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38123 Povo – Trento (Italy), Via Sommarive 14
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A RECONSTRUCTION STRATEGY BASED ON THE DORT METHOD FOR IMAGING FINITE DIMENSION SCATTERERS

Davide Franceschini, Massimo Donelli, Andrea Rosani, and Andrea Massa

Department of Information and Communication Technology,
University of Trento, Via Sommarive 14, 38050 Trento, Italy
Phone: +39 0461 882067, Fax: +39 0461 882093
Email: andrea.massa@ing.unitn.it, {manuel.benedetti, andrea.rosani}@dit.unitn.it

Abstract. This paper presents a two stage strategy for the electromagnetic imaging of unknown lossless profiles. The first step is aimed at providing the localizations and an estimate of the number of the regions-of-interest (*RoI*s) where the scatterers are supposed to be located through the application of the *DORT* method enhanced with a suitable processing of the information associated with the eigenvalues of the time reversal operator. Towards this aim, a suitable procedure is developed and applied in an extended and unsupervised fashion for localizing the finite-dimension scatterers under test. Successively, the second stage is devoted to reconstruct the unknown scatterer distributions by means of an iterative multi-resolution non-linear inversion performed only in the RoIs determined during the previous step.

Keywords: Microwave Imaging, Electromagnetic Inverse Scattering, Time-Reversal, DORT

1. INTRODUCTION

Electromagnetic inversion techniques are based on the assumption that the reconstruction of unknown objects lying in an investigation domain can be obtained through the analysis of the electromagnetic interactions between interrogating sources and targets under test. In particular, when electromagnetic fields at microwave frequencies are considered, several non-trivial issues as non-linearity, non-uniqueness, and ill-posedness [1] should be carefully addressed. Some of such drawbacks can be overcome avoiding the solution of the full inverse scattering problem and only aiming at detecting or localizing the targets under test. For example, this guideline is pursued in [2] and [3]. Moreover, the method of decomposition of the time reversal operator (DORT) has been successfully applied [4] for localizing unknown objects. Such a methodology allows one to synthesize an incident wave focusing on a selected target by performing a time reversal operation on a set of scattered field measures. Undoubtedly, the advantages of this technique are manifold since it significantly simplifies the imaging procedure because of the reduced computational costs and its robustness to the measurement noise.

However, the DORT also presents some intrinsic limitations since it turns out to be very effective for small scatterers [4] and only provides the localization of the targets under test. As a matter of fact, no quantitative information about the characteristics of the dielectric profile under test can be obtained. On the contrary, several applications of electromagnetic imaging require a detailed retrieval of the distributions of both permittivity and conductivity parameters. Unfortunately, such a requirement involves the solution of a full non-linear inverse scattering problem by means of computationally-expensive deterministic or stochastic procedures (see [5] and the references cited therein).

In order to avoid the use of numerical iterative techniques with large discretization grids (thus excessive computational costs and local-minima problems), this contribution presents a reconstruction strategy based on two steps. During the first one, an enhanced version of the DORT method (aimed at extending its range of applicability when the "point-like scatterer" condition does not hold true) is employed to define positions and extensions of the RoIs, while in the second step the quantitative reconstruction of the RoIs is obtained through an iterative multi-resolution optimization algorithm [6].

2. MATHEMATICAL FORMULATION

Let us suppose that an unknown object located in an investigation domain D_I and described in terms of an object function distribution $\tau(x, y)$ has to be reconstructed. By assuming a multi-view/multi-illumination acquisition setup, the scenario under test is probed by a set of V different incident fields $E_{inc}^{v}(x, y)$, $v = 1,...,V$, and the inverse scattering process is carried out starting from a set of scattered electric field samples, $E_{scatt}^{v}(x_m, y_m), m = 1,..,M^{(v)}$, collected in an external measurement domain D_M.

In order to better exploit the information available from inversion data, the reconstruction problem is solved through a two-stage strategy, whose key-features can be summarized as follows.

Step 1: RoIs definitions by means of an enhanced DORT-based technique

At the first step the *RoI*s lying in D_I are determined exploiting an enhanced version of the *DORT* method. According to the standard procedure presented in [4], the set of eigenvalues of the time reversal operator, $\{\lambda_p, p = 1,..., P\}$, and their corresponding eigenvectors, $\{\underline{\upsilon}_p, p = 1,..., P\}$, are determined. For point-like targets, the number of nonvanishing eigenvalues is directly related to the number of scatterers (i.e., the number of *RoI*s). Unfortunately, such a rule does not hold true in many applications of practical interest, thus a suitable method has been developed for fruitfully exploiting the information on the objects provided by the dominant set of *Q* eigenvalues. Such a set is defined by analyzing the step-like behaviour of $\{\lambda_p, p = 1,..., P\}$ and identifying the threshold value η_λ. Accordingly, the dominant set of eigenvalues turns out to be $\{\lambda_p, p = 1,..., Q\}$, being $\lambda_q > \eta_\lambda$, and the corresponding eigenvectors can be selected and exploited to generate *Q* fields

$$\widetilde{E}_p(x, y) = \frac{\omega\mu_0}{4}\sum_{m=1}^{M}\upsilon_{m,p} H_o^{(1)}(k_o\rho_m) \qquad p = 1,..., Q \tag{1}$$

where ω is the angular frequency, $\upsilon_{m,p}$ the m-th component of the p-th eigenvector, $H_o^{(1)}$ the first-kind zero-th order Hankel function, k_0 the background wave number and $\rho_m = \sqrt{(x_m - x)^2 + (y_m - y)^2}$.

Successively, the information about the $i = 1,..., I$ *RoI*s [i.e., the values of the coordinates, $\left(x_{RoI}^{i}, y_{RoI}^{i}\right)$, and the dimensions, $L_{D(RoI)}^{i}$] is obtained by processing a thresholded map, $E^{TH}(x, y)$, of the superposition

$$E_{\sup}(x,y)=\sum_{p=1}^{Q}\tilde{E}_p(x,y) \quad (2)$$

of the *Q* synthesized fields $\left[\tilde{E}_p(x,y),\, p=1,\dots,Q\right]$, while $E^{TH}(x,y)$ is obtained as

$$E^{TH}(x,y)=\begin{cases}0 & \text{if } E_{\sup}(x,y)<\eta_E\\ \max\left[E_{\sup}(x,y)\right] & \text{otherwise}\end{cases} \quad (3)$$

in which η_E is a threshold heuristically-determined.

Step 2. Quantitative reconstruction of the unknown contrast

Starting from the information on the detected *RoI*s, the second step consists in a reconstruction procedure aimed at retrieving a finite dimensional representation of the object function. Towards this end and in order to better exploit the information content of the scattered data, the *IMSA* [6] is applied. At each step of the *IMSA*, the reconstruction problem is recast into an optimization one by defining a multi-resolution cost function to be minimized by means of an effective deterministic or stochastic algorithm.

3. NUMERICAL RESULTS

Let us consider a homogeneous dielectric ($\tau_{ref}=0.5$) scatterer of square ($L_{ref}=\lambda$ sided) cross-section. Such a cylinder [Fig. 1(*a*)] is located in a square investigation domain of side $L_{D_I}=4\lambda$ illuminated by a set of $V=8$ plane waves impinging from equally-spaced directions. The data are collected on a circular observation domain ($R_{obs}=2{,}6\lambda$ in radius) at $M=37$ equally-distributed measurement points. The result of the enhanced time-reversal step (*Step 1*) is given in [Fig. 1(*b*)] where the thresholded map of the synthesized is shown as well as the estimate of the *RoI* obtained by properly processing the field distribution.

Successively the RoI is discretized in $N=144$ equal subdomains and the reconstruction is carried out by means of the IMSA [Fig. 1(*c*)]. Eventually, for comparison purposes, Fig. 1(*d*) shows the results obtained with a standard conjugate gradient-based optimization algorithm, but without exploiting the information on the RoIs and choosing a discretization grid such that the same resolution accuracy of the previous experiment is assured in the whole investigation domain ("*bare*" approach).

Another set of numerical experiments has been carried out in order to study the effectiveness of the proposed approach in dealing with scenarios characterized by multiple *RoI*s. Therefore, the configuration of Fig. 2(*a*) concerned with two homogeneous ($\tau_{ref}=0.5$) cylinders ($L_{ref}=0.7\lambda$-sided) located at $x_c^{(1)}=-y_c^{(1)}=-1.0\lambda$, $x_c^{(2)}=-y_c^{(2)}=1.0\lambda$ has been considered. Moreover, the parameters of the probing setup are the same used in the previous experiment.

Let us first present the results obtained in the noise free case. Thanks to the proposed strategy, two *RoI*s centred at $x^{(1)}=-0.94\lambda$, $y^{(1)}=0.95\lambda$ and $x^{(2)}=0.97\lambda$, $y^{(1)}=-0.96\lambda$ have been estimated (the dimensions of the *RoI*s are $L^{(1)}=0.86\lambda$ and $L^{(2)}=0.85\lambda$, respectively). Then, such *RoI*s have been imaged and the reconstructions achieved through the two-step

DORT method are shown in [Fig. 2(*b*)]. The retrieved contrast is clearly more accurate than that obtained using the standard "bare" approach [Fig. 2(*c*)].

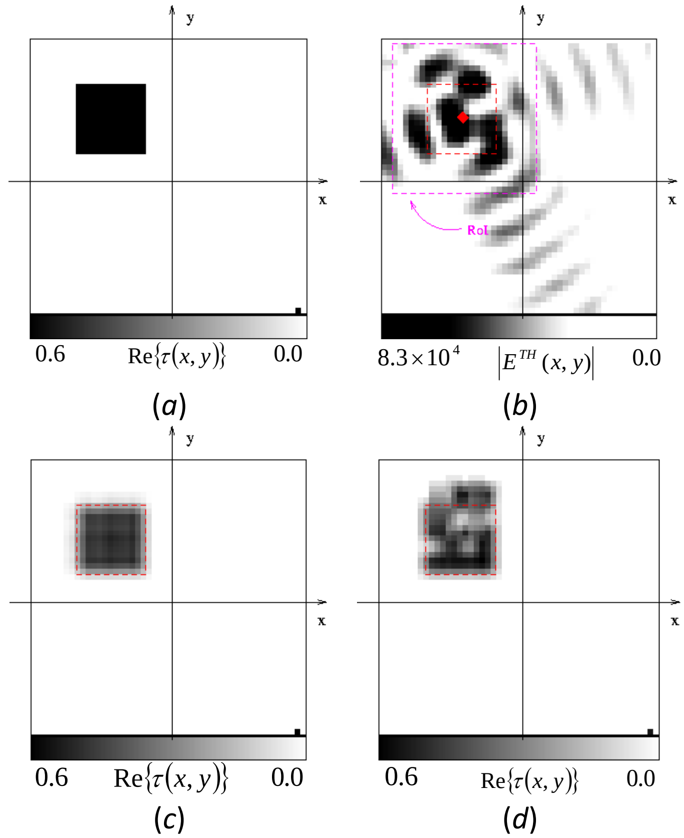

Figure 1. Square dielectric cylinder. (*a*) Reference distribution of the object function. (*b*) Map of the thresholded electric field associated to the first *Q* singular values. Reconstructed profile by means of the proposed two step strategy (*c*) and the standard approach (*d*).

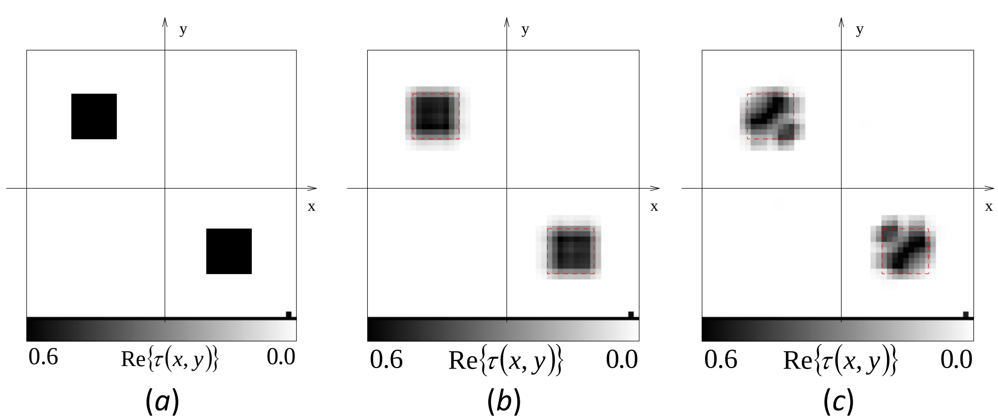

Figure 2. Multiple scatterers. (*a*) Reference profile. Reconstructed profile (noise free data) by means of the proposed two step strategy (*b*) and the standard approach (*d*).

Since the use of thresholds in the first step of the proposed method could be critical when dealing with noise-contaminated measures, the robustness of the two-step algorithm has been analyzed blurring the problem data by means of an additive a gaussian noise.

As expected, the DORT-based approach is not insensitive to the noise, even though the spectrum of the eigenvalues (as shown in Fig. 3) does not show significant differences compared to the noiseless situation. Therefore, this recurrent step-like behaviour has suggested to set the threshold value to $\eta_{\lambda} = 10^{-4}$.

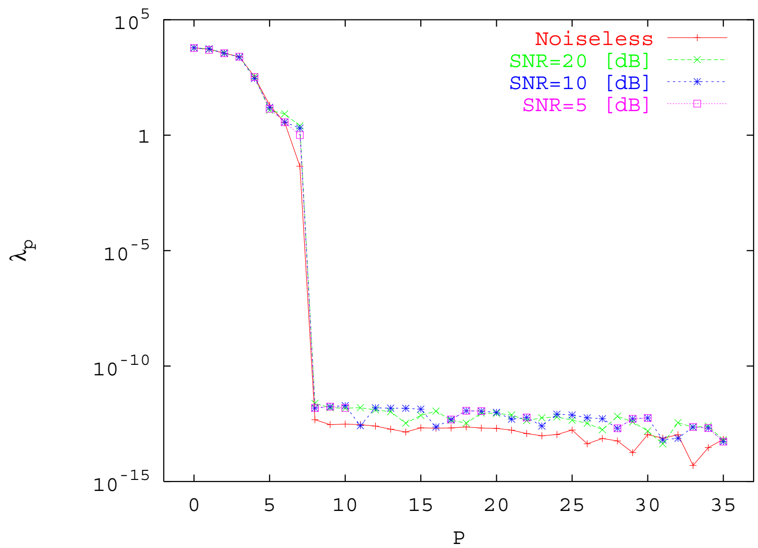

Figure 3 Multiple scatterers. Behavior of the eigenvalues spectrum for different *SNR* values.

As far as the reconstruction accuracy is concerned, the results of the numerical analysis that has been carried out are reported in Tab. I in terms of the corresponding error figures (for the definition of the reconstruction errors please refer to [6]). The performance of the approach decreases when corrupted data are considered, but the values of Tab. I confirm a satisfactory robustness of the two-step method, being $\chi_{tot} < 5.3\%$ for $SNR > 5\ dB$.

	χ_{tot}	$\chi_{\mathrm{int}}^{(1)}$	$\chi_{\mathrm{int}}^{(2)}$	χ_{ext}
Noiseless	*3.47*	*2.99*	*3.01*	*2.07*
$SNR = 20\,dB$	*3.71*	*3.59*	*3.18*	*2.67*
$SNR = 10\,dB$	*4.02*	*4.27*	*4.00*	*3.71*
$SNR = 5\,dB$	*5.32*	*4.49*	*4.74*	*4.51*

Table I. Multiple scatterers. Values of the error figures for the proposed two step strategy when the data are blurred by means of a gaussian noise.

4. CONCLUSIONS

In this paper, a two-step strategy based on the *DORT* method (originally devoted to detect point-like scatterers) has been presented for reconstructing finite-dimension objects. A set of representative results demonstrated that the proposed technique is able to overcome the standard reconstruction approach guaranteeing a good accuracy in the retrieval process.

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