A Multi-Component Approach to the Seismic Characterization of Real Multi-Layered Media

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Abstract
The dispersion behaviour of ground roll has been exploited in recent decades for the imaging and acoustic characterization of the shallow subsurface, often using the Multichannel Analysis of Surface Waves (MASW) method. The effectiveness of the method in providing an accurate shear wave profile of a soil deposit and the identification of seismic event largely depends on the degree of interference from other noise sources, unrelated to the primary signal of interest. In MASW a wavefield transformation in the frequency-wavenumber (*f-k*) domain is usually performed exploiting vertical components of a seismic event, picked up at different positions, theoretically allowing the identification of different wave-types and higher Rayleigh wave (R-wave) modes.
In this work a multi-component approach to seismic characterization of the shallow subsurface is proposed. The body wave content is lowered and the Rayleigh wave content is enhanced by simultaneously exploiting the vertical and horizontal components of the seismic event in the direction of propagation of the ground roll (i.e. the direction of the receivers array), in the time domain. The effects of this multi-component approach are investigated in detail by finite element modelling and with respect to an *in-situ* experimental study. In general, the approach was found to be beneficial in enhancing the Rayleigh wave content of the seismic signal, hence improving mode separation in the *f-k* domain. It augments the range of detectable frequencies actually excited by ground roll and improves the resolution of the *f-k* transformation by a factor of two.

Keywords: Multichannel Analysis of Surface Waves (MASW), Frequency-Wavenumber (*f-k*) transformation, higher modes Rayleigh wave, multi-component approach, Finite Element Model (FEM).

1. Introduction

Multichannel Analysis of Surface Waves (MASW) is a seismic wave method which involves the measurement of Rayleigh waves propagating along the surface of a medium and exploits their dispersive behaviour for in-depth analysis. The set-up configuration consists of a source of seismic energy and multiple receivers (typically 24, but sometimes 48 or more) on the ground surface spaced equally along a survey line [1, 2]. The source offset and the spacing between receivers are chosen according to the wavelength and hence the depth of investigation. The seismic energy is recorded simultaneously by all the receivers. MASW typically uses either a continuous source like a vibrator or an impulsive source like a sledgehammer [3]. The experimental data is sometimes analyzed in the time domain, but more frequently transformed from the time-space domain into the frequency-wavenumber (*f-k*) domain. Theoretically, the transformation of the wavefield from the time-space into the frequency-wavenumber domain allows the identification of different wave-types and higher Rayleigh modes [4-6], which significantly affect the determination of the actual phase velocity, as well as near-field and far-field effect [3]. In the common practice, the spectrum obtained with the vertical signal is inverted, with the aim to obtain the shear wave profile. Sometimes the inversion is ambiguous and the solution is not unique. In an attempt to minimize the ambiguity and to face all the problems related to the nonuniqueness of the solution, joint analysis are performed inverting the spectrum obtained with the horizontal and vertical traces, or exploiting the spectral ratio between the horizontal and vertical components of the background microtremor field (HVSR)[7]. In practice, the reliability of surface-wave methods lies in their capability to minimize noise inclusion [8], defined as everything other than the desired signal.

1.1 Frequency-Wavenumber Transformation

The *f-k* transformation is essentially a two-dimensional Fourier Transform which transforms the vertical space-time domain representation $u(x,t)$ of a seismic event into the frequency-wavenumber representation $U(f,k)$: the time information is transformed into frequency components and the spatial information is transformed into wavenumber components. The peaks of the $U(f,k)$ spectrum are associated with the energy maxima and hence to the vibrational modes of propagation. The transformation is a well-established method and its properties are reported in many works [4, 9-12].

The resolution of the image in the *f-k* domain is inversely proportional to the length of the signal in time and to the length of the sensor line in space. The resolution is a key factor in separating the different modal contributions: often a peak in the *f-k* domain is not associated with a single mode, but is rather a superposition of several different modes. This can then lead to erroneous interpretation of the results.

In this work a new signal processing method involving both vertical and horizontal ground roll displacements is proposed, reducing the effect of noise and improving the global resolution of the *f-k* spectrum by a factor of two. The benefits of the multi-component approach are investigated with the aid of numerical and experimental investigations.

2. Multi-Component Method for Signal Processing

The aim of the new multi-component method is to enhance the R-wave content of a signal and the global resolution of the *f-k* transform. To achieve this, the mutual product between the vertical and the horizontal component in the time domain $p(x,t)$ is calculated before the double Fourier Transform, as follows:

$$p(x,t) = u(x,t) \cdot w(x,t) \tag{1}$$

Where $u(x,t)$ and $w(x,t)$ are the vertical and horizontal time-space representations.

This processing method arises from the observation of displacements induced by a seismic event: horizontal displacement of Rayleigh wave is $\pi/2$ out of phase from the vertical displacement, and the resulting motion is an ellipse with retrograde rotation, while body wave components have in-phase behaviour. Rayleigh wave energy is usually stronger than other wave components or events, such as body waves, background noise or backscatter waves, when a vertical excitation is used [13].

The product between vertical and horizontal signals in time domain can be seen as a convolution between the spectra in the frequency domain: by multiplying the time histories every wave component that is strong in both signals will be enhanced in the convoluted spectrum, conversely energy that is weak in the input signals will be weaker in the output spectrum. Thus, adopting the product between the vertical and horizontal displacements has a dual benefit: it lowers content of the noise and unwanted waves and it enhances the Rayleigh wave content. We can express the vertical and horizontal components of the Rayleigh waves as:

$$u(x,t) = A \cdot \mathrm{e}^{\mathrm{i}(\omega t - kx)} \tag{2}$$

$$w(x,t) = B \cdot \mathrm{e}^{\mathrm{i}(\varphi t - lx + \frac{\pi}{2})} \tag{3}$$

Where A, B are the amplitude of the sinusoids, ω, φ are angular frequencies, k, l their respective wavenumbers. If we make the assumption of $\omega = \varphi$, $k = l$ and we multiply the two quantities together:

$$p(x,t) = u(x,t) \cdot w(x,t) = A \cdot \mathrm{e}^{\mathrm{i}(\omega t - kx)} \cdot B \cdot \mathrm{e}^{\mathrm{i}(\omega t - kx + \frac{\pi}{2})} = A \cdot B \cdot \mathrm{e}^{\mathrm{i}2(\omega t - kx + \frac{\pi}{4})} \tag{4}$$

The equation shows that, although a doubling of the time and space frequencies occurs and a phase shift of $\pi/2$ is introduced, the dispersion relationship is otherwise unaffected.
In order to restore the original time and space variation, the matrix needs to be stretched by a factor of two in both time and space. This then has the effect of increasing the resolution by a factor of two in frequency and wavenumber domains, once the Fourier transforms are performed.
The gain in resolution is accompanied by a proportional reduction in the breadth of the $P(f,k)$ spectrum: the aliasing will occur for half the values obtained with a single-component survey. This drawback can be easily overtaken by adopting a fairly small temporal and spatial discretization, bearing in mind that aliasing is primarily governed by Nyquist's sampling criterion [12].

3. Numerical Simulations

For the forthcoming simulations a finite/infinite element model (FEM) is assembled through the Abaqus/CAE software, in a single vertical plane, with the aim of studying the wave propagation in two-layered systems. The model is semi-elliptical in shape and each layer is considered to be elastic and isotropic.
Following the work of Zerwer [14], since we only are interested in surface wave measurements, the graded mesh progressively increases in the downward vertical direction. The mesh elements are smaller near the surface, where Rayleigh wave propagates, with an element size equal to l_{max}. Infinite elements are applied to the boundaries of the model, although the duration of the simulation with respect to the size of the model always ensures the absence of reflected waves. Infinite elements behave as many infinitesimal dashpots which are oriented normally and tangentially with respect to the boundary. The analysed profile consists of a two-layered system simulating a soil deposit with irregular stratification (i.e. the velocity does not increase with depth). Parameters are shown in Table 1. The following plane strain elements are utilized in the forthcoming simulation: CPE3 and CINPE4.

3.1 Model Constraints, Limitations and Attenuation

With finite elements methods, two discretization constraints should be adopted in order to achieve appropriate spatial and temporal resolution. The spatial condition assures that a sufficient number of points in space are sampled in order to recreate the wave, or in other word that the element size l_{max} is small enough (it is the analogue of the Nyquist criterion in the time domain) [14, 15].

$$f_{max} \le \left(\frac{1}{8} \div \frac{1}{5}\right) \frac{V_S}{l_{max}} \tag{5}$$

where V_S is the shear velocity and f_{max} is the maximum frequency. The denominator is usually chosen to be well within the Nyquist limit.

The temporal constraint must be set after the spatial, to ensure that the wave front does not travel faster than the time step Δt. This is achieved using the Curant condition [14, 16, 17], here rearranged for two-dimensional problems:

$$\Delta t_{\max} \leq \frac{1}{V_P \sqrt{\frac{1}{l_{\max}^{2}}}} \tag{6}$$

where V_P is the compressional wave velocity.

Damping in numerical simulations is usually expressed in terms of the mass damping α and the stiffness damping β, as follows [14, 16-18]:

$$\xi = \frac{1}{2\omega}\alpha + \frac{\omega}{2}\beta \tag{7}$$

Where ξ is the damping ratio and ω is the angular frequency of excitation.

It can be noticed that the damping ratio varies with the frequency of excitation. The values of α and β are usually selected, according to engineering estimations, such that the critical damping ratio is set at two known frequencies.

The parameters of the model are reported in Table 1. The mechanical parameters have been chosen to represent typical soils. The upper layer is stiffer than the underlying half-space, simulating a soil deposit with irregular stratification, where the fundamental mode is not dominant but higher modes play a relevant role.

Table 1. Parameters of the FEM model

Poisson ratio, ν	0.33
Mass damping, α	0
Stiffness damping, β	$0.025/200 \cdot 2\pi$
Mass density, ρ	$2000\text{Kg} \cdot \text{m}^{-3}$
Element size at the surface, $l_{\min}$	0.05m
Semi-major axis	10m
Semi-minor axis	7.5m
Time step, $\Delta t_{\max}$	$5 \cdot 10^{-5}$sec
Duration of the simulation T	0.10sec
Thickness upper layer	1m
Thickness half-space	∞
Young's Modulus upper layer	300MPa
Young's Modulus half-space	100MPa

The load used in the simulation is depicted in Figure 1: it consists of a short transient impulse.

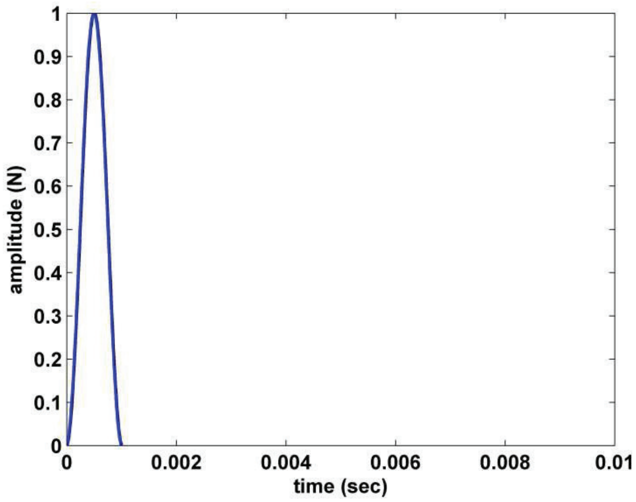


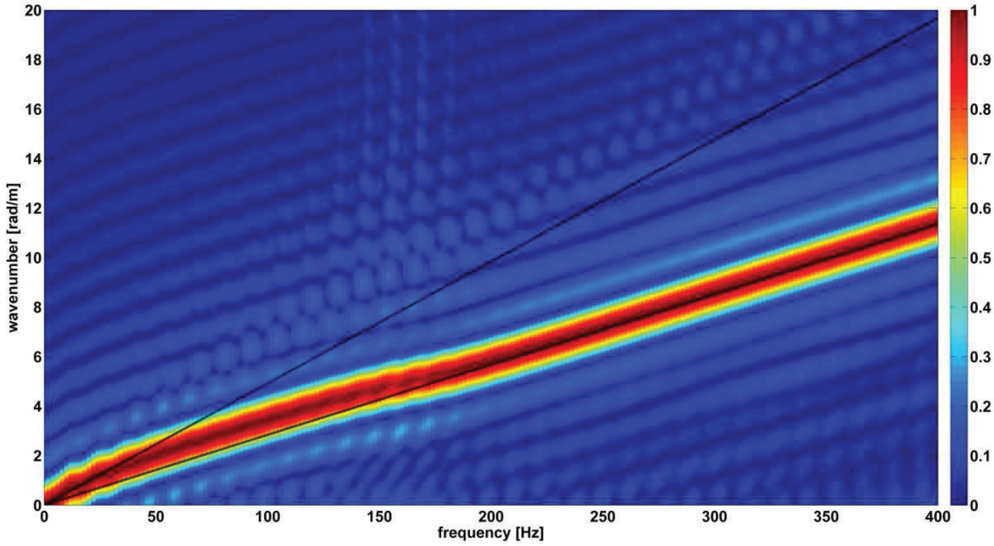
Figure 1. Time history of the load used in the simulation.

3.2 Benefits of Multi-Component Method

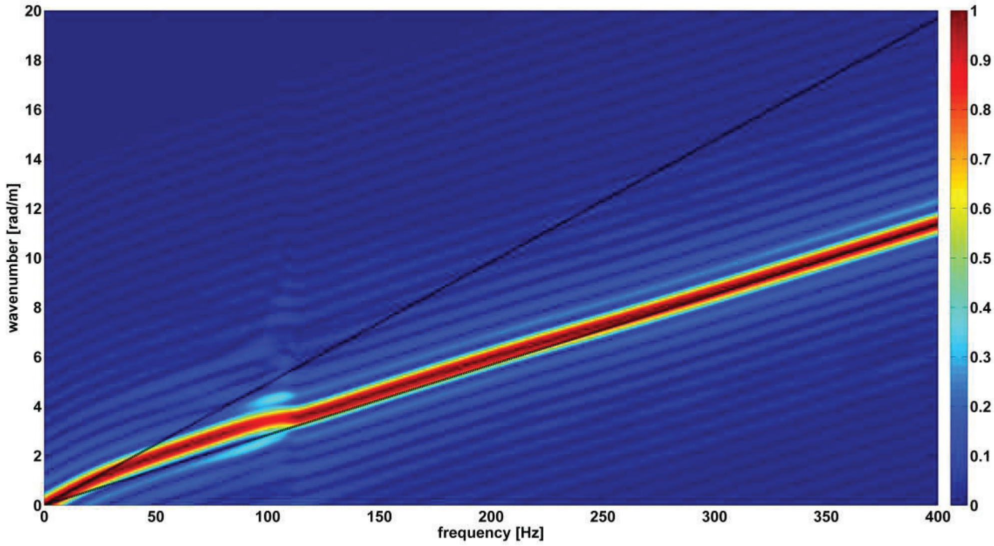
The benefits of the multi-component method are shown by comparing the resolution of the *f-k* spectra obtained with the numerical simulation and by looking at the signals in the time domain.

Figure 2a depicts the *f-k* spectrum obtained compiling the vertical signals coming from 100 points equally spaced by 0.05m, while Figure 2b depicts the *f-k* spectrum obtained compiling the same number of signals using the multi-component method. In both cases the slope of the two black lines indicates the velocity of the direct Rayleigh wave in the upper layer and in the half-space, as calculated from the constitutive relationships [19]. The *f-k* obtained with the analytical solution is depicted in Figure 3: the analytical solution of the wave propagation problem in a layered medium used in this work is based on the use of stiffness matrices in the frequency-wavenumber domain [20].

From a comparison between the two *f-k* spectra of Figure 2 it is possible to notice that the multi-component method leads to a better resolution and allows the possibility to distinguish different propagation modes. In fact, the resolution increases by a factor of two, as can be perceived from the energy peaks being narrower. Moreover, the spectrum of Figure 2b is closer to the *f-k* spectrum obtained with the exact solution of Figure 3. With the multi-component approach is possible to find out the presence of a mode jump, i.e. an osculation point where the energy jumps from a mode to another (or where the dispersion curves of different modes come close to each other) at a frequency of approximately 130 Hz . The existence of a mode jump is confirmed by looking at the analytical *f-k* spectrum (Figure 3). It seems impossible to recognize different modes of propagation or to localise the mode jump by only looking at the spectrum of Figure 2a, which rather seems to describe a single apparent continuous mode of propagation. The damping ratio used for the analytical solution is the same used in the FEM simulations.



(a)



(b)

Figure 2. Normalised *f-k* spectrum with 100 vertical traces (a) and with 100 multi-component traces (b). The slope of the two black lines indicates the velocity of the Rayleigh wave in the upper layer and in the half-space. The spectrum obtained with the traditional method seems to describe e a single mode of propagation, while is possible to notice a mode jump at the frequency of about 130 Hz when the multi-component method is utilized.

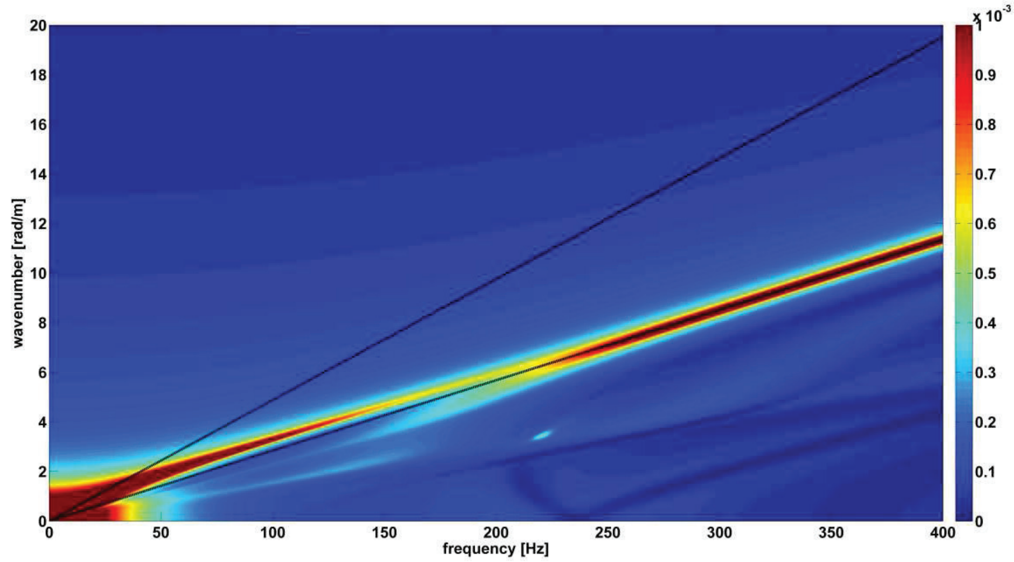


Figure 3. Analytical solution in the normalised *f-k* domain, the slope of the two black lines indicates the velocity of the direct Rayleigh wave in the upper layer and in the half-space. It is possible to notice a mode jump at the frequency of about 130 Hz.

Figure 4a depicts the vertical, horizontal and multi-component displacements picked at a surface point 1.75m far from the source, obtained from the finite element model. The shift of $\pi/2$ between the horizontal and vertical displacements descending from the retrograde elliptical motion is evident, since horizontal displacements peak when the vertical displacements are zero. Moreover, the difference between Rayleigh wave amplitude and body waves amplitude increases when the multi-component approach is adopted. In fact, compressional and shear energy in the multi-component signal is minimal.
The amplitude of the *f-k* spectra obtained with 100 signals at the frequency of 100Hz, which can be seen as a slice of the *f-k* spectrum at a certain frequency, is displayed in Figure 4b. The peaks of the *f-k* spectra are associated to the energy maxima and hence to the vibrational modes of propagation. The figure reports also the amplitude of the *f-k* spectrum obtained with the analytical solution. With the multi-component method the half-width between the neighbouring minima is halved, i.e. the resolution is doubled, with respect to the resolution of the *f-k* spectrum obtained using the vertical components. In addition, multi-component method leads to an improvement in the accuracy: the spectra obtained with the multi-component method and with the analytical solution peak at approximately the same wavenumber, while the position of the peak of the spectrum obtained with vertical traces seems inaccurate. Figure 4 clearly shows the improvement in the resolution and the ability in discerning different modes of propagation of the multi-component approach.

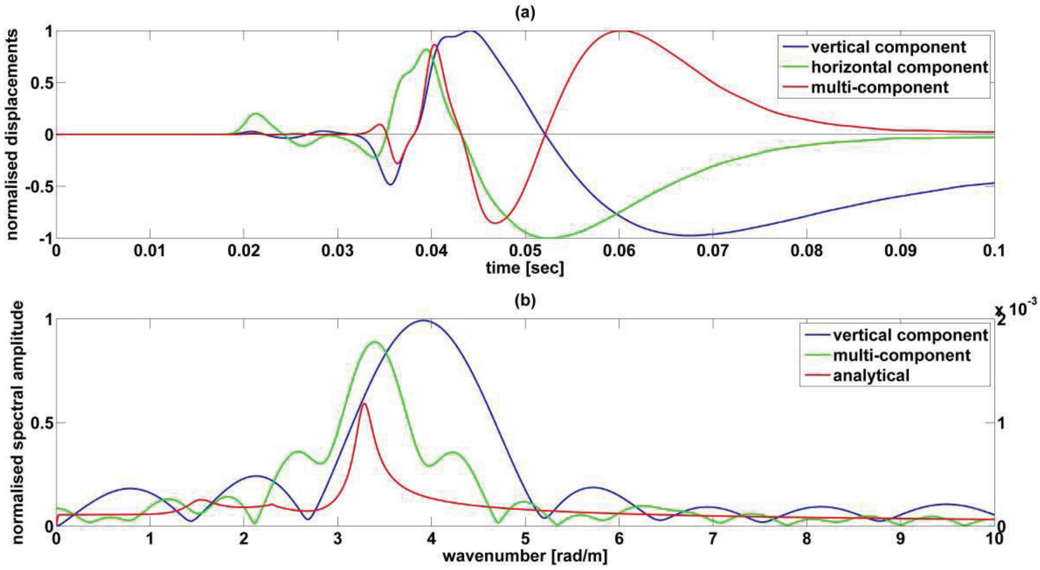


Figure 4. Normalised displacements picked at a surface point 1.75m far from the source (a) and normalised spectral amplitude of the *f-k* spectrum at 100 Hz (b), from numerical simulation. The multi-component trace contains minimal body wave energy and leads to a more accurate estimation of the dispersive behaviour.

4. Preliminary *in-situ* Experiments

Here the effects of the multi-component approach on the *f-k* transformation are evaluated with respect of an *in-situ* experiment on soil.

The experimental set-up consists of a source and an array of 21 tri-axial geophones, disposed as in Figure 5, covering a length L of 5.00m. The data was acquired using a ProSig P8020 data acquisition unit and a laptop. The tests was repeated and recorded 5 times with a sample frequency of 8kHz and duration of 3sec, under the same input conditions, and then averaged in the frequency domain.

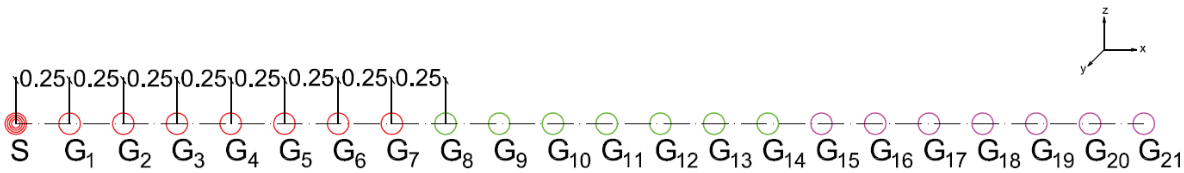


Figure 5. Experimental set-up, G refers to geophone, S refers to source and the number identifies the position. Distances are shown in metres.

The source consisted of a 4-oz metallic mallet striking on a circular aluminium plate of 0.15m diameter and 1.5cm thick. The data acquisition was triggered with respect to the hammer impact.

F-k spectra were obtained utilizing the vertical traces from the geophones and also utilizing the vertical and horizontal traces, in a multi-component sense as described previously. They are depicted in Figure 6. At a first glance it is possible to notice that the spectrum obtained with multi-component traces (6b) has narrower energy peaks, i.e. a better resolution. This *f-k* spectrum is also better defined in the low frequency range (frequencies lower than 35Hz), where is possible to notice clear energy peaks; it is not possible to extract this information from the spectrum obtained from vertical traces (6a).

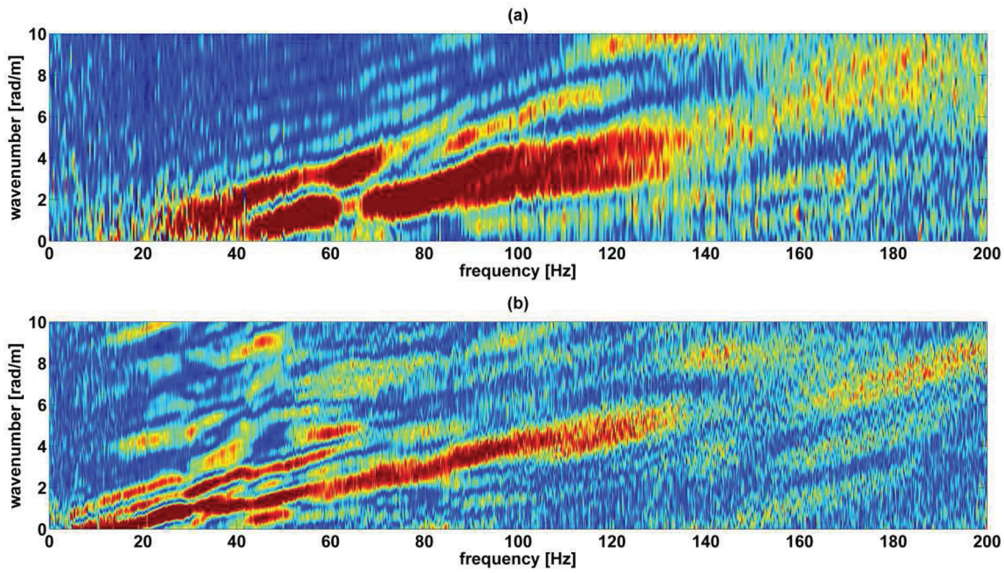


Figure 6. Normalised *f-k* spectrum with vertical traces (a) and with multi-component traces (b) coming from the experimental session. The spectrum obtained with multi-component traces has narrower peaks and is better defined for low frequencies (under 35 Hz).

5. Conclusions

The multi-component approach to seismic data proposed here has been found to be beneficial in enhancing the Rayleigh wave content of a seismic signal, lowering the content of noise, defined as unwanted waves.

In the frequency-wavenumber spectrum obtained via the multi-component method, the energy peaks are sharper (i.e. the bandwidth of each peak is smaller), allowing for a better separation of different normal Rayleigh wave modes and improving the accuracy of the survey. Moreover, it has been proven to increase the detection of energy peaks at low frequencies (lower than 35Hz) and to augment the resolution of the *f-k* spectrum by a factor of two in both domains.

Thus the method could be advantageous in such cases when the area to survey is difficult to access, small, or whenever a long deployment of sensors is impossible. The method could be particularly beneficial in soil deposits where the stiffness is not regular, i.e. when higher modes play a relevant role in the wave propagation and the fundamental one is not anymore the mode carrying more energy, and to survey bigger depths.

It has proven to be a valid multimode separation technique, able to enhance the Rayleigh energy extracted from a dispersion measurement by doubling the resolution with respect to the classical single-component testing. It also avoids considering lateral inhomogeneity and variations from the main assumption of horizontality of layers that are likely to happen if the surveyed area increases.

Acknowledgements

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