Leverage and evolving heterogeneous beliefs in a simple agent-based financial market

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Leverage and evolving heterogeneous beliefs in a simple agent-based financial market

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Abstract

Recent research has acknowledged the crucial role of financial intermediaries' balance sheet variables – namely, wealth and leverage – in the dynamics of asset prices. In this paper we use a prototypical "small-type" artificial financial market model with heterogeneous interacting traders to pin down how asset prices are affected by the complex interaction between balance sheet constraints and the endogenous evolution of trading rules.

Keywords: Agent-based model, Financial markets, Leverage cycle
JEL classification: C63, D53, G12, G18

1. Introduction

The last two decades has witnessed an explosion of studies using agent-based computer simulations to analyze the dynamics of financial markets (LeBaron, 2006; Iori and Porter, 2014). Asset pricing models based on the complex evolution of beliefs and trading strategies in a population of heterogeneous investors have proved extremely successful to replicate a large number of stylized facts (LeBaron, 2006; Cont, 2007; Lux, 2009), assess the performance of alternative policy actions in controlled environments (Westerhoff and Dieci, 2006; Fricke and Lux, 2016; Chen, 2016), and evaluate the impact on prices and returns of different behavioral biases (Shimokawa *et al*., 2007; Li *et al*., 2014; Pruna *et al*., 2017).

As it became clear that price movements and the financial health of intermediaries reinforce each other in molding amplification mechanisms conducive to market disruptions (Adrian and Shin, 2010; Krishnamurty, 2010), in the aftermath of the 2007-08 global financial crisis this literature has been enriched by models dealing with leverage through an explicit modeling of traders' balance-sheet dynamics (Friedman and Abraham, 2009; Thurner *et al*., 2012; Fischer and Riedler, 2014; Aymanns and Farmer, 2015; Aymanns *et al*., 2016). The key insights emerged from this research are that leverage by itself is a sufficient condition to generate fat tails and excessive volatility in market returns, and that although qualitatively different modes of behavior are admissible – i.e.: *i*) stable convergence, *ii*) irregular large endogenous fluctuations and *iii*) globally unstable fluctuations – calibrations based on real data put firmly the model into regime *ii*).

The value added of this paper is twofold. First, we show that a richness of results can be achieved by means of a basic "small-type" agent-based financial market model, much simpler than the large and sophisticated models referred to above. In addition to an increased easiness of reproducibility and controllability, our analysis sheds also a sharper light on how market outcomes are ultimately driven by the interaction between balance sheet constraints and the endogenous evolution of beliefs among market participants. When combined with the possibility that investors can repeatedly choose from a range of trading strategies and switch to the more successful one, leverage turns out to be a double-edged sword: it adds to extreme price movements on the one hand, but it facilitates price discovery on the other one.

The remainder of the paper is organized as follows. In Section 2 we present the model. Section 3 illustrates simulation results in terms of stylized facts and the interaction between price dynamics, leverage and trading strategy choices. Section 4 concludes.

2. The model

The model is a prototypical 2-type computational asset pricing model with heterogeneous adaptive beliefs and leverage, largely inspired to e.g. Chiarella *et al.* (2009), Hommes and Wagener (2009), Franke and Westerhoff (2012) and Thurner *et al.* (2012). For a comprehensive defense of the distinct modeling choices we refer to the original sources.

A fixed number N of traders operate in the market for a financial asset, whose unitary price at day $t \in (1, T)$ is P_t. After the trading day has closed, the wealth of a representative trader $i \in (1, N)$ is:

$$W_{i,t} = P_t Z_{i,t} + C_{i,t} \tag{1}$$

where Z_i denotes the total amount of the asset holding on his balance sheet, while C_i is the cash position. We assume that the latter can be negative. This occurs thanks to an unmodelled banking sector extending credit to the trader at an exogenously given interest rate, that for the sake of simplicity is normalized to 0. Borrowing, which is fully collateralized by the asset, is then defined as $L_{i,t} = max[-C_{i,t}, 0]$, while the leverage ratio becomes:

$$\lambda_{i,t} = \frac{P_t Z_{i,t}}{W_{i,t}}. \tag{2}$$

The leverage ratio cannot exceed a fixed threshold λ^{max}, that we postulate equal for all traders. As shown by Shin (2010), this restriction can be justified alternatively on the basis of a microprudential regulatory constraint, a VaR-based risk management approach to control exposure to the stock market, or a limit margin imposed by funding suppliers. Irrespective of its underlying rationale, the ceiling leverage ratio introduces a critical non-linearity in the actual orders executed by leverage-constrained traders. Under certain conditions, it could even transform an *ex-ante* desired demand for the asset into an *ex-post* forced supply.

Let $D_{i,t}$ and $Q_{i,t}$ the day t's notional and effective net demand for the asset expressed by trader i, where both quantities will be determined below. The law of motion of the asset holding is given by:

$$Z_{i,t} = Z_{i,t-1} + Z_{i,t}^{add} \tag{3}$$

where:

$$Z_{i,t}^{add} = \begin{cases} D_{i,t} & if \quad \lambda_{i,t} < \lambda^{max} \\ Q_{i,t} & if \quad \lambda_{i,t} \geq \lambda^{max} \end{cases} \tag{4}$$

When choosing market actions, each trader can resort to two alternative strategies. If acting as a value investor or *fundamentalist*, he exploits his knowledge of the asset's exogenous and fixed fundamental value F, and places orders in accordance to the belief that the market price will to converge to it. By using small letters to define the logarithm of relevant variables, the expectation of the price at day $t + 1$ made by a fundamentalist is given by:

$$E_t^F(p_{t+1} - p_t) = \alpha(f - p_t) \tag{5}$$

where $\alpha > 0$ measures the expected speed of convergence of the market price to its true underlying value, while the notional demand for the asset placed just before the day $t+1$ opening bell rings is:

$$D_t^F = \varphi E_t^F(p_{t+1} - p_t) = k^F(f - p) + \varepsilon_t^F \tag{6}$$

with $\varphi > 0$, and $k^F = \varphi\alpha$ measuring the aggressiveness of the value-based trading strategy. The error term $\varepsilon_t^F \sim N(0, \sigma_F)$ accommodates errors in placing orders or other kinds of noisy behavior affecting fundamentalists' demand.

As an alternative, a trader can place his notional orders by behaving as a trend follower or *chartist*. In this case, the deviation of future prices is expected to grow over time at a constant rate $\beta > 0$:

$$E_t^C(p_{t+1} - p_t) = \beta(p_t - p_{t-1}) \tag{7}$$

so that the notional demand coming from a chartist, for $\vartheta > 0$, reads as:

$$D_t^C = \vartheta E_t^C(p_{t+1} - p_t) = k^C(p_t - p_{t-1}) + \varepsilon_t^C \tag{8}$$

with $k^C = \vartheta\beta$ and $\varepsilon_t^C \sim N(0, \sigma_C)$ admitting an identical interpretation to that advanced for the fundamentalist strategy.

The market order implied by the notional demand can be executed only if $\lambda_{i,t} < \lambda^{max}$. If this condition is violated, the net demand expressed by a constrained trader is obtained by exploiting the definition of leverage ratio and the balance sheet identity as:

$$Q_{i,t} = Z_{t-1}(\lambda^{max} - 1) + \lambda^{max} \frac{C_{t-1}}{P_t}. \tag{9}$$

In line with the adaptive beliefs system approach popularized by Brock and Hommes (1997; 1998), agents are allowed to switch between the two admissible strategies according to their relative past profitability. The underlying dynamics is governed by a function measuring the profits obtained by following strategy $J \in (F, C)$:

$$A_t^J = (P_t - P_{t-1})D_{t-2}^J + mA_{t-1}^J \qquad (10)$$

where it is recognized that the orders submitted in $t–2$ are executed at the price recorded at the end of trading day $t–1$, while the parameter $0 < m < 1$ tunes the memory of the fitness process. Finally, the transition-probability matrix between embraced strategies from $t–1$ to t is given by:

		t	
		F	C
$t–1$	F	π_t^F	π_t^C
	C	π_t^F	π_t^C

where strategy switching probabilities are calculated according to a multinomial logit model (Manski and McFadden, 1981) with $\gamma \geq 0$ tuning the intensity of choice:

$$\pi_t^J = \frac{exp\left(\gamma A_t^J\right)}{\sum_{J\in(F,C)} exp\left(\gamma A_t^J\right)}. \qquad (11)$$

Once all the orders of fundamentalists and chartists have been derived, a market maker takes an offsetting long or short position to assure that the excess demand at the end of day $t+1$ equals zero, so that the closing price is:

$$P_{t+1} = P_t + d\left(\frac{\sum_{i\in(1,N)} H_{i,t}^{add}}{N}\right) + \varepsilon_t^P \qquad (12)$$

where $d > 0$ defines the reaction speed of the price impact function, and $\varepsilon_t^P \sim N(0, \sigma_P)$ takes into account the market-mediated effects associated to noisy trading.

Starting from initial conditions for the cash position $C_{i,1}$, the market price $P_1 = F$ and a random assignment of asset holdings and trading strategies across the whole population of traders, during each trading day $t \geq 2$ the model proceeds along the following steps:

1. Define the appropriate trading strategy for each trader $i \in (1, N)$.
2. Calculate the individual net (positive or negative) notional demand for the asset and the associated borrowing.
3. If the leverage constraint binds, adjust the market order to assure that the level of the leverage ratio belongs to admissible values.
4. The market maker adjusts the market excess demand and announces the new price.
5. Each trader $i \in (1, N)$ calculates his new wealth position.

6. If $W_i \leq 0$, the trader defaults. Set his final wealth to 0, and after two periods replace him with another trader endowed with standard initial conditions.
7. For any viable trader, calculate the relative fitness of alternative strategies.

3. Simulation results

Table 1 reports the parameter values used in benchmark simulations. This specification is in line with standard settings adopted in the literature recalled at the beginning of Section 2. The only point that deserves to be stressed is that chartists are assumed to be more "volatile" than fundamentalists, both as regards the sensitivity with which they adjust their orders to market signals, and the noise affecting their behavior (Franke and Westerhoff, 2012).

Symbol	Description	Value
N	Number of traders	1000
T	Simulation periods (days)	5000
F	Fundamental price (log)	0
λ^{max}	Maximum leverage	12
k^F	Order reaction of fundamentalists	0.05
k^C	Order reaction of chartists	0.1
d	Price impact sensitivity	1
m	Memory in updating fitness	0.985
γ	Weight in probability of choosing strategy	400
C_1	Initial cash endowment	0.5
σ_F	Noise in fundamentalists' demand	0.01
σ_C	Noise in chartists' demand	0.05
σ_P	Noise in the market maker's behavior	0.01

Table 1. Benchmark simulation parameters.

Figure 1 shows the output from a representative simulation run. The same sequence of random draws for stochastic shocks employed here will be used in several occasions later on, in order to guarantee a comparability of results. The data shown covers a time span of 5000 trading days, while a burn-in initial interval of 2000 periods has been discarded to get rid of transients. Panel *a*. shows the time series for the log price. It is characterized by large dislocations from the (log) fundamental value (held fixed at zero) and occasional boom-and-bust dynamics, in which a speculative bubble is followed by a deep recession. Panel *b*. shows trading volumes, here proxied by traders' orders. Note that volumes spring up when the price is far from the fundamental value, due to the fact that when this occurs trading opportunities increase. The dynamics of the leverage ratio is shown in Panel *c*., where we report the mean and median values of its cross-section distribution at each single trading day. It appears that the skewness of the distribution increases before a bust occurs. The inequality of balance sheet sizes across the population of traders, measured in terms of the cross-sectional coefficient of variation,

looms large as a burst episode gains momentum (Panel *e*.). As regards returns, Panel *d*. shows the presence of clustered volatility, Panel *f*. highlights the existence of fat tails in the returns' distribution, while Panel *h*. displays the absence of autocorrelation for level returns and a significant and slowly declining autocorrelation for absolute returns, respectively. Finally, the decoupling of the asset price from its fundamental value is associated with a predominance of chartist traders, while mean-reversion with a large fraction of fundamentalists (Panel *g*.). All these facts are in line with the empirical evidence (Cont, 2010).
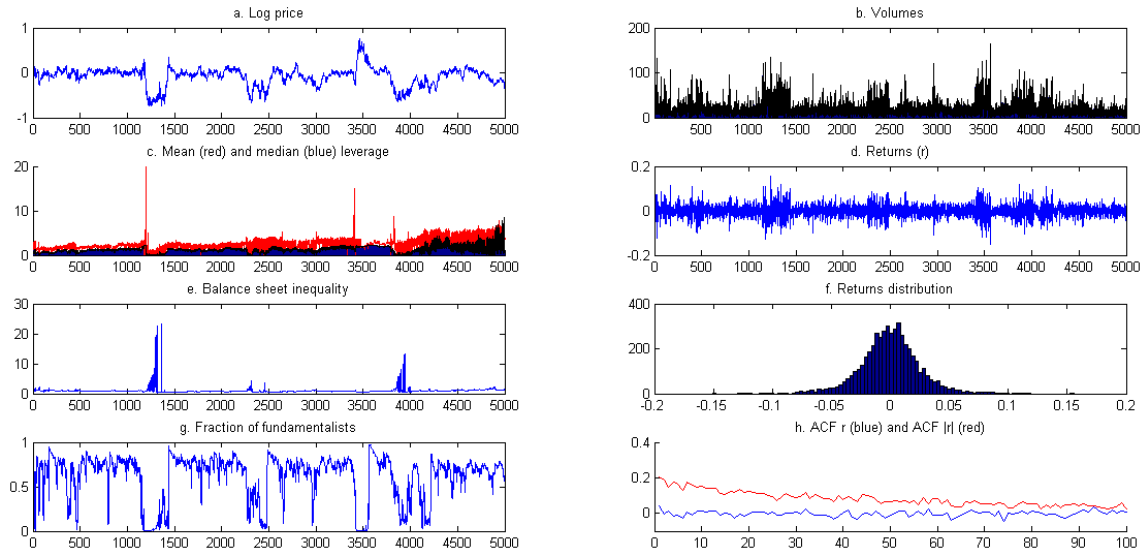
Figure 1. Simulation results. Price, returns, volumes and share of fundamentalists.
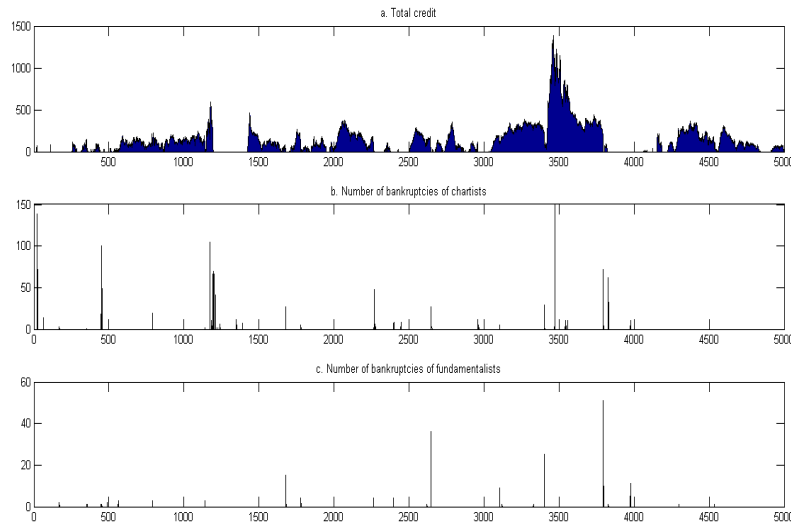
Figure 2. Simulation results. Borrowing and default rates.

The total amount of new credit borrowed by traders is clearly pro-cyclical, as shown in Panel *a*. of Figure 2. The chartist strategy is slightly more risky, as can be appreciated by looking at the failure counts in Panels *b*. and *c*. Figure 3 presents additional results

obtained by averaging over 100 Montecarlo repetitions. Panel *a*. shows that – as in real data – the autocorrelation of volumes is significantly positive and decays slowly, while by employing the same methodology as in Ding *et al*. (1993) we detect long memory in absolute returns (Panel *c*.). We also find evidence of scaling behavior in returns by estimating the generalized Hurst exponent proposed by Di Matteo *et al*. (2005) for the first two moments of the distribution, as maximum leverage λ^{max} is allowed to vary between 2 and 20 (Panels *b*. and *d*.). Furthermore, since H(1) ≠ H(2) for all leverage limits higher than 2, we also find evidence of multi-fractality when leverage is permitted.
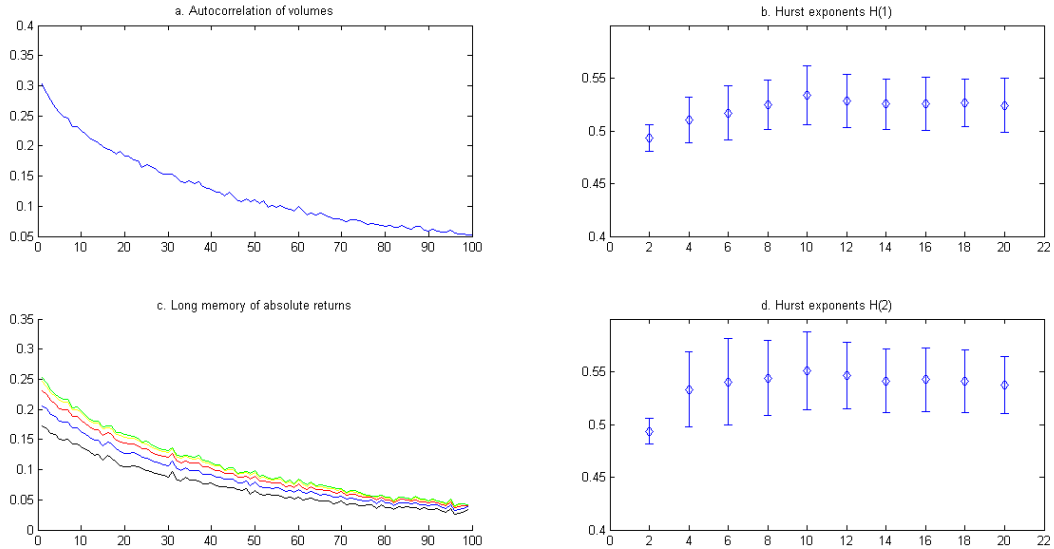
Figure 3. Simulation results, Montecarlo averages. H(1) and H(2) are the Hurst exponents for the mean and the variance of returns, respectively, and in Panels *b*. and *d*. the horizontal axis measures values of λ^{max}. Panel *c*. reports the autocorrelation function of $|r|^d$ for $d \in$ (1, 0.8, 0.6, 0.4, 0.2) from high to low.

The impact of the interaction between trading strategies and the leverage constraint can be appreciated by looking at the four panels of Figure 4. In particular, we compare along several dimensions the market outcomes resulting from the benchmark simulation (red) with a simulation in which the maximum amount of allowed leverage is restricted to $\lambda^{max} = 2$ (blue). The price distortion, measured as the absolute deviation of the market price from its fundamental, is systematically higher when leverage (hence borrowing) is repressed (Panel *a*.). Note that this ensues from a systematic undervaluation of the market price, that over the whole simulation period ranges between 30% and 70%. The trading volume in the repressed market is much lower (Panel *b*.), while as in Thurner *et al*. (2012) leverage causes fat tails (Panel *c*.).[1] Combining these findings, we conclude that leverage implies that fat tails and higher market efficiency come hand in hand. The reason can be understood by looking at Panel *d*., where it is shown that limiting the maximum leverage admissible to investors dramatically affects the endogenous evolution of trading strategies. As we inhibit the use of borrowing to lever on market signals, the fitness associated to different strategies

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[1] And cluster volatility too. Proof of this finding is not shown to save space, but available upon request.

ends up to coincide, so that the fractions of trader types remains basically constant and equal to ½ irrespective of the value assumed by the parameter measuring the intensity of choice γ.
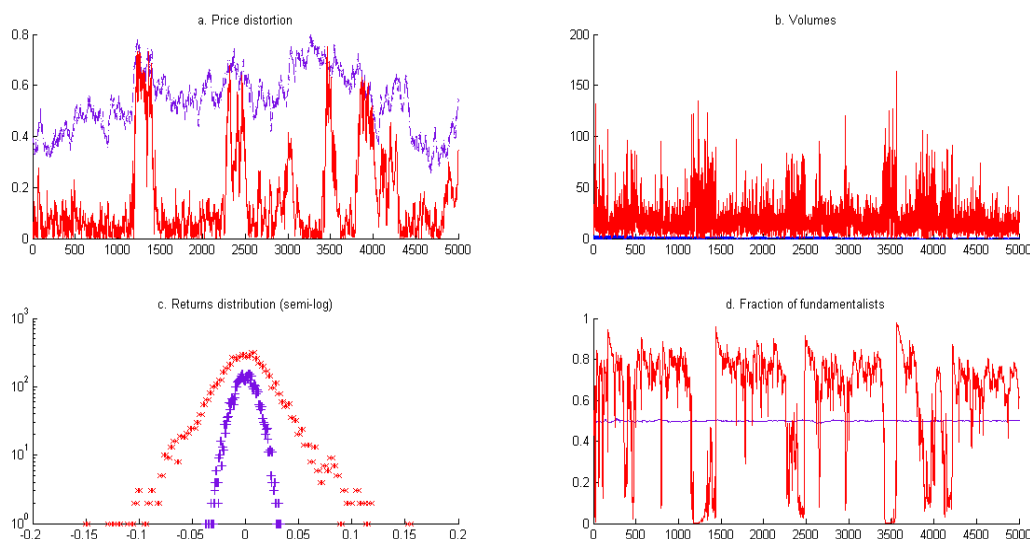

Figure 4. Simulation results

In order to disentangle the influence on asset pricing of balance sheet variables from that of the endogenous choice of trading strategy, we use time series obtained from 100 Montecarlo repetitions with $\lambda^{max} = 12$ to perform a panel regression of absolute returns and the price distortion on intermediaries' mean leverage, the fraction of active fundamentalists and the interaction between the two. To obtain more stable results, data has been transformed to quarterly observations.

	$|r|$	Price distortion
Constant	1.1841***	2.2338***
	(4.7417)	(6.0864)
λ	-0.1168***	-0.2261***
	(-3.4248)	(-4.9239)
Fund	-1.2037****	-2.1528***
	(-4.1016)	(-5.5581)
$\lambda \times Fund$	0.1315***	0.2456***
	(3.2999)	(4.6009)
$\overline{R^2}$	0.1991	0.4970
F-stat	90.3492	362.6906

Table 2. Panel estimates on 100 Montecarlo repetitions, quarterly series. EGLS random effects with White cross-section standard errors and covariance. *t*-statistics in brackets. ***,**,*: significant at the 1, 5 and 10 percent level, respectively.

Notice that in this experiment the leverage constraint is only occasionally binding, meaning that the typical pyramiding and depyramiding effects associated to borrowing can be appreciated although margin calls play a minor role. As expected, a higher fraction of fundamentalists contributes to limit both the volatility of returns and price distortion. Somehow less conceivably, the leverage ratio exerts similar effects. The presence of a significant interaction between the two variates, however, allows us to recognize also that when the fundamentalist belief is adopted by a fraction of traders higher than about 90%, the impact of a higher leverage on volatility and distortion changes sign. As soon as the market environment becomes conducive to an efficient price discovery due to a preponderant presence of traders targeting the true fundamental value, higher values of leverage tend to foster the risk of going bankrupt even for very small price movements. This in turn contributes to increase the volatility of prices and the degree of price distortion. All in all, one can fully appreciate the role of leverage on asset prices only in combination with knowledge of the trading strategies adopted from time to time by investors.

So far the leverage constraint (λ^{max}) has been assumed to remain fixed along a simulation run. As discussed in Geanokoplos (2010), it is common to observe that varying margin calls decrease the maximum amount of leverage when market conditions deteriorate significantly. To assess this, in Figure 5 we compare three different market price series[2] when the admissible maximum leverage is decreased to 4 as soon as the market price falls below its fundamental by 20% (black), 15% (red) or 10% (blue), respectively, while it returns to its original value of 12 when the price reverts to its mean by crossing from below the corresponding threshold.
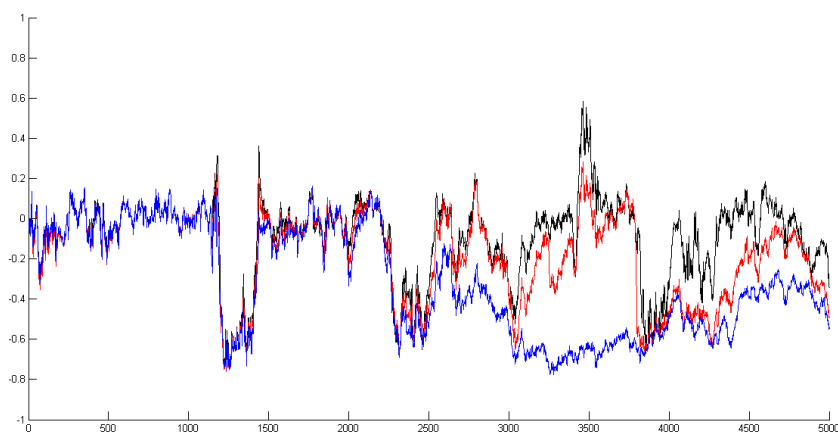

Figure 5. Simulation results. Comparable time series for the log price with different thresholds to activate a conservative margin call.

Simulations show that when conservative margin calls enter into the scene only for seriously disrupted market conditions (i.e., 20% below fundamental), the time profile of market price remains basically unaffected although the threshold is crossed in several occasions.[3] When margin calls are triggered for a less severe decoupling of the market

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[2] Comparability is assured by using in the three runs the same seed of benchmark simulations.

[3] The point can be appreciated by a comparison with Panel *a*. of Figure 1.

price from its fundamental value, however, burst episodes can become sustained, or even transform themselves into a depressed market. The reason for this latter phenomenon is that the once the trigger is reached the price cannot escape from a level lower than 10% from *F* due to the endogenous inhibition of switches between strategies, so that the maximum leverage remains persistently set at 4. A policy targeted to minimize risk at the individual level leads to a systemic disruption.

4. Conclusions

A full understanding of the role of leverage in financial markets can be achieved only by taking into account its impact on the endogenous evolution of trading strategies among heterogeneous investors. By using a basic agent-based asset pricing model with fundamentalist and chartist traders, the paper shows that leverage plays a double role: it contributes to generate extreme market movements – whose typical signature is fat tails and clustered volatility of returns – *and* facilitates price discovery, by allowing traders to fully exploit market signals and switch between trading strategies as their fitness vary. How regulators should stand in front of such a complex interaction is a matter left for further research.

References

Adrian, T. and Shin, H. (2010), Liquidity and leverage, *Journal of Financial Intermediation*, **19**:418-437.

Aymanns, C., Caccioli, F., Farmer, D. and Tan, W. (2016), Taming the Basel leverage cycle, *Journal of Financial Stability*, **27**:263-277.

Aymanns, C. and Farmer, D. (2015), The dynamics of the leverage cycles, *Journal of Economic Dynamics and Control*, **50**:155-179.

Brock, W. and Hommes, C. (1997), A rational route to randomness, *Econometrica*, **65**:1059-1095.

Brock, W. and Hommes, C. (1998), Heterogeneous beliefs and routes to chaos in a simple asset pricing model, *Journal of Economic Dynamics and Control*, **22**:1235-1274.

Chen, H. (2016), A Tobin tax only on sellers, *Finance Research Letters*, **19**:83-89.

Chiarella, C., Dieci, R. and He, X.-Z. (2009), Heterogeneity, market mechanisms and asset price dynamics, in Hens, T. and Schenk-Hoppé, K. (eds), *Handbook of Financial Markets*. North-Holland.

Cont, R. (2007), Volatility clustering in financial markets: empirical facts and agent-based models, in Teyssière, G. and Kirman, A. (eds), *Long memory in Economics*. Springer.

Cont, R. (2010), Stylized properties of asset returns, in Cont, R. (ed.), *Encyclopedia of Quantitative Finance*. Wiley.

Di Matteo, T., Aste, T. and Dacorogna, M. (2005), Long-term memories of developed and emerging markets: Using the scaling analysis to characterize their stage of development, *Journal of Banking and Finance*, **29**:827-851.

Ding, Z., Granger, C. and Engle, R. (1993), A long memory property of stock market returns and a new model, *Journal of Empirical Finance*, **1**:83-106.

Fischer, T. and Riedler, J. (2014), Prices, debt and market structure in an agent-based model of the financial market, *Journal of Economic Dynamics and Control*, **48**:95-120.

Franke, R. and Westerhoff, F. (2012), Structural stochastic volatility in asset pricing dynamics: estimation and model contest, *Journal of Economic Dynamics and Control*, **36**:1193-1211.

Geanokoplos, J. (2010), The leverage cycle, in Acemoglu, K., Rogoff, K. and Woodford, M. (eds), *NBER Macroeconomics Annual 2009*. University of Chicago Press.

Hommes, C. (2006), Heterogeneous agent models in economics and finance, in Tesfatsion, L. and Judd, K. (eds), *Handbook of Computational Economics*, *vol.* 2. North-Holland.

Krishnamurthy, A. (2010), Amplification mechanisms in liquidity crises, *American Economic Journal: Macroeconomics*, **2**:1-30.

Iori, G. and Porter, J. (2014), Agent-based modeling for financialmarkets, in Chen, S.-H., Kaboudan, M. and Du, Y.-R. (eds), *The Oxford Handbook of Computational Economics and Finance*. Oxford University Press.

LeBaron, B. (2006a), Agent-based computational finance, in Tesfatsion, L. and Judd, K. (eds), *Handbook of Computational Economics*, *vol.* 2. North-Holland.

LeBaron, B. (2006b), Agent-based financial markets: matching stylized facts with style, in Colander, D. (ed.), *Post Walrasian Macroeconomics*. Cambridge University Press.

Li, B., Shang, W., Li, H., Huo, L. and Xu, S. (2014), Disposition effect in an agent-based financial market model, *Procedia Computer Science*, **31**:680-690.

Lux, T. (2009), Stochastic behavioral asset-pricing models and the stylized facts, in Hens, T. and Schenk-Hoppé, K. (eds), *Handbook of Financial Markets*. North-Holland.

Manski, C. and McFadden, D. (1981), *Structural Analysis of Discrete Data with Econometric Applications*. MIT Press.

Shimokawa, T., Suzuki, K. and Misawa, T. (2007), An agent-based approach to financial stylized facts, *Physica A*, **379**:207-225.

Shin, H. (2010), *Risk and Liquidity*. Oxford University Press.

Thurner, S., Farmer, D. and Geanokoplos, J. (2012), Leverage causes fat tails and clustered volatility, *Quantitative Finance*, **12**:695-707.

Westerhoff, F. and Dieci, R. (2006), The effectiveness of Keynes-Tobin transaction taxes when heterogeneous agents can trade in different markets: a behavioral finance approach, *Journal of Economic Dynamics and Control*, **30**:293-322.