Comparing the Performance of Optimally Tuned Dynamic Vibration Absorbers with Very Large or Very Small Moment of Inertia

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Abstract

In this article, the performance of a two degree-of-freedom Dynamic Vibration Absorber (DVA) with very large or very small moment of inertia is studied. Although it has been shown previously that an optimally tuned DVA with a negligibly small moment of inertia marginally outperforms the optimally tuned DVA with a very large moment of inertia, the physical reasons for this have not been made clear. Using a simplified model of the stiffness elements of the DVA, it is shown that the two sets of parallel combinations of stiffness and damping elements of the DVA with negligibly small moment of inertia effectively act in series, rather than in parallel as in the other case. Further, it is shown that the stiffness and damping elements can be represented as a single stiffness and a single damping element whose properties are frequency dependent. This frequency dependency means that there is additional freedom in choosing the optimum stiffness and damping of the DVA, which results in better performance.

1. Introduction

Since the Dynamic Vibration Absorber (DVA) was proposed by Ormondroyd and Den Hartog about a century ago [1], there has been much research on this topic. It has often been modelled as a translational single-degree-of-freedom (SDOF) mass, spring, damper system. However, a single mass DVA can have as many as three DOFs in translation or six DOFs including rotation, and recently some studies on such absorbers have been carried out. Zuo and Nayfeh [2] optimized the stiffness and damping values for a multi-degree-of-freedom (MDOF) DVA attached to a MDOF system and showed that the single mass DVA can suppress multiple resonance peaks of the structure to which it is attached. Jang and Choi [3] derived the dynamic equations of two bodies in space and extracted the conditions for stiffness, mass, and moment of inertia values for the suppression of multiple peaks. Zuo and Nayfeh [4] also recently investigated the suppression of the resonance peak of a SDOF system using a DVA which had translational and rotational DOFs. The performance of the DVA was characterized by the H_2 and H_∞ norms of the displacement of the SDOF system. They calculated the optimum values for the two sets of springs and dampers, and moment of inertia of the DVA mass, and showed that the 2DOF DVA was more effective than the optimised SDOF DVA according to these metrics. In particular, Zuo and Nayfeh showed that, even if the DVA has negligible inertia but is allowed to rotate, then it performs marginally better than a DVA which has a very large moment of inertia. It was stated in [4] that the reason for this improved performance is that the 2DOF DVA can be represented by a third-order rather than a second-order transfer function. The aim of this article is to provide a physical explanation as to why the 2DOF DVA, with negligible moment of inertia, outperforms the DVA with a very large moment of inertia, in minimizing the H_2 norm of the host SDOF system response.

2. Description of the problem

The system of interest is shown in Fig. 1. A 2DOF DVA with a rotational and translational degree-of-freedom is attached to a base-excited main structure, represented by an undamped translational SDOF system of mass m_s and stiffness k_s. The DVA has mass, m_a and moment of inertia J_a, and is connected through a rigid, mass-less link to two sets of springs and dampers k_1, c_1 and k_2, c_2 respectively. The distances from the mass centre to the two sets of springs and dampers are given by d_1 and d_2 respectively. If the moment of inertia tends to infinity then the DVA behaves as a translational SDOF system as the mass is prevented from rotating. Alternatively, if the moment of inertia tends to zero, the DVA still behaves as a SDOF system as the sets of springs and dampers are connected directly to each other, but only through the system geometry rather than the inertia of the DVA mass. This is explored further in this section.

To aid interpretation of the behavior of the two DVAs, a simplified model of the system of springs and dampers in the DVA is sought. To achieve this, they are considered separately from the DVA mass, as shown in Fig. 2a, and harmonic excitation of the form $Fe^{j\omega t}$ and $Xe^{j\omega t}$ is assumed. The force F, is applied at the position where the system of springs and dampers is connected to the DVA mass. The relationship between the displacement at this position and the displacement at the positions where the springs and dampers are connected is given by

$$X = \frac{d_1 X_2 + d_2 X_1}{d_1 + d_2} \tag{1}$$

The relationship between the force F, and the forces transmitted to the springs and dampers is given by

$$F = F_1 + F_2 \tag{2}$$

If it assumed that the moment of inertia of the absorber is negligible, then

$$F_1 d_1 + F_2 d_2 = 0 \tag{3}$$

Noting that the receptance of the spring and damper system is given by X/F, Eqns. (1-3) can be combined to give

$$\alpha = \frac{X}{F} = \frac{1}{q_2 K_1} + \frac{1}{q_1 K_2} \tag{4}$$

where $K_1 = \frac{F_1}{X_1} = j\omega c_1 + k_1$ and $K_2 = \frac{F_2}{X_2} = j\omega c_2 + k_2$ are the dynamic stiffnesses of each set of parallel combinations of springs and dampers, and $q_i = \left(\frac{d_1 + d_2}{d_i}\right)^2$, where ($i = 1,2$). Eqn. (4) represents a system of dynamic stiffnesses connected in series such as that shown in Fig 2b, where $\tilde{k}_1 = q_2 k_1$, $\tilde{k}_2 = q_1 k_2$, $\tilde{c}_1 = q_2 c_1$, and $\tilde{c}_2 = q_1 c_2$. An absorber with this combination of stiffness and damping elements has been considered previously by Snowdon [6] and Zuo [7]. Note that q_1 and q_2 play the role of amplifying or reducing the size of the coefficients. When $d_2/d_1 \to 0$, $q_2/q_1 \to \infty$ and $\alpha \cong 1/K_2$ and when $d_2/d_1 \to \infty$,

$q_2/q_1 \to 0$ and $\alpha \cong 1/K_1$. When $d_1 = d_2$, $q_1 = q_2 = 1/4$ and the two sets of dynamic stiffness contribute equally to the receptance; additionally, if both sets of stiffnesses and damping are the same, then the total dynamic stiffness is simply twice that of one of the sets of elements (Also, if the system is prevented from rotating, as it would if the DVA mass had infinite moment of inertia, then the total dynamic stiffness would simply be twice that of the individual elements regardless of the ratio d_2/d_1).

Another way of viewing the series combination of Fig. 2b., which is helpful when comparing the reasons why the two absorbers behave differently, is to consider it as a parallel combination of stiffness and damping where the coefficients $k(\omega)$, and $c(\omega)$ are frequency dependent, as shown in Fig. 2c. The coefficients are given by

$$k(\omega) = \mathrm{Re}\left\{\frac{F}{X}\right\} = \frac{\left(\tilde{c}_2^2\tilde{k}_1 + \tilde{c}_1^2\tilde{k}_2\right)\omega^2 + \tilde{k}_1\tilde{k}_2\left(\tilde{k}_1 + \tilde{k}_2\right)}{\left(\tilde{c}_1 + \tilde{c}_2\right)^2\omega^2 + \left(\tilde{k}_1 + \tilde{k}_2\right)^2} \tag{5a}$$

$$c(\omega) = \mathrm{Im}\left\{\frac{-F}{\omega X}\right\} = \frac{\tilde{c}_1\tilde{c}_2\left(\tilde{c}_1 + \tilde{c}_2\right)\omega^2 + \tilde{c}_1\tilde{k}_2^2 + c_2'\tilde{k}_1^2}{\left(\tilde{c}_1 + \tilde{c}_2\right)^2\omega^2 + \left(\tilde{k}_1 + \tilde{k}_2\right)^2} \tag{5b}$$

(The prime superscript in the nominator should be replaced to ~; I cannot access to this)

To determine the response of the system to base excitation with the DVA attached, the dynamic stiffness approach can be used. The transmissibility of motion is given by

$$\frac{X_s}{X_e} = \frac{k_s}{K_s + K_{DVA}} \tag{6}$$

where X_s and X_e are the Fourier transforms of the main structure and base displacements respectively. The dynamic stiffnesses of the DVA and the main structure are given respectively by

$$K_{DVA} = \frac{1}{\dfrac{1}{k(\omega)+j\omega c(\omega)} - \dfrac{1}{\omega^2 m_a}}, \qquad K_s = k_s - \omega^2 m_s \tag{7a,b}$$

The performance measure of the system is the H_2 norm as in [4], which can be calculated by

$$\|H\|_2 = \sqrt{\frac{1}{2\pi}\int_{-\infty}^{\infty}\left|\frac{X_s}{X_e}\right|^2 d\omega} \tag{8}$$

3. Comparison of the performance of the two DVAs

The characteristics and performance of the system in Fig. 1 are now investigated. Of interest are the cases when the moment of inertia of the absorber is very large or very small, but for simplicity the cases considered here are when $J_a = \infty$ and $J_a = 0$. To determine the optimum parameters in each case the H_2 norm is calculated numerically using Eq. (8). An exhaustive numerical search for the optimum stiffness and damping parameters is undertaken to determine the values that give the minimum H_2 norm. These are also checked with values in the literature [4,5]. For the simulations, d_1 is set to be equal to d_2 and $m_s = 1$ kg and $k_s = 1$ Nm^{-1} as shown in Tab. 1. This does not affect the generality of the results since a variation of the stiffnesses is equivalent to a change in their position. The optimization leads to the values given in Tab.1. The resulting absolute displacement of the mass of the main structure X_s for a given displacement of the base X_e, i.e. the modulus of the transmissibility of the system $|X_s/X_e|$ is shown in Fig. 3 as a function of non-dimensional frequency, ω/ω_s, where $\omega_s = \sqrt{k_s/m_s}$ is the natural frequency of the main structure. It can be seen that for the case when the moment of inertia is negligible then the transmissibility is marginally less, over the frequency range near the peaks, than for the case when the moment of inertia is very large. The corresponding values of the H_2 norm are given in Tab. 1.

The question as to why the two systems give a different performance can be answered by studying the equivalent stiffness models for the two systems. For the case when the moment of inertia of the DVA is very large, there is effectively one stiffness and one damping value that can be adjusted as the two sets of springs and dampers tend to act as one because there is no rotation. Moreover the values of the stiffness and damping coefficients are independent of frequency. With the case of the DVA with a negligibly small moment of inertia, the system of stiffnesses and dampers can be reduced to a single stiffness and damper, but very importantly in this case, they are frequency dependent as depicted in Fig. 2c. The frequency dependence cannot be chosen arbitrarily but can be adjusted by choosing appropriate values of k_1, c_1, k_2, c_2, (may be replaced to k_1, c_1, k_2, and c_2.) This gives additional freedom in choosing the optimum stiffness and damping for the DVA. The optimum normalised frequency dependent stiffness and damping properties corresponding to Fig 2c. are plotted in Figs. 4a and 4b as a function of non-dimensional frequency respectively. It can be seen that the stiffness is relatively small at low frequencies and rises at high frequencies. In fact, it is equivalent to the serial combination of the two springs $\tilde{k}_1$ and $\tilde{k}_2$ at low frequencies and to $\tilde{k}_2$ at high frequencies. It is interesting that the optimized value of $\tilde{c}_2 = 0$, but there is a finite value of $\tilde{c}_1$. This is the same finding as reported in references [4] and [7]. The frequency dependent damping values correspond to $\tilde{c}_1\left(\frac{\tilde{k}_2}{\tilde{k}_1+\tilde{k}_2}\right)^2$ at low frequencies decreasing to zero at high frequencies.

Of course, the DVAs consist of a mass as well as the stiffness and damping elements. To examine the difference between the dynamic behaviour of the two DVAs discussed above their normalized apparent mass is plotted for the modulus and phase in Figs. 5a and 5b respectively for the optimised parameters. It can be seen that although the stiffness and damping characteristics are quite different, their dynamic behavior is quite similar in the frequency region close to $\omega/\omega_s = 1$.

4. Conclusions

This article has investigated the performance of two DVAs - one which has a negligible moment of inertia and one which has a very large moment of inertia – attached to a translational SDOF main structure. It has been shown that the DVA with negligible moment of inertia is dynamically equivalent to a DVA with frequency dependent stiffness and damping. When comparing the H_2 norm of the response of the main structure the DVA with negligible moment of inertia marginally outperforms the DVA with a large moment of inertia. It is suggested that the effective frequency dependency of the DVA stiffness and damping is responsible for this.

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Figures

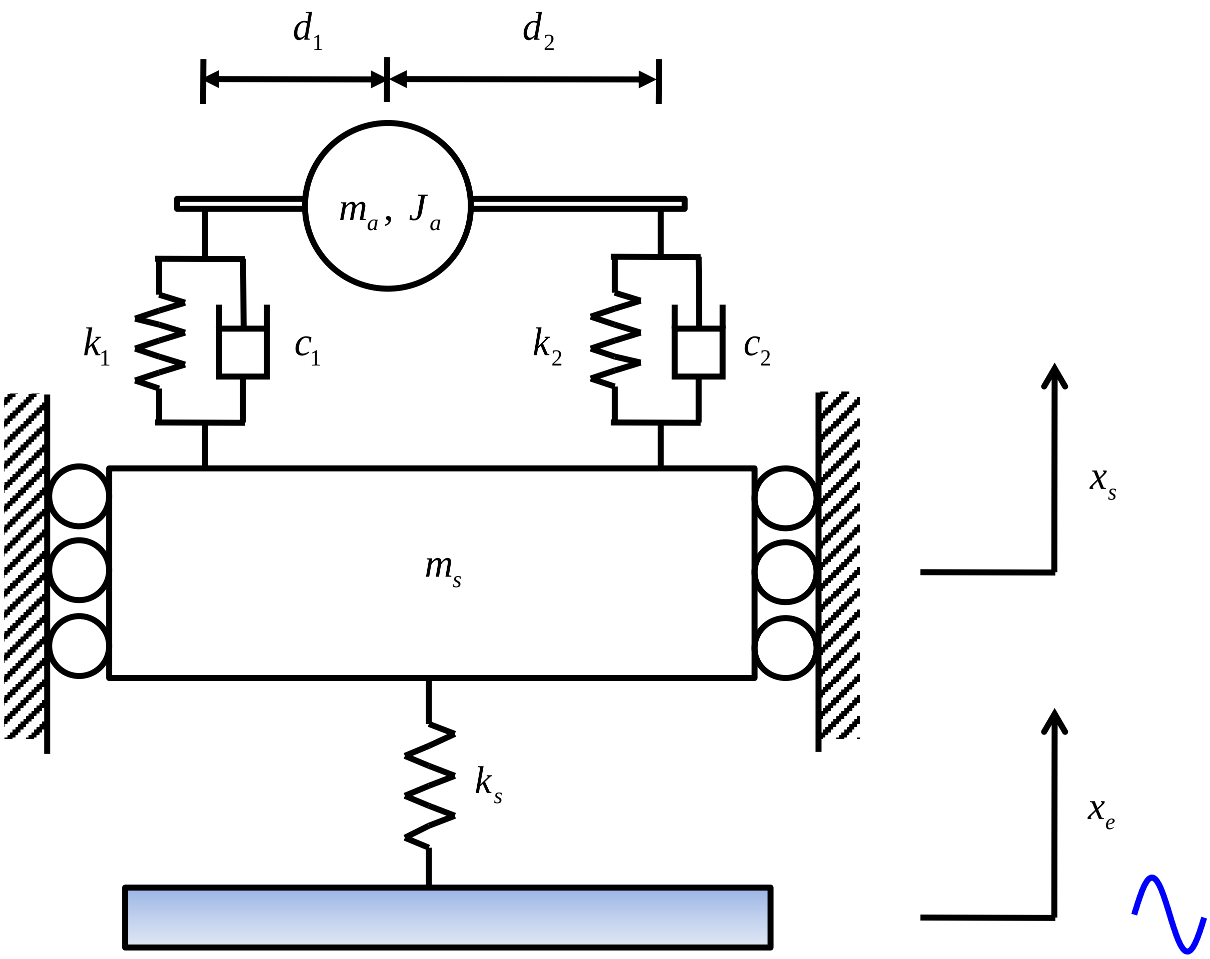

Fig. 1. Base-excited SDOF system with 2DOF DVA attached.

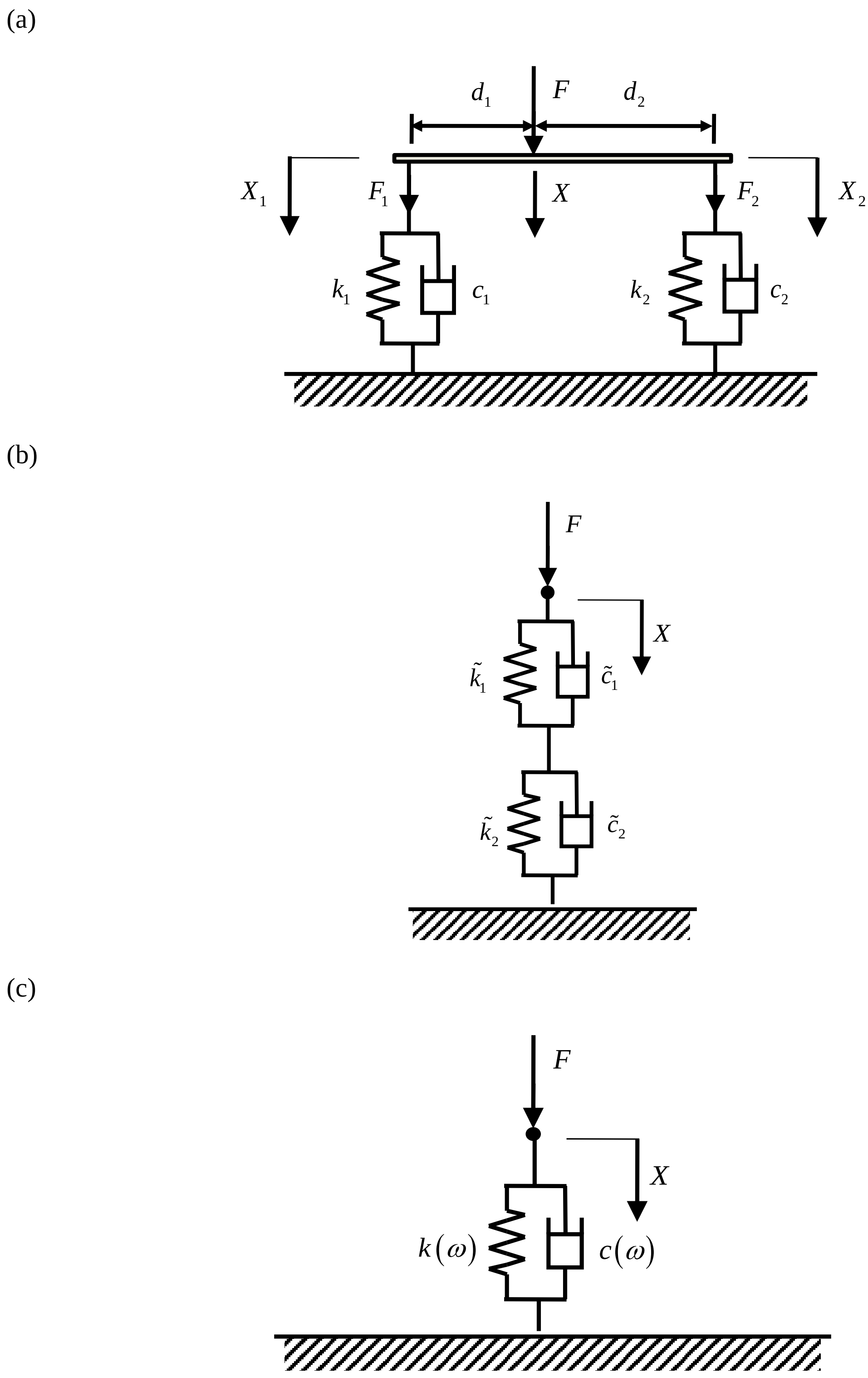

Fig. 2. Models for springs and dampers (a) original model, (b) equivalent series model, (c) equivalent parallel model using frequency dependent spring and damper

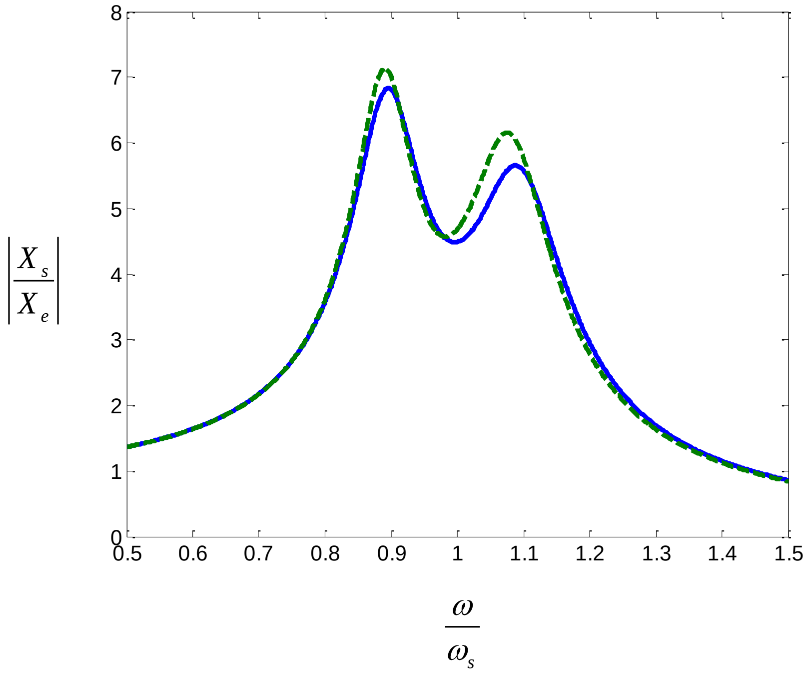

Fig. 3. Transmissibility of the system between the base and main structure; DVA with $J_a = 0$. (solid line), DVA with $J_a = \infty$ (dotted line).

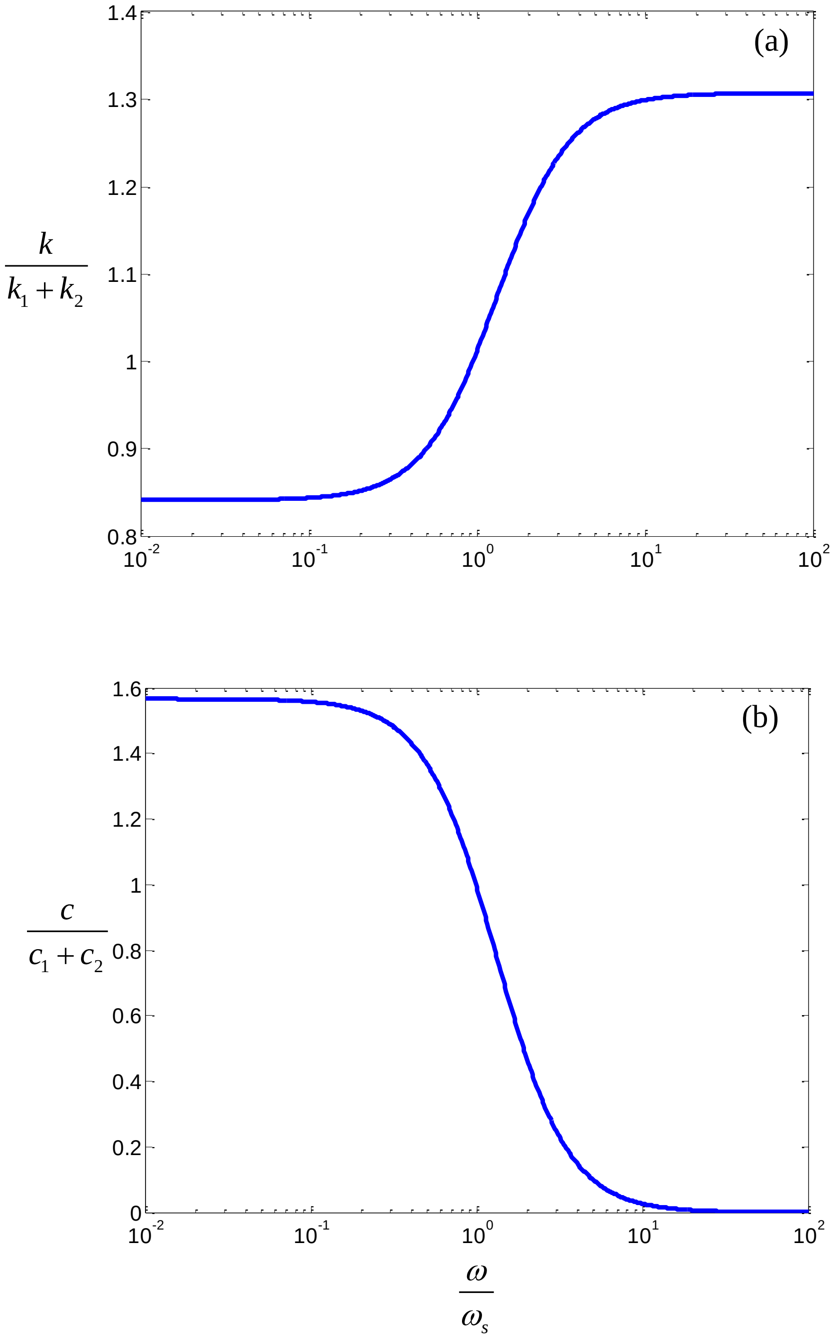

Fig. 4. Non-dimensional equivalent optimum absorber stiffness and damping as a function of non-dimensional frequency for a mass ratio of 5%. (a) stiffness, (b) damping.

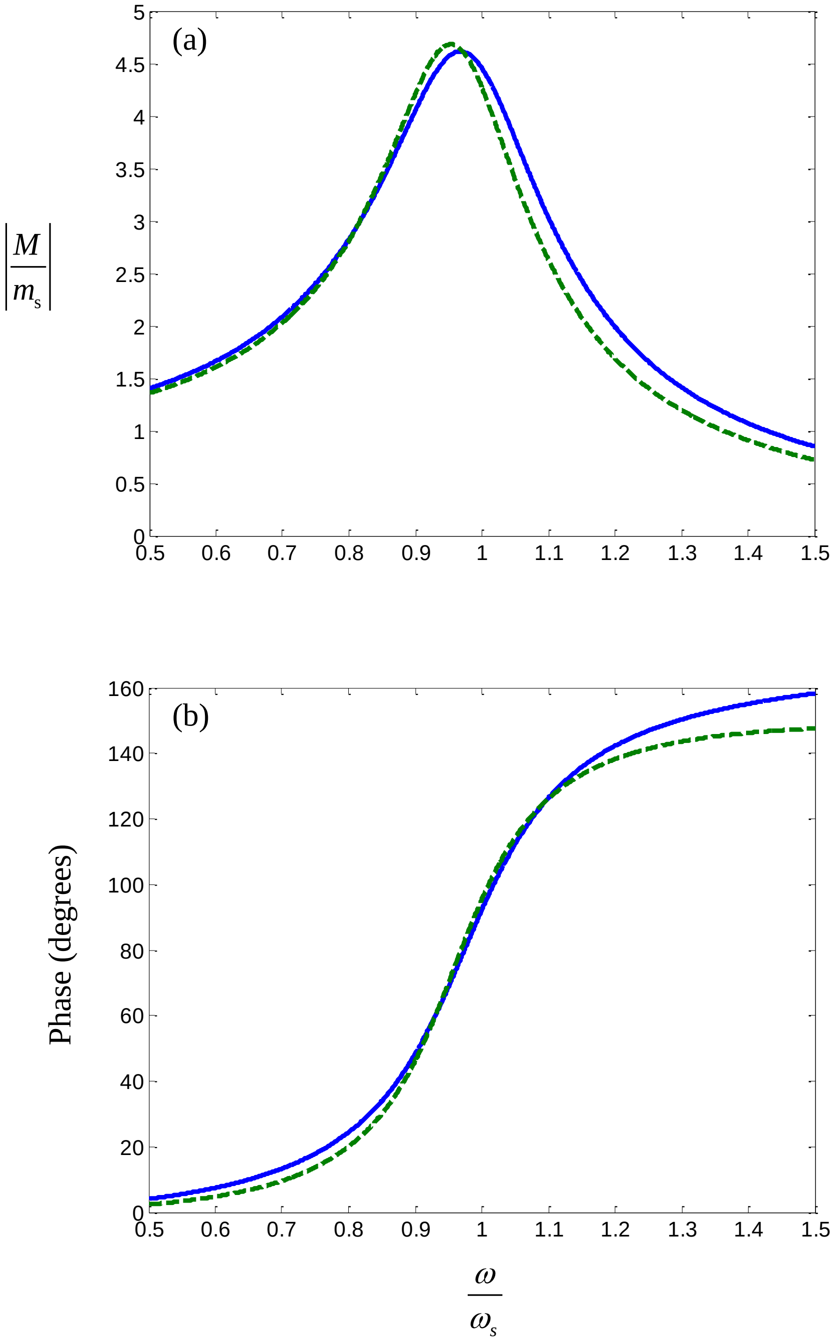

Fig. 5. Normalised apparent mass of the absorber (a) Magnitude and (b) phase. DVA with $J_a = 0$ (solid line), DVA with $J_a = \infty$ (dotted line).

Tables

	Main structure	
m_s	1 kg	
k_s	1 Nm^{-1}	
	DVA (Optimal Values)	
	Very Large Moment of Inertia	**Negligible Moment of Inertia** $d_1 = d_2$
m_a	0.05 kg	0.05 kg
J_a	∞ kgm^2	0 kgm^2
$k_1 + k_2$	0.0465 Nm^{-1}	
$c_1 + c_2$	0.0106 Nsm^{-1}	
k_1		0.0274 Nm^{-1}
c_1		0.0330 Nsm^{-1}
k_2		0.0152 Nm^{-1}
c_2		0 Nsm^{-1}
$\lVert H \rVert_2$	2.1087	2.0660

Tab. 1. Physical parameters used in the simulations