Some considerations about vertical ground motions modelling in earthquake engineering

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Abstract

One of the current main challenges in Geotechnical and Structural Engineering is the analysis of the vertical component of site ground motion. In engineering practice, a simplified formulation of the Biot's equations is usually employed to model the coupled hydro-mechanical behaviour of saturated soils, namely the u–p formulation that neglects some terms of fluid inertial forces. This is in contrast with more refined formulations such as the u–U formulations that takes all inertial terms into account. The aim of this work is the validation of the u–p formulation as compared with the u–U formulation by means of numerical simulations, which are performed for different levels of permeability and different dynamic motions. The results are analysed in terms of frequency content and amplification rate, discussing the limits of applicability of the u–p formulation with respect to the u–U formulation.

1 Introduction

In Geotechnical and Structural Engineering there is an increasing interest for the analysis of the vertical component of site ground motion. In fact, it is well known that damages to buildings and structures during an earthquake may arise from the horizontal component as well as from the vertical component of site ground motion. For instance, a number of seismic protection systems are developed for the design of special constructions (i.e. petrochemical plants and storage systems Larkin, 2018, Carta & al., 2016) although they are focused on horizontal seismic actions, despite these constructions may undergo detrimental effects in the vertical direction. Nevertheless, the effects of the vertical component of site ground motion has not yet been thoroughly investigated and only simplified formulations are usually employed in the current practice when performing finite element modelling, whereas several investigations are available for the horizontal component. In particular, modelling of the coupled hydro-mechanical behaviour of saturated soils under static and dynamic conditions is commonly performed by means of the well known u–p formulation of the Biot's equations; an alternative choice is the u–U formulation which is rarely employed in engineering practice due to its much higher computational costs.

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Table 1: Material properties of the components of the two-phase medium. Properties referred to the solid phase and to the fluid phase are denoted with subscripts "s" and "f" respectively.

Parameter	Symbol	Value
Density	ρ	$2020\,\mathrm{kg\,m^{-3}}$
Porosity	n	0.4
Young modulus	E	$1200\,\mathrm{MPa}$
Poisson ratio	ν	0.3
Density	ρ_s	$2700\,\mathrm{kg\,m^{-3}}$
Bulk modulus	K_f	$2.1771\,\mathrm{GPa}$
Density	ρ_f	$1000\,\mathrm{kg\,m^{-3}}$

Despite being widely employed, the u–p formulation is based on a set of simplifications that limit its range of validity in terms of maximum frequency content of input motions, of thickness and of permeability of the soil layers (Zienkiewicz & al., 1980). Furthermore, the current formulation of such validity limits is lacking in the distinction between shear and longitudinal wave propagation.

The aim of this work is to present a novel theoretical validation of the widely diffused u–p formulation as compared to u–U formulation of the Biot's equations for the analysis of the vertical component of site ground motion in the dynamic regime. The proposed validation is based on two case studies, namely: the propagation of a single longitudinal pulse and the seismic response of a soil layer subjected to a registered vertical seismic ground motion applied at the soil base. In this way, the original validation of Zienkiewicz & al. (Zienkiewicz & al., 1980), that was based on a single frequency soil motion applied at the the top surface of a soil layer, is here extended to the more general case of pulse propagation and seismic ground motion. To this purpose, the results obtained with the u–p formulation implemented as a user-defined subroutine in a commercial finite element code (Abaqus Unified FEA®) are compared with those obtained with the u–U formulation implemented in an in-house finite element code (Gajo & al., 1994). In particular, a parametric study is performed in order to investigate the vertical site response as a function of the soil permeability, the soil layer thickness, and the soil state conditions. The results and the comparisons are provided in terms of the frequency content, the type of the seismic site ground motion, and the amplification function. Finally, the limits of applicability of the u–p as compared with u–U formulations for applications in Geotechnical Earthquake Engineering are discussed.

2 Methods

2.1 Field equations

The well known u–p formulation for the dynamic behaviour of saturate porous media can be expressed by the following set of equations (Zienkiewicz & al., 1980) for

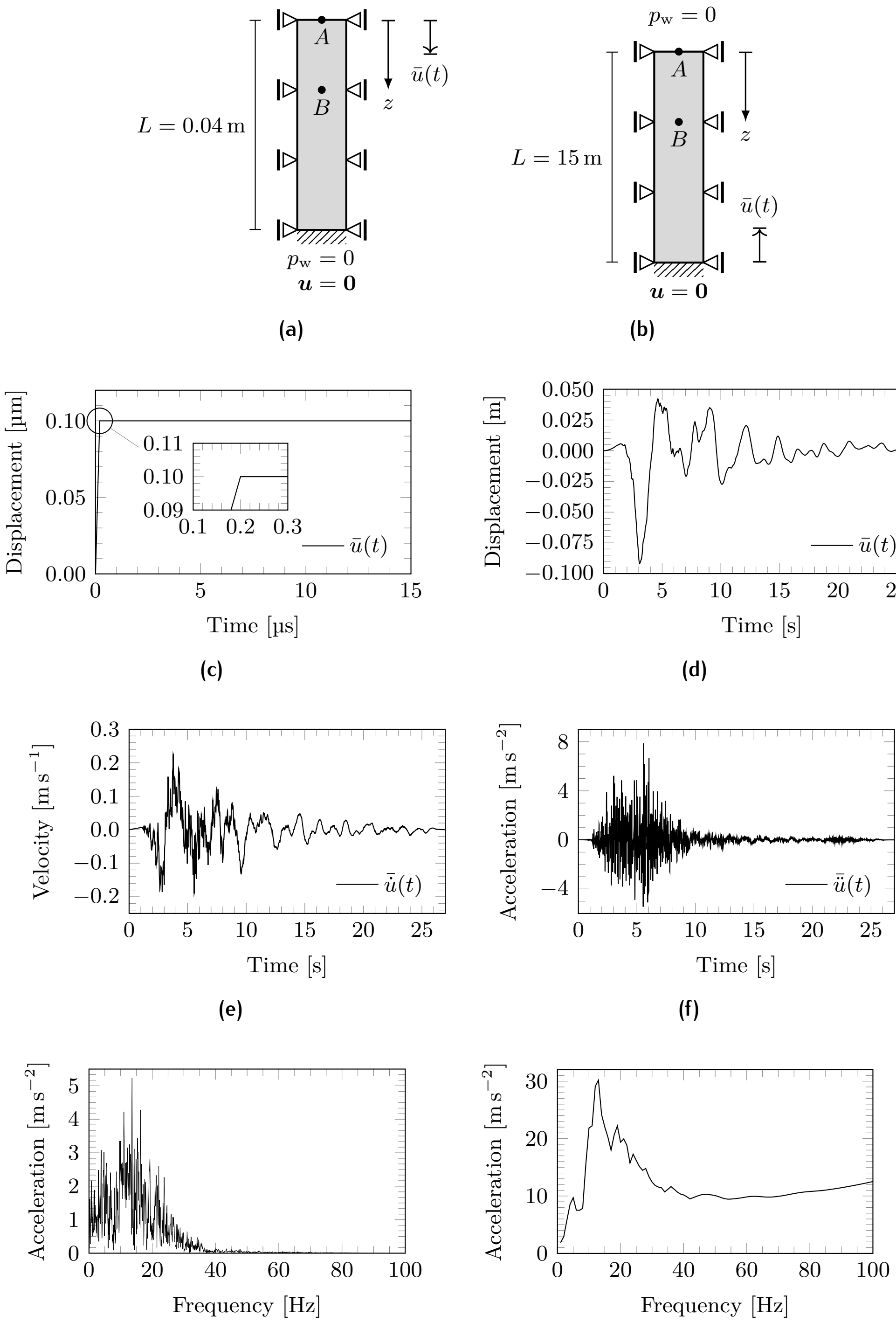

Figure 1: Reference schemes and loading time histories for the numerical simulations for the two case studies. (a) Geometry reference for the first case study and its vertical displacement time-history at the top surface (c). (b) Geometry reference for the second case study. (d)-(f): Vertical displacement, velocity, and acceleration records of the ground motion for the second case study (Christchurch earthquake, 2011, NZ); Fourier transform of the vertical acceleration (g) and vertical acceleration response spectrum at the bottom surface (h) for the second case study.

a linear-elastic soil response

$$\sigma_{ij} = \sigma'_{ij} - \delta_{ij} p \,, \tag{1a}$$

$$\varepsilon_{ij} = \frac{1}{2}(u_{i,j} + u_{j,i}) \,, \tag{1b}$$

$$\mathrm{d}\sigma'_{ij} = D_{ijkl}(\mathrm{d}\varepsilon_{kl} - \mathrm{d}\varepsilon^0_{kl}) \,, \tag{1c}$$

$$\sigma_{ij,j} + \rho g_i = \rho \ddot{u}_i \,, \tag{1d}$$

$$\dot{\varepsilon}_{ii} + \frac{K_\mathrm{D}}{g}\left[-\frac{1}{\rho_\mathrm{f}} p_{,i} + g_i - \ddot{u}_i\right]_{,i} = \dot{p}\frac{n}{K_\mathrm{f}} \,, \tag{1e}$$

where u_i is the displacement of the solid skeleton, K_D is the Darcy permeability coefficient, n is the porosity, ρ is the density of the whole porous medium, ρ_f is the pore fluid density, D_{ijkl} is the elastic stiffness tensor, K_f is the bulk modulus of the pore fluid and g_i is $i-$th component of the gravity acceleration, having modulus g, ε^0_{kl} is the initial (creep or thermal) strain. It can be noted that the fluid mass balance equation (1e) includes inertial forces due to pore fluid. The effects of these forces are discussed in the next section. The u–p formulation is implemented as a user-defined, 2D, finite element (through a UEL subroutine) in the commercial finite element code Abaqus Unified FEA®. Eight node finite element are used for the discretisation of the solid displacements, whereas four node elements are used for the pore pressures.

The governing equations for the u–U formulation are given by (Gajo & al., 1994, Gajo, 1995)

$$D_{ijkl}\varepsilon_{kl} + (\alpha - n)^2 Q(\varepsilon_{jj})_{,i} + n(\alpha - n)Q(U_{j,j})_{,i} + (1-n)\rho_\mathrm{s} g_i - (1-n)\rho_\mathrm{s}\ddot{u}_i$$
$$-\rho_\mathrm{a}(\ddot{u}_i - \ddot{U}_i) - \frac{n^2}{k}(\dot{u}_i - \dot{U}_i) = 0 \,, \tag{2a}$$

$$n(\alpha - n)Q(\varepsilon_{jj})_{,i} + n^2 Q(U_{j,j})_{,i} + n\rho_\mathrm{f} g_i - n\rho_\mathrm{f}\ddot{U}_i - \rho_\mathrm{a}(\ddot{U}_i - \ddot{u}_i) - \frac{n^2}{k}(\dot{U}_i - \dot{u}_i) = 0 \,, \tag{2b}$$

where U_i is the absolute displacement of the pore fluid, $k = K_\mathrm{D}/(g\rho_\mathrm{f})$, $\alpha = 1$ and $1/Q = n/K_\mathrm{f}$, since the solid constituent is assumed incompressible for the sake of simplicity, ρ_a is the added mass of pore fluid which is neglected here for the sake of consistency with u–p formulation, ρ_s is the density of the solid constituent. The u–U formulation is implemented in an in-house 1D FEM code (Gajo & al., 1994), in which both the solid and the pore fluid displacements are approximated with quadratic elements.

2.2 Numerical simulations

Two case studies on the transient response of a finite length, saturated soil column subjected to longitudinal dynamic excitation are analysed. Both u–p and u–U formulations are employed and linear elastic isotropic material properties are assumed, as summarized in table 1. The soil column is laterally constrained, so that lateral displacements and horizontal strains are equal to zero. Since the response of the system is thought as an incremental response, no gravity, null initial stress state, and null pore pressure are assumed.

In the first case study, the soil column has a length of $0.04\,\mathrm{m}$ and the system is discretised with 800 elements. The bottom surface of the soil column is constrained, whereas no water flux is allowed at the top, bottom, and lateral surfaces, as shown in figure 1a. A time-dependent longitudinal displacement is applied at the top surface to represent the impulse, which generates a longitudinal wave in the soil column. The

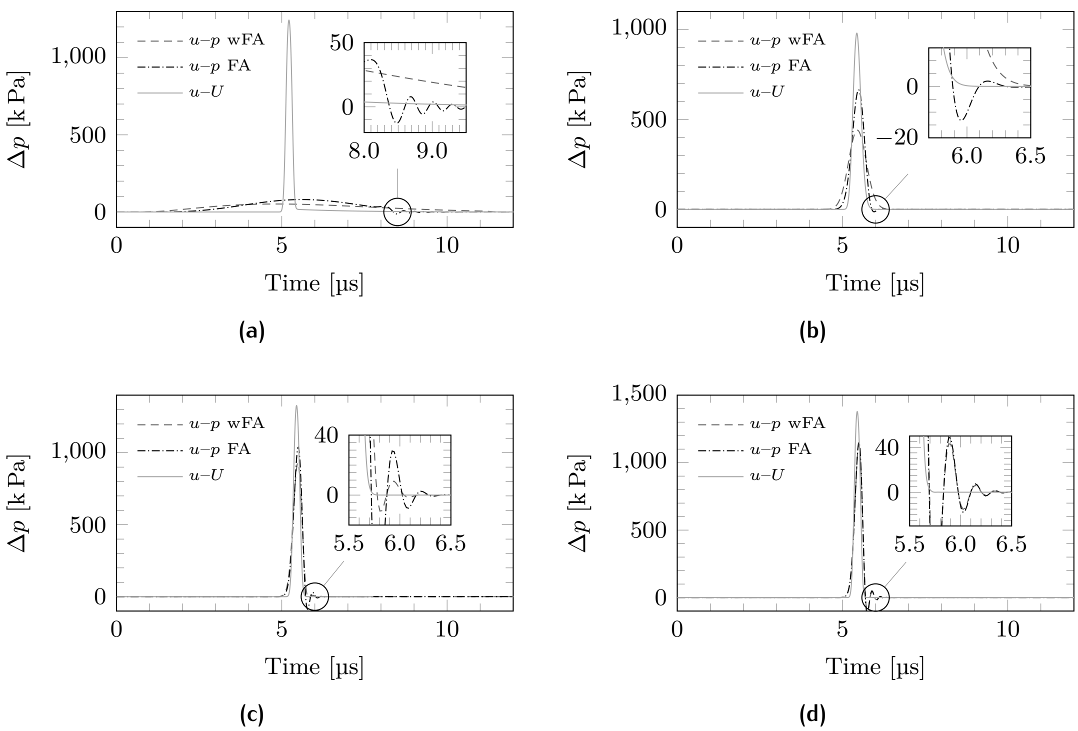

Figure 2: Comparison between u–p and u–U formulations for the first case study for different levels of permeability. (a) $K_{\mathrm{D}} = 1 \times 10^{-5}\,\mathrm{m\,s^{-1}}$, (b) $K_{\mathrm{D}} = 1 \times 10^{-7}\,\mathrm{m\,s^{-1}}$, (c) $K_{\mathrm{D}} = 1 \times 10^{-8}\,\mathrm{m\,s^{-1}}$, (d) $K_{\mathrm{D}} = 1 \times 10^{-9}\,\mathrm{m\,s^{-1}}$. Labels "FA" and "wFA" mean, respectively, with and without fluid inertial force in the mass balance equation (1e).

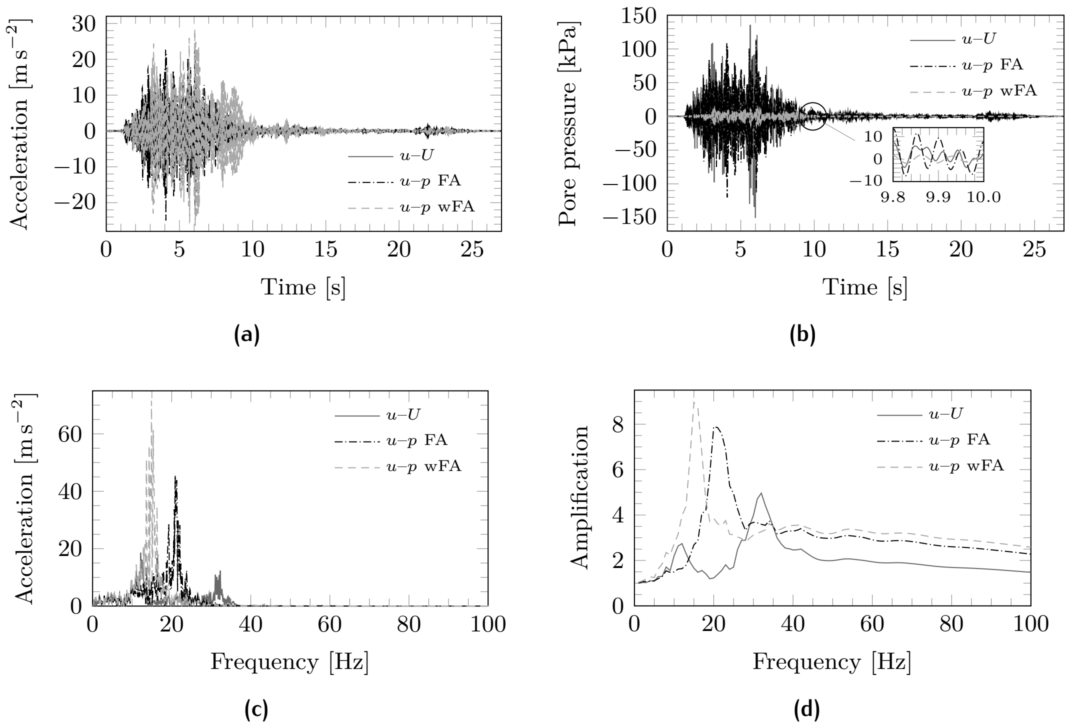

Figure 3: Comparison between u–p and u–U formulations for the second case study with permeability $K_\mathrm{D} = 1\,\mathrm{m\,s^{-1}}$. (a) Vertical acceleration at $z = 0\,\mathrm{m}$, (b) pore pressure at $z = 5\,\mathrm{m}$, (c) Fourier transform of the vertical acceleration, (d) vertical acceleration amplification.

total time of the simulation is equal to $15\,\mathrm{\mu s}$ and the displacement at the top surface of the soil column is linearly increased of $0.1\,\mathrm{\mu m}$ within the interval $[0\,\mathrm{\mu s}, 0.2\,\mathrm{\mu s}]$ and then kept constant; the time step is chosen equal to $0.0025\,\mathrm{\mu s}$.

In the second case study, the soil column has a length of $15\,\mathrm{m}$ and is discretised with 30 elements; the top surface of the soil column is free, and the fluid pressure is equal to zero, as shown in figure 1b. A prescribed longitudinal displacement is applied at the bottom surface, which represents the vertical component of the Christchurch earthquake (2011, NZ) (Han & al., 2018). No water flux is allowed at the bottom and at the lateral surfaces. The time step of the simulation is chosen equal to $2.5 \times 10^{-3}\,\mathrm{s}$.

3 Results and discussion

The results are provided in terms of water pore pressure measured at a specific depth (point B in figures 1a and 1b) for both case studies, and of vertical acceleration measured at the top of the soil column (point A in figure 1b) for the second case study, only.

The results for the first case study are illustrated in figure 2 for various permeabilities. The results of the u–p formulation are provided both for the cases in which the pore fluid inertial forces in the mass balance equation (1e) is neglected and is taken into account. These results of the u–p formulation are compared with those obtained with the u–U formulation that was validated against an analytical solution (Gajo and Mongiovì, 1995). It can be observed that for large permeabilities ($K_\mathrm{D} = 1 \times 10^{-5}\,\mathrm{m\,s^{-1}}$ in this case), the results obtained with u–p formulation are completely unreliable, showing a sort of diffusion phenomenon. The amplitude of the pore pressure pulse

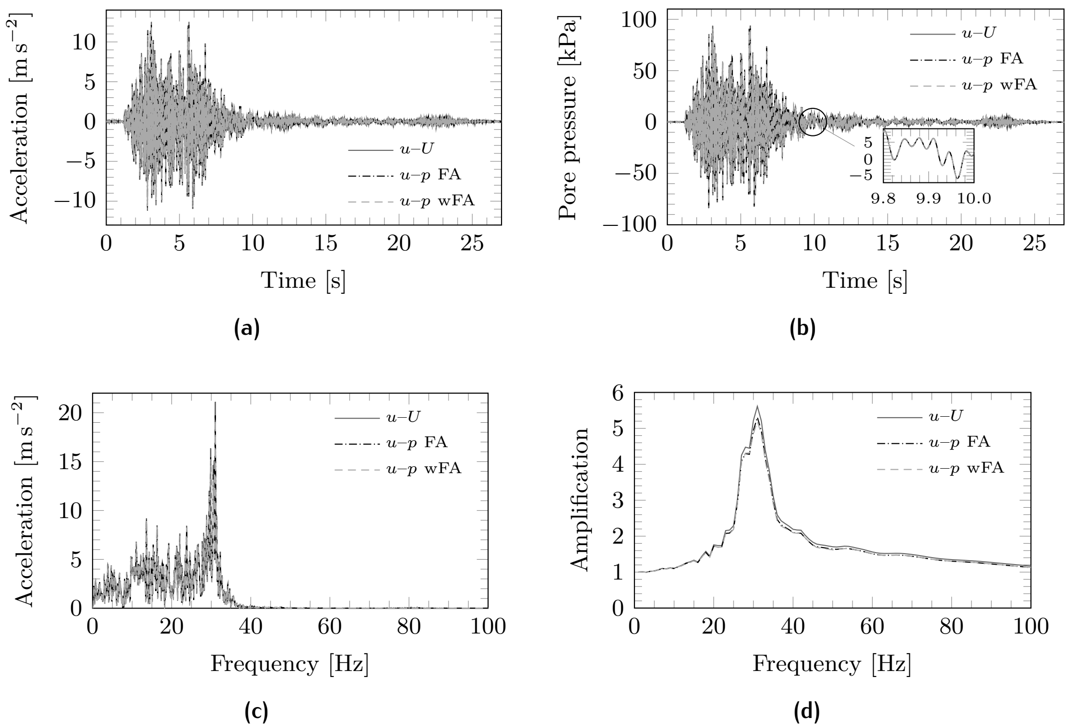

Figure 4: Comparison between u–p and u–U formulations for the second case study with permeability $K_{\mathrm{D}} = 1 \times 10^{-4}\,\mathrm{m\,s^{-1}}$. (a) Vertical acceleration at $z = 0\,\mathrm{m}$, (b) pore pressure at $z = 5\,\mathrm{m}$, (c) Fourier transform of the vertical acceleration, (d) vertical acceleration amplification.

evaluated with u–p formulation is much smaller than that evaluated with u–U formulation. With the decrease of permeability, the results of u–p formulation become closer to those of u–U formulation, with the pore pressure tending to the form of a Dirac δ-function in the time domain. For permeabilities smaller than $K_{\mathrm{D}} = 1 \times 10^{-8}\,\mathrm{m\,s^{-1}}$, the results are almost superposed to each other. In addition, the pore fluid inertial forces in the mass balance equation give important effects for the largest permeabilities, leading to an increase of the amplitude of the pore pressure pulse. It is worth noting that the above mentioned permeability values generally depend on the frequency content of the input signal, on the propagation length, and on the stiffness of the porous solid.

In the second case study, the pore pressure is evaluated at a depth of $5\,\mathrm{m}$, and the time history of the vertical accelerations at the ground surface are illustrated in figures 3-4. It can be observed that the results of u–p formulation are much different with respect to u–U formulation for the highest permeabilities, namely for $K_{\mathrm{D}} = 1 \times 10^{0} \div 1 \times 10^{-3}\,\mathrm{m\,s^{-1}}$, with discrepancies decreasing with decreasing permeability. In terms of amplification factors, the discrepancies between the two formulations for $K_{\mathrm{D}} = 1 \times 10^{-3}\,\mathrm{m\,s^{-1}}$ can be less than 5% for a wide range of frequencies ($f \leq 25\,\mathrm{Hz}$ and $f \geq 40\,\mathrm{Hz}$), but are above 10% when referring to the amplification peak and its neighbourhood ($25\,\mathrm{Hz} \leq f \leq 40\,\mathrm{Hz}$), whereas, for $K_{\mathrm{D}} \leq 1 \times 10^{-4}\,\mathrm{m\,s^{-1}}$ the two formulations provide practically superposed results.

It is worth observing that the discrepancies between the two formulations for $K_{\mathrm{D}} \geq 1 \times 10^{-3}\,\mathrm{m\,s^{-1}}$ concern equally the pore pressures, the accelerations, and the amplification factors. Therefore, the limits of validity of u–p formulation that can be deduced from the comparisons shown in figures 3-4 are the same for pore pressures, accelerations, and amplification factors. Furthermore, neglecting pore fluid inertial

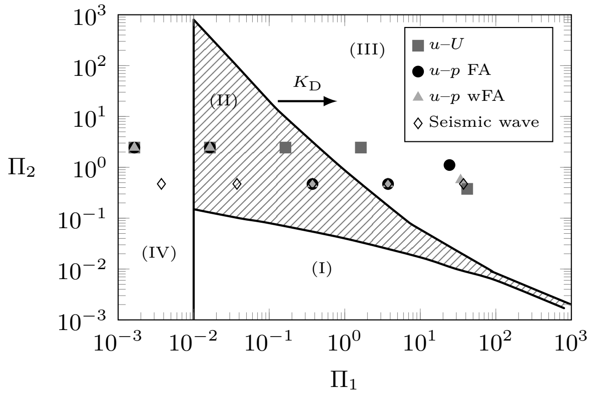

Figure 5: Comparison between u–p and u–U formulations based on the zones of applicability proposed by Zienkiewicz & al. (Zienkiewicz & al., 1980) According to their work, zone (I) denotes the zone of slow phenomena, whereas (II) and (III) denote, respectively, the zone of moderate speed and the zone of fast phenomena; zone (IV) corresponds to the zone of undrained behaviour.

forces in the mass balance equation (1e) leads to the less consistent results with u–U formulation for $K_\mathrm{D} \geq 1 \times 10^{-3}\,\mathrm{m\,s^{-1}}$, although the pore fluid inertial forces have negligible effects for the lowest permeabilities ($K_\mathrm{D} \leq 1 \times 10^{-4}\,\mathrm{m\,s^{-1}}$), namely in the range where the u–p formulation is more reliable.

The results shown in figures 3-4 are finally compared with the ranges of validity of u–p formulation that were proposed by Zienkiewicz & al. (Zienkiewicz & al., 1980), as shown in figure 5. The results are plotted in terms of two non-dimensional quantities Π_1 and Π_2 defined as follows

$$\Pi_1 = \frac{K_\mathrm{D} V_\mathrm{c}^2}{g\beta\omega L^2}\,, \tag{3a}$$

$$\Pi_2 = \frac{\omega^2 L^2}{V_\mathrm{c}^2}\,, \tag{3b}$$

where V_c is the compression wave velocity (assumed equal to $1869.26\,\mathrm{m\,s^{-1}}$), β is the ratio between the fluid density and the total density, and the angular frequency ω is deduced from the frequency associated with the largest amplitude in the Fourier series transform of the acceleration history either evaluated at the top surface or measured on the seismic ground motion. It can be noted that, according to Zienkiewicz & al. (Zienkiewicz & al., 1980), the u–p formulation is expected to be unreliable only for $K_\mathrm{D} \geq 1 \times 10^{-1}\,\mathrm{m\,s^{-1}}$, whereas its use would be permitted for $K_\mathrm{D} \leq 1 \times 10^{-2}\,\mathrm{m\,s^{-1}}$. This is not however fully consistent with the comparisons shown in figures 3-4, where the discrepancies between u–p and u–U formulations in terms of amplification ratio can be larger than 10% for $K_\mathrm{D} = 1 \times 10^{-2} \div 1 \times 10^{-3}\,\mathrm{m\,s^{-1}}$, for the soil conditions and soil layer thickness considered in this work.

4 Conclusions

Two case studies on the transient response of a finite length, saturated soil column subjected to longitudinal dynamic excitation are considered for the validation of u–p formulation as compared to u–U formulation.

The results of the first case study concern the propagation of a longitudinal wave and allow for the determination of permeability ranges in which u–U and u–p formulations can lead to the same outcomes, together with an estimate of the error between the formulations.

The results of the second case study show that in the case of a seismic ground motion, which encompasses a large number of frequencies associated with different amplitudes, the outcomes of u–p and u–U formulations can lead to validity ranges that are slightly different from those identified by Zienkiewicz & al. (Zienkiewicz & al., 1980) and, therefore, this work paves the way for a novel insight into the modelling of soils under seismic actions, using u–p formulation.

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