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Multi-period portfolio decision analysis: A case study in the infrastructure management sector

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ABSTRACT

This paper presents an approach to select and plan the optimal execution of potential investment activities. The model is composed by a computational part, in the form of a combinatorial optimization problem, coupled with a preference elicitation module used to capture subjective judgments. In particular, the structure of the elicitation module draws from portfolio decision analysis and Multi-Attribute Value Theory and shows how their use can be integrated with a multi-period optimization problem with activities durations and constraints on their overlaps. The problem formulation was inspired by a real-world infrastructure management case in the energy distribution sector and tested on a dataset of more than three hundred activities of improvement of infrastructure conditions. Finally, the approach proposed in this paper is validated by analyzing its results and its robustness concerning the input data of the real-world case study.

1. Introduction

Choosing and scheduling multiple investment activities is a complex task with strategic importance and, as such, is less and less often left to the intuition of decision makers. For this reason, we are witnessing an increase in the use of analytic tools, the so-called prescriptive analytics [1], to support and justify decisions.

The knapsack problem is possibly the foremost optimization problem used to choose a discrete subset of alternatives from a reference set [2] according to a given optimality criterion. Despite its potential complexity, the basic problem can nowadays be solved efficiently [3]. Many extensions of the knapsack problem have been proposed to model, for instance, the case of multiple choices [4], multiple periods [5] and multiple objectives [6]. The search for specific algorithms to solve specific extensions of the knapsack problem is, however, limited by the fact that realistic models are often complicated by several constraints such as time duration of activities [7], variation of the budget limit over time [8] and precedence relations between projects [9]. The complexity of certain variants leads scholars to use heuristic [10] and meta-heuristic [11] methods to solve the problem.

Given its important role in the objective function, the *a priori* attribution of values to elements of the reference set—the so-called “prioritization” procedure—has been extensively addressed in the literature, but mostly following merely economical targets. For example, Chen and Askin [12], Koç et al. [13] and Sakka et al. [14] used the Net Present Value to quantify the values of projects, and for Liu and Wang [15], the profit of each project represents the discriminating value of prioritization.

Recently, the newer and wider concept of *Portfolio Decision Analysis* (PDA) has emerged [16]. According to Salo et al. [17], PDA is “the application of decision analysis to the problem of selecting a subset or portfolio from a large set of alternatives”. Namely, PDA aims at coupling portfolio optimization problems, such as the knapsack and its variants, with results and good practices from decision analysis [18]. As such, the focus is shifted from the algorithmic part to a correct and holistic formulation of the problem, which should account for the subjective preferences of experts, their nonlinear and multi-criterial nature. PDA has been used, for example, to select infrastructural projects for the improvements of bridges’ conditions [19], projects for urban development [20], investments on ICT [21], in the energy field [22], in research and development field [23], and in environmental management [24], where an open-source *Python* MCDA library [25] was also created. As recalled by Mild et al. [19], PDA does not only offer a systematic approach to selection processes, but also enhances the transparency and the defensibility of decisions made by publicly owned companies. For an updated overview on PDA, one can refer to a recent survey [26].

It is worth noting that, albeit similar, multi-period PDA problems should not be confused with the well-known *resource constrained project scheduling* (RCPS) problems [27,28]. In RCPS problems, activities have a value only with respect to the attainment of a final goal. For example,

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the value of constructing the external walls of a house is strictly related to the fact that the final goal, building a house, is achieved. That is, parts of projects rarely have intrinsic values. On the contrary, in portfolio decision analysis, like in the knapsack problem, activities have intrinsic values.

In the PDA field, there is an important aspect related to possible biases that can affect the decision process. Behavioral decision theory explains the psychological aspects that can affect decisions and judgments. In particular, using a Multi-Criteria Decision Analysis (MCDA) method, as in this paper, the definition of weights can be the most sensitive part from a cognitive point of view. Some examples of biases which may affect our proposal are the *anchoring*, the *equalizing* and the *range insensitivity biases*. However, there are specific methods that help mitigate biases in this part of the process, e.g. *swing* and *trade-off* methods according to Morton and Fasolo [29]. The interested reader can refer to Montibeller and Von Winterfeldt [30] for a review of biases and debiasing methods.

So far, most of the real-world applications of PDA have been limited to one-shot decision problems, in the sense that the proposed applications regarded single-period selection problems and not planning problems. Even in the few cases when a planning horizon was taken into account (e.g. Barbati et al. [31] and Sarnataro et al. [20]), activities were assumed to have the same (or no) length, which leads to a simplified formulation at the cost of sacrificing realism. It appears that the time dimension has been, to some extent, neglected, and that, on this subject, there is space for improvements. Inspired by a real-world application, in this manuscript we address this issue.

The real-world application concerns infrastructure management for an energy distribution network. Specifically, this paper proposes a PDA based approach to a planning problem involving a set of potential investment activities. This is done by addressing a prioritization problem that needs to be adapted to particular demands and for which considering the profit alone would be misleading. The novelties of this approach, in comparison to previous research in PDA, are that (i) the optimization problem is multi-period, so in each period a choice must be made on which activities to start and (ii) activities have different durations, which create possible overlaps.

Given the recognized importance of involving experts in the prioritization phase, we consider a module to help elicitate the preferences of the experts and consider them within a MILP problem to plan the execution of investment activities. The module is based on *Multi-Attribute Value Theory* (MAVT, Keeney and Raiffa [32]). Alternative scoring methods, like AHP [33] and TOPSIS [34] were also considered. However, due to unresolved issues with these methods or too strict assumptions, it was judged that they would not have led to a sufficiently “defensible” decision. In the case of the AHP, we considered the following critical issues: the arbitrary association between linguistic labels to numerical evaluations [35], the questionable interpretation of weights [36], and the rank-reversal [37]. In the case of TOPSIS, the method’s tacit assumption of linearity of the value functions of different attributes is violated by our real-world application. We note that the use of MAVT in infrastructure management has already been corroborated in the literature, e.g. [38–40].

The rest of the paper is organized as follows. Section 2 introduces, in two separate subsections, the optimization problem whose solution is the optimal execution plan of activities, and the decision making technique used to estimate the values of activities. The explanation of the latter is kept sufficiently detailed to make it accessible also to readers unfamiliar with this field. Section 3 describes the application which inspired the theoretical framework and shows the results. Finally, Section 4 contains some reflections and conclusions.

2. Problem definition and model formulation

Similarly to multi-period knapsack problems, the problem described in this section considers a set of activities $A = \{1, \dots, n\}$ and a set of periods $T = \{1, \dots, m\}$ available for their execution. The set T will be equivalently called *time horizon*. In addition to the normal budget constraints, it is reasonable to assume that, in general, technical constraints may exist too. For example, there could be limits to the number of activities that can be executed simultaneously, precedence relations between their executions, periods when some projects cannot be executed, deadlines, and, perhaps more importantly, different projects may have different durations. Every activity has a value that indicates the importance of an early execution. Section 2.1 presents how the value is used in the optimization problem and, after that, Section 2.2 describes the method used to estimate it. This method involved experts to properly define the preferences and the objectives of the problem.

2.1. Optimization problem

The goal of the optimization problem presented in this paper is to (optimally) select and schedule a number of activities, to facilitate early executions of activities with greater value. For this reason, we draw from similar approaches [9] and, in the following, the objective function is the sum of the discounted values of the planned activities. More precisely, we consider $x_{i,t}$ the binary variable equal to 1 if the execution of the ith activity starts in the tth period, and 0 otherwise, $v_i > 0$ the value of the ith activity, and $r > 0$ a suitable discount factor. Thus, the objective function is as follows:

$$\text{maximize} \quad \sum_{i\in A}\sum_{t\in T} \frac{v_i}{(1+r)^t} x_{i,t}. \tag{OBJ}$$

Turning our attention to the constraints, we acknowledge that each activity can be chosen, i.e. can begin, at most once within the time horizon T. Given the lack of resources, some activities may not be chosen, and therefore the constraint should be in the form of inequality,

$$\sum_{t\in T} x_{i,t} \le 1 \quad \forall i \in A. \tag{C1}$$

Once started, the ith activity must last its predetermined execution time, $l_i \in \mathbb{N}_+$. To formalize this constraint we need a new auxiliary binary variable $z_{i,k}$ equal to 1 if the ith activity is being executed in the kth period. The following constraint imposes that the beginning of an activity at time t ($x_{i,t} = 1$) implies its execution for l_i periods, ($z_{i,k} = 1\ \forall k = t, \dots, \min\{t + l_i - 1, |T|\}$),

$$l_i x_{i,t} \le \sum_{k=t}^{\min\{t+l_i-1,|T|\}} z_{i,k} \quad \forall i \in A,\ t \in T. \tag{C2}$$

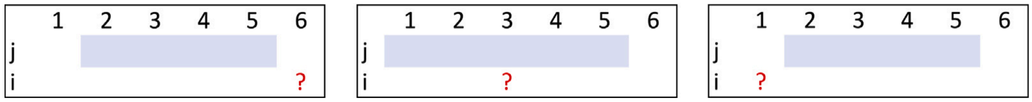
In particular, the min operator is used so that all the activities starting in T will also end in T.[1]

It is reasonable to assume that a periodic budget is assigned a priori for the execution of activities. For this reason, we assume that the time horizon, T, is partitioned in p subperiods $T_1, \dots, T_p$ each of which is associated to a given budget, $b_k > 0\ \forall k = 1, \dots, p$. Hence, if we consider $c_i > 0$ to be the cost of the ith activity and assume that it is paid at the beginning of the execution, the family of constraints can be written as,

$$\sum_{i\in A}\sum_{t\in T_k} c_i x_{i,t} \le b_k \quad k \in \{1, \dots, p\}. \tag{C3}$$

Similarly, the budget could be decomposed in terms of subsets of activities. For instance, it could be specified, by means of further constraints, that at least a fraction of budget be invested in activities of a certain type, or activities in a given geographical region. If we define

[1] This is a choice inspired by the case study presented later and it should not be considered binding.



(a) With $k = 6$: $\theta_{i,j,k} = 1$, $\sum_{t=1}^{\max\{k-l_j,1\}} x_{j,t} = 1$, and activity i *can* begin in period k.

(b) With $k = 3$: $\theta_{i,j,k} = 0$, $\sum_{t=1}^{\max\{k-l_j,1\}} x_{j,t} = 1$, and activity i *cannot* begin in period k.

(c) With $k = 1$: $\theta_{i,j,k} = 0$, $\sum_{t=1}^{\max\{k-l_j,1\}} x_{j,t} = 0$, and activity i *cannot* begin in period k.

Fig. 1. Constraint (C6): The values assumed by θ and the sum $\sum_{t=1}^{\max\{k-l_j,1\}} x_{j,t}$ govern the execution of activities with the constraint precedence. The three cases depicted show how the constraint allows the execution of task i, which must be preceded by j, only when both values are equal to 1 (a), while in (b) and (c) one of the two values is equal to 0, preventing task i from being scheduled before or in the process of executing activity j.

these subsets as $A_1, \dots, A_r \subset A$, each associated with a minimum budget $\tilde{b}_k > 0\ \forall k = 1, \dots, r$, we can formulate the constraint as

$$\sum_{i\in A_k}\sum_{t\in T} c_i x_{i,t} \geq \tilde{b}_k \quad k \in \{1,\dots,r\}. \tag{C4}$$

It is realistic to assume that there may be limitations on the number of activities which can be executed simultaneously, and this may depend on who is in charge of their execution. We assume that there is an indexed set of contractors/executors $E = \{1,\dots,q\}$ and that we can partition the set of activities into subsets, each associated to a given contractor. Namely, each $A_j \subset A$ contains the projects which should be executed by the jth contractor. In addition, given their limited capacities, the jth contractor can execute, at most, $u_j \in \mathbb{N}_+$ activities simultaneously. Said this, the related constraint is,

$$\sum_{i\in A_j} z_{i,t} \leq u_j \quad \forall j \in E,\ t \in T. \tag{C5}$$

In addition, there are precedence constraints between activities, e.g. activity i cannot start before j has been concluded. We call $\mathcal{R} \subset A\times A$ the set of pairs (i,j) such that $(i,j)\in\mathcal{R}$ indicates that i cannot be initiated before j is completed. In order to model this constraint we first define a binary indicator $\theta_{i,j,k}$ for all $(i,j)\in\mathcal{R}$, such that

$$\theta_{i,j,k} = \begin{cases} 1, & \text{if } k > l_j \\ 0, & \text{if } k \leq l_j \end{cases} \quad \forall k \in T.$$

With this new parameter, the constraint can now be defined as follows,

$$x_{i,k} \leq \theta_{i,j,k} \sum_{t=1}^{\max\{k-l_j,1\}} x_{j,t} \quad \forall (i,j)\in\mathcal{R} \quad k \in T. \tag{C6}$$

The rationale behind constraint (C6) is twofold:

- The planned start of i *cannot* be scheduled in the first l_j periods. In fact, this is impossible because there would not be enough time to execute j, even if this latter was scheduled at $t = 1$. In the constraint, this is guaranteed by $\theta_{i,j,k} = 0$ holding by definition whenever $k \leq l_j$, thus making $x_{i,k} \leq 0$ in (C6) for all the $k \leq l_j$.
- The planned start of i *can* happen when $k > l_j$ but only under the condition that j has already been executed. That is, the planned start of the jth activity should be scheduled at least l_j periods before the planned start of the ith activity.

Three representative examples of relevant combinations for the constraint (C6) are presented in Fig. 1.

Some activities have a deadline, which means that they must be executed and completed within a predefined period. We call $A^d \subset A$ the subset of such activities and $d_i \in T\ \forall i \in A^d$ their deadlines. Hence, it is necessary to impose that such activities start, at the latest, l_i periods before their deadlines. That is,

$$\sum_{t=1}^{d_i-l_i} x_{i,t} = 1 \quad \forall i \in A^d \tag{C7}$$

As we will see later, in the discussion of the application, some activities cannot be executed in some periods. We define $\mathcal{S} \subset A\times T$ the subset of pairs (i,t) such that, if $(i,t)\in\mathcal{S}$, then the ith activity cannot be executed at time t. Hence, the following family of constraints prevents the execution of some activities in some given periods,

$$z_{i,t} = 0 \quad \forall (i,t)\in\mathcal{S} \tag{C8}$$

It is also reasonable to assume that the optimization problem be run at regular intervals to consider new information and activities. In this case, the starting dates of some activities may have already been stipulated and cannot be altered. We defined $\mathcal{S}' \subset P\times T$ as the set containing all the pairs (i,t) such that the ith project must start in period t. Thus, the constraint implicitly removes $x_{i,t}$ from the set of variables by fixing its value as follows,

$$x_{i,t} = 1 \quad \forall (i,t)\in\mathcal{S}' \tag{C9}$$

Finally, we can reckon that the variables of this problem are $x_{i,j}, z_{i,j} \in \{0,1\}$, and (OBJ) together with (C1)–(C9) form a binary integer programming (BIP) problem, which can be solved by combinatorial optimization techniques. Let us note the flexibility of the optimization problem formulated above: some constraints can be removed whereas some additional ones can be added. To a large extent, our specific formulation reflects the real-world case study which required its formulation.

2.2. Determination of the values of investment activities

Values of activities, v_i's, play an important role inside the objective function (OBJ) of the optimization problem presented before: the greater v_i the higher the likelihood that the ith activity will be chosen and executed at an early date. Consequently, it is essential to estimate them with due diligence. To achieve this goal one has to consider the real context in which the problem arises: there is a finite set of activities that could be described in terms of their characteristics. Among the several methods of PDA, Multi-Attribute Value Theory (MAVT) was chosen to determine the values of projects. The framework of MAVT is appropriate for the situation of a real-world application in which only one objective cannot describe properly the situation. MAVT has a sufficiently general approach and takes into consideration that the objectives and the attributes are not easily identifiable without the involvement of experts and decision makers, due to the complexity and specificity of their preferences, necessities and aims. To evaluate correctly all the elements of our problem, we involved experts of the company. When there is this kind of collaboration, MAVT is widely used [26], thanks to the intuitive approach that allows DM's to easily appreciate results [41].

Formally, the typical MAVT problem considers a finite set of n alternatives $A = \{A_1, A_2, \dots, A_n\}$ and a finite set of m attributes or criteria, $C = \{C_1, C_2, \dots, C_m\}$ with $m \geq 2$. Attributes are features of the alternatives which are used to quantify the extent to which an objective has been achieved. Namely, attributes operationalize objectives by making their achievement measurable [42]. We call X_j the set of possible levels of the jth attribute, and $\underline{x}_j, \overline{x}_j \in X_j$ the least and the most desirable levels, respectively. If we assume that the chosen attributes are sufficient to adequately describe every alternative, then, each alternative $A_i \in A$ can be equivalently represented by a vector, $\mathbf{x}^i = (x_1^i, \dots, x_m^i) \in X_1 \times \cdots \times X_m$, whose jth component x_j^i is the jth attribute level of the ith alternative.

At this point, MAVT recommends the construction of a single-attribute value function, $v_j : X_j \to [0,1]$, for each attribute C_j with the requirements $v_j(\underline{x}_j) = 0$ and $v_j(\overline{x}_j) = 1$. Functions v_j's assign values to levels so that the greater its value, the better the level.

Once the m value functions are defined, their values must be aggregated by means of a function $v : [0,1]^m \to [0,1]$ returning a single representative value such that, given two vectors $\mathbf{x}$ and $\mathbf{y}$,

$$v\left(v_1(x_1), \dots, v_m(x_m)\right) \geq v\left(v_1(y_1), \dots, v_m(y_m)\right) \quad \Leftrightarrow \quad \mathbf{x} \succeq \mathbf{y}$$

where $\succeq$ is a weak preference relation, reading as "$\mathbf{x}$ is at least as good as $\mathbf{y}$". Under mild additional assumptions (i.e. preference independence [32] and measurability [18]) v has the following additive form,

$$v\left(v_1(x_1^i), \dots, v_m(x_m^i)\right) = \sum_{j=1}^{m} w_j v_j(x_j^i) \tag{1}$$

where w_j's, the *scaling constants* (or weights) of the attributes, are in the interval $[0,1]$ and sum up to 1, i.e. $\sum_{j=1}^m w_j = 1$.

It must be remarked that, in some cases, the additive form may not be able to describe the preferences and it may be necessary to use different representations or, in case of uncertainty, Multi Attribute Utility Theory [43].

There are many procedures to elicit the scaling constants $w_1, \dots, w_m$ from experts [44]: ratio method, swing procedure, and trade-off procedure [18, pp. 135–141], just to cite few of them. The trade-off method, which was used in the application case presented in this paper, requires prior knowledge of the value functions $v_1, \dots, v_m$, of the attributes. By interviewing experts, we determine pairs of alternatives (real or fictitious) that differed in the value of only two attributes, e.g.

$$\mathbf{x}^1 = (x_1, \dots, x_{i-1}, x_i^1, x_{i+1}, \dots, x_{j-1}, x_j^1, x_{j+1}, \dots, x_m)$$
$$\mathbf{x}^2 = (x_1, \dots, x_{i-1}, x_i^2, x_{i+1}, \dots, x_{j-1}, x_j^2, x_{j+1}, \dots, x_m)$$

and that gave the experts the same satisfaction level. With this information, and assuming additivity, we then know that

$$\sum_{j=1}^{m} w_j v_j(x_j^1) = \sum_{j=1}^{m} w_j v_j(x_j^2)$$

which can be simplified into

$$w_i v_i(x_i^1) + w_j v_j(x_j^1) = w_i v_i(x_i^2) + w_j v_j(x_j^2) \tag{2}$$

which, in turn, is a linear equation in the two variables w_i and w_j. With a properly chosen set of $m-1$ of such comparisons and equations, in combination with the normalization condition $\sum_{j=1}^m w_j = 1$, one obtains a system of equations with a unique solution.

While these are the minimum requirements to find a set of scaling constants, it is possible to ask the experts for more comparisons, thus increasing the number of equations in the system. In the literature [45, p. 290] [32, p. 123], eliciting more tradeoffs than necessary is seen as a strategy to obtain more reliable results. On the one hand, this procedure may increase the robustness of the results, on the other hand, it may yield an over-determined system of equations. In such cases, possible inconsistencies can be solved by asking the experts to rectify their initial statements or, if the extent of the inconsistencies is considered tolerable, one can solve a linear optimization problem to find the most suitable scaling constants given an overdetermined equation system, (see, e.g. [18, p. 143]).

3. Application to investment selection and planning

The real-world application inspiring the optimization model (OBJ)–(C9) refers to SET Distribuzione S.p.A. which is the main electric distribution network operator in the Province of Trento, Italy. This company manages 12,000 km of networks, more than 4000 electrical substations, and distributes electricity to over 160 municipalities. Energy is supplied to public organizations, private companies and individual citizens, for a total of over 330,000 customers. The goal of the company is to provide the best service ensuring continuity of the electricity supply and the achievement of power quality standards throughout the territory. A high level of service quality translates into a reduction in the number and duration of electricity supply interruptions.

To fulfill these objectives, SET focuses on planning, scheduling and executing interventions aimed at improving and maintaining the quality of the service, as well as at the technical adaptation to the energy demand, environmental requirements and regulatory prescriptions. The activities carried out by SET can be divided into three macro categories: new connections, quality improvement and load adaptation interventions.

New connections are technical solutions for the connection of users and producers to the local electricity network. The requests for the connection of users to the distribution network are linked, in number and quantity, to the dynamics of the overall development of the local economy. In addition to this general trend, SET must consider a second one, specific to the sector, which derives from the increase in the so-called "electrical penetration", or rather the transition from nonelectrical to electrical energy needs associated with industrial processes, human activities and services.

Interventions for service quality improvement are investments aimed at improving the quality of service for end users, finding their input in the standard objectives defined by local energy regulatory authorities. For SET, these are essentially programs to maintain the quality achieved for most of the medium voltage lines and municipalities served, with targeted improvement interventions only in some rural and mountain areas and some industrial areas, in order to improve the service for particularly sensitive users. In these areas, it is considered appropriate to further increase the quality of the service, in order to reduce the cases of prolonged and extended interruptions in the event of highly disturbed conditions. For these reasons, technical and regulatory measures are necessary to safeguard the safety and stability of the electricity system. Some examples are the increase in the degree of disconnection and re-powering of the network, automation of the network with neutral compensation technique, and the implementation of rings for counterfeeding.

Load adaptation interventions, such as enhancement of existing line sections or the construction of new lines from existing primary substations, are also part of quality improvement as they avoid network saturation in areas where electricity consumption has increased significantly.

Clearly, the mission of the company is to provide the best service given a limited amount of resources. Hence, it appears that monetary aspects are only relevant as constraints of the problem (limited resources) but not as goals of the single investment activities. Therefore, an objective function based merely on financial benefits would be misleading.

3.1. On the values of investment activities

Four interviews with two experts have been carried out. The experts involved are engineers of the Operations and Technological Innovation department of the company, but also users of the electrical service. To correctly develop the MAVT problem, our approach was inspired by value-focused thinking [42]: we talked to the experts to identify the fundamental objectives of the interventions. In the first interview, we asked for a description of the company and the types of activities. Thus, we collected data about the company, as described in Section 3, and identified the four fundamental objectives: maximize the number of users connected to the net, maximize the quality of the service,

Table 1
Details of attributes.

j	Attribute name	Measurement unit	X_j	$\underline{x}_j$	$\overline{x}_j$
1	Number of users	users	$\{100, \dots, 15000\}$	100	15000
2	Resilience improvement	number	$[0, 1]$	0	1
3	Quality improvement	verbal	{very low, low, medium, high, very high}	very low	very high
4	Setup time	months	$\{3, \dots, 24\}$	24	3

minimize service interruptions and minimize the delays in the execution of activities. Moreover, as mentioned before, financial profit was not considered an objective. After this phase, we proceeded with the definition of four attributes which contribute to the creation of the overall values of investment activities. Every colloquium was conducted in the presence of both experts, and all the provided answers were given after consensus was reached between the two of them. The identified attributes are:

Number of benefiting users: The value of an investment activity is related to the number of users who would benefit from the execution of the activity. Of course, the more benefiting users, the better.

Resilience improvement: the resilience is the capability to limit the extent, severity and frequency of system outages following an extreme event. The general formulation of the resilience index is a function of the return period of an extreme event and the number of users incurring a consequent service disruption. To calculate the resilience improvement, the company compares indices before and after the execution of an activity. A high level of resilience is achieved through a set of key measures effectively applicable before, during and after extreme events such as the increase of the resistance to stresses, which minimize failures frequency, or the preparation of rehabilitation methods, which minimize recovery time. From the engineering point of view, the main interventions that contribute to the increase of resilience of the electricity network are the replacement of overhead lines with underground lines because the main cause of stress for the electricity network is related to fallen trees.

Quality improvement: the quality of the service is given by the degree of satisfaction of general prerequisites and specific standards, established by local authorities, which are identified in performance parameters to be guaranteed in services provided to all users. Typical quality-intensive interventions concern, for example, investments to increase the load capacity of parts of the networks.

Setup time: represents the bureaucratic and technical period of time necessary to initialize an activity. In fact, all activities require a time frame for receiving the material and obtaining the necessary authorizations. Activities with short setup times are usually preferred over those with long ones so that potential delays are minimized.

The aforementioned attributes were discussed with two experts and then revised to verify their suitability and check the satisfaction of a number of well-known reasonable properties [46]. That is, attributes should be unambiguous, comprehensive, direct, operational and understandable. Table 1 shows the details of the four attributes.

For each attribute, a single value function v_j was identified using the Mid-Value Splitting Technique, or Bisection method [18, p. 119], and a strong involvement of the experts. The experts were asked to identify a value to divide the interval $[\underline{x}_j, \overline{x}_j]$ in two parts by identifying $x_{0.5}$ such that $v_j(x_{0.5}) = 0.5$. That is, a value such that a transition from $\underline{x}_j$ to $x_{0.5}$ be perceived as beneficial as a transition from $x_{0.5}$ to $\overline{x}_j$, i.e. $v_j(x_{0.5}) - v_j(\underline{x}_j) \approx v_j(\overline{x}_j) - v_j(x_{0.5})$. The same procedure was repeated to get the intermediate values $x_{0.25}$ and $x_{0.75}$ and iterated for all the attributes. To make the process smoother for experts, the discrete levels of the Quality improvement attribute were exchanged for numerical values in the range $[0, 1]$. Given this information, the piecewise linear value functions $v_1, \dots, v_4$ are shown in Fig. 2.

Resilience and Quality attributes have an S-shaped trend, which highlights how, given the scarcity of resources, extremely high or low improvements of these attributes are hard to justify. In the middle part, there is a greater increase that clarifies how the company privileges activities with a balance between cost and improvement of characteristics. The trends of Benefiting users and Setup time are simpler: the former shows that, for the company, it is essential to reach as many users as possible, and the shape of the value function for the latter is due to the preference for all those activities that can be quickly initialized. Note that, for the Setup time, the curve tends to become flat, which indicates that, as they increase, the Setup times become "almost equally bad".

Once the single-attribute value functions are defined, the values of the attribute weights, w_j's, can be calculated using the Trade-Off method explained in Section 2.2. First of all, experts were asked to make a classification in order of relevance of the identified attributes starting on the ground of sets $X_1, \dots, X_4$ and their boundary levels $\underline{x}_j$ and $\overline{x}_j$ for $j = 1, \dots, 4$. From this preliminary analysis, the number of benefiting users and the setup costs appeared to be the most and the least influential attributes, respectively. The experts were not comfortable saying more about Resilience and Quality: in their opinion, the weights of resilience and quality should have been approximately equal, but they added that saying that they are equal would have been a strong statement. As a sufficient number of judgments were already elicited, we agreed to leave this comparison unspecified, without trying other methods. Consequently, we could assume an order relation between some scaling constants

$$w_1 > w_2, \quad w_1 > w_3, \quad w_2 > w_4, \quad w_3 > w_4. \tag{3}$$

In the application case, four pairs of alternatives were submitted to the experts comparing, for each of them, two attribute values (e.g. alternative 1 with the best User value and the worst Resilience value is compared with alternative 2 with the best User value and the worst Resilience value). The experts expressed their preference on the best alternative for each pair and this was used to check the relations in (3). Given the same four pairs of alternatives, for each pair the experts were asked to lower the maximum value of the best alternative until it reaches a level of satisfaction that makes it equally preferable to the worst alternative in the pair. The so obtained four pairs of equivalent (in terms of value) attribute vectors are collected in Table 2.

The selection of the four pairs of alternatives was based on the four relations in (3) and considering only pairs which could be comfortably compared by the experts. The comparison between "Number of users" and "Setup time" was too hard to make, since the relevance of the benefiting users is much higher than the setup times and it was difficult to directly relate the two attributes. An opposite reason made the comparison between "Quality" and "Resilience" equally hard for the experts, who could not truly identify which one should have a greater weight. Note that, from the discussion with the experts, it was found that, for extremely low values of improvement of Resilience and Quality, the additive form of the value function does not hold, i.e. when both Quality and Resilience are at the lowest level, the associated investment activity has no value, regardless the number of benefiting users. Nevertheless, the cases in which the levels of Quality

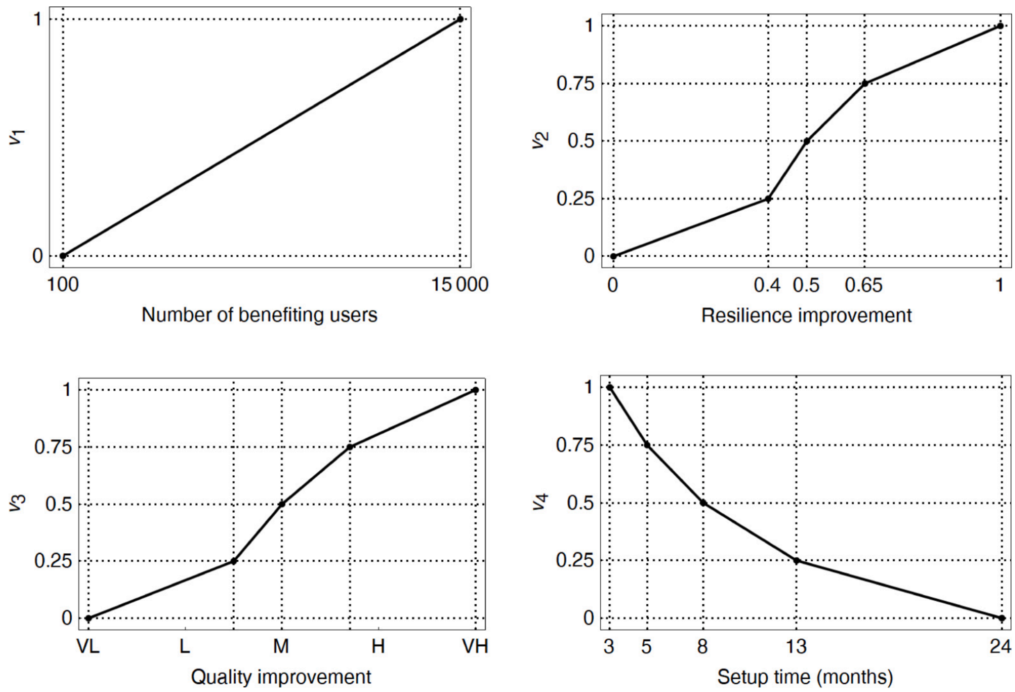


Fig. 2. Piecewise linear value functions for the four attributes.

Table 2
Trade-off method values and attribute Values: values that equal the two alternatives in terms of experts' satisfaction in each tested pair and the corresponding values obtained from the single attribute value functions.

	Attributes				Value functions			
	Users	Resilience	Quality	Time	$v_1(x_1)$	$v_2(x_2)$	$v_3(x_3)$	$v_4(x_4)$
Users–Resilience	3000	0	–	–	0.2	0	–	–
	100	1	–	–	0	1	–	–
Users–Quality	2000	–	very low	–	0.13	–	0	–
	100	–	very high	–	0	–	1	–
Resilience-Setup Time	–	0.7	–	24	–	0.79	–	0
	–	0	–	3	–	0	–	1
Quality-Setup Time	–	–	0.8	24	–	–	0.83	0
	–	–	very low	3	–	–	0	1

or Resilience are zero, are not realistic, because a job that improves neither quality nor resilience would not, in the first place, be included in the list of potential activities. Hence, in our context, the additive form of the value function is valid.

Based on the results in Table 2, relations between values of the scaling constants of the four attributes are sketched in Fig. 3. Let us note that the set of comparisons has two reference points, i.e. the attributes with the greatest and lowest values of the scaling constants, respectively, and that, recently, it was claimed that this "consider the opposite" strategy has a positive effect on the mitigation of the anchoring bias [47].

Formally, the system of equations obtained from the interviews with the experts was the following,

$$\begin{cases} w_1 \cdot v_1(3000) - w_2 \cdot v_2(1) = 0 \\ w_1 \cdot v_1(2000) - w_3 \cdot v_3(1) = 0 \\ w_2 \cdot v_2(0.7) - w_4 \cdot v_4(3) = 0 \\ w_3 \cdot v_3(0.8) - w_4 \cdot v_4(3) = 0 \\ w_1 + w_2 + w_3 + w_4 = 1 \end{cases} \implies \begin{cases} 0.20w_1 - w_2 = 0 \\ 0.13w_1 - w_3 = 0 \\ 0.79w_2 - w_4 = 0 \\ 0.83w_3 - w_4 = 0 \\ w_1 + w_2 + w_3 + w_4 = 1 \end{cases}$$

Such a system is overdetermined since there is not perfect consistency, i.e. referring to Fig. 3, one can see that $5 \cdot 1.26 \neq 7.7 \cdot 1.20$. However, in agreement with the experts, such inconsistency was considered tolerable and a compromise solution was deemed satisfactory. Thus we

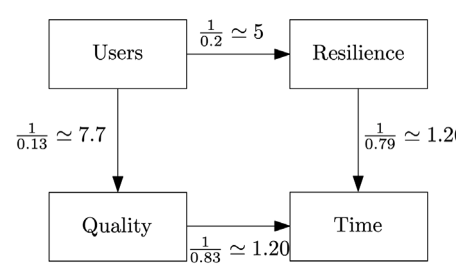


Fig. 3. Relations between weights: schematic diagram of the relationships between attribute weights. The arrows show the relationships between the values obtained from the single attribute value functions, thus obtaining the importance of one attribute with respect to another.

used a linear goal programming problem [18, p. 143] to find the scaling constants which seem to be the best fitting:

$$w_1^* = 0.69, \quad w_2^* = 0.14, \quad w_3^* = 0.09, \quad w_4^* = 0.08$$

which also confirm the order relation presented in (3). Using the scaling constants and the value functions, if we consider a generic activity associated with an attribute vector $\mathbf{x}_i$, then the additive multi-attribute value function returning the overall value of the activity is defined as

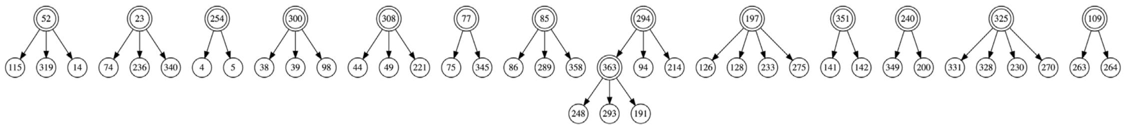


Fig. 4. Precedence relations between activities. Predecessors are double-circled.

Table 3
Budget and teams limits.

Contractor no.	1	2	3	4	5	6	7
Budget [M €/year]	1.5	1	1	1	1.5	1	1
Teams	2	1	1	1	2	1	1

follows

$$v_i := w_1^* v_1(x_1^i) + w_2^* v_2(x_2^i) + w_3^* v_3(x_3^i) + w_4^* v_4(x_4^i) \tag{4}$$

The values of activities, v_i's are utilized in the objective function (OBJ) of the optimization problem as described in Section 2.1.

3.2. Analysis of the solutions

The analysis of the model has been developed with the language AMPL and Gurobi 9.1.0 on a computer with an Intel Core i5 dual-core processor, 2.5 GHz, 8 GB of RAM, running macOS Catalina version 10.15.7. The aim is to validate the proper functioning of the model in terms of useful results for the final users and of usability (e.g. time of execution). The complete form of the optimization model is:

$$\begin{aligned}
\text{maximize} \quad & \sum_{i\in A}\sum_{t\in T} \frac{v_i}{(1+r)^t} x_{i,t} \\
\text{subject to} \quad & \sum_{t\in T} x_{i,t} \le 1 \quad \forall i \in A \\
& l_i x_{i,t} \le \sum_{k=t}^{\min\{t+l_i-1,|T|\}} z_{i,k} \quad \forall i \in A, t \in T \\
& \sum_{i\in A}\sum_{t\in T_k} c_i x_{i,t} \le b_k \quad k \in \{1,\dots,p\} \\
& \sum_{i\in A_k}\sum_{t\in T} c_i x_{i,t} \ge \tilde{b}_k \quad k \in \{1,\dots,r\} \\
& \sum_{i\in A_j} z_{i,t} \le u_j \quad \forall j \in E,\ t \in T \\
& x_{i,k} \le \theta_{i,j,k} \sum_{t=1}^{\max\{k-l_j,1\}} x_{j,t} \quad \forall (i,j) \in \mathcal{R} \quad k \in T \\
& \sum_{t=1}^{d_i-l_i} x_{i,t} = 1 \quad \forall i \in A^d \\
& z_{i,t} = 0 \quad \forall (i,t) \in S \\
& x_{i,t} = 1 \quad \forall (i,t) \in S' \\
& x_{i,t},\ z_{i,t} \in \{0,1\} \quad \forall i \in A, t \in T
\end{aligned}$$

The dataset provided by SET Distribuzione includes 368 potential investment activities for the improvement of the energy distribution network, each one with a different priority $v_i \in [0,1]$. All activities have a predetermined duration l_i and a cost c_i. The activities are executed by 7 contractors, each one with a maximum budget per year and a maximum number of teams that could be occupied simultaneously (Table 3).

Some activities have more characteristics: 5 have a planned start, 17 have a deadline, 107 cannot be performed in some seasons (winter or summer, or both), and, as shown in Fig. 4, there are 39 precedence relations between activities.

Note that precedence constraints are often defined on pairs of activities associated to different contractors, and this does not allow the problem to be decomposed into subproblems, one for each contractor.

A time horizon of 5 years, $T = 60$ months, was considered reasonable. Due to scarcity of resources—i.e. time, availability of working teams, and budget—it is not mandatory that all activities are chosen to be executed within the time horizon $[0, T]$. The aim is to search for the optimal scheduling of the chosen activities in accordance with the constraints.

The objective function, as explained in Section 2.1, maximizes the sum of activities values. In this way, the model plans the execution of activities and we expect to see activities scheduled with an order of priority levels decreasing in time. The model, maximizing the value of the objective function, produces the values of the two binary variables $x_{i,t}$ and $z_{i,t}$ that indicate, respectively, when an activity starts and when an activity is in execution. Values of $z_{i,t}$ were used to produce Gantt charts.

As a full presentation of the results of the analysis would be too space consuming,[2] we focus on two representative contractors (4 and 5) and analyze their schedules. Fig. 5(a) reports the schedule of contractor 4, which operates with 1 team, and we can appreciate that the value of planned projects decreases in time. That is, valuable projects are anticipated. There are no overlaps of activities (as there is only one available team) and all periods are used. The few activities which, given their values, may seem misplaced have been marked with capital letters. All these instances are justifiable considering the particular constraints affecting the respective activities, e.g. the activities which have one or more predecessors are planned to start after the execution of other activities, and therefore this tends to delay their beginning.

Fig. 5(b) represents the optimized schedule of contractor 5, which operates with 2 teams. Also in this case, values of planned activities are decreasing in time. Even in this case, some activities seem misplaced, but their position is again justifiable.

After visual confirmation of the reasonable scheduling, it is interesting to check how the model works in terms of the use of resources. In detail, we analyze the number of teams that work simultaneously every month and the use of budget through years.

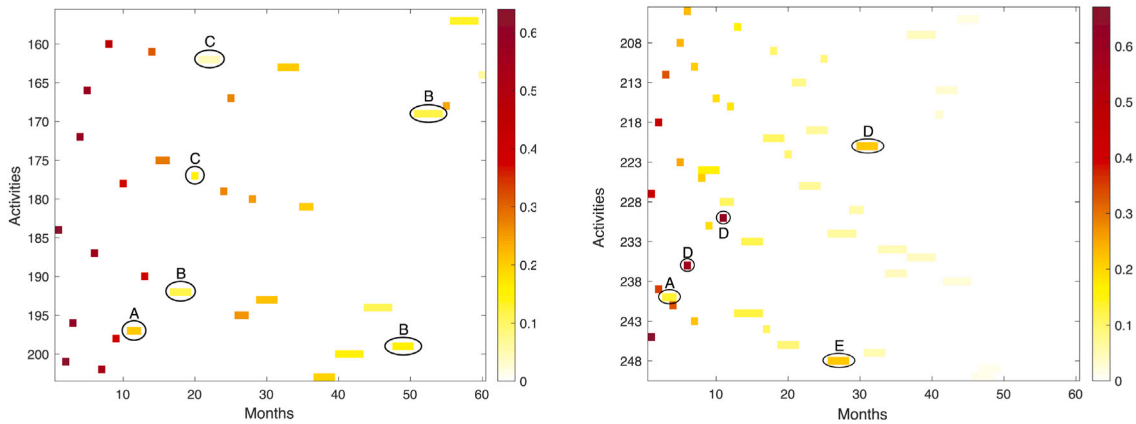
There is a maximum number of teams which could work simultaneously as described in Table 3. A total of 9 teams can be employed in the same month. Fig. 6(a) shows that there are no periods when a team is idle unless the contractor has already finished the assigned activities, as happens to contractors 1 and 5. We can then say that teams are used at the maximum of their capacities. This condition limits the use of budget: Fig. 6(b) shows that not all the available budget is spent every year because there are no more teams that can execute activities.

Reuse of the savings, to increase the number of performed activities employing more teams, or a reassignment of activities to contractors were not considered by the company. However, the results of the analysis could be used for this purpose.

Moreover, to deal with new information and uncertainty, e.g. unexpected changes in cost and duration of activities, the optimization problem can be run at regular intervals, following a rolling horizon approach, to adapt the schedule and keep the model flexible.

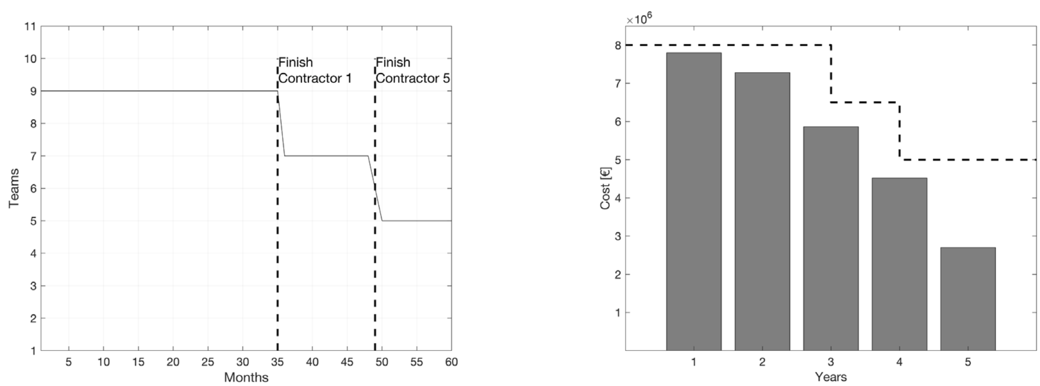
All these results validate the model ability to schedule activities considering their priorities. Additionally, the analysis of the usage of resources shows how the budget could be reallocated to improve the performance of the company.

[2] A full presentation, together with the AMPL files, is available on request from the authors.



(a) Scheduling of Contractor 4 (b) Scheduling of Contractor 5

Fig. 5. Scheduling and priorities of Contractors 4 and 5. The colored bar on the right side of the graph, represents the values collected by the activities each month. That is, the color of every horizontal bar corresponds to the value of priority normalized with respect to the duration of the associated activity, e.g. an activity with priority value $v = 0.60$ and execution time $l = 3$ months is represented as an horizontal bar covering 3 months with value $v = 0.2$. The identification numbers of the activities are reported on the ordinate axis. Circled bars indicate that the activity (A) is a predecessor of another activity executed by a different contractor (B) has a planned start (C) has a programmed deadline (D) has a predecessor (E) has a predecessor which is itself linked to a predecessor. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)



(a) Teams occupancy: number of teams employed every month of the time horizon. The two vertical lines mark the end of the activities for contractors 1 and 5, respectively

(b) Budget use: total cost per year of executed activities. The dashed line is the total available budget through years with two steps when contractors 1 and 5 end activities.

Fig. 6. Resources utilization within the time horizon.

The complexity of the problem, and the large number of constraints, make considering and optimizing the four objectives separately unfeasible. On the other hand, such a pragmatic impossibility of considering a multi-objective optimization problem and the ease with which experts could answer our questions corroborates the necessity and the feasibility of our approach, which is instead based on an *a priori* aggregation of attribute levels into a single representative value and a single objective function.

3.3. Sensitivity analysis

Sensitivity analysis has been carried out to assess the convergence towards the optimal solution and the robustness with respect to the parameter r. As far as the *convergence* is concerned, we believe that, to be operational, the algorithm should lead to near-optimal solutions in a reasonable amount of time. In fact, it should be possible to solve the problem multiple times to test different allocations with respect, for instance, to a varying budget. Namely, it should allow for if–then analysis and fine tuning. The Lagrangian duality gap, i.e. the distance between the primal and the dual problems evaluated at a given feasible solution, can be used to analyze the convergence of the algorithm. In particular, such gap shall decrease as the algorithm approaches the global optimum. Fig. 7 presents a graphical analysis of the relative Lagrangian duality gap as a function of the running time, after the first 20 min of computations. It can be seen that in around one hour the relative gap was reduced to 0.1% (blue line), which guarantees that the incumbent solution is near optimal. Some preliminary tests on the scalability of the problem with respect to the number of investment activities are also shown in Fig. 7, and further tests on larger (artificially created) instances of the problem verified its tractability up to 500 investment activities. To analyze other scenarios, we tested the model on a time horizon of 36 months. As shown in Fig. 7 (green line), with a shorter time horizon the model is slower to reach the same value of the relative gap. While in this latter case the number of variables is significantly reduced, the intrinsic problem of selecting a subset of alternatives for execution becomes more constrained and

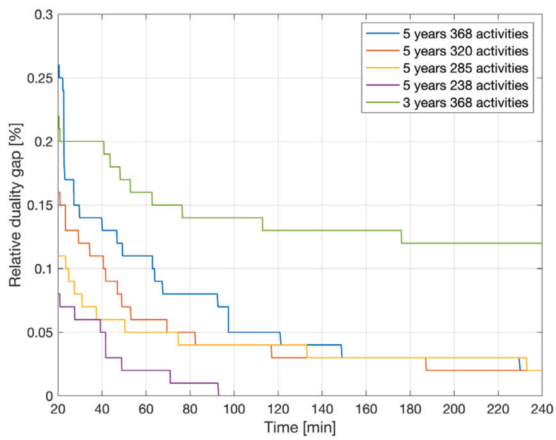


Fig. 7. Evolution of the relative duality gap: the blue line represents the gap trend for the complete set of 368 activities while the other lines represent instances of subsets with less activities. The green line represents the gap trend for the set of 368 activities in 3 years. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

Table 4
Comparisons between solutions obtained with different discount rates. Values in the table are the Jaccard similarity coefficients of the two sets of activities selected by optimizing with two different discount rates: the value 1 indicates that the two solutions contain the same activities. Between parentheses, we give the average difference, in months, between the start of the same activity in models with different values of discount rate.

	r			
r	0.01	0.02	0.04	0.10
0.01	1	0.98 (1.6)	0.99 (1.3)	0.97 (3.3)
0.02		1	0.99 (1.4)	0.97 (2.7)
0.04			1	0.97 (3.1)
0.10				1

thus harder to be solved. However, even in this case, in less than one hour the relative duality gap was reduced to 0.15%, which shows that the incumbent solution is nearly optimal. With the help of the experts, it was pragmatically established that the search could be stopped well before the optimum was reached, as long as the incumbent solution is ‘good enough’. This is in agreement with Herbert Simon’s idea of *satisficing* solution [48].

The other issue left to be determined was the influence of the discount rate $r > 0$, whose positive value induces anticipation of the execution dates of the most valuable activities, in the optimal solution. While it is certainly difficult to find a most suitable value for r, by solving multiple instances of the optimization problem keeping everything unchanged, except the discount rate, it was possible to reduce the importance of giving a very precise value to r. Table 4 shows results on the similarity of optimal solutions for $r \in \{0.01, 0.02, 0.04, 0.10\}$ and it can be seen that similar results were obtained for all instances.

In the worst case, as much as 97% of the selected activities were the same and the average distance between starting dates of common selected activities was only 2 months in a time horizon of 60 months. Hence, results indicate that the solution is not overly sensitive to the parameter r when this is chosen in the range $[0.01, 0.10]$.

4. Discussion and conclusions

In this paper, we proposed a model to select and plan the execution of investment activities of a publicly owned company entrusted with the management of an energy distribution network. This selection and planning process was done taking into account all the possible constraints, the characteristics of every activity, and assigning a value to each of them by means of MAVT, one of the foremost methodologies of PDA.

The first step was the definition of the values of investment activities through Multi-Attribute Value Theory. This process involved meetings with experts, whose contribution was fundamental to correctly define attributes and to test the suitability of scaling constants and value functions. This process permitted adaptation of the model to a real-world case. The obtained result and the use of MAVT are considered satisfactory due to the agreement of experts with the outcomes.

The second step consisted of solving a combinatorial optimization problem which uses as inputs the values assigned to every activity. The model takes into account several constraints, including the limits of available resources, and the characteristics of activities, e.g., the duration of each one. The model gives a satisfactory plan of execution in an adequate time, in this way it is a valid and efficient tool to support the scheduling decisions of the company.

The combination of the two steps gives a complete model which, with a few input data already in possession of the company, or that can be easily estimated by experts, offered a transparent decision support system.

The additive representation of preferences was suitable for the two experts in our real-world application. However, in other cases a non-additive aggregation model might be more correct. Nevertheless, even in this context, the model that we proposed, which uses the trade-off method for weight elicitation, can still be applied [49]. Moreover, in presence of uncertainty, Multi-Attribute Utility Theory (MAUT), instead of MAVT, is more appropriate. In our case, MAUT could be applied to consider the uncertainty of the Setup Time attribute. Uncertainty and non-additivity could be considered simultaneously as recently proposed for portfolio decision analysis with non-additive multi-attribute utility functions [43]. In addition, a broader representation of stakeholders with diverging preferences might provide additional insights, for instance by involving a larger number of stakeholders, including service users.

Salo et al. [50], possibly the main contributors and developers of PDA, discussed the general multi-criteria portfolio decision analysis problem and listed (i) the expansion of its knowledge base and (ii) its embodiment into organizational decision processes as future research directions. Although there is still a lot to do, we would like to think of our contribution as a step forward in these directions: we solved a novel multi-period PDA problem, based on a real-world case, with activities durations and constraints on their overlaps. Furthermore, we believe that the approach proposed in this paper is an example of a prescriptive analytic tool with a predictive valence, as its capacity of planning helps forecast the use of resources for the entire time horizon. Hence, it seems that the distinction between prescriptive and predictive analytics, which is certainly useful in introductory textbooks, is blurred in real-life applications.

Certainly, we hope that our contribution could encourage practitioners to use more non-trivial decision analysis techniques in a field, engineering, where the typical multi-criteria decision making application is instead often “based on simple scoring models” [51].

In the future, it would be relevant to explore the potential of multi-period PDA to select and plan the execution of activities whose aim is to help reach the so called sustainable development goals (SDGs) [52]: relevant SDGs can be used as objectives, and various indicators as attributes of investment activities. A significant help may come from recent advances on methods for the construction of composite indices [53].

CRediT authorship contribution statement

Gaia Gasparini: Methodology, Software, Validation, Formal analysis, Data curation, Writing – review & editing, Visualization. **Matteo Brunelli:** Conceptualization, Methodology, Software, Validation, Writing – review & editing, Supervision. **Marius Dan Chiriac:** Data curation, Investigation.

Declaration of competing interest

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