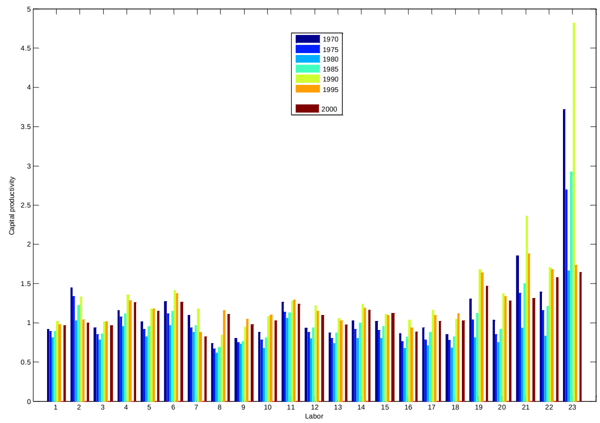


Fig.A.1.13 –Multi-factor productivity indicator based on Gossling's subsystems-FRA

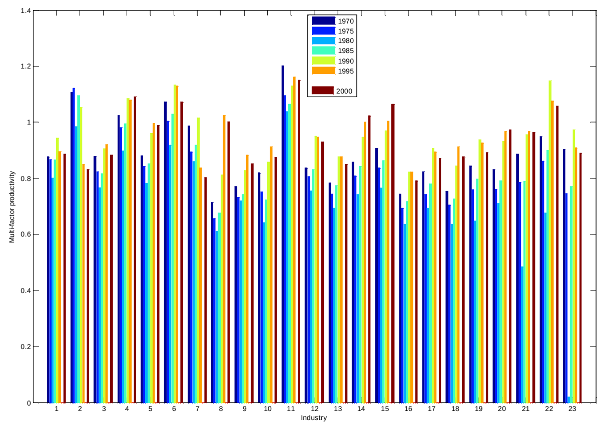


Fig.A.1.14 - Labor productivity indicator based on Gossling's subsystems - UK

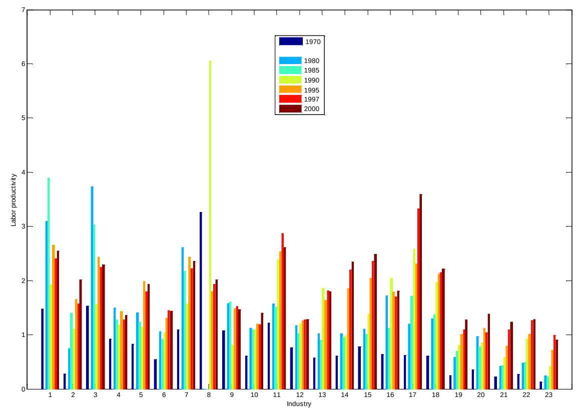


Fig.A.1.15 - Capital productivity indicator based on Gossling's subsystems - UK

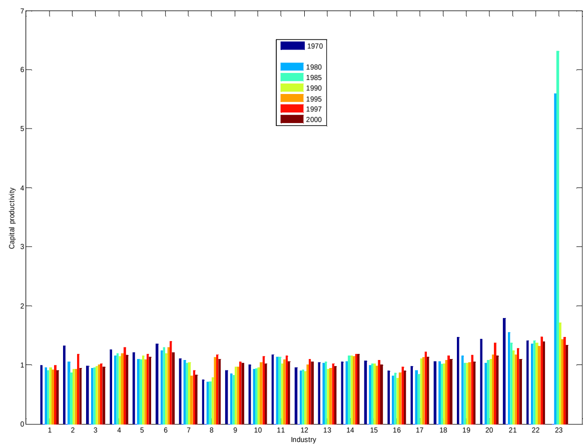
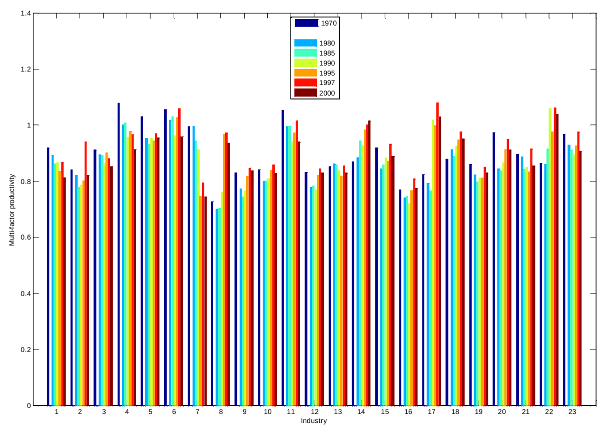


Fig.A.1.16 –Multi-factor productivity indicator based on Gossling's subsystems-UK



A.2 – PRODUCTIVITY IN THE ITALIAN REGIONS: DEVELOPMENT OF ALTERNATIVE INDICATORS BASED ON INPUT-OUTPUT TABLES

Tab.A.2.1 - Index based on physical quantities – 2001 (values)

Industry	Trentino	Sicily	Piedmont	Lombardy	Veneto	Campania	Emilia Romagna	Tuscany
Agriculture and Fishing	37.0	29.6	62.5	84.0	62.3	33.3	66.0	57.2
Extraction of minerals	110.6	980.8	660.1	447.4	910.3	539.8	1138.6	417.2
Mfr. of Food, Beverages and Tobacco	104.7	83.6	117.8	130.0	127.2	94.0	106.9	161.2
Mfr. of Textiles, Wearing Apparel, Leather	209.9	174.0	139.1	125.3	125.6	107.2	135.5	108.6
Mfr. of Wood and Wood Products	92.1	91.4	125.0	128.1	127.2	101.3	140.7	128.5
Mfr. of Paper Products, Printing and Publishing	197.3	144.4	151.8	126.9	152.8	132.1	156.3	157.0
Mfr. of Refined Petroleum	408.0	375.4	402.3	392.0	590.3	334.9	985.5	380.8
Mfr. of Chemicals and Man-Made Fibers Etc.	367.5	350.2	307.2	219.4	283.7	354.2	286.3	285.9
Mfr. of Rubber and Plastic Products	108.0	118.3	124.1	104.6	122.8	135.6	133.7	155.9
Mfr. of Other Non Metallic Mineral Products	105.9	113.0	172.7	157.5	137.3	119.1	134.9	130.7
Mfr. And Processing of Basic Metals	184.0	163.0	157.7	143.6	159.7	152.2	164.0	180.6
Mfr. of Machinery and Equipment n.e.c.	142.8	281.6	125.4	117.1	118.0	185.9	118.1	155.5
Mfr. of Electrical and Optical Equipment	310.7	170.8	153.1	136.5	141.0	139.3	162.5	186.1
Mfr. of Transport Equipment	417.1	262.8	123.7	215.5	242.1	159.1	197.8	245.7
Mfr. of Furniture, Mfr. n.e.c	227.6	145.5	106.6	115.9	87.9	120.8	139.1	88.1
Electricity, Gas and Water Supply	225.2	153.9	218.9	230.6	235.3	164.5	232.4	200.5
Construction	70.4	55.0	62.8	60.9	63.3	54.2	64.0	58.6
Wholesale and Retail Trade	63.6	70.9	78.6	85.4	79.1	66.4	78.8	73.9
Hotels and Restaurants	48.3	44.9	51.9	61.4	54.4	41.9	56.3	52.5
Transport, Post and Telecommunications	12.2	92.8	108.9	113.2	115.3	88.0	115.6	111.4
Financial Intermediation, Insurance	111.8	139.0	122.9	135.6	135.2	125.5	135.1	124.4
Computer, Research and Development, Consultancy	142.4	140.5	128.4	115.5	143.9	127.3	136.0	138.0
Public Administration	53.3	59.7	67.5	83.1	65.6	53.1	68.8	55.1
Education	39.3	38.2	37.4	39.1	38.2	36.0	42.1	43.3
Health Care Activities Etc.	46.7	53.1	51.3	51.9	53.3	62.0	50.1	53.9
Other Service Activities	58.5	42.3	46.0	48.7	46.9	33.8	44.4	44.8
Renting of Machinery and Equipment, Real Estate Activities	109.3	87.2	92.3	94.1	105.3	80.2	102.4	100.9

Tab.A.2.2 - Index based on physical quantities – 2004 (values)

Industry	Trentino	Sicily	Piedmont	Lombardy	Veneto	Campania	Emilia Romagna	Tuscany
Agriculture and Fishing	34.4	33.7	65.0	89.8	69.0	39.9	79.0	64.7
Extraction of minerals	114.0	944.0	681.0	471.8	978.2	567.3	1514.8	421.6
Mfr. of Food, Beverages and Tobacco	106.1	82.2	112.1	120.7	121.4	98.1	106.9	146.1
Mfr. of Textiles, Wearing Apparel, Leather	199.7	156.9	138.3	118.9	126.0	103.8	138.6	103.2
Mfr. of Wood and Wood Products	105.2	97.6	141.0	137.7	131.7	104.8	156.2	133.2
Mfr. of Paper Products, Printing and Publishing	179.7	136.2	147.6	121.0	147.7	132.9	157.5	146.8
Mfr. of Refined Petroleum	319.2	264.0	393.8	355.8	549.1	324.7	868.1	338.8
Mfr. of Chemicals and Man-Made Fibers Etc.	344.8	297.2	342.5	231.5	301.1	373.9	328.9	281.7
Mfr. of Rubber and Plastic Products	117.6	121.2	139.1	111.2	128.9	142.7	146.6	158.0
Mfr. of Other Non Metallic Mineral Products	106.4	115.6	180.7	158.3	145.7	126.1	145.4	130.8
Mfr. And Processing of Basic Metals	186.2	159.3	170.3	150.6	167.1	155.6	176.0	181.9
Mfr. of Machinery and Equipment n.e.c.	142.3	268.0	130.5	111.5	117.9	184.6	117.4	149.3
Mfr. of Electrical and Optical Equipment	296.1	160.2	161.6	126.7	138.2	135.8	163.1	177.7
Mfr. of Transport Equipment	399.5	266.9	130.1	194.5	223.9	152.9	197.8	227.4
Mfr. of Furniture, Mfr. n.e.c	257.8	155.7	116.6	120.1	89.0	124.8	149.5	92.6
Electricity, Gas and Water Supply	285.5	157.4	229.3	225.4	253.3	178.3	253.7	211.8
Construction	64.0	51.0	58.8	57.9	57.7	50.3	63.5	53.8
Wholesale and Retail Trade	64.9	70.7	77.0	82.2	80.4	65.0	77.8	76.2
Hotels and Restaurants	44.4	41.4	46.0	54.6	50.2	40.3	52.0	45.7
Transport, Post and Telecommunications	132.4	85.9	114.5	116.6	120.3	86.6	120.6	107.5
Financial Intermediation, Insurance	109.0	135.9	120.5	139.7	136.7	123.0	137.9	124.6
Computer, Research and Development, Consultancy	129.8	137.8	119.5	107.4	133.3	110.8	128.2	127.9
Public Administration	61.2	62.9	71.0	87.5	66.6	55.1	71.8	57.6
Education	40.1	38.5	38.1	39.4	38.7	36.5	42.8	44.2
Health Care Activities Etc.	46.3	58.8	50.4	50.7	52.8	63.5	52.5	52.8
Other Service Activities	56.4	40.6	45.3	46.5	45.5	30.8	42.6	44.0
Renting of Machinery and Equipment, Real Estate Activities	94.2	84.8	86.7	89.4	99.1	73.8	94.0	96.0

Tab.A.2.3 - Index based on production prices – 2001 (values)

Industry	Trentino	Sicily	Piedmont	Lombardy	Veneto	Campania	Emilia Romagna	Tuscany
Agriculture and Fishing	0.5726	0.5552	0.6477	0.6556	0.6597	0.5103	0.6732	0.5662
Extraction of minerals	0.9954	0.9591	1.0158	1.1099	1.0403	0.8736	0.9765	1.0615
Mfr. of Food, Beverages and Tobacco	1.1069	1.0052	1.237	1.1733	1.1516	0.9481	1.2088	1.0177
Mfr. of Textiles, Wearing Apparel, Leather	0.9904	0.8201	1.1844	1.0915	1.2153	0.9654	1.1379	1.2081
Mfr. of Wood and Wood Products	1.3078	0.8184	1.0255	0.9378	1.0488	0.9061	0.9747	1.0833
Mfr. of Paper Products, Printing and Publishing	1.3726	0.891	1.2792	1.1125	1.2203	0.9894	1.1398	1.226
Mfr. of Refined Petroleum	1.3198	1.3087	1.3217	1.3619	1.2887	1.136	1.2247	1.4375
Mfr. of Chemicals and Man-Made Fibers Etc.	1.3495	1.4815	1.5063	1.4835	1.5317	1.1973	1.5029	1.5161
Mfr. of Rubber and Plastic Products	1.125	0.9394	1.1845	1.1317	1.1928	0.9702	1.1444	1.1718
Mfr. of Other Non Metallic Mineral Products	1.0676	0.9079	1.0804	1.0427	1.1097	0.8987	1.1785	1.0616
Mfr. And Processing of Basic Metals	1.074	0.8565	1.0628	1.1235	1.0958	0.9187	1.0669	1.0211
Mfr. of Machinery and Equipment n.e.c.	1.1373	1.0355	1.1899	1.1663	1.194	1.0058	1.2352	1.1442
Mfr. of Electrical and Optical Equipment	1.0511	1.0386	1.1613	1.1145	1.1244	0.9918	1.1672	1.1058
Mfr. of Transport Equipment	1.0361	1.1284	1.3988	1.3299	1.3844	1.24	1.4219	1.3377
Mfr. of Furniture, Mfr. n.e.c	1.1506	0.9285	0.899	1.1547	1.0395	0.8658	1.2407	0.8877
Electricity, Gas and Water Supply	1.6288	1.2679	1.2035	1.2429	1.4493	1.0089	1.5011	1.3994
Construction	0.9379	0.8405	0.9379	0.8786	0.9594	0.8347	0.9695	0.8822
Wholesale and Retail Trade	0.7504	0.7894	0.9093	0.9178	0.9285	0.8004	0.9381	0.8637
Hotels and Restaurants	0.7777	0.7574	0.8072	0.7812	0.8214	0.7163	0.8723	0.7466
Transport, Post and Telecommunications	0.7947	0.9162	0.97	0.98	0.9642	0.9204	0.9679	0.932
Financial Intermediation, Insurance	0.6405	0.7917	0.8937	0.8777	0.904	0.7302	0.8951	0.8388
Computer, Research and Development, Consultancy	0.9574	0.9581	1.0134	0.9656	1.0867	0.8898	1.1022	1.0242
Public Administration	0.7178	0.6269	0.6737	0.6674	0.6781	0.6173	0.6793	0.6533
Education	0.5504	0.473	0.5318	0.5274	0.5427	0.4567	0.5492	0.5252
Health Care Activities Etc.	0.5418	0.619	0.6562	0.6487	0.6755	0.6363	0.6736	0.6466
Other Service Activities	0.7669	0.5427	0.6815	0.7101	0.6867	0.5711	0.6791	0.6499
Renting of Machinery and Equipment, Real Estate Activities	0.6935	0.579	0.6004	0.5916	0.6266	0.5382	0.6299	0.6081

Tab.A.2.4 - Index based on production prices – 2004 (values)

Industry	Trentino	Sicily	Piedmont	Lombardy	Veneto	Campania	Emilia Romagna	Tuscany
Agriculture and Fishing	0.5877	0.5479	0.6482	0.6464	0.6943	0.5102	0.714	0.5619
Extraction of minerals	1.0471	1.0183	1.132	0.9464	1.1347	0.9155	1.0227	1.0387
Mfr. of Food, Beverages and Tobacco	1.1023	0.9984	1.2518	1.1331	1.1446	0.9484	1.2486	0.9958
Mfr. of Textiles, Wearing Apparel, Leather	0.9813	0.7542	1.1182	1.0321	1.1812	0.9078	1.121	1.1678
Mfr. of Wood and Wood Products	1.3882	0.8104	1.0678	0.9638	1.06	0.908	1.002	1.1132
Mfr. of Paper Products, Printing and Publishing	1.2985	0.923	1.2016	1.0932	1.1712	1.028	1.1321	1.2101
Mfr. of Refined Petroleum	1.4161	1.4272	1.4562	1.3183	1.3612	1.2726	1.3638	1.4267
Mfr. of Chemicals and Man-Made Fibers Etc.	1.4128	1.4833	1.5072	1.4521	1.4744	1.275	1.6125	1.4575
Mfr. of Rubber and Plastic Products	1.1262	0.9137	1.2116	1.142	1.1809	0.971	1.1789	1.1853
Mfr. of Other Non Metallic Mineral Products	1.0908	0.9452	1.1318	1.0375	1.1225	0.9137	1.2171	1.0478
Mfr. And Processing of Basic Metals	1.0648	0.8305	1.0891	1.1316	1.108	0.9672	1.085	1.0258
Mfr. of Machinery and Equipment n.e.c.	1.121	0.9837	1.2232	1.1568	1.1692	1.0187	1.1995	1.0911
Mfr. of Electrical and Optical Equipment	0.9625	0.9452	1.1634	1.0794	1.0743	0.9586	1.1071	1.0291
Mfr. of Transport Equipment	0.9421	1.0597	1.4495	1.3288	1.3591	1.2509	1.3973	1.2831
Mfr. of Furniture, Mfr. n.e.c	1.134	0.8761	0.8926	1.1385	0.9981	0.8324	1.2183	0.868
Electricity, Gas and Water Supply	1.769	1.2427	1.2475	1.1526	1.5063	0.9989	1.6474	1.4096
Construction	0.937	0.8134	0.9501	0.8506	0.9335	0.8344	1.0051	0.8525
Wholesale and Retail Trade	0.7662	0.7834	0.9272	0.9063	0.9288	0.8083	0.9334	0.8745
Hotels and Restaurants	0.7742	0.7548	0.8189	0.7484	0.8196	0.7198	0.848	0.7301
Transport, Post and Telecommunications	0.8589	0.8483	0.9875	0.9817	0.9738	0.918	0.9411	0.8943
Financial Intermediation, Insurance	0.6744	0.7962	0.9126	0.9001	0.9291	0.7558	0.9314	0.8516
Computer, Research and Development, Consultancy	0.967	0.968	0.9888	0.9533	1.065	0.8511	1.0952	0.9946
Public Administration	0.76	0.6293	0.6998	0.6724	0.7014	0.6348	0.6952	0.6612
Education	0.5683	0.4742	0.5466	0.5307	0.5557	0.4518	0.5809	0.5357
Health Care Activities Etc.	0.5707	0.6737	0.6674	0.6444	0.6779	0.7029	0.7005	0.6531
Other Service Activities	0.8052	0.5257	0.6897	0.7022	0.6874	0.563	0.6743	0.6467
Renting of Machinery and Equipment, Real Estate Activities	0.6921	0.6001	0.6113	0.5957	0.6435	0.5445	0.6501	0.6174

Tab.A.2.5 - The Velupillai-Fredholm-Zambelli index

	2001	2004
Global	0.98	0.94
Trentino	0.66	0.69
Sicily	0.58	0.57
Piedmont	0.67	0.66
Lombardy	0.67	0.66
Veneto	0.68	0.69
Campania	0.56	0.56
E-Romagna	0.69	0.70
Tuscany	0.65	0.65

Fig.A.2.1 - Production prices – Agriculture and Fishing 2001

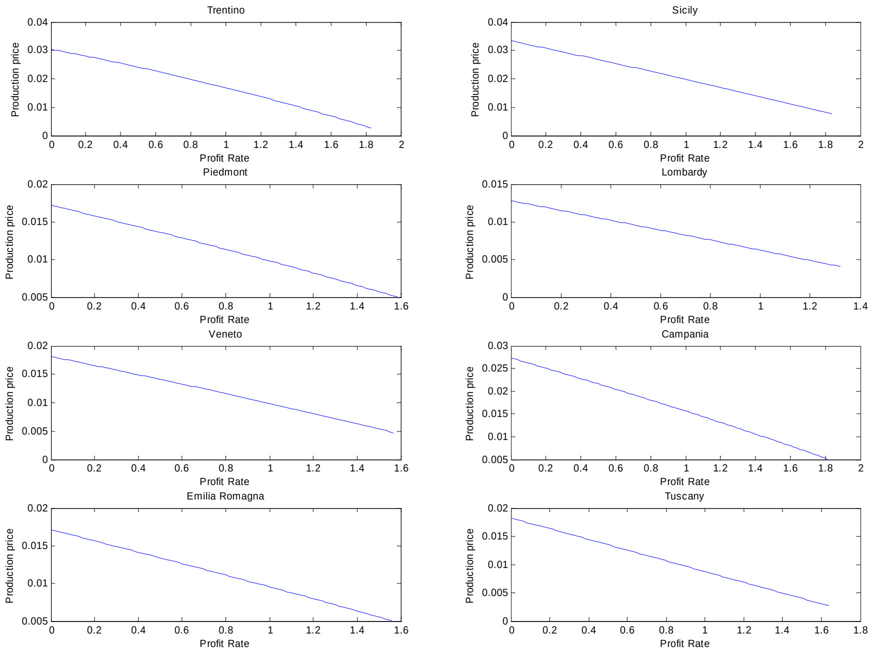


Fig.A.2.2 - Production prices – Agriculture and Fishing 2004

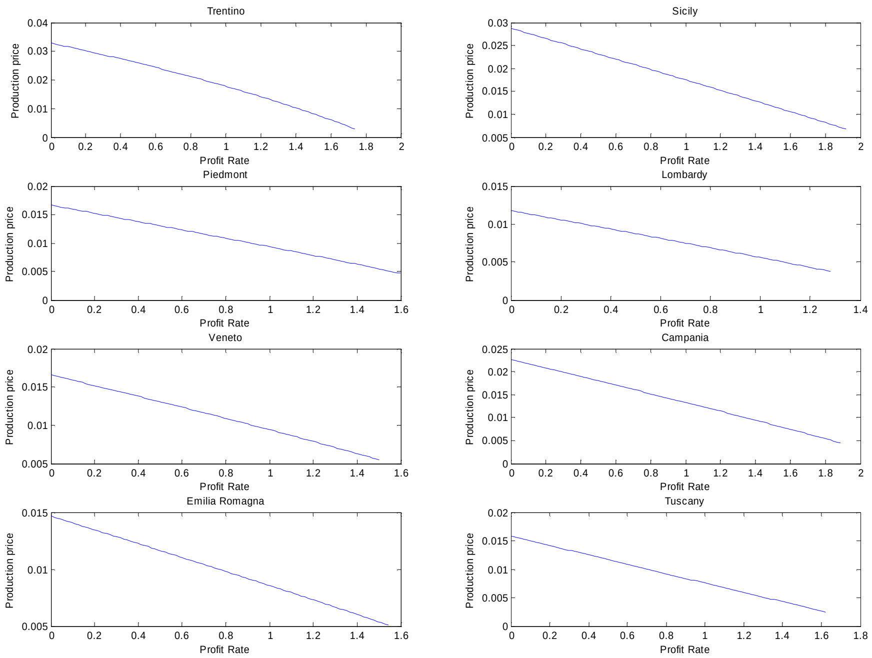


Fig.A.2.3 - Production prices – Extraction of minerals 2001

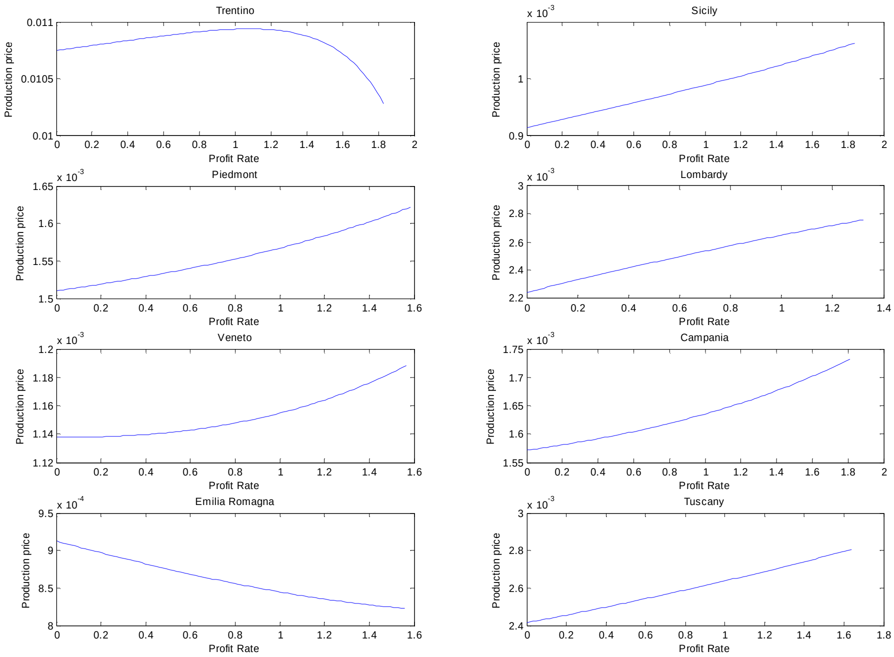


Fig.A.2.4 - Production prices – Extraction of minerals 2004

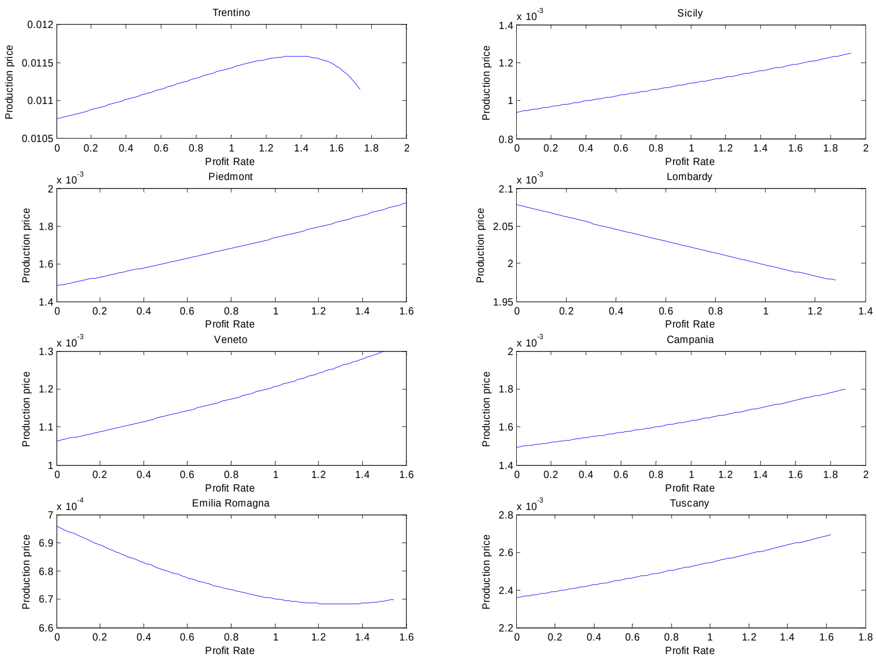


Fig.A.2.5 - Production prices – Mfr. of Food, Beverages and Tobacco 2001

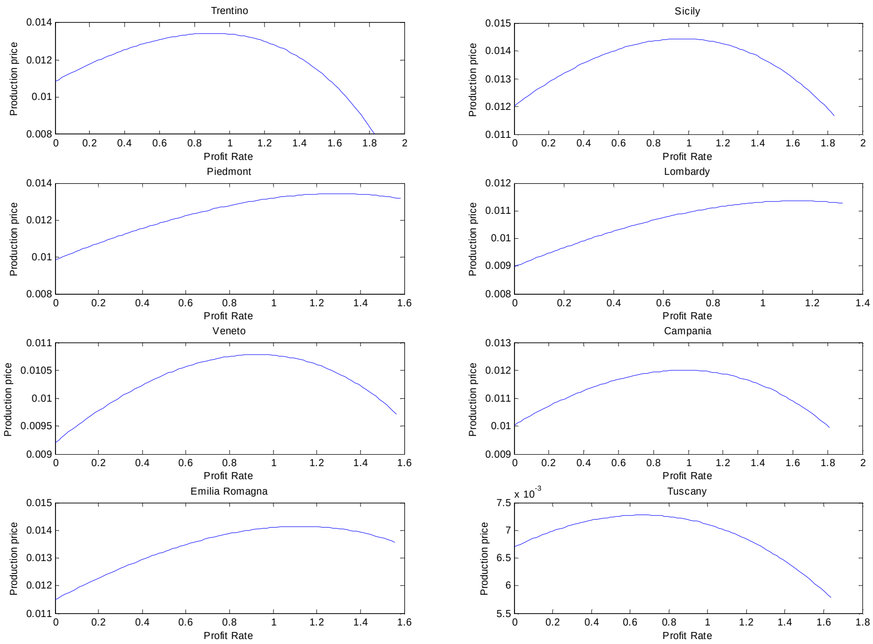


Fig.A.2.6 - Production prices – Mfr. of Food, Beverages and Tobacco 2004

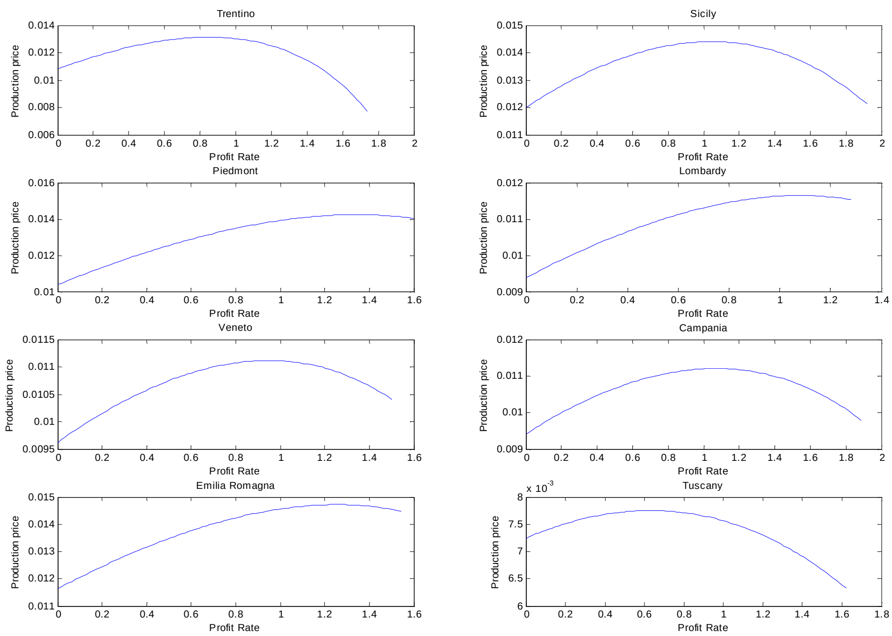


Fig.A.2.7 - Production prices – Mfr. of Textiles, Wearing Apparel, Leather 2001

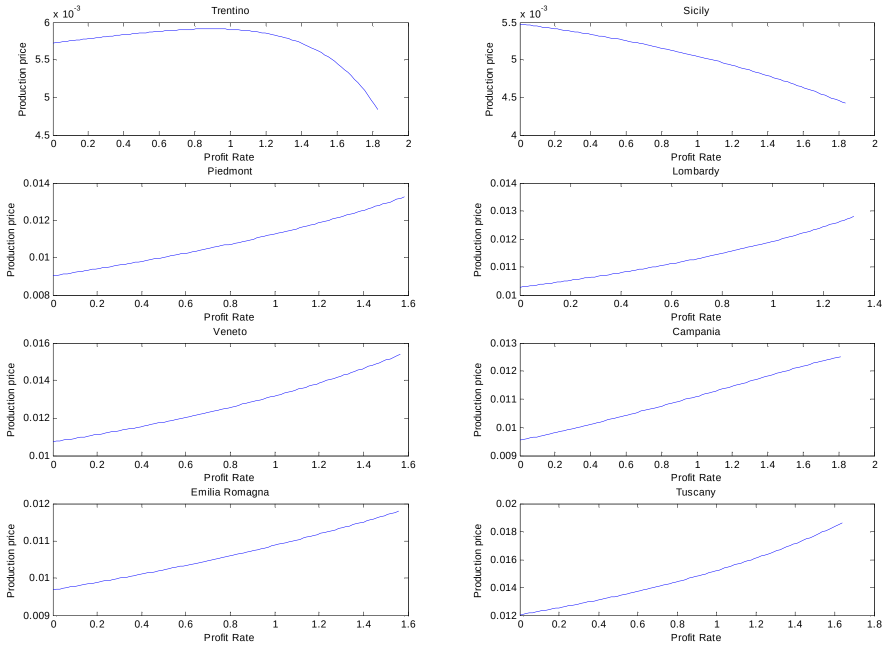


Fig.A.2.8 - Production prices – Mfr. of Textiles, Wearing Apparel, Leather 2004

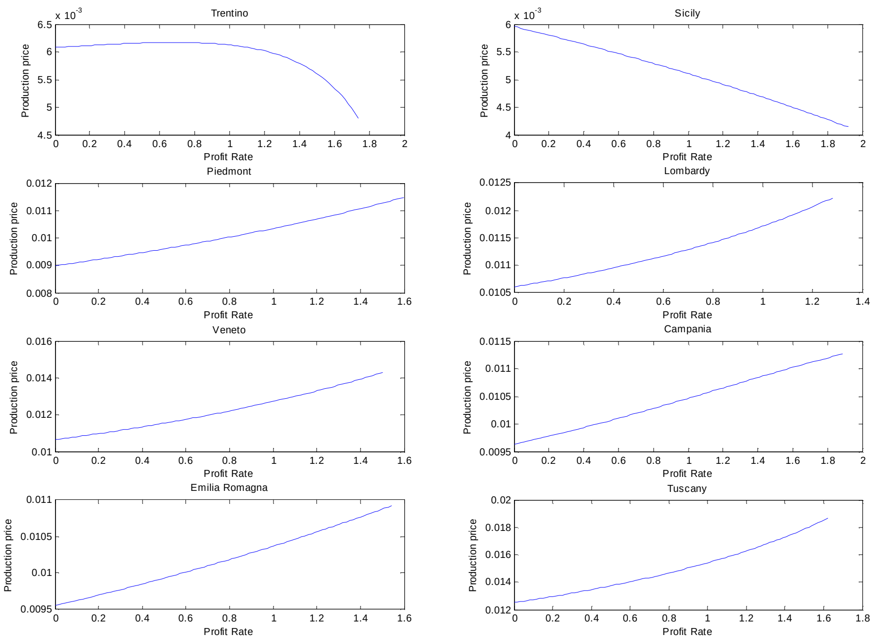


Fig.A.2.9 - Production prices – Mfr. of Wood and Wood Products 2004

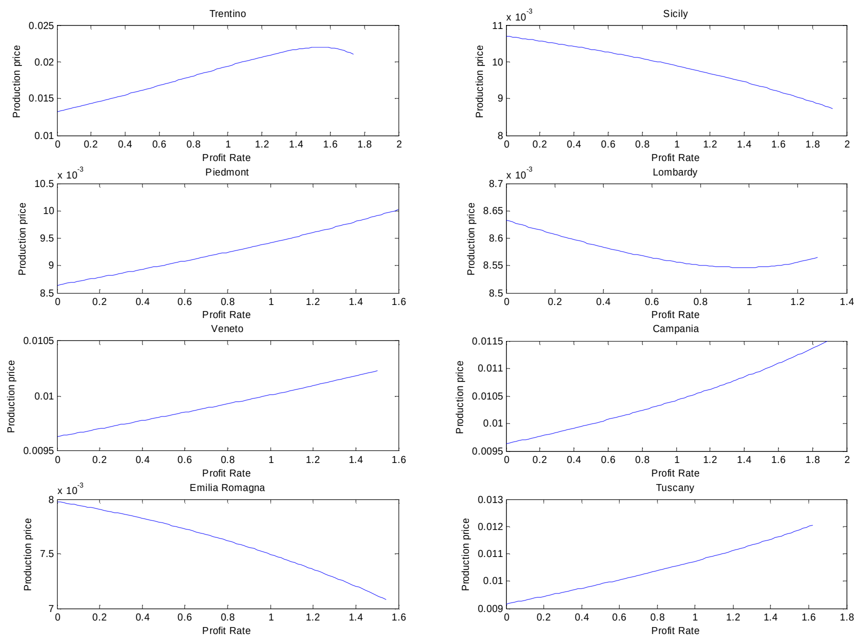


Fig.A.2.10 - Production prices – Mfr. of Wood and Wood Products 2004

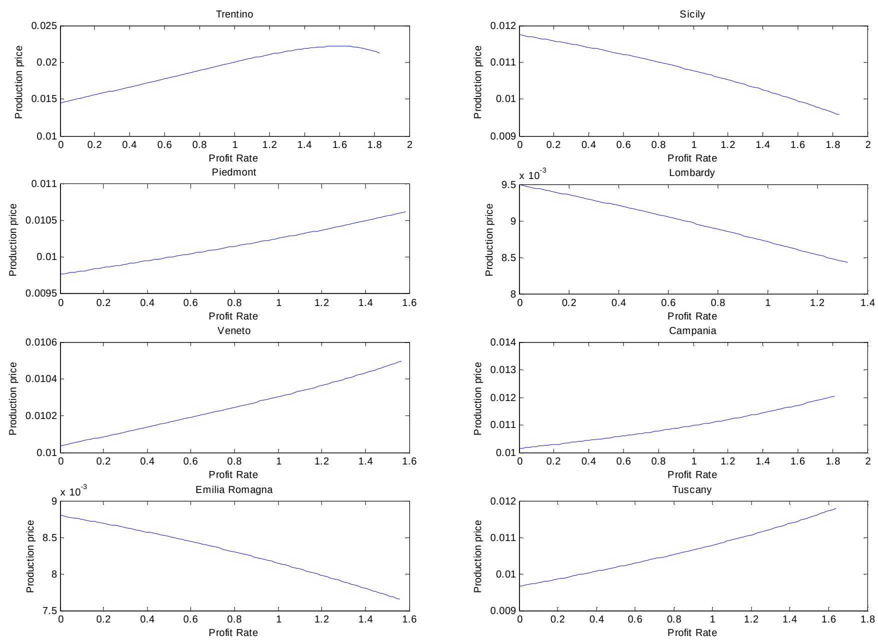


Fig.A.2.11 - Production prices – Mfr. of Paper Products, Printing and Publishing 2001

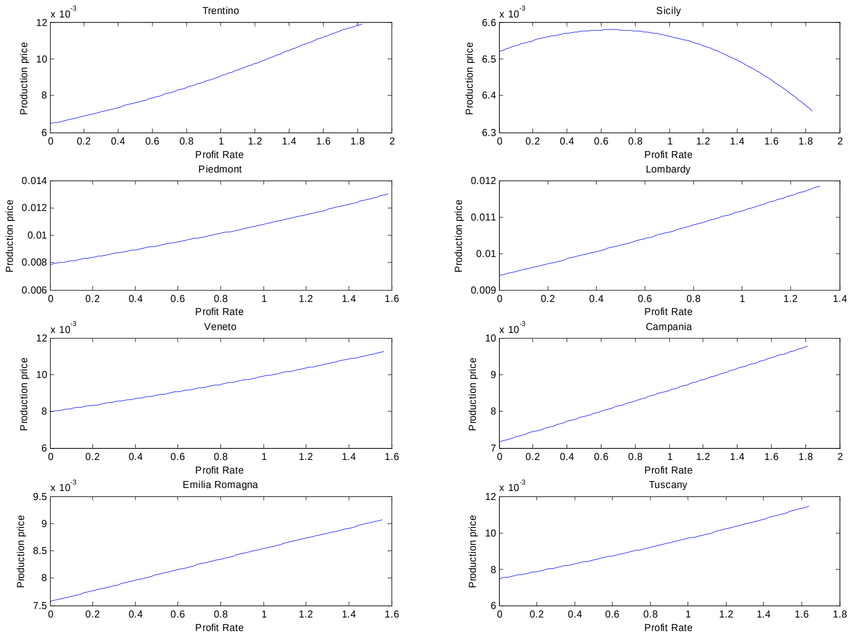


Fig.A.2.12 - Production prices – Mfr. of Paper Products, Printing and Publishing 2004

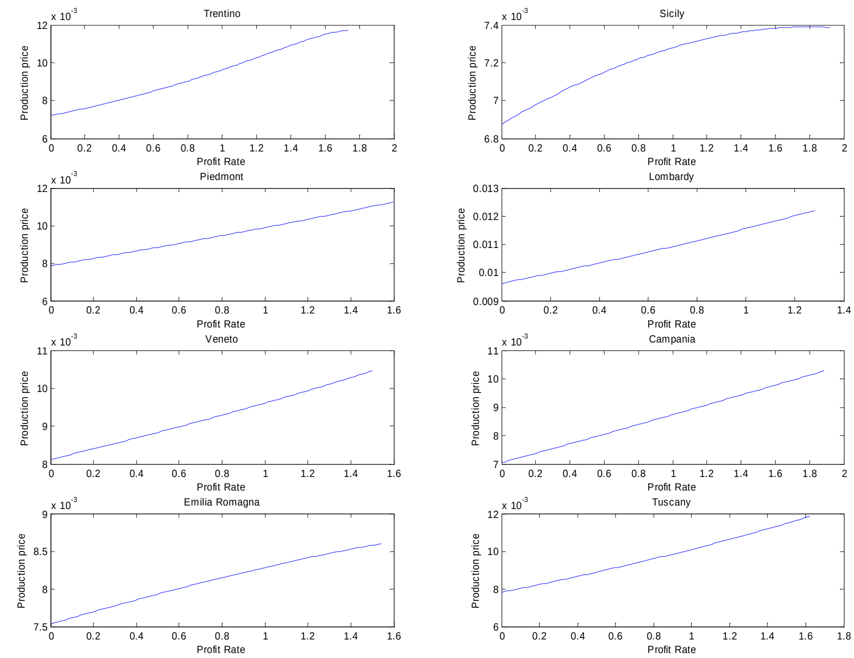


Fig.A.2.13 - Production prices – Mfr. of Refined Petroleum 2001

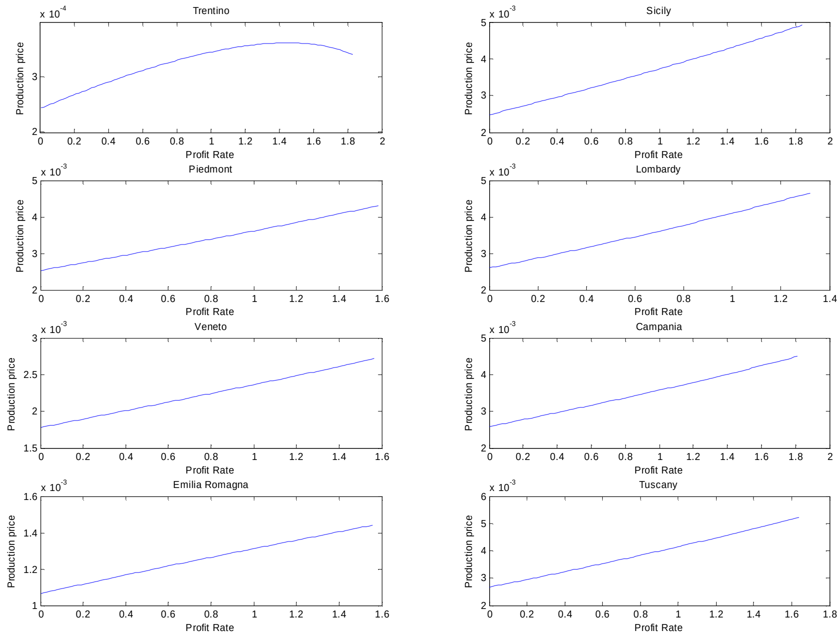


Fig.A.2.14 - Production prices – Mfr. of Refined Petroleum 2004

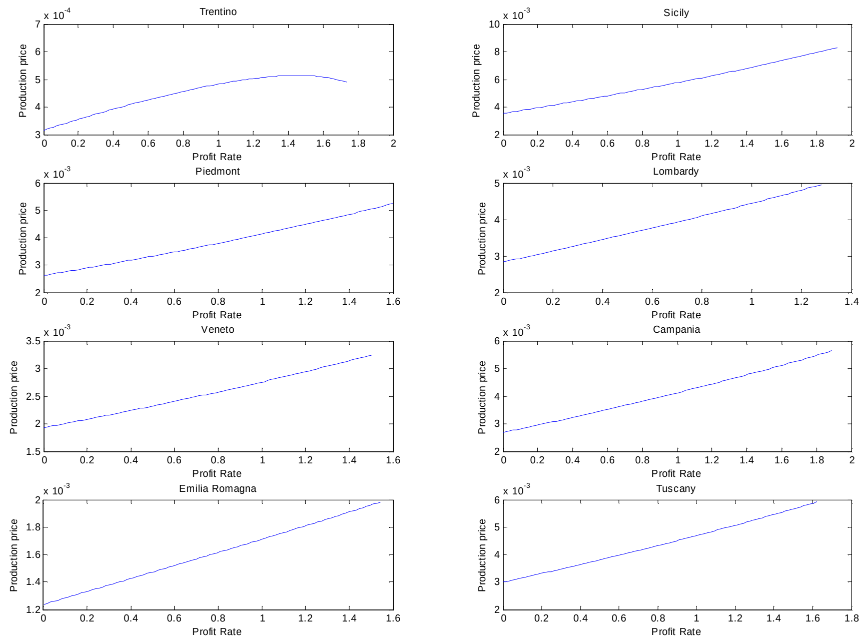


Fig.A.2.15 - Production prices – Mfr. of Chemicals and Man-Made Fibers Etc. 2001

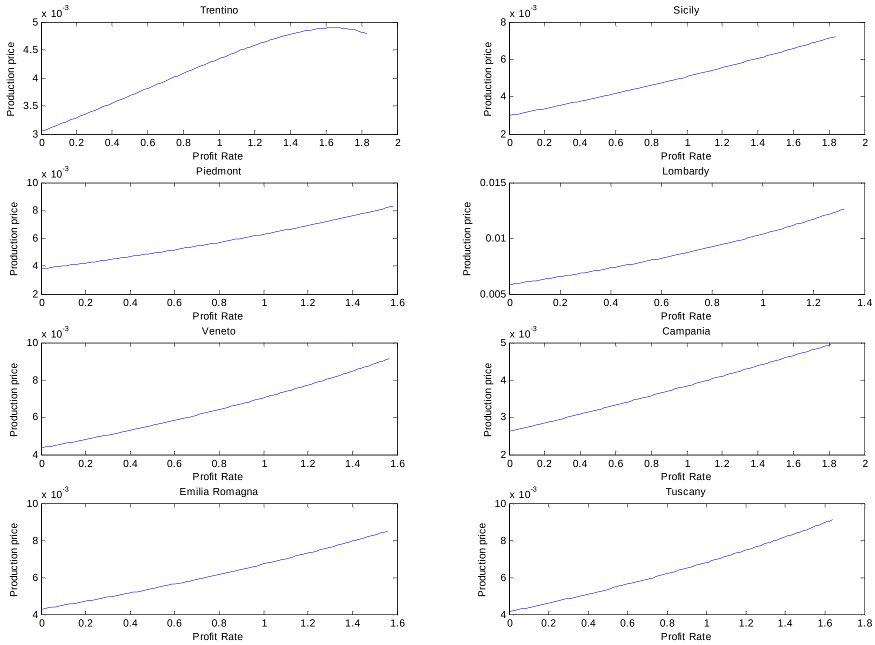


Fig.A.2.16 - Production prices – Mfr. of Chemicals and Man-Made Fibers Etc. 2004

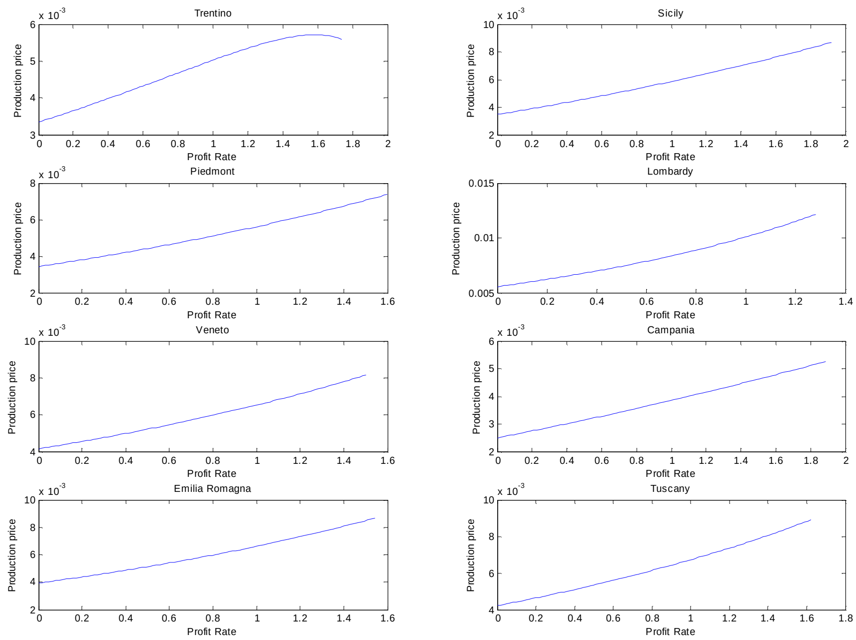


Fig.A.2.17 - Production prices – Mfr. of Rubber and Plastic Products 2001

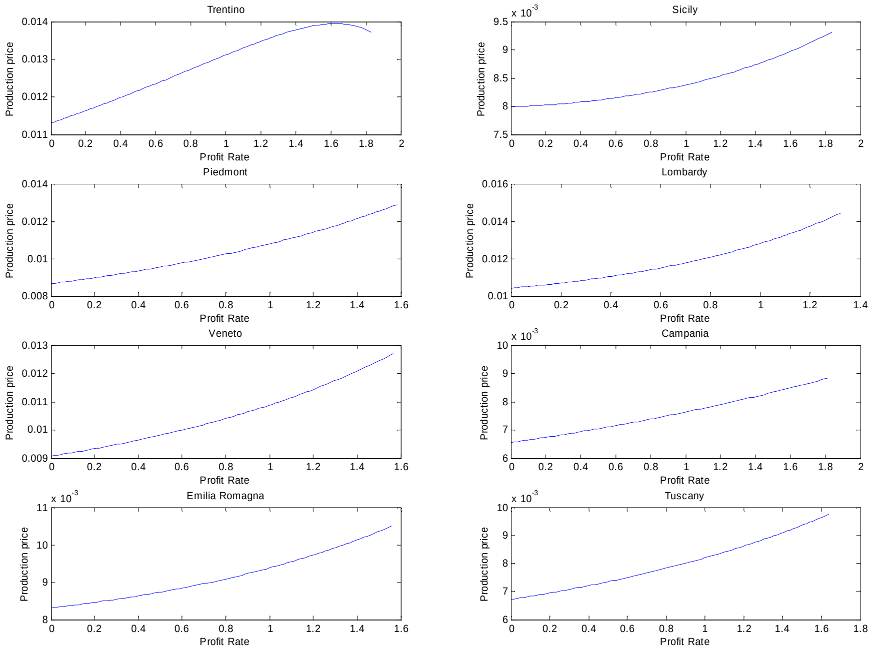


Fig.A.2.18 - Production prices – Mfr. of Rubber and Plastic Products 2004

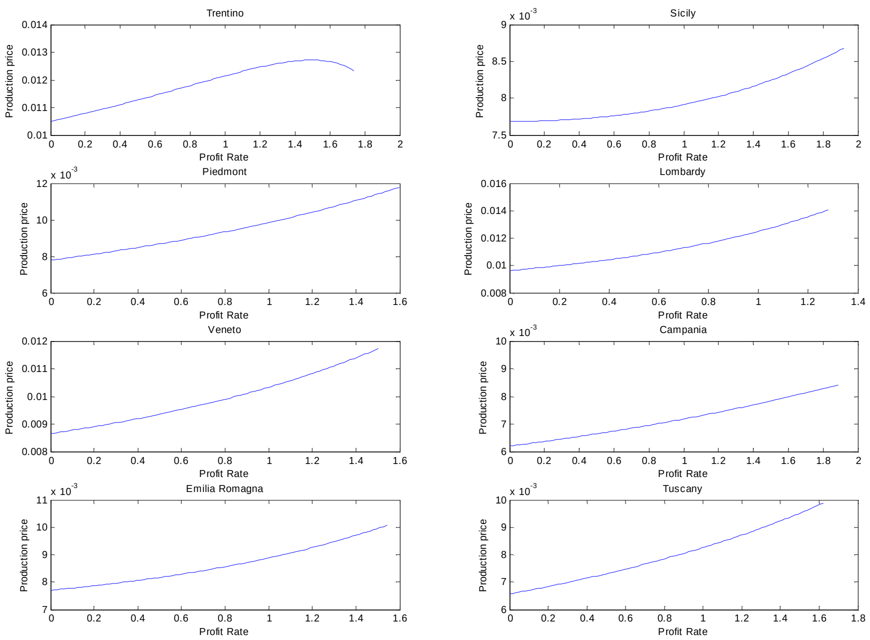


Fig.A.2.19 - Production prices – Mfr. of Other Non Metallic Mineral Products 2001

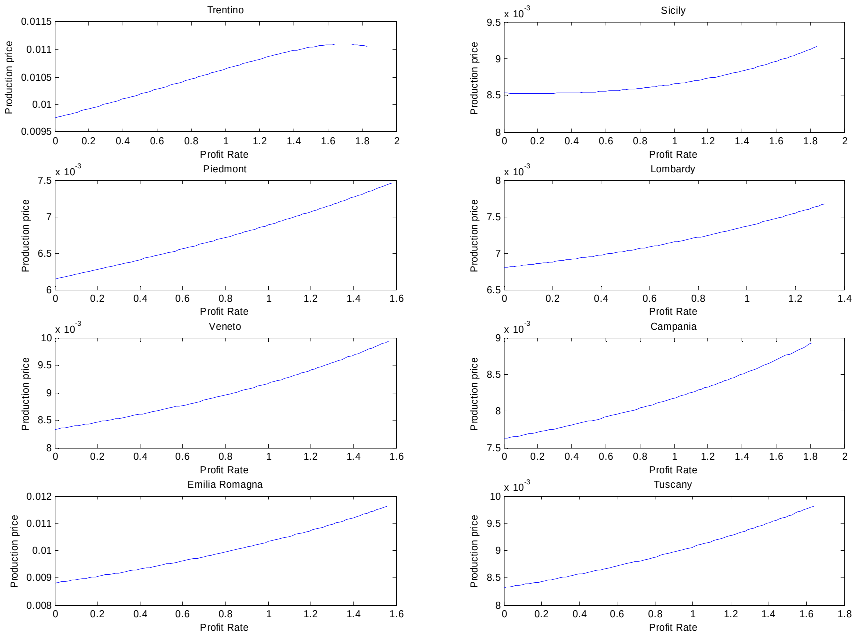


Fig.A.2.20 - Production prices – Mfr. of Other Non Metallic Mineral Products 2004

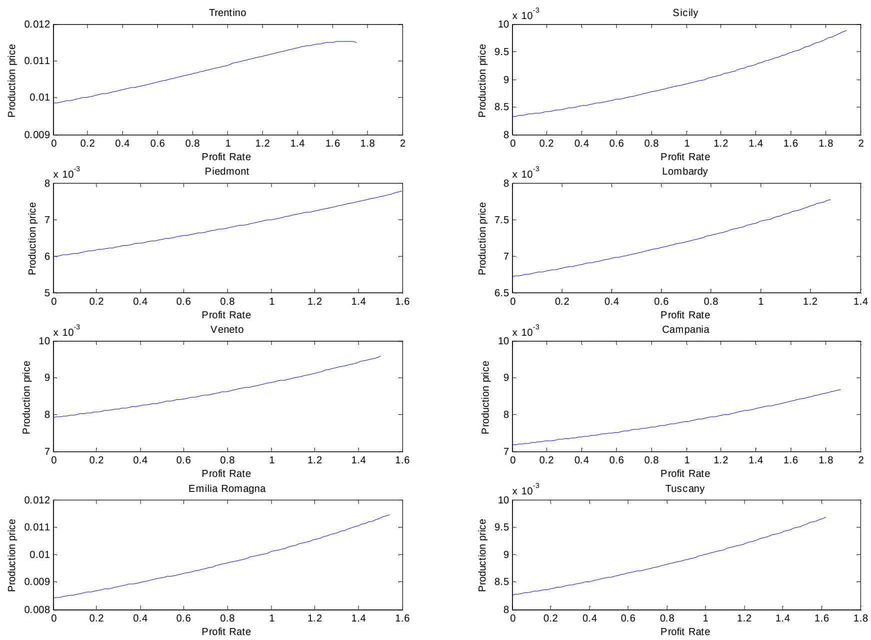


Fig.A.2.21 - Production prices – Mfr. and Processing of Basic Metals 2001

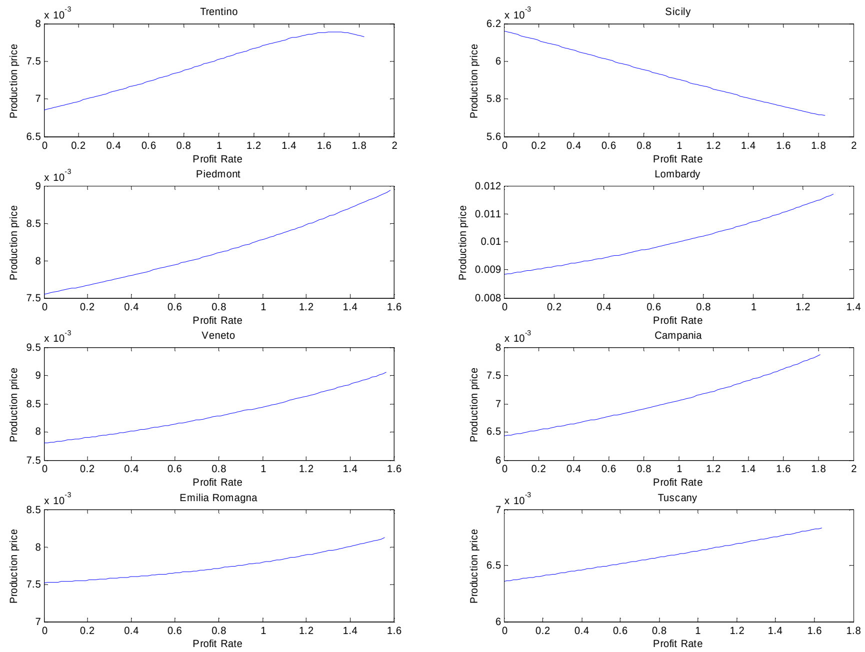


Fig.A.2.22 - Production prices – Mfr. and Processing of Basic Metals 2004

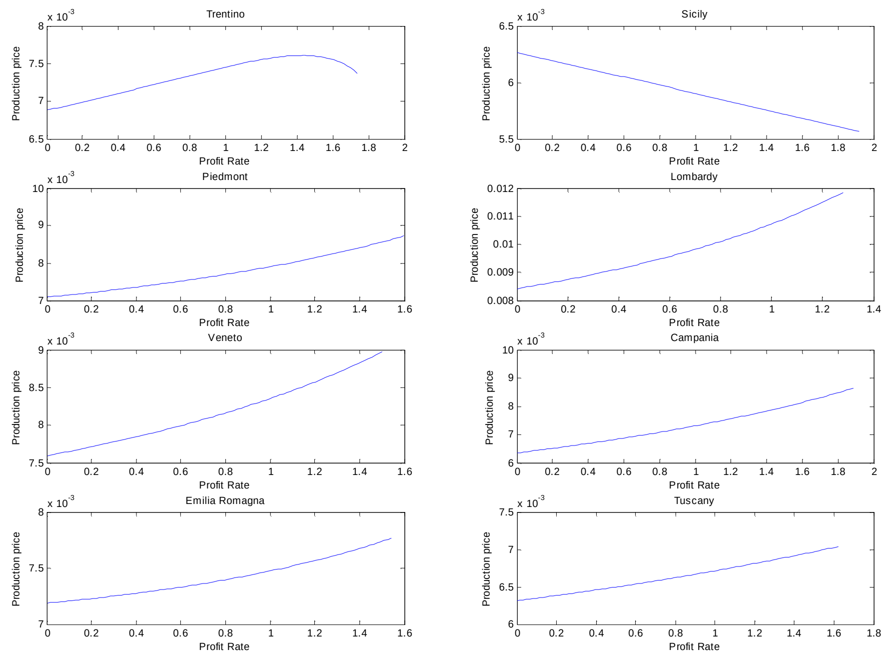


Fig.A.2.23 - Production prices – Mfr. of Machinery and Equipment n.e.c. 2001

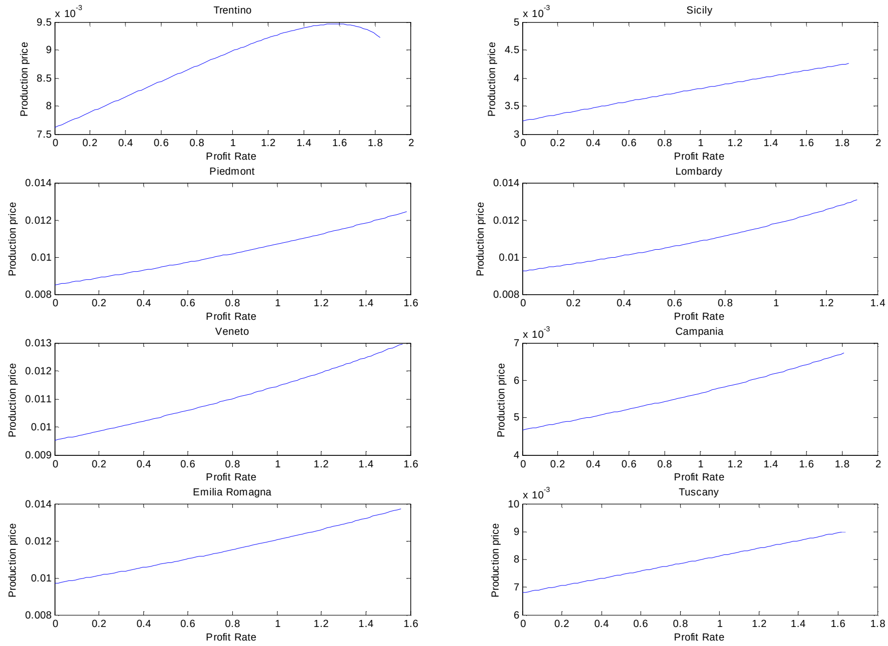


Fig.A.2.24 - Production prices – Mfr. of Machinery and Equipment n.e.c. 2004

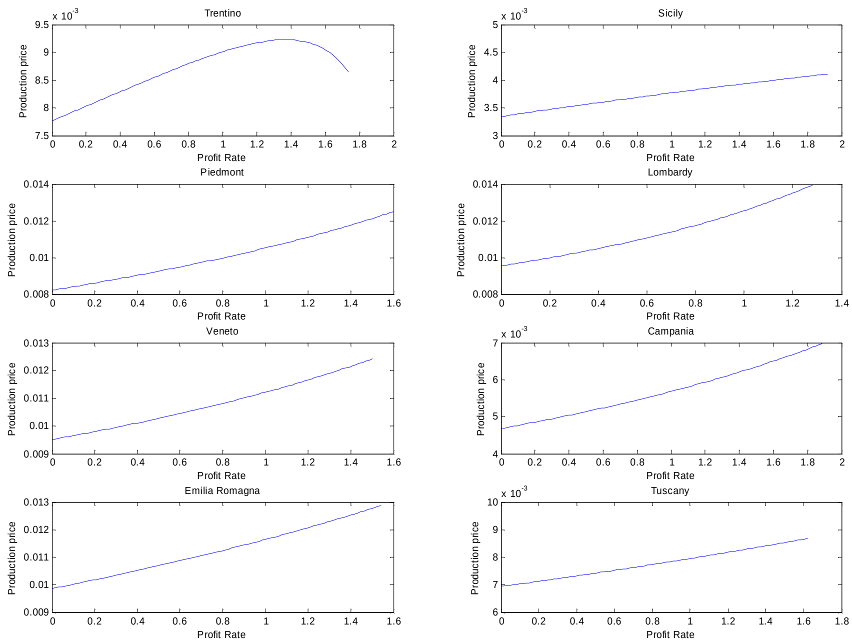


Fig.A.2.25 - Production prices – Mfr. of Electrical and Optical Equipment 2001

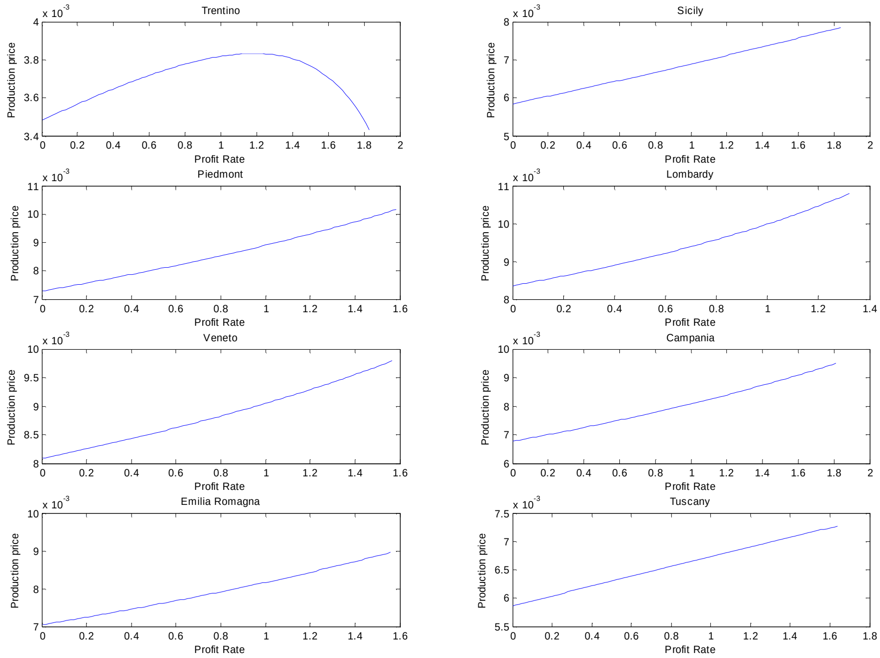


Fig.A.2.26 - Production prices – Mfr. of Electrical and Optical Equipment 2004

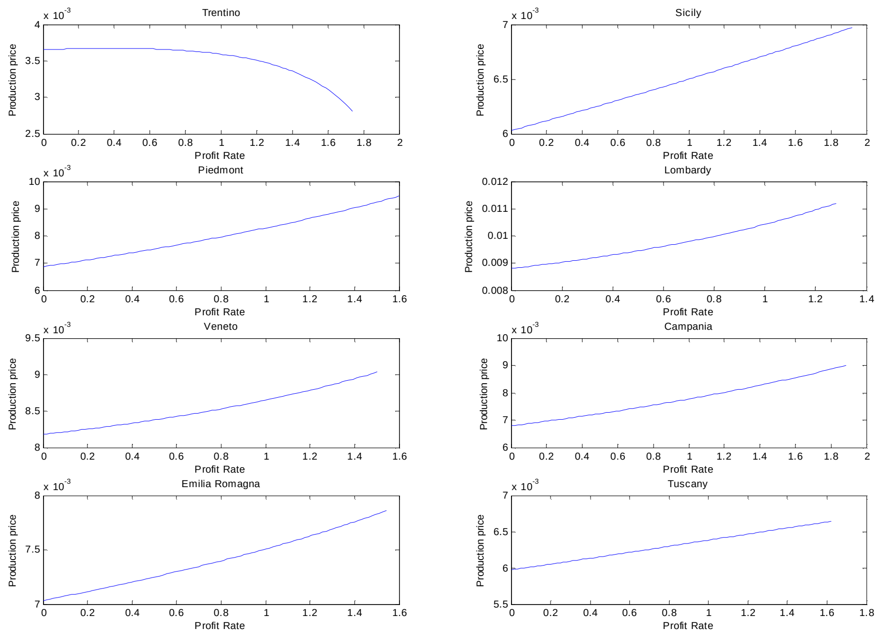


Fig.A.2.27 - Production prices – Mfr. of Transport Equipment 2001

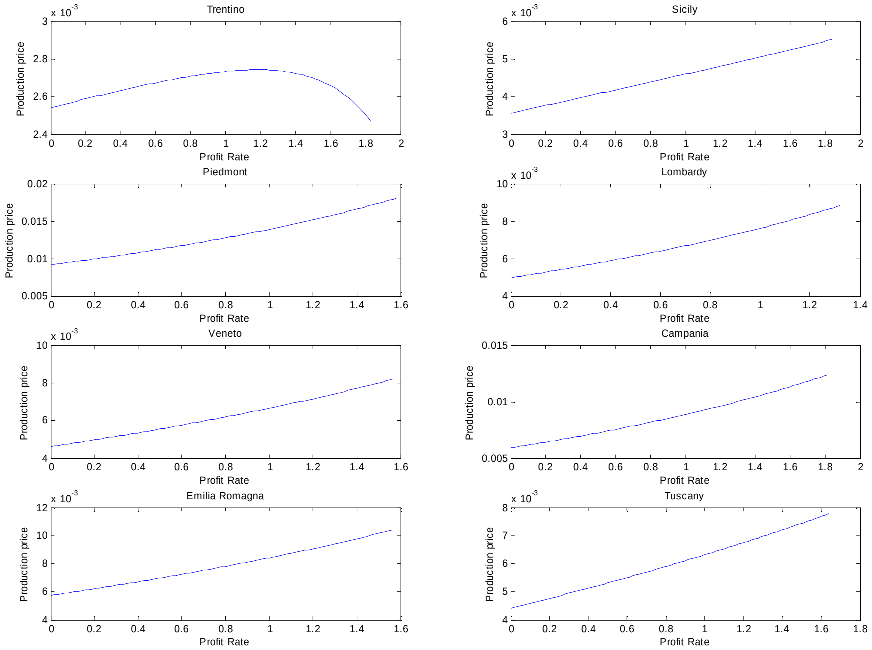


Fig.A.2.28 - Production prices – Mfr. of Transport Equipment 2004

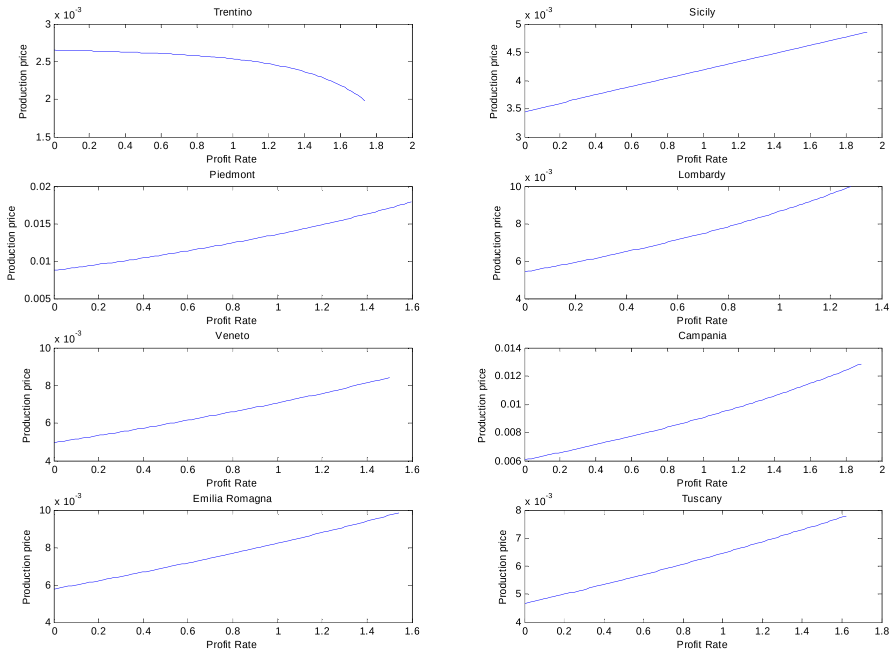


Fig.A.2.29 - Production prices – Mfr. of Furniture, Mfr. n.e.c. 2001

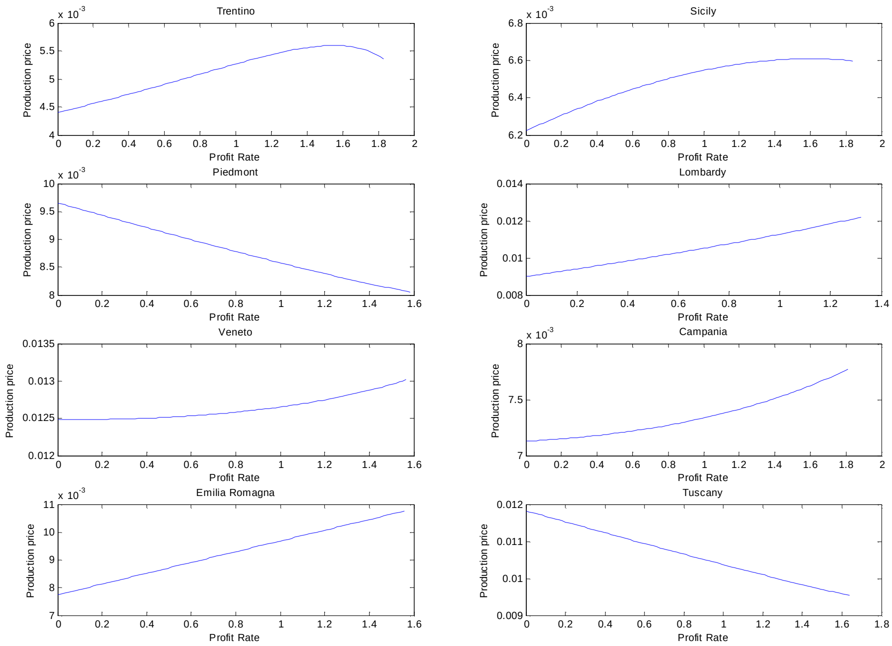


Fig.A.2.30 - Production prices – Mfr. of Furniture, Mfr. n.e.c. 2004

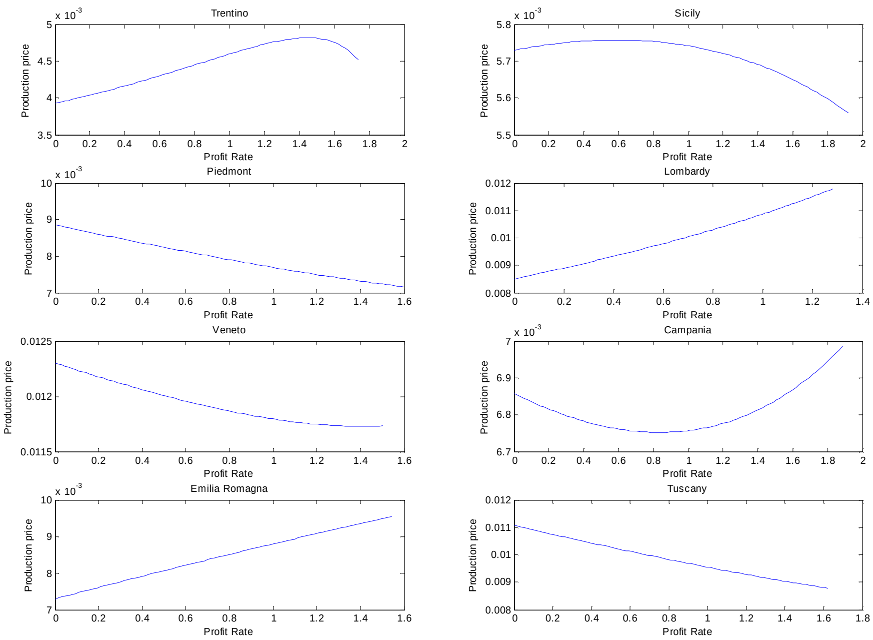


Fig.A.2.31 - Production prices – Electricity, Gas and Water Supply 2001

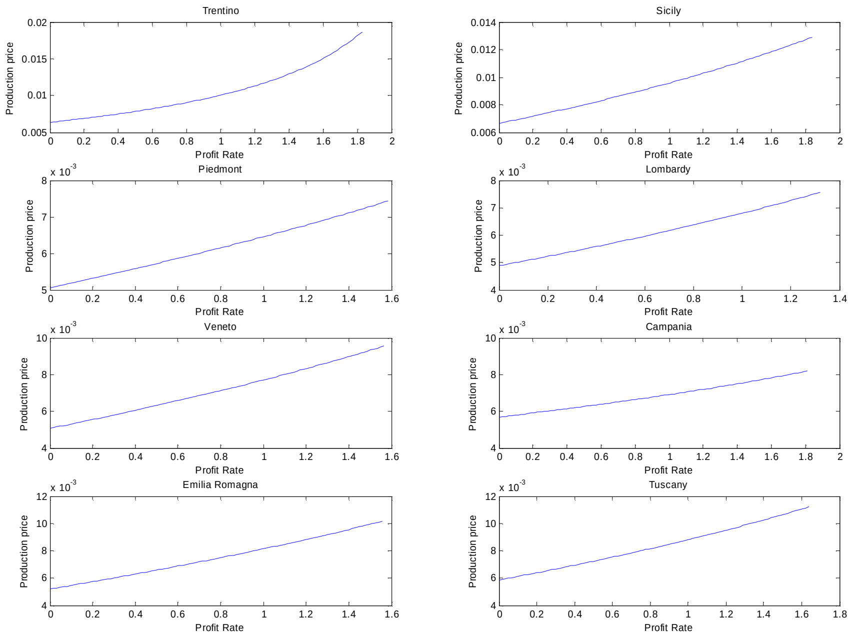


Fig.A.2.32 - Production prices – Electricity, Gas and Water Supply 2004

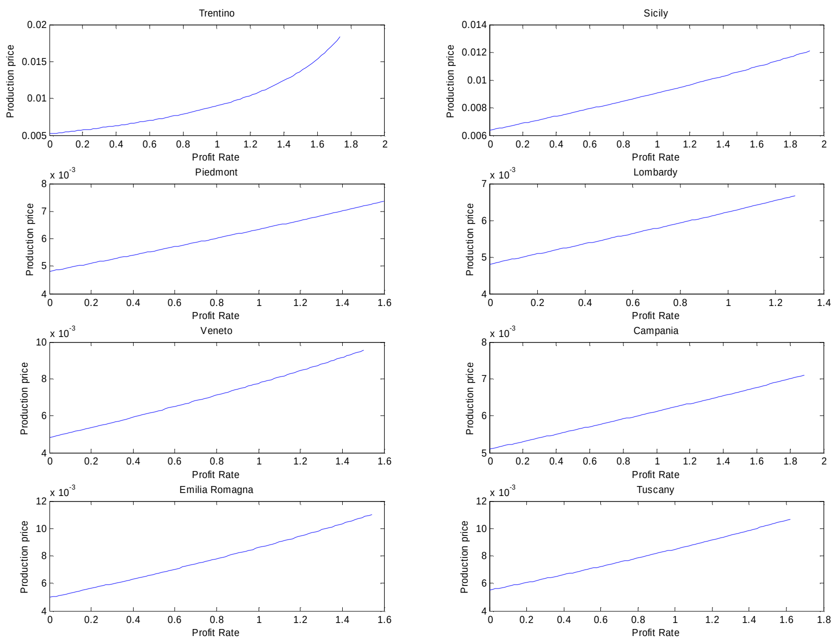


Fig.A.2.33 - Production prices – Construction 2001

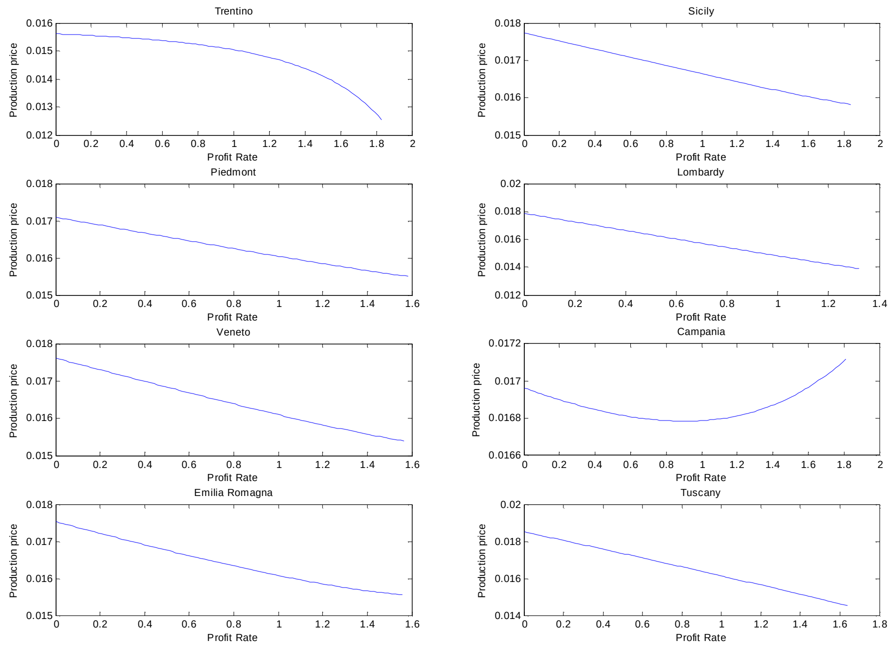


Fig.A.2.34 - Production prices – Construction 2004

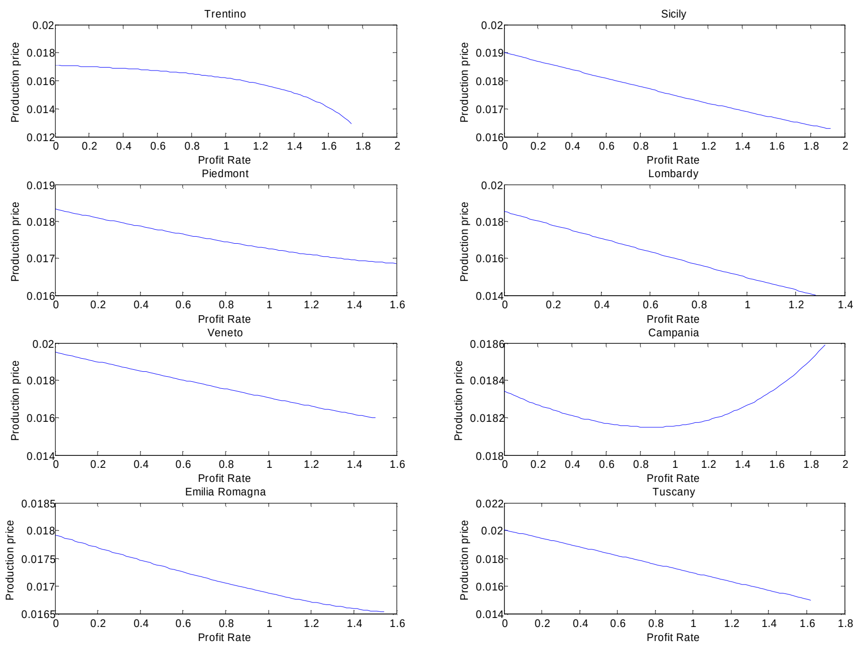


Fig.A.2.35 - Production prices – Wholesale and Retail Trade 2001

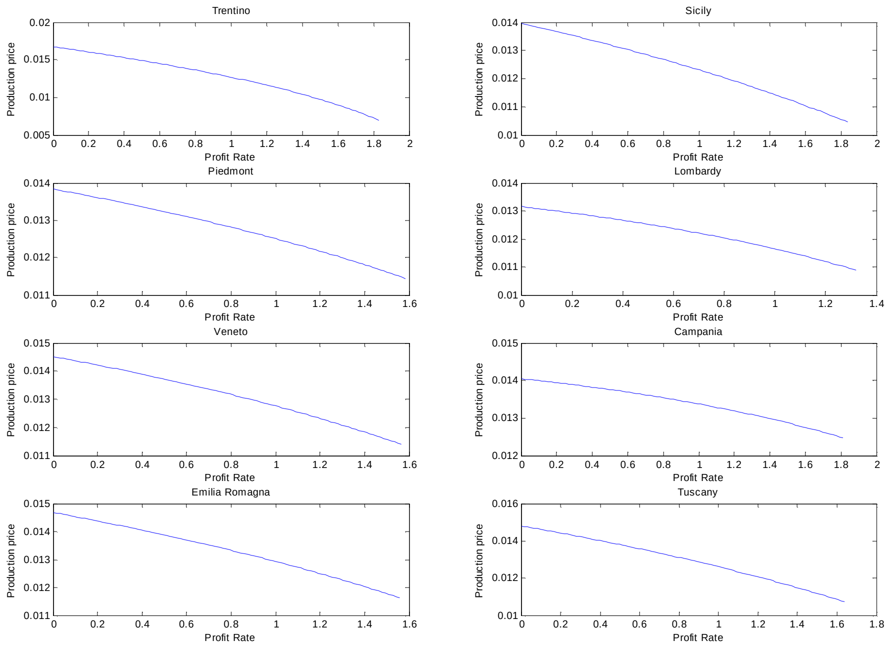


Fig.A.2.36 - Production prices – Wholesale and Retail Trade 2004

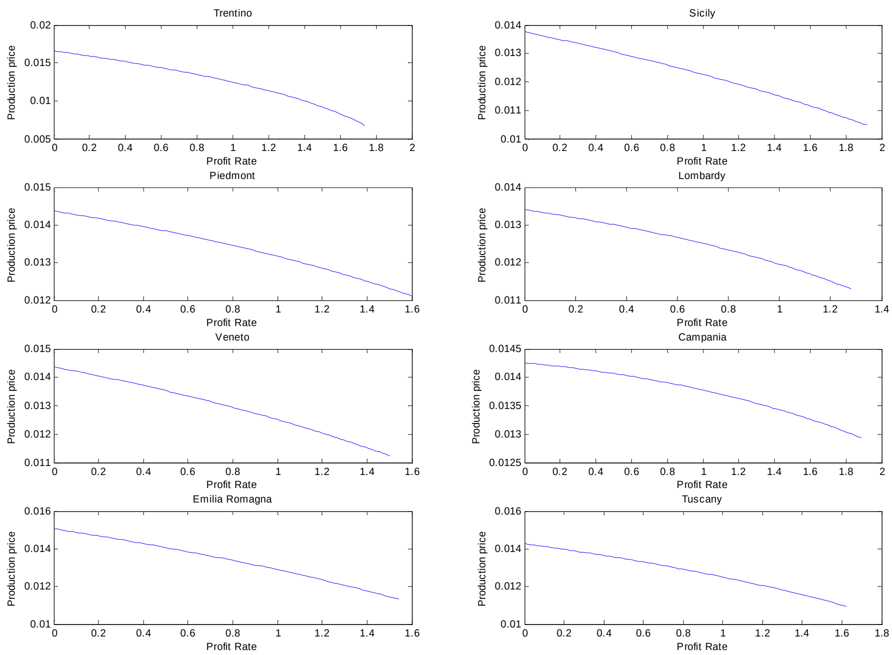


Fig.A.2.37 - Production prices – Hotels and Restaurants 2001

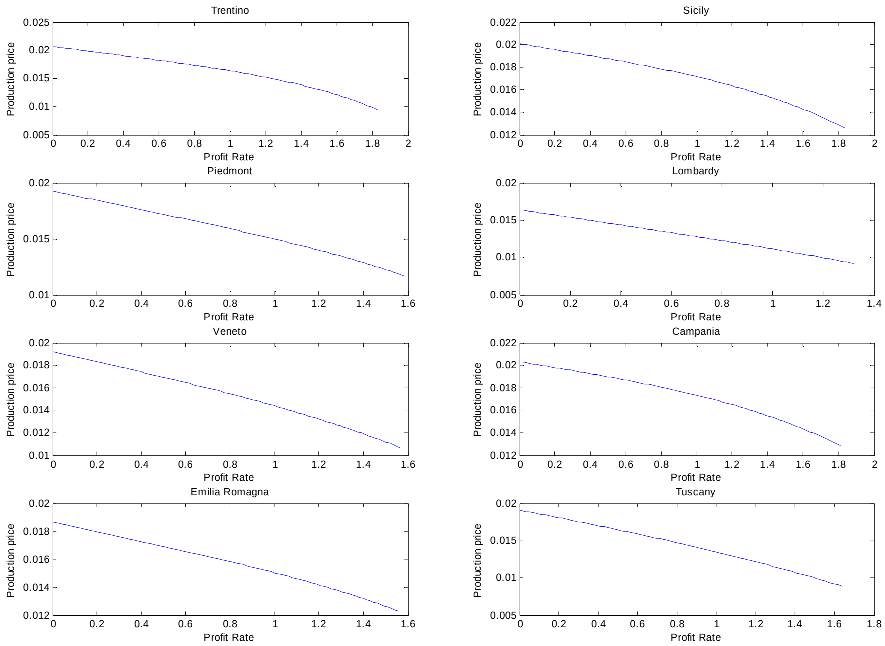


Fig.A.2.38 - Production prices – Hotels and Restaurants 2004

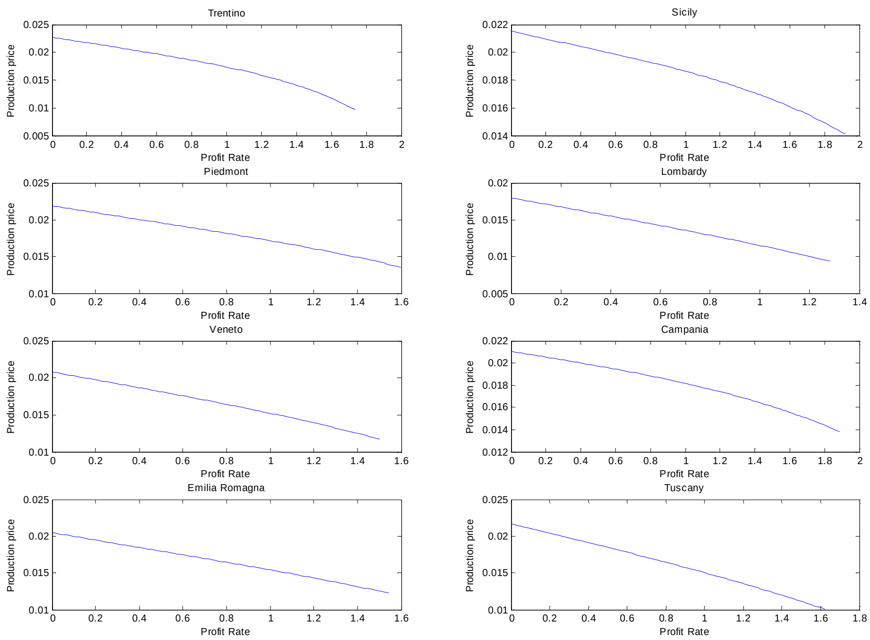


Fig.A.2.39 - Production prices – Transport, Post and Telecommunications 2004

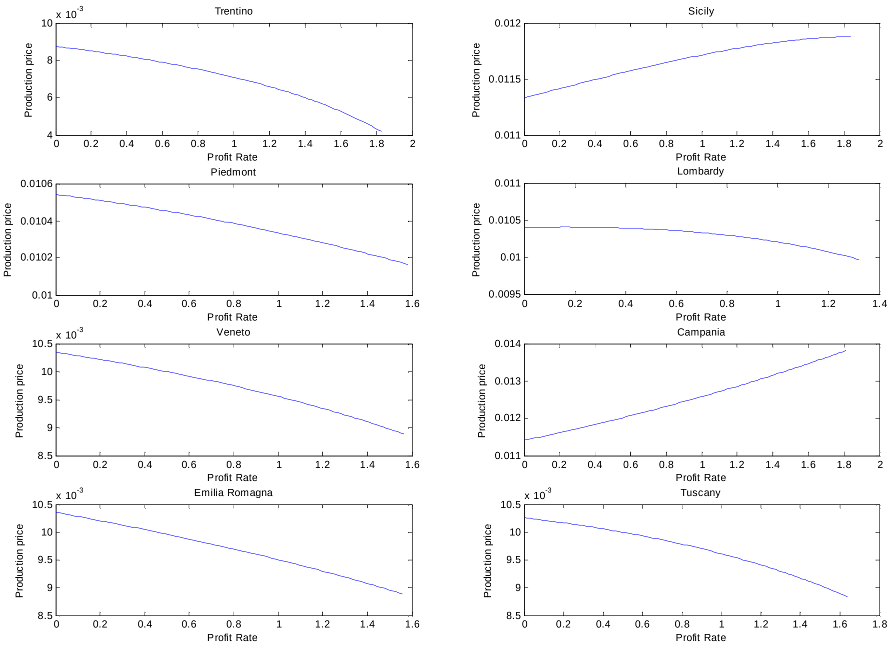


Fig.A.2.40 - Production prices – Transport, Post and Telecommunications 2004

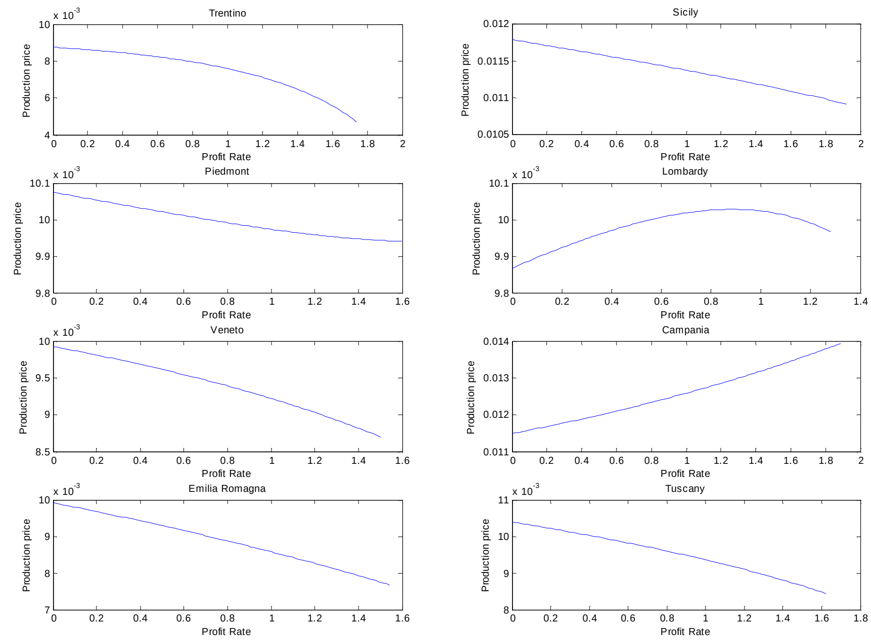


Fig.A.2.41 - Production prices – Financial Intermediation, Insurance 2001

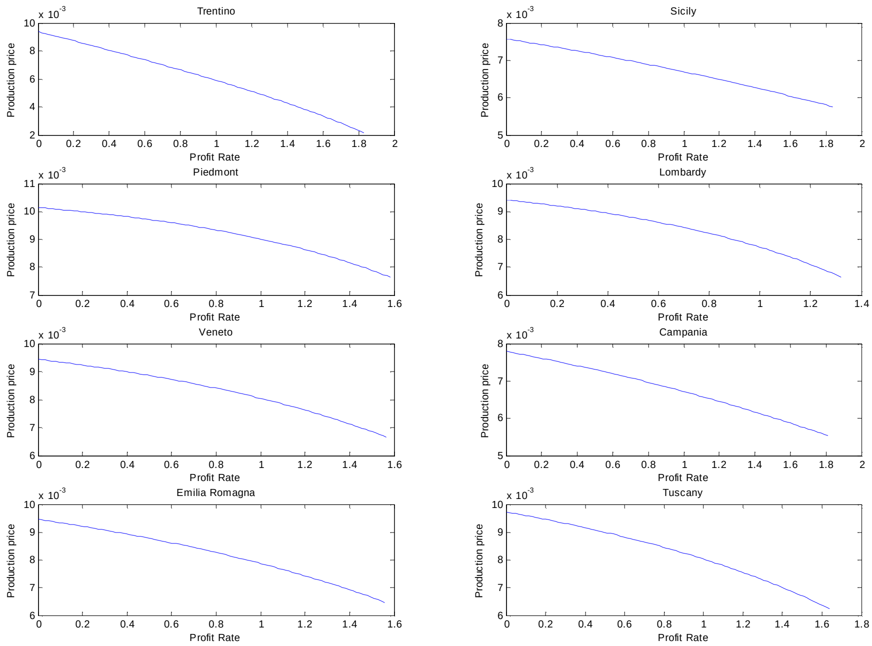


Fig.A.2.42 - Production prices – Financial Intermediation, Insurance 2001

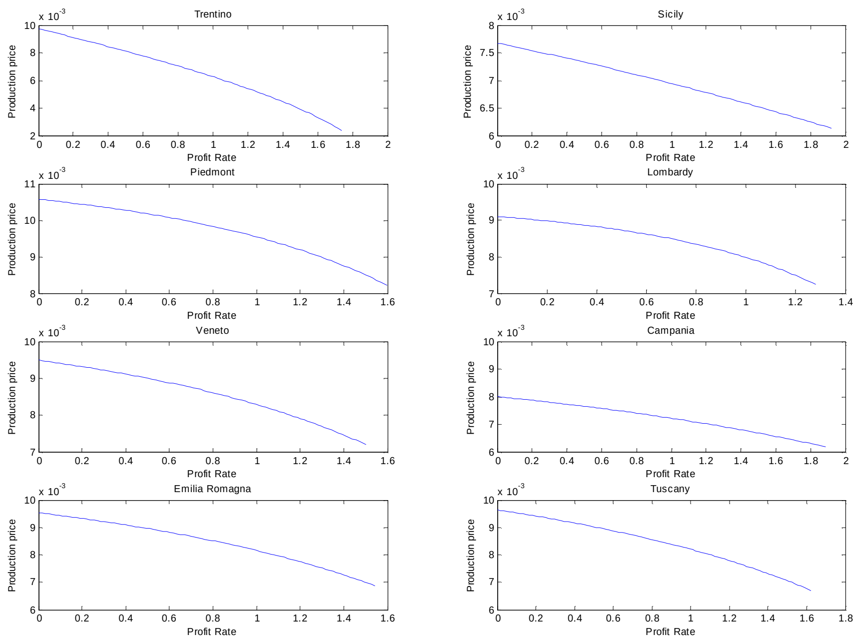


Fig.A.2.43 - Production prices – Computer, Research and Development, Consultancy 2001

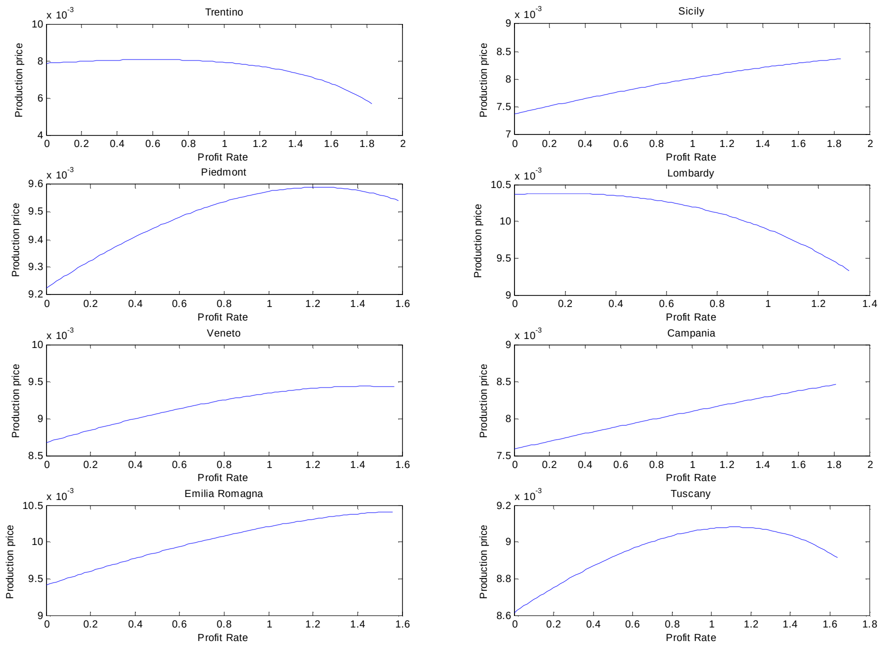


Fig.A.2.44 - Production prices – Computer, Research and Development, Consultancy 2004

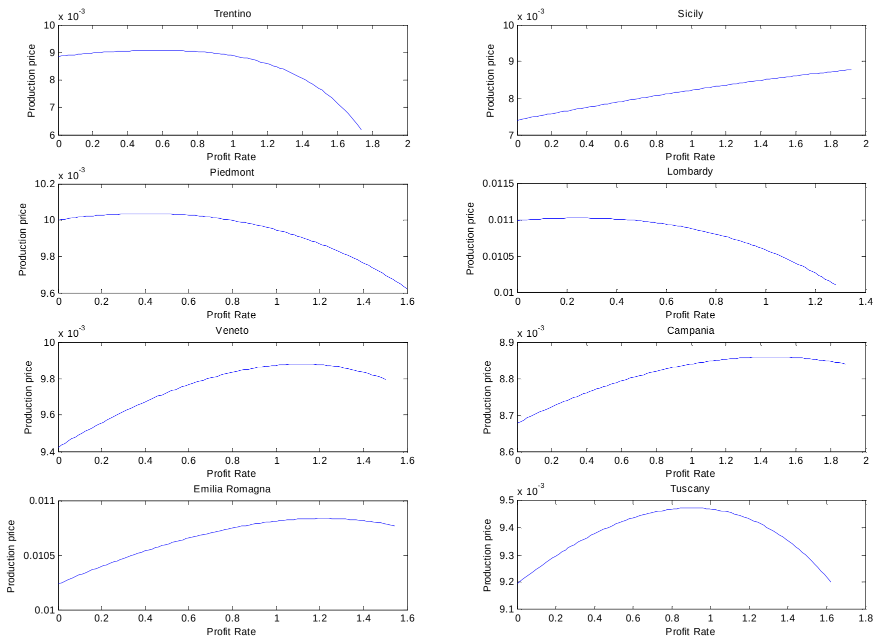


Fig.A.2.45 - Production prices – Public Administration 2001

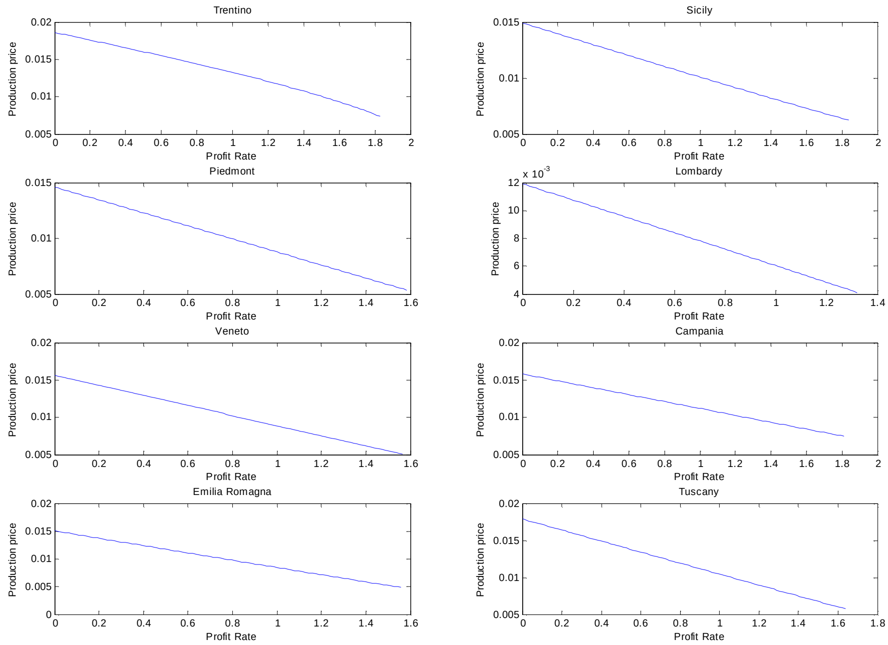


Fig.A.2.46 - Production prices – Public Administration 2004

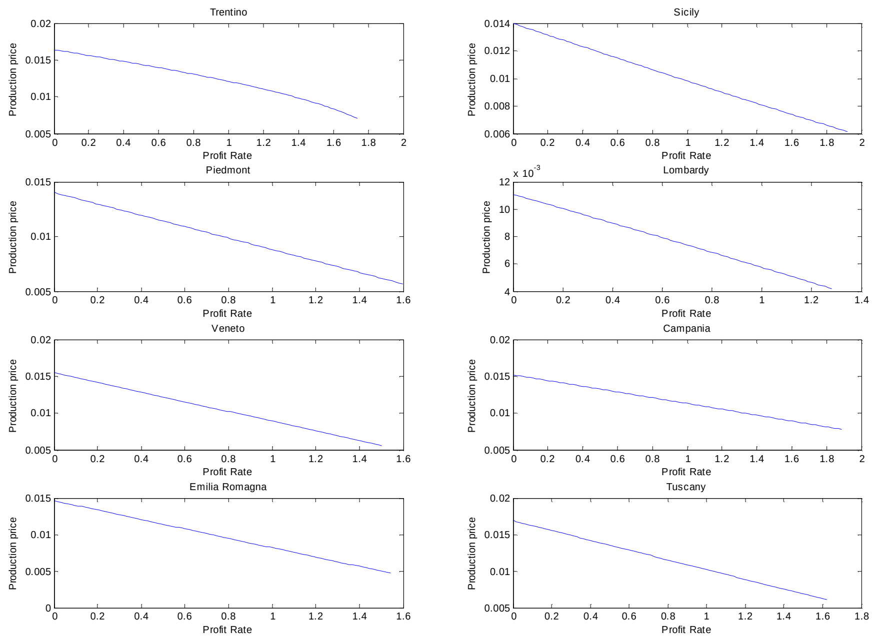


Fig.A.2.47 - Production prices – Education 2001

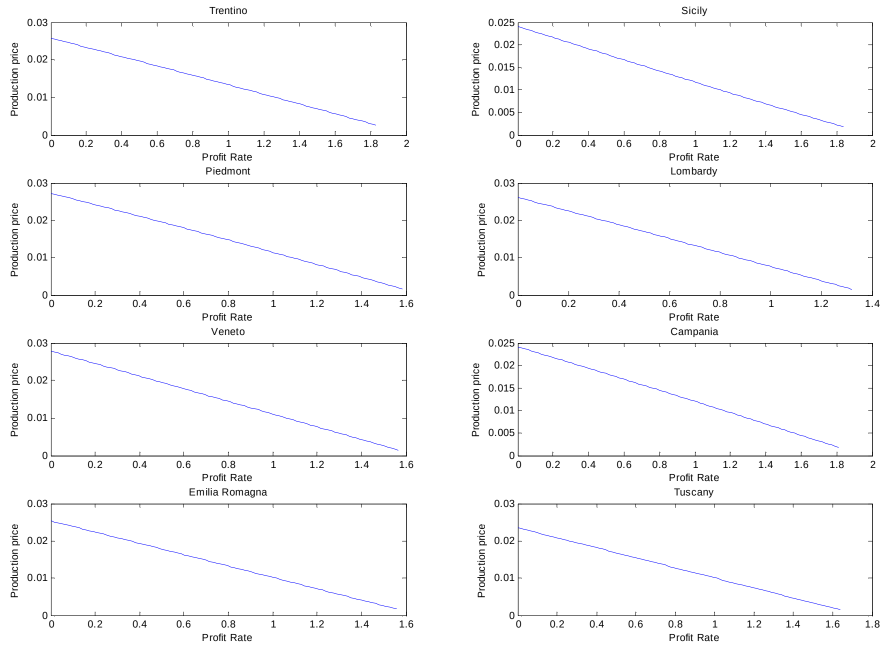


Fig.A.2.48 - Production prices – Education 2004

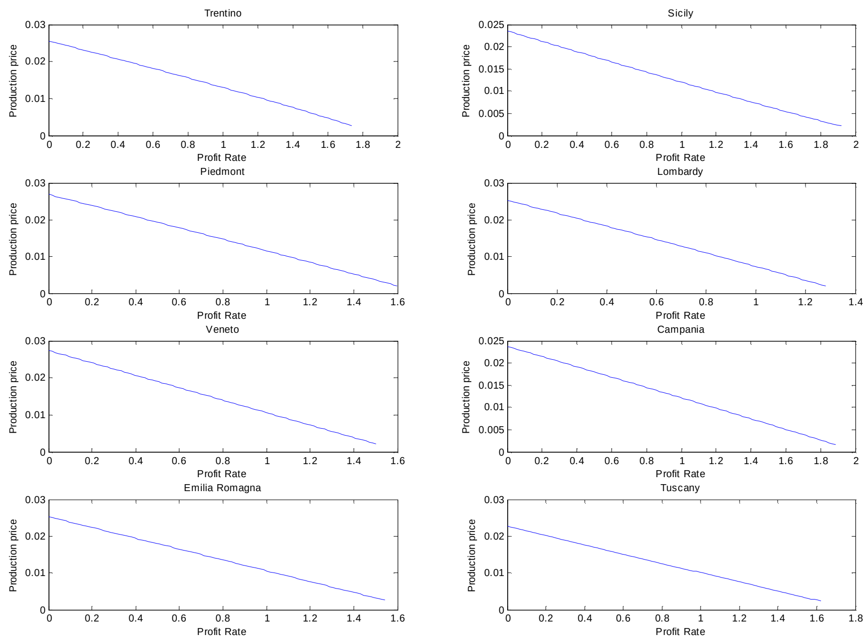


Fig.A.2.49 - Production prices – Health Care Activities Etc. 2001

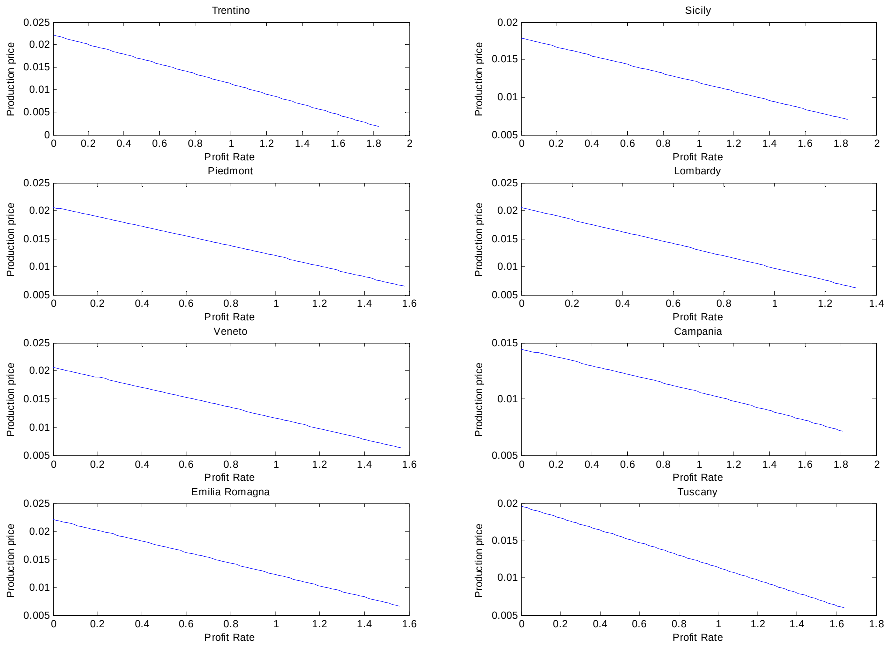


Fig.A.2.50 - Production prices – Health Care Activities Etc. 2004

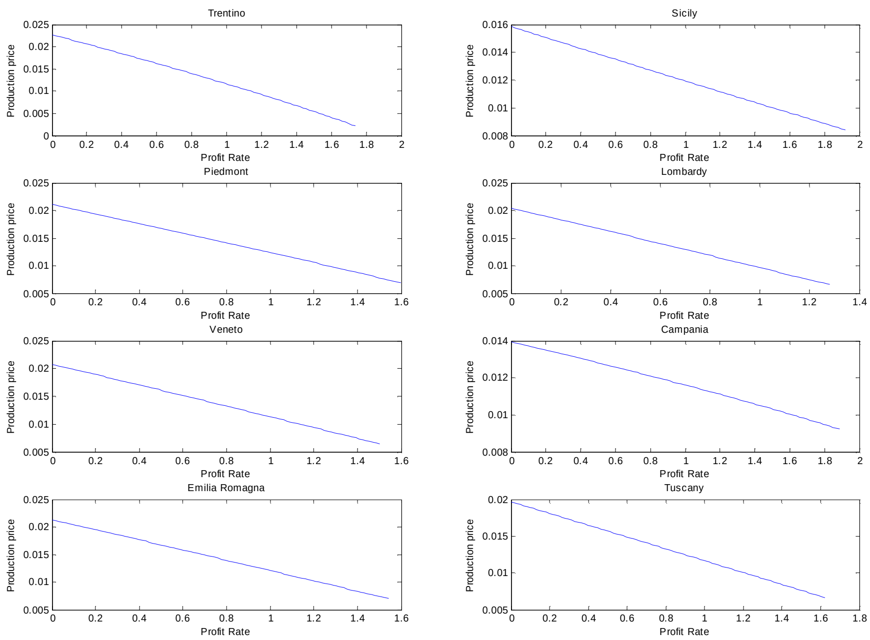


Fig.A.2.51 - Production prices – Other Service Activities 2001

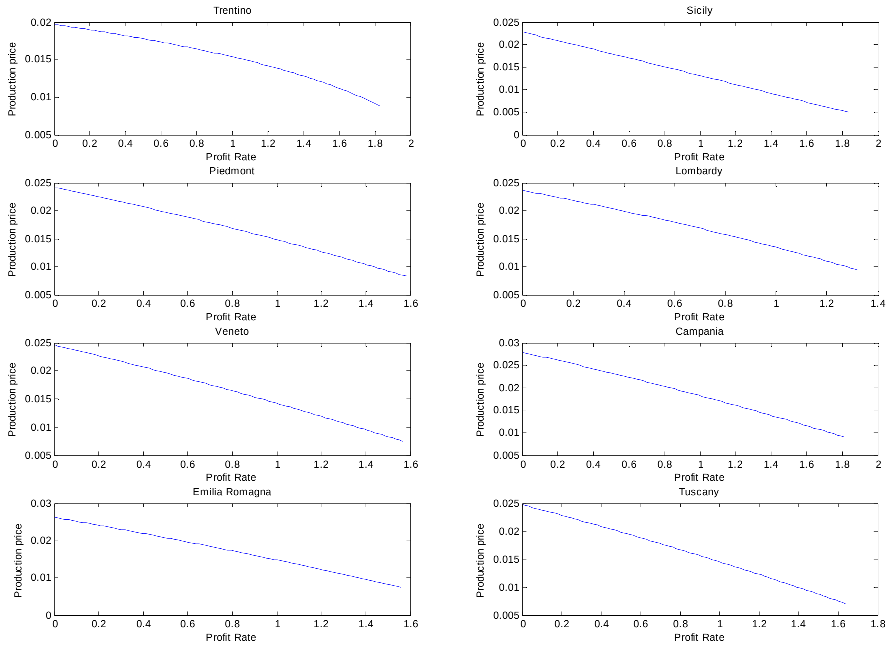


Fig.A.2.52 - Production prices – Other Service Activities 2004

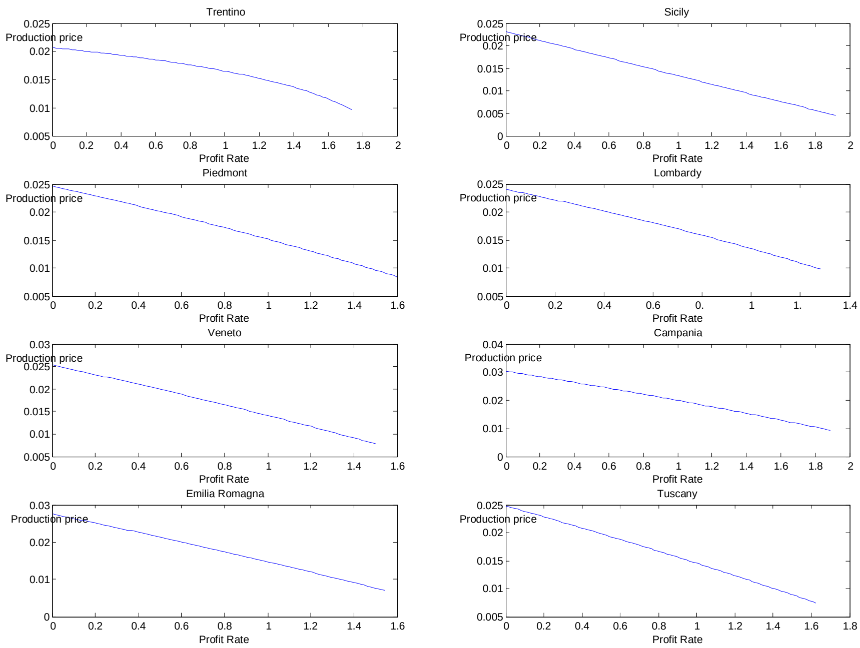


Fig.A.2.53 - Production prices – Renting of Machinery and Equipment, Real Estate Activities 2001

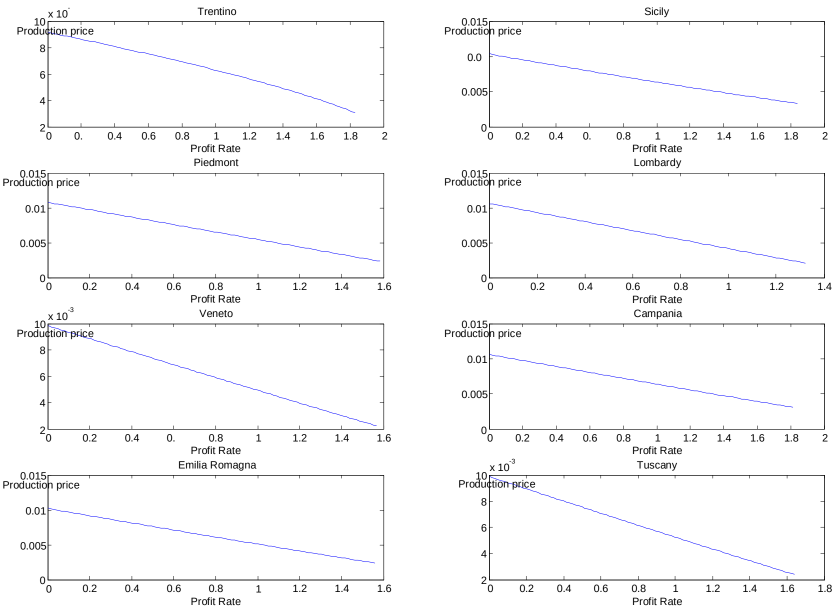


Fig.A.2.54 - Production prices – Renting of Machinery and Equipment, Real Estate Activities 2004

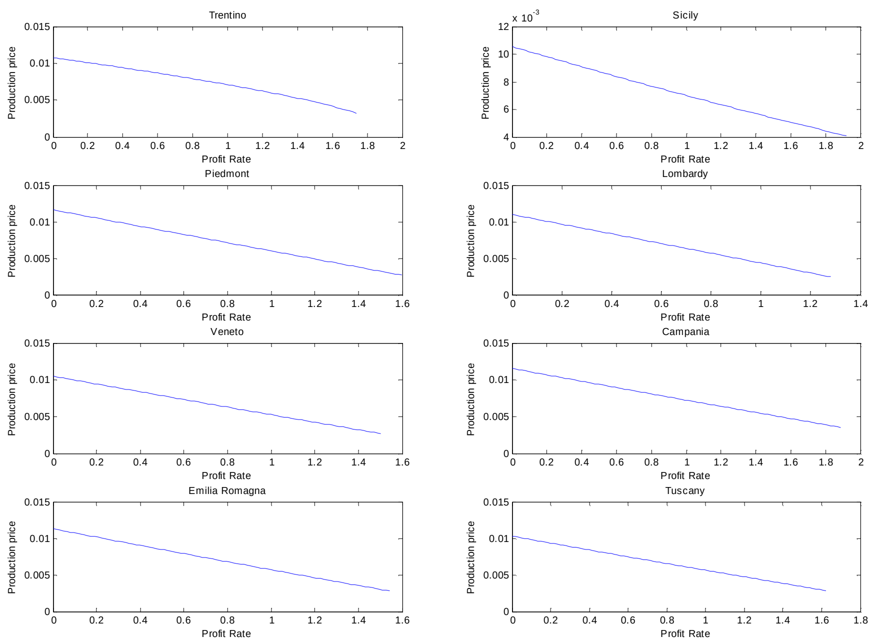


Table A.2.6. Classification of Production prices into four groups - 2001 (Monotonic-decreasing [↓], montonic-increasing [↑], parabolic[↓↑], and reverse parabolic[↑↓]). In the last column, classification of industries in UNIFORM (The Production prices of at least six regions belong to the same group) and MIXED (otherwise).

Industry	**Trentino**	**Sicily**	**Piedmont**	**Lombardy**	**Veneto**	**Campania**	**Emilia Romagna**	**Tuscany**	**Industry classification**
Agriculture and Fishing	↓	↓	↓	↓	↓	↓	↓	↓	**Uniform**
Extraction of minerals	↑↓	↑	↑	↑	↓↑	↑	↓	↑	**Mixed**
Mfr. of Food, Beverages and Tobacco	↑↓	↑↓	↑↓	↑↓	↑↓	↑↓	↑↓	↑↓	**Uniform**
Mfr. of Textiles, Wearing Apparel, Leather	↑↓	↓	↑	↑	↑	↑	↑	↑	**Uniform**
Mfr. of Wood and Wood Products	↑↓	↓	↑	↓	↑	↑	↓	↑	**Mixed**
Mfr. of Paper Products, Printing and Publishing	↑	↑↓	↑	↑	↑	↑	↑	↑	**Uniform**
Mfr. of Refined Petroleum	↑↓	↑	↑	↑	↑	↑	↑	↑	**Uniform**
Mfr. of Chemicals and Man-Made Fibers Etc.	↑↓	↑	↑	↑	↑	↑	↑	↑	**Uniform**
Mfr. of Rubber and Plastic Products	↑↓	↑	↑	↑	↑	↑	↑	↑	**Uniform**
Mfr. of Other Non Metallic Mineral Products	↑↓	↓↑	↑	↑	↑	↑	↑	↑	**Uniform**
Mfr. And Processing of Basic Metals	↑↓	↓	↑	↑	↑	↑	↑	↑	**Uniform**
Mfr. of Machinery and Equipment n.e.c.	↑↓	↑	↑	↑	↑	↑	↑	↑	**Uniform**
Mfr. of Electrical and Optical Equipment	↑↓	↑	↑	↑	↑	↑	↑	↑	**Uniform**
Mfr. of Transport Equipment	↑↓	↑	↑	↑	↑	↑	↑	↑	**Uniform**
Mfr. of Furniture, Mfr. n.e.c	↑↓	↑↓	↓	↑	↑	↑	↑	↓	**Mixed**
Electricity, Gas and Water Supply	↑	↑	↑	↑	↑	↑	↑	↑	**Uniform**
Construction	↓	↓	↓	↓	↓	↓↑	↓	↓	**Uniform**
Wholesale and Retail Trade	↓	↓	↓	↓	↓	↓	↓	↓	**Uniform**
Hotels and Restaurants	↓	↓	↓	↓	↓	↓	↓	↓	**Uniform**
Transport, Post and Telecommunications	↓	↑	↓	↑↓	↓	↑	↓	↓	**Mixed**
Financial Intermediation, Insurance	↓	↓	↓	↓	↓	↓	↓	↓	**Uniform**
Computer, Research and Development, Consultancy	↑↓	↑	↑↓	↑↓	↑↓	↑	↑	↑↓	**Mixed**
Public Administration	↓	↓	↓	↓	↓	↓	↓	↓	**Uniform**
Education	↓	↓	↓	↓	↓	↓	↓	↓	**Uniform**
Health Care Activities Etc.	↓	↓	↓	↓	↓	↓	↓	↓	**Uniform**
Other Service Activities	↓	↓	↓	↓	↓	↓	↓	↓	**Uniform**
Renting of Machinery and Equipment, Real Estate Activities	↓	↓	↓	↓	↓	↓	↓	↓	**Uniform**

Table A.2.7. Classification of Production prices into four groups - 2004 (Monotonic-decreasing [↓], montonic-increasing [↑], parabolic[↓↑], and reverse parabolic[↑↓]). In the last column, classification of industries in UNIFORM (The Production prices of at least six regions are the same group) and MIXED (otherwise)

Industry	**Trentino**	**Sicily**	**Piedmont**	**Lombardy**	**Veneto**	**Campania**	**Emilia Romagna**	**Tuscany**	**Industry classification**
Agriculture and Fishing	↓	↓	↓	↓	↓	↓	↓	↓	**Uniform**
Extraction of minerals	↑↓	↑	↑	↓	↑	↑	↓↑	↑	**Mixed**
Mfr. of Food, Beverages and Tobacco	↑↓	↑↓	↑↓	↑↓	↑↓	↑↓	↑↓	↑↓	**Uniform**
Mfr. of Textiles, Wearing Apparel, Leather	↑↓	↓	↑	↑	↑	↑	↑	↑	**Uniform**
Mfr. of Wood and Wood Products	↑↓	↓	↑	↓	↑	↑	↓	↑	**Mixed**
Mfr. of Paper Products, Printing and Publishing	↑	↑	↑	↑	↑	↑	↑	↑	**Uniform**
Mfr. of Refined Petroleum	↑↓	↑	↑	↑	↑	↑	↑	↑	**Uniform**
Mfr. of Chemicals and Man-Made Fibers Etc.	↑↓	↑	↑	↑	↑	↑	↑	↑	**Uniform**
Mfr. of Rubber and Plastic Products	↑↓	↑	↑	↑	↑	↑	↑	↑	**Uniform**
Mfr. of Other Non Metallic Mineral Products	↑↓	↑	↑	↑	↑	↑	↑	↑	**Uniform**
Mfr. And Processing of Basic Metals	↑↓	↓	↑	↑	↑	↑	↑	↑	**Uniform**
Mfr. of Machinery and Equipment n.e.c.	↑↓	↑	↑	↑	↑	↑	↑	↑	**Uniform**
Mfr. of Electrical and Optical Equipment	↑↓	↑	↑	↑	↑	↑	↑	↑	**Uniform**
Mfr. of Transport Equipment	↓	↑	↑	↑	↑	↑	↑	↑	**Uniform**
Mfr. of Furniture, Mfr. n.e.c	↑↓	↑↓	↓	↑	↓	↓↑	↑	↓	**Mixed**
Electricity, Gas and Water Supply	↑	↑	↑	↑	↑	↑	↑	↑	**Uniform**
Construction	↓	↓	↓	↓	↓	↓↑	↓	↓	**Uniform**
Wholesale and Retail Trade	↓	↓	↓	↓	↓	↓	↓	↓	**Uniform**
Hotels and Restaurants	↓	↓	↓	↓	↓	↓	↓	↓	**Uniform**
Transport, Post and Telecommunications	↓	↓	↓	↑↓	↓	↑	↓	↓	**Uniform**
Financial Intermediation, Insurance	↓	↓	↓	↓	↓	↓	↓	↓	**Uniform**
Computer, Research and Development, Consultancy	↑↓	↑	↑↓	↑↓	↑↓	↑↓	↑↓	↑↓	**Uniform**
Public Administration	↓	↓	↓	↓	↓	↓	↓	↓	**Uniform**
Education	↓	↓	↓	↓	↓	↓	↓	↓	**Uniform**
Health Care Activities Etc.	↓	↓	↓	↓	↓	↓	↓	↓	**Uniform**
Other Service Activities	↓	↓	↓	↓	↓	↓	↓	↓	**Uniform**
Renting of Machinery and Equipment, Real Estate Activities	↓	↓	↓	↓	↓	↓	↓	↓	**Uniform**