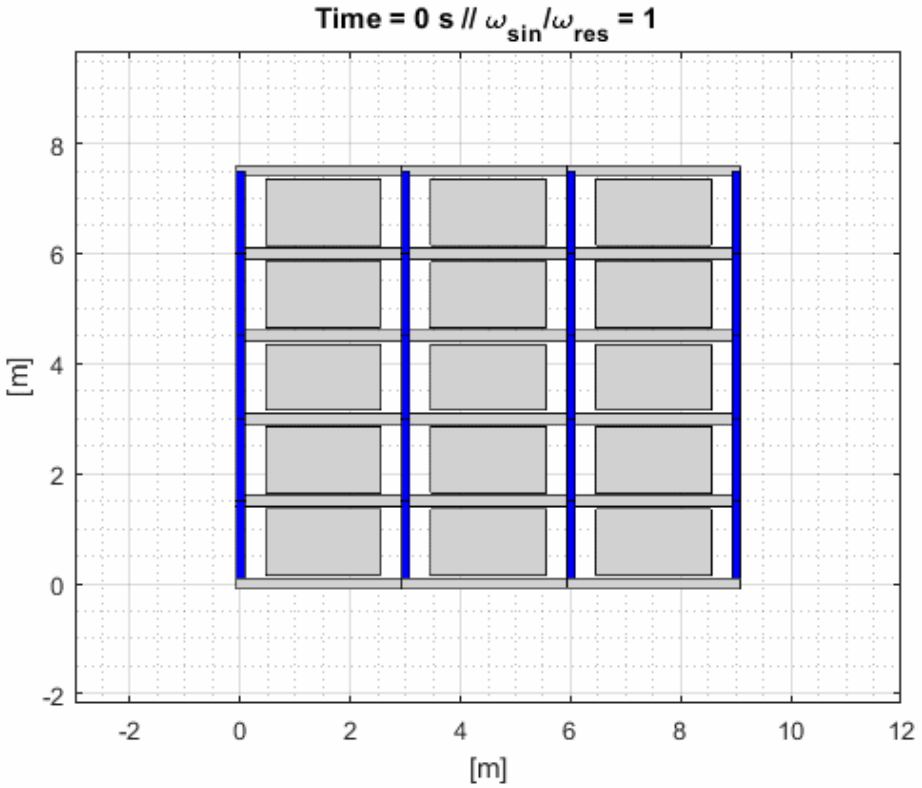
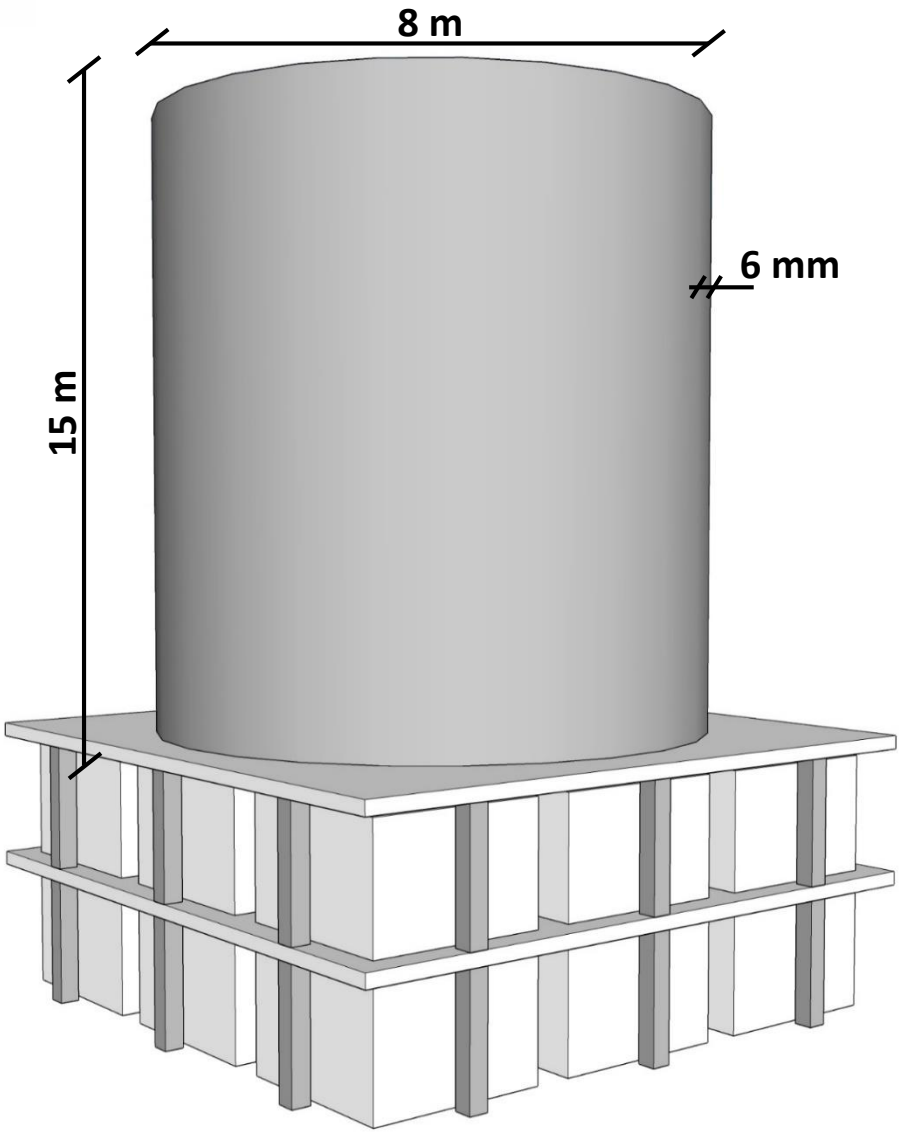
# Metamaterial-based Foundation System endowed with Non-Linear Oscillators

Moritz Wenzel, Roberto Andreotti, Oreste S. Bursi

Department of Civil, Environment and Mechanical Engineering,
University of Trento, Via Mesiano 77, 38123, Trento, Italy.

Moritz.wenzel@unitn.it
roberto.andreotti@unitn.com
oreste.bursi@unitn.it

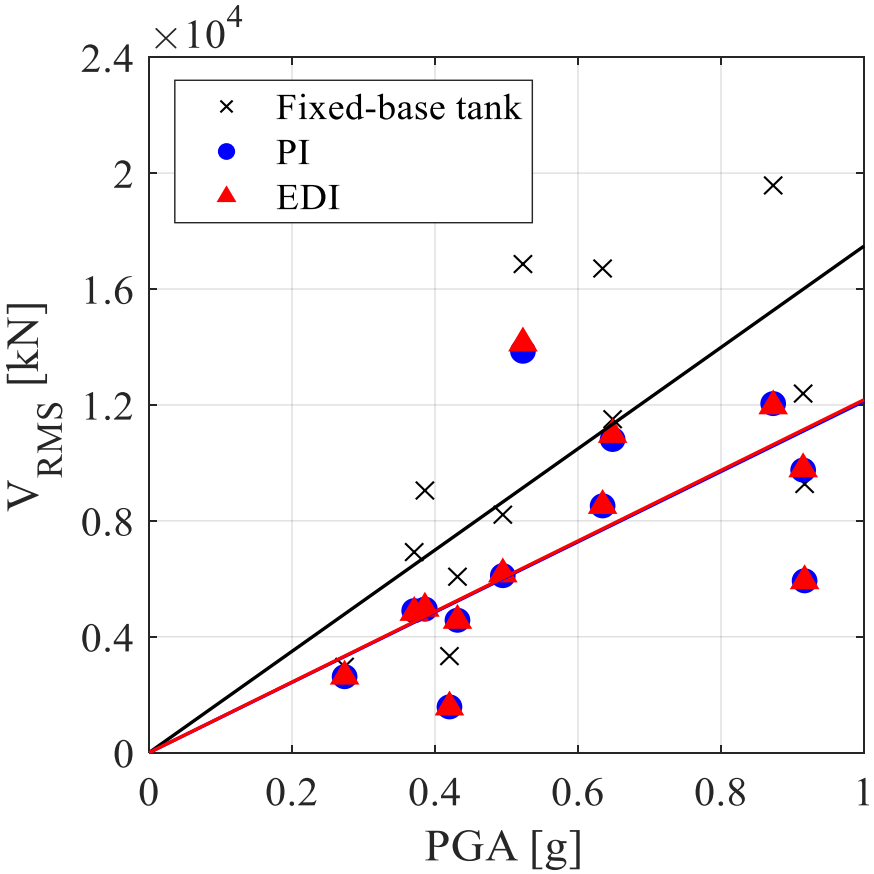


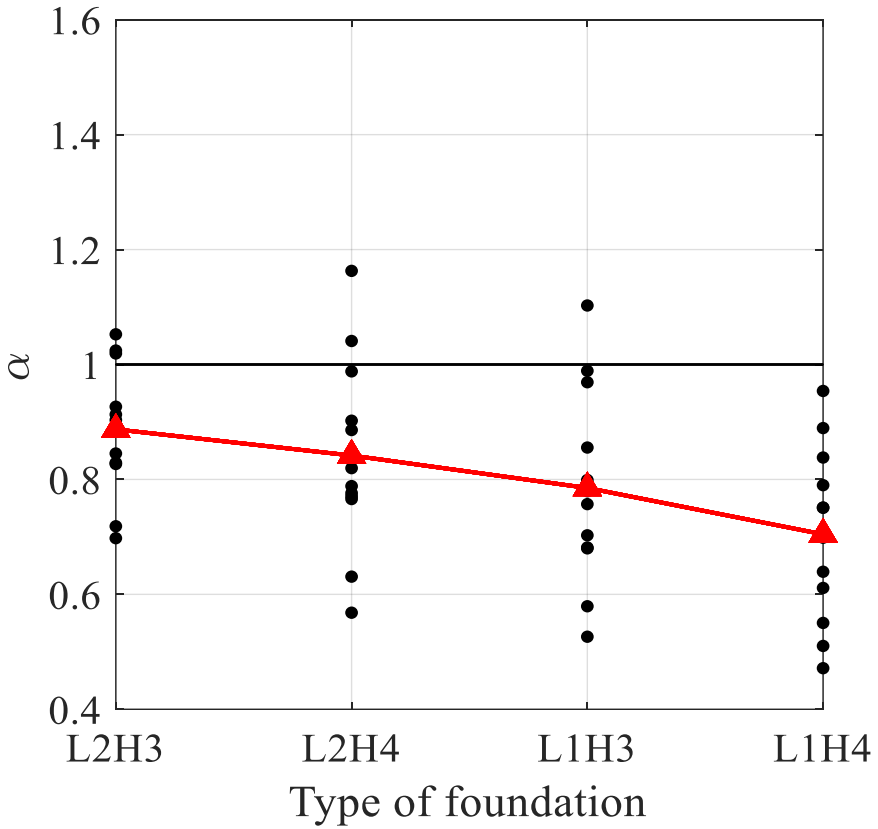




# Foundation

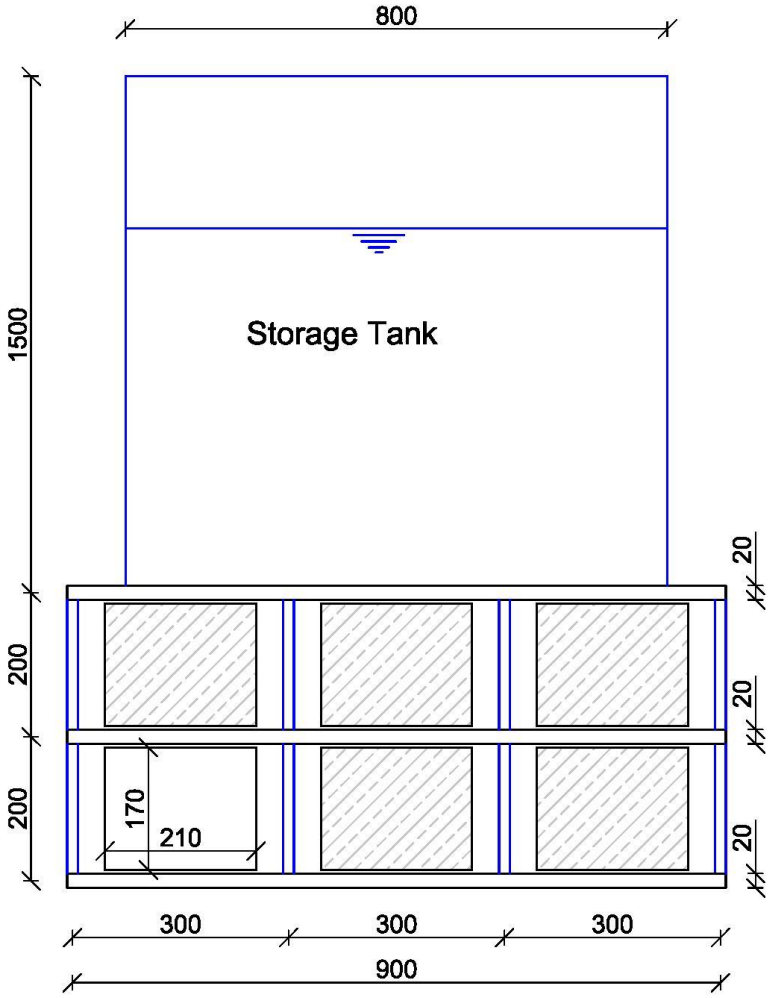
# Coupled System

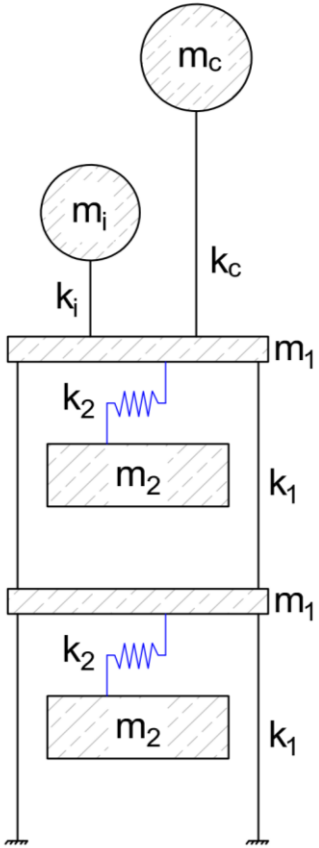
## Time History Analysis

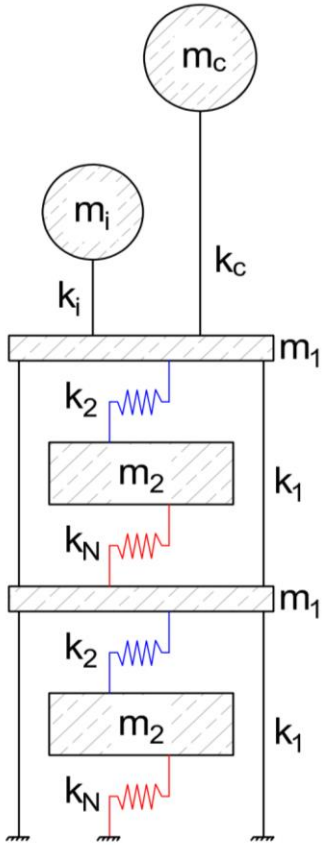


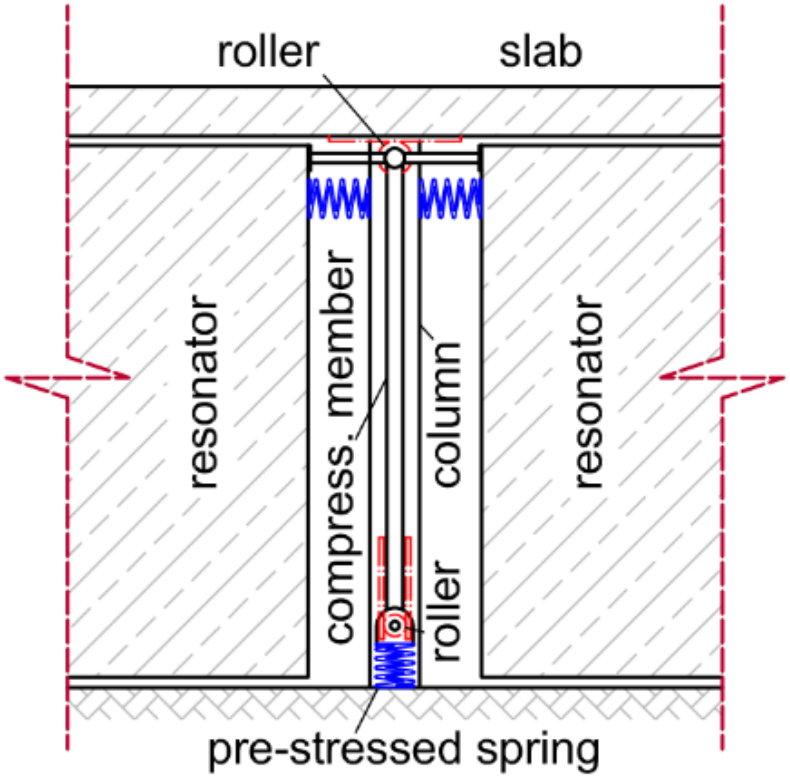
# Negative Stiffness

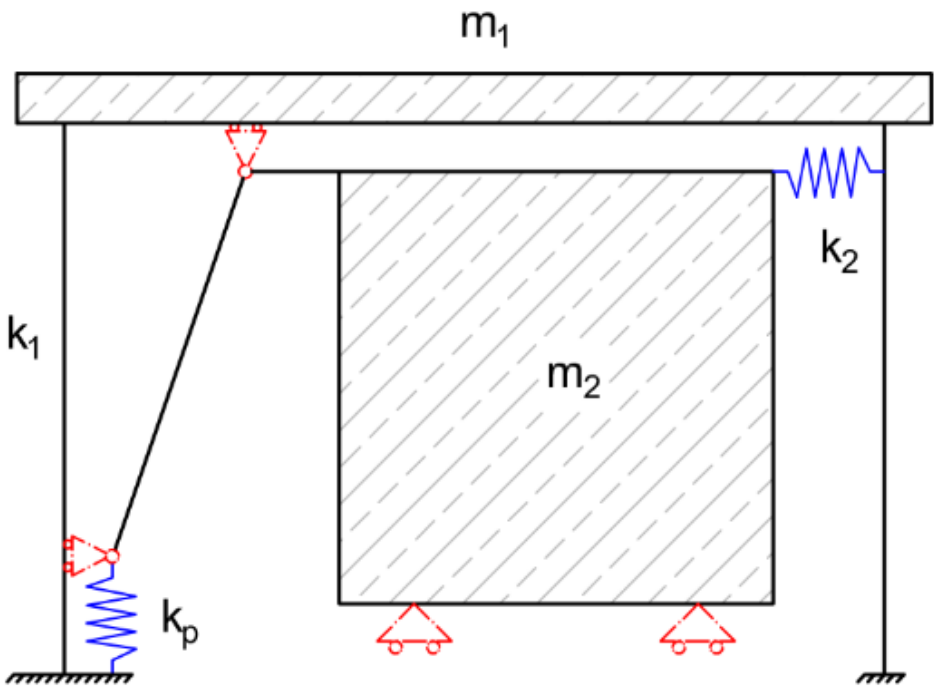


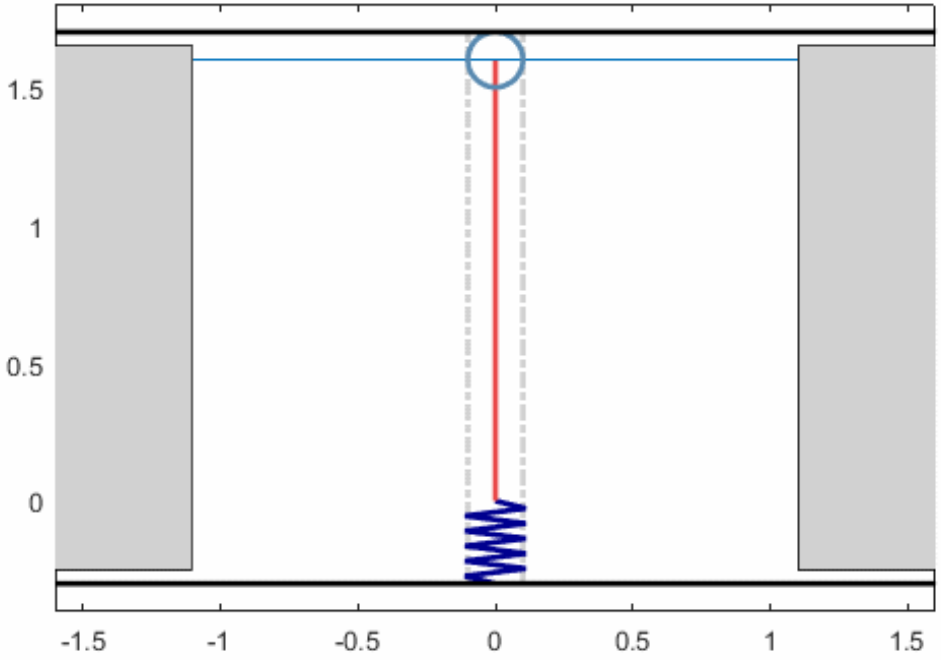



# Mechanism


roller
slab
resonator
compress. member
column
roller
resonator
pre-stressed spring

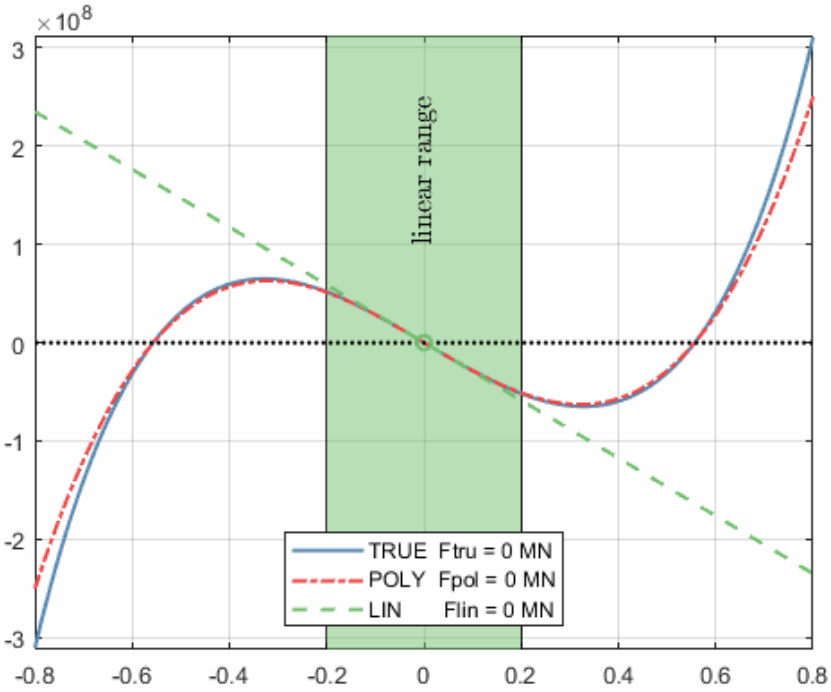

m1
k1
k2
m2
kp

# P-Δ Relationship

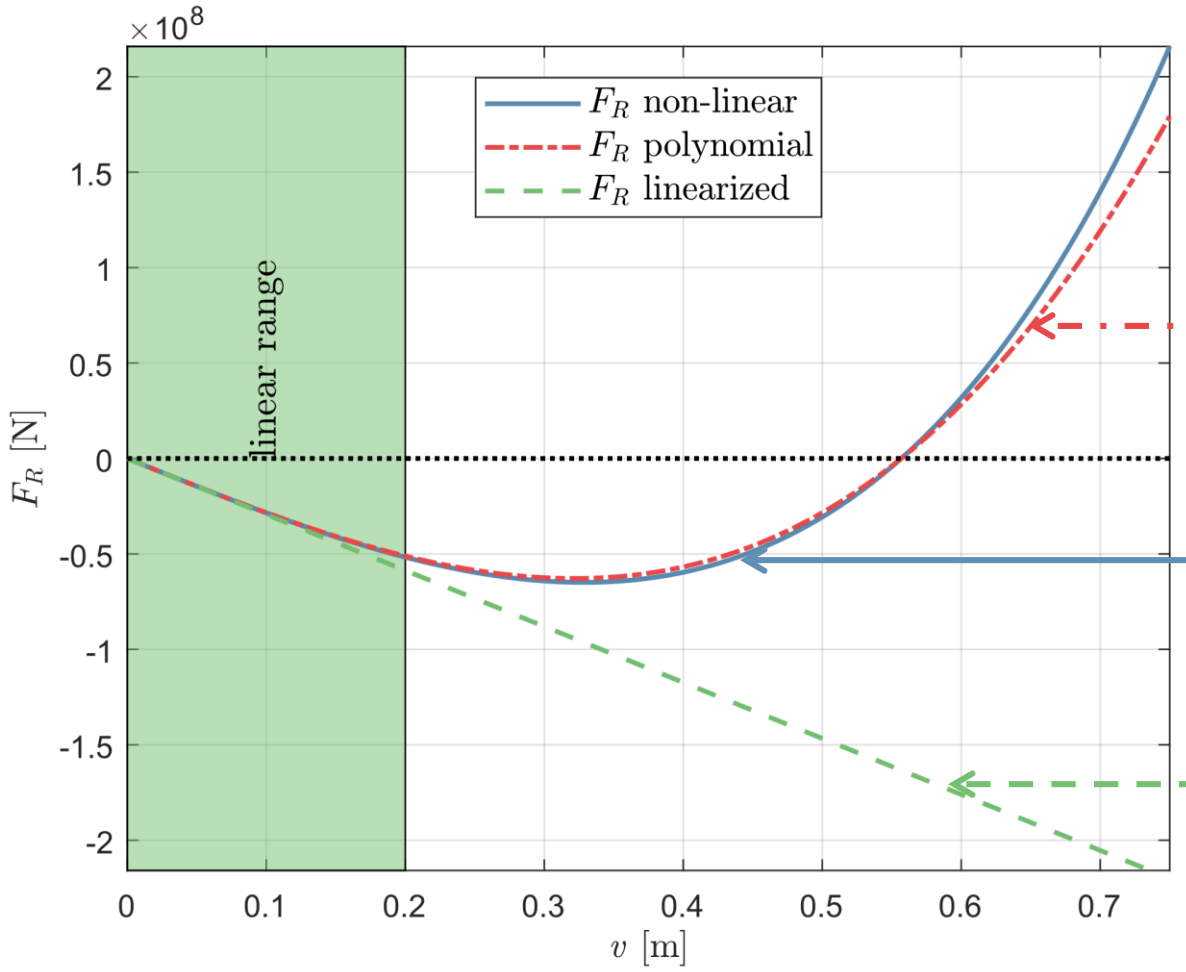



**Mechanism in Motion**

**Force on Resonators**



# P-Δ Relationship

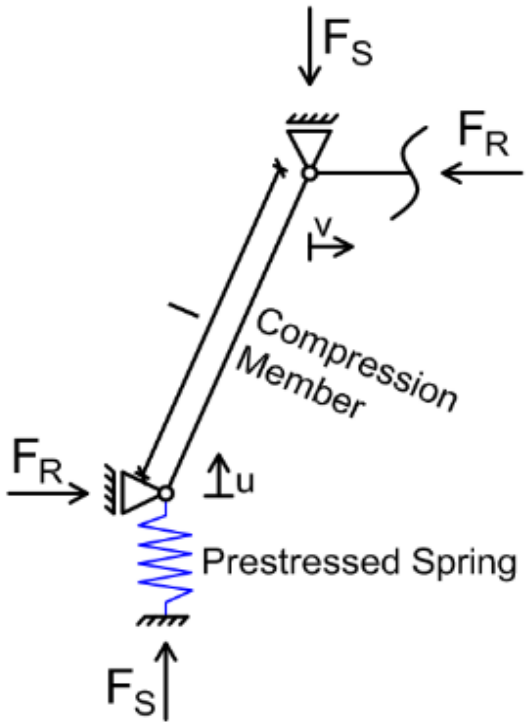



$$Fr(v) = v \cdot a_{NSE} + v^3 \cdot b_{NSE}$$

$$Fr(v) = \frac{v\left(-P + k_p\left(l - l\sqrt{1 - \frac{v^2}{l^2}}\right)\right)}{l\sqrt{1 - \frac{v^2}{l^2}}}$$

$$Fr(v) = v \cdot a_{NSE}$$

# P-Δ Relationship




**True Relationship**

$$F_{TRU}(v) = \frac{v\left(-P + k_p\left(l - l\sqrt{1 - \frac{v^2}{l^2}}\right)\right)}{l\sqrt{1 - \frac{v^2}{l^2}}}$$

**Polynomial Approximation**

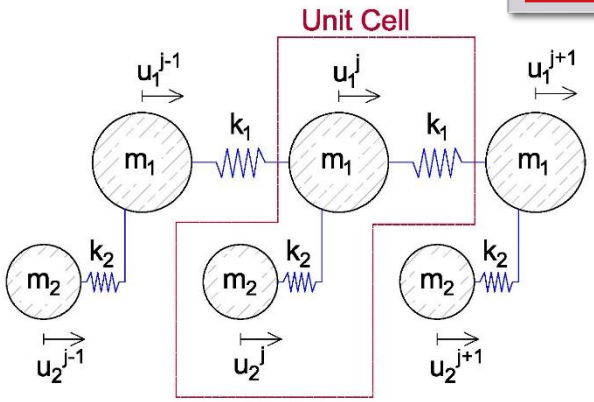
$$F_{POL}(v) = v \cdot a_{NSE} + v^3 \cdot b_{NSE}$$

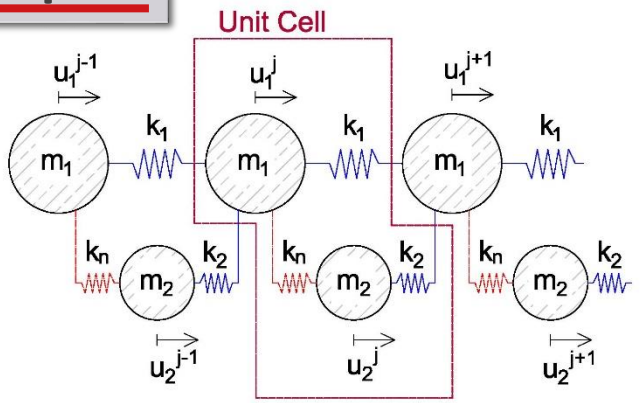
$$a_{NSE} = -\frac{P}{l} \qquad b_{NSE} = \frac{{k_P}^2}{l(2 \cdot k_P \cdot l - P)}$$

**Linearized Path**

$$F_{LIN}(v) = v \cdot a_{NSE}$$

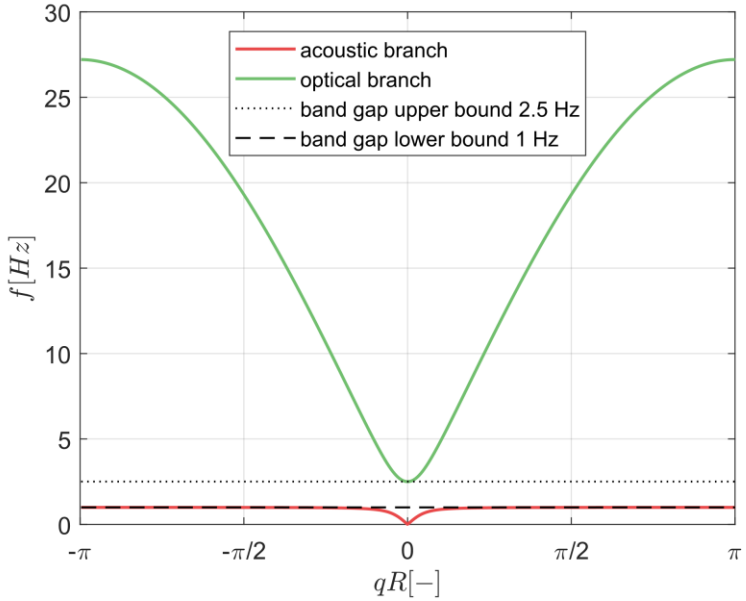
# Band Gaps

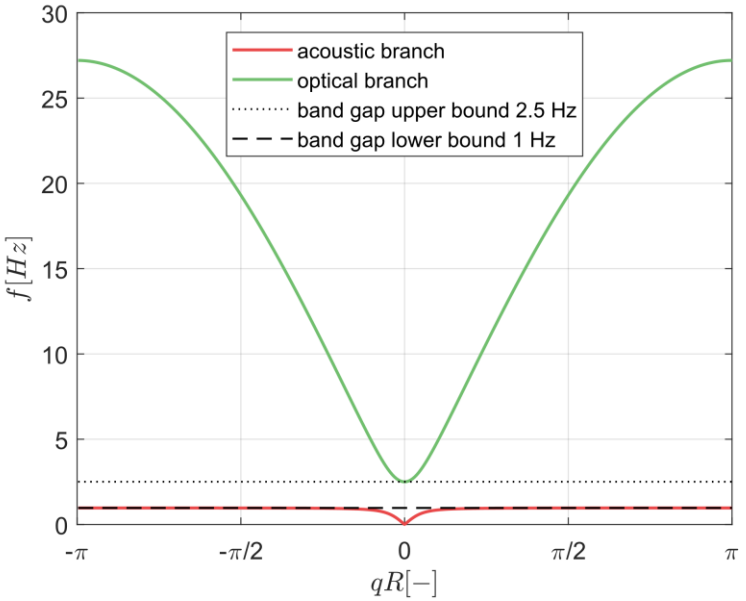



$$m_1 m_2 \omega^4 - [(m_1 + m_2)k_2 + 2m_2 k_1(1 - \cos(qR))]\omega^2 + 2k_1 k_2(1 - \cos(qR)) = 0$$

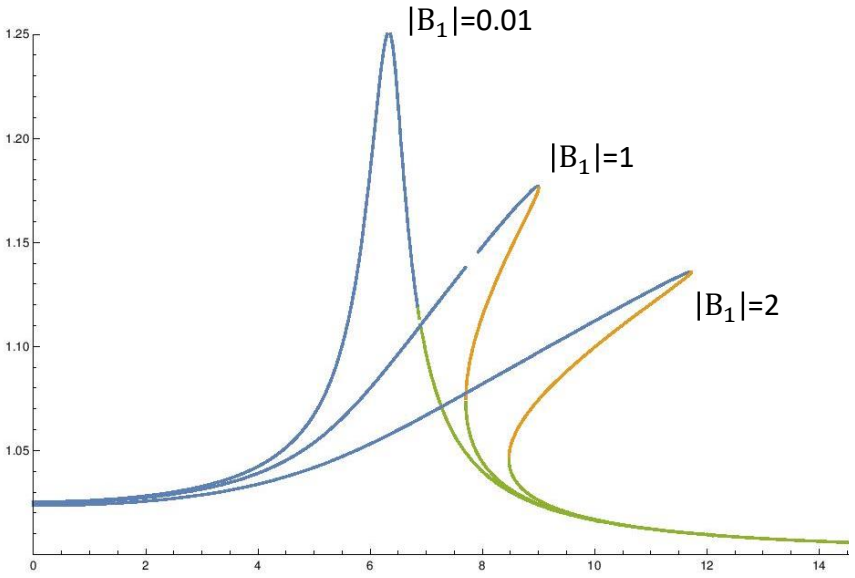
$$m_1 m_2 \omega^4 - [-k_n m_1 - 2k_1 m_2 - k_n m_2 - k_2(m_1 + m_2) + 2k_1 m_2 \cos(qR)]\omega^2 + 4[k_2 k_n + k_1(k_2 + k_n)] \sin\left(\frac{qR}{2}\right) = 0$$

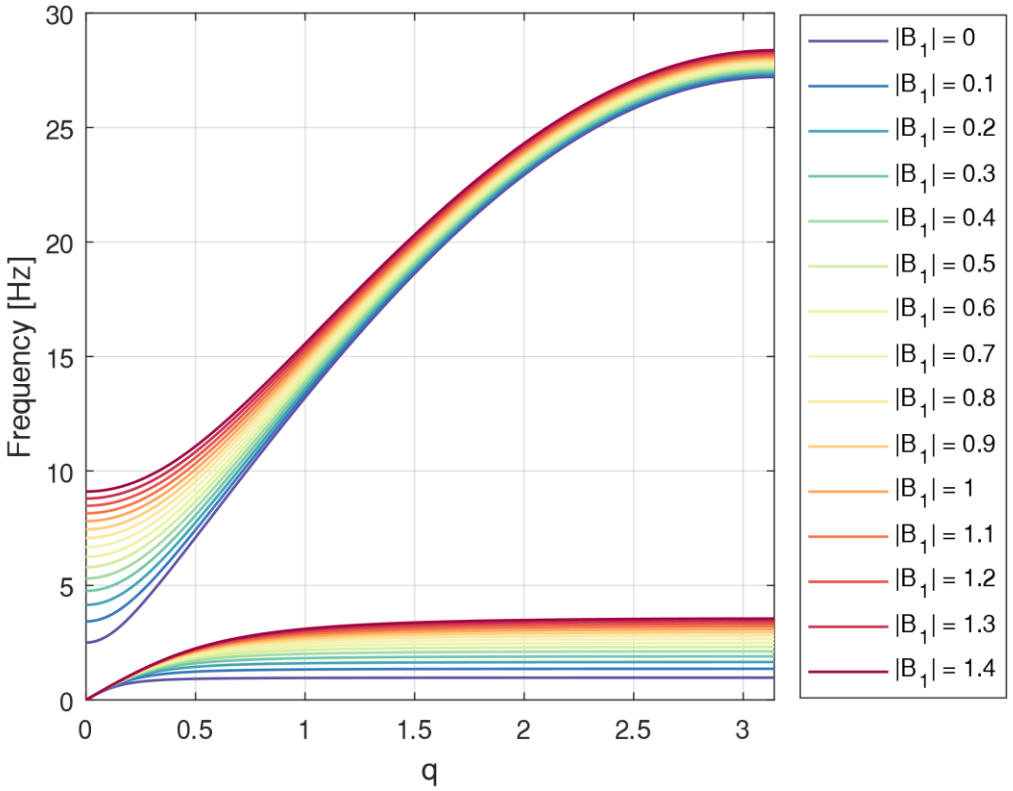


# Band Gaps

## Harmonic Balance Method – Poly. Non-Linearity

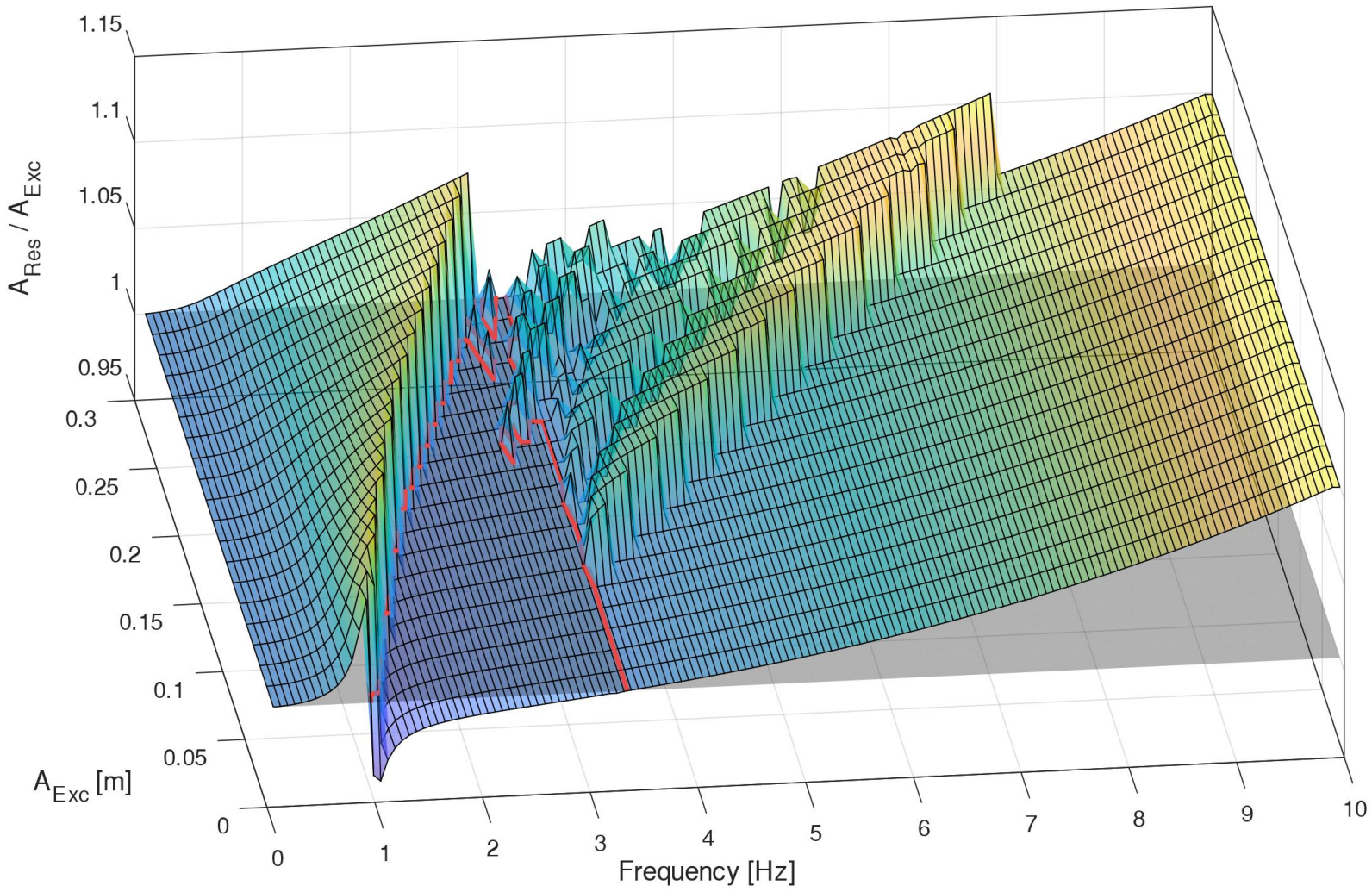



$$Cos(q) = \frac{2k_1k_2 - k_2m_1\omega^2 - 2k_1m_2\omega^2 - k_2m_2\omega^2 + m_1m_2\omega^4 + a(2k_1 + 2k_2 - (m_1 + m_2)\omega^2) + 3bB_1\bar{B}_1(2k_1 + 2k_2 - (m_1 + m_2)\omega^2)}{2(a(k_1 + k_2) + 3bB_1\bar{B}_1(k_1 + k_2) + k_1(k_2 - m_2\omega^2))}$$

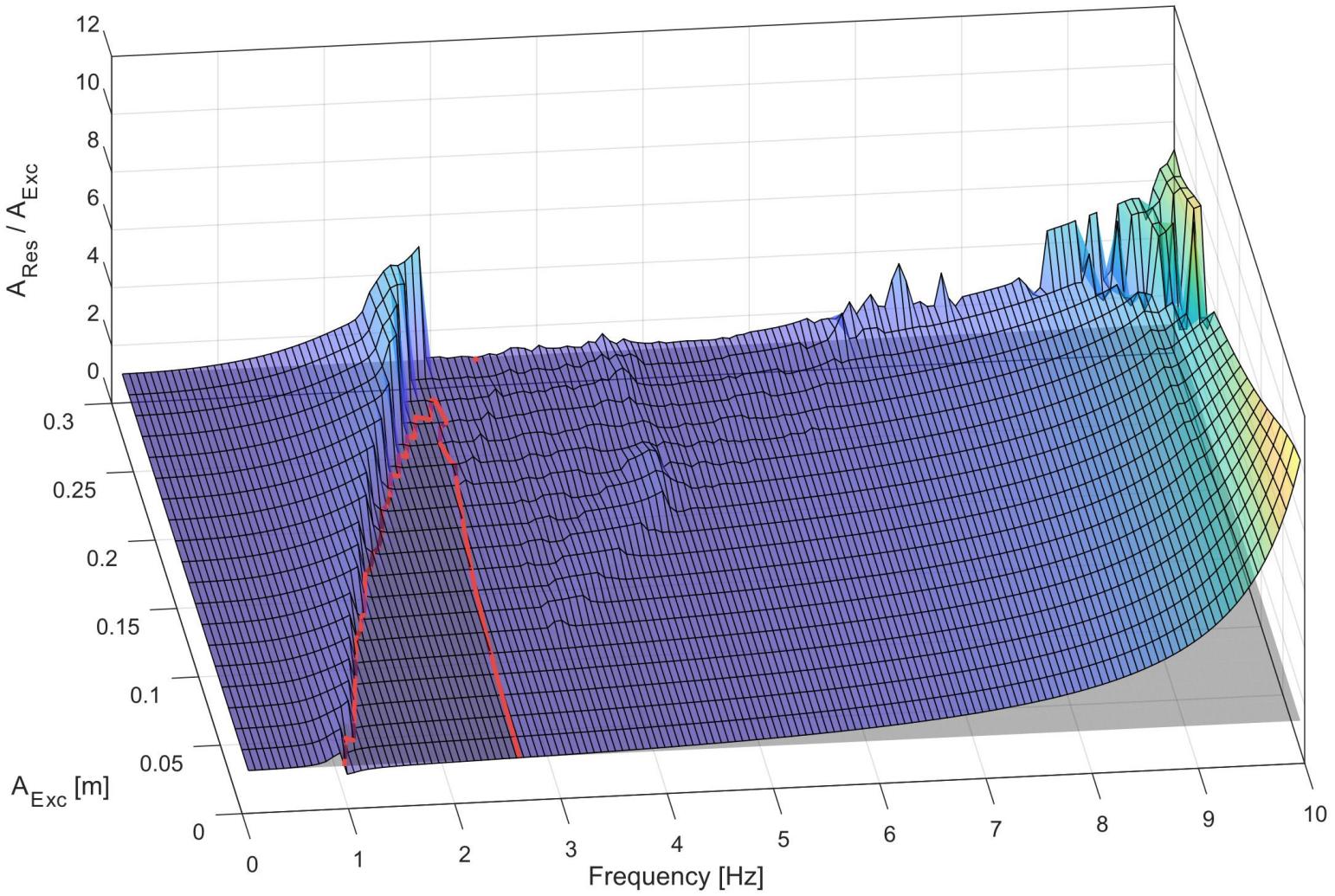
**Layers: 1**

# FRF Numerical



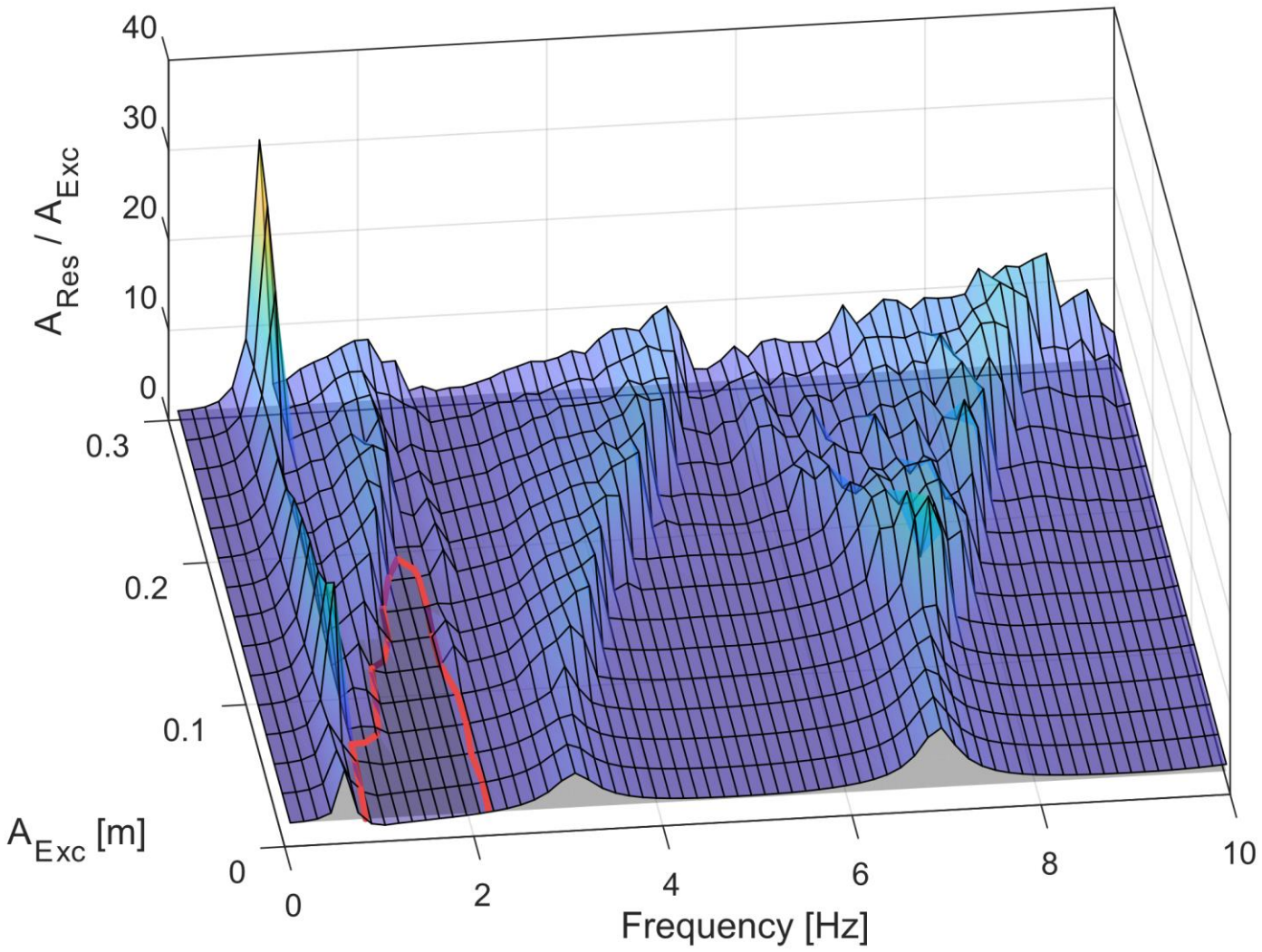
**Layers: 4**

# FRF Numerical



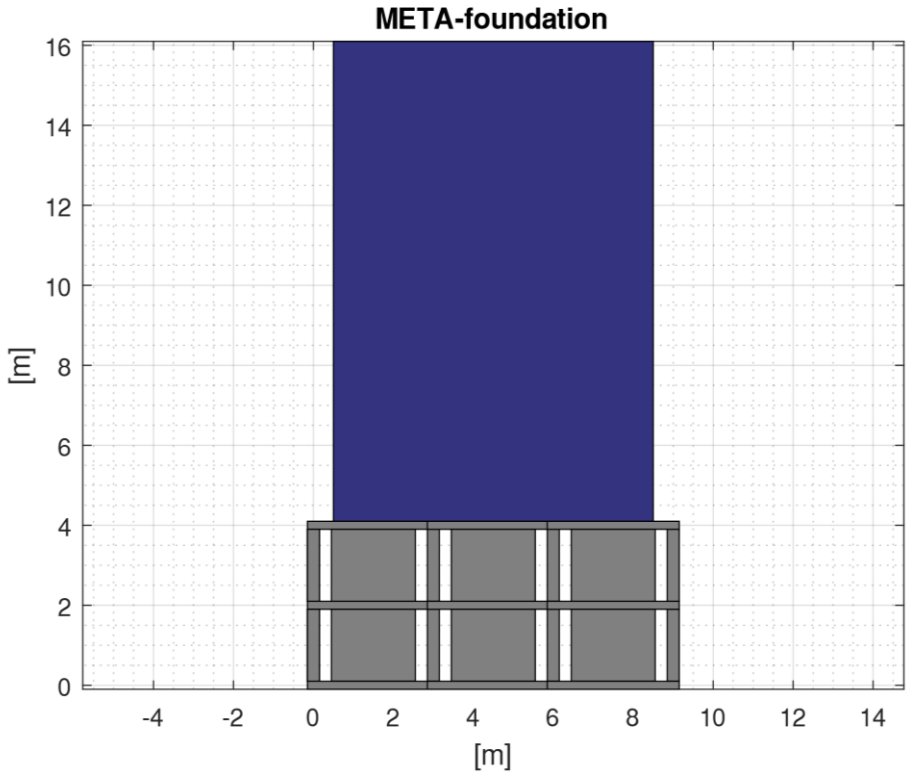
**Layers: 20**

# FRF Numerical

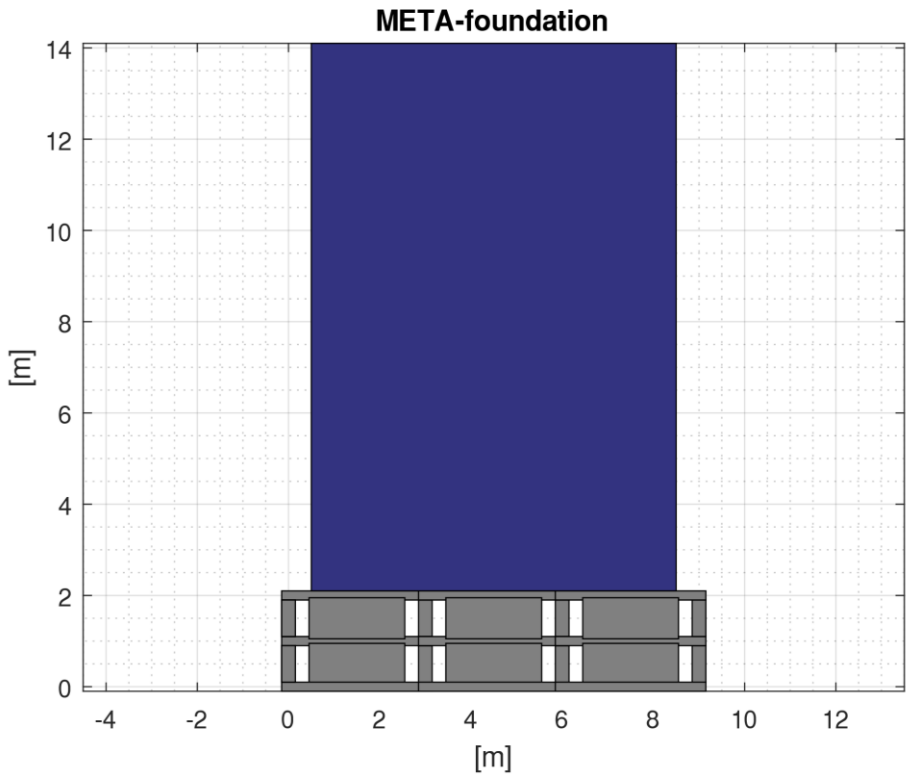


# Time History Analysis

## Original System

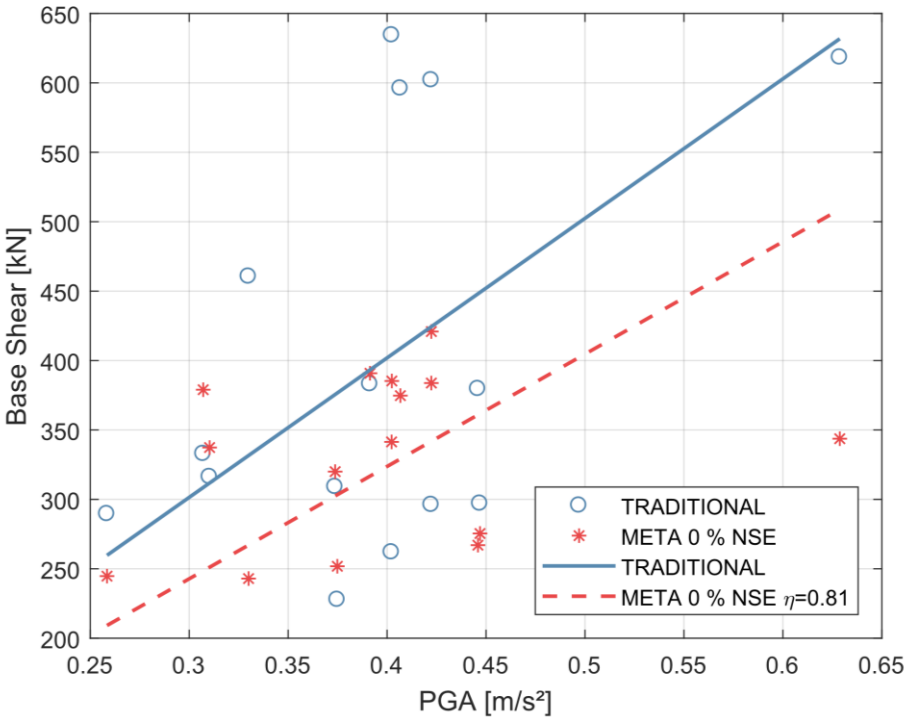



## NSE enhanced System



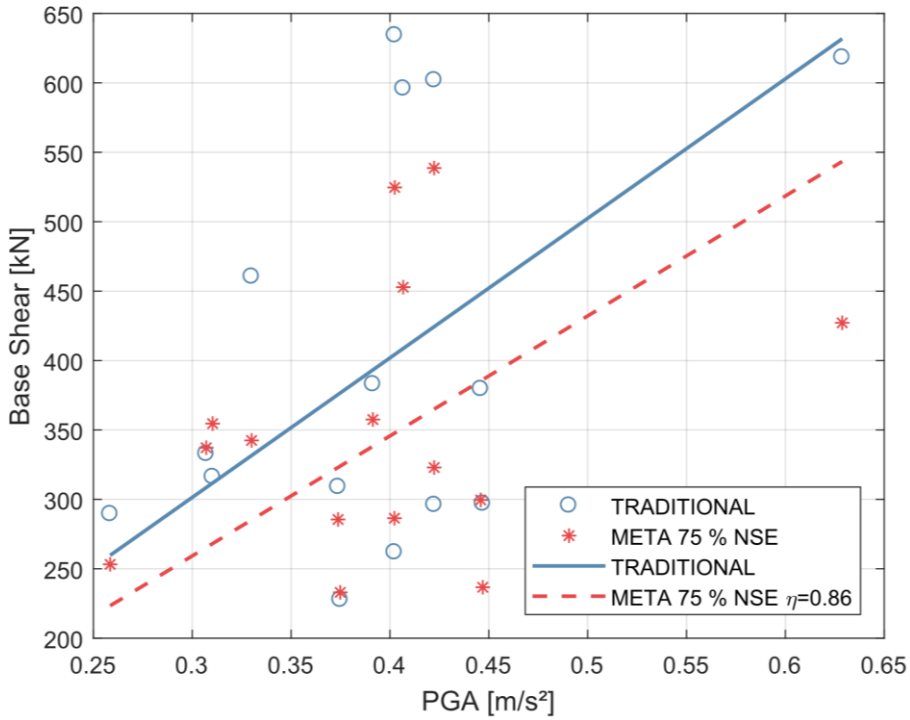
# Time History Analysis

## Original System



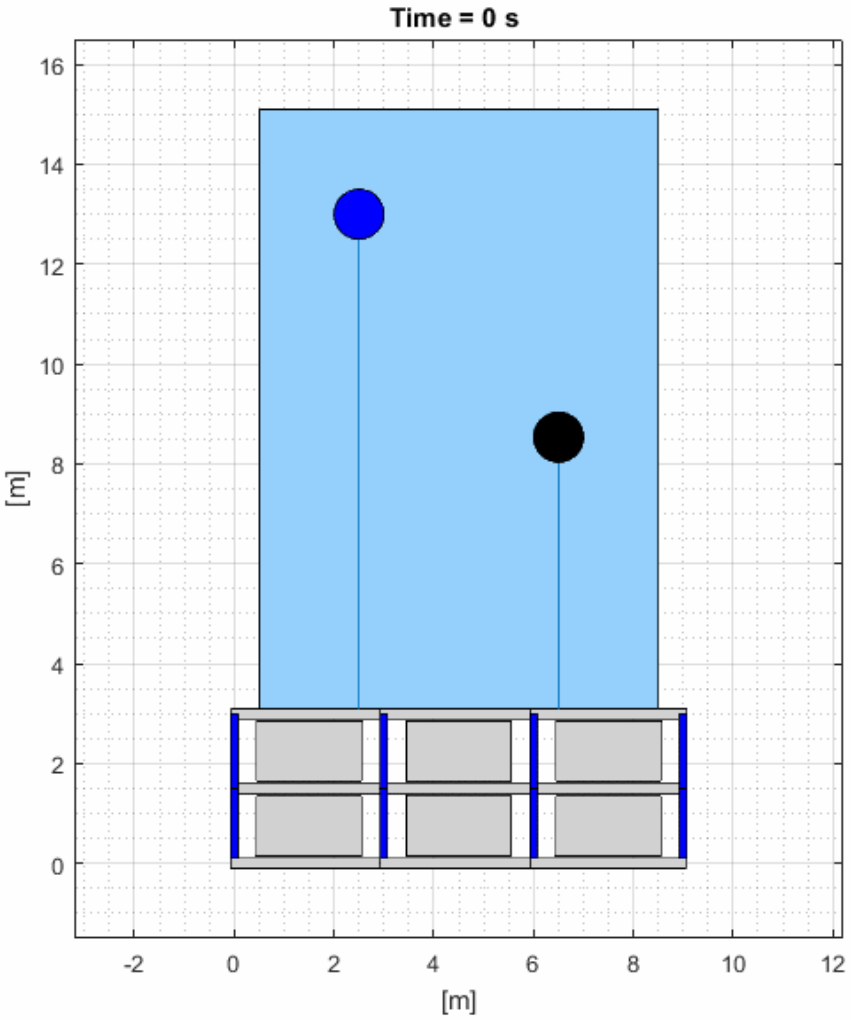

**η = 0.81**

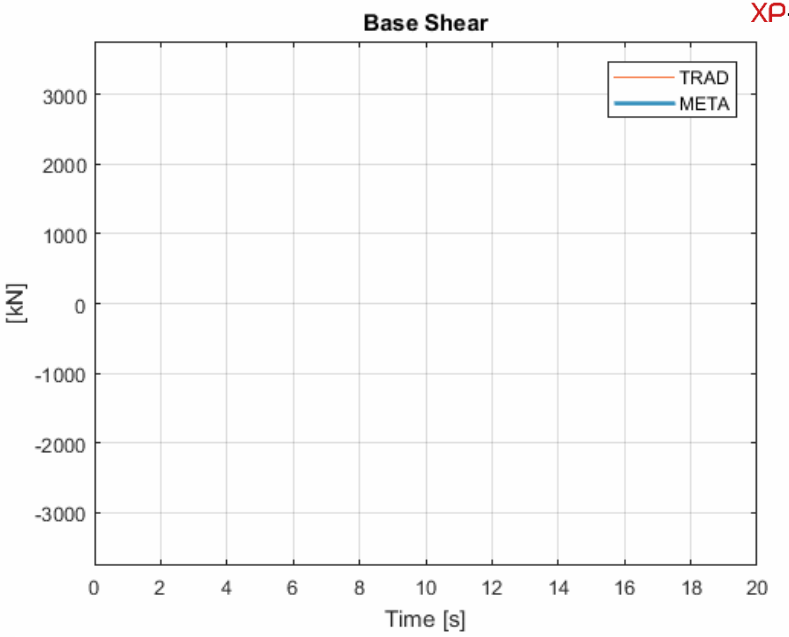
## NSE enhanced System

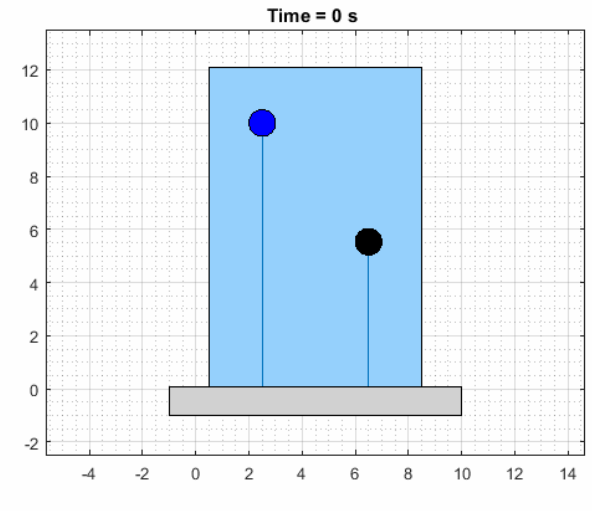



**η = 0.86**

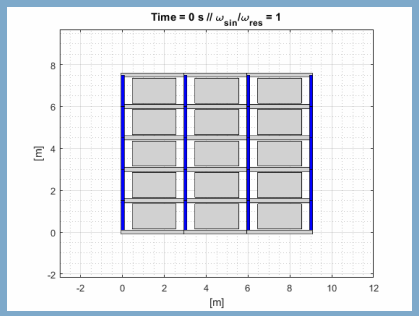
# Coupled System



XP-resilience




Acknowledgements:

XP-Resilience
Ref: 721816

https://r.unitn.it/en/dicam/xp-resilience

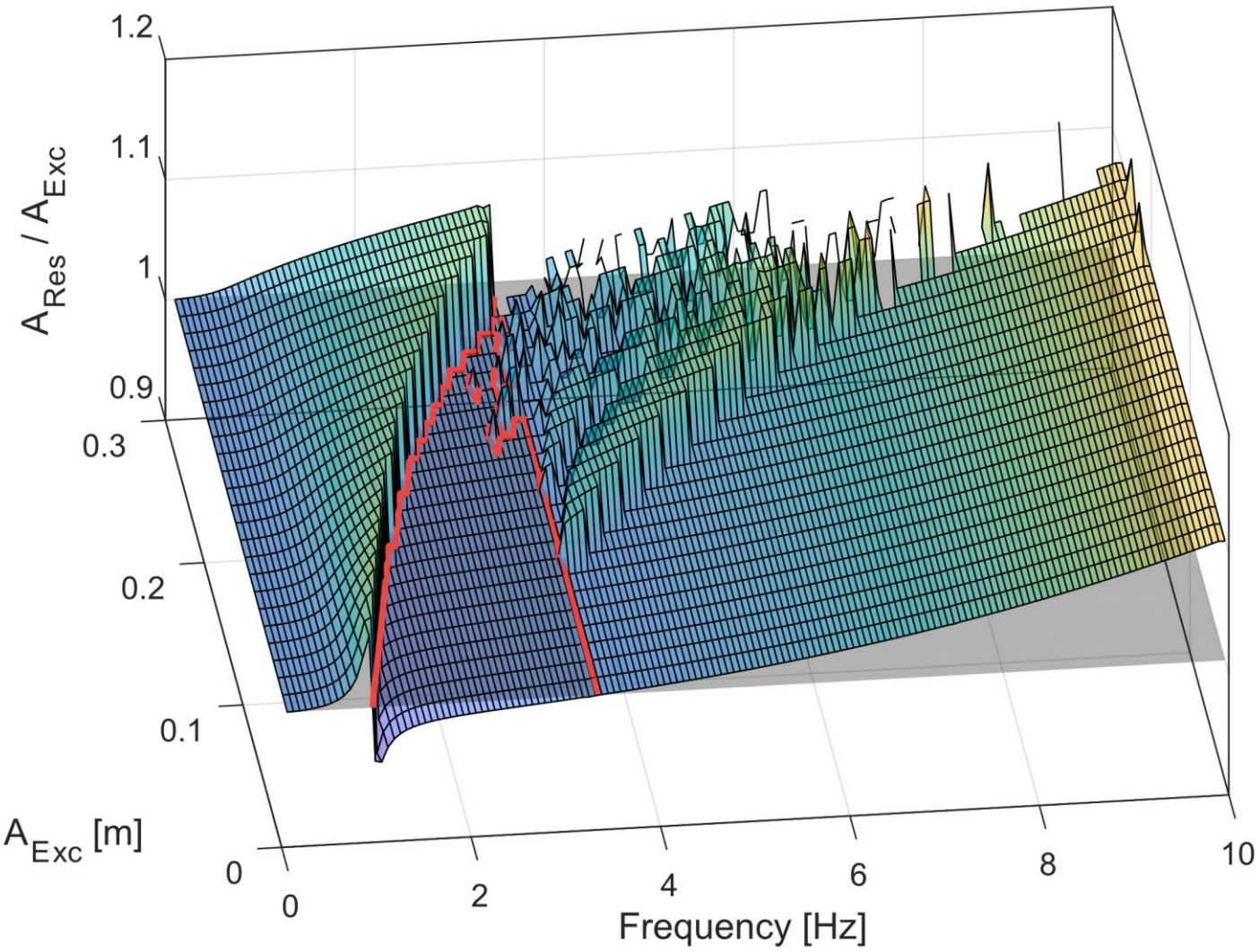
# Conclusion

- Metafoundation **feasible**
- **Negative Stiffness** element improves the performance

- **Economic design?**
- Details of the **Resonator mechanism 3D**
- Application to **different Structures**

**Layers: 1**

# FRF Numerical



# Optimization

Fourier Transformation of EOMs

$$\boldsymbol{M}\,(-\omega^2\boldsymbol{X}(\omega)) + \boldsymbol{C}\,(i\omega\boldsymbol{X}(\omega)) + \boldsymbol{K}(\boldsymbol{X}(\omega)) = \boldsymbol{P}(\omega)$$

$$(-\boldsymbol{M}\omega^2 + i\omega\boldsymbol{C} + \boldsymbol{K})\,\boldsymbol{X}(\omega) = \boldsymbol{P}(\omega)$$

Transmission Matrix

$$\boldsymbol{H}(\omega) = (-\boldsymbol{M}\omega^2 + i\omega\boldsymbol{C} + \boldsymbol{K})^{-1}$$

$\boldsymbol{H}(\omega)$ numerical Optimization

$$\boldsymbol{H}(\omega, k_2, c_2) = (-\boldsymbol{M}\omega^2 + i\omega\boldsymbol{C}(c_2) + \boldsymbol{K}(k_2))^{-1}$$

Power Spectral Density - Response

$$\boldsymbol{S_Q}(\omega, k_2, c_2) = |\boldsymbol{H}(\omega, k_2, c_2)|^2 \boldsymbol{S_{\ddot{u}_g}}$$

Variance of the Response

$$\sigma_{qi}^2(k_2, c_2) = \int_{-\infty}^{+\infty} S_{QI}(\omega, k_2, c_2)\, d\omega$$

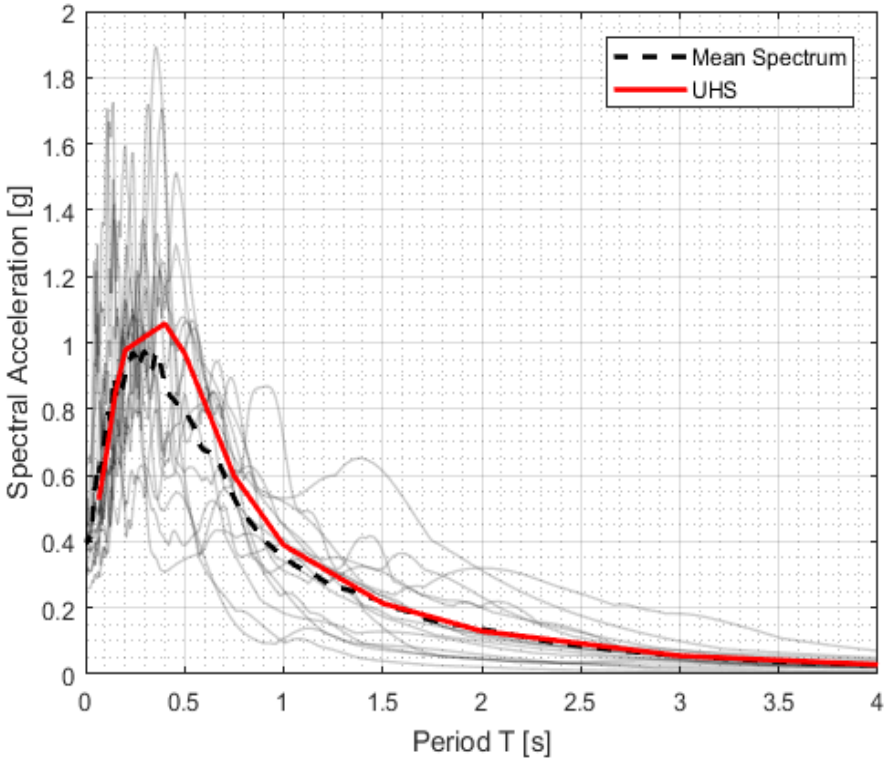
Performance Index

$$PI(k_2, c_2) = \frac{\sigma_{meta}^2(k_2, c_2)}{\sigma_{trad}^2}$$

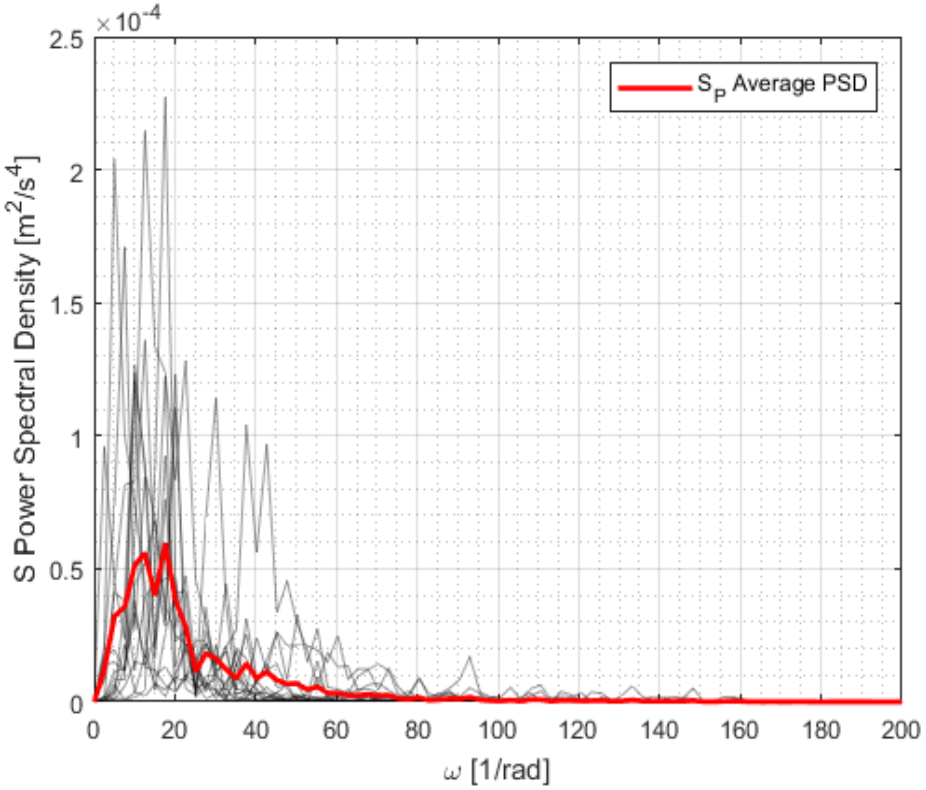
# Optimization

## Ground Motion Model

### Uniform Hazard Spectrum

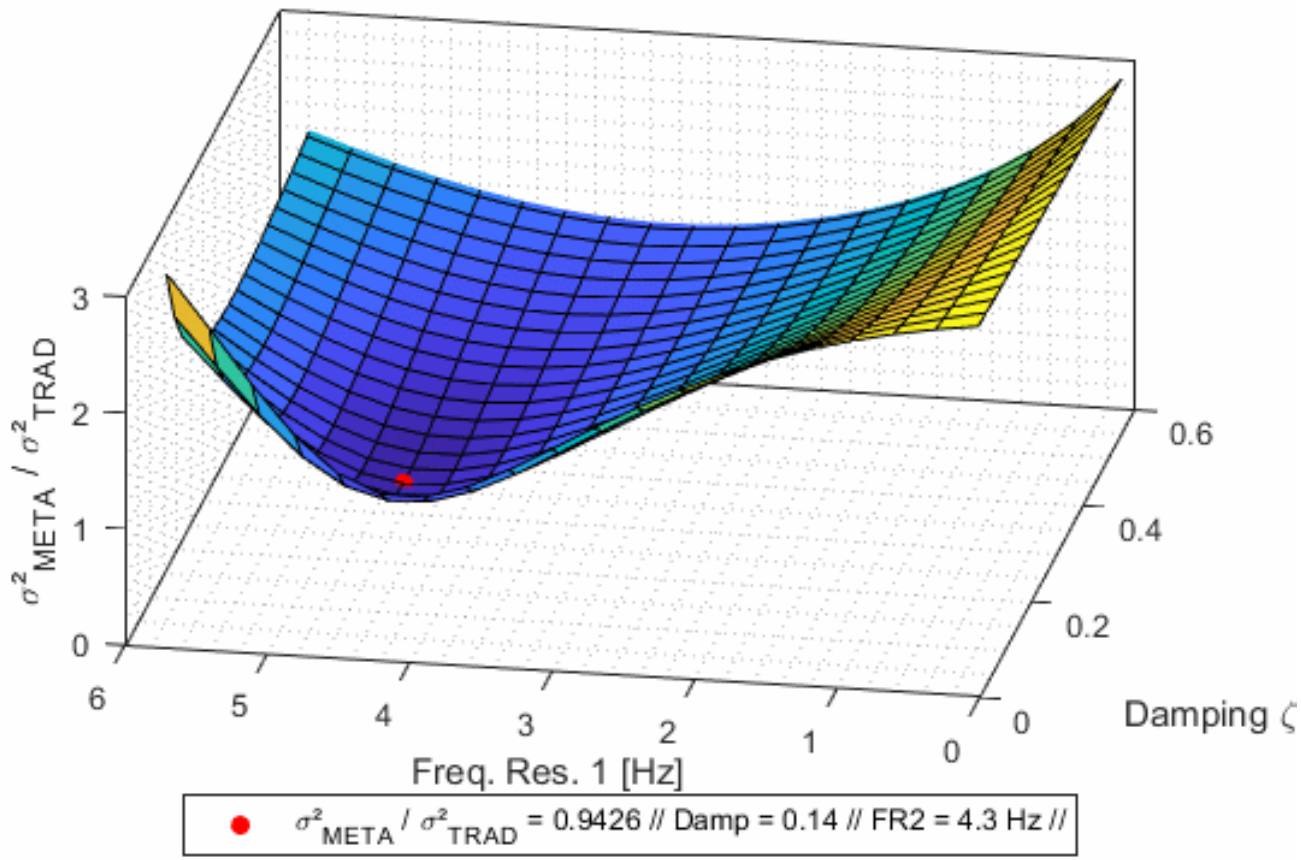



### Welch's Average PSD



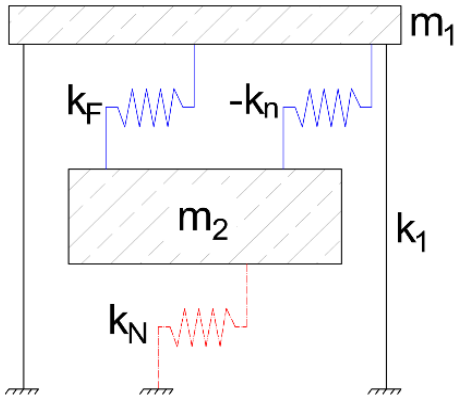
# Optimization

## Optimization Plane



# Motivation

# Stability Issue




$$\frac{1}{k_1}+\frac{1}{k_n}+\frac{1}{k_F}+\frac{1}{-k_n}\geq 0$$

$$k_n \geq \frac{k_F}{2}-\sqrt{\frac{{k_F}^2}{4}+k_1 k_F}$$

$$k_n = -\frac{P}{l}$$