

Non-contact acoustic characterisation of the dynamic patterns at the free surface of shallow turbulent flows

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Abstract

A recent experimental study characterised the patterns at the free surface of shallow subcritical turbulent flows over a homogeneously rough bed. Detailed wave probe array measurements indicated the pattern of the frequency-wave number spectra at a range of Froude Numbers. Acoustic scattering data was collected simultaneously. The present analysis demonstrates the application of an acoustic technique to obtain the same water surface wave characterisation remotely. The technique reconstructs the free surface elevation based on the measurement of the airborne scattered acoustic field with an array of transducers. Using this method, the dispersion relation of the free surface patterns is measured for a range of flow conditions in a laboratory flume. The study shows the potential of acoustic non-contact measurements for the detailed monitoring of the hydraulic flow conditions in shallow sub-critical turbulent flows.

1. Introduction

Non-contact techniques to monitor the flow velocity and depth of shallow free surface flows are often cheaper than traditional flow measurements, as they require less maintenance. The performance of optical methods such as the Large-Scale Particle Image Velocimetry is limited by the illumination and seeding conditions (Muste et al, 2008). Airborne microwave Doppler sensors are limited to relatively highly turbulent flows, where short water waves with the wavelength of the order of a few millimetre allow sufficient backscattering of the incident signal (Costa et al., 2006).

A recent experimental study in a laboratory flume (Dolcetti et al., 2016) has demonstrated the link between the temporal and spatial scales of the free surface and the main hydraulic quantities of shallow turbulent flows, namely the mean flow depth and the mean surface velocity. This study showed the frequency-wavenumber spectra of the surface elevation in two orthogonal directions and for a wide range of sub critical flow conditions over a rough bed, measured with two linear arrays of conductance wave probes. The dispersion relation of the water surface depends on the mean flow depth and mean surface velocity, and was shown to provide a complete characterisation of the surface dynamics.

The measurement of the acoustic field scattered by the water surface with arrays of ultrasonic transducers allows the reconstruction of the surface elevation in space and in time based on the so-called stationary phase method (Nichols et al., 2013). The present study shows that these measurement allow the remote characterisation of the dispersion relation of the water surface elevation of shallow turbulent flows at moderate Froude numbers.

2. Experimental setup and flow conditions

The measurements were performed in a 12.6 m long and 0.459 m wide rectangular laboratory flume. By use of a downstream control, the flow mean depth H was maintained homogeneous along the flume. The flume bed was covered with three layers of hexagonally packed plastic spheres with the diameter $d_s = 25.4$ mm. Six sub critical flow conditions were tested, with the largest Froude number of 0.52. At higher Froude numbers, the reconstructed surface elevation

showed large peaks which have been associated with a limit of the stationary phase approximation, as discussed by Nichols et al. (2013). The parameters of the tested flow conditions are summarised in Table 1, where s is the channel slope, U_0 is the mean surface velocity, σ is the measured standard deviation of the surface elevation, k_0 is the wavenumber of the stationary waves, and F and Re are the Froude and Reynolds Number based on the mean depth and mean surface velocity, respectively. Details of the measurements of these quantities are reported by Dolcetti et al. (2016).

Table 1: Experimental flow conditions

Flow condition	H (mm)	s	U_0 (m/s)	F	Re	σ (mm)	k_0 (rad/m)
1	42.2	0.001	0.19	0.30	8.0×10^3	0.05	-
2	72.9	0.001	0.35	0.41	2.5×10^4	0.33	89.7
3	101.0	0.001	0.41	0.41	4.1×10^4	0.29	63.6
4	42.2	0.002	0.30	0.47	1.3×10^4	0.23	131.5
5	101.3	0.002	0.49	0.49	4.9×10^4	2.05	45.1
6	43.0	0.003	0.34	0.52	1.5×10^4	0.37	99.7

3. Measurement of the surface dispersion relation with arrays of ultrasonic transducers

The acoustic setup comprised of a set of 7 TR-89B Series Type 40 piezoelectric transducers manufactured by Massa Products Corporation. The transducers were installed at the distance of 8 m from the flume inlet, and were aligned along the flume centreline with a constant spacing of 35 mm. The height of the array of transducers above the mean water surface was regulated before each measurement, and was equal to 200 mm. The central transducer acted as an acoustic source. It was driven as a source with a sinusoidal signal at the frequency of 39 kHz. The remaining 6 transducers recorded the acoustic pressure field scattered by the rough water surface. The recordings were performed with the sampling frequency of 500 kHz for a duration of 5 minutes. The analytic signal was determined every 0.01 s from the Hilbert transform of the digitised signal and averaged over the time of 1 ms. The analysis was performed on 29 data segments with the length of 10 seconds. The results shown are the average over the 29 segments.

According to the stationary phase approximation, the surface elevation at the stationary phase point with co-ordinates (x'_j, y'_j) can be determined as

$$\zeta(x'_j, y'_j, t) = -q_z^{-1} \{ \Im [\log (P_j(t)/P_0(t))] - \langle \Im [\log (P_j(t)/P_0(t)) \rangle_t \}, \quad (1)$$

where q_z is a geometric factor, \Im represents the imaginary part, and $\langle \rangle_t$ represents time-averaging. $P_j(t)$ and $P_0(t)$ represent the analytic signal recorded by the j -th transducer and the input of the source, respectively, obtained by a Hilbert transform. If the spatial gradient of the surface is small, the stationary phase point has approximately the co-ordinates $x'_j \approx x_j/2$, $y'_j \approx 0$, where x_j is the distance of the transducer from the source and the x -direction is parallel to the streamwise direction. With the setup used here, the surface elevation is reconstructed at 6 locations $x_j/2$ with the spacing of $\Delta x = 17.5$ mm and with the maximum distance of 105 mm.

The space-time correlation function of the surface elevation along the direction x is defined as

$$W_x = \sum_j \sum_t \langle \zeta(x'_j, 0, t) \zeta(x'_t, 0, t + \tau) \rangle_t / \sigma_j \sigma_t, \quad (2)$$

where $\zeta(x'_j, 0, t)$ and $\zeta(x'_t, 0, t)$ represent the water surface elevation reconstructed with the j -th and with the t -th transducer, respectively, τ is a time-lag, and σ_j is the standard deviation of $\zeta(x'_j, 0, t)$ in time. The frequency-wavenumber spectrum $S_x(k_x, \omega)$ in the direction x is found from a two-dimensional Fourier transform of W_x in space and in time as a function of the streamwise wavenumber k_x and the radian frequency ω . The spectra had the wavenumber resolution of 29.9 rad/m and the Nyquist wavenumber of 179.5 rad/m. The integral of S_x along the wavenumber axis was subtracted at each frequency in order to remove the dependence of the spectrum on the frequency.

4. Results

Examples of the frequency-wavenumber spectra of the surface elevation measured with the proposed acoustic technique for conditions 1, 3, 4, and 6, are shown in Fig. 1. Dolcetti et al. (2016) compared these spectra with the dispersion relation of gravity capillary waves propagating in a flow with a power function time-averaged streamwise velocity profile with exponent $n = 1/3$. This relation expresses the frequency of the gravity-capillary waves, ω , as a function of their

wavenumber, k , and direction of propagation, θ , and is described by

$$\omega^2 = \frac{(g + \gamma k^2 / \rho) k^2 p}{(kU_0 \cos(\theta) / \omega - 1)[(kU_0 \cos(\theta) / \omega - 1) - npkU_0 \cos(\theta) / \omega]}, \quad (3)$$

where g is the gravity constant, γ is the surface tension, ρ is the water density, and p can be found by integrating numerically an initial value problem suggested by Fenton (1973) (equation 18). In the conditions where U_0 is larger than the minimum phase velocity of gravity capillary waves, $c_m \approx 0.23$ m/s, Dolcetti et al. (2016) found that the surface pattern is dominated by stationary waves with the wavenumber k_0 . These waves propagate against the flow with the velocity equal to the advection by the flow, so that they appear as static. In these conditions, Dolcetti et al. (2016) observed a radial pattern of waves with the wavenumber k_0 propagating in all directions. In the condition 1, where $U_0 < c_m$, Dolcetti et al. (2016) observed non-dispersive waves propagating at the velocity U_0 described by the relation $\omega = k_x U_0$. In all flow conditions, gravity capillary waves propagating downstream which followed equation (3) with $\theta = 0$ were also observed.

All patterns described by Dolcetti et al. (2016) can be observed in Fig. 1. Specifically, the presence of non-dispersive waves in the condition 1 and of the radial pattern of waves in the remaining conditions are confirmed. The proposed equations underestimate the frequency of the radial pattern slightly in condition 6 and more visibly in condition 4 (Fig. 1c). This was already observed by Dolcetti et al. (2016) based on measurements with arrays of conductance wave probes. With respect to the results presented by Dolcetti et al. (2016), the spectra obtained with the acoustic method show a cut-off at the wavenumber of approximately 100 rad/m (wavelength of approximately 62.8 mm), and a larger amplitude near the origin. Both phenomena are caused by the limited ability of the stationary phase method to resolve short waves, which causes aliasing at low wavenumbers.

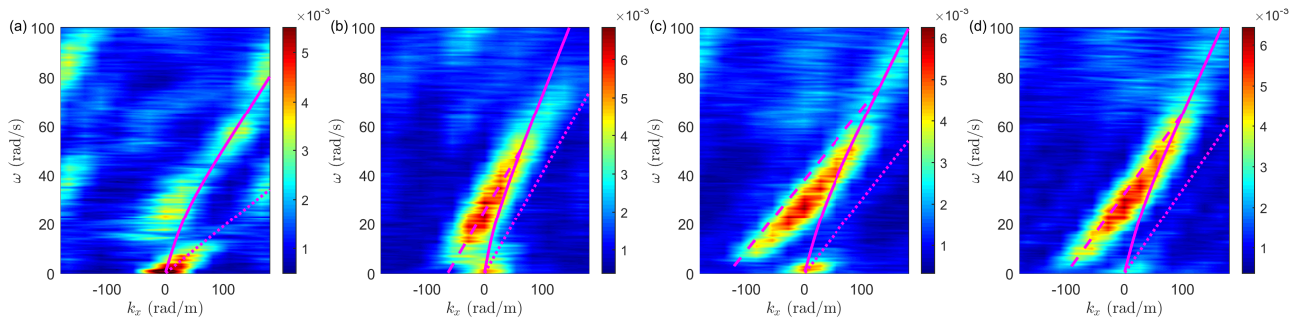


Figure 1: Examples of the frequency-wavenumber spectra obtained remotely with the proposed acoustic technique for (a) condition 1, (b) condition 3, (c) condition 4 and (d) condition 6. (solid) downstream propagating gravity waves, (dashed) radial pattern of gravity waves, (dotted) non-dispersive waves.

5. Conclusions

The results presented here have shown that the dispersion relations of the free surface of shallow turbulent flows can be measured remotely using a stationary phase method applied to arrays of ultrasonic transducers. These relations depend on the characteristic hydraulic quantities of the flow, namely the mean flow depth and the mean surface velocity. Hence, the measurements could be used potentially in order to estimate these quantities remotely. Due to its application limited to moderate Froude Numbers and small surface fluctuations, the method could complement remote measurement techniques based on Doppler which need the surface to be rougher. The limitations with short waves and very rough surfaces could be overcome by alternative techniques, such as the one based on holography proposed by Krynkin et al. (2016).

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