Optimisation of off-road motorcycle suspensions

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Abstract

Off-road motorcycle suspensions are subjected to severe solicitations like large jumps and bumps. Its optimisation requires assessing comfort and handling for which several indices can be found in the literature. However these indices were developed for other vehicles and it is not clear how they behave assessing large obstacles. To investigate this, four comfort indices and four handling indices are used to assess the performance of a motorcycle modelled with two degree of freedom under continuos and impulsive off-road excitations. It is found that the best suited indices are the Vibration Dose Value for comfort and the eighth order mean value of the contact force for handling. Additionally, these indices are used to optimise the damping on a five degree of freedom model of a motorcycle, with which is found that on each road scenario, the optimal damping is in between the damping for optimal comfort and optimal handling, and the final selection depend on the relative preference between both objectives.

1 Introduction

Off road motorcycles are subjected to severe solicitations, like large jumps and bumps. Their suspension system have a longer stroke and different configuration of spring, damper and geometry compared to other motorcycles to reduce impacts on the rider and keep traction. The design and optimisation of its parameters is done mainly by experimental tests based on the evaluation of the rider. Numerical optimisations can reduce the number of experimental tests, however an accurate definition or selection of performance indices is needed since they determine the sense in which the system is optimal

In general, the assessment of the performance of vehicles considers comfort and handling characteristics. Comfort refers to the isolation of the rider from road inputs and handling refers to the easiness to perform manoeuvres, like turning, accelerating, and braking. Comfort is commonly assessed by the accelerations on the rider and handling by the contact force or by performance on specific manoeuvres. For both, several indices can be found on the literature. However these indices were developed for other vehicles and it is not clear how they behave on the special conditions of off-road. Specifically, for off-road motorcycles the indices need to describe, at least, the high impacts in landings to protect the driver, and the large variation in the contact force with the ground since is a requisite to perform any manoeuvre.

In this paper we aim to determine an objective measurement of the performance of off-road motorcycles to carry out suspension optimisations with special focus on damping. To achieve this goal, in section 2 we perform a literature review from where we select eight relevant indices; in section 3 we describe the motorcycle and road models used to perform a parametric analysis of the selected indices in off-road conditions; in section 4 we evaluate the indices and select the best two indices which are used on section 5 to present an example of suspension optimisation on a motorcycle model of five degree of freedom; finally in section 6, we highlight the main conclusions.

2 Performance assessment

Comfort and handling are subjective perceptions of the rider that describe the performance of vehicles. The following objective measurements are used to assess performance.

Comfort is measured by the accelerations on the rider when driving over a certain obstacle or road. Specifically, the maximum value [1, 2, 3], the root-mean-square [4, 5, 6, 7] and the standard deviation [8], are considered as indices. Additionally, standards ISO 2631-1, used in [9, 10, 11, 12], BS 6841, used in [13, 14] and VDI 2057, used in [15], provide methodologies to assess comfort from the accelerations, and [16] states that, in off road vehicles, all of these provide a good correlation between the objective measurement and the subjective perception of the riders.

In a different approach, [17] defines an index based on the displacement transmissibility for stationary conditions and on the peak suspension force for transient conditions.

Conversely, handling is measured by diverse approaches which can be grouped in: 1) analysis of the contact force in a rough road or event, and 2) behaviour in specific manoeuvres. The first group focus on the contact force of the tyre since this force is a requisite to perform any manoeuvre. Specifically, the history of the contact force is assessed by the root-mean-square [3, 6, 7], the standard deviation [8, 15] and the time spent without contact or "flying time" [1]. On the second group, handling is assessed directly on specific manoeuvres, which include single lane change [18, 19], slalom [20], lateral input and U-turns [19], and braking [21].

The selection of indices for off-road motorcycle optimisation is not straight forward given the large variety of indices used to describe comfort and handling. Except for Vliet's confort indices, none of them is designed for off-road conditions, hence it is not clear how they behave with large amplitude obstacles. In order to select the appropriate indices, a comparison between the most relevant is performed.

2.1 Comfort assessment methodology

Continuos excitation The following four comfort indices are selected for the analysis. Considering that every standard is equally representative of comfort [16], ISO2631 is chosen because of its availability, from where two indices are selected. These are compared to the indices used by Vliet [17] to optimise off-road motorcycles, given that it was used for the same purpose as need in this study.

ISO 2631-1, defines methods for the measurement and evaluation of periodic, random and transient human whole-body vibrations. The measurements are based on the accelerations on the interface between the source of vibration and the human body in seated, standing and recumbent positions. The acceleration signal is weighted by frequencies to consider that human sensitivity to acceleration depends on the frequency of excitation.

The evaluation method depends on the magnitude of peaks and transients that the signal contains. For smooth signals, the basic method is suggested which calculates the root-mean-square of the frequency-weighted acceleration, and is defined as comfort index 1, equation (1), while for more severe and fluctuating signals, the Vibration Dose Value (VDV) is suggested. Its division by $T^{1/4}$ is defined as comfort index 2, equation (2), to compare it to C_1 in the same scale.

$$C_1 \equiv \Psi_{a_w} = \left[\frac{1}{T} \int_0^T a_w^2(t) dt \right]^{1/2} \tag{1}$$

$$C_2 \equiv \frac{VDV}{T^{1/4}} = \frac{\left[\int_0^T a_w^4(t) dt \right]^{1/4}}{T^{1/4}} \tag{2}$$

where $a_w(t)$ is the frequency-weighted acceleration, and T is the duration of the measurement.

The standard provides two criteria to detect severe accelerations: 1) if the crest factor, which is defined as the ratio between the maximum instantaneous peak value of the frequency-weighted acceleration and its root-mean-square value, is larger than nine, equation (3), and 2) if the ratio between the two indices is larger than 1.75, equation (4).

$$c_f \equiv \frac{\max(|a_w(t)|)}{\Psi_{a_w}} = \frac{\max(|a_w(t)|)}{C_1} > 9 \tag{3}$$

$$r_1 \equiv \frac{VDV}{T^{1/4}\Psi_{a_w}} = \frac{C_2}{C_1} > 1.75 \tag{4}$$

Before ISO 2631-1, Vliet [17] evaluated the performance of an off road motorcycle in stationary and transient situations. The stationary condition was evaluated by the transmissibility TR_d between the displacement of chassis with respect to the displacement of the ground. He evaluated the transmissibility at the natural frequency, and at 8 times this frequency, according to equation (5), where D_1 is a weighting factor. It was argued that this definition is to consider that the changes in transmissibility with damping are opposite at resonance and high frequencies. When the system is linear, the displacement, and acceleration transmissibility (TR_a) coincide, making this index a measurement of comfort.

$$C_{3,s} = Tr_{d,\omega_n} + D_1 Tr_{d,8\omega_n} \tag{5}$$

Impulsive and transient For impulsive excitation, two considerations are made. Firstly, each signal is considered only until the end of the event, since integrating over longer periods of time (before or after this event) will change the magnitudes of indices C_1 and C_2 because of the term T in equations (1) and (2).

Secondly, index C_3 is not representative of this condition since there is no base movement to define a transmissibility. Instead, Vliet [17] compares an optimal force F_{opt} transmitted by the suspension with respect to the peak force F_p. The optimal force was defined as the constant force, that working through the complete suspension stroke, would dissipate all the kinetic energy of the system at the moment of impact in the first compression, equation (6).

$$F_{opt} = \frac{mv_i^2}{2l_{max}} \tag{6}$$

This performance index also assess comfort given that the force acting in the sprung mass is proportional to the acceleration. To consider the minimisation of the index as comfort improvement, the inverse of the original index is defined as the third comfort index on the transient condition, equation (7).

$$C_{3,t} = \frac{F_p}{F_{opt}} \tag{7}$$

2.2 Handling assessment methodology

For handling, only the contact force is studied since its understanding is the basis for the analysis of manoeuvres. The root-mean-square value, Ψ_x and standard deviation σ_x are the most commonly used indices in literature, and are related by equation (8), [22], where μ_x is the mean value.

$$\Psi_x^2 = \mu_x^2 + \sigma_x^2 \tag{8}$$

Since both show similar information, only the root-mean-square is considered, and is defined as a handling index, equation (9), where N is the number of elements of the signal of the contact force f_2.

Additionally, to study the influence of impacts the fourth and eighth order averages are considered since they would amplify the peak values which are undesirable. Each of them is defined as a handling index, equations (10) and (11).

Finally, the time spent flying is added to the comparison, given that in off-road riding the loss of contact is recurrent and no discussion is found in literature relating some of the statistical evaluation of the force with the loss of contact. It is considered with respect to the total time simulated, T, according to equation (12).

$$H_1 = \Psi_{f_2} = \left(\frac{\sum {f_2}^2}{N}\right)^{0.5} \tag{9}$$

$$H_2 = \left(\frac{\sum {f_2}^4}{N}\right)^{0.25} \tag{10}$$

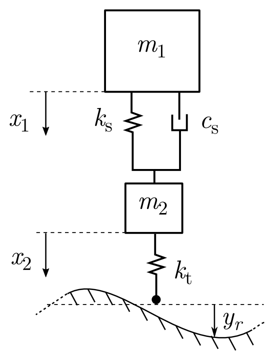
$$H_3 = \left(\frac{\sum {f_2}^8}{N}\right)^{0.125} \tag{11}$$

$$H_4 = \frac{\sum t_{f_2=0}}{T} \tag{12}$$

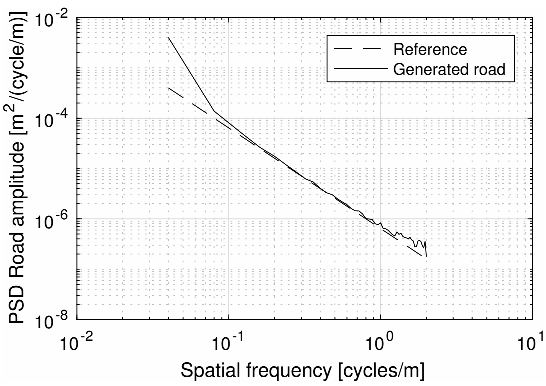
For transient evaluation, the signal of force history is truncated at the end of the event and the same four handling indices are calculated and compared.

3 Methodology

In order to perform a parametric analysis of these indices a simplified model is used as a compromise between accuracy and simplicity. The methodology consist in analysing the motorcycle subjected to continuos and impulsive excitations with the indices described above on different cases. Specifically, the continuos excitations considered are driving on two road at two velocities to compare slow and fast transit. On the other hand, the impulsive excitation considered are vertical landings from two heights. The parameter of interest is the damping factor of the bounce mode, which is varied from 0.1 to 1 for each case.



(a) Two degree of freedom model of the motorcycle



(b) Road spectrum

Figure 1: Representative time domain results

3.1 Mathematical Model

The motorcycle model considered has two degree of freedom Fig (1a). In-plane motion is assumed, detachment from the ground and suspension and tyre stroke limits are considered, and ground deformation and relative motion between the rider and the motorcycle are neglected. The equations of motion of this system are well known to be equations (13).

$$\begin{aligned} m_1\ddot{x}_1 &= -f_s(x_1 - x_2, \dot{x}_1 - \dot{x}_2) + m_1 g \\ m_2\ddot{x}_2 &= -f_t(x_2 - y_r) + f_s(x_1 - x_2, \dot{x}_1 - \dot{x}_2) + m_2 g \end{aligned} \tag{13}$$

The road considered for the analysis has a random, ergodic and stationary profile. It is modelled according to ISO 8608 which describes the random road profiles by the power spectral density (PSD) of the vertical displacement of the road. It describes the amplitude of the road with respect to the spatial frequency by a line in the log log scale, equation (14).

$$G_d(n) = G_d(n_0)\left(\frac{n}{n_0}\right)^{-k_r} \tag{14}$$

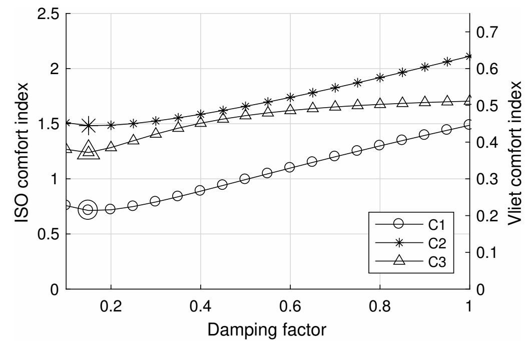
where n is the spatial frequency, G_d is the vertical displacement PSD, n_0 is a reference spatial frequency (= 0.1 cycles/m) and k_r is the exponent of the fitted PSD also called waviness. The parameter $G_d(n_0)$ describes the severity of the irregularities of the road and defines road classes in the standard.

For the simulation, one road profile is generated based on the procedure described by [23]. Figure (1b) show the reference power spectral density used to generate the profile, which corresponds to a road class B, and the power spectral density of the generated road. The second road profile is generated by amplifying eight times the first road, corresponding to a road class E.

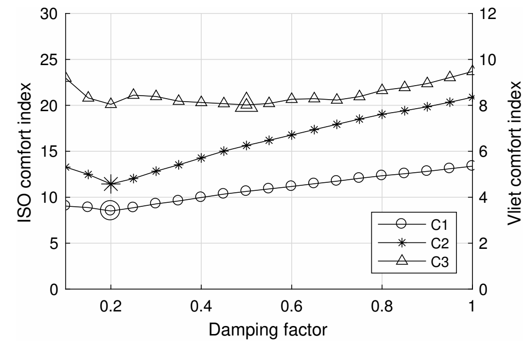
4 Results

Continuos excitation Figure (2a) and (2b) show the variation of the comfort indices 1, 2 and 3 with the damping factor for the cases indicated. It can be seen that C_1 and C_2 have the same variation with damping and both have minimum on the same abscissa. The crest factor and the ratio between indices does not reach their corresponding thresholds on any of the cases simulated, indicating that the vibrations induced by transiting on a rough road are not severe enough to require C_2. Differently, C_3 is minimised with the same damping as the other indices if the excitation is low, but with a higher damping if the excitation is intense and the variation with damping is not as smooth smooth as with the other indices.

Figure (3a) and (3b) show the variation of the handling indices for the cases indicated. It can be seen that on the smooth road, the three statistical indices are minimised with a damping factor of 0.35 while the fraction of time flying is null for every case. Similarly, in the harsh road, all the indices show a minimum at damping factor 0.4 including the time spent flying. It is worth noting that the flying time and the root-mean-square of the contact force show a similar behaviour.

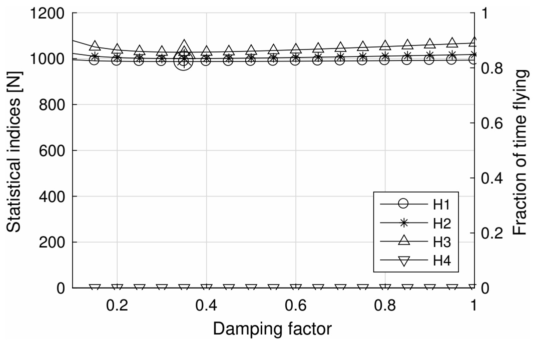


(a) Comfort indices - Driving on class B road at 10 m/s

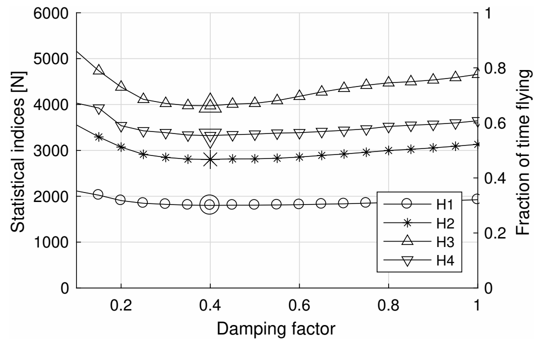


(b) Comfort indices - Driving on class E road at 40 m/s

Figure 2: Comfort indices with continuos excitation. C1 is the rms value of the frequency-weighted acceleration, C2 is the modified VDV, C3 is Vliet's stationary index. Larger markers show the minimum.



(a) Handling indices - Driving on class B road at 10 m/s



(b) Handling indices - Driving on class E road at 40 m/s

Figure 3: Handling indices with continuos excitation. H1 is the rms value of the contact force, H2 and H3 are the 4th and 8th order mean values of the contact force respectively, and H4 is the fraction of time spent flying. Larger markers show the minimum.

Impulsive excitation Figure (4a) and (4b) shows that the relations between comfort indices are different than with continuos excitation. C_1 and C_2 have minimums on different abscissas, mainly because C_1 underestimate the severity of the shocks that occur when the suspension hits the end of the compression stroke when the damping is low. This situation is revealed by the crest factor which is higher than 9 at low damping and the ratio r_1 which is larger than 1.75 for every damping, both indicating that C_2 should be used. C_3 also shows minimums on different abscissas compared to the other indices. However it is interesting to note that using a different approach, this index also reveals that at high landing velocity the impacts are more severe with low damping.

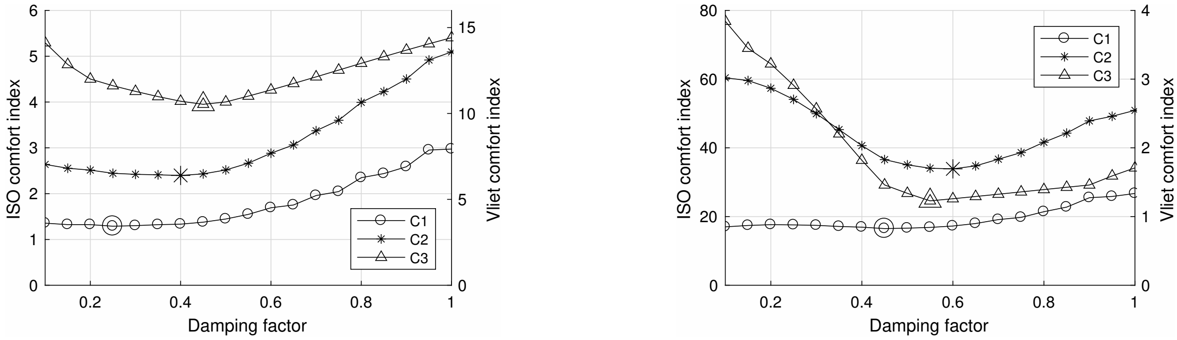
Figure (5a) and (5b) reveal that handling indices also have different behaviour with impulsive excitation. On low impact, the second and fourth order averages show a minimum with 0.1 damping, while the eighth order, has a minimum at 0.4. Conversely on high impact, second order is minimised with 0.1; fourth order, with 0.55; and eighth order, with 0.6. On the other hand, the fraction of time spent flying H_4, is null for the low impacts, and increases for the lower damping as the impact increase.

4.1 Discussion

From the continuos excitation situations, $C_{3,s}$ is discarded for further use because with intense excitation the variation is irregular between damping factors 0.2 and 0.7. This implies that with small changes in the input the minimum could have a significant change of abscissa, which makes it inappropriate for optimisation purpose. On the other hand, the variation of C_1 and C_2 is smooth and both show minimums with the same damping, making them equally useful for optimisations.

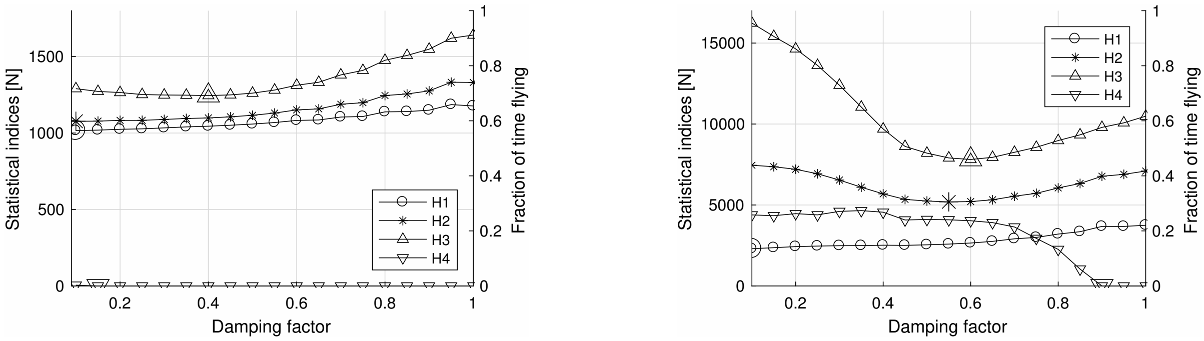
H_4 is not useful for optimisation if the road is smooth, given that it can not differentiate performance if there is no loss of contact. Conversely, H_1, H_2, and H_3 show the same minimum, consequently are equally useful.

From the impulsive excitation situations, C_2 is required for the analysis according to both criteria, consequently C_1 is discarded. $C_{3,t}$ can be used as well, although it indicates different minimums than C_2.



(a) Comfort indices - Landing at 1 m/s (from 51 mm) (b) Comfort indices - Landing at 8 m/s (from 3200 mm)

Figure 4: Comfort indices with impulsive excitation. C1 is the rms value of the frequency-weighted acceleration, C2 is the modified VDV, C3 is Vliet's transient index. Larger markers shows the minimum.



(a) Handling indices - Landing at 1 m/s (from 51 mm) (b) Handling indices - Landing at 8 m/s (from 3200 mm)

Figure 5: Handling indices with impulsive excitation.H1 is the rms value of the contact force, H2 and H3 are the 4th and 8th order mean values of the contact force respectively, and H4 is the fraction of time spent flying. Larger markers shows the minimum.

H_1 is discarded because it achieve minimums with the lowest damping in both cases which is not representative of reality since it is known that a certain damping is required to have a good handling. For the same reason, H_2 is not useful since it is minimised with the lowest damping on the low impact. Conversely, H_3 show minimum on 0.4 and 0.6 which is more likely to represent reality.

Considering the above discussion, C_2 and H_3 are selected as measurements of performance for off-road because they are the only indices representative of continuos and impulsive excitations.

5 Optimisation of a five-degree-of-freedom motorcycle

The selected indices, C_2 and H_3, are used to optimise the front and rear damping coefficients of a motorcycle modelled by five degrees of freedom to provide an example of use. The model of the motorcycle consist on the chassis, swing-arm, and both wheels. The degrees of freedom are longitudinal x_s and vertical z_s translations, pitch μ of the centre of gravity of the chassis, and the relative motions of the front z_f and rear α_r wheels with respect to the chassis, as shown in Figure (6). The equations of motion are derived using MBSymba [24].

The contact point of the wheels are assumed fixed with respect to the motorcycle, which means that they do not move forward or backward because of road irregularities. The road profile and landing velocities are the same as used previously. On each situation, damping factor of the bounce mode between 0.1 and 1 are considered. This value is used to scale down the front $c_{f,c}$ and rear $c_{f,r}$ damping which critically damps the bounce mode to obtain the front c_f and rear c_r damping for each simulation, as exemplified for specific cases on Table 1. The comfort index is calculated considering the vertical acceleration of the centre of gravity of the chassis, while the handling index is calculated for each wheel.

Figures (7a) and (7b) show the variation of comfort and handling with damping under light and severe continuos excitation, respectively. The higher comfort is achieved with a damping factor of 0.2 and 0.25 of the bounce mode in light and severe excitation respectively. Conversely, in light excitation, the front contact force is optimised with a damping factor of 0.4, and the rear with 0.5, while under severe excitation, the front is optimised with 0.5 and the rear with 0.3.

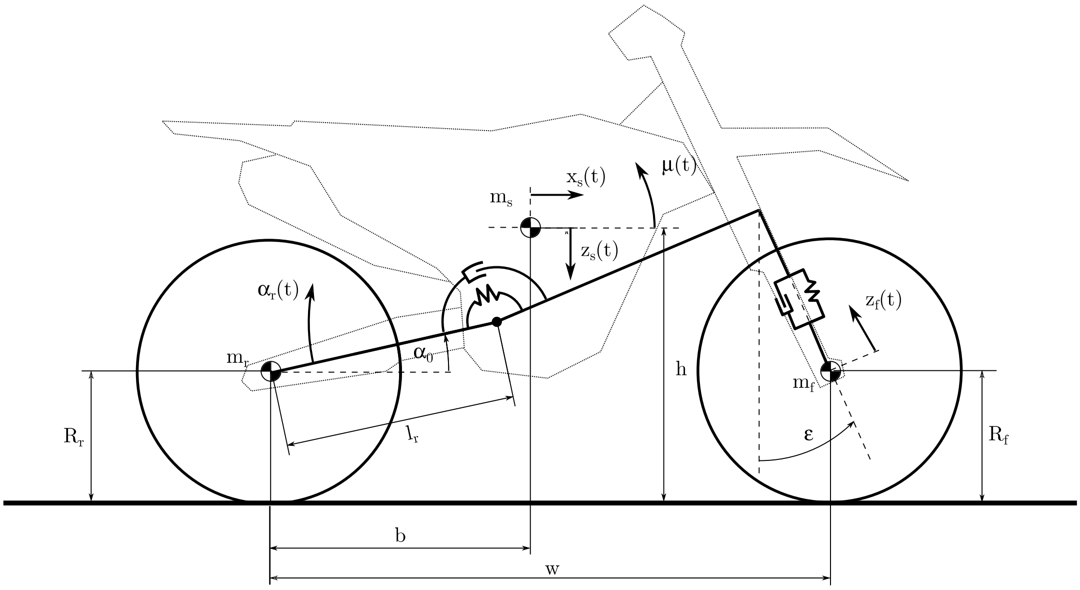
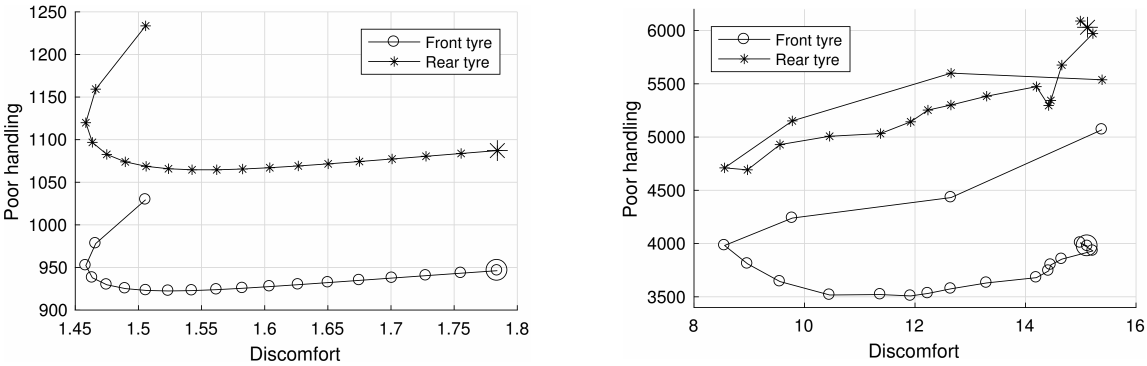


Figure 6: Five degree of freedom model of the motorcycle. x_s and z_s are the longitudinal and vertical translation of the center of gravity of the chassis, μ its pitch rotation, and z_f and α_r are the relative displacements of the front and rear suspensions, respectively.

Table 1: Selected front c_f and rear c_r damping coefficients for the corresponding damping factor of the bounce mode ζ used in the simulations. Dampings for $\zeta = 1$ correspond to front $c_{f,c}$ and rear $c_{f,r}$ critical damping coefficients of the bounce mode.

ζ	c_f (Ns/m)	c_r (Ns/m)
0.1	172	193
0.4	689	771
0.7	1206	1350
1.0	1723	1928



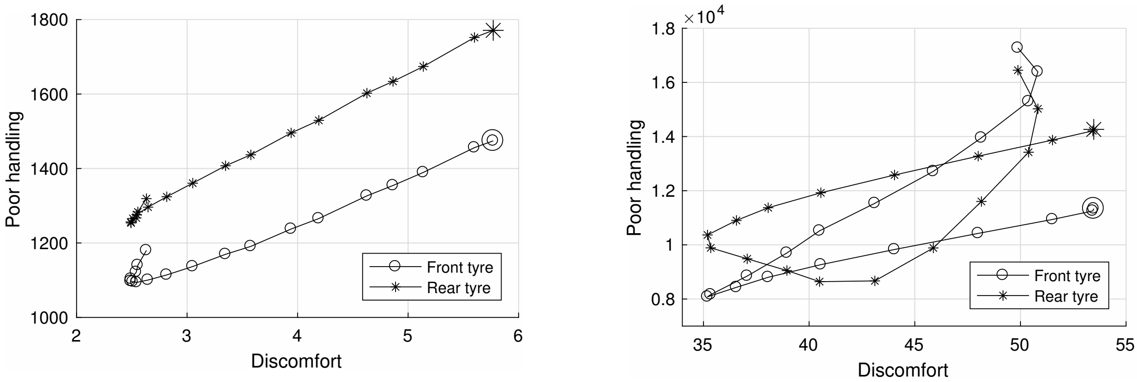
(a) Driving on class B road at 10 m/s

(b) Driving on class E road at 40 m/s

Figure 7: Handling vs comfort indices with continuos excitation. The damping factor starts on 1 on the larger marker decreasing with a step of 0.05.

Figures (8a) and (8b) show the variation of comfort and handling with damping under impulsive excitations. Comfort is optimised with a damping factor of 0.3 and 0.6 on light and severe impulsive excitation respectively. On the other hand, in light excitation, the front contact force is optimised with 0.40 and rear with 0.25 while under severe excitation, the front with 0.6 and the rear with 0.4.

The values of damping factors given above define the limits in between which there is a trade off between comfort and handling. This mean that in this range of damping, a change in damping will improve one objective and worsen the other. Additionally, moving way from this limits, this means, using very low or very



(a) Landing at 1 m/s (from 51 mm)

(b) Landing at 8 m/s (from 3200 mm)

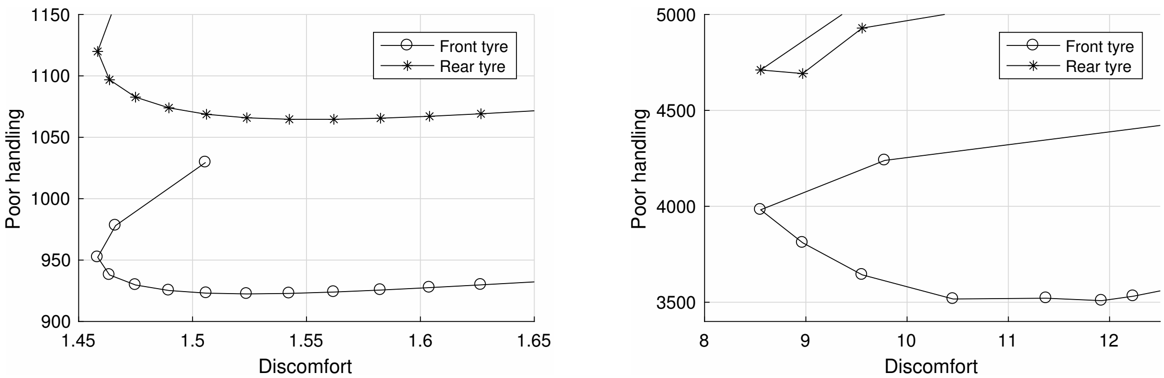
Figure 8: Handling vs comfort indices with transient excitation. The damping factor starts on 1 on the larger marker decreasing with a step of 0.05.

high damping, will worsen both objectives. This can be observed graphically on handling vs comfort plots. If the slope is negative there is a trade-off between objectives, while if it is positive, both objectives are worsened.

Figures (9) and (10) show that the the trade-off region, and relation by which the objectives are traded-off, are different on each situation. For example in Figure (9a) the region is from damping factor 0.2 to 0.5, where most of comfort is gained close to 0.5 and most handling is gained close to 0.2. Differently, in Figure (9b) the optimal region for the rear is narrow (0.25 - 0.3), while for the front is wider (0.25 - 0.5). Most of comfort is gained close to 0.5 while close to 0.25 only front handling is improved. In Figure (10a), the optimal region is between 0.25 and 0.4. Inside this region the objectives barely improve, while outside, any change in damping greatly worsen both objectives. Lastly, Figure(10b) show that the optimal region for the front is a single point (0.6) while for the rear is a region between 0.4 and 0.6 with an almost constant trade-off relation.

Given that there is no damping that optimises all the objectives in all situations, the final choice will depend on the situation and preferred objective. For example, if the motorcycle will be used mainly on good roads with ocasional small jumps, and comfort is preferred, a damping factor between 0.2 and 0.3 of the bounce mode would be the choice. Differently, if the application is racing on severe roads, where handling is preferred over comfort, the choice would be a damping factor between 0.35 and 0.55.

Finally, the above analysis shows that the conflict between comfort and handling reported by [14] for four-wheeled off-road vehicle also exist in off-road motorcycles, which also agrees with experimental knowledge of motorcycles [25].



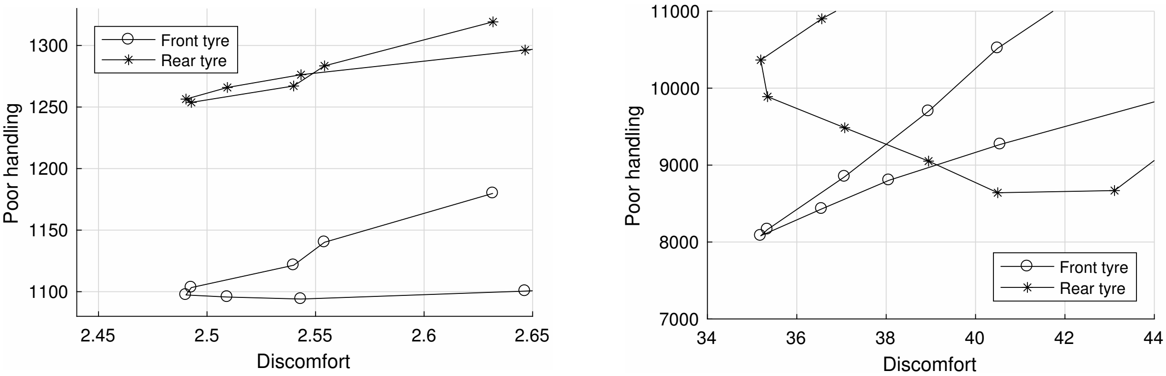
(a) Driving on class B road at 10 m/s

(b) Driving on class E road at 40 m/s

Figure 9: Handling vs comfort indices with continuos excitation. The damping factor starts on 1 on the larger marker decreasing with a step of 0.05.

6 Conclusion

From the literature review four comfort indices where selected, namely root-mean-square of frequency weighted acceleration, Vibration Dose Value and Vliets' indices; and four handling indices that evaluates the contact



(a) Landing at 1 m/s (from 51 mm)

(b) Landing at 8 m/s (from 3200 mm)

Figure 10: Handling vs comfort indices with transient excitation. The damping factor starts on 1 on the larger marker decreasing with a step of 0.05.

force, namely root-mean-square, fourth and eighth order averages and flying time. The comparison in continuos and transient excitations, indicates that the best procedure to assess comfort is by the Vibration Dose Value, and handling by the eighth order average of the contact forces.

Using these indices to measure the performance of the motorcycle under an specific excitation, it is found that comfort and handling are maximised with different damping. These two damping are the limits of an interval in which one objective improves while the other worsens if the damping is changed, and outside which, both objectives are worsened. Consequently, the optimal damping for each situation is inside this interval. Considering this interval and choosing the relative importance between both objectives, it is posible to select the optimal damping for each situation.

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