Operations Research Perspectives 8 (2021) 100179

Contents lists available at ScienceDirect

Operations Research Perspectives

journal homepage: www.elsevier.com/locate/orp

Combinatorial abilities and heuristic behavior in online search environments

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ARTICLE INFO

Keywords:
Combinatorial sequence
Search engine
Ranking credibility
Online evaluations
Uncertainty

ABSTRACT

Consider the problem faced by a decision maker (DM) who must select the order in which to evaluate the unknown alternatives displayed by an online search engine. DMs do not know the distribution of the realizations that result from clicking on an alternative and must therefore account for the differences that may exist between the ranking provided by the engine and their subjective potential evaluations. The current paper formalizes the information retrieval incentives of DMs through a combinatorial function that incorporates the position of the results displayed by the search engine and the order in which they are evaluated. This function defines a benchmark framework measuring the ability of search engines to identify the preferences of DMs and the capacity of the latter to assimilate and evaluate the information provided by the engines. We compare the cumulative frequencies derived from the implementation of evaluation models of varying complexity with the average traffic shares and click through rates of the alternatives ranked within the first page of Google results. A seemingly paradoxical result is obtained when simulating the information acquisition behavior of DMs after performing an online search, namely, artificial agents require more complex combinatorial abilities than actual DMs to approximate better the heuristic choices of the latter in online evaluation environments.

1. Introduction

The rankings delivered by online search engines condition to a large extent the preferences and choices of decision makers (DMs) due to the trust placed by the users in the corresponding orders, despite their complete lack of knowledge regarding the procedure implemented to generate the rankings [7,11,13]. Eye-tracking technology has been used to illustrate the fact that consumers scan the results in the order provided by the engine, fixating on the highest-ranked ones [10]. Epstein and Robertson [5] review the empirical literature describing the substantial effect that online search rankings have on the preferences and decisions of DMs.

The limited capacity of DMs to retrieve and assimilate information has been consistently illustrated across different scientific domains [14]. DMs are incapable of computing all 3,628,800 permutations resulting from the first ten results ranked and select a subset of them on which to base their evaluation processes. It may therefore seem plausible to assume that DMs simply follow the order of the ranking when evaluating the alternatives. Given an average of two clicks per search [1], this behavior should lead to identical percentages on the first two alternatives ranked by the search engine.

That is, if the first two alternatives deliver the subjective utility requested, the retrieval process should end with the DM having clicked just on the first and second alternatives. Note that if all DMs do so based on the assumption that, given their ranking positions, the first two alternatives deliver the highest utility, then we should observe all DMs clicking on the first two alternatives and ending their searches right after.

The behavior observed would therefore consist of each alternative receiving half of the total clicks and accounting for half of the average traffic share. The click through rates (CTRs) should be equal to 100% for each alternative, since they are both clicked by DMs on a per-search basis. However, this is not what is observed [3,4], but a high proportion of clicks concentrated on the top two results and decreasing percentages as we move towards the bottom of the page.

From an intuitive viewpoint, the only requirement defined on the evaluation process should be the fact that DMs lack information regarding the realizations of the alternatives but trust the order

This work was supported by the Open Access Publishing Fund of the Free University of Bozen-Bolzano.
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https://doi.org/10.1016/j.orp.2021.100179
Received 22 April 2020; Received in revised form 12 October 2020; Accepted 12 January 2021
Available online 14 January 2021

displayed by the search engine. This requirement follows from the uncertainty inherent to the evaluation of the alternatives and the limited capacity of DMs to acquire and process information. Given the fact that DMs do not know the distribution of the realizations that result from clicking on an alternative, a uniform density should be assigned to the potential set of evaluations. Moreover, the intervals defining the domains of the corresponding probability functions should reflect the order of the ranking provided by the engine.

A considerable causality problem arises when interpreting the data retrieved from search engines regarding the behavior of users. On one hand, it may be assumed that the ranking delivered by the search engine satisfies the preferences of DMs. This assumption implies that any divergence from the suggested alternatives represents frictions on the evaluation capacities of DMs, generally dealt with by psychologists and neuroscientists. On the other hand, it may be assumed that search engines try to elicit the preferences of DMs from their search keywords and the frictions observed highlight their inability to do so. A combination of any of these scenarios could also be considered, with a higher emphasis placed on any of them depending on a series of preselected factors.

Thus, even though DMs have consistently stated their trust in the rankings delivered by search engines [9] and psychologists have emphasized the volatility inherent to the preferences of consumers [8], the dominant causal effect cannot be identified, especially if we want to avoid observer bias-related phenomena. The current paper defines a benchmark framework to measure the frictions arising between the ranking provided by a search engine – and its ability to identify the preferences of users –, and the capacity of DMs to assimilate and evaluate information.

More precisely, we formalize the sequential evaluation patterns of the alternatives ranked by a search engine. We illustrate the capacity of pairwise comparisons to approximate the actual online search behavior of users and how increments in the complexity of the evaluation mechanism provide better approximations. This finding constitutes a somehow paradoxical result, since a better approximation of the frequencies observed is obtained when endowing DMs with a higher degree of combinatorial complexity. The complexity of the combinatorial problem increases considerably as we add alternatives to the permutations since we must consider both the order in which the alternatives are observed and the different domains of the evaluation intervals.

To the best of our knowledge, the sequential evaluation structure introduced in the current paper is completely novel, while being based on a basic set of assumptions consistent with rational information acquisition processes [12]. The remaining of the paper proceeds as follows. Section 2 introduces the main assumptions required to develop the sequential structure defined in Section 3 and enhanced in Section 4. Section 5 simulates several numerical evaluations and compares them with the results obtained by different empirical studies. Section 6 concludes and suggests potential extensions.

2. Basic assumptions

We formalize and analyze the optimal behavior that should be exhibited by a DM deciding in which order to observe the results displayed in the first page of an online search engine. The only assumption required to formulate the retrieval process is the fact that DMs trust the engine to provide them with a ranking that reflects their subjective preferences [6]. As a result, the subjective utility assigned to each alternative before browsing through the results should be determined by the order in which they are displayed, with the first one receiving the highest value and the others continuing in descending order.

As described in the introduction, DMs are constrained in their information acquisition and assimilation capacities and are therefore prone to define heuristic mechanisms that simplify their evaluation processes [15]. Fig. 1 illustrates a direct consequence from this fact, namely, the average number of terms per online search query in the United States both in 2017 and 2020. DMs have increased the complexity of their searches from mainly of one-word queries in 2017 to two-word queries in 2020. Despite this tendency, a three-word upper limit prevails across periods, with queries containing three words or less accounting for 80% of total searches in both years.

We will consider two evaluation scenarios, accounting for pairwise and triple comparisons across alternatives based on the ranking positions displayed by the engine. Pairwise comparisons between

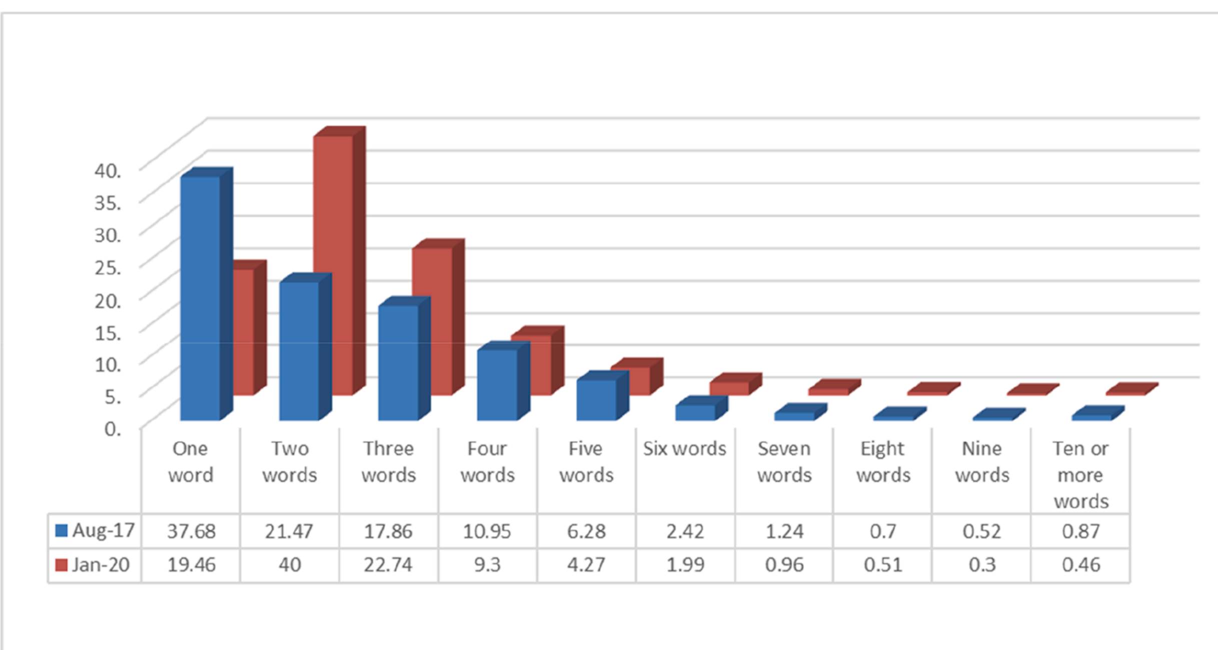


Fig. 1. Average number of search terms for online search queries in the United States: August 2017 vs. January 2020.
Data retrieved from Statista (https://www.statista.com/statistics/269740/number-of-search-terms-in-internet-research-in-the-us/). Original source: keyworddiscovery.com.

alternatives constitute a standard heuristic mechanism implemented in evaluation techniques such as the Analytical Hierarchy Process and several other decision-making models [16]. The increase in complexity derived from the incorporation of a third alternative to evaluate and compare constitutes the main focus of analysis of the current paper.

As a measure of complexity of the combinatorial process, we have considered the number of permutations that may be defined using the alternatives provided within the first page of results and their grouping in ordered categories. For instance, the pairwise evaluation scenario requires accounting for two sets of ordered alternatives. The number of permutation categories that must be defined increase to a total of six when considering the evaluation of three alternatives. The complexity of the process increases until a total of 3,628,800 permutations is reached when incorporating to the analysis the ten alternatives displayed by the engine. The inherent complexity of the corresponding categorization process is summarized in Table 1.

Let $p^i \in X$ denote the ranking position of alternative i, while $p^j, p^k \in X$ correspond to the positions of alternatives j and k, with $i \neq j \neq k$. Each alternative receives a numerical evaluation according to its ranking position as displayed by the engine. Empirical analyses generally focus on the first ten alternatives delivered by the engine [5], thus, a descending categorization of the alternatives is defined by assigning a value of 10 to the first and a value of 1 to the last.

It should be emphasized that the results described do not rely on these numerical values but on the relative ranking positions assigned by the engine.

In the following, we will use x^i to denote the value assigned to the ranking position p^i of alternative i. Furthermore, we will let the real interval $[0,\ y^{iM}]$ be the set of potential realizations that can be observed when the ith link is clicked by the DM and $y^i \in [0,\ y^{iM}]$ denote one of these potential realizations. The superscript M denotes the maximum value of the interval being considered.

As stated above, we will be working under the assumption that $x^i = 10 - (i-1)$, for $i = 1,\ldots,10$, so that the alternatives ranked in the first ten positions by the engine are assigned a decreasing value from 10 to 1. However, the value scale used to categorize the ranking order assigned by the engine can be any scale based on ten decreasing values.

To better reflect the ranking position of the alternatives, we assume that for i, j and k such that $i \neq j \neq k$ and $x^i > x^j > x^k$, we have $y^{iM} > y^{jM} > y^{kM}$. We further simplify notations and calculations by assuming that $y^{iM} = x^i$ for every alternative i.

Finally, Y will denote the set of all potential realizations, that is, the set of all y^i with $i = 1,\ldots,10$, where y^i is the potential realization that can be observed after clicking on the i-th link. Hence, $Y = \bigcup_i [0, y^{iM}] = [0, y^{1M}]$. Under the assumption that for each alternative i, $y^{iM} = x^i$, the set Y reduces to the interval $[0, 10]$.

Fig. 2 provides a graphical representation of the notations and assumptions introduced above, summarizing the basic framework of analysis.

In order to illustrate numerically the information retrieval and evaluation processes, it will be assumed that the DM is endowed with a utility function $u : Y \to R$, representing his subjective preferences on Y, and a probability function $f : Y \to [0,\ 1]$, describing his beliefs about the distribution of potential realizations when evaluating an alternative.

2.1. Uncertain evaluations

Given the ranking position of an alternative, $p^i \in X$, and its value, x^i, on a 10 to 1 scale, the DM defines the set of potential evaluations that may be retrieved within the interval $[0, y^{iM}]$. That is, the DM considers an interval of feasible realizations to be derived from clicking on a link and evaluating an option in detail. This assumption follows from the importance placed by cognitive scientists on the subjective perception of DMs when evaluating an alternative [2].

Given the uncertainty inherent to the actual characteristics of each alternative, with DMs being required to click on a ranking item to obtain additional information about a set of unknown potential realizations, we maximize information entropy by defining a uniform density over the potential realizations within $[0, y^{iM}]$. That is, $\forall i = 1,\ldots,10$, we define the following density function:

$$f_i(y^i) = \begin{cases} \dfrac{1}{x^i} & \text{if } y^i \in [0, y^{iM}] = [0, x^i] \\ 0 & \text{otherwise} \end{cases} \tag{1}$$

The main results obtained are independent of the density chosen and hold when considering any alternative probability function, such as a normal. Figs. 4 and 5, which illustrate the distribution of probability through intervals $[0, y^{iM}]$ and $[0,\ y^{jM}]$, provide additional intuition regarding this statement.

As stated earlier, we have assumed that $x^i = y^{iM}$, a notational choice that aims at simplifying the description of the model. For the same reason, the utility function on Y is assumed to be the identity function, that is: $\forall y \in Y,\ \ u(y) = y$.

Note that alternatives located higher in the ranking may provide the DM with a lower utility than those in lower positions, though such an event occurs with a decreasing probability as we move down the ranking. Fig. 3 presents three graphical examples within a two-observation setting, namely, when the DM clicks on two alternatives per search query.

3. Heuristic value functions

Let x^i, x^j and x^k be the values reflecting the ranking positions of alternatives i, j and k, with $i \neq j \neq k$. The value functions defined in terms of the potential realizations of the different alternatives are determined by the order in which the latter are observed as well as their corresponding positions in the ranking.

The heuristic scenario considers the realizations associated with the ranking position p^i relative to those associated with the ranking position p^j, while accounting for the corresponding uncertainty on the side of the DM. Two potential evaluation settings must be considered, a scenario whose complexity increases considerably when incorporating a third potential alternative to the analysis, since the resulting framework must account for a total of six potential evaluation settings.

3.1. The $x^i \leq x^j$ setting

The initial evaluation case corresponds to the $x^i \leq x^j$ setting, with the first alternative clicked being ranked in a lower position by the search engine. We define the following value function for the pairs (y^i, y^j) of potential realizations that can be observed by the DM.

$$V\left(x^i, x^j, y^{iM}, y^{jM}\right) = \int_0^{y^{iM}} \frac{1}{x^i} \left[\int_0^{y^i} \frac{1}{x^j} \left(y^i\right) dy^j + \int_{y^i}^{y^{jM}} \frac{1}{x^j} \left(y^j - c^j\right) dy^j \right] dy^i, \quad y^{iM} \leq y^{jM} \tag{2}$$

The value function V assigns an expected utility value to each pair $(y^i,$

Table 1
Evaluation scenarios and number of permutation categories.

Number of alternatives evaluated	1	2	3	4	5	6	7	8	9	10
Permutation categories	1	2	6	24	120	720	5040	40,320	362,880	3,628,800

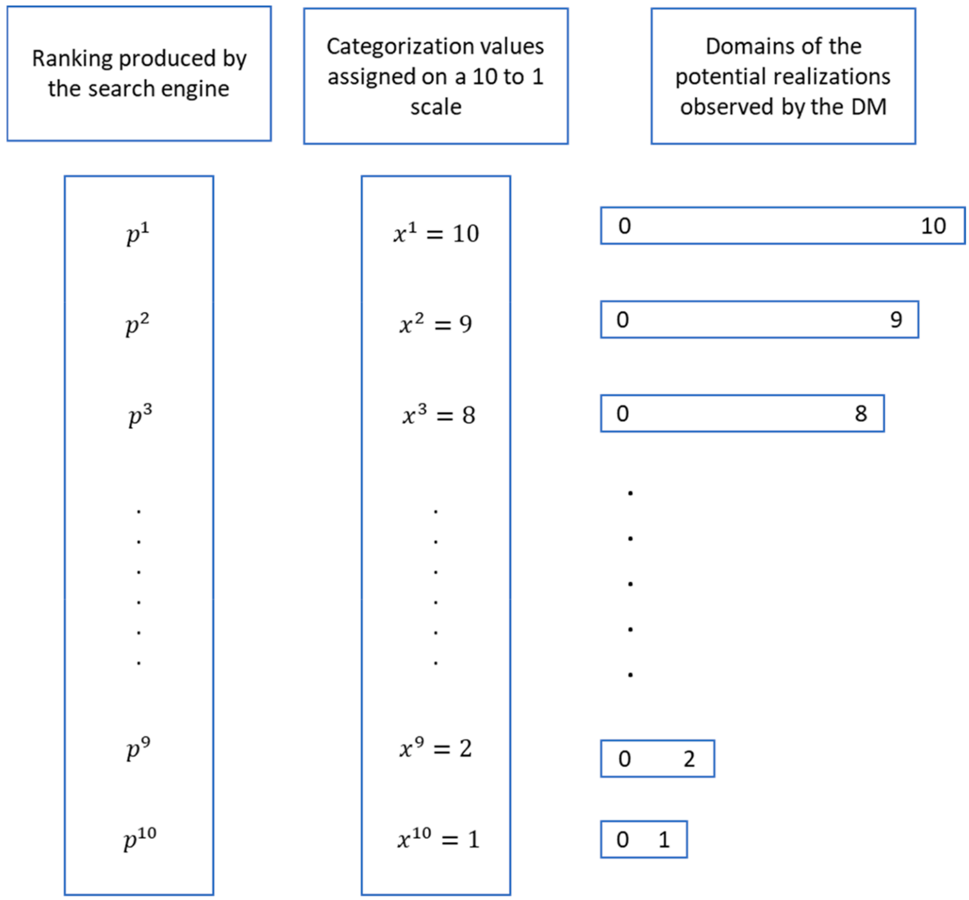


Fig. 2. Basic framework of analysis: notations and assumptions.

y^j). The quantities $\frac{1}{x^i}$ and $\frac{1}{x^j}$ represent the density functions associated with y^i and y^j, respectively.

Note that the rank differences between alternatives i and j imply that $y^{iM} \leq y^{jM}$. In this regard, the second alternative may improve upon the utility provided by the initial one, while incurring a penalty cost of c^j, or it may not, implying that the initial alternative provides the highest utility. That is, after observing y^i, the realization y^j may belong to either $[0, y^i]$ or $[y^i, y^{jM}]$. The integrals $\int_0^{y^i} \frac{1}{x^j}(y^i)dy^j$ and $\int_{y^i}^{y^{jM}} \frac{1}{x^j}(y^j - c^j)dy^j$ formalize the expected payoffs when $y^j \in [0, y^i]$ and $y^j \in [y^i, y^{jM}]$, respectively.

Fig. 4 shows the domains of the potential realizations y^i and y^j when considering the $x^i \leq x^j$ setting, illustrating how the value of y^j can vary with respect to an already observed value of y^i.

The penalty cost has been included to reflect the inefficiency in the search process of the DM, as well as to distinguish the expected payoffs received from evaluating two alternatives in a different order. That is, without a penalty cost, the order in which the alternatives composing each pair are evaluated does not have an effect on the expected utility.

3.2. *The $x^i > x^j$ setting*

Similarly to the above scenario, the $x^i > x^j$ setting requires modifying the value function so as to adapt it to the domains within which each alternative is defined

$$V(x^i, x^j, y^{iM}, y^{jM}) = \int_{y^{jM}}^{y^{iM}} \frac{1}{x^i}[y^i]dy^i + \int_0^{y^{jM}} \frac{1}{x^i}\left[\int_0^{y^i} \frac{1}{x^j}(y^i)dy^j + \int_{y^i}^{y^{jM}} \frac{1}{x^j}(y^j - c^j)dy^j\right]dy^i, \quad y^{iM} > y^{jM} \tag{3}$$

Note that the domain of y^i is now allowed to exceed that of y^j, a possibility accounted for by the first right hand side term. As was the case in the previous setting, y^{jM} defines the upper limit of the y^j domain, leading the second evaluation to either improve upon the initial one, at a cost of c^j, or to provide a lower utility. More precisely, the second right hand side term comprises the payoffs relative to the cases where $y^j \in [0, y^i]$ and $y^j \in [y^i, y^{jM}]$.

Fig. 5 illustrates the domains of the potential realizations y^i and y^j when considering the $x^i > x^j$ setting. In particular, it is shown how the value of y^j varies with respect to the observed value of y^i.

4. Incorporating a third alternative

The value function defined by DMs must incorporate two distinct combinatorial effects: those derived from the order in which alternatives are evaluated and those following from the order in which the alternatives are ranked. In all cases, the DMs must account for the uncertainty inherent to the potential realizations of each alternative and adapt to the relative limits of the different domains defined by the ranking position and the order of evaluation. Moreover, the increments in utility (relative

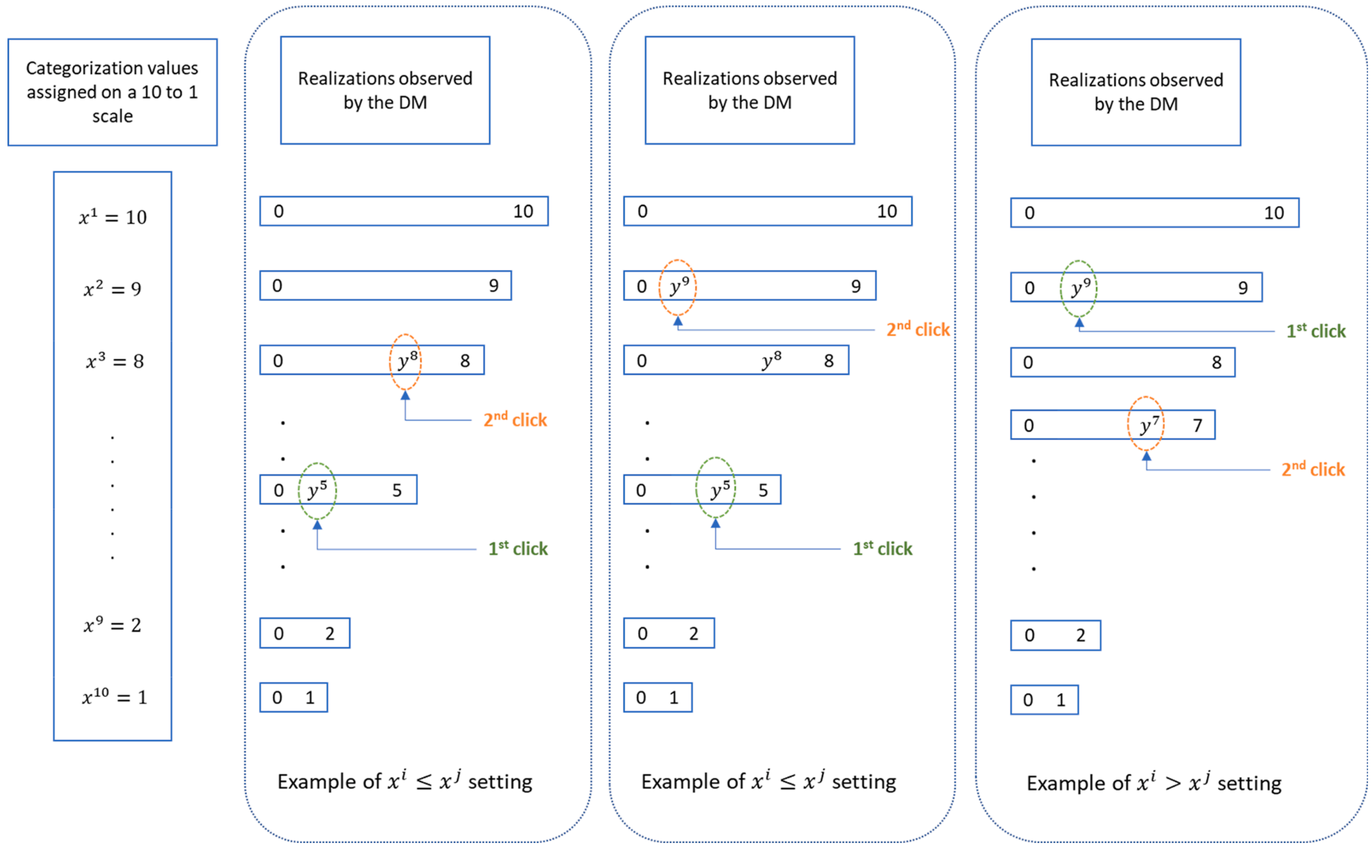


Fig. 3. Examples of observed realizations after two clicks.

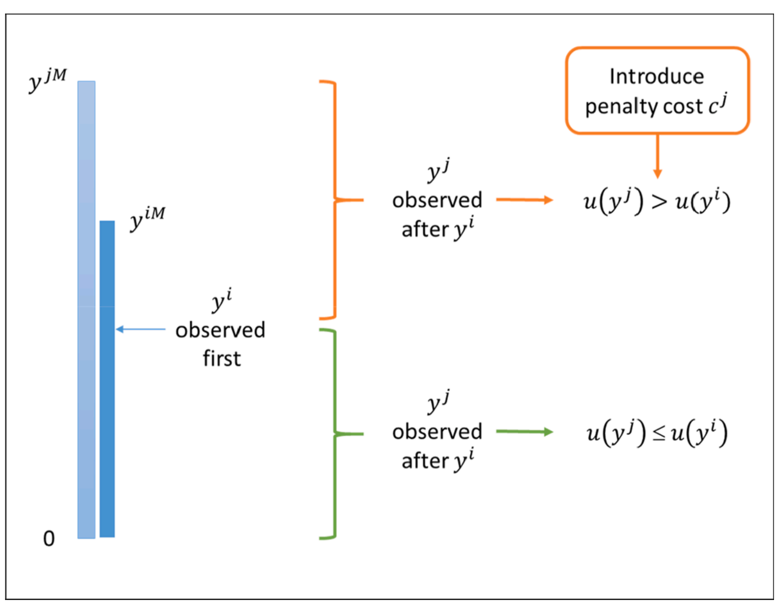


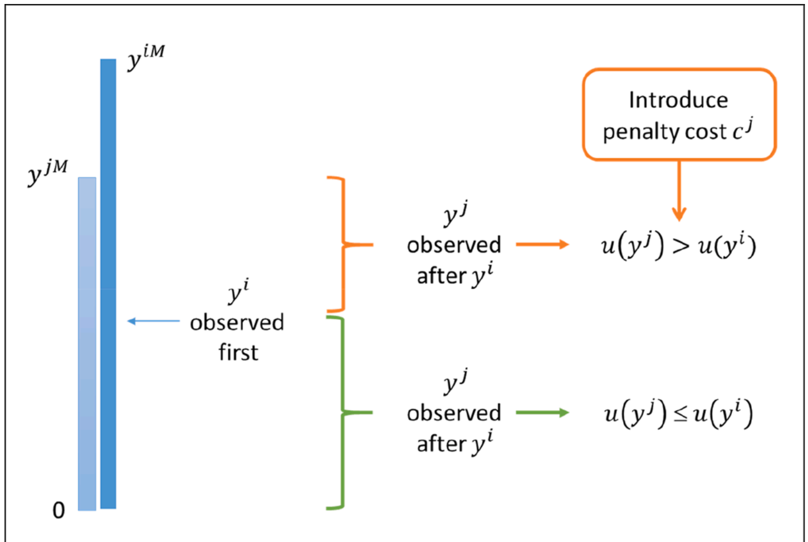
Fig. 4. Relative positions of the potential realizations y^i and y^j in the $x^i \leq x^j$ setting.

to the initial evaluation) following from the second and third alternative must include the respective incremental penalty costs of c^j and $2c^j$.

We describe below the whole set of permutations defining the function $V(x^i, x^j, x^k, y^{iM}, y^{jM}, y^{kM})$. A total of six different settings conditioned by the relative ranking position of the alternatives and the corresponding uncertain evaluation intervals will be defined.

For instance, consider the initial $x^i \leq x^j \leq x^k$ setting. In this case, part of the domain of y^j surpasses the upper limit defined by y^{iM} as x^j improves upon x^i. This very same possibility applies to the domain of y^k relative to both y^{iM} and y^{jM}. The whole set of possibilities is described within the right hand side of Eq. (4), where the terms have been adapted to account for the limits of the supports of the different density functions.

In particular, the first term corresponds to the case where the initial evaluation delivers the highest utility, while the second term describes the case where the third evaluation delivers the highest utility at a cost of $2c^j$. Note that both cases take place within the scenario where the



(a). Possible values of y^j when the observed value of y^i is below y^{jM}

(b). Possible values of y^j when the observed value of y^i is above y^{jM}

Fig. 5. Relative positions of the potential realizations y^i and y^j in the $x^i > x^j$ setting.

second evaluation underperforms relative to the first and the third one. The third and fourth terms are located within the interval where the second evaluation delivers a higher utility than the first one, with the third alternative delivering a lower and higher utility than the second, respectively.

All in all, the relative ranking position of the alternatives constitutes an important determinant of the expectations defining the value function, with the incorporation of a third evaluation requiring the following combinatorial possibilities – whose intuition and description follow the ones just provided – to be considered

(a) The $x^i \leq x^j \leq x^k$ setting

$$V\left(x^i, x^j, x^k, y^{iM}, y^{jM}, y^{kM}\right) = \int_0^{y^{iM}} \frac{1}{x^i} \left[\begin{array}{l} \displaystyle\int_0^{y^i} \frac{1}{x^j} \left[\int_0^{y^j} \frac{1}{x^k} \left(y^i\right) dy^k + \int_{y^i}^{y^{kM}} \frac{1}{x^k} \left(y^k - 2c^j\right) dy^k \right] dy^j + \\ \displaystyle\int_{y^i}^{y^{jM}} \frac{1}{x^j} \left[\int_0^{y^j} \frac{1}{x^k} \left(y^j - c^j\right) dy^k + \int_{y^j}^{y^{kM}} \frac{1}{x^k} \left(y^k - 2c^j\right) dy^k \right] dy^j \end{array} \right] dy^i \tag{4}$$

(b) The $x^i \leq x^k \leq x^j$ setting

$$V\left(x^i,x^j,x^k,y^{iM},y^{jM},y^{kM}\right)=$$
$$\int_0^{y^{iM}} \frac{1}{x^i}\left[\begin{array}{l} \int_0^{y^i} \frac{1}{x^j}\left[\int_0^{y^i} \frac{1}{x^k}\left(y^i\right)dy^k+\int_{y^i}^{y^{kM}} \frac{1}{x^k}\left(y^k-2c^j\right)dy^k\right]dy^j+ \\ \int_{y^i}^{y^{kM}} \frac{1}{x^j}\left[\int_0^{y^j} \frac{1}{x^k}\left(y^j-c^j\right)dy^k+\int_{y^j}^{y^{kM}} \frac{1}{x^k}\left(y^k-2c^j\right)dy^k\right]dy^j+\int_{y^{kM}}^{y^{jM}} \frac{1}{x^j}\left(y^j-c^j\right)dy^j \end{array}\right]dy^i \tag{5}$$

(c) The $x^k \le x^i \le x^j$ setting

$$V\left(x^i,x^j,x^k,y^{iM},y^{jM},y^{kM}\right)=$$
$$\int_{y^{kM}}^{y^{iM}} \frac{1}{x^i}\left[\int_0^{y^i} \frac{1}{x^j}\left(y^i\right)dy^j+\int_{y^i}^{y^{jM}} \frac{1}{x^j}\left(y^j-c^j\right)dy^j\right]dy^i+$$
$$\int_0^{y^{kM}} \frac{1}{x^i}\left[\begin{array}{l} \int_0^{y^i} \frac{1}{x^j}\left[\int_0^{y^i} \frac{1}{x^k}\left(y^i\right)dy^k+\int_{y^i}^{y^{kM}} \frac{1}{x^k}\left(y^k-2c^j\right)dy^k\right]dy^j+ \\ \int_{y^i}^{y^{kM}} \frac{1}{x^j}\left[\int_0^{y^j} \frac{1}{x^k}\left(y^j-c^j\right)dy^k+\int_{y^j}^{y^{kM}} \frac{1}{x^k}\left(y^k-2c^j\right)dy^k\right]dy^j+\int_{y^{kM}}^{y^{jM}} \frac{1}{x^j}\left(y^j-c^j\right)dy^j \end{array}\right]dy^i \tag{6}$$

(d) The $x^k \le x^j \le x^i$ setting

$$V\left(x^i,x^j,x^k,y^{iM},y^{jM},y^{kM}\right)=$$
$$\int_{y^{jM}}^{y^{iM}} \frac{1}{x^i}\left[y^i\right]dy^i+\int_{y^{kM}}^{y^{jM}} \frac{1}{x^i}\left[\int_0^{y^i} \frac{1}{x^j}\left(y^i\right)dy^j+\int_{y^i}^{y^{jM}} \frac{1}{x^j}\left(y^j-c^j\right)dy^j\right]dy^i+$$
$$\int_0^{y^{kM}} \frac{1}{x^i}\left[\begin{array}{l} \int_0^{y^i} \frac{1}{x^j}\left[\int_0^{y^i} \frac{1}{x^k}\left(y^i\right)dy^k+\int_{y^i}^{y^{kM}} \frac{1}{x^k}\left(y^k-2c^j\right)dy^k\right]dy^j+ \\ \int_{y^i}^{y^{kM}} \frac{1}{x^j}\left[\int_0^{y^j} \frac{1}{x^k}\left(y^j-c^j\right)dy^k+\int_{y^j}^{y^{kM}} \frac{1}{x^k}\left(y^k-2c^j\right)dy^k\right]dy^j+\int_{y^{kM}}^{y^{jM}} \frac{1}{x^j}\left(y^j-c^j\right)dy^j \end{array}\right]dy^i \tag{7}$$

(e) The $x^j \le x^i \le x^k$ setting

$$V\left(x^i,x^j,x^k,y^{iM},y^{jM},y^{kM}\right)=$$
$$\int_{y^{jM}}^{y^{iM}} \frac{1}{x^i}\left[\int_0^{y^i} \frac{1}{x^k}\left(y^i\right)dy^k+\int_{y^i}^{y^{kM}} \frac{1}{x^k}\left(y^k-2c^j\right)dy^k\right]dy^i+$$
$$\int_0^{y^{jM}} \frac{1}{x^i}\left[\begin{array}{l} \int_0^{y^i} \frac{1}{x^j}\left[\int_0^{y^i} \frac{1}{x^k}\left(y^i\right)dy^k+\int_{y^i}^{y^{kM}} \frac{1}{x^k}\left(y^k-2c^j\right)dy^k\right]dy^j+ \\ \int_{y^i}^{y^{jM}} \frac{1}{x^j}\left[\int_0^{y^j} \frac{1}{x^k}\left(y^j-c^j\right)dy^k+\int_{y^j}^{y^{kM}} \frac{1}{x^k}\left(y^k-2c^j\right)dy^k\right]dy^j \end{array}\right]dy^i \tag{8}$$

(f) The $x^j \le x^k \le x^i$ setting

$$V\left(x^i,x^j,x^k,y^{iM},y^{jM},y^{kM}\right)=$$
$$\int_{y^{kM}}^{y^{iM}} \frac{1}{x^i}\left[y^i\right]dy^i+\int_{y^{jM}}^{y^{kM}} \frac{1}{x^i}\left[\int_0^{y^i} \frac{1}{x^k}\left(y^i\right)dy^k+\int_{y^i}^{y^{kM}} \frac{1}{x^k}\left(y^k-2c^j\right)dy^k\right]dy^i+$$
$$\int_0^{y^{jM}} \frac{1}{x^i}\left[\begin{array}{l} \int_0^{y^i} \frac{1}{x^j}\left[\int_0^{y^i} \frac{1}{x^k}\left(y^i\right)dy^k+\int_{y^i}^{y^{kM}} \frac{1}{x^k}\left(y^k-2c^j\right)dy^k\right]dy^j+ \\ \int_{y^i}^{y^{jM}} \frac{1}{x^j}\left[\int_0^{y^j} \frac{1}{x^k}\left(y^j-c^j\right)dy^k+\int_{y^j}^{y^{kM}} \frac{1}{x^k}\left(y^k-2c^j\right)dy^k\right]dy^j \end{array}\right]dy^i \tag{9}$$

5. Numerical evaluations and cumulative frequencies

Figs. 6 to 8 illustrate the expected utility obtained from the evaluation of the triples corresponding to different sets of permutations. In all figures, the expected utility of the triples where the initial evaluation corresponds to the first alternative in the ranking is represented by a blue circle, an orange square is used when the initial evaluation corresponds to the second alternative in the ranking, and a green diamond when it is the third one. Fig. 6 presents all triples simultaneously, while Figs. 7 and 8 illustrate the first and second subcases, and the first and third subcases, respectively. The ordering in expected utility terms generated by the different evaluation patterns can be clearly observed in these figures.

Two empirical studies are used to contrast the validity of the simulations. Chitika [3] computed the average traffic percentages of the first 15 ranking results provided by Google from a sample of several million impressions in US and Canada that were retrieved in May 2013. More recently, Dean [4] analyzed the CTR of the organic results displayed in the first page of Google from a sample of five million searches. The percentages obtained by these studies are summarized in the second columns of Tables 2 and 3, respectively.

Table 2 compares the average traffic percentages of the first ten search results with the cumulative frequencies obtained from implementing both evaluation settings and computing the corresponding deciles. Note that an implicit assumption behind the computation of the cumulative frequencies is that each evaluation is equivalent to a search click. In this regard, Table 3 applies the same analysis to the average traffic shares derived from the CTRs reported by Dean [4]. In addition to the intuitive visual analysis derived from a direct comparison of the rankings, the squared Euclidean distance has been computed to illustrate the dissimilarities existing among the different evaluation categories.

Note how the average traffic shares described by Chitika [3] fit within the second decile of the pairwise evaluation setting and the first decile of the triple one – both of which display relatively small Euclidean distances –. The simplicity of search queries illustrated in Fig. 1, with one-word queries constituting the most prominent category in 2017, provides a suitable complement to this finding. That is, pairwise and triple evaluation settings can be used to categorize the search behavior exhibited by DMs when the information retrieval incentives are constrained by a relatively low number of search terms.

Consider Fig. 1 again and note the substantial increase exhibited by the two- and three-word categories in 2020. The average traffic shares obtained by Dean [4] illustrate that as DMs become more sophisticated and perform searches based on a larger number of words, the second decile of the triple evaluation scenario provides a more suitable fit. That is, as DMs perform more complex searches, they are willing to consider a larger number of potential alternatives – some of which correspond to

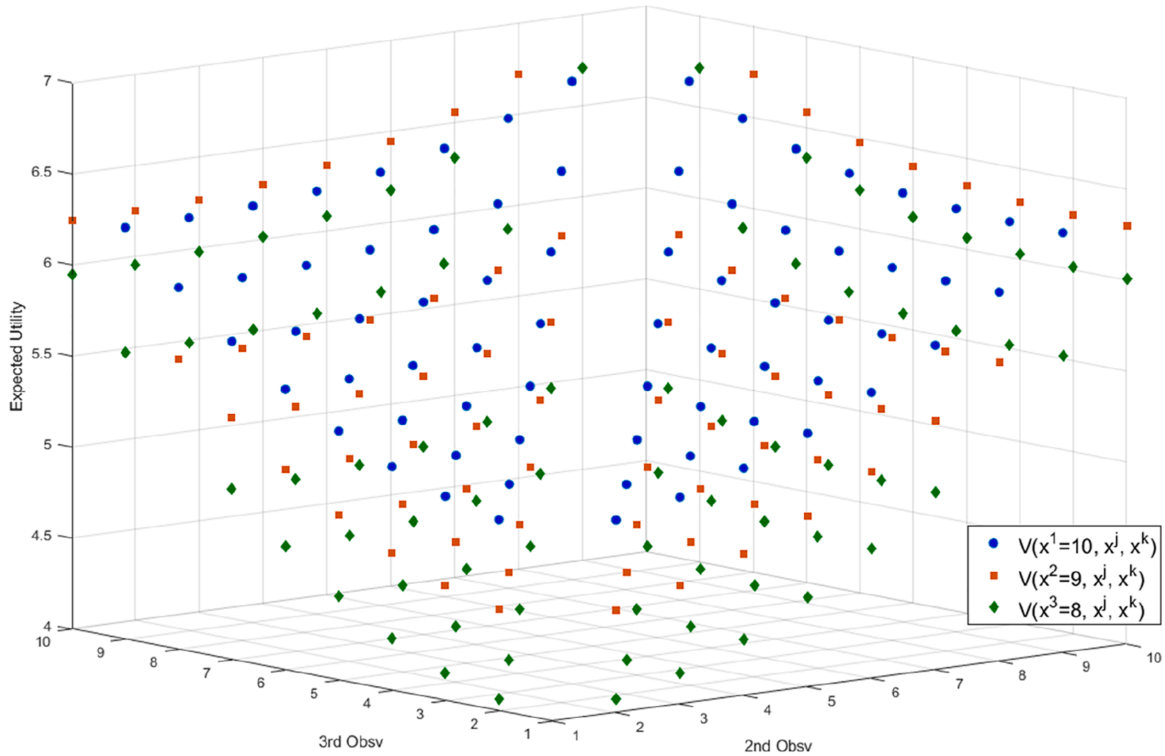


Fig. 6. Expected utility from triples of clicks (x^i, x^j, x^k) with $c^j = 0.1$ when $x^i = x^1$, $x^i = x^2$ and $x^i = x^3$.

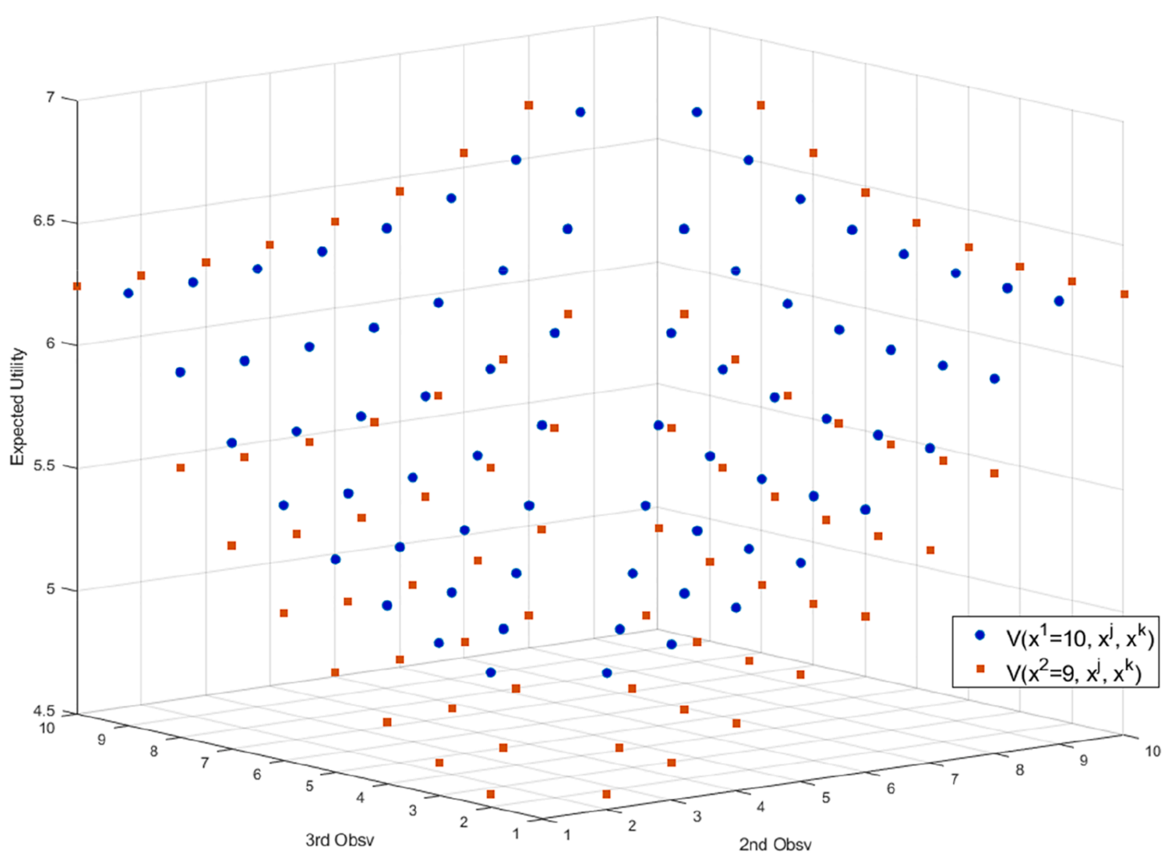


Fig. 7. Expected utility from triples of clicks (x^i, x^j, x^k) with $c^j = 0.1$ when $x^i = x^1$ and $x^i = x^2$.

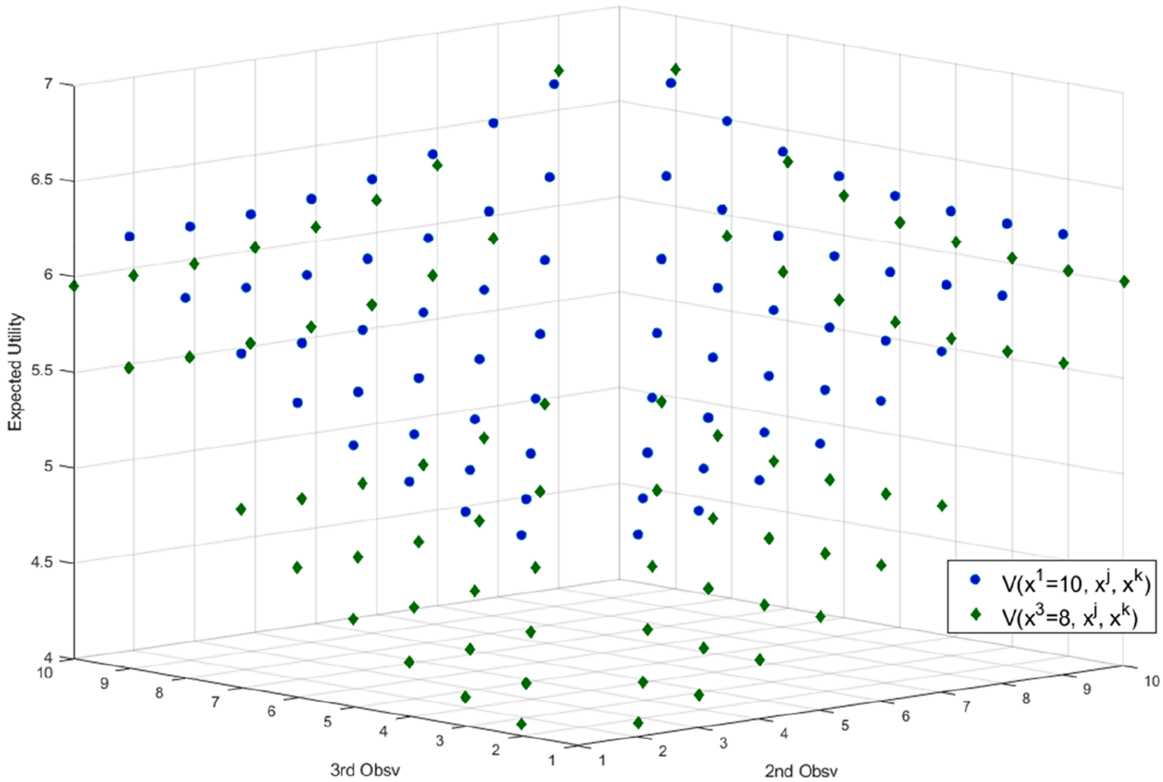


Fig. 8. Expected utility from triples of clicks (x^i, x^j, x^k) with $c^j = 0.1$ when $x^i = x^1$ and $x^i = x^3$.

Table 2
Comparing cumulative frequencies with the average traffic shares from Chitika [3].

		Pairwise		Triple	
Google ResultPage Rank	AverageTraffic Share	First D9 pairs18 clicks	Second D18 pairs36 clicks	First D72 triples216 clicks	Second D144 triples432 clicks
1	32.5	38.9	36,1	33,3	28,7
2	17.6	22.2	19,4	22,2	15,7
3	11.4	22.2	11,1	13,9	15,7
4	8.1	11.1	11,1	5,6	11,6
5	6.1	5.6	8,3	5,6	6
6	4.4	0	5,6	5,6	6
7	3.5	0	5,6	4,2	4,6
8	3.1	0	2,8	3,7	4,2
9	2.6	0	0	3,2	3,9
10	2.4	0	0	2,8	3,5
11–15	3.5	–	–	–	–
Squared Euclidean Distance		**242**	**49**	**37**	**57**

links located in lower ranking positions –.

From a causality perspective, the performance of queries including a larger number of search terms may be assumed to reflect the intention of DMs to consider a larger number of alternatives among those provided by the search engine within the first page of results. Alternatively, we could also conclude that as DMs become more sophisticated, the search engine distributes potentially suitable alternatives through a wider set of ranked elements.

All in all, ranking similarities increase when incorporating a third observation to the combinatorial approach, particularly when considering the first and second deciles within the scenarios derived from Chitika [3] and Dean [4], respectively. The similarities are also substantial between rank categories when considering the second decile from the pairwise setting, which highlights the considerable importance of the heuristic mechanisms applied by DMs when dealing with complex evaluation environments. However, such similarities vanish when shifting to the scenario derived from Dean [4], with triples performing substantially better than pairs.

We conclude by highlighting the higher similarities exhibited by the triples when compared to the pairs in both scenarios, while being aware of the fact that computing these permutations imposes a substantial burden on the DMs, who are unable to implement such complex structures when evaluating alternatives. In this regard, adding a fourth alternative to the value function would probably deliver better ranking approximations within both scenarios. However, the conclusions derived from the current analysis would remain unchanged.

6. Conclusion

We have formalized the evaluation process of the different alternatives composing the ranking of an online search engine. The corresponding value functions have been designed to determine the order in which to evaluate the alternatives, characterized by the uncertain intervals following from the ranking positions displayed by the engine. These functions define a benchmark framework measuring the ability of search engines to identify the preferences of DMs and the capacity of the

Table 3
Comparing cumulative frequencies with the CTR from Dean [4].

Google Result Page Rank	CTR	Average Traffic Share*	Pairwise First D9 pairs18 clicks	Pairwise Second D18 pairs36 clicks	Triple First D72 triples216 clicks	Triple Second D144 triples432 clicks
1	31,7	26.96	38.9	36.1	33.3	28.7
2	24,7	21.00	22.2	19.4	22.2	15.7
3	18,7	15.90	22.2	11.1	13.9	15.7
4	13,6	11.56	11.1	11.1	5.6	11.6
5	9,5	8.08	5.6	8.3	5.6	6
6	6,2	5.27	0	5.6	5.6	6
7	4,1	3.49	0	5.6	4.2	4.6
8	3,1	2.64	0	2.8	3.7	4.2
9	3	2.55	0	0	3.2	3.9
10	3	2.55	0	0	2.8	3.5
11–15	–	–	–	–	–	–
Squared Euclidean Distance			**250**	**127**	**90**	**42**

* We must highlight the fact that Dean [4] does not provide the average traffic shares but the CTRs obtained from 5,000,000 search queries. Given the definition of click through rate as the ratio of users who click on a link to the number of total users who perform a search, we can easily derive the corresponding average traffic shares by performing the following calculations for $i = 1,\ldots,10$:1. Total number of clicks on alternative i = CTR of alternative i times 5,000,000.2. Total number of clicks $= \sum_i$Total number of clicks on alternative i.3. Average traffic share of alternative i = Total number of clicks on alternative i / Total number of clicks.

latter to assimilate and evaluate the information provided by the engines.

The complete uncertainty faced by the users, other than their belief in the ranking provided by the search engine, implies that DMs must account for the relative domains and potential realizations of all the alternatives being evaluated when computing the value functions. The increasing complexity of this combinatorial process has been illustrated formally and numerically throughout the paper.

We conclude by noting that the evaluation scenario introduced can be adjusted to formalize interactions across alternatives within uncertain environments in multi-criteria decision making techniques such as TOPSIS. From a strategic viewpoint, Epstein and Robertson [5] demonstrated empirically what they defined as the search engine manipulation effect, namely, the design of search rankings so as to modify the decisions of undecided voters. The current model could be applied to determine the basic requirements in terms of information acquisition for different disclosure strategies to be viable.

Finally, further heuristic developments would be required to incorporate more complex combinatorial scenarios into the analysis, such as the unification of all the domains within a common framework while adjusting the shape of the probability density to account for the ranking position of the different alternatives and their effects on the expectations of the DM.

CRediT authorship contribution statement

Debora Di Caprio: Conceptualization, Methodology, Formal analysis, Writing - original draft, Writing - review & editing. **Francisco J. Santos-Arteaga:** Conceptualization, Methodology, Formal analysis, Writing - original draft, Writing - review & editing.

Declaration of Competing Interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Acknowledgement

This work was supported by the Open Access Publishing Fund of the Free University of Bozen-Bolzano.

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