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Rolling motion: experiments and simulations focusing on sliding friction forces

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Summary. — The paper presents an activity sequence aimed at elucidating the role of sliding friction forces in determining/shaping the rolling motion. The sequence is based on experiments and computer simulations and it is devoted both to high school and undergraduate students. Measurements are carried out by using the open source Tracker Video Analysis software, while interactive simulations are realized by means of Algodoo, a freeware 2D-simulation software. Data collected from questionnaires before and after the activities, and from final reports, show the effectiveness of combining simulations and Video Based Analysis experiments in improving students' understanding of rolling motion.

1. – Introduction

As is well known, rolling motion is a complex phenomenon whose full comprehension involves the combination of several fundamental physics topics, such as rigid body dynamics, friction forces, and conservation of energy. For example, to deal with collisions between two rolling spheres in an appropriate way requires that the role of friction in converting linear to rotational motion and vice-versa be taken into account (Domenech and Casasús, 1991; Mathavan *et al.* 2009; Wallace and Schroeder, 1988).

In this paper we present a sequence of activities aimed at spotlighting the role of friction in rolling motion in different situations. The sequence design results from a careful

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analysis of textbooks and of research findings on students' difficulties, as reported in the literature. A series of experiments based on video analysis is used to highlight selected key concepts and to motivate students in their exploration of the topic. Interactive simulations, which can be modified on the fly by students, are used to stimulate autonomous investigation in inquiry activities. Measurements are performed through the Tracker Video Analysis open source tool; while interactive simulations are designed and run within the freeware 2D simulation environment Algodoo (`http://www.algodoo.com/`). The sequence of activities is aimed at both high school and undergraduate students and has been proposed for preliminary testing to twenty student teachers (ST). In particular, in the first trial, the following research question was investigated: Is a combination of real experiments and interactive simulations effective in sustaining students' understanding of rolling motion?

2. – Students' difficulties

Many papers have investigated students' ideas on the relationship between friction and rolling motion, and identified typical difficulties. In the following we summarize the problematic aspects which we tried to address in the activity sequence.

a) The idea of relative motion to understand the kinematic of the rolling body. Students have great difficulty distinguishing between the velocity of a point on a rigid wheel, ball or cylinder as measured with respect the centre of mass, or the ground (Rimoldini and Singh, 2005).

b) Sliding friction forces and their relation with the rolling motion on a horizontal plane. Several studies show that the role played by sliding friction forces in shaping the motion of rolling bodies is in some cases underestimated and in others overestimated by students. For example, they do not recognize the action of kinetic friction force in producing the transition from sliding to rotational motion, and, as a consequence, do not realize that kinetic friction force does work. In other cases the role of friction is overestimated. For example, few students recognize that when the condition of rolling without slipping is satisfied a sphere is not slowed by sliding friction forces (Rimoldini and Singh, 2005). Moreover, from Close *et al.* (2013) we know that for many students a body cannot rotate or roll in absence of friction because they think that a torque is necessary to maintain rotation.

c) The role of friction in rolling on an inclined plane. Students have difficulties in explaining rolling motion along an incline (Rimoldini and Singh, 2005; Close *et al.* 2013). Some of them are convinced that pure rolling motion along an incline is governed by static friction only. Moreover often they think that a sphere cannot simply slide along a frictionless incline, while for the case in which friction is present, some of them believe that the sphere would remain at rest for small inclination angle, while others expect the sphere to roll without slipping for all angles.

3. – Description of the teaching sequence

3·1. *Didactical choices.* – We made a few fundamental decisions regarding the design of the teaching sequence which can be summarized as follows:

a) Propose activities based on a combination of real experiments and interactive simulations. Measurements are performed through the Tracker Video Analysis

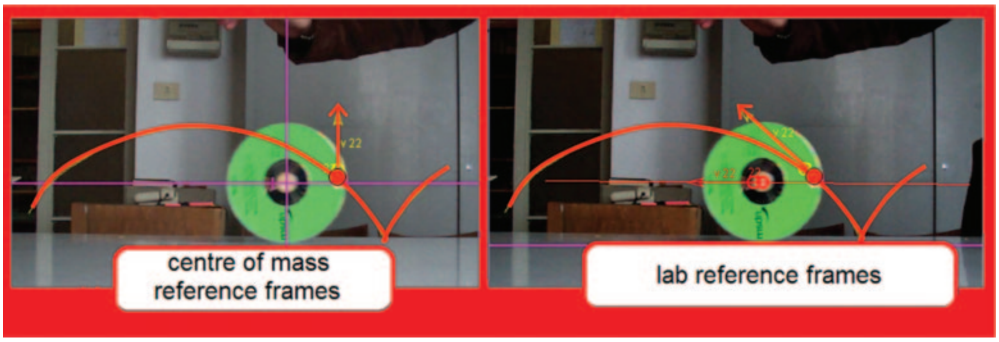

Fig. 1. – In the snapshot the velocity vectors in the centre of mass and lab reference frames of a point on the edge of the disk are shown at the same instant.

open source tool; while interactive simulations are designed and run within the freeware 2D simulation environment Algodoo.

b) Let students perform the experimental and modelling activities in small groups. Students are guided through sequenced activities to make observations that they can use as the basis for their models.

c) Engage students in the step-by-step process of constructing a qualitative model that they can use to predict and explain the behaviour of rolling bodies. When some degree of formalization becomes necessary, only basic mathematics is used, always in tight connection with qualitative reasoning.

d) Encourage autonomous exploration of a complex problem starting from an initial motivating question (specifically, in this case, the question concerns collision between two rolling spheres). Students analyze the problem de-structuring it into subproblems that they try to solve, by designing Algodoo simulations. Such approach requires students to plan a solution through a sequence of steps while keeping in mind the global issue, and leads them to a thorough exploration of the relationship between friction and rolling motion. Moreover, observing students work and discuss in groups during this activity provides us insight on the role that modelling activity has in scaffolding students' knowledge.

3˙2. *The activities*. – Main steps of the activity sequence are:

i) Measuring and analyzing, through Tracker Video Analysis, the motion of a rolling spool, made by two CDs connected by a wooden cylinder.

To help students realize that the trajectories, and the velocity vectors of a point on the spool measured in the centre of mass and lab reference frames are different, we propose them to record the motion of the spool through a digital video camera, and then to analyse the video with Tracker. The velocity of a point on the edge of the disk is tracked in both the reference frames (fig. 1). Students are invited to compare their predictions with experimental results. The typical cycloidal trajectory in the lab frame is observed and compared with data obtained from Algodoo simulation of the same system.

ii) Using interactive simulations, the role of friction in the dynamics of the rolling disk is studied, when no other accelerating force or momentum are applied (fig. 2). Students analyze the motion of a disk which is initially sliding on a rigid horizontal frictionless surface and only has a translational velocity. Then the disk encounters a second rough

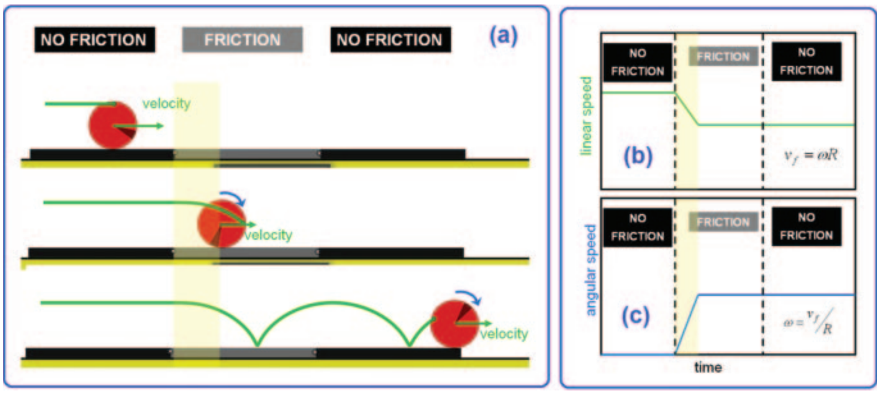

Fig. 2. – (a) Trajectory of a point on the edge of a disk when it passes from a pure translational motion to a pure rolling motion on a rough plane and then moves in a pure rolling motion on a third frictionless plane. (b), (c) Graphs of the linear and angular speeds as functions of time before, during and after the transition to pure rolling motion. Two phases of the motion on the rough plane are clearly shown, the first one in which friction is operating and doing work, and the second one in which the rolling condition has been reached, and friction force vanishes.

surface (grey plane). Working with the simulation, the students realize that kinetic friction force on the rough plane produces a decrease in the linear velocity of the disk and an increase in its angular velocity, until finally slipping stops, and pure rolling begins. Therefore simulations help students recognize the null role of kinetic friction in rolling without sliding, and show that, in absence of external forces, no sliding friction (either static or dynamic) acts on a disk that is already in pure rolling condition. We focus students' attention on the fact that, in the first time instants after the disk enters the rough plane, the linear and angular velocities are not yet related by the relation $v = \omega R$, since the disk rolls and slips at the same time. In fig. 2, v and ωR are plotted as functions of time. Students can also verify that if a third, frictionless plane is inserted in succession (the black one on the right in fig. 2), the disk continues to roll without slipping although no friction force acts on it and no change in the trajectories and velocities can be observed.

While working with the simulation, students usually raise two questions: a) how can the sliding friction force disappear? b) if the sliding friction force disappears, what causes the torque providing the rotation? The first question reveals a limited understanding of friction as a force that adjusts in magnitude to exactly balance the applied force; the second one shows that students hold a naïve idea of the relation between rotation and torque, similar to the ingenuous idea of force as necessary for movement (diSessa 1993).

It is useful to point out with students that a friction force appears a) when two surfaces in contact are in relative motion with respect to one another, or b) when a force attempts to produce relative motion between two surfaces in contact. Neither of the two conditions occurs when the disk is rolling without slipping and no friction force acts on it. It is of course helpful here to remind students that, when a body rolls without slipping, the point of contact with the surface is always instantaneously at rest with respect to the surface itself.

iii) Rolling motion with an additional force applied (fig. 3). Using both videos (captured by students and analyzed with Tracker) and simulations, students investigate the motion of a disk rolling down a plane at different inclination angles and focus on the

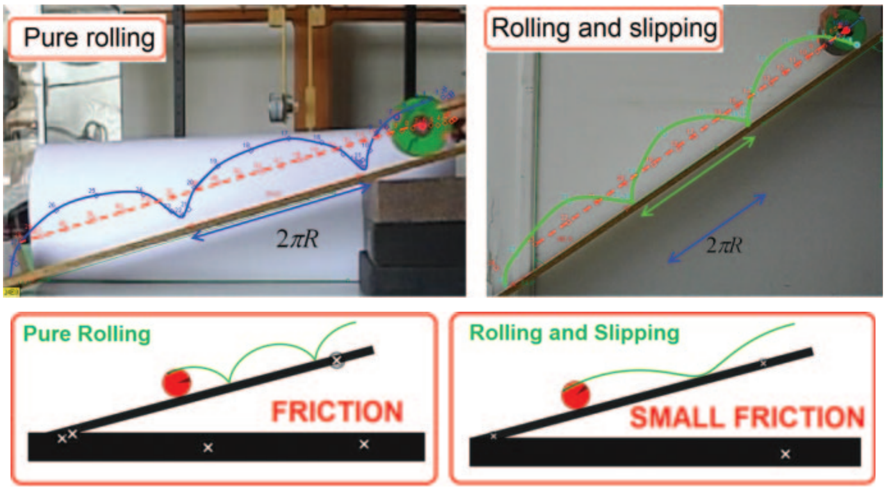

Fig. 3. – Students capture on video the motion of a disk along an inclined plane. They vary the inclination angle to investigate the differences between the cases of pure rolling, and rolling with slipping. The different measured trajectories of a point on the edge for pure rolling (small tilting angle, distance covered equal to $2\pi R$) and for rolling with slipping (large tilting angle, distance covered greater than $2\pi R$) are reported. Experimental results are compared with simulations, in which students can modify both the slope and the friction coefficient.

differences between pure rolling and rolling with slipping. They identify the pure rolling condition using the trajectories of a point on the disk edge, but also by verifying if the values of the linear velocity v and of the angular velocity ω of the geometrical centre satisfy the condition $v = \omega R$; in these conditions, students compute the total mechanical energy of the disk and verify its conservation, thus confirming that static friction does no work.

iv) Inquiry activity: collisions between two balls. To engage students in an inquiry activity, we propose them to study the collision between two rolling balls. An ingenuous approach to the problem of colliding spheres assumes that no rolling occurs and disregards the effects of friction forces immediately after the collision. However, the description of "real" collisions requires that both rotational and translational motion be taken into account together with the role of friction in converting one into the other (Close *et al.* 2013; Hierrezuelo and Carnero, 1995).

We start from an experimental activity in which we ask students to observe and compare the elastic collision between two identical carts on a guide, with the one between two identical rolling spheres (fig. 4). Students first examine the elastic collision between the carts, one of which is initially at rest. A quantitative analysis of the collision is carried out by recording the carts' motion and analysing the videos.

Using Tracker, students can verify that the results of the experiment are in agreement with the laws of conservation of momentum and energy. Next we propose to students a video of the collision between two identical steel spheres, one of which is initially at rest. We stop the video one instant before the collision, asking students to make predictions about the following evolution. Contradicting students' predictions, the projectile ball does not stop after the collision. In order to explain this unexpected result students are invited to explore several variants of the experiment by designing and manipulating Algodoo simulations.

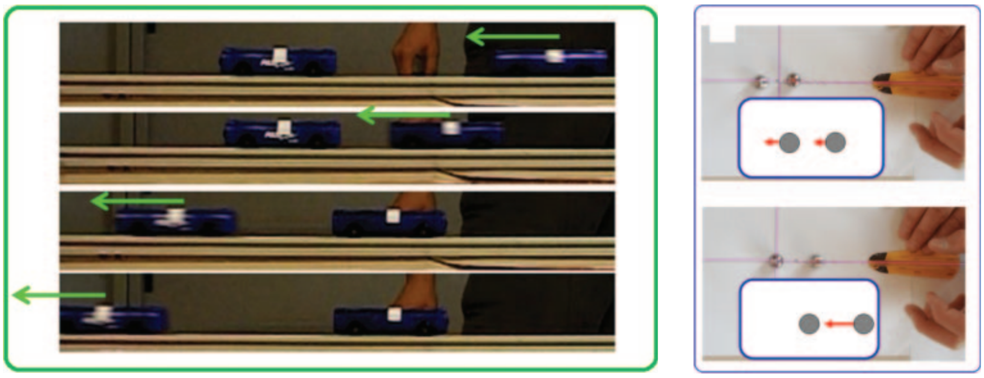

Fig. 4. – Left: Elastic collision between two identical carts, one of which is initially at rest. Right: Video of the collision between two identical steel spheres, one of which is initially at rest.

TABLE I. – *Collisions simulated by students during the activity.*

Projectile motion	μ_P	μ_T	v_P before	ω_P before	v_P after	ω_P after	v_T after	ω_T after
i) Translating without friction	$=0$	$=0$	$\neq 0$	$=0$	$=0$	$=0$	v_P before	$=0$
ii) Translating and rotating with friction between projectile and plane	$\neq 0$	$=0$	$\neq 0$	$\neq 0$	$\neq 0$	$\neq 0$	v_P before	$=0$
iii) Translating without friction. There is friction between target and plane	$=0$	$\neq 0$	$\neq 0$	$=0$	$=0$	$=0$	$v_T < v_P$	$\neq 0$
iv) Translating and rotating with friction between projectile and plane and between target and plane	$\neq 0$	$\neq 0$	$\neq 0$	$\neq 0$	$\neq 0$	$\neq 0$	$v_T < v_P$	$\neq 0$

Working in groups with the modelling software, students decompose the initial complex problem into sub-problems to analyze the role of different factors and then construct correlations between them. The main steps of this exploration are summarized in table I, where we report the different cases which students modelled during the activity. The strategy followed by each group was different, but the steps reported in table I were common to all groups.

The first simulation (the spheres lie on a frictionless plane, and the projectile ball slides without rolling) reproduces the same condition as the cart collision; in fact in this case the projectile ball stops after the collision. In the second simulation the projectile ball approaches with a rolling motion. This helps students recognize that in the case of head-on collision the target ball only acquires the translational momentum of the projectile ball, while angular momentum is not transferred. (In the following activities they can

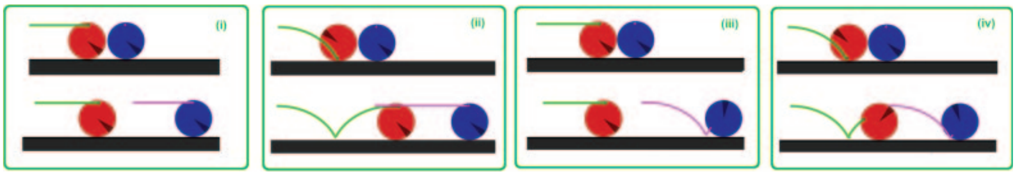

Fig. 5. – Some examples of Algodoo simulations created by students.

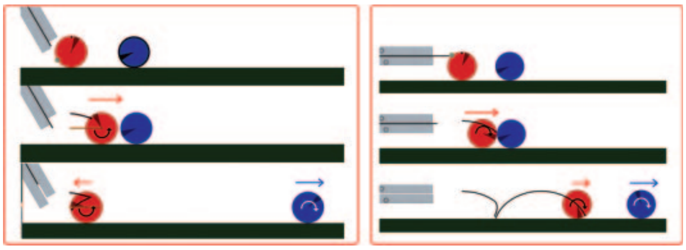

Fig. 6. – Shots with top or bottom spin and consequences on the motion after the collision.

also verify, by changing the friction coefficient between the two spheres, that angular momentum can only be transferred if the spheres exert a friction force one onto the other during the collision.) In the third case the target ball is placed on a plane with friction. After the collision this ball departs sliding on the plane, but along the motion kinetic friction produces a decrease in linear velocity and an increase in rotational velocity, until the pure rolling condition is reached. The fourth case models the real situation initially observed, which now students are able to reconstruct and explain, based on previous analysis.

In a further autonomous investigation some students simulate particular billiard shots. For example, they studied how the motion of the cue ball and the result of the collision vary by hitting the cue ball over or below its centre (see figs. 5 and 6).

Other simulations developed by students concern the analysis of the billiard ball dynamics in cushion-ball impacts. In these cases the direction of the momentum after the impact depends on the spin of the ball and the simulation allows emphasizing the role of the friction force due to the cushion in determining such direction (see fig. 7).

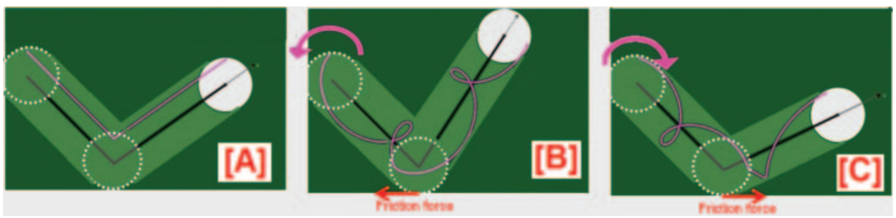

Fig. 7. – Left- or right-hand spin applied to the cue-ball to increase or reduce the angle of reflection and its velocity after impact with a cushion.

4. – Results

The sequence was tested with a group of 20 graduate students (ST) attending a course for qualification as mathematics and physics teachers. Students performed the experimental and simulation activities in groups of three and completed the sequence in three sessions of 2 hours each. Assistants worked as facilitators, giving support where necessary. Our sources of data on students' progress and ideas included two questionnaires, worksheets filled in during the experimental activities, discussions during and after the experiments, answers to written questions, and a final report in which they elaborated on what elements of the proposed sequence they considered essential. In the following we refer in particular to data collected from the students' final report, and from the two questionnaires, one given before the activities (pre-test) and the other after the end of the sequence (post-test). Some questions were drawn from the literature, to make it possible a comparison of our students' results with those obtained in a different context.

4·1. *Qualitative results.* – In their reports ST mainly focus on the autonomous investigation with Algodoo simulations about the collisions between rolling spheres. Students understood well that rolling motion results from the composition of translational and rotational motion, and that in elastic collisions between two balls only the linear motion of the projectile is transferred to the target, while the angular momentum is not transferred:

One of them writes "*The momentum of the ball projectile is entirely transferred to the target ball, but … rotational momentum is not transmitted*".

Students are aware of the role of friction (kinetic friction) in the transition from sliding to rotational motion:

> "*the behavior of the target ball (after the collision) is no doubt due to friction between it and the table; sliding friction force is opposed to velocity, slowing the translational motion and, since it is not applied on the center, also causes an angular acceleration to the sphere*".

Students acknowledge that friction can play a motive role:

> "*the ball does not have a linear velocity immediately after the impact, but is still rotating; then, under the effect of friction force, angular velocity decreases while linear velocity increases*".

Students were effectively engaged in decomposing the complex problem of the collision between rolling balls

> "*When a steel ball collides with a second ball in a central bump, how do they behave? This is a seemingly simple question, but the answer is not obvious and especially dense of physical knowledge. To answer correctly you should ask: is the surface on which the cue ball is located frictionless? Is the plane where the target ball is located before the collision frictionless?* […]".

Students highlighted the role played by software in their learning process

> "*The software plays a significant role in this activity, because it allows us to freely and easily check the parameters in the game, in order to test our predictions*".

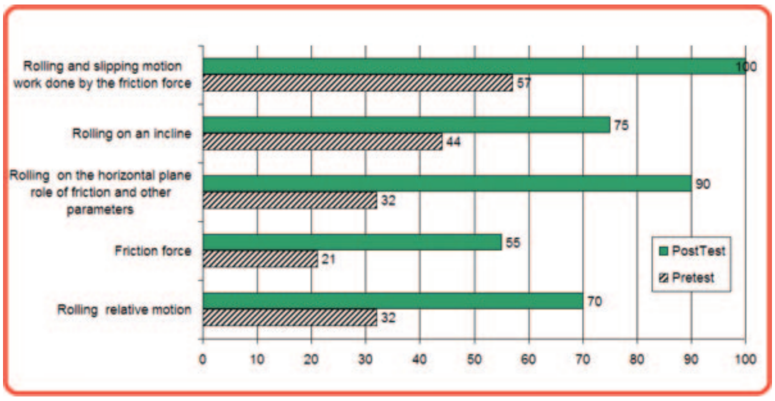

Fig. 8. – Comparison between pre- and post-test results analyzing the questions of multi-choice test by concepts.

4·2. *Quantitative results.* – The questionnaires were not meant to be comprehensive and cover all topics involved with rolling and rotational motion, but to focus on basic concepts underlying our teaching sequence: rolling and relative motion, rolling on the horizontal plane and role of friction, rolling on an incline and work done by the sliding friction force in the presence of rolling and slipping motion.

In the following, we summarize the most relevant conclusions we drew from the pre-test to present a global picture of students' ideas before the activity sequence.

a) Students had great difficulty in distinguishing between the speeds of different points on a rigid wheel with respect to the centre of the wheel or ground. Only 32% of the ST identify the exact direction of the speed of a point on the edge of the wheel in rolling motion. The analogous result in Rimoldini and Singh (2005) was 38%.

b) Only a small fraction of ST recognized that a marble rolling without slipping across a rigid horizontal floor is not slowed by friction (32% *vs.* 25% of Rimoldini and Singh (2005)).

c) More than 40% of the ST were not convinced that a sphere on a frictionless inclined plane slides without rolling (42% of ST *vs.* 44% in Rimoldini and Singh (2005)) while, in the case with friction, 32% believed that the sphere remains at rest for a small inclination angle, and 26% that the sphere rolls without slipping for all angles.

d) Only 42% of ST were convinced that pure rolling motion along an incline is governed by static friction. Moreover only a small fraction (21%) recognized that the value of the friction force has to be less or equal to the product between the static friction coefficient and the normal force.

e) More than 40% of ST did not recognize that the kinetic friction force does work on a sphere which is initially sliding on a rough horizontal plane, and makes the transition to pure rolling motion.

On the whole, in the post-test the percentage of incorrect answers was, for our students, below 25%. This result alone is an indication that the sequence created a fruitful environment for the students' learning, enabling them to address their initial difficulties. In fig. 8 we compare pre- and post-test results for items related to the same concepts. More in detail:

a) Answers to the post-test confirmed an improvement of students' understanding of the kinematic of rolling motion and in their capability to distinguish between the speeds of different points with respect to the centre of the wheel or the ground. 70% of the ST correctly answered a question about the velocity of three different points on a rolling wheel with respect to the road, compared to 57% in Rimoldini and Singh (2005).

b) After the sequence a large percentage of ST (89%) was able to recognize that a marble rolling without slipping across a rigid horizontal floor is not slowed down by friction. Most students (79%) also understood that no friction force is acting when a body rolls without slipping along an horizontal plane.

c) 70% of ST recognized that whether a sphere moving down along an incline undergoes pure rolling, or rolling with slipping, depends both on the static friction coefficient and on the inclination angle.

d) 58% of ST answered correctly that the magnitude of the friction force is not necessarily equal to, but lesser or equal than, the product between the static friction coefficient and the normal force.

e) 75% of ST recognized the role played by kinetic friction force on a sliding sphere in making it reach the pure rolling condition on a rough horizontal plane.

5. – Conclusions

Video analysis based activities were used to highlight experimental situations in which the relationship between friction and rolling is especially complex, or leads to counter-intuitive results. Interactive simulations were essential for exploring multiple variations of a given physical situation, and provided the ideal environment for a guided inquiry activity.

Analysis of qualitative data on students' reasoning suggests that this approach allowed students to obtain a richer and more precise understanding of the subject. Comparison of pre and post test results shows that students obtained sensible performance improvements, and overcame many common difficulties. This seems to confirm that a sequence design based on a combination of real experiments and interactive simulations is effective in sustaining students' understanding of rolling motion.

REFERENCES

[1] Close H. G., Gomez L. S. and Heron P. R. L., *Am. J. Phys.*, **81** (2013) 458.
[2] diSessa A. A., *Cognition Instruct.*, **10** (1993) 105.
[3] Domenech A. and Casasús E., *Phys. Educ.*, **26** (1991) 186; Hierrezuelo J. and Carnero C., *Phys. Educ.*, **30** (1995) 177.
[4] Mathavan S., Jackson M. R. and Parkin R. M., *Am. J. Phys.*, **77** (2009) 788.
[5] Phommarach S., Wattanakasiwich P. and Johnston I., *Phys. Educ.*, **47** (2012) 189.
[6] Rimoldini L. G. and Singh C., *Phys. Rev. ST Phys. Educ. Res.*, **1** (2005) 1.
[7] Wallace R. E. and Schroeder M. C., *Am. J. Phys.*, **56** (1988) 815.