



Full Length Article

Honeycomb metamaterial panel with core grid of wire-masses resonators for tuneable vibration and sound radiation bandgap

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ABSTRACT

Controlling flexural vibrations and sound radiation in thin-walled structures subjected to time-varying tonal excitations remains a major engineering challenge. While acoustic metamaterials have demonstrated significant potential for vibration attenuation, most tuneable designs rely on complex architectures that hinder practical implementation. This study proposes a lightweight composite panel integrating a simple honeycomb-core elastic metamaterial capable of generating a continuously tuneable flexural vibration bandgap. The proposed concept consists of a tensioned wire routed in a continuous zigzag path through the honeycomb cells, with concentrated masses attached at the centre of each cell. Each wire-mass subsystem acts as a base-excited resonator whose natural frequency is controlled by the wire tension. The resulting periodic array of identical resonators enables continuous tuning of the resonance frequency and the associated flexural vibration stopband without modifying the structural layout. The design methodology is presented and validated through simulations and experimental testing of a prototype panel. The results demonstrate the formation of both flexural vibration and sound radiation stopband with maximum attenuations of 25 dB and 20 dB respectively. By varying the wire tension, the stopband can be shifted over a frequency range of approximately 300 Hz, allowing adaptive suppression of time-varying tonal disturbances. The proposed metamaterial combines structural simplicity, low mass, ease of implementation, and robust, broadband-tuneable dynamic behaviour, providing an effective strategy for adaptive vibration and noise control in lightweight engineering structures operating under variable excitation conditions, including aerospace, automotive, marine, and industrial applications.

1. Introduction

To reduce the ecological footprint and the operation costs of surface and air transportation vehicles [1], new designs have emerged encompassing stiff lightweight wall structures [2–5] and lightweight fuel-efficient powertrains, such as the new generation of turboprop and turbofan aircraft engines [6,7] and the current low-displacement car engines [8,9], which, however, pose new challenges, particularly for the control of strong low-frequency tonal noise transmission to the interior of the vehicles [10]. Indeed, the

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combination of stiff-lightweight wall structures and strong low-frequency tonal disturbances from the powertrain is proving quite demanding for the design of vehicles with low levels of interior noise, particularly in cases where the frequency of the tonal excitation may vary over a wide range during operation [11–14]. In general, lightweight partitions inherently have low sound insulation capacity (e.g. mass law [15]). Moreover, partitions with a high stiffness to mass ratio are characterised by two concurrent phenomena that are further detrimental to sound insulation at low/medium audio frequencies. Firstly, a downward shift of the coincidence frequency where the wavelengths of structural and acoustic vibrations coincide such that the sound transmission/radiation becomes particularly effective [15]. Secondly, a widening of the low-frequency band where the flexural modal overlap is low and thus the spectrum of the structure flexural response is characterised by well separated resonance peaks where the sound transmission is enhanced too [16]. Overall, the combination of these two effects results into high noise transmission/radiation at low/medium audio frequencies, which require additional sound proofing treatments on the partitions, like for example mass/damping/stiffness add-ons, double panel architectures with sound insulation blankets, vibration absorbers, etc. [17–19]. However, these treatments tend to be less effective at low and medium audio frequencies unless bulky solutions are adopted, at the expense of significant weight penalties. Moreover, they cannot be set to track and control time-varying tonal excitations or to compensate for changes in the dynamic response of the structure and of the treatment itself, due for example to temperature changes, imposed loads, inertial effects generated by strong accelerations.

In this respect, during the past two decades, there has been quite a lot of research work on acoustic metamaterials [20] (often called elastic metamaterials too) where distributed structures, such as thin panels, are enriched with a regular grid of vibration absorbers [21,22], which generate both interference-type and resonance-type stop band effects (also called bandgaps) [21], that is, a significant drop of the vibration response, and thus of the sound radiation/transmission, within limited frequency bands [23]. Interference-type, often called Bragg-type, stop band effects rely on the destructive interference of the waves transmitted and reflected by a regular array of discontinuities on the main structure, such as for examples little block masses or lumped springs, mechanical constraints and geometric features of the structure itself [24–29]. Instead, resonance-based stop bands depend on the vibration effects generated on the wave motion of the main structure by a regular array of resonators [30]. In the literature [31], structures showing interference-type stop bands only, are typically known as sonic materials or phononic structures, where the unit cell forming the periodic structure is called sonic crystal or phononic crystal. Instead, structures showing also resonance-based stop bands are normally referred to as acoustic metamaterials or elastic metamaterials. As discussed in [32], in general, phononic structures show interference stop band effects at high audio frequencies. Moreover, the stop band attenuation is somewhat limited unless heavy inclusions are adopted. On the contrary, the stop band of acoustic metamaterials can be conveniently set at low audio frequencies and significant attenuations can be generated by somewhat lightweight resonator inclusions [33–38]. These properties make acoustic metamaterials of great interest for practical applications such as surface and air transportation vehicles [20]. Indeed, recently, a few successful demonstrators have been proposed for machinery enclosures [39], rectangular ducts [40], car firewall panels [41,42], car ceiling panels [43], whole car bodies [44], sandwich partitions [45], sandwich panels [46,47], aircraft trim panels [48,49].

Classical acoustic metamaterials offer appealing solutions for the reduction of tonal vibration and sound transmission through stiff-lightweight panel-like components of vehicles. Indeed, they can generate remarkable vibration and sound radiation cuts, although only over a narrow and fixed frequency band. In this respect, the feasibility of online tuning of the band gap centre frequency has been considered in [50,51]. Possibly, the first attempts to develop tuneable acoustic metamaterials relied on materials made with piezoelectric phononic crystals [52], which encompass piezoelectric transducers connected to tuneable electrical shunts, normally composed by series/parallel Resistive-Inductive (RL) circuits [53,54]. Here the resonance stop band frequency is controlled by the electro-mechanical resonance produced by the inductive effect of the shunt and the capacitive effect of the piezoelectric transducer [55,56]. The effectiveness of the shunted piezoelectric patch is normally enhanced by adding a negative capacitive effect in the circuit, which is typically synthesised with an active circuit that can be conveniently used to tune the resonance frequency effect [57] (switching shunts and digital shunts have also been developed for these purposes [52]). Early beam-like and plate-like experimental demonstrators of tuneable piezoelectric acoustic metamaterials were presented in Refs. [58–61]. Although these studies proved experimentally the feasibility and tuneability of band gaps, their practical implementation remains challenging as they require adaptive shunt circuits encompassing operational amplifiers, which, to have a realistic metamaterial, should be miniaturised into small low-weight and low-power consumption units that can be embedded/printed on each piezoelectric patch. Alternative solutions based on tensioned membranes with centre masses were proposed too, where the resonance effect of each membrane-mass was adapted by varying the tensioning of the membrane. Here a few tensioning solutions were proposed, which, for example, relied on air pressurization of sealed double membrane arrangement [62] and on small membrane disk held by tiny beams whose stiffness can be modulated via in-plane stress [63–65]. Finally, tuneable metamaterials were also proposed which rely on temperature-sensitive material [66], shape memory alloys [67,68], magnetic and magnetorheological transducers [69,70].

In this article, a simple tuneable acoustic metamaterial design is proposed and investigated, which consists of a composite panel with a two-dimensional regular grid of tuneable resonators encased into the honeycomb core structure. The resonators are made with a single wire in tension, which is passed through tiny holes opened at the centres of opposite faces of the honeycomb hexagonal elements in such a way as to form a zigzag layout over the whole area of the honeycomb structure. The two ends of the wire are wound around wire tensioners fixed at diagonally opposite corners of the structure. An array of small masses is fitted along the wire in correspondence of the middle point of each hexagonal element of the honeycomb structure. In this way, each hexagon contains an equally tensioned wire with a fixed small mass attached at its centre. The transverse vibration of each wire-mass segment mimics that of a mass–spring–damper resonator, whose resonance frequency can be tuned by adjusting the wire tension. Thus, the honeycomb panel results equipped with a regular grid of equal resonators whose fundamental resonance frequency can be suitably varied in such a way as to generate a tuneable resonance stop band effect. Compared with other solutions, the design proposed in this paper allows the simultaneous and uniform tensioning of all wire sections across the 53 hexagonal bays, thereby allowing all 53 resonators to be tuned

simultaneously and consistently using only a pair of tensioning reels. Overall, the implementation of wire–mass resonators in a honeycomb panel represents a particularly advantageous combination: honeycomb panels require simple, lightweight, and tuneable vibration absorbers, while their hexagonal core bays provide a natural means of accommodating such devices without substantially compromising the lightweight construction of the panel.

A similar concept had been proposed by Yu and Lesieutre [46], who fitted in the hexagonal bays of a honeycomb panel resonators formed by small beams with a centre block mass. Miao *et al.* [71] also considered a similar configuration, although the seismic masses of the absorbers embedded within the hexagonal cells were suspended on soft helical springs to produce low-frequency bandgaps. Alternatively, Jin *et al.* [72] investigated a honeycomb structure composed of square cells, which readily accommodated small absorbers consisting of a cantilever beam with a tip block mass. A similar concept was investigated by Gao *et al.* [73], although in their design the square cells contained multiple absorbers, each consisting of a folded beam with a tip mass, arranged so as to make full use of the space available within each cell. Finally, Song *et al.* [74] also investigated bandgap formation in honeycomb metamaterial panels, but considered simpler cylindrical resonators consisting of a soft elastic disc topped by a heavy disc, which were mounted on the outer surface of the panel. In all cases, these designs permit only fixed tuning of the resonators and thus provided a vibration stop-band effect at a single prescribed frequency. This work instead proposes a simple and lightweight solution that allows the resonance frequencies of the resonators to be uniformly tuned online using a pair of wire-tensioning drums, such that the stop-band effect can be continuously shifted over a wide frequency range to suitably track and control the effect of tonal disturbances with varying excitation frequency.

It is interesting to note that an early concept of a periodic structure exhibiting a vibration bandgap was introduced by Lord Rayleigh, who, in 1887 [75], studied transverse wave propagation in a tensioned wire periodically loaded with point masses. In this context, it should be emphasised that the present study focuses on the formation of a vibration bandgap in the flexural response of a honeycomb panel equipped with a tensioned wire interlaced through the honeycomb core structure, with point masses attached to the wire at the midpoints of the hexagonal cells. In this configuration, each wire–mass subsystem acts as a local mass–spring resonator, thus forming the periodic array of vibration absorbers typically employed in acoustic metamaterials [33–38].

It is envisaged that the proposed smart panel can be successfully employed in soundproofing applications such as aircraft and helicopter fuselage trim panels, automotive firewalls and side/ceiling panels, partitions in yacht compartments, machinery enclosures and in general wall structures of enclosures. In particular, this elastic metamaterial is expected to provide a viable solution for controlling tonal disturbances whose frequency varies over time, as is the case with vehicle powertrains. In this application, the panel should be equipped with an electromechanical drum tensioner controlled by a feedback system that, based on real-time measurements of the tonal disturbance, for example, obtained using a tachometer, adjusts the resonance frequency of the wire–mass resonators embedded in the honeycomb core structure. In this way, the resonators can be tuned to resonate at the frequency of the tonal disturbance, thereby generating a stop-band effect precisely at the target frequency.

This article provides two principal contributions, supported by theoretical formulations, simulation results, and experimental measurements. First, it provides a thorough description and analysis of the working principles of the proposed honeycomb panel with the array of tuneable wire-masses vibration absorbers. Second, it presents numerical and experimental demonstrations of the practical tunability of the vibration and sound radiation bandgap effect over a wide frequency band using a prototype panel. To this end, the article is structured into three main parts. To start with, Section 2 presents a detailed description of the proposed adaptive metamaterial, with particular emphasis on the design of the honeycomb composite panel and the sizing of the tensioned wire–masses assembly used to form a grid of tuneable resonators. Next, to provide initial insights into the formation of tuneable bandgaps, Section 3

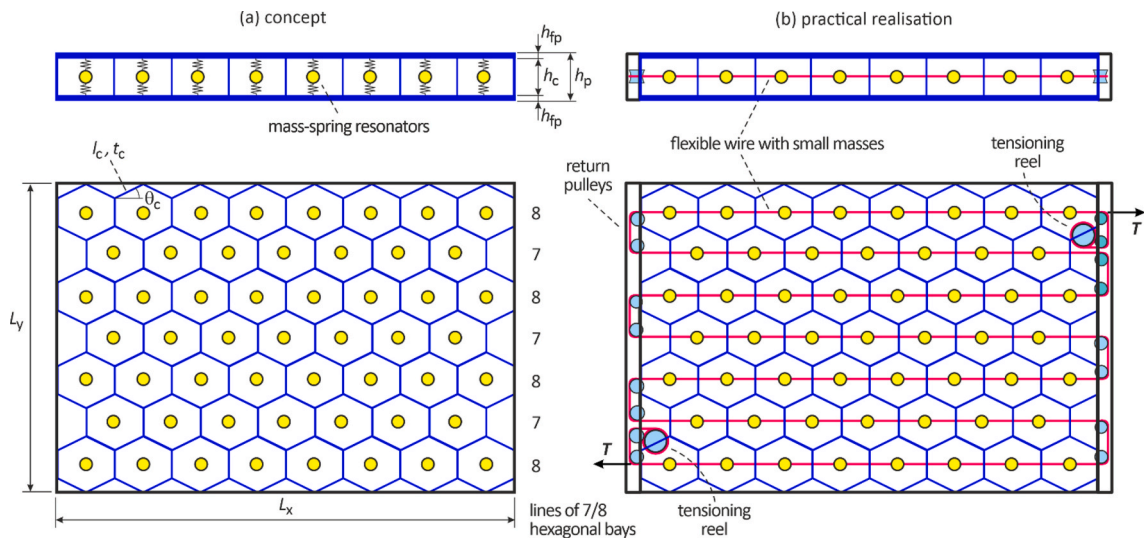


Fig. 1. Elastic metamaterial formed by a composite panel with a grid of resonators encased in the bays of the core honeycomb material: a) concept, b) practical realisation.

presents numerical results on the flexural vibration and sound radiation generated by equivalent homogeneous panels with different grids of spring-mass resonators. Then, Section 4 discusses the flexural response and sound radiation of a prototype honeycomb panel, focusing on the effectiveness of the tensioned wire-masses resonators in generating tuneable flexural vibration bandgaps over a wide frequency range. Finally, Section 5 presents the main conclusions of the study.

2. Panel acoustic metamaterial with tensioned wire-masses

To start with, this section presents and discusses the working principles and the dimensioning procedures of the proposed tuneable composite panel, which, as depicted in Fig. 1a, is formed by a core honeycomb structure with two thin faceplates. The hexagonal bays of the honeycomb structure are fitted with small spring-mass systems forming a regular grid of equal resonators that generate a resonance stop band effect in the spectrum of the total flexural response of the panel [30,33–38].

As depicted in Fig. 1b, the spring-mass components of the resonators are given by a single elastic wire in tension with equally spaced point masses. The wire is interlaced through the rows of hexagonal bays of the core honeycomb material. More specifically, it is passed through small holes from one face to the opposite face of the hexagonal honeycomb structures. This, combined with return pulleys at the ends of each row of hexagonal bays, creates a zigzag pattern across the entire honeycomb structure. The two ends of the interlaced wire are strained with turning tensioning reels such that its transverse stiffness can be regulated over a significant range. As shown in Fig. 1b, the wire is fitted with point masses at the centre of each hexagonal bay of the core structure. In this way, each hexagon holds an equally tensioned elastic wire with a small mass at its centre. The transverse vibration of the constrained wire-mass in each hexagonal bay replicates that of a mass-spring-damper resonator whose natural frequency can be suitably tuned by straining the wire (here the damping effect is principally due to wire material damping). Overall, the honeycomb panel is equipped with a regular grid of identical resonators whose fundamental resonance frequency can be suitably varied by adjusting the wire tension. Consequently, the vibration stopband effect generated by the grid of resonators can be swiftly shifted in a given frequency range to track and control the effects of tonal disturbances with varying excitation frequency.

The following two subsections present the design details of the composite honeycomb panel and tensioned wire assembly. The dimensions of the composite panel are derived from the fundamental principles and laws presented by Johnson and Sims [76] and by Gibson [77]. The sizing of the multi-bay tensioned wire with attached point masses is based on a transfer matrix model [78,79], which is used to investigate how the wire tension and material properties of both the wire and the point masses affect the first few natural frequencies and mode shapes associated with its transverse vibrations.

2.1. Design of the honeycomb panel

The design of the honeycomb panel was carried out through an iterative procedure, the main steps of which are briefly outlined below. The resulting dimensions, materials, and other relevant design parameters of the panel are summarized in Table 1. For practical reasons, it was decided from the outset to manufacture the core honeycomb structure from a thermoplastic polymer PA12 (commonly known as Nylon) using additive manufacturing technology, specifically an HP Multi Jet Fusion (MJF) 3D printer [80]. Furthermore, to allow visual inspection of the tensioned wire-masses assembly passing through the hexagonal cells of the honeycomb core structure, transparent and rigid PMMA face sheets were employed (PMMA is also a thermoplastic material commonly known as acrylic glass or Perspex). The panel dimensions were selected so that the specimen could be mounted on the open side of a rigid box test rig available in the laboratory, which hosted the shaker used to excite the panel in bending. A dedicated clamping frame was designed and manufactured for this purpose. The lateral members of the frame were equipped with small pulleys to ensure uniform tensioning of the wire passing through the rows of hexagonal core cells.

The design of the honeycomb panel was implemented in two phases. To start with, the flexural stiffness and the equivalent Young's modulus of the homogeneous panel were determined using the dimensions, material properties, and whole-panel reference data reported in Table 1, assuming simply supported boundary conditions. This is a reasonable hypothesis for the experimental rig used in the second part of this study where the panel was mounted on a rectangular frame with somewhat rounded edges.

i) Recalling that the surface of the panel is $S_p = L_x L_y$, the mass per unit surface was derived as follows:

Table 1
Geometry, materials and whole honeycomb panel reference data.

Subject	Parameter	Value
Geometry	Dimensions	$L_x \times L_y = 414 \times 314 \text{ mm}^2$
	Total thickness	$h_p = 12 \text{ mm}$
	Core Thickness	$h_c = 10 \text{ mm}$
	Faceplates Thickness	$h_{fp} = 1 \text{ mm}$
	Angle of honeycomb edges	$\theta_c = 30 \text{ deg}$
Materials	PA12 (Nylon)	Honeycomb density
		$\rho_c = 1010 \text{ kg/m}^3$
		Honeycomb Young's modulus
	PMMA (Perspex)	$E_c = 1650 \text{ N/mm}^2$
		Faceplate density
Whole-panel	Design reference values	$\rho_{fp} = 1180 \text{ kg/m}^3$
		Faceplate Young's modulus
		$E_{fp} = 3000 \text{ N/mm}^2$
		Total weight
		$M_p = 0.6 \text{ kg}$
		Fundamental natural frequency
		$f_1 = 200 \text{ Hz}$

$$m_p = M_p / S_p. \quad (1)$$

ii) The flexural stiffness B_p of an equivalent homogeneous panel was then derived from the following expression assuming its fundamental natural frequency for flexural vibrations is given by $f_1 = 200\text{Hz}$ and $r_1 = r_2 = 1$

$$f_r = \frac{1}{2\pi} \sqrt{\frac{B_p}{m_p} \left[\left(\frac{r_1 \pi}{L_x} \right)^2 + \left(\frac{r_2 \pi}{L_y} \right)^2 \right]}. \quad (2)$$

iii) Hence, taking $\nu_p = 0.33$, the Young's modulus of the equivalent panel was calculated from the following formula:

$$B_p = \frac{E_p h_p^3}{12(1 - \nu_p^2)}, \quad (3)$$

where $h_p = h_c + 2h_{fp}$ is the thickness of the whole honeycomb panel.

At this point the length and thickness of the hexagonal segments of the honeycomb structure were derived iteratively with the following steps assuming the material and geometrical properties of the core honeycomb and faceplates reported in Table 1.

iv) Considering the material (E_c, E_{fp}) and dimensional (h_c, h_{fp}) properties given in Table 1 and the Young's modulus of the equivalent panel E_p derived above, the core face plate thickness ratio $\gamma = h_c/h_p$ was derived from the following expression given by Johnson and Sims [76]:

$$E_p = E_c(\gamma)^3 + E_{fp}(1 - \gamma^3). \quad (4)$$

v) Next, for a given layout of the hexagonal honeycomb structure, defined by the number of hexagonal cells in the x and y directions and a given angle θ_c of each bay edges, the length l_c of each bay edges were derived from geometrical considerations (see Fig. 1b).

vi) Then, the thickness t_c of the honeycomb hexagonal structure was derived from the following formula given by Gibson in [77]:

$$E_c = E_{fp} \left(\frac{t_c}{l_c} \right)^3 \frac{\cos(\theta_c)}{(1 + \sin(\theta_c)) \sin^2(\theta_c)}. \quad (5)$$

vii) The mass of the honeycomb panel was finally calculated, recalling that, for the given layout, the honeycomb core structure accounts for a total number of hexagonal cells N_c and thus a total number of edges N_e

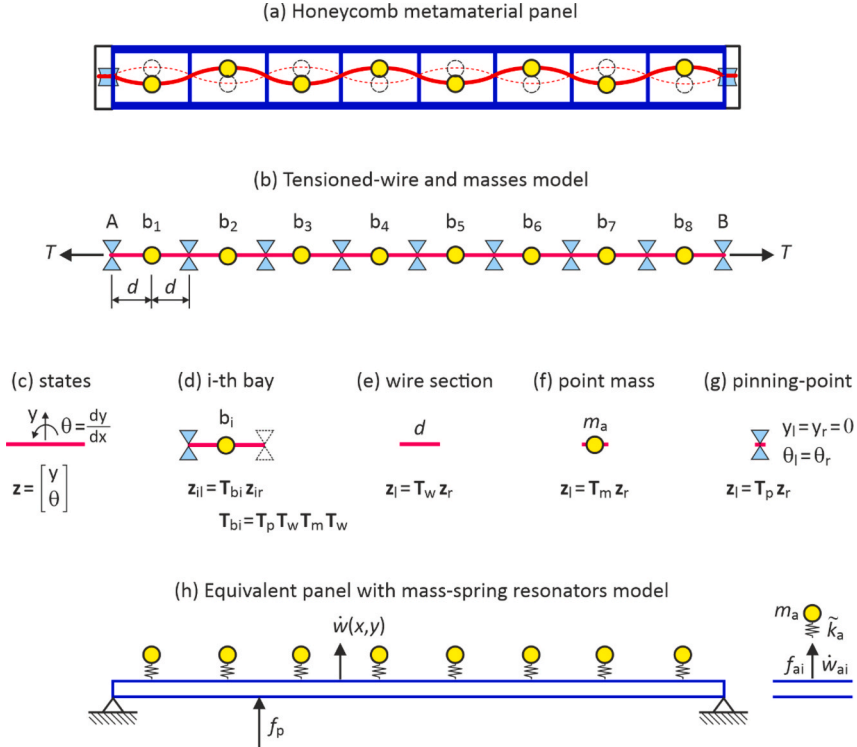


Fig. 2. Honeycomb metamaterial panel with tensioned wire and block masses. a) Transverse section with array of 8 hexagonal bays traversed by a stretched wire equipped with block masses. b) Model for the transverse vibration of one row of the tensioned wire and block masses. c-g) Components employed in the transfer-matrix model. h) Model of the equivalent homogeneous panel with mass-spring resonators.

$$M_p = 2S_p h_{ip} \rho_{ip} + N_e l_c h_c t_c \rho_c. \quad (6)$$

The steps iv), v), vi), vii) were repeated for a few layouts of the hexagonal honeycomb structure (i.e. for a few combinations of hexagonal bays in x and y directions) until the thickness of the honeycomb structure converged to a value such that the total mass of the panel was close to the reference value $M_p = 0.6\text{kg}$. As shown in Fig. 1b, the final geometry of the core honeycomb structure is formed by 7 rows with either 8 or 7 hexagonal cells having edges of length $l_c = 28.3\text{mm}$ and thickness $t_c = 2\text{mm}$. Thus, it encompasses a total number of hexagonal cells $N_c = 8 \times 4 + 7 \times 3 = 53$ and a total number of edges $N_e = 8 \times 16 + 9 \times 4 + 8 \times 4 = 188$.

2.2. Design of the tensioned wire-masses structure

As depicted in Fig. 1b, the core elastic metamaterial proposed in this article is made up of rows of hexagonal bays traversed by a stretched wire having small masses at the centre of each bay. Fig. 2a shows a lateral section of one of these arrays, emphasizing how the transverse vibration of the wire is pinned at the crossing points by the side faces of each hexagonal bay. The resulting transverse vibration of the tensioned wire-masses assembly is of paramount importance for assessing whether the proposed design produces a regular grid of local vibration resonators enclosed within the hexagonal bays of the honeycomb core structure. Hence, to start with, this section presents a mathematical model for the derivation of the transverse vibration of the tensioned wire-masses assembly passing through an array of hexagonal bays. Then, numerical results are presented for the first few natural frequencies and modes shapes of the transverse vibrations. The analysis is focused both on the effects produced by the wire tensioning and the wire and masses material properties. As will be shown below, the transverse vibration of the wire-masses structure depends on the applied tension, as well as on both the wire mass per unit length and the masses of the small blocks attached to the wire. The axial stiffness of the wire influences only the elongation that should be imposed on the wire to produce a given tension. Thus, it defines the tensioning actuator and tensioning mechanism that should be employed to generate the desired strain, and hence the desired tension, in the wire.

To investigate the transverse vibration of the tensioned wire and masses assembly, a transfer matrix model was developed [78,79], which is briefly discussed here. The model assumes that the two-pulleys systems used to turn the wire from one row of hexagonal bays to the neighbouring one (see Fig. 1b) firmly constrain the vibration of the wire such that the vibration in each row is decoupled from the other rows. Hence, as shown in Fig. 2b, the model considers only a single wire line encompassing either 7 or 8 bays, that is 7 or 8 masses. Moreover, it supposes a weak coupling between the transverse vibration of the tensioned wire-masses assembly and the flexural vibration of the core structure. Accordingly, the wire is assumed to be pinned at the locations where it passes through the side faces of the honeycomb structure. Hence, the line of wire-masses is modelled as an array of bays that, as shown in Fig. 2d, encompass: i) a pinning point ii) a plain wire section of length d , iii) a block mass m_a and iv) a wire section of length d . Transfer matrices are thus employed to relate the transverse vibration states on the right-hand and left-hand sides of these components.

As depicted in Fig. 2c, for a wire in tension, the state of the transverse vibration is defined by the transverse displacement $y(x, t)$ and the cross-section rotation $\theta(x, t) = \partial y(x, t) / \partial x$. Assuming time-harmonic vibrations, the two states can be written in the form $y(x, t) = \text{Re}\{y(x, \omega)e^{j\omega t}\}$ and $\theta(x, t) = \text{Re}\{\theta(x, \omega)e^{j\omega t}\}$, where $y(x, \omega)$ and $\theta(x, \omega)$ are the complex amplitudes of the displacement and rotation, ω is the circular frequency and $j = \sqrt{-1}$. The transfer matrix model developed for this study, relates the complex amplitudes of the states at two positions along the wire-masses-constraints structure, which are gathered into the following state vector

$$\mathbf{z}(x, \omega) = \begin{bmatrix} y(x, \omega) \\ \theta(x, \omega) \end{bmatrix}. \quad (7)$$

More specifically, the state vectors at the right-hand and left-hand sides of the components that make up the wire-masses-constraints line structure can be related by a 2×2 transfer matrix $\mathbf{T}$ such that

$$\mathbf{z}_r(\omega) = \mathbf{T}(\omega)\mathbf{z}_l(\omega), \quad (8)$$

where $\mathbf{z}_r(\omega) = \mathbf{z}(x_r, \omega)$ and $\mathbf{z}_l(\omega) = \mathbf{z}(x_l, \omega)$. Imposing congruence condition between the states at the right and left ends of neighbouring components, a global transfer matrix that relates the states at the left and right terminations of the whole line can then be found by simple multiplication of the consecutive transfer matrices of each element along the wire-masses-constraints line.

The transfer matrix for a segment of the wire (Fig. 2e) is first derived here recalling that the transverse vibration of a tensioned wire is governed by the following wave equation [81]:

$$\frac{\partial^2 y(x, t)}{\partial x^2} - \frac{1}{c^2} \frac{\partial^2 y(x, t)}{\partial t^2} = 0, \quad (9)$$

where $y(x, t)$ is the transverse displacement at position x of the wire, $c = \sqrt{T/\rho}$ is the speed of propagation of the transverse wave along the wire. Here, T is the tension applied to the wire and $\rho = \mu A_w$ is its mass per unit length of the wire, which is given by the product of the wire density μ and the wire cross section area $A_w = \pi D_w^2/4$, where D_w is the diameter of the wire. For time-harmonic motion, the transverse displacement can thus be written as follows

$$y(x, t) = \text{Re}\{A^+ e^{j(\omega t - kx)} + A^- e^{j(\omega t + kx)}\} = \text{Re}\{a^+(x, \omega)e^{j\omega t}\} + \text{Re}\{a^-(x, \omega)e^{j\omega t}\}, \quad (10)$$

where

$$a^+(x, \omega) = A^+ e^{-jkx}, \quad (11)$$

$$a^-(x, \omega) = A^- e^{+jkx}, \quad (12)$$

are the complex amplitudes at position x of respectively the positive-going and negative-going transverse waves having amplitude A^+ and A^- (here $k = \omega/c$ is the wavenumber). These two parameters represent the states of the wave propagation, which can be gathered into the following wave state vector

$$\mathbf{w}(x, \omega) = \begin{bmatrix} a^+(x, \omega) \\ a^-(x, \omega) \end{bmatrix}. \quad (13)$$

If, as shown in Fig. 2e, a section of the wire with length d is considered, then the wave state vectors at the right-hand and left-hand sides are linked by the following matrix expression

$$\mathbf{w}_r(\omega) = \mathbf{R}_w(d, \omega) \mathbf{w}_l(\omega), \quad (14)$$

where $\mathbf{w}_r(\omega) = \mathbf{w}(x_r, \omega)$, $\mathbf{w}_l(\omega) = \mathbf{w}(x_l, \omega)$ and

$$\mathbf{R}_w(d, \omega) = \begin{bmatrix} e^{-jkd} & 0 \\ 0 & e^{+jkd} \end{bmatrix}. \quad (15)$$

Now, moving back to Eq. (10) the displacement state $y(x, t)$ can be readily derived from the wave states $a^+(x, \omega)$ and $a^-(x, \omega)$. Also, the cross-section rotation state $\theta(x, t) = \partial y(x, t) / \partial x$ can be derived from the wave states recalling that, the shear stress at position x of the wire is given by

$$V(x, t) = -T \frac{\partial y(x, t)}{\partial x}, \quad (16)$$

where T is the tension applied to the wire and, for time-harmonic vibrations, $V(x, t) = \text{Re}\{V(x, \omega)e^{j\omega t}\}$. Hence, recalling Eq. (10)

$$V(x, t) = \text{Re}\{jkTA^+ e^{j(\omega t - kx)} - jkTA^- e^{j(\omega t + kx)}\} = \text{Re}\{jkTa^+(x, \omega)e^{j\omega t}\} - \text{Re}\{jkTa^-(x, \omega)e^{j\omega t}\}. \quad (17)$$

Thus, considering a position x along the wire, the displacement/rotation state vector defined in Eq. (7) can be derived from the wave state vector defined in Eq. (13) with the following matrix relation

$$\mathbf{z}(x, \omega) = \mathbf{S}(\omega) \mathbf{w}(x, \omega), \quad (18)$$

where

$$\mathbf{S}(\omega) = \begin{bmatrix} 1 & 1 \\ jkT & -jkT \end{bmatrix}. \quad (19)$$

Hence, the displacement/rotation state vectors at the right-hand and left-hand sides of the wire section of length d can be linked by the following matrix expression

$$\mathbf{w}_r(\omega) = \mathbf{T}_w(d, \omega) \mathbf{w}_l(\omega), \quad (20)$$

where

$$\mathbf{T}_w(d, \omega) = \mathbf{S}(\omega) \mathbf{R}_w(d, \omega) \mathbf{S}(\omega)^{-1}. \quad (21)$$

Recalling Euler's identities $e^{\pm jkd} = \cos(kd) \pm j \sin(kd)$, after some mathematical manipulations, the transfer matrix for the wire segment results given by

$$\mathbf{T}_w(d, \omega) = \begin{bmatrix} \cos(kd) & -\frac{1}{kT} \sin(kd) \\ kT \sin(kd) & \cos(kd) \end{bmatrix}. \quad (22)$$

Now, moving to the block mass (Fig. 2f), the total shear stress $V(x, t)$ applied at the left-hand and right-hand side of the mass must be balanced by the inertia of the mass itself, so that, the following relation is obtained

$$V(x_r, t) - V(x_l, t) = m_a \frac{\partial^2 y(x, t)}{\partial t^2} \Big|_{x=x_l, x_r}. \quad (23)$$

Hence, the displacement/rotation state vectors at the right-hand and left-hand sides of the block mass m_a are related by the following matrix expression

$$\mathbf{w}_r(\omega) = \mathbf{T}_m(m_a, \omega) \mathbf{w}_l(\omega), \quad (24)$$

where

$$\mathbf{T}_m(m_a, \omega) = \begin{bmatrix} 1 & 0 \\ -\frac{m_a \omega^2}{T} & 1 \end{bmatrix}. \quad (25)$$

Finally, assuming that, as depicted in Fig. 2a, the point where the wire passes through a small hole in the side faces of the hexagonal bay enforces the following constraints $y_r = y_l = 0$ and $\theta_r = \theta_l$, the displacement/rotation state vectors at the right-hand and left-hand sides of the pinning constraint (Fig. 2g) are linked by the following relation

$$\mathbf{w}_r(\omega) = \mathbf{T}_p(m_a, \omega) \mathbf{w}_l(\omega), \quad (26)$$

where the transfer matrix is given by

$$\mathbf{T}_p = \begin{bmatrix} 1 & 0 \\ -\frac{k}{T} & 1 \end{bmatrix}. \quad (27)$$

For simplicity, as shown in Fig. 2b, the wire-masses assembly has been subdivided into bays

$b_1, b_2, \dots, b_N$, which, as depicted in Fig. 2d are composed by a pinning point, a wire section, a block mass and a wire section, such that the transfer matrix for the i -th bay is given by

$$\mathbf{T}_{bi}(\omega) = \mathbf{T}_w(\omega) \mathbf{T}_m(\omega) \mathbf{T}_w(\omega) \mathbf{T}_p(\omega). \quad (28)$$

Hence, the transfer matrix for the whole set of 8 bays can be derived by multiplying the transfer matrices of each bay with a final transfer matrix for the remaining pinning point missing in the last bay:

$$\mathbf{T}(\omega) = \mathbf{T}_p(\omega) \mathbf{T}_{b8}(\omega) \mathbf{T}_{b7}(\omega) \dots \mathbf{T}_{b1}(\omega). \quad (29)$$

For the rows encompassing only 7 bays, the global transfer matrix is given by the same formula having only 7 transfer matrices $\mathbf{T}_{bi}(\omega)$. Now, the transfer matrix formulation for the whole wire and 8 masses passing through 9 pinning points can be rewritten in the following extended form

$$\begin{bmatrix} y_B(\omega) \\ \theta_B(\omega) \end{bmatrix} = \begin{bmatrix} T_{11}(\omega) & T_{12}(\omega) \\ T_{21}(\omega) & T_{22}(\omega) \end{bmatrix} \begin{bmatrix} y_A(\omega) \\ \theta_A(\omega) \end{bmatrix}. \quad (30)$$

Since $y_B(\omega) = y_A(\omega) = 0$ and $\theta_{A(\omega)} \neq 0$, $\theta_{B(\omega)} \neq 0$, the following condition must be satisfied

$$T_{12}(\omega) = 0, \quad (31)$$

where the dependence on frequency is shown to underline the fact that Eq. (31) is actually a frequency equation that gives indeed the natural frequencies $\omega_{n1}, \omega_{n2}, \omega_{n3}, \dots$ of the tensioned wire assembly. Once the natural frequencies are calculated, the vibration natural

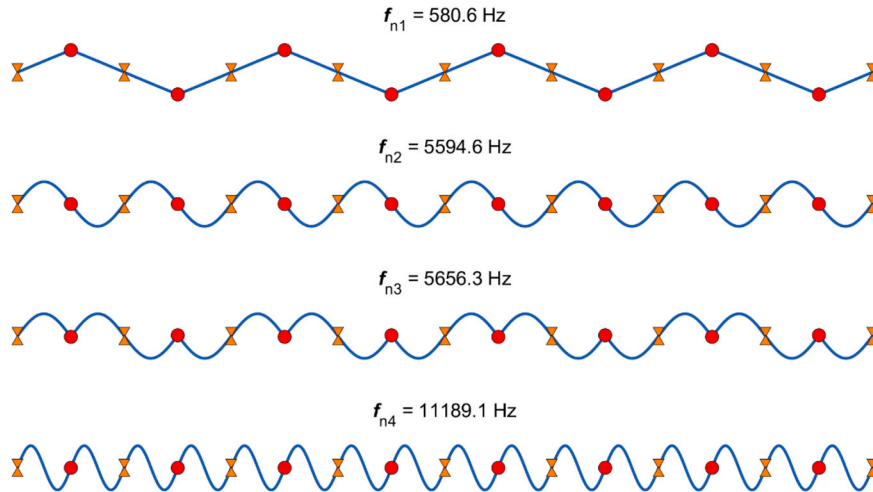


Fig. 3. First four natural frequencies and natural modes of a 8-spans tensioned wire with point masses characterised by the reference values $T_{ref} = 250\text{N}$, $m_{a,ref} = 1.4\text{g}$, $\rho_{ref} = 5 \frac{\text{g}}{\text{m}}$.

modes can be reconstructed to within a constant by substitution of the natural frequency into the transfer matrix formulation calculated along the whole length of the wire.

Fig. 3 shows the first four natural modes of a reference wire-mass configuration with 8 bays, which is characterized by the reference parameters $T_r = 250\text{N}$, $m_{a,r} = 1.4\text{g}$, $\rho_r = 5 \frac{\text{g}}{\text{m}}$. The first mode exhibits an almost triangular shape within each bay, which preserves the slope continuity at the pinning points. The transverse inertial effect of the lumped masses is sufficiently large to stretch the wire segments such that the mode shape has an almost rectilinear profile between adjacent masses. The higher-order modes exhibit smoother and more rounded shapes while still preserving the slope at the pinning points. In the odd modes (Modes 1 and 3), the masses occupy anti-nodal positions, where the modal amplitude reaches its maximum value. Consequently, the slope of the mode shape is discontinuous at the locations of the masses owing to the inertial forces exerted by the oscillating masses. In contrast, in the even modes (Modes 2 and 4), the masses are located at nodal points of the mode shapes. Since the masses do not undergo motion at these locations, they do not generate inertial forces, and therefore no discontinuity in the slope is observed across their positions.

For a fixed number, N_b , and length, $2d$, of the hexagonal bays, the natural frequencies and the mode shapes depend on three parameters of the wire-masses assembly, that is: the tension applied to the wire, T , the point masses, m_a , and the mass per unit length of the wire, ρ . Figs. 4, 5, 6 show how the first three natural frequencies of an 8-bays wire-masses structure vary when these three parameters are altered in turn from the reference values $T_r = 250\text{N}$, $m_{a,r} = 1.4\text{g}$, $\rho_r = 5 \frac{\text{g}}{\text{m}}$. To start with, Fig. 4 shows that the three natural frequencies sensibly grow as the tension T of the wire is raised. For instance, when the tension is raised from 50N to 500N the first natural frequency shifts from 260Hz to 821Hz, with an increment 561Hz. Similar effects are found for the second and third natural frequencies, which are confined at much higher frequencies comprised between 2530–8000Hz and 5020–15870Hz with increments of 5470Hz and 10850Hz respectively. Fig. 5 highlights that, when the masses arranged along the wire are raised, the fundamental natural frequency undergoes a significant drop of 500Hz comprised between 930Hz and 400Hz. The higher order modes are instead characterized by significantly smaller drops of about 140Hz and 70Hz respectively. Finally, Fig. 6 highlights that when the mass per unit length ρ of the wire is raised from 1 to $5 \frac{\text{g}}{\text{m}}$, the fundamental natural frequency drops only by 15Hz, whereas the second and third natural frequencies drops respectively by about 5260Hz, from 9605 to 4350Hz, and by about 10560Hz, from 19157 to 8599Hz.

The working principle of the core metamaterial is based on the resonant response of the first mode of the tensioned wire with point masses. Indeed, the triangular transverse vibration mode across each bay of the honeycomb material reproduces the dynamic response of a mass-spring resonant system, so that the panel results equipped with a regular grid of equal resonators. Hence, the focus of the analysis presented in this section is on the first mode. The results presented in Figs. 4–6 suggest that, in general, the fundamental natural frequency of the wire-masses structure is controlled by the tension applied to the wire and the mass of the blocks fitted on the wire. The mass per unit length of the wire has instead a little effect for realistic values, that is for realistic material densities and dimension (i.e. diameter) of the wire. Thus, to generate a given shift of the fundamental natural frequency of the wire-masses structure, and thus of the array of resonators, two strategies can be employed. The first imposes upfront the likely tensioning range ΔT of the wire and then derives the mass m_a of the blocks that would ensure the fundamental natural frequency lies in the desired range Δf_{n1} . Instead, the second, sets upfront the mass m_a of the blocks and then derives the tensioning range ΔT necessary to have the desired range of the fundamental natural frequency Δf_{n1} . Here, it should be emphasized that the larger is the mass of the blocks fitted along the wire the higher is the stopband effect generated by the array of resonators. Hence, the design of the wire-masses must identify the better compromise for the tensioning range ΔT and block masses m_a that would guarantee the desired range of the fundamental natural frequency Δf_{n1} for realistic values of the tensioning range ΔT and block masses m_a . In particular, the tensioning range must be within the limits imposed by the tensioning actuator and mechanism. Also, the block masses must be within the limits imposed by the total weight the panel could have.

Fig. 7 shows the shape of the first natural mode of a 8-spans tensioned wire with point masses characterised by a) the reference values $T_r = 250\text{N}$, $\rho_r = 5 \frac{\text{g}}{\text{m}}$, $m_{a,r} = 1.4\text{g}$, b) the reference values with $T = 2T_r$, c) the reference values with $m_a = 0.05m_{a,r}$, d) the reference values with $\rho = 20\rho_r$. Contrasting graph b) with the reference profile graph a), it is noted that when the tension is doubled, the shape of the natural mode remains the same with the typical triangular profile within each hexagonal bay. Also, comparing graph c) with graph a), it is found that when the mass of the blocks fitted along the wire is significantly reduced by a factor $1/20$ then the shape of the

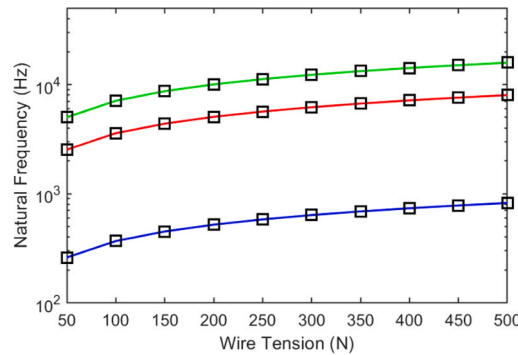


Fig. 4. First (blue line), second (red line), third (green line) natural frequencies with respect to wire tension T , assuming $m_a = m_{a,r} = 1.4\text{g}$, $\rho = 5 \frac{\text{g}}{\text{m}}$.

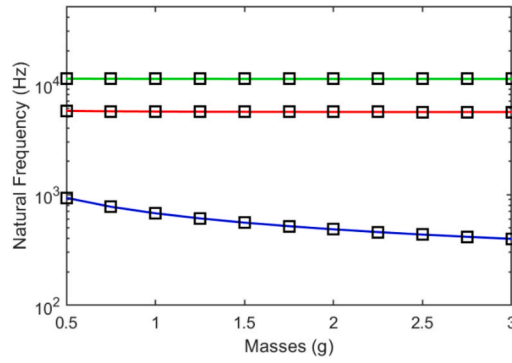


Fig. 5. First (blue line), second (red line), third (green line) natural frequencies with respect to masses m_a , assuming $T = T_r = 250\text{N}$, $\rho = \rho_r = 5 \frac{\text{g}}{\text{m}}$.

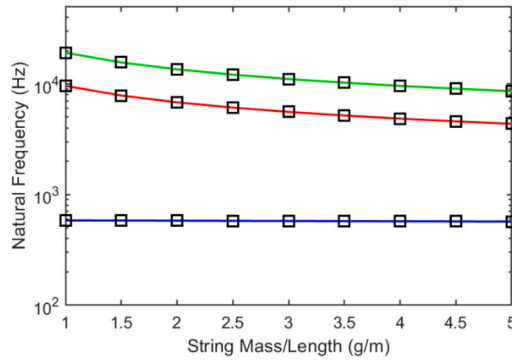


Fig. 6. First (blue line), second (red line), third (green line) natural frequencies with respect to wire mass per unit length ρ , assuming $T = T_r = 250\text{N}$, $m_a = m_{a,r} = 1.4\text{g}$.

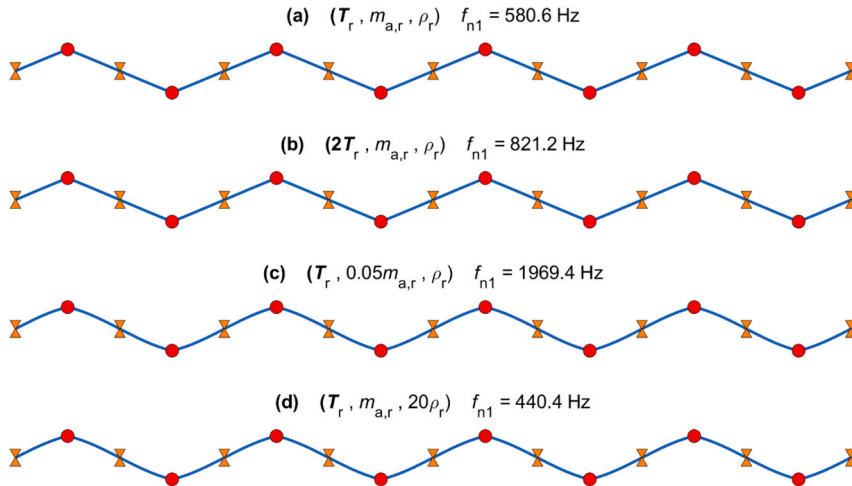


Fig. 7. First natural frequency and natural mode of a 8-spans tensioned wire with point masses characterised by a) the reference values $T_r = 250\text{N}$, $\rho_r = 5 \frac{\text{g}}{\text{m}}$, $m_{a,r} = 1.4\text{g}$, b) the reference values with $T = 2T_r$, c) the reference values with $m_a = 0.05m_{a,r}$, d) the reference values with $\rho = 20\rho_r$.

fundamental mode is rounded in proximity of the block masses. This is because the transverse inertial effect of the masses is not sufficient to pull enough the wire such that it has a rectilinear shape. Finally, contrasting graph d) with graph a), it is noted that when the mass per unit length of the wire is increased substantially, then the mode shows a rounded shape. In general, it can be concluded that the fundamental mode shape of the wire-masses system is characterised by triangular shapes within each bay unless the ratio between the mass of the blocks fitted on the wire and the mass per unit length of the wire, m_a/ρ , is very small.

Lastly, Fig. 8 shows the shapes and natural frequencies of the fundamental mode for the two wire-masses configurations present in

the proposed honeycomb panel, which, as can be seen in Fig. 1, encompass lines with either 8 or 7 hexagonal bays. The result presented in Fig. 8 confirms that, since the lengths of the bays remains the same, the fundamental natural frequency and mode shape are equal for the two configurations apart from the fact that one mode show 8 triangular profiles and the other only 7 triangular profiles.

The analysis presented in this section was employed for the design of the wire-masses structure to be fitted in the honeycomb panel described in the previous section. The design was implemented by incremental approximations until the block masses and wire mass per unit length were fixed to $\rho_r = 5 \frac{\text{g}}{\text{m}}$, $m_a = 2.5\text{g}$. In this way, with tensions comprised between $T = 50\text{N}$ and $T = 350\text{N}$, the fundamental natural frequency of 8-bays and 7-bays lines of wire-masses was set between $f_{n1} = 400\text{Hz}$ and $f_{n1} = 700\text{Hz}$. These can be seen as the resonance frequencies of mass-spring resonator accommodated in each hexagonal bay of the honeycomb structure, having a seismic mass $m_a = 2.5\text{g}$ and a stiffness $k_a = m_a \omega_a^2$, where $\omega_a = 2\pi f_{n1}$ and f_{n1} depends on the tension T applied to the wire.

3. Panel acoustic metamaterial: time-harmonic vibration and sound radiation response

The time-harmonic flexural response and sound radiation of the elastic metamaterial panel depicted in Fig. 1 are now investigated for a tonal transverse force excitation and different configurations of the resonators. The following two subsections first present the simplified mathematical model that has been developed to generate the simulation results and then present the analysis of the simulated spectra of the time-averaged total flexural responses and total sound radiation per unit force excitation.

Here, the objective is to provide fundamental insight into the vibration and sound-radiation bandgap effects occurring in the 400–700 Hz frequency range as increasingly dense arrays of absorbers with progressively higher inherent damping are considered. Therefore, the analysis is performed on an equivalent panel that reproduces the principal vibration features of the prototype honeycomb panel investigated in Section 4, most notably the resonant responses associated with the low-order flexural modes that govern the panel vibration up to 1 kHz. To this end, a simplified model is developed in which the honeycomb panel is replaced by a homogeneous panel with smeared density and equivalent bending stiffness properties (e.g., see Ref. [82] for more details on the smeared approach). This approach accurately captures the low-frequency flexural response of the panel with the array of resonators.

It is important to emphasize that, in the low-frequency range between 400 and 700 Hz considered in this study, the regular grid of resonators generates a stop-band effect through Bragg scattering. In fact, at such low frequencies, the flexural response of the panel under consideration is characterized by wavelengths much longer than the characteristic dimension of the grid of resonators, that is, approximately the distance between adjacent resonators. Moreover, the panel response is largely governed by stationary flexural wave patterns resulting from wave reflections at its boundaries. Indeed, the finite dimensions of the panel strongly influence its low-frequency vibration response in the frequency range where the bandgap effect occurs. Under these conditions, as thoroughly discussed by Sugino et al. [83], a dispersion analysis based on Bloch wave propagation would not provide conclusive information regarding the vibration-control effects generated by the grid of resonators. Instead, the vibration response should be analysed in terms of the modal behaviour of the finite structure, namely through the superposition of the resonant responses associated with its low-order flexural modes. This approach is therefore adopted in the following subsections.

3.1. Mobility-impedance model

This section presents the mobility-impedance model developed to calculate numerically the spectrum of the total flexural vibration response and the total sound radiation of the panel per unit force excitation. The formulation is derived considering the model depicted in Fig. 2h, where the honeycomb panel shown in Fig. 1 is exemplified as a homogeneous material having equivalent flexural stiffness E_p and smeared density $\rho_p = M_p/(S_p h_p)$.

The formulation assumes time-harmonic displacements, velocities and forces given in the form $w(t) = \text{Re}\{w(\omega)e^{j\omega t}\}$, $\dot{w}(t) = \text{Re}\{\dot{w}(\omega)e^{j\omega t}\}$, $f(t) = \text{Re}\{f(\omega)e^{j\omega t}\}$, where $w(\omega)$, $\dot{w}(\omega)$, $f(\omega)$ are their complex amplitudes, ω is the circular frequency and $j = \sqrt{-1}$. The column vector with the complex velocities at the base of the N absorbers $\dot{\mathbf{w}}_a(\omega) = [\dot{w}_{a1}(\omega) \ \cdots \ \dot{w}_{aN}(\omega)]^T$ can be derived with the following mobility matrix expression [84,85]:

$$\dot{\mathbf{w}}_a(\omega) = \mathbf{Y}_{ap}(\omega)\mathbf{F}_p + \mathbf{Y}_{aa}(\omega)\mathbf{f}_a(\omega), \quad (32)$$

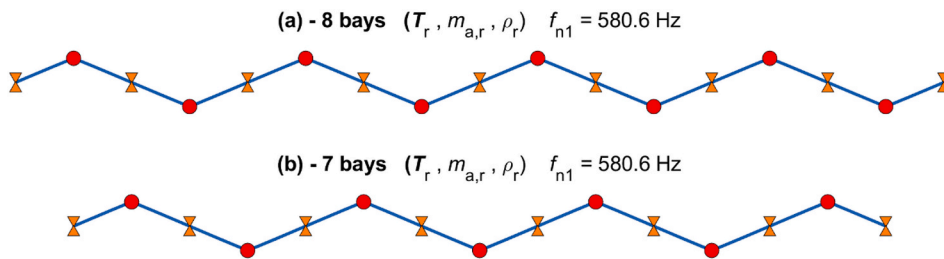


Fig. 8. First natural frequency and natural mode of a 8-spans and 7-spans tensioned wire with point masses characterised by the reference values $T_r = 250\text{N}$, $\rho_r = 5 \frac{\text{g}}{\text{m}}$, $m_{a,r} = 1.4\text{g}$.

where the column vector $\mathbf{f}_a = [f_{a,1}(\omega) \ \cdots \ f_{a,N}(\omega)]^T$ contains the complex amplitudes of the forces at the base of the absorbers and F_p is the amplitude of the primary force excitation, which was assumed equal to $f_p(t) = \text{Re}\{F_p e^{i\omega t}\} = F_p \cos(\omega t)$. Also, $\mathbf{Y}_{ap}(\omega)$ is a column vector with the plate mobility functions between the positions of the absorbers and the position of the primary force and $\mathbf{Y}_{aa}(\omega)$ is a square matrix with the plate mobility functions between the positions of the absorbers. The mobility functions in these two matrices are given by

$$Y_{ap,i}(\omega) = \frac{\dot{w}_{a,i}(\omega)}{f_p(\omega)} = \boldsymbol{\phi}_{a,i}^T \Omega(\omega) \boldsymbol{\phi}_p, \quad (33)$$

$$Y_{aa,ij}(\omega) = \frac{\dot{w}_{a,i}(\omega)}{f_{a,j}(\omega)} = \boldsymbol{\phi}_{a,i}^T \Omega(\omega) \boldsymbol{\phi}_{a,j}. \quad (34)$$

Here, the $\boldsymbol{\phi}_{a,i}(\omega) = [\phi_1(x_{a,i}, y_{a,i}) \ \cdots \ \phi_R(x_{a,i}, y_{a,i})]^T$, $\boldsymbol{\phi}_{a,j}(\omega) = [\phi_1(x_{a,j}, y_{a,j}) \ \cdots \ \phi_R(x_{a,j}, y_{a,j})]^T$, $\boldsymbol{\phi}_p(\omega) = [\phi_1(x_p, y_p) \ \cdots \ \phi_R(x_p, y_p)]^T$ are column vectors containing the mass-normalised modal amplitudes at the absorber and primary excitation positions, which, for a simply supported panel are given by

$$\phi_r(x, y) = 2 \sin\left(\frac{r_1 \pi x}{L_x}\right) \sin\left(\frac{r_2 \pi y}{L_y}\right), \quad (35)$$

where $r_1 = 1, 2, \dots$, $r_2 = 1, 2, \dots$ are the modal indices of the r -th mode. Also, $\Omega(\omega)$ is a diagonal matrix with the resonance terms

$$\Omega_{rr}(\omega) = \frac{j\omega}{M_p [\omega_r^2 - \omega^2 + j2\xi_r \omega_r \omega]}, \quad (36)$$

where ξ_r , ω_r are respectively the damping ratio and natural frequency for the r -th natural mode of the plate:

$$\omega_r = \sqrt{\frac{E_p h_p^2}{12\rho_p (1 - \nu_p^2)}} \left[\left(\frac{r_1 \pi}{L_x}\right)^2 + \left(\frac{r_2 \pi}{L_y}\right)^2 \right]. \quad (37)$$

In this equation, ρ_p , E_p , ν_p are the smeared density, equivalent Young's modulus of elasticity and equivalent Poisson ratio of the honeycomb panel. Also L_x , L_y , h_p , are the surface dimensions and the equivalent thickness of the rectangular panel. The vector of the forces and velocities at the base of the absorbers are related by the following impedance matrix expression

$$\mathbf{f}_a(\omega) = -\mathbf{Z}_{aa}(\omega) \dot{\mathbf{w}}_a(\omega), \quad (38)$$

where $\mathbf{Z}_{aa}(\omega)$ is a diagonal matrix whose i -th term is given by the base impedance of the i -th vibration absorber, that is:

$$Z_{aa,ii}(\omega) = \frac{j\omega m_a \tilde{k}_a}{(\tilde{k}_a - \omega^2 m_a)}. \quad (39)$$

Here m_a and $\tilde{k}_a$ are the absorber seismic mass and the complex stiffness that accounts for material losses, which is thus given by $\tilde{k}_a = k_a(1 + j\eta_a)$, where k_a is the stiffness of the spring and η_a is the loss factor. Now, substituting Eq. (38) into Eq. (32) gives

$$\dot{\mathbf{w}}_a(\omega) = \hat{\mathbf{Y}}_{ap}(\omega) F_p, \quad (40)$$

where $\hat{\mathbf{Y}}_{ap}(\omega) = [\mathbf{I} + \mathbf{Y}_{aa}(\omega) \mathbf{Z}_{aa}(\omega)]^{-1} \mathbf{Y}_{ap}(\omega)$. Thus, Eq. (38) becomes

$$\mathbf{f}_a(\omega) = -\mathbf{Z}_{aa}(\omega) \hat{\mathbf{Y}}_{ap}(\omega) F_p. \quad (41)$$

The time-average total flexural vibration of the panel is expressed with respect to the total flexural kinetic energy, such that, for time-harmonic vibrations,

$$K(\omega) = \frac{1}{T} \int_{t_1}^{t_1+T} \int_{S_p} \frac{1}{2} \rho_p h_p \dot{w}(x, y, t)^2 dS_p dt = \frac{1}{4} \rho_p h_p \int_{S_p} |\dot{w}(x, y, \omega)|^2 dS, \quad (42)$$

where $\dot{w}(x, y, t)$ is the velocity at position (x, y) of the plate, whose complex amplitude is given by

$$\dot{w}(x, y, \omega) = \boldsymbol{\phi}^T(x, y) \mathbf{b}_p(\omega) F_p + \boldsymbol{\phi}^T(x, y) \mathbf{b}_a(\omega) \mathbf{f}_a(\omega). \quad (43)$$

Here $\boldsymbol{\phi}(x, y) = [\phi_1(x, y) \ \cdots \ \phi_R(x, y)]^T$ is a column vector with the modal amplitudes at position (x, y) . Also,

$$\mathbf{b}_p(\omega) = \Omega(\omega)\boldsymbol{\phi}_p, \quad (44)$$

$$\mathbf{b}_a(\omega) = \Omega(\omega)\boldsymbol{\Phi}_a, \quad (45)$$

with $\boldsymbol{\Phi}_a = [\boldsymbol{\phi}_{a,1} \ \cdots \ \boldsymbol{\phi}_{a,N}]$ ($\boldsymbol{\phi}_p$ and $\boldsymbol{\phi}_{a,i}$ were defined above for Eqs. (33), (34)). Thus, substituting Eq. (43) into Eq. (32) and recalling Eq. (41), leads to

$$K(\omega) = \frac{1}{4}m_p\mathbf{B}_p^H(\omega)\mathbf{B}_p(\omega)F_p^2, \quad (46)$$

where $\mathbf{B}_p(\omega) = \mathbf{b}_p(\omega) - \mathbf{b}_a(\omega)\mathbf{Z}_{aa}(\omega)\hat{\mathbf{Y}}_{ap}(\omega)$ and $()^H$ is the Hermitian transpose matrix operator.

As shown in [15], assuming the panel lies on a baffle, for time-harmonic vibrations, the time-average total sound power radiation panel can be calculated from the following integral over the surface of the panel:

$$P(\omega) = \frac{1}{T} \int_{t_1}^{t_1+T} \int_{S_p} p(x,y,t) \dot{w}(x,y,t) dS_p dt = \frac{1}{2} \text{Re} \left\{ \int_{S_p} p(x,y,\omega) \dot{w}^*(x,y,\omega) dS_p \right\}, \quad (47)$$

where $p(x,y,t) = \text{Re}\{p(x,y,\omega)e^{j\omega t}\}$ is the acoustic pressure at position (x,y) on the surface of the panel and $p(x,y,\omega)$ is its complex amplitude. Considering the Rayleigh integral [15], this expression can be rewritten in terms of the panel velocity only

$$P(\omega) = \frac{1}{2} \text{Re} \left\{ \frac{j\omega^2\rho_0}{2\pi c} \int_{S_p} \int_{S'_p} \dot{w}(x',y',\omega) \frac{e^{-jkR}}{kR} \dot{w}^*(x,y,\omega) dS'_p dS_p \right\} \quad (48)$$

where $k = \omega/c$ is the acoustic wavenumber and c, ρ_0 are the speed of sound and the density of air. Also, $R = \sqrt{(x-x')^2 + (y-y')^2}$ is the distance between (x,y) and (x',y') positions. Thus, recalling Eqs. (43) to (45), the total the time-average total sound power radiation panel results given by

$$P(\omega) = \mathbf{B}_p^H(\omega)\mathbf{A}(\omega)\mathbf{B}_p(\omega)F_p^2, \quad (49)$$

where $\mathbf{A}(\omega)$ is the fully populated *power transfer matrix* whose m,n element is given by [15]

$$A_{m,n}(\omega) = \frac{\omega^2\rho_0}{4\pi c} \int_{S_p} \int_{S'_p} \phi_m(x',y',\omega) \frac{\sin(kR)}{kR} \phi_n(x,y,\omega) dS'_p dS_p \quad (50)$$

and $\phi_m(x',y',\omega)$, $\phi_n(x,y,\omega)$ are the amplitudes of the m -th and n -th flexural modes at positions (x,y) and (x',y') respectively.

3.2. Spectral analysis: simulation results

This section presents and discusses the 0–1kHz spectra of the panel total flexural kinetic energy and total sound power radiation per unit force excitation simulated with the model presented above assuming the transverse force excitation is located at $x_f = 0.75L_x$, $y_f = 0.45L_y$. For the honeycomb panel considered in this study, below 1 kHz the flexural modal overlap is fairly low [16]. As a result, the flexural vibration response at each frequency is governed by either a single resonant response or the superposition of a few resonant responses associated with the lower-order flexural modes of the panel [15]. Thus, to provide a relatively uniform excitation of the entire set of modes resonating below 1 kHz, the forcing point was carefully selected so as to avoid the nodal lines of the lower-order flexural modes of the panel. In this way, the spectrum of the total flexural response of the plain panel would show quite a uniform sequence of resonance peaks or multi-resonance crests, with no antiresonance troughs that could be confused with the bandgap generated by the array of resonators. The honeycomb panel is modelled as a simply supported homogeneous panel with the equivalent thickness and the smeared density and equivalent Young's modulus and Poisson's ratio reported in Table 1.

As anticipated above, the model encompasses the principal features of the flexural response of the prototype honeycomb panel built for this study (e.g., the resonant responses of the low-order flexural modes) and models the wire-masses resonators in each hexagonal bay of the core material simply with a mass-spring lumped parameter model. Moreover, it does not replicate the exact boundary conditions imposed on the panel by the test rig, which are bound to vary significantly in between measurements depending on the locking strength of the frame structure and changes in temperature. Hence, the results presented below should not be directly compared with the measurement results presented in Section 4, but should rather be regarded as background indications of the likely formation of bandgaps for given arrangements of the vibration absorbers.

The analysis considers three aspects. Firstly, the generation of a band gap centred at 500Hz with increasingly denser arrays of vibration absorbers characterised by grids with 4×4 , 5×5 , 6×6 , 7×7 , 8×8 , 9×9 absorbers having total seismic mass equal to 22% of that of the plain honeycomb panel and damping ratio of 2%. Secondly, the formation of a bandgap centred at 500Hz with a 8×8 grid of absorbers having total seismic mass equal to 22% of that of the plain panel and increasingly larger damping effects charac-

terised by 2%, 5%, 10%, 20% damping ratios. Thirdly, the making of bandgaps at 400, 500, 550, 600Hz, 650, 700Hz with the grid of $8 \times 7 = 56$ absorbers positioned at the centres of the hexagonal bays of the honeycomb panel depicted in Fig. 1 having total seismic mass is 22% of that of the plain honeycomb panel and damping ratio 2%. For simplicity, the first two analyses consider only the vibration response of the panel, whereas the third considers both its vibration response and sound radiation.

To start with, Fig. 9 shows that the configurations with 4×4 , 5×5 , 6×6 , 7×7 , 8×8 , 9×9 grids of absorbers generate respectively a -10 , -16 , -22 , -24 , -26 , -28 dB vibration bandgaps at 500 Hz. The bandgaps are characterised by a V-shaped trough that becomes increasingly deeper as the number of absorbers is raised. The bandgaps are accompanied by a reorganisation of the resonant response of the low order flexural modes. Moving to Fig. 10, it is noted that the bandgap produced by the 8×8 grid of absorbers tends to fall from -30 dB to -8 dB as the damping ratio of the absorber is brought from 2% to 20%.

Lastly, Fig. 11 shows the spectra of the time-averaged total flexural kinetic energy (left-hand plots) and total radiated sound power (right-hand plots) for the plain honeycomb panel (solid black line) and the honeycomb panel equipped with a grid of 53 absorbers positioned at the centres of the hexagonal bays shown in Fig. 1. Focusing first on the vibration and sound radiation of the plain honeycomb panel, it can be observed that the spectrum of the total radiated sound power exhibits a different layout than that of the total flexural kinetic energy. This is not surprising, since, at frequencies below acoustic coincidence [15], which, for the panel considered here, lies well above the 1 kHz frequency range, low-order modes can exhibit significantly different sound radiation efficiencies (e.g., see Ref. [15]). In particular, even modes are generally inefficient sound radiators because they are characterised by equal portions of the panel surface vibrating out of phase. As a result, destructive interference between the sound fields radiated by

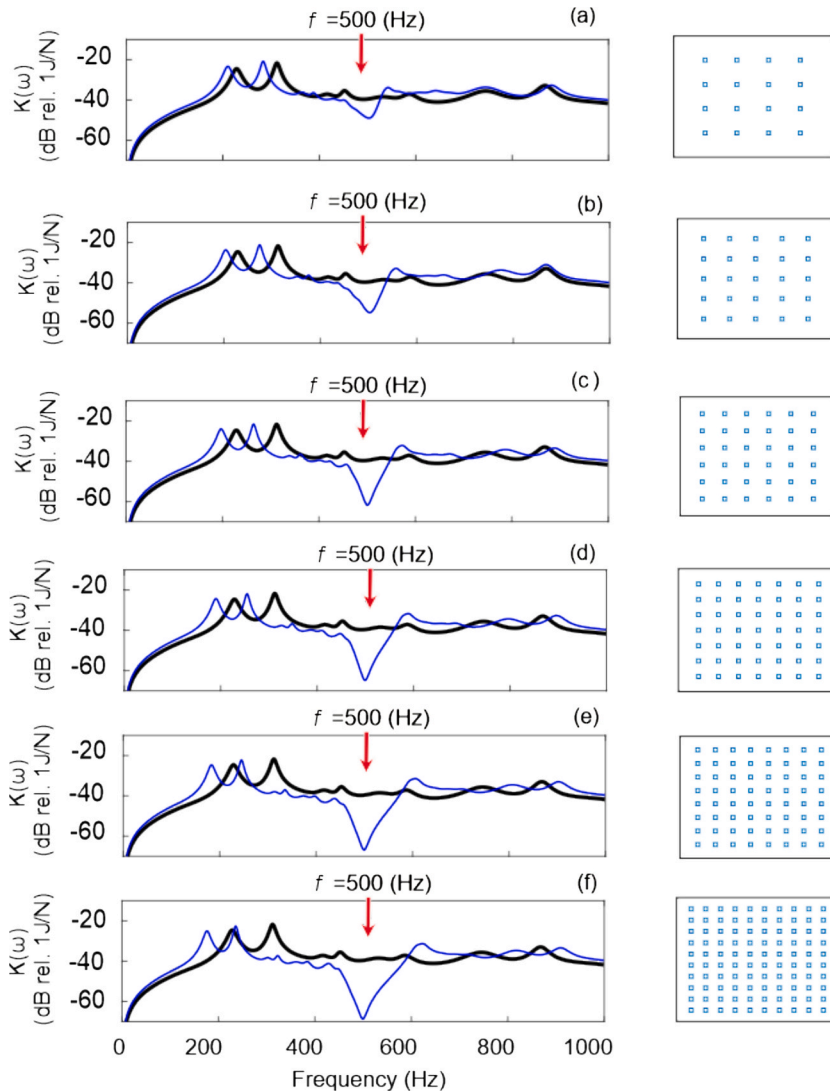


Fig. 9. Spectra of the total flexural kinetic energy of the honeycomb panel without (thick black lines) and with grids of 4×4 , 5×5 , 6×6 , 7×7 , 8×8 , 9×9 absorbers (faint blue lines) having total seismic mass equal to 22% of the plain honeycomb panel, resonance frequency 500Hz and damping ratio of 2%.

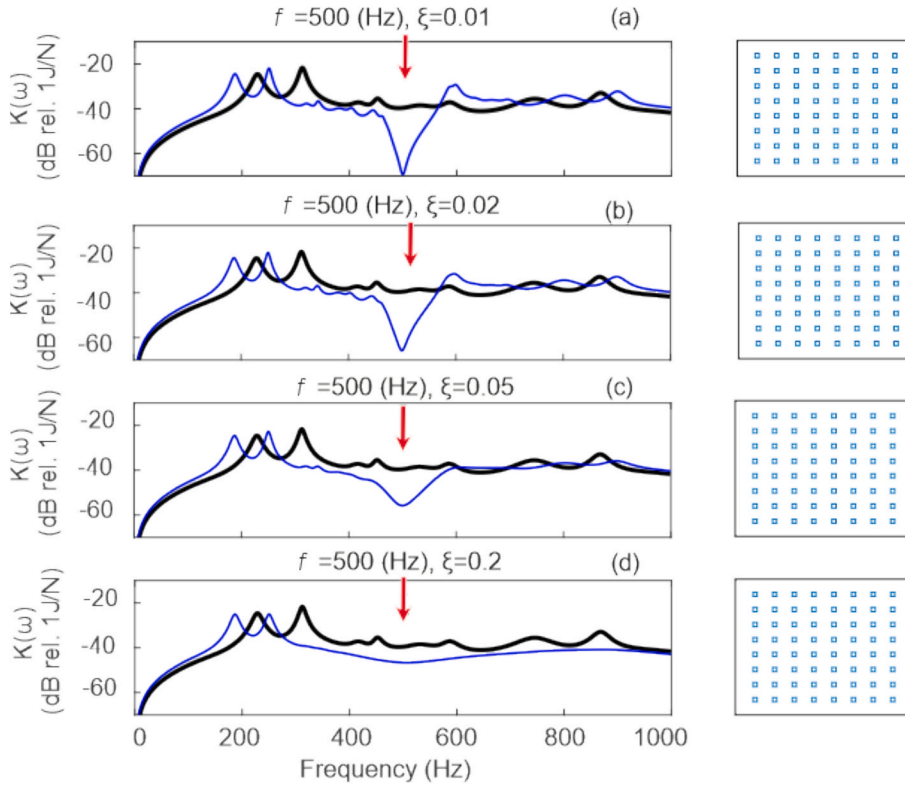


Fig. 10. Spectra of the total flexural kinetic energy of the honeycomb panel without (thick black lines) and with a 8×8 grid of absorbers (faint blue lines) having total seismic mass equal to 22% of the plain honeycomb panel and fundamental resonance frequency at 500 Hz.

these regions leads to a relatively low overall far-field sound radiation. By contrast, low-order odd modes, particularly the (1,1) mode, are efficient sound radiators because their transverse motion produces an in-phase volumetric displacement of the panel surface, resulting in effective sound radiation. Overall, all this results into a reorganisation of the amplitudes of the resonance peaks in the spectrum of the sound radiation, which are in fact controlled by the sound radiation efficiency of the resonant flexural mode [16].

Turning now to the vibration and sound radiation of the honeycomb panel equipped with the grid of 53 absorbers positioned at the centres of the hexagonal bays shown in Fig. 1, the left-hand side column of plots in Fig. 11 shows that pronounced vibration bandgaps are generated at 400, 500, 550, 600, 650, 700 Hz centre frequencies, with reductions in the flexural vibration response that progressively raise from 25 to 30 dB with increasing target frequency. The top width of these V-shaped bandgaps also raises from about 100 Hz to about 200 Hz with increasing target frequency. In parallel, the right-hand side column of plots in Fig. 11 shows that the sound radiation is also characterised by these bandgaps, although the associated reductions are less pronounced and lies between 20 and 25 dB. Interestingly here the deeper bandgaps of about 25 dB are obtained at the low target frequencies of 400, 500, 550, 600 Hz. Moreover, the top width of the V-shaped bandgaps are quite similar and equal to 100 Hz for all target frequencies. The reduced amplitude of the sound radiation bandgaps is likely to be due to the mechanisms that governs the sound radiation at frequencies below coincidence frequency [16]. In fact, as can be noticed from Eqs. (49) and (50), at each frequency, the total sound radiation by the baffled panel builds up from the self-radiations of each flexural mode and the mutual-radiations of all pairs of flexural modes (in fact the power transfer matrix in Eq. (50) is fully populated). As discussed in [16], at resonance frequencies where the vibration response is controlled by the resonant flexural mode, the sound radiation is bound to be controlled by the radiation efficiency of the resonant mode itself. For odd modes it will be quite large whereas for even modes it will be lower. Instead, at off-resonance frequencies the sound radiation is dictated by both the self- and the mutual-radiation efficiencies of the flexural modes that mainly contributes to the flexural response at the given frequency. As shown in [16] the mutual radiation efficiencies can be either positive or negative depending on frequency, which indicates that the sound radiations due to the interactions of pairs of modes could contribute constructively or destructively to the total sound radiation. Thus, it is expected that the lower amplitudes of the sound radiation bandgaps at the target frequencies of 400, 500, 550, 600, 650, 700 Hz may be caused by the fact that, at these frequencies, the array of resonators rearranges the flexural vibration field in such a way the destructive mutual sound radiation effects are less effective. Overall, the vibration and sound radiation spectra depicted in Fig. 11 confirm that the grid of 53 vibration absorbers generate substantial vibration and sound radiation bandgaps at the frequencies where the absorbers are tuned to resonate.

For completeness, the effectiveness of a regular grid of $8 \times 7 = 54$ absorbers was also investigated, which showed that the grid of $8 + 7 + 8 + 7 + 8 + 7 + 8 = 53$ absorbers positioned at the centres of the hexagonal bays of the honeycomb panel produce quite the same vibration and sound radiation bandgaps. This confirms that the proposed honeycomb panel, with the wire-mass absorbers encased

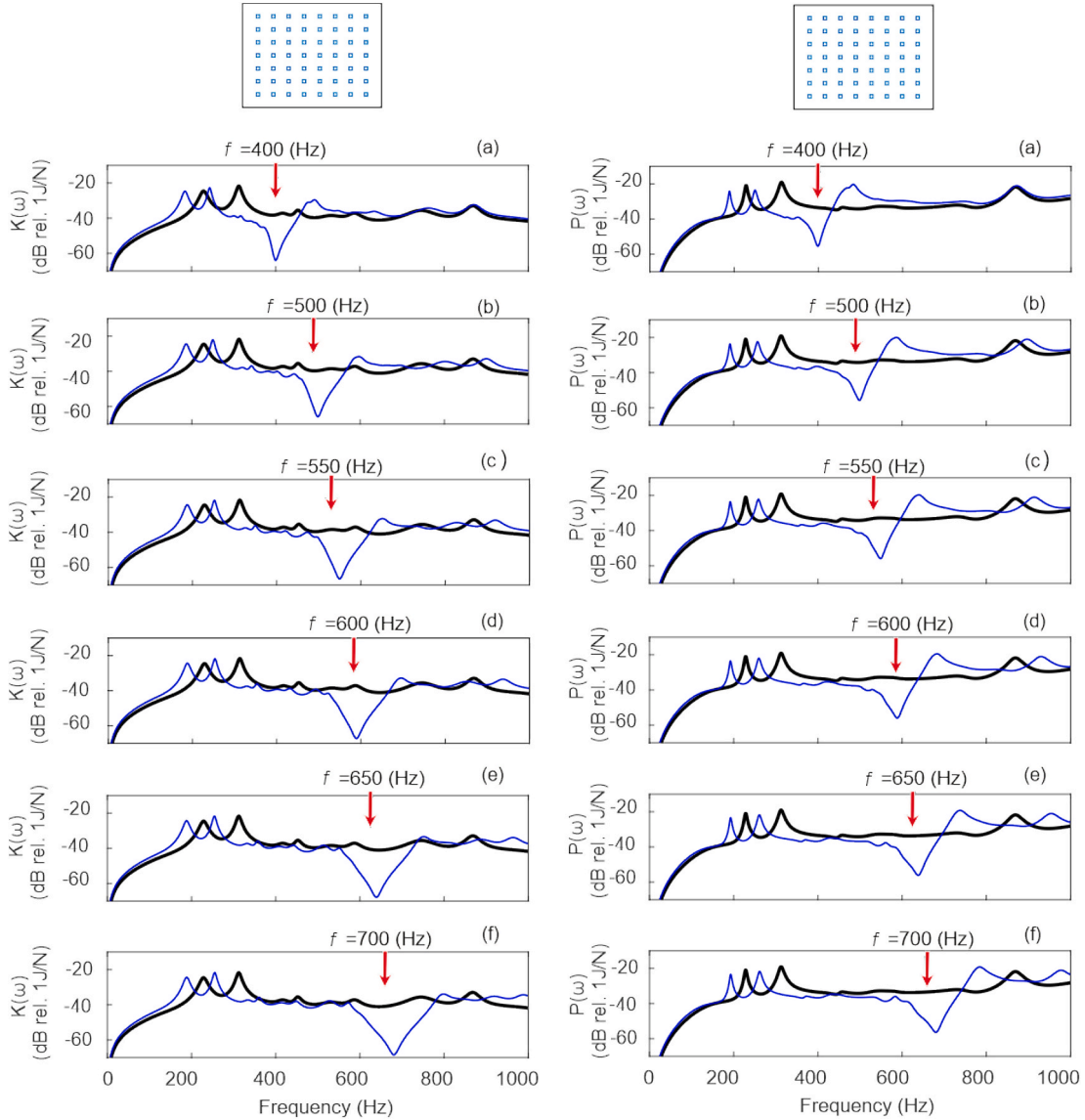


Fig. 11. Simulated spectra of the time-averaged total flexural kinetic energy (left hand side plots) and total sound power radiation (right hand side plots). Black line plane panel, blue lines panel with $8 \times 7 = 56$ wire-masses resonators tuned at a) 400, b) 500, c) 600, d) 700, e) 750, f) 800 Hz.

within the honeycomb bays, can be regarded in all respects as an acoustic metamaterial that effectively generates vibration and sound radiation bandgaps at the resonance frequency of the array of wire-mass resonators.

4. Experimental results

This Section presents an experimental investigation of the flexural response and sound radiation of a prototype honeycomb panel with the acoustic metamaterial core formed by wire-mass resonators, based on the designs of the plain honeycomb composite panel and tensioned wire-masses assembly presented in Section 2. The analysis focuses on the effectiveness of the tensioned wire-masses resonators in generating tuneable flexural vibration and sound radiation bandgaps over a wide frequency range. The experimental results presented in this section are bound to provide the conclusive validation of the proposed concept and of the theoretical framework developed in the paper.

4.1. Prototype panel and experimental rig

Fig. 12a shows the prototype honeycomb panel mounted on a rigid frame encompassing the return pulleys and tensioning reels used to strain the wire interlaced through the rows of hexagonal bays of the core honeycomb material. As discussed in Section 2.2, the

panel was made with Perspex faceplates having thickness 1 mm. Also, the 10 mm core honeycomb structure encompassing hexagonal structures with edges $l_c = 28.3\text{mm}$ and thickness $t_c = 2\text{mm}$ was 3D printed using PA12 material.

As described in Section 2.2 and shown in Fig. 12a,b, the resonators were made with a tensioned wire passing by the hexagonal bays of the honeycomb material in such a way as to form the interlaced pattern depicted in Fig. 1b. The wire was made of steel with diameter 1mm. The block masses were small spheres made of lead having diameter 8mm and weight 2.5g. The materials and dimensions of the wire and small masses were initially selected based on the parametric results presented in Figs. 4–6 of Section 2.3. They were then refined through a trial-and-error procedure, considering commercially available wires and spheres, so that the resonance frequency of the wire–masses array of resonators could be set within the range of 400–700 Hz.

As shown in Fig. 13a, the frame of the panel was fixed on a heavy box structure, which hosted the electro-magnetic shaker used to generate a transverse point force excitation at position $x_f = 0.75L_x$, $y_f = 0.45L_y$. The shaker was connected to the panel via a lightweight stinger equipped with a force cell used to detect exactly the force excitation. As anticipated in Section 3.2, the position of the excitation point was chosen so as to consistently excite all the flexural modes resonating below 1 kHz, such that the spectrum of the total flexural response of the plain panel was characterized by a relatively uniform sequence of single resonance peaks or multi-resonances crests, without antiresonance troughs that could be confused with the bandgap generated by the wire-mass resonator array.

As can be seen in Fig. 13b, the transverse velocity generated by the flexural vibration was measured at a dense grid of $N_m = 15 \times 10 = 150$ points with a scanner laser vibrometer. Based on a trial-and-error procedure, the selected measurement grid was found to guarantee a sufficiently accurate full-field characterization of the flexural vibration of the panel up to 1 kHz. More specifically, mobility Frequency Response Functions (FRFs)

$$Y_m(\omega) = \frac{\dot{w}_m(\omega)}{f_p(\omega)}, \quad (51)$$

were measured between the grid of $N_m = 150$ points and the force excitation. Here $\dot{w}_m(\omega)$ and $f_p(\omega)$ are respectively the complex amplitudes of the time-harmonic velocities, $\dot{w}_m(t) = \text{Re}\{\dot{w}_m(\omega)e^{j\omega t}\}$ and of the primary force, $f_p(t) = \text{Re}\{f_p(\omega)e^{j\omega t}\}$. The total flexural vibration of the panel was then derived in terms of the time-averaged total flexural kinetic energy, which recalling Eq. (42) was approximated with the following summation

$$K(\omega) \cong \frac{M_p}{4N_m} \sum_{m=1}^{N_m} |Y_m(\omega)|^2 F_p^2, \quad (52)$$

where F_p is the amplitude of the time-harmonic excitation.

Recently, optical methods have become established as a reliable and effective approach for measuring sound radiation from vibrating structures, using either laser vibrometry [86–88] or optical cameras [89–91]. Thus, the accurate laser vibrometer measurements performed to assess the overall vibration response of the prototype honeycomb panel with the wire-masses resonators have also been used to obtain its total radiated sound power. More specifically, recalling Eqs. (48) and (51), the time-averaged total radiated sound power has been derived from the following expression:

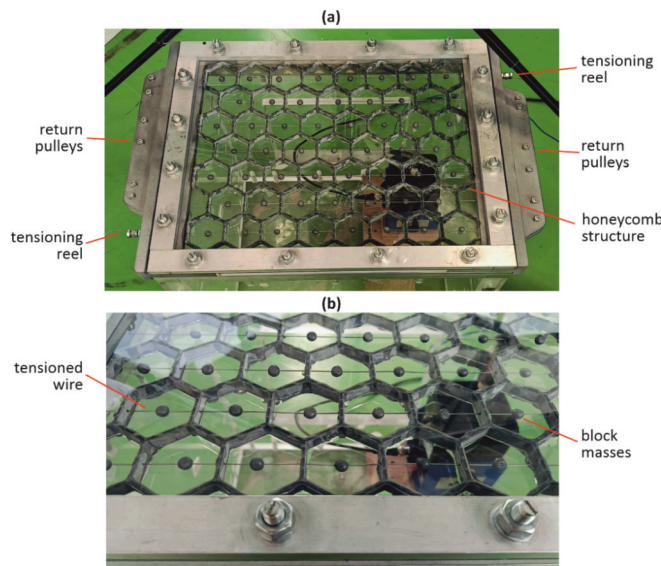


Fig. 12. a) Prototype honeycomb panel mounted on a rigid frame encompassing the return pulleys and tensioning reels. b) Detailed view of the tensioned wire and masses resonators in each hexagonal bay of the honeycomb core material.

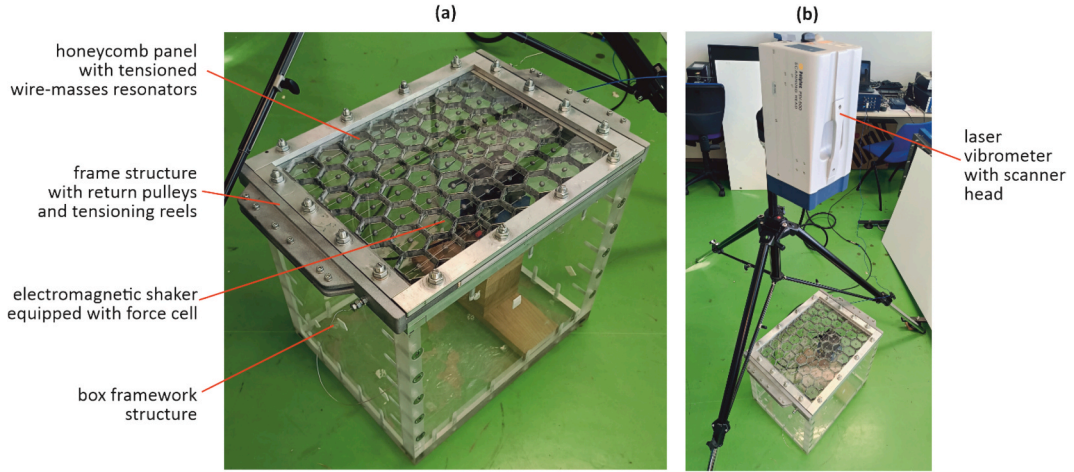


Fig. 13. a) Box framework structure used to hold the panel and to host the shaker for the point force excitation. b) Laser vibrometer with scanning head used to measure the flexural vibration of the panel.

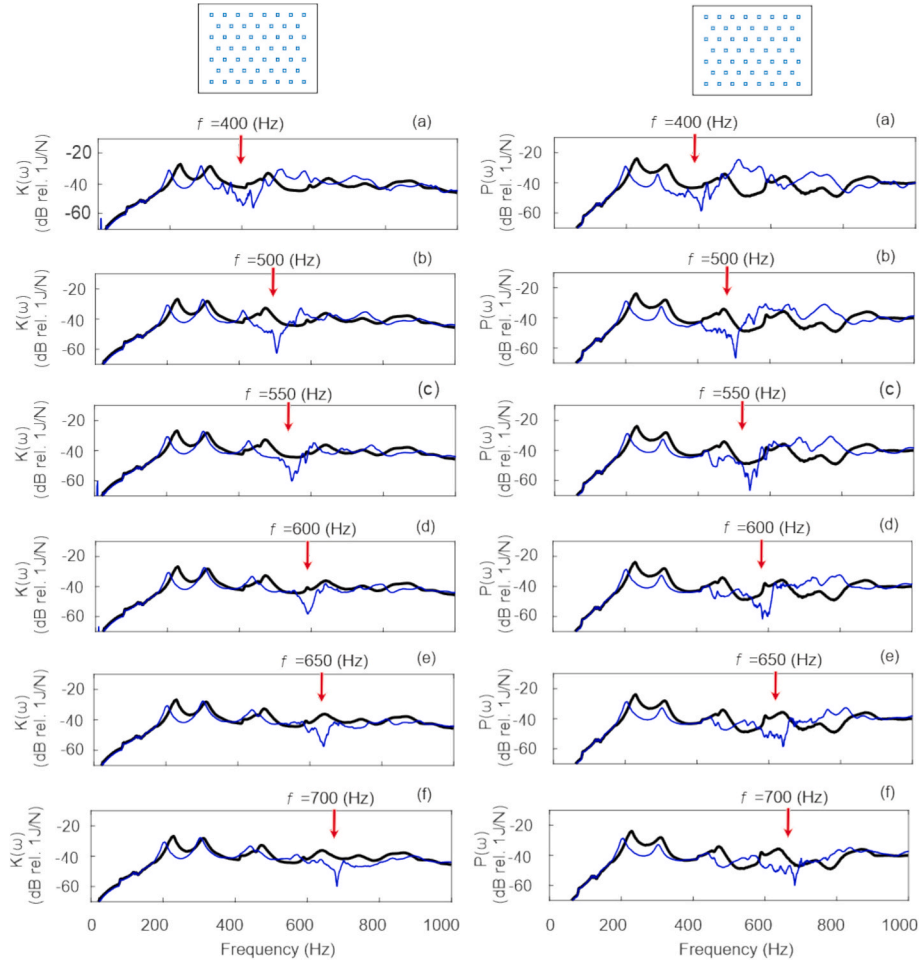


Fig. 14. Measured spectra of the time-averaged total flexural kinetic energy (left hand side plots) and total sound power radiation. Black line plane panel, blue lines panel with $8+7+8+7+8+7+8 = 53$ wire-masses resonators tuned at a) 400, b) 500, c) 600, d) 700, e) 750, f) 800 Hz.

$$P(\omega) \cong \frac{1}{2} \operatorname{Re} \left\{ \frac{j\omega^2 \rho_0 A_e}{2\pi c} \sum_{m'=1}^{N_m} \sum_{m=1}^{N_m} Y_m(\omega) \frac{e^{-jkR_{mm'}}}{kR_{mm'}} Y_m^*(\omega) \right\} F_p^2, \quad (53)$$

where $()^*$ is the complex conjugate operator, A_e is the area of the grid of rectangular elements centred at the $N_m = 15 \times 10 = 150$ grid of vibration measurement points and $R_{mm'} = \sqrt{(x_m - x_{m'})^2 + (y_m - y_{m'})^2}$.

4.2. Measured spectrum of the total flexural kinetic energy and total sound power radiation

Fig. 14 shows the spectra of the panel time-averaged flexural kinetic energy (left hand side column) and time-averaged total radiated sound power (right hand side column) obtained from the vibration measurement taken with the laser vibrometer as detailed in the previous Section 4.1 for the plane panel (black spectra) and for the panel equipped with the $N_c = 53$ resonators (blue spectra) tuned at 400, 500, 550, 600, 650, 700 Hz.

Focusing first on the vibration and sound radiation responses of the plain honeycomb panel, the measured results confirm the main trends predicted by the numerical simulations. In particular, the sound radiation spectrum exhibits a different spectral layout due to the uneven contributions to the sound radiation by the low order flexural modes of the panel. Turning to the honeycomb panel equipped with the grid of 53 vibration absorbers, the experimental measurements also confirm the formation of significant vibration and sound radiation band gaps around the absorber tuning frequencies. Considering first the vibration spectra in the left-hand side plots of Fig. 14, at the target frequencies of 400, 500, 550, 600, 650, 700 Hz, the prototype panel generates vibration bandgaps comprised between 12 and 25 dB. The top amplitude of the V-shaped bandgaps is significantly narrower than that predicted in the simulations and, apart from the bandgap at 400 Hz, it is of the order of 20 Hz. In parallel, the sound radiation spectra in the right-hand side plots of Fig. 14, show that the prototype panel generates at the target frequencies of 400, 500, 550, 600, 650, 700 Hz sound radiation bandgaps comprised between 12 and 20 dB.

The measured vibration and sound radiation spectra reported in Fig. 14 consistently exhibit smaller bandgap effects than those observed in the simulated spectra presented in Fig. 11. These discrepancies are primarily attributed to the challenges associated with the laboratory fabrication of the honeycomb panel incorporating the interlaced wire-mass absorbers. In particular, interlacing the wire-mass systems through the hexagonal bays of the honeycomb core required considerable care, especially in terms of accurately positioning the masses halfway across the bays and drilling sufficiently small holes to ensure that the wires were effectively pinned at the bay edges. Overall, the home-made panel exhibited some geometrical imperfections, which prevented the wire-mass absorbers in the individual honeycomb bays from being tuned to exactly the same frequency, despite the uniform tension applied to the wire throughout the entire honeycomb structure. As a result, the resonance stop-band mechanism of the 53 wire-mass resonators was not exactly focused at the target frequency, but was instead spread over neighbouring frequencies, resulting in a less pronounced bandgap. It is expected that the quality of the laboratory-made metamaterial honeycomb panel could be significantly improved through appropriate industrial manufacturing processes, potentially enabling the 20–30 dB stop-band attenuation predicted by the simulations.

Nevertheless, these initial experimental results have demonstrated that the proposed honeycomb panel equipped with an array of wire-mass absorbers can generate significant stopband effects in both the flexural vibration and sound radiation of the panel over a significant frequency range of 300 Hz comprised between 400 and 700 Hz. To achieve a self-contained system in which the wire tension is continuously adjusted online to track and control a time-varying tonal disturbance, the panel should be equipped with a pair of miniature electromechanical wire-tensioning mechanisms. These mechanisms could be accommodated within the corner hexagonal cells of the honeycomb panel and used to adjust the tension of the elastic wire supporting the absorber masses. In addition, a microcontroller, supplied with a reference signal associated with the tonal disturbance and implementing a simple frequency-tracking algorithm, should be integrated with the electromechanical tensioning mechanisms to continuously adjust the resonance frequency of the wire-mass absorbers. This would enable the bandgap effect to be dynamically tuned to the frequency of the unwanted time-varying tonal disturbance.

5. Conclusions

This work has introduced a tuneable acoustic metamaterial panel featuring a honeycomb composite structure incorporating a periodic network of wire-mass resonators interlaced through the hexagonal cells. The proposed concept enables adaptive vibration attenuation by tuning the resonance frequency of the resonators through the applied tensile load. Numerical and experimental investigations consistently demonstrate the formation of vibration and sound radiation bandgaps at prescribed frequencies, confirming the effectiveness of the proposed design. The experimental prototype achieved vibration and sound radiation attenuations of 12–25 dB and 12–20 dB respectively, validating the capability of the concept to provide tuneable control of vibration and noise radiation generated by time-varying tonal disturbances.

The proposed smart panel is envisaged as a promising solution for soundproofing applications in the aerospace, automotive, marine, and industrial sectors, including fuselage trim panels, vehicle firewalls, compartment partitions, and machinery enclosures. In particular, its tunable resonant behavior makes it suitable for controlling tonal disturbances with time-varying frequencies, such as those generated by vehicle powertrains. By integrating an electromechanical tensioning system with real-time feedback control, the wire-masses resonators could be continuously tuned to the disturbance frequency, thereby maintaining the stop-band effect at the target frequency.

The construction of the prototype highlighted several practical challenges that should be addressed in future implementations. The manufacturing procedure adopted in this study relied on manually interlacing the wire through the honeycomb core and positioning a concentrated mass at the centre of each cell before bonding the second face sheet. Although effective as a proof of concept, this approach proved labour-intensive and sensitive to assembly tolerances, particularly with respect to the accurate positioning of the seismic masses, which directly influences the dynamic response of the tensioned wire-masses structure. Also, the use of a commercially available 1 mm diameter steel wire required relatively high tensile loads to achieve the desired tuning range. These loads are sustained by the in-plane stiffness of the composite panel and may ultimately be limited by structural stability considerations, including the onset of panel buckling. This aspect identifies an important design trade-off between tuning capability and structural integrity.

Despite these challenges, the proposed metamaterial successfully demonstrated tuneable stop bands over a frequency tuning range approaching 300 Hz with vibration and sound radiation attenuations of up to 25 and 20 dB respectively. These results confirm the feasibility of the concept and its potential for adaptive vibration control in lightweight composite panels. Future work will focus on improving both the resonator design and the tuning mechanism. In particular, the use of tailored wire materials and active tensioning systems, such as piezoelectric actuation or thermally controlled shape memory alloy wires, could provide a more efficient, compact, and reliable means of adjusting the resonance frequency of the wire-masses structure while reducing the required tensile loads and facilitating practical implementation. In addition, a microcontroller implementing a simple frequency-tracking algorithm for the time-varying tonal disturbance should be developed to dynamically adjust the wire tension, thereby maintaining the vibration and sound radiation bandgap at the disturbance frequency.

CRediT authorship contribution statement

P. Gardonio: Writing – review & editing, Investigation, Conceptualization. **F. Codutti:** Investigation, Conceptualization. **L. Ortis:** Writing – review & editing, Investigation, Conceptualization. **E. Rustighi:** Writing – review & editing, Investigation, Conceptualization. **C. Malacarne:** Investigation, Conceptualization. **M. Perini:** Conceptualization, Investigation.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Data availability

Data will be made available on request.

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