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Off-road motorcycle tyre force estimation

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Abstract. To improve the performance of off-road motorcycles, tyre contact forces need to be estimated. Differently to on-road motorcycle contact force estimators where the rider can be considered attached to the chassis, an off-road motorcycles estimator needs to model the rider dynamics, since driving in standing position provides significant isolation from motorcycle motion. In this article, we use a novel simple rider model to approximate the interaction with the motorcycle when no sensing on him is possible. In particular, the novelty is to keep the pitch angle of the rider constant by means of an active torque on the hip. A tyre force estimator is built using this rider model coupled to a five degree-of-freedom motorcycle, which is verified with a virtual experiment performed on Working Model 2D, showing promising results.

1. Introduction

In vehicle dynamics, the tyre contact forces are of interest to implement control systems and assess performance. It is known that direct measurement of them is not cost-effective nor reliable [1], thus estimation techniques are required.

Examples of contact force estimators for motorcycles can be found for on-road. Teerius [2], and Slimi [3], have developed state estimators for lateral dynamics of motorcycles, with which, the slip angles are estimated, and with a tyre model, the forces are estimated. However, the estimation of contact forces based on tyre models is subjected to uncertainty from the differences between the tyre characterisation and driving conditions [4]. In off-road driving, this differences are frequently expected due to changes in road material while driving, thread wear, and tyre temperature and pressure. To avoid the uncertainty from a tyre model, we opt to use an inverse dynamic model of the motorcycle to estimate the tyre contact forces.

In addition to the motorcycle, in off-road riding, the rider needs to be modelled as a separate body from the motorcycle because of the large relative motion that exist between them. Research on off-road bicycles [5], [6], have modelled the rider detached from the frame because arms and legs provide significant vibration isolation to the rider [6]. They consider the rider as a multibody system, with passive spring and damper elements connecting hip to foot, and shoulder to hand. The degrees of freedom considered are rotation and vertical translation for the torso, while the horizontal motion is constrained. However, in a novel approach, we consider rotation of the torso constrained by a feedback torque applied at the hip, while the horizontal motion is free, because it matches better the riding style in off-road.

In this article, we present a procedure to estimate the contact forces in off-road motorcycles. The estimator consist of a motorcycle and rider model, which are described in section 2; the procedure of the calculation is explained in section 3; a verification of the estimator using virtual data is shown in section 4, and finally the main conclusions are summarized.

2. Multibody model

Multibody modelling may be performed numerically or symbolically. On one hand, numerical simulations are well suited for final design stages [7] since detailed simulations for specific loading conditions can be performed. On the other hand, symbolic modelling is well suited for early design stages, or to develop simple and effective control models. In fact, it captures not only the essential behaviour of the system, but also detailed modelling is possible. However it requires a highly trained user [7]. Since our aim is to detect the essence of the system with a simple model, we use symbolic modelling with MBSymba [8], [7], [9], which is an add-on for Maple [10], for the automatic generation of equations of motion.

2.1. Motorcycle Model

The dynamics of the motorcycle on a plane involve multiple moving parts, however as it is a common practice in motorcycle dynamics analysis, it can be represented by four rigid bodies: ① rear wheel, ② swing arm, ③ chassis, and ④ front wheel, Figure (1).

The motion is described by five degrees of freedom: horizontal, vertical and rotational displacement of the chassis, x_s, z_s, μ, front suspension compression, z_f and rear suspension compression measured as the rotation of the swing arm with respect to the chassis, α_r, and are collected on the array

$$\mathbf{q_m} = [x_s, z_s, \mu, z_f, \alpha_r]^T. \tag{1}$$

The selection of these coordinates is not unique and other combinations could have been made. This set was selected because it can be related directly to inertial sensors on the chassis and displacement potentiometers on the suspensions.

Forces There are two sources of external forces acting on the motorcycle: the rider and the ground.

- The rider exert forces through his arms and legs over the handlebar and foot-pegs, respectively. They are collected in

  $$\mathbf{f_r} = [f_{lt}, f_l, f_a]^T, \tag{2}$$

  where f_a is the force along his arm, and f_l and f_{lt}, through his legs, and are calculated using the rider model detailed on the next section.
- Over each tyre, the ground exert a normal contact force (N_f, N_r), and under braking or acceleration, tangential contact forces (Q_f, Q_r) are also exerted. The contact point position is described in term of angles η at the front and ζ at the rear, Figure (1). The magnitude of the forces are collected in

  $$\mathbf{f_t} = [N_f, N_r, Q_f, Q_r]^T. \tag{3}$$

Additionally, for the derivation of the equations of motions it is necessary to describe the suspension forces. The front suspension force S_f acts between the chassis and front wheel along the fork, and the rear suspension torque, TSr, act between the chassis and swing arm. They are described in terms of relative position and velocity of the suspension ends. They are collected in

$$\mathbf{f_s} = [S_f, TS_r]^T. \tag{4}$$

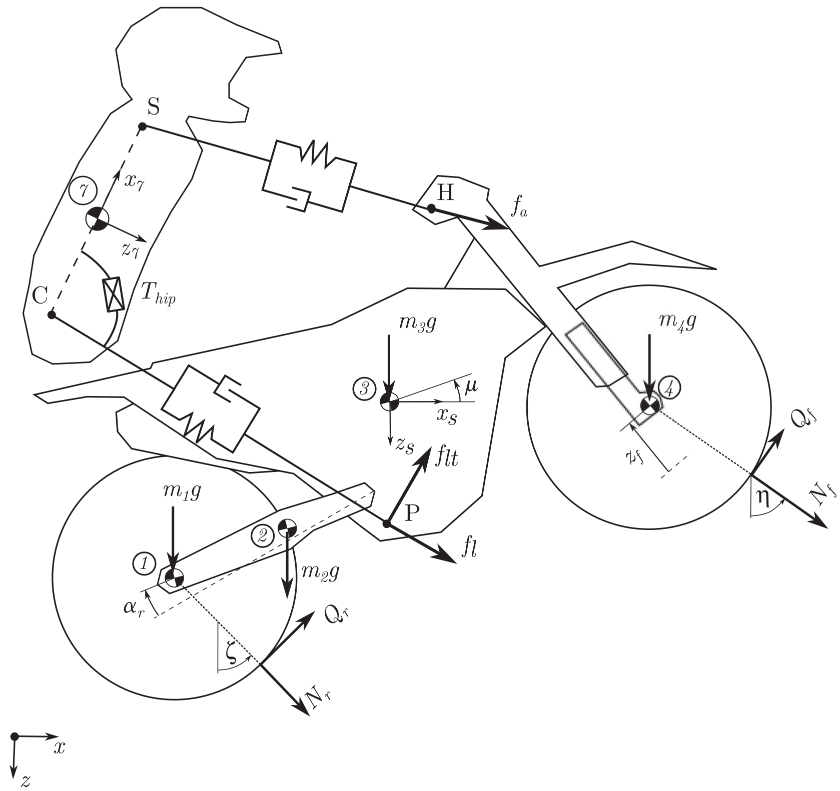

Figure 1: Motorcycle and rider models. The five bodies numbered are: 1) rear wheel, 2) swing arm, 3) chassis, 4) front wheel, and 7) rider. The motion of the motorcycle is described by horizontal, vertical and rotational displacement of the chassis, x_s, z_s, μ, front suspension compression, z_f and rear suspension compression, α_r, while the motion of the rider is described by x_7, z_7. Over the rider, act the motorcycle forces shown at the foot-peg (point P) and handlebar (point H), while over the bike act the reaction of these forces (not shown) and the normal N and tangential Q contact forces over each tyre. Additionally, on the rider the rotational action of shoulder and hip is replaced by an active torque at the hip.

Equations of motion and solution Using Newton-Euler approach, five equations of motion are derived:

- On motorcycle, ① + ② + ③ + ④: translation along x, z, and rotation on y around chassis centre of mass.
- On rear wheel and swing arm ① + ②: rotation on y around main pin of swing arm.
- On front wheel ④: translation along fork z_f.

They can be organised as

$$\mathbf{M_m}(\mathbf{q_m})\ddot{\mathbf{q}}_\mathbf{m} + \mathbf{M_m}'(\mathbf{q_m}, \dot{\mathbf{q}}_\mathbf{m}) = \mathbf{A_{wm}}(\mathbf{q_m})g + \mathbf{A_s}\mathbf{f_s} + \mathbf{A_r}(\mathbf{q_m})\mathbf{f_r} + \mathbf{A_t}(\mathbf{q_m})\mathbf{f_t}, \tag{5}$$

where $\mathbf{M_m}$ is the motorcycle mass matrix; $\mathbf{M_m}'$ is a pseudo-mass matrix containing products of velocities; $\mathbf{A_{wm}}$, $\mathbf{A_s}$, $\mathbf{A_r}$, $\mathbf{A_t}$ are matrices that project the weight, suspension force, rider force and tyre force vectors, respectively, into the generalized coordinates.

In this work it is assumed that, the position, velocities and accelerations are measured by sensors, i.e. $\mathbf{q_m}, \dot{\mathbf{q}}_\mathbf{m}, \ddot{\mathbf{q}}_\mathbf{m}$, are known; the suspension forces can be estimated from these signals;

while the rider forces, from a rider model. Therefore, the differential Equation (5), becomes algebraic with the tyre force vector as the unknown. The equations are rearranged as

$$\mathbf{A_t}(\mathbf{q_m})\mathbf{f_t} = \mathbf{b_1}(\mathbf{q_m}, \dot{\mathbf{q}}_\mathbf{m}, \ddot{\mathbf{q}}_\mathbf{m}, \mathbf{f_s}, \mathbf{f_r}), \tag{6}$$

where vector $\mathbf{b_1}$ collects all the known terms of Equation (5). Matrix $\mathbf{A_t}$ is not square, since there are five equations and four unknowns, therefore it is not invertible. To calculate the best fit of $\mathbf{f_t}$ in the five equations, the system is solved by the least squares method as

$$\mathbf{f_t} = (\mathbf{A_t}^T\mathbf{A_t})^{-1}\mathbf{A_t}^T\mathbf{b_1}. \tag{7}$$

In doing this derivation, the contact angles η and ζ were neglected because these angles are small on frequently driven terrains like gravel, dirt or sand roads since irregularities are small compared to the wheel radius. By neglecting the contact angle, any acceleration measured on the chassis x-coordinate (local forward direction), will be attributed to a braking force, and not to an horizontal component of the normal force, as will happen if a large obstacle is hit. Since this situation is not expected to occur frequently, the evaluation of the contact force of the motorcycle over a long section should not be affected significantly by this error.

2.2. Rider Model

The human body is a structure with multiple degrees of freedom, heterogeneous mass distribution and controlled by a combination of passive and active elements between its segments. A comprehensive description of its motion is complex and laborious. The introduction of some simplifications under certain constraints reduce significantly the modelling and computation time, without losing the description of the fundamental characteristics of the phenomena.

To this end, the human body is usually considered as a collection of fifteen rigid bodies: head, upper and lower torso, two upper and two lower arms, two hands, two upper and two lower legs and two foot. They are connected by fourteen joints: cervical vertebrae as one group (neck), thoracic and lumbar vertebrae as another group, two shoulders, two elbows, two wrists, two hips, two knees and two ankles.

To this model we introduce further simplifications. First, the muscular action of arms and legs is divided into linear and rotational forces. On one hand, the total force of arms and legs is simplified to passive springs and dampers, since this behaviour have been identified on research on human bodies under similar conditions. Indeed, [6], estimated the arms and leg stiffness and damping of a human body mounted on bicycles and subjected to sinusoidal excitation. Also, it is explained in [11] that during landings, (which is similar to impacts received when driving standing-up) the first 50 to 80 milliseconds immediately after the impact, the reaction force is passive. On the other hand, the rotational action of hip and/or shoulder is required to keep the torso at a specific angle. To simplify modelling, shoulder torque is neglected because it is expected to be much smaller compared to the hip counterpart. The reason for this assumption is that the muscles responsible for hip rotation (glutes, psoas and illacus) are larger and more powerful than the muscles responsible for shoulder rotation (pectoralis major, latissimus dorsi and teres major), since the former are responsible for keeping the torso vertical, which involves around half of body weight, while the latter are for arm motions which are around 5 % of body weight each.

In addition to the muscular simplifications, the following motions are neglected because they are small compared to the relative translation between the torso and chassis. First, head, upper and lower torso show small relative motion among them, and are considered on the torso. Second, hand and foot, are considered as point masses on handlebar and footpeg. Third, the displacement of arms and legs masses is neglected, because the masses are small compared to the torso, and upper arm and leg masses are added to the torso, while, lower arm and legs, to the handlebar and footpeg.

Finally, a constraint on the torso has to be added, because it has three degrees of freedom, and forces of arm and legs provide two constraints, leaving one motion free. In the cyclist models of [5] and [6], the horizontal translation of the rider has been constraint. However, we have observed that in off-road motorcycle driving, the motorcycle is pushed forward and backwards with respect to the rider to overcome different obstacles, while the head and torso are kept at about a constant angle, therefore we opt to constraint the torso rotation. This is achieved by a torque in the hip which cancels-out any resultant torque between arms and legs.

Summing up these simplifications, the rider is reduced to one body connected to the motorcycle by two spring-damper elements, plus an internal torque at the hip, as shown on Figure (1).

The generalized coordinates used to describe the motion of the rider are: displacement along the hip-shoulder direction, x_7, and perpendicular to it pointing to the motorcycle, z_7. They are collected in

$$\mathbf{q_r} = [x_7, z_7]^T. \tag{8}$$

The selection of this coordinates is because they can be related directly to an inertial sensor attached to the rider's torso, as done equivalently on the motorcycle.

The inputs to the rider is the motion of the chassis of the motorcycle, described by

$$\mathbf{u_r} = [x_s, z_s, \mu, \dot{x}_s, \dot{z}_s, \dot{\mu}]^T, \tag{9}$$

which is a subset of $\mathbf{q_m}$ and its derivative.

Leg and arm forces Three forces act on the rider, as mentioned on the motorcycle model. The two forces that arise from the spring and damper elements which are directed along them, and the third force arise from the internal torque on the hip, and act perpendicular to the leg. These forces are expressed as functions of the generalized coordinates $\mathbf{q_r}$ and the inputs $\mathbf{u_r}$, to remove them from the equations of motions, as follows.

First, the springs and dampers are considered linear with their length and rate of change, as on [5] and [6], i.e.

$$\begin{aligned} f_l &= k_l(l_{l0} - l_l) + c_l \dot{l}_l \\ f_a &= k_a(l_{a0} - l_a) + c_a \dot{l}_a, \end{aligned} \tag{10}$$

where the length of arm and legs, l_l and l_a, and their rate of change, $\dot{l}_l$ and $\dot{l}_a$ are described by the length of vectors $\mathbf{v_l}$ and $\mathbf{v_a}$, which depend on $\mathbf{q_r}$ and $\mathbf{u_r}$ as

$$\begin{aligned} l_l &= ||\mathbf{v_l}|| = g_1(\mathbf{q_r}, \mathbf{u_r}) \\ l_a &= ||\mathbf{v_a}|| = g_2(\mathbf{q_r}, \mathbf{u_r}). \end{aligned} \tag{11}$$

Second, given that it is assumed that the hip torque cancells-out the resultant torque between arms and legs, which is written as

$$0 = T_{hip} + \mathbf{r_{7c}} \times (\mathbf{f_l} + \mathbf{f_{lt}}) + \mathbf{r_{7s}} \times \mathbf{f_a} \tag{12}$$

where $\mathbf{r_{7c}}$ and $\mathbf{r_{7s}}$ are the vectors from torso centre of mass to shoulder and hip, respectively, and that from rotations on the leg around the hip, the torque can be written in terms of f_{lt} as,

$$T_{hip} = f_{lt} l_l, \tag{13}$$

the force f_{lt} can be expressed in terms of f_l and f_a, which depend only on $\mathbf{q_r}$ and $\mathbf{u_r}$.

Equation of motion and output Using Newton approach, two equation of motion are derived, one for each coordinate of $\mathbf{q_r}$, and are arranged as

$$\mathbf{M_r}(\mathbf{q_r})\ddot{\mathbf{q}}_\mathbf{r} + \mathbf{M_r}'(\mathbf{q_r}, \dot{\mathbf{q}}_\mathbf{r}, \mathbf{u_r}) = \mathbf{A_{wr}}g + \mathbf{A_m}(\mathbf{q_r}, \dot{\mathbf{q}}_\mathbf{r}, \mathbf{u_r})\ \mathbf{f_r}(\mathbf{q_r}, \dot{\mathbf{q}}_\mathbf{r}, \mathbf{u_r}), \tag{14}$$

where $\mathbf{M_r}$ is the rider mass matrix; $\mathbf{M_r}'$ is a pseudo-mass matrix containing products of velocities and the motorcycle motion; $\mathbf{A_{wr}}$, $\mathbf{A_m}$, are matrices that project the weight and rider forces, into the generalized coordinates. In this differential equation, the only known variables are the inputs $\mathbf{u_r}$, therefore it has to be integrated to known the motion of the rider. To solve the system by a Runge-Kutta integrator, it is reduced to a first order differential equation as

$$\begin{aligned} \dot{\mathbf{v}}_\mathbf{r} &= \mathbf{A_2}^{-1}\mathbf{b_2}(\mathbf{v_r}, \mathbf{q_r}, \mathbf{u_r}) \\ \dot{\mathbf{q}}_\mathbf{r} &= \mathbf{v_r}. \end{aligned} \tag{15}$$

After solving for the states of the rider $\mathbf{q_r}$, the magnitude of legs and arm forces acting on the motorcycle are retrieved from Equations (10) - (13) as

$$\mathbf{f_r} = \mathbf{f_r}(\mathbf{q_r}, \dot{\mathbf{q}}_\mathbf{r}, \mathbf{u_r}). \tag{16}$$

The codes for the derivation of the equations of the motorcycle and rider models can be found in the Appendix.

3. Estimator structure

Figure (2) shows an overview of the estimator structure. The estimator reads motorcycle kinematics, which are the states of the system, to estimate the contact forces at the tyres through a procedure described next.

It starts from the source of the motorcycle kinematics data, which can be experimental or virtual. Experimentally, some of the motorcycle states are measured with commonly available sensors, which are accelerometers, gyroscopes and displacement potentiometers. Afterwards, the signals are filtered and fused to improve the exactitude of measurements, and to estimate the remaining motorcycle states. The processing of experimental signals is not treated in this article. Conversely, virtual data is an alternative to the experimental data, and is only used to test the estimator, as explained on section 4.

The procedure continues with the estimator itself, which consist of two consecutive blocks: rider model and motorcycle model. First, the rider model reads some of the motorcycle states, to estimate the arm and leg forces by forward dynamics. Second, these forces, plus the motorcycle states, are passed to the motorcycle model to estimates the tyre contact forces by inverse dynamics.

In particular, the rider model block, reads the position and velocity of the motorcycle chassis, Equation (9), and solves the differential equation, Equation (14), to determine the state of the rider $\mathbf{q_r}$. Then, using the calculated states of the rider and its inputs, it calculates the leg and arm forces $\mathbf{f_r}$ using Equation (16) which is the output of the block.

The motorcycle block, reads the rider forces and the motorcycle states to solve Equation (6) by least squares method, Equation (7), to estimate the normal and tangential components of the tyre contact forces, which finishes the procedure.

4. Verification of model

The equations of rider and motorcycle models are verified using data from a virtual experiment, since measurements are noise free and it is possible to measure the contact forces directly. To this end, the motorcycle states are measured and passed to the estimator block to calculate rider and contact forces which are compared to the forces measured directly in the virtual simulation.

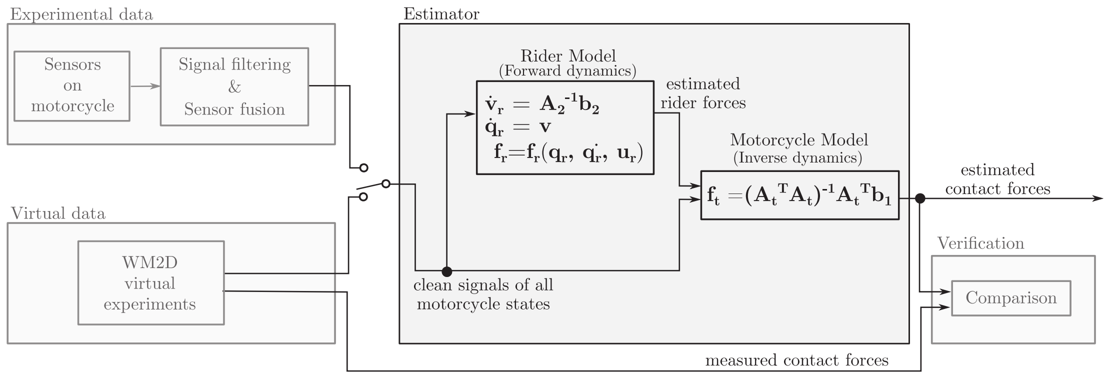

Figure 2: Estimator structure.

The virtual experiment is carried out in Working Model 2D, [12], Figure (3), with a model which has additional details than the one described in section 2, making it a closer representation to reality. On one hand, the motorcycle model adds wheel rotational inertias and moving contact points of the wheels. On the other hand, the rider model considers the length and masses of upper and lower arms and legs, and the hip torque over the torso angle is controlled by a PD feedback controller, as

$$T_{hip} = K_p(e - \theta_{7p}) + K_d\dot{e}, \tag{17}$$

where K_p and K_d are the proportional and derivative constants, $e = \theta_7(t) - \theta_{7,ref}$, is the absolute angle error e of the torso, $\theta_{7,ref}$ is the desired angular position of the torso and θ_{7p} is an angular preload which avoids a different resting position than the desired.

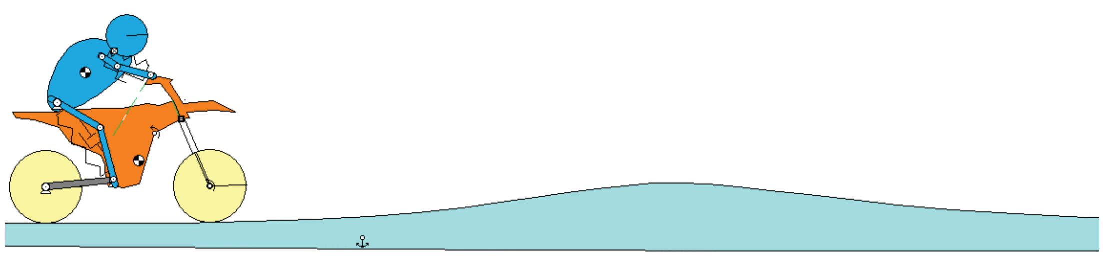

Figure 3: Virtual experiment of motorcycle with rider braking as it transit over a bump in Working Model 2D. Motorcycle is modelled with four bodies; rear brake torque 20 Nm from $t =$ 0.1 to 2 s, front brake torque 20 Nm from $t = 0.1$ to 2.5 s; rider has five bodies with three degrees of freedom; torque at hip is applied by a PD controller to mantain the torso angle; bump is 0.3 m high and 7 m long; initial velocity 5 m/s.

4.1. Results

Figure (4), shows that the estimated output of the rider model is in correspondence with the virtual measurement. The discrepancies on the forces are originated on the difference between the estimated motion of the rider and the one that occurred on the experiment. This is expected since the rider models of the estimator and the virtual experiment are different as explained in the introduction of section 4. The different mass distribution and hip torque controllers can explain this different motion.The other possible source of discrepancy is that either arm and/or leg reached total extension in the virtual experiment, since contrarily, the rider model does not have a limit, however it was checked that it did not occur in this situation.

With respect to the hypothesis of considering horizontal motion of the rider, it showed to be significant, since it is checked that it is of the same order of magnitude than the vertical motion. Indeed, the amplitude of the horizontal motion is 0.45 m, while the vertical is 0.41 m. Nonetheless, to validate this hypothesis, a real-life experiment is required.

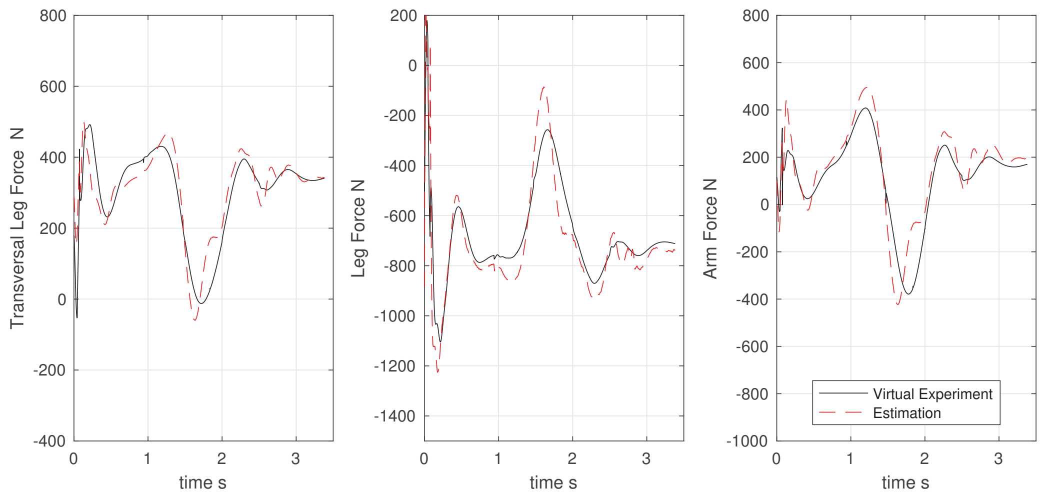

Figure 4: Comparison of virtual and estimated leg and arm forces

Figure (5), shows that the estimated vertical contact forces are in good agreement with the virtual measurements as well, however there is a discrepancy on the horizontal force when it transits through the bump. The differences arise from the two simplifications done between the virtual experiment and the motorcycle model: tyre contact point always at the bottom and wheel inertias. Indeed, in a simulation in which the wheel inertia was removed in the virtual experiment, and the moving contact point was added to the motorcycle model, the results match perfectly. This result suggests that if large bumps are expected recurrently and the horizontal force is of particular interest, then the contact point needs to be considered in the model, but if the normal force is the main interest, this estimator would be enough. Nonetheless, a parametric analysis is needed to quantify and assess the effect of these simplifications on the estimations.

5. Conclusion

In this article an estimator of tyre contact forces on off-road motorcycle is presented. Modelling the rider as one body that keeps the pitch angle constant by means of an active torque at the hip showed promising preliminary results, since the motion exhibited while braking over a bump resembles the one observed in reality. For the future, we aim to do a parametric study of the simplifications done such as the fixed torso angle, rider lumped into one mass, neglecting wheel inertias, and fixed contact points on the wheel. Afterwards, we envisage an experimental validation of the estimator.

Acknowledgments

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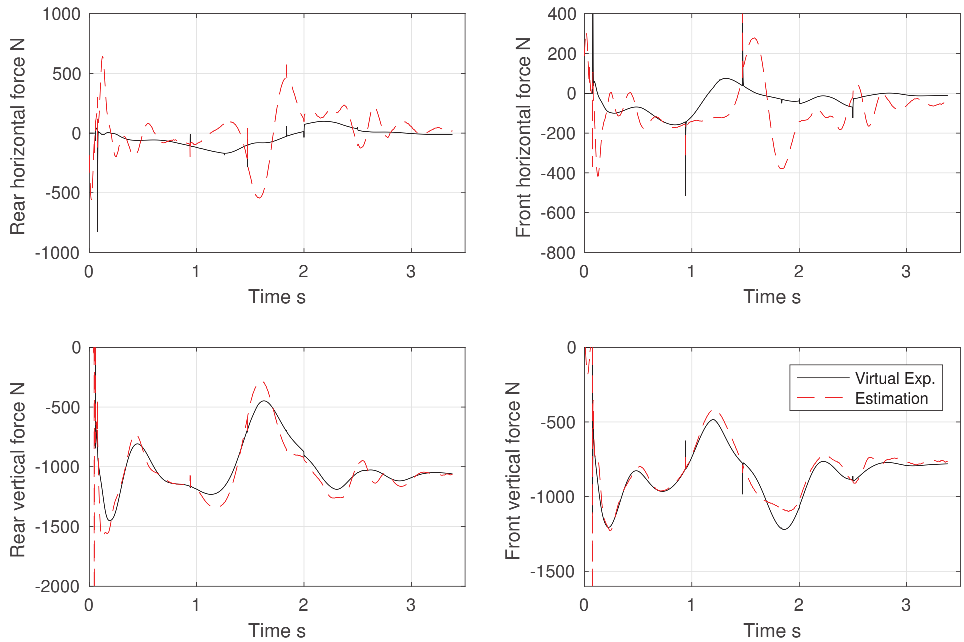

Figure 5: Comparison of virtual and estimated contact forces. On the left, rear wheel forces, on the right, front wheel forces.

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Table 1: List of symbols: sub-index i refers to f or r for front or rear; j, to a or l for arm or leg; and k, to m or r for motorcycle or rider.

C	Hip point	g_2	Function for arm length
H	Handlebar point	k_j	Stiffness of body j
$\mathbf{M_k}$	Mass matrix of model k	l_j	Length of body j
N_i	Normal contact force on tyre i	l_{j0}	Initial length of body j
P	Footpeg point	m_n	Mass of body number n
Q_i	Tangential contact force on tyre i	$\mathbf{q_k}$	Coordinates of model k
S	Shoulder point	$r_{a,b}$	Vector from point a to point b
S_f	Front suspension force along fork	x_7	Rider displacement along H-S direction
TS_r	Rear suspension torque on swing arm	x_s	Horizontal displacement of chassis
T_{hip}	Hip torque	z_7	Rider displacement perpendicular to x_7
c_j	Damping of body j	z_f	Fork compression
f_j	Force along body j	z_s	Vertical displacement of chassis
$\mathbf{f_r}$	Forces of motorcycle over rider	α_r	Rear suspension compression angle
$\mathbf{f_s}$	Suspension forces	ϵ	Caster angle
$\mathbf{f_t}$	Tyre contact forces	ζ	Rear contact angle
f_{lt}	Force perpendicular to leg	η	Front contact angle
g_1	Function for leg length	μ	Angular displacement of chassis

Table 2: Inertial parameters and initial positions of bodies.

Body	Mass kg	Inertia $kg\ m^2$	x_i m	z_i m
1	18	0	0	-0.3
2	0	0.28	0.279	-0.328
3	81	23	0.768	-0.507
4	12	0	1.35	-0.3
7	75	0	0.33	-1.22

Table 3: Additional inertial parameters and initial positions used in rider of virtual experiment.

Body	Mass kg	Inertia $kg\ m^2$	x_i m	z_i m
Torso	41.2	1.85	0.33	-1.22
Upper leg	16.5	0.3	0.25	-0.89
Lower leg	9.7	0.2	0.51	-0.55
Upper arm	4.5	0.04	0.52	-1.32
Lower arm	4	0.1	0.75	-1.24

Table 4: Stiffness and damping of rider and motorcycle.

Component	Stiffness N/m	Damping Ns/m
Arm	23100	515
Leg	74000	1350
Front susp.	10320	533
Rear susp.	15420	752

Table 5: Parameters of the hip torque PD controller of virtual experiment.

Parameter	Value	
K_p	-2000	Nm
K_d	-500	Nm
$\theta_{7,ref}$	-0.79	rad
θ_{7p}	0	rad

Appendix - Maple / MBSymba models

Rider Model

```
> interface(displayprecision=3, rtablesize=50):
```

Coordinates

```
> q := [x__7(t), z__7(t)]:
> u := [x__s(t), z__s(t), mu(t)]:
> PDEtools[declare](q,u,prime=t,quiet); alias(C=cos,S=sin):
```

Frames and points

```
> T__R := ground:
> _gravity := make_VECTOR(ground,0,0,g):
> T__3 := T__R * translate(x__s(t) + x__03, 0, z__s(t) + z__03 ) * rotate('Y',
mu(t)) :
> T__7 := T__R * translate(x__07, 0, z__07) * rotate('Y', theta__07 ) * translate(x__7(t
, 0, z__7(t)):
> G7 := make_POINT(T__7, g7x, 0, g7z):
> P__3h := make_POINT(T__3, x__3h, 0, z__3h):  handlebar
> P__3p := make_POINT(T__3, x__3p, 0, z__3p):  footpeg
> P__7s := make_POINT(T__7, x__7s, 0, 0):  shoulder
> P__7c := make_POINT(T__7, x__7c, 0, 0):  hip (cadera)
> PS := project(P__7s, T__R): shoulder point in ground coordinates
> PC := project(P__7c, T__R): cadera (hip) point in ground coordinates
> ua := < uax, 0, uaz, 0 > / l__arm:
> va := <   0, 1,   0, 0 > :
> wa := < wax, 0 , waz, 0 > / l__arm:
> T__arm := < ua | va | wa | < comp_X(PS), 0, comp_Z(PS), 1 > >:
> ul := < ulx, 0, ulz, 0 > / l__leg:
> vl := <   0, 1,   0, 0 > :
> wl := < wlx, 0, wlz, 0 > / l__leg:
> T__leg := < ul | vl | wl | < comp_X(PC), 0, comp_Z(PC), 1 > >:
```

Bodies and forces

```
> rider := make_BODY(G7, m[7], 0, I[7], 0):
> arm_force := make_FORCE(T__arm, 0, 0, F__a, PS, rider):
> leg_force := make_FORCE(T__leg, F__lt, 0, F__l, PC, rider):
> hip_torque := make_TORQUE(T__R, 0, F__lt * l__leg, 0, rider ):
```

Implicit equations of motion

```
> collect_list := [ diff(x__7(t),t,t), diff(z__7(t),t,t), m[7], g, F__a, F__l]:
> rider_system := {rider, arm_force, leg_force, hip_torque}:
> eqnsN := newton_equations( rider_system ):
> dyn_eqnsN1 := collect([comp_X(eqnsN), comp_Z(eqnsN)], collect_list):
> eqnsN_Y := comp_Y( euler_equations(rider_system, G7)):
> flt1 := F__lt = collect(solve(%, F__lt), collect_list):
> dyn_eqnsN0 := collect(subs(%, dyn_eqnsN1), collect_list):
```

Expressions for arm and leg lengths, vectors, unit vectors and forces

```
> v__sh := project(join_points(P__7s, P__3h), T__R): Shoulder to handlebar
> v__cp := project(join_points(P__7c, P__3p), T__R): Cadera to footpeg
> l__armi := sqrt(dot_prod(v__sh,v__sh)):
> l__legi := sqrt(dot_prod(v__cp,v__cp)):
> lengthi := l__arm = l__armi, l__leg = l__legi, v__arm = diff(l__armi,t), v__leg
= diff(l__legi,t):
> v__shx := project(rotate('Y',Pi/2, v__sh ),ground):
> v__cpx := project(rotate('Y',Pi/2, v__cp ),ground):
> unit_vectors := uax = comp_X(v__shx), uaz = comp_Z(v__shx), wax = comp_X(v__sh),
waz = comp_Z(v__sh), ulx = comp_X(v__cpx), ulz = comp_Z(v__cpx), wlx = comp_X(v__cp),
wlz = comp_Z(v__cp):
> forces := F__a = c__a * v__arm + k__a * (l__arm - l__arm0 - l__armp),
> F__l = c__l * v__leg + k__l * (l__leg - l__leg0 - l__legp) :
```

Explicit equations of motion

```
> subs(forces,dyn_eqnsN0): subs(lengthi,%): subs(unit_vectors,%): dyn_eqns:=%:
```

Initial conditions in rider fram

```
> P7 := project(origin(T__7), T__R):
> V7 := velocity(P7, T__R):
> P7_c := [ comp_X(P7), comp_Z(P7) ] :
> V7_c := [ comp_X(V7), comp_Z(V7) ] :
> Pg70 := [ x__g70, z__g70]:
> Vg70 := [vx__g70, vz__g70]:
> P7_Pg := op(combine( solve(P7_c - Pg70 , [op(q)] ), trig)):
> op( combine(solve(V7_c - Vg70 , [op( diff(q,t)) ]))):
> V7_Vg := subs( P7_Pg, %):
> X7i := < rhs(op(1,%%%)); rhs(op(2,%%%)); rhs(op(1,%)); rhs(op(2,%)) >:
```

First order formulation

```
> fo_vars, fo_eqns := first_order(dyn_eqns, q, t):
> A2, b2 := GenerateMatrix(fo_eqns, diff(fo_vars,t)):
```

Motorcycle forces over rider - f__r

```
> fl_tr := project(leg_force,T__R):
> fa_tr := project(arm_force,T__R):
> Flx1 := comp_X(fl_tr):
> Flz1 := comp_Z(fl_tr):
> Fax1 := comp_X(fa_tr):
> Faz1 := comp_Z(fa_tr):
> f8_i := [Flx1, Flz1, Fax1, Faz1]:
> subs(flt1, f8_i): subs(forces, %): subs(unit_vectors, %): subs(lengthi,
%): fr := %:
```

Motorcycle Model

```
> interface(displayprecision=3, rtablesize=50):
> with(LinearAlgebra):
> q := [x__s(t),z__s(t), mu(t), z__f(t), alpha__r(t)]:
> PDEtools[declare](q,u,prime=t,quiet):
> alias(C=cos,S=sin):
```

Frames:

```
> T__R := ground:
> _gravity := make_VECTOR(ground,0,0,g):
> T__3 := T__R * translate(b,0,-h) * translate(x__s(t), 0, z__s(t) )
> * rotate('Y',mu(t)) :
> T__3 * translate(-b, 0, h - R__r) * rotate('Y', alpha__r0) * translate(l__r,0,0):
> P__S := project( origin(%), T__3):
> T__2 := T__3 * translate(comp_X(P__S), 0, comp_Z(P__S))
> * rotate('Y', alpha__r0 - alpha__r(t)) * translate(-l__s, 0, 0):
> T__1 := T__2 * translate(-(l__r - l__s), 0, 0):
> T__4 := T__3 * translate(w - b, 0, h - R__f) * rotate('Y', epsilon)
> * translate(0, 0, -z__f(t) ):
```

Bodies

```
> rear_wheel := make_BODY(T__1, m__1, 0, 0, 0): show(rear_wheel):
> swing_arm := make_BODY(T__2, 0, 0, I__2, 0): show(swing_arm):
> chassis := make_BODY(T__3, m__3, 0, I__3, 0): show(chassis):
> front_wheel := make_BODY(T__4, m__4, 0, 0, 0): show(front_wheel):
```

Points

```
> cp_front := project(CoM(front_wheel),T__R) + make_VECTOR(T__R, 0, 0, R__f):
> cp_rear  := project(CoM(rear_wheel ),T__R) + make_VECTOR(T__R, 0, 0, R__r):
> P__3h := make_POINT(T__3, x__3h, 0, z__3h): handlebar
> P__3p := make_POINT(T__3, x__3p, 0, z__3p): footpeg
```

Forces

```
> front_cont := make_FORCE(ground, N__fx, 0, N__fz, cp_front, front_wheel):
> rear_cont := make_FORCE(ground, N__rx, 0, N__rz, cp_rear , rear_wheel):
> leg_force := make_FORCE(T__R, -flx, 0, -flz, P__3p, chassis):
> arm_force := make_FORCE(T__R, -fax, 0, -faz, P__3h, chassis):
> front_susp := make_FORCE(T__4, 0, 0, S__f, origin(T__4), front_wheel, chassis):
> rear_susp := make_TORQUE(T__2, 0, TS__r, 0, swing_arm, chassis):
```

Equations of motion of all motorcycle

```
> all_bike := {rear_wheel, swing_arm, chassis, front_wheel, rear_cont, front_cont,
leg_force, arm_force}:
> eqnsN__A := newton_equations(all_bike):
> eqnsN__Ax := comp_X(eqnsN__A):
> eqnsN__Az := comp_Z(eqnsN__A):
> eqnsE__A := euler_equations(all_bike, origin(T__3)):
> eqnsN__Ay := combine(comp_Y(eqnsE__A),'trig'):
```

Equation of motion of Front Wheel

```
> front     := {front_wheel, front_cont, front_susp}:
> eqnsN__4  := newton_equations(front):
> eqnsN__4z := collect(comp_Z(project(eqnsN__4,T__4)),m__4):
```

Equation of motion of Rear Wheel + Swing arm

```
> rear_swing_arm     := {rear_wheel, swing_arm, rear_cont, rear_susp}:
> eqnsE__12  := euler_equations(rear_swing_arm, P__S):
> eqnsE__12y := combine(comp_Y(eqnsE__12),'trig'):
```

Solution

```
> all_eqns := [eqnsN__Ax, eqnsN__Az, eqnsN__Ay, eqnsN__4z, eqnsE__12y]:
> x__m := [N__fx, N__fz, N__rx, N__rz ]:
> GenerateMatrix(all_eqns, x__m):
> A1:= combine(%[1],'trig'):
> b1:= -%%[2]:
```