# Experiments and models about the force between permanent magnets: asymptotic analysis of a difficult problem

Pasquale Onorato[1], Massimiliano Malgieri[2]

[2] Department of Physics, Università degli Studi di Trento, Povo (TN), Italy.
[2] Department of Physics, Università degli Studi di Pavia, Pavia, Italy

## Abstract

We propose a simple experiment which allows students to explore quantitatively the magnetic interaction between neodymium cylindrical magnets. The experiment employs a precision digital balance, two screws with known thread pitch and two transparent tubes to measure the repulsive force between two magnets as a function of their distance. Different measurements are performed, focusing on the behaviour of the interaction force at short and large distances and the role of the magnets' aspect ratio. We discuss the comparison between theoretical expectations resulting from conceptually simple approximate models and experimental results.

The experiments employ inexpensive materials and address a relevant topic in the physics curriculum; thus, they are appropriate for the undergraduate physics laboratory, for advanced high school students, and in the context of teacher education and in-service training to enhance students' knowledge of magnetism.

## 1. Introduction

Many researches in last decades has shown how experimental activities are significant sources of knowledge in the physics teaching/learning practice, since they offer students the chance to contextualize abstract concepts and to challenge their naïve beliefs [1-3].In the case of electromagnetic phenomena, the main difficulties encountered by learners are clearly identified in the literature [4-7] and in recent years several studies were published focusing on experimental activities as a means to enhance students' conceptual understanding [8-15].

Authors have proposed in the past a number of experiments on the magnetic interaction between permanent magnets; e.g. employing magnets supported by a balance[15-1920] or placing magnets within a glass tube[20-21]. In many cases experiments a magnetic force proportional to $1/r^2$ was found[20-21][1], others authors discussed the asymptotic $1/r^4$ behaviour [17-19] and few papers [22] measured the behaviour of the interaction force when the distance is smaller than the transverse dimension (radius) of the magnets.

In this paper, we present a simple and handy experiment aimed at exploring quantitatively the magnetic interaction by using low-cost materials. Our aim is improving results of previous experiments [17-19] where the authors investigated the relationship between magnetic force with respect to the distance of separation between two identical disc or coaxial cylindrical magnets. The experiment uses a digital precision balance, two screws of known thread pitch and two transparent tubes. This simple setup allows a comparison between the experimental results and the predictions provided by theoretical models of magnetic interaction in the asymptotic regions. Motivating questions posed to students at the beginning of the activity concern studying according to which

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[1] However, the approximate inverse square law interaction fails both when the distance between the magnets increases and when the magnets are in close proximity with each other.

laws the interaction force between two permanent magnets behaves a) for very short distances and b) for very long distances.

The complexity of calculating the force between two cylindrical magnets makes the full theoretical solution substantially inaccessible for students. Even when computed, the analytical expression [17] and [22]is expressed as an integral of which a numerical solution is then provided. Such complexity leads us to analyse the system in its asymptotic limits, studying separately the trends at very short and very long distances to see whether intuitive understanding of the behaviour can be gained in limit cases. This approach is often followed by scientists when faced with complex problems, and requires students to engage in reflection not only to understand what happens in a certain limit, but also in relation to what other quantities or dimensions in the problem the limit is reached. In this sense it is also useful to discuss what happens for magnets with different aspect ratios, where the relevant dimensional scales may be different from one case to the other[22].

The activities described in this paper were tested with 30 students from a laboratory course for perspective physics teachers at the University of Trento. The course is centred on electromagnetism, and is organized in such a way that for most topics in the subject we propose a laboratory activity typically meant to address and discuss some of the most relevant learning difficulties reported in the literature. In the case of this particular experiment, the main motivation was that very little attention is usually given in standard textbooks and courses on electromagnetism to the interaction between permanent magnets[17], whereas quite powerful magnets are nowadays cheaply available for teachers to perform simple and meaningful experiments with their students. Another motivation was given by the difficulties previous cohorts had showed in understanding intuitively the domain of validity of a dipole approximation, both in earlier versions of the experiment hereby discussed, and in the analysis on the interaction of electrically charged objects using a balance scale [23-24]. Both in the case of this experiment, and more generally in our course, we adopt a Predict-Observe-Explain strategy [25]: each experiment is preceded by the request of a prediction through one or more questions investigating students' initial conceptions which in some cases (such as in this study) are also used as pre-test. After the experiment, students discuss between them and with teachers the compatibility between their predictions and the experimental results. In the case of this study, students also answer a post-test identical to the pre-test several weeks after the experimental activity, to investigate long term learning retention.

## 2. The magnet-magnet interaction

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It is quite hard to compute the total interaction force between two cylindrical magnets, and the mathematical techniques needed are not accessible to undergraduate students. For two cylindrical magnets with their magnetic dipole aligned, the force can be computed analytically using elliptic integrals[26]. An analytical expression was discussed in ref.[27]. The expression describes the force between two cylindrical magnets of the same radius $R$ with their dipoles aligned on the same axis $z$, which is also the central axis of the cylinders, in the assumption of uniform magnetization $M$. In these conditions no forces exist along the other axes $x$ and $y$, while the force acting along $z$ has the form

$$F_z = -8\pi K_d R^2 \int_0^{\infty} \frac{J_1^2(q)}{q} \sinh(q\tau_1)\sinh(q\tau_2)\, e^{-q\varsigma} d\varsigma \tag{1}$$

where $\tau_i=d_i/(2R)$, $i=1, 2$, are the aspect ratios of the two cylinders, which we have assumed to have the same radius $R$ but possibly different heights $d$; $\zeta=Z/R$ is the scaled distance between the centers of the two cylinders (where $Z$ is the ordinary distance), $J_1(q)$ is a modified Bessel function of the first kind and we have introduced the magnetostatic energy constant $K_d=\mu_0 M^2/2$ for convenience of notation.

Of course for an expression such as Eq. (1) any form of intuitive understanding is impeded. Thus, we make two fundamental choices:

1) We aim at studying experimentally asymptotic behaviours for short and long distances

2) We use as a theoretical lens appropriate simplified models which can help students to gain conceptual understanding of the problem. In particular, we choose the route of electrostatic analogies i.e. replacing magnets with electric dipoles or more complex charge distributions according the Gilbert model[28].

The Gilbert model assumes that the force between two magnets is due to magnetic charges near the poles repelling or attracting each other in the same manner as the Coulomb force between electric charges. In the Gilbert model a magnetic H-field is produced by magnetic charges that are 'smeared' around each pole This model works even close to the magnet, where the magnetic field depends heavily on the detailed shape and magnetization of the magnet.

Approaching the computation of the force between two cylindrical magnets by using the Gilbert model we conclude that formally, the field can be expressed as a multipole expansion: a dipole field, plus a quadrupole field, plus an octupole field, etc. Obviously thinking in terms of asymptotic behaviours (multipole expansion) when we analyse the interaction between two cylindrical magnets we must consider 3 different length scales, the radius $R$, the height$d$ and the distance between the magnets $x$.

With this schematization we can review the interaction for the different distance ranges.

**Contact force and interaction at small distances.** When the distance, $x$, between the magnets is small (i.e. $x << R$ and$x << d$ ) the magnitude of the force between two very close "magnetic surfaces" is given in the case that$d >> R$ by

$$F^{\infty}_{stick} = \frac{1}{2\pi\mu_0}\left(\frac{m^2}{d^2}\right)\frac{1}{R^2}, \tag{2}$$

where $m$ is the magnetic moment. The force Eq. (2) is written in analogy with the force acting between the plates of a capacitor, where a charge $Q = {}^{m}/_{d}$ is uniformly smeared on each plate,

$$F^{electric}_{stick} = \frac{1}{2\pi\varepsilon_0}\left(\frac{Q^2}{R^2}\right).$$

In a more general case the contact force is reduced for the case of large radii with respect to the length of the magnet [29],

$$F^{R}_{stick} \approx F^{\infty}_{stick}\frac{d}{\sqrt{R^2+d^2}}.$$

The behaviour of the force in proximity of the contact can be obtained using multipole expansion and it is obtained (for$x \ll R$ and$x \ll d$ )

$$F_{x<R}(x) \approx F^{R}_{stick}\left(1-\frac{x}{\alpha R}\right). \tag{3}$$

This trend is similar to the one of two uniformly charged disks at very short distance (see Fig.1 bottom).

**Intermediate and long distance region.** n the intermediate distance region a well known expression can be derived from Gilbert's electrostatic model, assuming 4 interacting pointlike charges [20].

$$F(x) = F_{stick}^{\infty}\frac{d^2+R^2}{d^2}R^2\left(\frac{1}{x^2}+\frac{1}{(x+2d)^2}-\frac{2}{(x+d)^2}\right) \equiv A\left(\frac{1}{x^2}+\frac{1}{(x+2d)^2}-\frac{2}{(x+d)^2}\right) \quad (4)$$

The same expression can also be derived from low order expansion of the Bessel functions in Eq. (1) according to their definitory power series [22]. In the limit of very long distances, Eq. (4) provides the *1/r*$^4$behaviour which is typical of dipole-dipole interactions, but with different constants depending on the magnets' aspect ratio:

***Long Magnets*** FromcEq. (4) in the limit of large distances$x \gg d$ , and for $d \gg R$

$$F_{\infty}(x) = F_{stick}^{\infty}\frac{6d^2R^2}{x^4} \quad (4.b)$$

***Flat Magnets*** From Eq. (4) in the limit of large distances $x \gg R$ , and for$R \gg d$

$$F_{\infty}(x) = F_{stick}^{\infty}\frac{6R^4}{x^4} \quad (4.c)$$

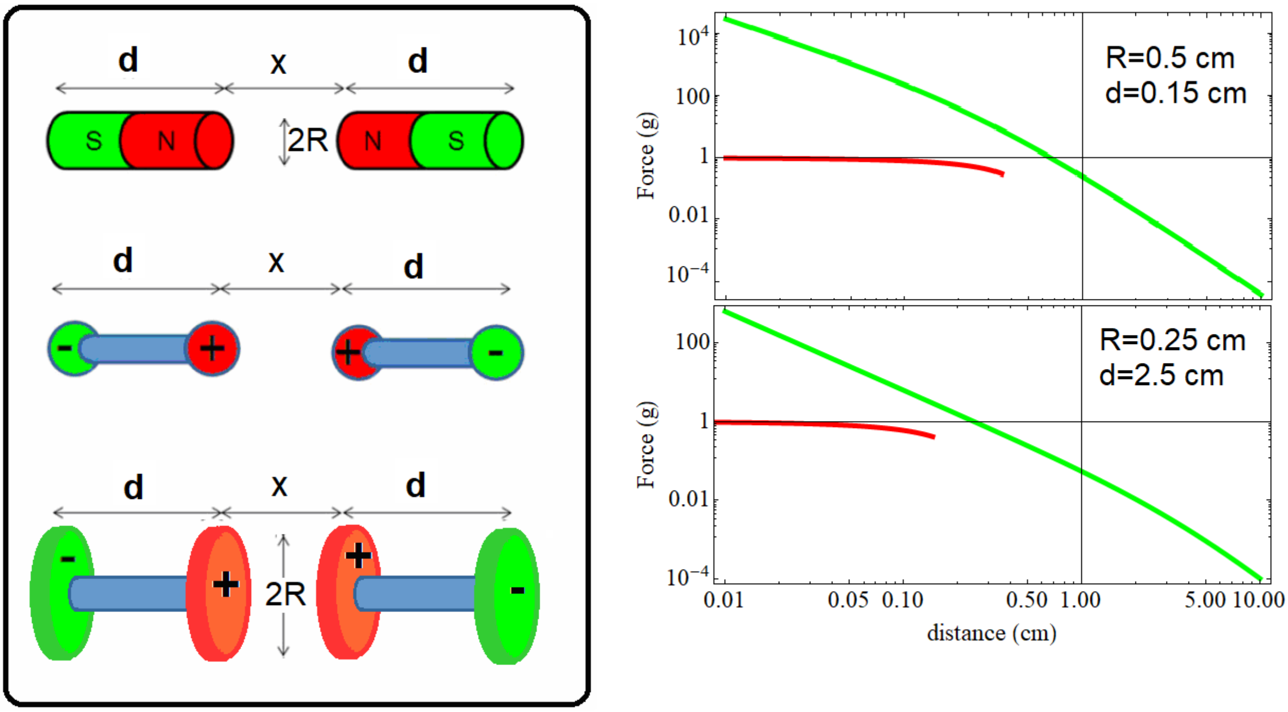

**Figure 1**(Left) The two models used in the Gilbert approximation, the charged disks approximation (when $x \ll 2R$ ) and the pointlike charge approximation ($x \gg 2R$ ).(Right) Force versus distance calculated according the Gilbert model. Red line: the multipole expansion near the contact. Green line: the dipole approximation according Eq.(4) with the asymptotic power law behaviour corresponding to the dipole-dipole interaction. This asymptotic regime is reached for short flat magnets (top panel).

## 3. Experimental procedure and data analysis

An electronic digital balance (with an accuracy of *0.01g*), a transparent tube and a simple screw which we previously calibrated by measuring the thread pitch ($0.076 \pm 0.001\ \mathrm{cm}$ ) were used in this experiment to explore the magnetic force as a function of distance between two identical neodymium cylindrical magnets, as shown in Figure 2. One of the magnets was taped onto the balance pan, and the other was fixed on the end of a diamagnetic screw, approximately 15 cm long. A support frame held the screw suspended above the precision electronic scale.
The magnets were then aligned in such a way to have a repulsive interaction, and a plexiglass cylinder[2], slightly wider than the magnets, was used to maintain the alignment during the experiment. Data measurements were performed in the interval of *0* to *6 cm* of separation between the two magnets the distance was measured by counting the number of turns of the screw, starting from the bottom i.e. when the magnets are nearly in contact one to the other.

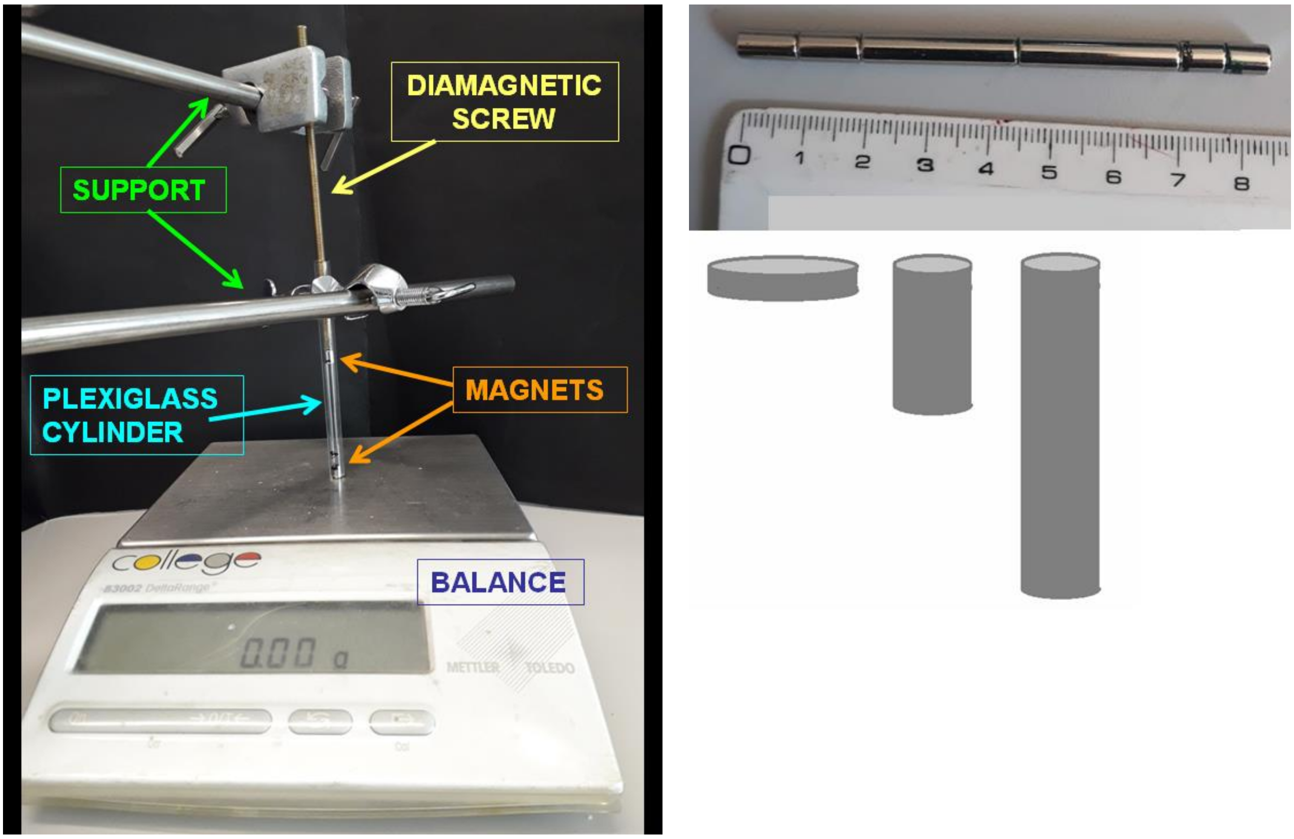

**Figure 2.** The experimental setup, showing the digital balance. Right the cylindrical magnets. The bottom magnet was taped onto the balance pan and in general the magnet could influence the measuring results of the electronic digital balance. In order to demonstrate that the effects of the magnets on the measuring results of the balance are negligible we perform the measurements of the repulsive force in two cases by putting two magnets at fixed distance one from the other. In the first case the bottom magnet was taped onto the balance pan while in the second case the bottom magnet is put on a non magnetic cylinder 4.5 cm height. The force measured in the two cases differs less than the 0.5% by showing a negligible magnetic interference on the measuring results of the electronic digital balance.

The setup described above was used to carry out two series of measurements for three different pairs of magnets. In each series the contact distance, *x* was varied from 50 to 1mm. At the end of each series the setup was disassembled and then reassembled again to prevent systematic errors.

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[2] We employed two plexiglass tubes. The large tube needed for thin discs has internal diameter $D \approx 10\ mm$ and height $h \approx 7.5\ cm$ , the narrow tube has internal diameter $D \approx 5.7mm$ and height $h \approx 10\ cm$

The reliability of the results of the magnetic dipole moment of the disc magnet was tested by performing the experiment in the same conditions for five trials.

The repulsive forces obtained are shown in figures 3, 4 and 5, where the graphs report the magnitude of magnetic force versus distance of separation between the two magnets in log-log plot scale.

We show the measured Force for three different aspect ratio, thin discs ($h_1$=1.55 ± 0.05mm and 2$R_1$=9.90±0.05mm) $\tau_i$=0.16, short cylinders ($h_2$ = 10.15 ±0.05mmand2$R_2$ = 4.85 ±0.05mm),$\tau_2$=2.09and a long cylinder ($h_2$ = 25.35 ±0.05mmand2$R_2$ = 4.85 ±0.05mm),$\tau_2$=5.23.

**Thin discs:** The first example of graphs from five trials of data measurements of magnetic force as a function of distance between disc magnets is shown in figure 3.

Starting from Eq.(3) we found the curve that has the best fit to the series of data points corresponding to the first measures at very short distance ($x < R$) (red line in fig 3). Thus we obtain $F^{R}_{stick} = 420 \pm 20\ \mathbf{g}$ ; $\alpha R = 0.50 \pm 0.05\ \mathbf{cm}$ and $F^{\infty}_{stick} \approx F^{R}_{stick} \frac{R}{d} = 1360 \pm 70\ \mathbf{g}$

As the distance increases $x >> d$ the force varies inversely as the fourth power of the distance between the two magnets with a power exponent *n*=4. Thus according Eq.(4.c),

$$F_{\infty}(x) = F^{\infty}_{stick} \frac{6R^{4}}{x^{4}} = \frac{B}{x^{4}}$$

we found the value of *B*, so that the curve has the best fit to the series of data points corresponding to distances between 2 cm and 5 cm. The best fit value is $B = 514 \pm 4\ \mathbf{g\ cm^{2}}$ with coefficient of determination, $\rho$ squared, $\rho^{2} = 0.99922$ . It follows that we can compare the extrapolated values $F^{\infty}_{stick} = \frac{B}{6R^{4}} \approx 1370 \pm 80\ \mathbf{g}$ from the large distance fit and $F^{\infty}_{stick} = 1360 \pm 70\ \mathbf{g}$ from the short distance fitwe have very good agreement between the two values.

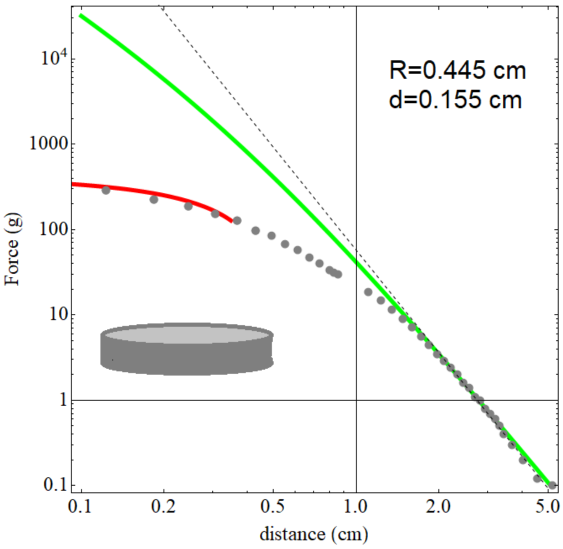


**Figure 3.** Force versus contact distance for thin discs ($h_1$=1.55 ± 0.05 mm and 2$R_1$=9.90±0.05mm). (Gray Circles): measured values; (continuous red line):Multipole expansion at small distances; (continuous green line) Point-like dipole approximation Eq.(4) (dashed line): asymptotic point dipole model Eq.(4.c).

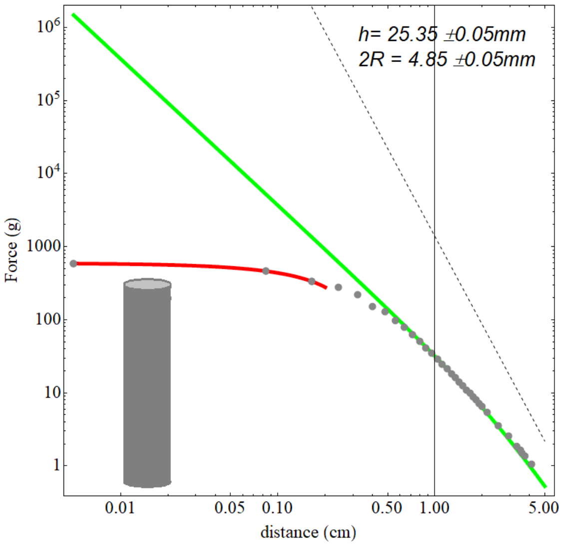
**Long cylinders:**The second example of graphs from trials of data measurements of magnetic force as a function of distance between two long cylindrical magnets is shown in Figure 4.

Starting from Eq.(3) we found the curve that has the best fit to the series of data points corresponding to the first measures at very short distance ($x < R$ and $x < d$ ) (red line in Fig.4). Thus we obtain $F^{R}_{stick} = 595 \pm 10\ \mathbf{g}$ and $\alpha R = 0.38 \pm 0.02\ \mathbf{cm}$ with the coefficient of determination$\rho^2 = 0.99986$

As the distance increases and$x \gg d$ the force varies according Eq.(4) while the asymptotic dipole/dipole model of interaction, Eq.(4.b), is not reached with our experimental setup.

Through a best fit procedure in the range of data points between 0.7 cm and 5 cmwe found the value of $A = F^{\infty}_{stick} \dfrac{d^2 + R^2}{d^2} R^2 = 37 \pm 2\ \mathbf{g\ cm^2}$ , with coefficient of determination$\rho^2 = 0.99989$ . It follows that we can compare the extrapolated values $F^{\infty}_{stick} = A \dfrac{d^2}{(d^2 + R^2)R^2} \rightarrow F^{\infty}_{stick} \approx 580 \pm 40\ \mathbf{g}$ , from the large distance fit and $F^{R}_{stick} = 595 \pm 10\ \mathbf{g}$ from the short distance fit, observing that the two results overlap significantly.

**Figure 4.** Force versus contact distance for thin discs ($h_2$=25.35±0.05mm *and* 2$R_2$=4.85±0.05mm) (Gray Circles): measured values; (continuous red line):Multipole expansion at small distances; (continuous green line) Point-like dipole approximation Eq.(4) (dashed line): asymptotic point dipole model Eq.(4.b).



**Short cylinders:**The third example of graphs from trials of data measurements of magnetic force as a function of distance between two short cylindrical magnets is shown in figure 5.

Starting from Eq.(3) we found the curve that has the best fit to the series of data points corresponding to the first measures at very short distance ($x < R$ and $x < d$ ) (red line in fig 4). Thus we obtain $F^{R}_{stick} = 450 \pm 10\mathbf{g}$ and $\alpha R = 0.40 \pm 0.02\mathbf{cm}$ . As the distance increases $x \gg d$ the force varies according Eq.(4) while the asymptotic dipole/dipole model of interaction, with the force behaving as ${}^{\mathbf{1}}\!/_{r^{\mathbf{4}}}$ , is not reached.

Through a best fit procedure in the range of data points between 0.5 cm and 4 cmwe found the value of $A = F^{\infty}_{stick} \dfrac{d^2 + R^2}{d^2} R^2 = 26.7 \pm 0.3\mathbf{g\ cm^2}$ , with coefficient of determination $\rho^2 = 0.9992$ . Thus we can compare the extrapolated values $F^{\infty}_{stick} = A \dfrac{d^2}{(d^2 + R^2)R^2} \rightarrow F^{\infty}_{stick} \approx 410 \pm 80g$ , from the large distance fit and $F^{R}_{stick} = 450 \pm 10g$ from the short distance fit, observing again that the two results overlap significantly.

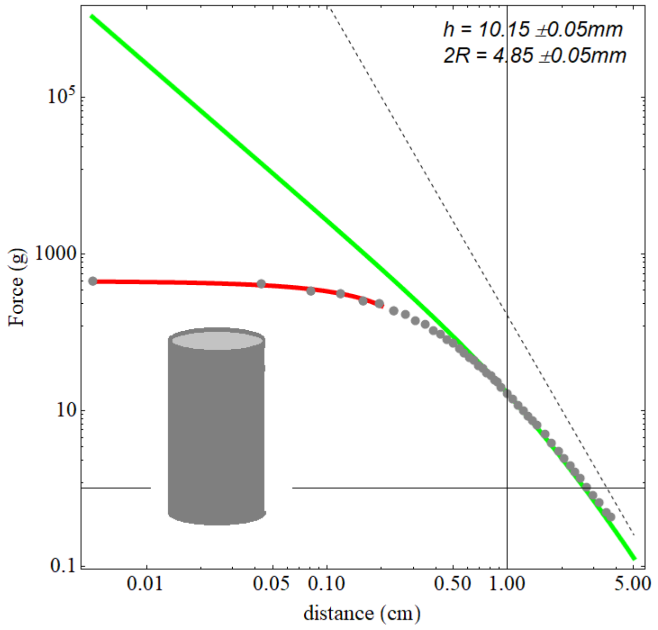
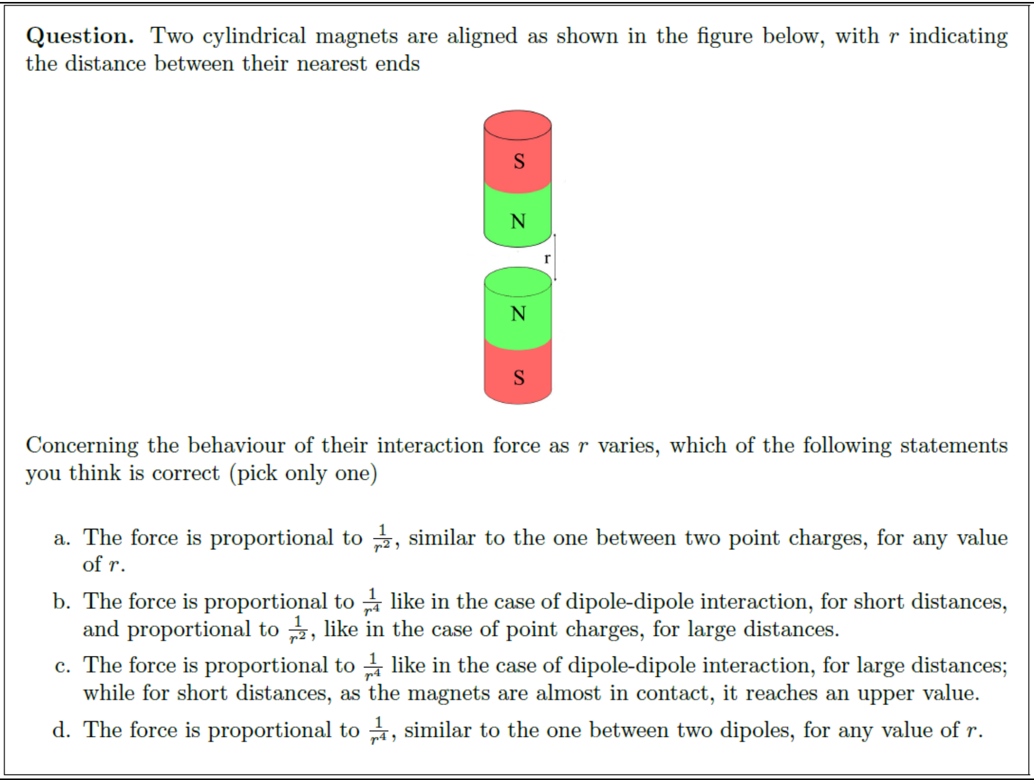


**Figure 5.** Force versus contact distance for short cylinders(*$h_2$* = 10.15 ±0.05mm *and* $2R_2$ = 4.85 ±0.05mm*)* (Graycircles): measured values; (continuous red line):Multipole expansion at small distances; (continuous green line): Point-like dipole approximation Eq.(4) (dashed line): asymptotic point dipole model $^1/_{r^4}$ as in Eqs. (4.b) and (4.c).

The agreement between the parameters obtained in each experiment for the short and intermediate/long distance separately shows the effectiveness of both the experiment and the theoretical approach.

## 4. Educational results

In the prediction/pre-test phase the item reported in Figure was given to students:



**Question.** Two cylindrical magnets are aligned as shown in the figure below, with $r$ indicating the distance between their nearest ends

Concerning the behaviour of their interaction force as $r$ varies, which of the following statements you think is correct (pick only one)

a. The force is proportional to $\frac{1}{r^2}$, similar to the one between two point charges, for any value of $r$.

b. The force is proportional to $\frac{1}{r^4}$ like in the case of dipole-dipole interaction, for short distances, and proportional to $\frac{1}{r^2}$, like in the case of point charges, for large distances.

c. The force is proportional to $\frac{1}{r^4}$ like in the case of dipole-dipole interaction, for large distances; while for short distances, as the magnets are almost in contact, it reaches an upper value.

d. The force is proportional to $\frac{1}{r^4}$, similar to the one between two dipoles, for any value of $r$.

**Figure 5** The item concerning the magnet magnet interaction force belongs to both the surveys administered to students before and after the experimental activities

The most striking result of this pre-test was that a majority of students (16/30) showed the alternative idea that the point charge model is valid in this case for large distances and the dipole model for short distances. The correct answer *c.* was chosen by one third of students (10/30). Other distractors were not very relevant. The full results, along with those of the delayed post-test, are reported in Table 1.

At the end of the activity, judging from discussion and the written reports of students groups, all students, or at least all groups, seemed to have understood the asymptotic behaviour of magnetic forces. A large part of the discussion with groups involved understanding how to determine *a priori* the respective domains of validity of the charged plates model and the dipole model.

**Table 1** – Students answers to the item reported in Figure 5

| Answer | Pre-test (N=30) | Delayed Post-test (N=26) |
|---|---|---|
| a. | 2 | 0 |
| b. | 16 | 8 |
| c. | 10 | 17 |
| d. | 2 | 1 |

However, the misconception showed a certain degree of robustness, reappearing in almost one third (8/26) students in the delayed post-test. This time however, a majority of students (17/26 or 65%) provided the correct answers for the behaviour of the magnetic force with distance.

## Conclusions

In this paper we proposed a simple experiment meant to validate simplified and asymptotic models exploring quantitatively the magnetic force between permanent magnets, a paradigmatic case where the asymptotic analysis of a difficult problem is effective. We note that the true asymptotic regime for large distances can only be reached for thin disc magnets, while for magnets of different aspect ratio the expression Eq.(4) gives a reasonably good approximation for intermediate distances as reported by other authors.The experiment employs simple and inexpensive materials, addresses a relevant topic in the physics curriculum, and allows students to practice not so common laboratory skills.

The activities were tested with 30 undergraduate students from a laboratory courseat the University of Trento in less than 1 hour. Students preformed the experiment and data analysis autonomously,working in large groups (Figure 7). Students also answered to some questions before, during and after the experimental activities.

From their answers given before the activity, we observe that students, although familiar with the behaviour of real magnets, often think that the force diverges when the magnets are in contact. More generallythey apply formulas usingpower law relationships with no concern about the range of validity of each approximation.Finally, the misconception that the magnetic force is always proportionalto $1/r^2$ [20-21] is common among our students. Before the experiments, only one third of our students applies the correct the power law formula for dipole-dipole interaction. In fact, from the pre-test we identified a more general critical point in students' ideas, concerning the nature and validity ofasymptotic behaviours. Students are often unable to understand the domain of validityof an approximate model; when they consider a dipole-dipole approximation, they sometimes wonder on whether it applies at short or large distances; and are confused by the simultaneous presence of different length scales such as $R$, $d$….

After performing the experiment, students recognized that the force does not diverge when the magnets are put in contact and carried out a detailed data analysis based on asymptotic behaviours. The latter quantitative analysis can produce an effective understanding of the complex relationship between force and distance. After the activity, two thirds of our students provided an effective conceptual explanation of the magnetic force between two identical magnets as a function of the distance.

Even though students had already gone through previous undergraduate lab courses, they were unfamiliar with drawing relationships in log-log or semilogarithmic scales,and appreciated the usefulness of such techniques to transform power laws into linear relationships.The laboratory activity represents an appropriate context to practice with such lab competencies, which are explicitly mentioned in the AAPT recommendations for the undergraduate physics laboratory curriculum [30].

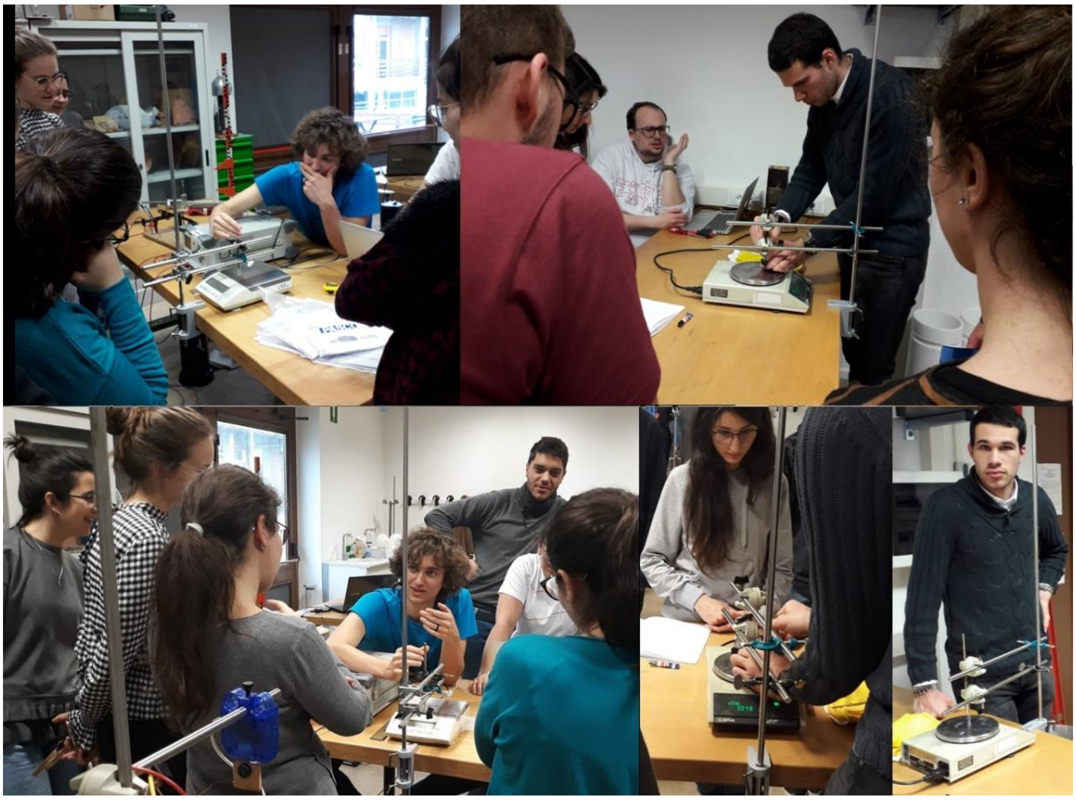


**Figure 7** Students during the experimental activities

In summary, considering the relevance of the topic and the relative straightforwardness of the experimental apparatus and procedure, the theoretical-experimental activities proposed here can offer a valuable contributeto promote the successful conceptual understanding of magnetic phenomena in an appropriate learning environment, and to enhance students' laboratory skills.

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