Operations Research Perspectives 6 (2019) 100106

Contents lists available at ScienceDirect

Operations Research Perspectives

journal homepage: www.elsevier.com/locate/orp

The role of anticipated emotions and the value of information in determining sequential search incentives

Debora Di Caprio[a,b], Francisco J. Santos-Arteaga[c,*], Madjid Tavana[d,e]

[a] Department of Mathematics and Statistics, York University, Toronto M3J 1P3, Canada
[b] Polo Tecnologico IISS G. Galilei, Via Cadorna 14, 39100 Bolzano, Italy
[c] Faculty of Economics and Management, Free University of Bolzano, Piazza Università 1, 39100 Bolzano, Italy
[d] Business Systems and Analytics Department, Distinguished Chair of Business Systems and Analytics, La Salle University, Philadelphia, PA 19141, USA
[e] Business Information Systems Department, Faculty of Business Administration and Economics, University of Paderborn, D-33098 Paderborn, Germany

ARTICLE INFO

Keywords:
Information acquisition
Anticipated emotions
Regret
Value of information
Certainty equivalent

ABSTRACT

We define a novel information acquisition model that accounts explicitly for the influence of positive and negative anticipated emotions in the evaluation and selection incentives of decision makers (DMs). The model focuses on the value assigned by the DMs to the information being acquired and its capacity to prevent regrettable decisions within a forward-looking sequential environment. We introduce a novel definition of value of information accounting for the two main uses that DMs can derive from it, namely, verifying the optimality or suboptimality of a potential decision and preventing the regret that may arise from a suboptimal decision. In particular, DMs would regret a decision whenever rejecting [accepting] an alternative that should have actually been accepted [rejected]. Our formal information acquisition model allows to account for the subjective relative importance assigned by the DMs to the verification and regret value of information. Moreover, we illustrate how the incentives defining the sequential information retrieval process of DMs are determined by the relative width of the domains on which the different characteristics describing the alternatives are defined.

1. Motivation

Beyond individual choice problems, which constitute the main focus of the current paper, the consistency of decision-making processes is essential for the performance of businesses, with information playing a fundamental role [9]. In particular, the quantity and quality of the information available have a direct effect on the quality of the decisions made in an organization [46]. At the same time, project managers tend to overestimate their decision-making capabilities, which prevents them from considering quality improvements in their information selection and choice processes [22,42]. This bias may result in potentially wrong judgments that could be prevented if managers were to modify their information acquisition criteria. Moreover, the process of information retrieval is inherently strategic [36,67], with managers selecting the information that they deem to be more useful from the large amounts available [45,64]. In this regard, Joho et al. [28] concluded that accounting for future realizations in advance is a fundamental task of information acquisition processes that should be incorporated in formal models. Therefore, dynamically consistent information retrieval processes should aim at selecting the most valuable information while preventing regrettable choices on the side of managers/consumers/decision makers (DMs).

2. Literature review

When undertaking a search process, DMs generally aim at decreasing the uncertainty implicit in the purchase of a product or evaluation of an alternative [33]. The literature has identified three main sources of uncertainty inherent to any search process, namely, the uncertainty following from the distribution of product characteristics [32], the uncertainty associated with the credibility and communication abilities of the information providers [16], and the one triggered by the limited cognitive capabilities of the DMs [35]. Intuitively, the first type of uncertainty gives place to the search process, the second relates to the potential utility or regret that may arise from considering the advice of third parties, and the third type relates to the limited

* Corresponding author at: Faculty of Economics and Management, Free University of Bolzano, Piazza Università 1, 39100 Bolzano, Italy.
E-mail addresses: dicaper@mathstat.yorku.ca, debora.dicaprio@istruzione.it (D. Di Caprio), fsantosarteaga@unibz.it (F.J. Santos-Arteaga), tavana@lasalle.edu (M. Tavana).
URL: http://tavana.us/ (M. Tavana).

https://doi.org/10.1016/j.orp.2019.100106
Received 3 August 2018; Received in revised form 2 February 2019; Accepted 13 March 2019
Available online 14 March 2019

capacity of the DMs to acquire and assimilate information. In this regard, selecting products or services in online settings implies facing a higher level of uncertainty given the complexities inherent to their evaluation. This is particularly the case when selecting experience products, since consumption is required for the DMs to provide an evaluation [30].

Nowadays, DMs face an overload of cheap information together with limited assimilation capacities and time constraints, both of which have important effects on their information acquisition and choice behaviour [11,54]. Beyond their cognitive constraints, the limited capacity of DMs to evaluate and compare alternatives through their characterizing categories is also related to the fact that time constraints shorten the information evaluation process [66] and force DMs to concentrate on a subset of the information available [40]. For instance. Cleveland and Ellis [12] reviewed multiple knowledge management studies and highlighted time constraints on the side of the information seekers and trust on the contributors as important factors determining knowledge transmission. Zimmer and Henry [71] focused on recommender systems and the acquisition of information from others, with information quality becoming essential when considering interpersonal sources.

Incrementing the amount of information available about the characteristics of an alternative increases the precision of the decisions while imposing a larger demand on the cognitive capacities of the DMs [17]. In dealing with this type of complexity, the literature has followed the guidelines of bounded rationality and introduced heuristic satisficing rules as a direct alternative to the formal requirements of expected utility theory [6,56]. These rules are introduced to balance the cognitive effort required on the side of the DMs and the accuracy of the decisions being made [20,31,49]. In this regard, Woudstra et al. [65] found empirical support for the application of a cost-benefit model of information retrieval under time pressure, determined by the accessibility and quality of the information source. Similarly, Janssen et al. [26] emphasized that the retrieval of big data from sources with heterogeneous qualities requires governance mechanisms to ensure a basic level of quality and prevent regrettable decisions.

These time and quality restrictions become particularly relevant in the case of organizational management. For example, Schulz et al. [53] developed a framework to identify and select relevant reports while noting that consumers, chief executives, CIOs, and CEOs generally deal with information based on a limited number of characteristics selected and categorized to determine their final choices. Moreover, given the strategic condition of the decisions made by project managers and their reliance on the information available, Eweje et al. [18] emphasized the substantial importance that the confidence of managers has for the decisions taken. These authors found that the extent to which project managers feel in control determines the amount and quality of the information retrieved, which, at the same time, may lead to potentially regrettable decisions. This is particularly important when evaluating projects at their early stages, where dynamically consistent evaluation criteria should be defined – and preserved throughout the entire project so as to increase its likelihood of success [50].

It should be noted that the information retrieval model introduced in the current paper can be adapted to incorporate a quality variable so as to differentiate across information sources [59,61]. However, we will focus on the value assigned by the DM to the information acquired and its capacity to prevent regrettable decisions within a forward-looking sequential environment.

2.1. On cognitive constraints and subjective emotions

An immediate conclusion derived from the literature described above is that a substantial amount of factors can be considered when analyzing the reasons for DMs to regret a given decision, including their limited capacity to observe all potential information sources and assimilate the one acquired. In this regard, Fig. 1 focuses on the relationship arising between time allocation and the value of the products involved in the decisions being made. The data – retrieved in 2017 – illustrate that when purchasing products online DMs worldwide do not spend a considerable amount of time acquiring information, thought the time spent on a decision increases with the value of the product being purchased.

Moreover, when considering recent data from the U.S. population, we observe in Fig. 2 that a substantial percentage of purchases are bought on impulse across all age groups. This result serves as a complement to the percentages described in Table 1, where half the U.S. consumers consulted in 2014 regretted their impulse purchases. The fundamental role played by emotions when purchasing a product on impulse is evident in Fig. 3 where divergent states of mind such as excitement, boredom and sadness are shown to govern the impulsive behavior of U.S. consumers.

The satisfaction derived from a purchase together with the effect from the subsequent increment or decrement in utility relative to its expected value have been regularly analyzed in the psychology literature [2]. These authors illustrated how whenever quality is below the value expected it has a larger effect on satisfaction than the one obtained when expectations are exceeded. In this regard, Lauraéus et al. [34] showed that the satisfaction derived from a purchase is determined by several factors, including the commitment of the DM to the search and the uncertainties tackled.

Psychologists have consistently emphasized the importance that subjective emotions and, in particular, regret have for the behavior of DMs [25,35,47,54]. The behavioral consequences that follow from the capacity of DMs to anticipate regrettable choices have been consistently highlighted in the related literature [19,62,68]. This branch of the psychology literature focuses on the materialization of the outcomes derived from a decision, which are themselves a source of emotion that may vary from surprise and happiness to disappointment and regret [35,44,69,70]. Consequently, the subsequent evaluation of the outcomes is conditioned by these emotions.

The importance placed on the subjective perception of DMs when evaluating the characteristics of potential choices has led cognitive sciences to the forefront of the current research on decision making [5,10]. This literature generally focuses on the effect that the characteristics of DMs have when determining their perception of the different alternatives [3,57]. For instance, personality traits have been recently shown to determine the information acquisition behavior of DMs [1]. However, despite the substantial amount of evidence presented by cognitive scientists, decision theoretical counterparts formalizing the process of information retrieval and its effects on the potential choices of DMs remain mainly unstudied in the literature [51].

The approach followed in the current paper considers an information acquisition process where DMs value information insomuch as it prevents them from making a suboptimal choice that they may regret afterwards. In this regard, the intuition for the expected utility model determining the information acquisition behavior of DMs can be related to the classical economic approach to regret [39]. The standard regret theoretical model in economics considers a utility function modified to account for the expected satisfaction obtained when choosing a given alternative, A, and rejecting another, B. In this case, denoting by x and y the consequences derived from choosing alternative A and B, respectively, the DM is endowed with the following utility function

$$u(x, y) = v(x) + R(v(x) - v(y)) \tag{1}$$

where $v(x)$ stands for the utility derived from choosing alternative A absent any consideration about B. $R(v(x) - v(y))$ accounts for any potential regret or utility gain derived from choosing A, with the expression $v(x) - v(y)$ representing the utility difference from having chosen A instead of B. As is generally the case in economic theory, the function R satisfies $R'(\cdot) > 0$ and $R''(\cdot) < 0$.

We extend this approach beyond the unique characteristic defining each alternative and design a sequential structure accounting for the

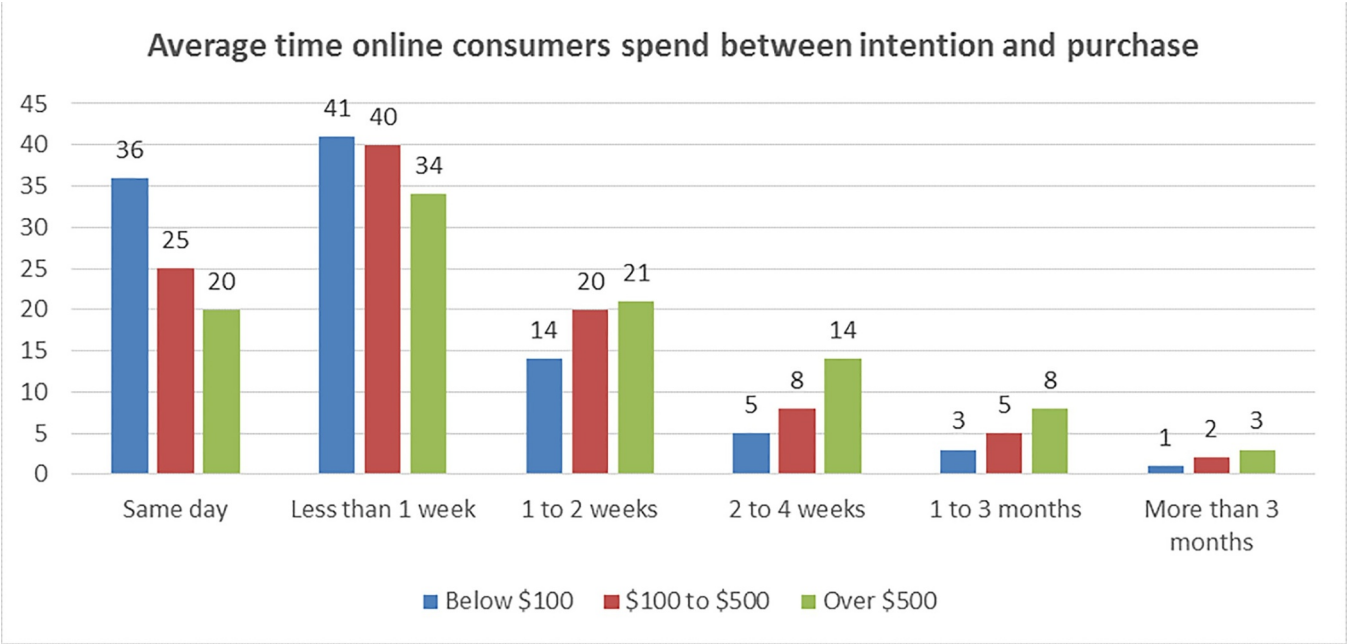


Fig. 1. Average time online consumers spend between intention and purchase by product value. Source: KPMG online survey conducted worldwide (51 countries) in 2017. The sample consisted of 18,430 respondents 15 to 70 years old. Consumers made at least one online purchase in the previous 12 months and were within the top 65% of income earners.

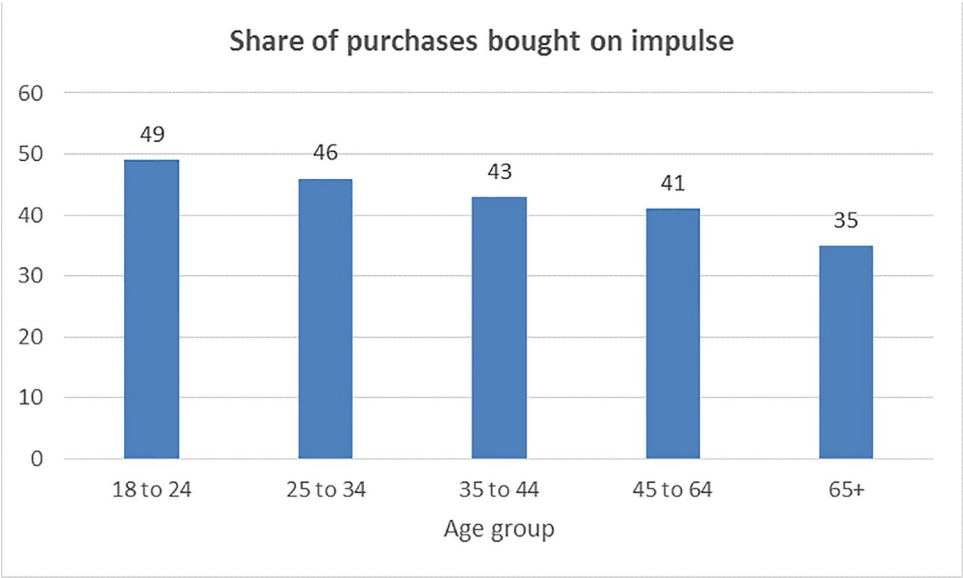


Fig. 2. Share of purchases bought on impulse in the United States as of 2018 by age group. Source: Thredup. The sample consisted of 2000 respondents18 years and older.

Table 1
Share of U.S. consumers regretting an impulse purchase in 2014.
Source: CreditCards.com. Survey conducted in the United States from November 6 to 9, 2014 via telephone interview. The sample consisted of 1000 respondents18 years and older.

Gender	Share of respondents
Female	52
Male	46

relative importance assigned to the different types of regret and verification outcomes disclosed by information. The behavior that follows from this dynamic structure is determined by the value of information when used to prevent regret or validate the utility derived from a given potential choice.

3. Contribution

The model introduced in the current paper complements and extends the psychological approach of Bagozzi et al. [4], who performed four empirical studies to illustrate the influence of both positive and negative anticipated emotions (AEs) in the purchase and non-purchase decisions of DMs. These authors suggested that research should be

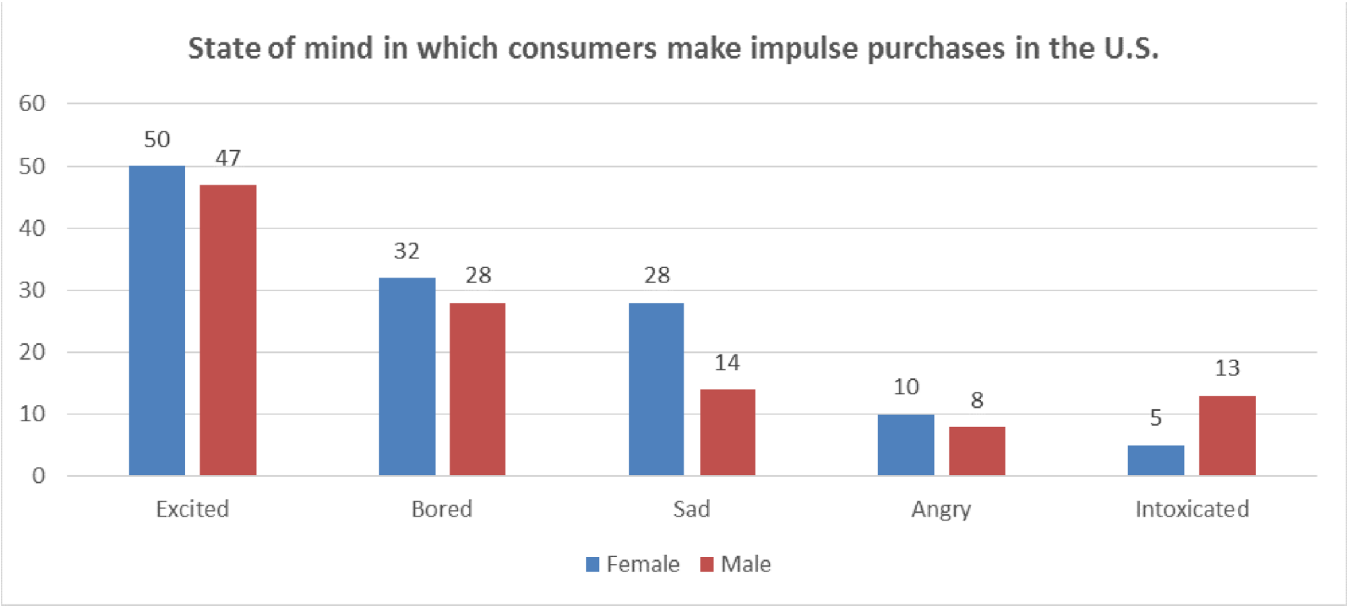


Fig. 3. State of mind in which U.S. consumers make impulse purchases.
Source: CreditCards.com. Survey conducted in the United States from November 6 to 9, 2014 via telephone interview. The sample consisted of 1000 respondents 18 years and older.

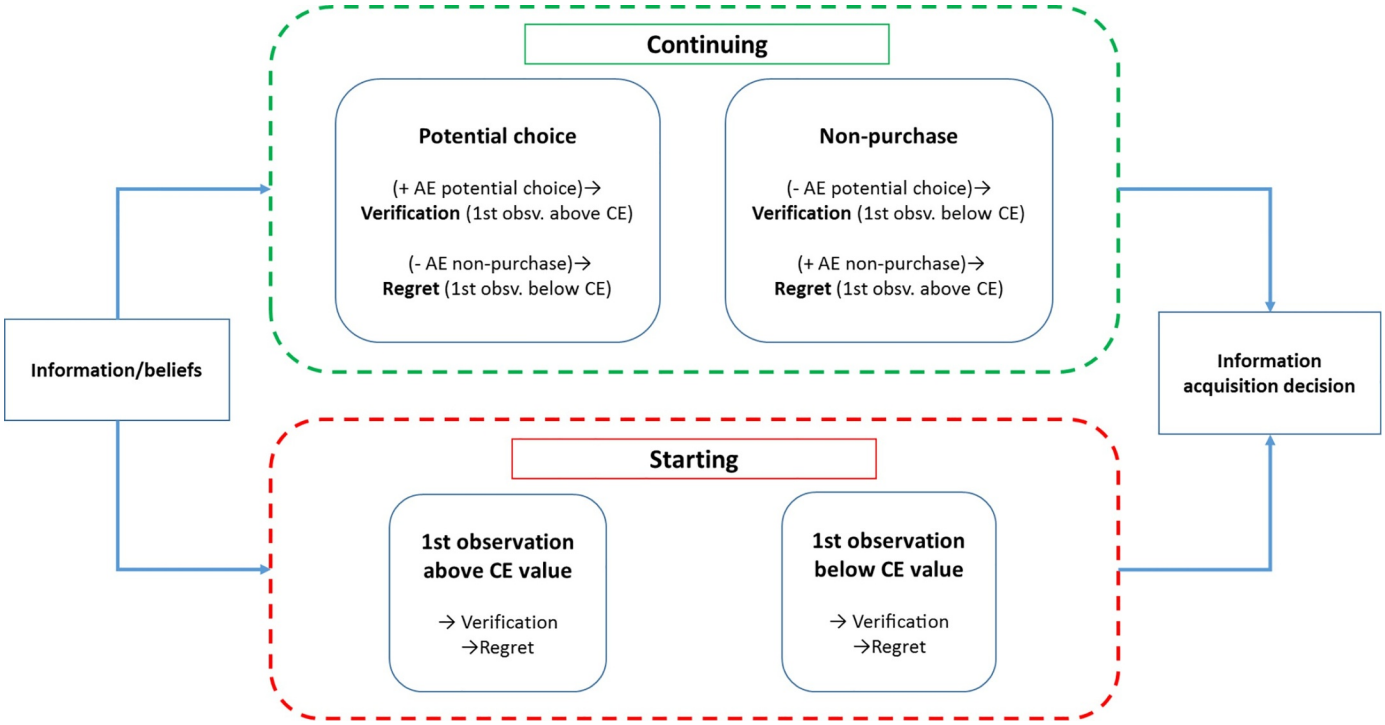


Fig. 4. The proposed information acquisition setting accounting for positive and negative anticipated emotions.

conducted to understand the role of forward-looking emotions in decision processes, while providing an empirical justification for this statement. In particular, they clustered the AEs of DMs in two different categories:

- AEs motivating purchase:
  - positive AEs derived from an expected pleasing purchase
  - negative AEs derived from the missed opportunities of a non-purchase decision
- AEs motivating non-purchase:
  - negative AEs derived from a disappointing purchase
  - positive AEs derived from the goodness resulting from a non-purchase decision

We define a formal information acquisition model that incorporates these four effects into the evaluation and decision framework of DMs. To do so, we introduce a novel definition of value of information accounting for the two main uses that can be derived from it by a DM: verifying the optimality of a potential decision and preventing the regret that may arise from a suboptimal decision [29,52]. Therefore, information will be considered valuable by a DM if it

- verifies the optimality of a given alternative/potential choice
- prevents the DM from rejecting an optimal alternative/potential choice
- verifies the suboptimality of a given alternative
- prevents the DM from selecting a suboptimal alternative

That is, information is valuable if it either allows the DM to verify the optimality or suboptimality of a partially observed alternative or prevents the DM from regretting the rejection or selection of a partially observed alternative. In this regard, the DM would regret a decision whenever rejecting [accepting] an alternative that should have actually been accepted [rejected].

The DM uses the information acquired either to continue observing additional characteristics from a partially observed alternative constituting a potential choice or to start observing characteristics from a new alternative. When continuing, the reference alternative is defined by the certainty equivalent (CE) value of the corresponding characteristics. When starting, the reference alternative is the highest between the previous partially observed one and the CE value.

Fig. 4 merges the fourfold AE setting described in Fig. 4 of Bagozzi et al. [4] with the information acquisition environment introduced in the current paper. In particular, our continuation incentives embed the AE framework described by Bagozzi et al. [4], which is extended through the incorporation of the starting incentives considered by the DM. Note that both types of incentives must be defined when acquiring information before the DM makes any decision.

Our formal information acquisition model allows to account for the subjective relative importance assigned by the DM to the verification and regret value of information. Moreover, we will illustrate how the incentives defining the sequential information retrieval process of the DM are determined by the relative width of the domains on which the different characteristics describing the alternatives are defined.

The remainder of the paper proceeds as follows. Section 4 provides background definitions and concepts. Section 5 introduces the proposed information acquisition selection problem. Section 6 defines the reference utility functions together with some of their properties. Section 7 introduces the value of information as a tool to avoid regrettable choices. Section 8 provides several numerical simulations illustrating the main results. Matlab has been used to perform all the numerical simulations analyzed throughout this section. Section 9 concludes and suggests potential extensions.

4. Background definitions and concepts

Let P be a nonempty set. A *preference relation* $\succsim$ on P is a binary relation on P satisfying reflexivity, completeness and transitivity. A *utility function representing a preference relation* $\succeq$ *on* P is a function U: $P \to R$ such that:

$$\forall\, p, p' \in P \quad p \succeq p' \Leftrightarrow U(p) \geq U(p') \tag{2}$$

where the symbol $\geq$ denotes the standard partial order on the reals.

Henceforth, D will denote a generic DM and Γ a given set of alternatives.

Following Di Caprio et al. [15] and Tavana et al. [60], we assume that each alternative is described by two categories of characteristics, that is, two sets of characteristics in which all the characteristics of the alternatives are partitioned. Note that adding further categories to the analysis would require considering an additional dimension per category through which the DM should generate potential combinations defining the resulting value of information. Representing these objects and performing comparisons to determine the behavior of DMs would require adding further restrictions so as to focus on the two main categories assumed to be the main determinants of his behavior.

At the same time, the partition of all the characteristics in a first and second category is assumed to reflect the order of dominance among characteristics assigned by D, that is, the values taken by the characteristics in the first category are more important to D than those taken by the characteristics in the second. To simplify the presentation, we will write first and/or second characteristic in place of first and/or second category.

We let X and Y be the sets of all possible values that can be taken by the first and second characteristic, respectively, and identify each alternative in Γ with a pair (x, y) belonging to the Cartesian product $X \times Y$.

In this kind of identification, the factor spaces are usually assumed to be compact, connected and separable spaces, endowed with a preference relation represented by utility functions which are continuous with respect to the assigned topology. See, among others, Debreu [13] and Di Caprio and Santos-Arteaga [14].

Following the classical economic approach to consumer information demand [63], we restrict our attention to the case where each factor space is a closed and bounded (hence, compact and connected) non-degenerate subinterval of $[0, +\infty)$. Thus, we let:

$$X = [x_m, x_M] \quad \text{and} \quad Y = [y_m, y_M] \tag{3}$$

with $x_m, x_M, y_m, y_M \in R$, $x_m \neq x_M$, $y_m \neq y_M$, and assume both endowed with the Euclidean topology. Moreover, in line with the classical approach of the theory of choice under uncertainty [21,41], D defines

- the standard linear order $<$ on R as his preference relation on both X and Y;
- two strictly increasing continuous utility functions u: $X \to$ R and v: $Y \to$ R to represent his preferences on X and Y, respectively;
- the following relation $\succeq_{u+v}$ as his preference relation of on $\Gamma = X \times Y$:

$$\forall\, (x_1, y_1),\ (x_2, y_2) \in X \times Y,\ (x_1, y_1) \succeq_{u+v} (x_2, y_2) \overset{def}{\Leftrightarrow} u(x_1) + v(y_1) \geq u(x_2) + v(y_2); \tag{4}$$

- two independent continuous probability density functions μ: $X \to [0, 1]$ and η: $Y \to [0, 1]$ to express his subjective "beliefs" that an element in X or in Y is the value of the first or second characteristic, respectively, of a randomly selected alternative;
- the CE values $c_X = u^{-1}(E_X)$ and $c_Y = v^{-1}(E_Y)$ as the values to assign to the unknown first and second characteristic of an alternative, respectively, where:

$$E_X = \int_X \mu(x) u(x) dx \quad \text{and} \quad E_Y = \int_Y \eta(y) v(y) dy. \tag{5}$$

Finally, in line with Simon [55], we assume that D is allowed to check a limited number of characteristics of a fixed subset of alternatives, $\bar{\Gamma}$.

Remark 1. (a) Without loss of generality, we can assume that $Support(\mu) = X$ and $Support(\eta) = Y$, that is, the support of the probability density functions coincide with the set of actually available values for the characteristics.

(b) The CE value c_X is the element of X whose utility $u(c_X)$ equals the expected utility value induced by μ, i.e., E_X. Similarly, c_Y is the elements of Y whose utility $v(c_Y)$ equals the expected utility value induced by η, i.e., E_Y. In our framework, both c_X and c_Y exist and are unique due to the continuity and strict increasingness of u and v, respectively.

(c) Note that $E_X + E_Y$ is the utility value that D associates to any randomly chosen alternative and also the main reference value against which to compare an alternative whose characteristics are either completely or partially known. ■

Henceforth, $\bar{A}$ will denote the *reference alternative* delivering the expected utility value $E_X + E_Y$, that is, a random alternative in Γ whose characteristics are both unknown.

Table 2
Main notations.

D	Decision maker (DM)
Γ	Set of all the alternatives
$\bar{\Gamma} = \{A_1, A_2\}$	Set of alternatives checked by D; A_1 and A_2 are the first and second alternative checked by D
$X = [x_m,\ x_M]$	Set of all possible values that can be assigned to the 1st characteristic
$Y = [y_m,\ y_M]$	Set of all possible values that can be assigned to the 2nd characteristic
$u,\ v$	Strictly increasing continuous utility functions on X and Y, respectively
$\succeq_{u+v}$	Preference relation on $X \times Y$ induced by $u + v$
$\mu,\ \eta$	Subjective "beliefs" of D on X and Y, respectively
$c_X,\ c_Y$	Certainty equivalent value induced by (μ, u) and (η, v), respectively
$E_X = u(c_X),\ E_Y = v(c_Y)$	Expected utility value induced by μ and η, respectively
x_i	Value of the 1st characteristic of A_i; $i = 1,\ 2$
y_i	Value of the 2nd characteristic of A_i; $i = 1,\ 2$
$\bar{A}$	Any random alternative; the utility of $\bar{A}$ is $E_X + E_Y$
y_x	Unique value that the 2nd characteristic of an alternative with 1st characteristic x must take to deliver a total utility value $E_X + E_Y$; $v(y_x) = E_X + E_Y - u(x)$.
Option (I)	*Continuing with* A_1: check the value y_1 of the 2nd characteristic of A_1
Option (II)	*Starting with* A_2: check the value x_2 of the 1st characteristic of A_2
α	Regret level fixed by D; $\alpha \in [0, 1]$
$EV^{I}_{total}(x_1, \alpha),\ EV^{II}_{total}(x_1, \alpha)$	Total expected information value relative to Option (I) and Option (II) at regret level α
$EV^{I}_{\substack{regret\\prevent}}(x_1),\ EV^{II}_{\substack{regret\\prevent}}(x_1)$	Regret-preventing value expected to be delivered by Option (I) and Option (II)
$EV^{I}_{\substack{choice\\confirm}}(x_1),\ EV^{II}_{\substack{choice\\confirm}}(x_1)$	Choice-confirming value expected to be delivered by Option (I) and Option (II)
$P^+(x_1),\ P^-(x_1)$	Intervals of integration used to define $EV^{I}_{\substack{regret\\prevent}}(x_1)$ and $EV^{I}_{\substack{choice\\confirm}}(x_1)$
$Q^+(x_1),\ Q^-(x_1)$	Intervals of integration used to define $EV^{II}_{\substack{regret\\prevent}}(x_1)$ and $EV^{II}_{\substack{choice\\confirm}}(x_1)$

The main notations introduced in this sections and those that will be introduced in the following ones are summarized in Table 2.

5. Proposed information acquisition selection problem

We consider the problem of a DM, D, who must decide on which one of two alternatives in Γ to focus when acquiring information sequentially.

Due to the dominance of the first characteristic over the second (see Section 2), the first piece of information to be acquired by D is the value of the first characteristic of any of the two given alternatives. Afterwards, D has to decide whether to check the second characteristic of the same alternative, or to check the first characteristic of the other alternative. This is formalized as follows.

Let A_1 and A_2 denote the two alternatives indexed in the order they are checked by D. That is, $\bar{\Gamma} = \{A_1, A_2\}$. Clearly, this order is purely random since there is no reason for any of the two alternatives to be checked before the other. Moreover, for $i = 1,\ 2$, let x_i and y_i be the values of the first and second characteristic of A_i, respectively. Thus, once the value x_1 has been observed, D has the following two options:

Option (I). *Continuing with* A_1: check the value y_1 of the 2nd characteristic of A_1
Option (II). *Starting with* A_2: check the value x_2 of the 1st characteristic of A_2

Assume that D's problem is exactly to establish which of these two options is the best one. Whether D chooses A_1, A_2 or any random alternative $\bar{A}$ will be clear after the information corresponding to the best of the options above has been acquired. Indeed, after acquiring the second piece of information, D can choose the alternative that presents the higher total utility (if both its characteristics are known) or expected utility (if only one of its characteristics is known).

In other words, we do not worry about what D will choose at the end, but about how D needs to use the information so as to avoid a regrettable final choice. Fig. 5 presents a graphical representation of the information acquisition selection problem faced by D and the potential choices that each of the two options would lead to in a utility maximization setting.

6. Reference utility values and cut-off values for the characteristics

As explained in the previous sections (see, in particular, Remark 1(c)), D considers $E_X + E_Y$ as the reference utility value against which to compare an alternative whose characteristics are either completely or partially known.

Thus, the value of the information that D will acquire selecting either Option (I) or Option (II) relates to the possibility that there exists a value of the second characteristic of A_1 such that D is indifferent between A_1 and $\bar{A}$, that is, a value $y \in Y$ such that $u(x_1) + v(y) = E_X + E_Y$.

More in general, we need to consider the set of all values x for which there exists y_x such that $u(x) + v(y_x) = E_X + E_Y$, that is:

$$S \overset{def}{=} \{x \in X\colon \exists\, y_x \text{ such that } v(y_x) = E_X + E_Y - u(x)\}.$$

Note that $S \neq \emptyset$, since $c_X \in S$. Also, it is not difficult to see that S does not necessarily coincide with X, that is, it might exist x such that for all $y \in Y$, $u(x) + v(y) \neq E_X + E_Y$. Furthermore, we have the following results.

Proposition 1. S is a closed subinterval of X, that is, $\exists s_m, s_M \in X$ such that $S = [s_m,\ s_M]$.

Proof. Let x', x'' be two elements of S and consider $x' < x < x''$. Then, $u(x') < u(x) < u(x'')$, from which it follows that $\exists\, y_{x'},\ y_{x''} \in Y$ such that $v(y_{x''}) = E_X + E_Y - u(x'') < E_X + E_Y - u(x) < E_X + E_Y - u(x') = v(y_{x'})$. Since the range of v is connected, $\exists y$ such that $v(y) = E_X + E_Y - u(x)$. Hence, $x \in S$. By the arbitrary choice of x it follows that S is an interval. To show that S is also closed, consider a sequence $\{x_n\}_{n \in \mathrm{N}}$ in S converging to an element x. It suffices to show that $x \in S$. Since u is continuous, the sequence $\{u(x_n)\}_{n \in \mathrm{N}}$ converges to $u(x)$. Thus, $\{E_X + E_Y - u(x_n)\}_{n \in \mathrm{N}}$ converges to $E_X + E_Y - u(x)$. At the same time, for each x_n there exists y_{x_n} such that $v(y_{x_n}) = E_X + E_Y - u(x_n)$. Hence, we have a sequence in Y whose image $\{v(y_{x_n})\}_{n \in \mathrm{N}}$ converges to

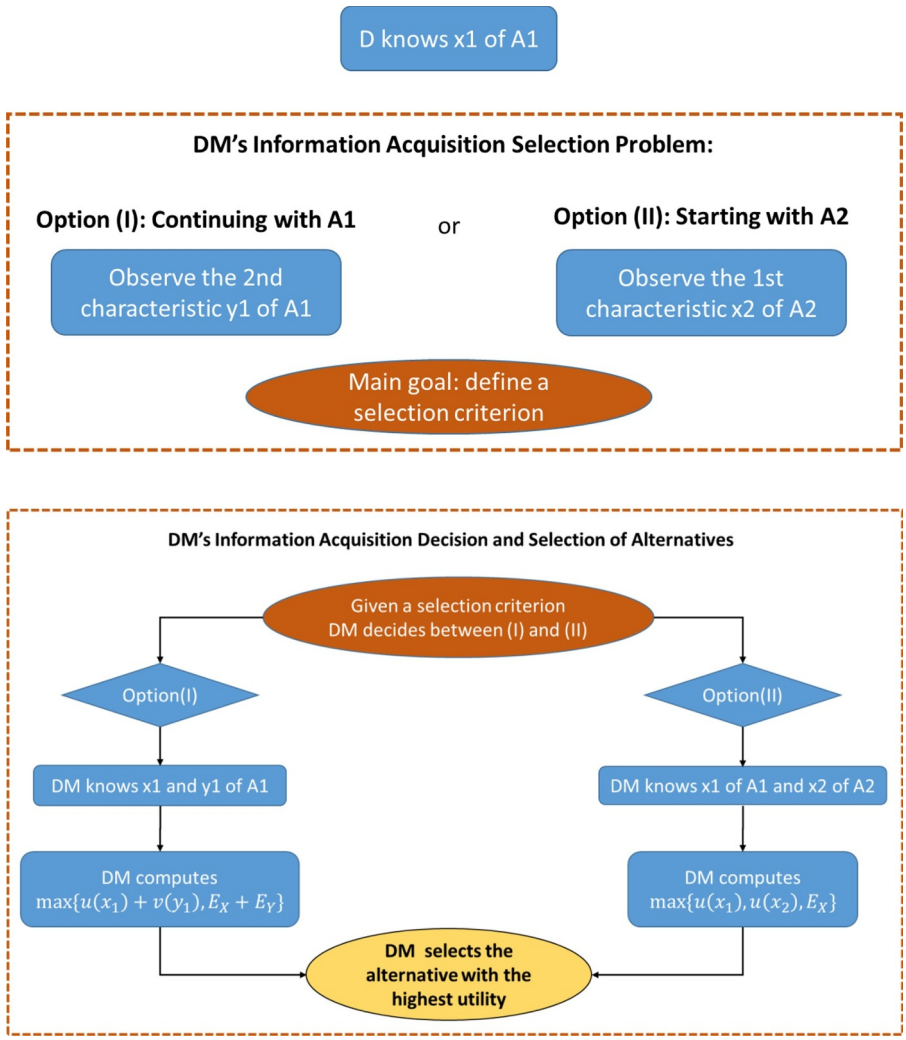


Fig. 5. Information acquisition and alternative selection problem faced by the DM.

$E_X + E_Y - u(x)$. By the continuity of v, $\exists\, \hat{y} \in Y$ such that $\{y_{x_n}\}_{n\in\mathbb{N}}$ converges to $\hat{y}$ and $v(\hat{y}) = E_X + E_Y - u(x)$. Therefore, $x \in S$. ■

Proposition 2. *For every $x \in S$, there exists a unique $y_x \in Y$ such that $v(y_x) = E_X + E_Y - u(x)$.*

Proof. If follows from u and v being strictly increasing. ■

Let φ: $X \to Y$ be the function defined as follows:

$$\varphi(x) = \begin{cases} y_M & \text{if } x \in [x_m,\ s_m) \\ y_x & \text{if } x \in S \\ y_m & \text{if } x \in (s_M,\ x_M] \end{cases} \tag{6}$$

By Proposition 2, the function φ is well-defined. This function associates to each value of the first characteristic of an alternative the unique value that the second characteristic should take for the alternative to be "equivalent" to $\bar{A}$. Note that the function φ is decreasing since higher values of x_1 require lower realizations of y to achieve the utility level provided by $\bar{A}$.

Thus, given the observed value $x_1 \in X$, $\varphi(x_1)$ works as a cut-off value that can be used to partition Y into two subsets $P^+(x_1)$ and $P^-(x_1)$ containing all the values y of the second characteristic that must be associated to x_1 to obtain a total utility $u(x_1) + v(y)$ respectively higher or lower than $E_X + E_Y$. In symbols:

$$P^+(x_1) \stackrel{def}{=} \{y \in Y\colon u(x_1) + v(y) \geq E_X + E_Y\} \tag{7}$$

and

$$P^-(x_1) \stackrel{def}{=} \{y \in Y\colon u(x_1) + v(y) < E_X + E_Y\}. \tag{8}$$

By Eqs. (6)–(8), it follows that:

$$P^+(x_1) = \{y \in Y\colon v(y) \geq v(\varphi(x_1))\} = [\varphi(x_1),\ y_M] = \begin{cases} \varnothing & \text{if } x_1 \in [x_m,\ s_m) \\ [y_{x_1},\ y_M] & \text{if } x_1 \in S \\ [y_m,\ y_M] & \text{if } x_1 \in (s_M,\ x_M] \end{cases} \tag{9}$$

$$P^-(x_1) = \{y \in Y\colon v(y) < v(\varphi(x_1))\} = [y_m,\ \varphi(x_1)) = \begin{cases} [y_m,\ y_M) & \text{if } x_1 \in [x_m,\ s_m) \\ [y_m,\ y_{x_1}) & \text{if } x_1 \in S \\ \varnothing & \text{if } x_1 \in (s_M,\ x_M] \end{cases} \tag{10}$$

Given $x_1 \in X$, the counterpart of the value $\varphi(x_1)$ when considering the set X is naturally given by max $\{x_1, c_X\}$. Indeed, if $x_1 < c_X$, then all the values $x > c_X$ deliver a utility higher than E_X while all the values $x < c_X$ provide less utility than E_X. Similarly, if $x_1 > c_X$, then all the values $x > x_1$ deliver a utility higher than $u(x_1)$ while all the values $x < x_1$ provide less utility than $u(x_1)$. Thus, we can partition X into two subsets, $Q^+(x_1)$ and $Q^-(x_1)$, containing all the values of the first characteristic that deliver a utility respectively higher or lower than max $\{u(x_1), E_X\}$. In symbols:

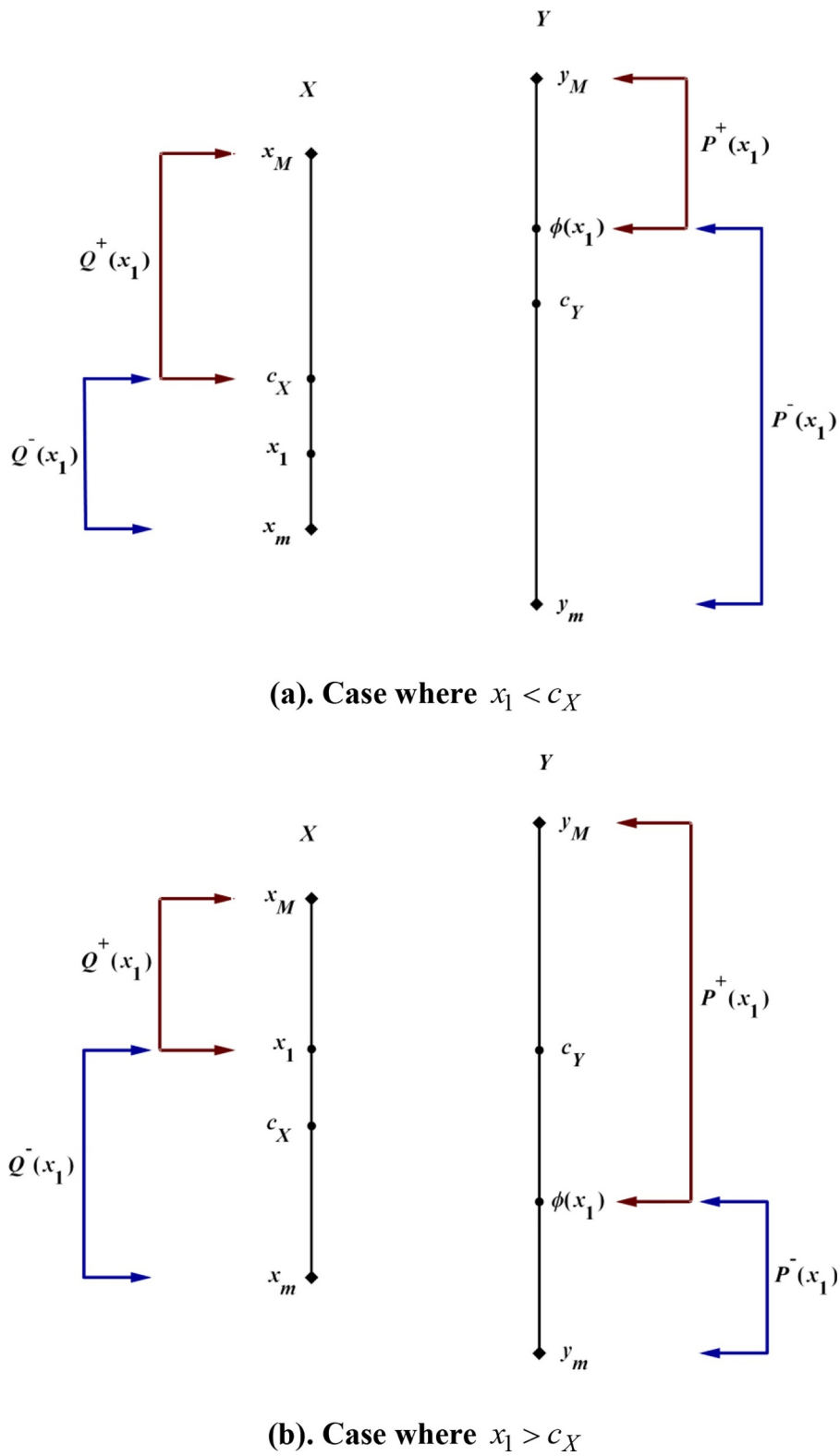


Fig. 6. Partition $\{Q^+(x_1), Q^-(x_1)\}$ of X and partition $\{P^+(x_1), P^-(x_1)\}$ of Y.

$$Q^+(x_1) \stackrel{def}{=} \{x \in X: u(x) \geq \max\{u(x_1),\ E_X\}\} = \{x \in X: x \geq \max\{x_1,\ c_X\}\} = [\max\{x_1,\ c_X\},\ x_M] \tag{11}$$

and

$$Q^-(x_1) \stackrel{def}{=} \{x \in X: u(x) < \max\{u(x_1),\ E_X\}\} = \{x \in X: x < \max\{x_1,\ c_X\}\} = [x_m,\ \max\{x_1,\ c_X\}). \tag{12}$$

Fig. 6 shows both the partition $\{Q^+(x_1), Q^-(x_1)\}$ of X and the partition $\{P^+(x_1), P^-(x_1)\}$ of Y in the case when $x_1 < c_X$ (Fig. 6(a)) and when $x_1 > c_X$ (Fig. 6(b)). An equivalent presentation of the sets $P^+(x_1)$, $P^-(x_1)$, $Q^+(x_1)$ and $Q^-(x_1)$ is provided by Di Caprio et al. [15] while Santos-Arteaga et al. [51] propose an extension to the multidimensional setting.

Remark 2. Note that the evaluation framework described, with the

characteristics of the CE alternative defining the reference values to improve upon, applies directly to situations where the DM short-lists a set of fully or partially observed alternatives among which to choose. In other words, the DM is assumed to have a given set of standards in mind and use the information acquisition process to select a group of alternatives that provide a utility higher than a random choice. This scenario fits particularly well with the sequential information acquisition processes applied in online search environments, characterized by the time pressure faced by DMs when browsing through large amounts of information available at negligible costs [7,38]. The potential regret defining the tradeoffs between two sequentially observed alternatives can be used to determine the dynamic structure of the resulting decision model [24,54]. Fig. 4 can now be revisited as a summary of the decision framework characterizing the information acquisition model described in the following sections. ■

7. Introducing the value of information as a tool to avoid regrettable choices

In order to define a sound selection criterion that D can use to identify the best option between Option (I) and Option (II), we need to formalize the concept of "valuable information" and introduce a suitable evaluation method capable to assess the value of acquiring a new piece of information.

To establish whether or not a piece of information is valuable and measure its value we build on the criterion introduced by Santos-Arteaga et al. [51]. According to this criterion, a new information has a value only if, by acquiring it, D changes the decision that he would have taken without it.

Definition 1. (Assumption 5 in [51]) *A new piece of information is valuable if it induces a reversal in D's potential final choice.*

We introduce the following extension of Definition 1.

Definition 2. *A new piece of information is valuable if it either allows D to make a non-regrettable choice or prevents a regrettable one, or both.*

Intuitively, information is considered valuable if it helps D selecting the alternative providing the highest utility, while preventing him from either choosing an alternative that turns out to be worse than $\bar{A}$, or rejecting an alternative that turns out to be better than $\bar{A}$.

In other words, information is valuable if either one or both of the following conditions are satisfied:

- Choice-confirming: information allows D to confirm that a given alternative provides a utility higher or lower than $E_X + E_Y$. The verification quality of information described within Fig. 4 in terms of the value of the first observation relative to the reference CE alternative accounts for its choice-confirming value.
- Regret-preventing: information prevents D from either choosing an alternative whose utility turns out to be less than $E_X + E_Y$, or rejecting an alternative whose utility turns out to be more than $E_X + E_Y$. In this case, the regret entries introduced within Fig. 4– also in terms of the value of the first observation relative to the reference CE alternative – are the ones accounting for the regret-preventing value of information.

Note that verification together with regret determine the evaluation of the continuing and starting options described in Fig. 4, emphasizing the important role played by both information qualities when defining the subsequent behaviour of D.

To complete Definition 2, we introduce a parameter α that is intended to reflect the relative value that D derives from the information when it allows him to reverse his originally intended choice (regret-preventing information). Clearly, the value of $(1-\alpha)$ will correspond to the relative importance assigned to the choice-confirming role of the information.

Definition 3. *We define the "regret level" of DM D as the value $\alpha \in [0, 1]$ subjectively assigned by D to weight the importance that he gives to the information satisfying the regret-preventing condition. At the same time, the value $(1-\alpha)$ weights the importance that D assigns to confirming a non-regrettable choice and will be called "confirmation level".*

As can be inferred from the description of the main characteristics highlighted in the literature review section, the confidence of DMs on the information providers constitutes one of the main features determining their search incentives. In particular, consumers have been shown to rely on word of mouth advice when provided by a credible information source [37]. The same intuition applies to online environments through the use of online reviews. In both cases, information is acquired so as to improve the outcome from the decisions being made [23], with the sources selected determining the evaluations performed and decisions made by the DMs [48].

Thus, the value of α could be assumed to behave as a proxy for the subjective level of trust placed by the DM on the information source, with a low level leading to a higher weight being assigned to the potentially regrettable outcomes derived from the decision. In this regard, a considerable amount of surveys has been conducted describing the behavior of DMs relative to the expectations generated on the potential outcomes obtained from the search process. Figs. 7 and 8 illustrate how when purchasing products online U.S. consumers rely considerably on the information retrieved from online reviews. Moreover, as described in Fig. 9, the opinions of friends, family and other consumers generate larger trust than those of independent reviewers, highlighting the substantial role played by subjectivity and emotions when evaluating the products being purchased.

We focus now on the value to assign to valuable information. In our setting, D needs to evaluate Option (I) and Option (II), that is, to assess the expected value of the information that would be provided by each of these options if selected by D.

7.1. Expected information value of option (I)

To compute the expected value of the information corresponding to Option (I), that is, the expected value of checking the second characteristic y_1 of the alternative A_1, we reason as follows.

Case (I.a): Suppose that $x_1 < c_X$.

If the second characteristic y_1 of A_1 is such that $u(x_1) + v(y_1) > E_X + E_Y$, then rejecting A_1 would be regrettable since it provides a utility higher than $E_X + E_Y$. That is, choosing A_1 would be better than choosing $\bar{A}$.

Thus, we can compute the regret-preventing value that D expects to derive from selecting Option (I) as follows:

$$EV^{I}_{\substack{regret\\prevent}}(x_1) = \int_{P^{+}(x_1)} \eta(y)(u(x_1) + v(y) - E_X - E_Y)\,dy. \tag{13}$$

If the second characteristic y_1 of A_1 is such that $u(x_1) + v(y_1) < E_X + E_Y$, then A_1 would be confirmed to be a suboptimal choice providing a lower utility than $\bar{A}$. That is, D would prefer a random alternative $\bar{A}$ to the completely observed alternative A_1.

Therefore, we can compute the choice-confirming value that D expects to derive from selecting Option (I) as follows:

$$EV^{I}_{\substack{choice\\confirm}}(x_1) = \int_{P^{-}(x_1)} \eta(y)(E_X + E_Y - u(x_1) - v(y))\,dy. \tag{14}$$

Case (I.b): Suppose that $x_1 \geq c_X$.

If the second characteristic y_1 of A_1 is such that $u(x_1) + v(y_1) > E_X + E_Y$, then A_1 is confirmed as a non-regrettable choice, which implies that choosing A_1 would be better than choosing $\bar{A}$.

This allows us to compute the choice-confirming value that D expects to derive from selecting Option (I) as follows:

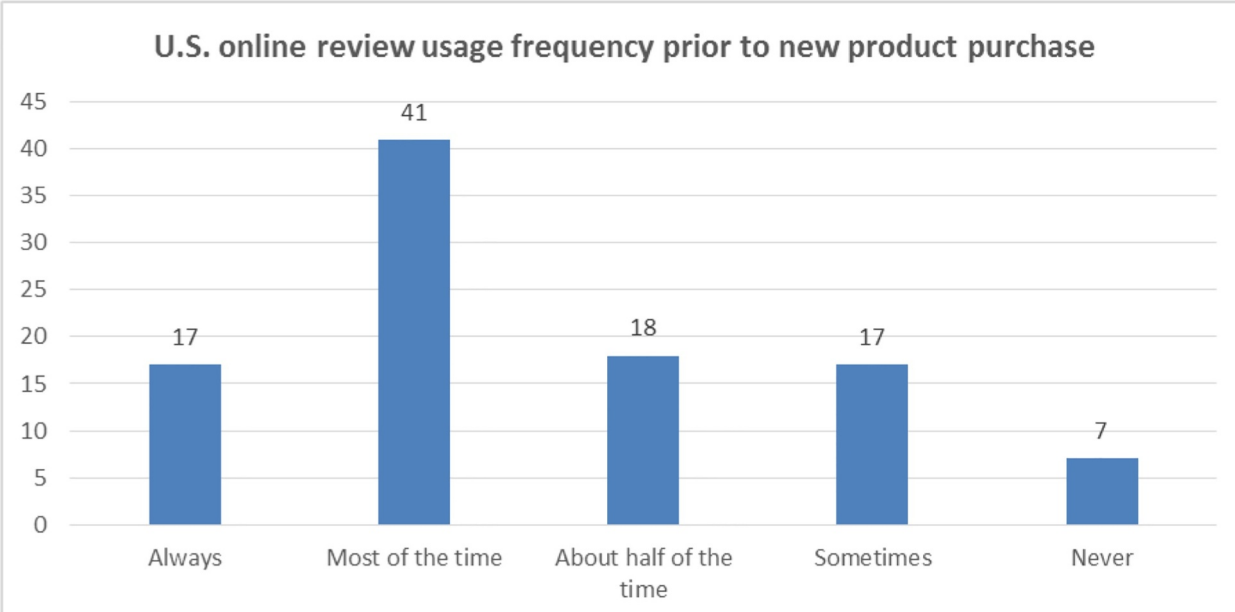


Fig. 7. When looking for a new product, how frequently do you read online reviews before purchasing the product? Source: Worldpay; Socratic Technologies. Survey conducted online in the United States on April 2017. The sample consisted of 501 respondents 25 years and older.

$$EV^{I}_{\substack{choice\\confirm}}(x_1) = \int_{P^{+}(x_1)} \eta(y)(u(x_1) + v(y) - E_X - E_Y)\,dy. \tag{15}$$

If the second characteristic y_1 of A_1 is such that $u(x_1) + v(y_1) < E_X + E_Y$, then rejecting $\bar{A}$ to choose A_1 is a regrettable choice; choosing A_1 would be worse than selecting $\bar{A}$, and is a sub-optimal choice prevented by the new information.

Hence, we can compute the regret-preventing value that D expects to derive from selecting Option (I):

$$EV^{I}_{\substack{regret\\prevent}}(x_1) = \int_{P^{-}(x_1)} \eta(y)(E_X + E_Y - u(x_1) - v(y))\,dy. \tag{16}$$

Given Definitions 2 and 3, Cases (I.a) and (I.b) above yield the following proposition.

Proposition 3. *Let x_1 be the value observed by D for the first characteristic of A_1. Acquiring the information described by Option (I) is valuable from both a regret-preventing and choice-confirming viewpoint. Furthermore, given a regret level $\alpha \in [0, 1]$, the expected information value that D associates to Option (I) is the weighted sum of the corresponding regret-preventing and choice-confirming values, that is:*

$$EV^{I}_{total}(x_1, \alpha) = \alpha \cdot EV^{I}_{\substack{regret\\prevent}}(x_1) + (1 - \alpha) \cdot EV^{I}_{\substack{choice\\confirm}}(x_1) \tag{17}$$

where:

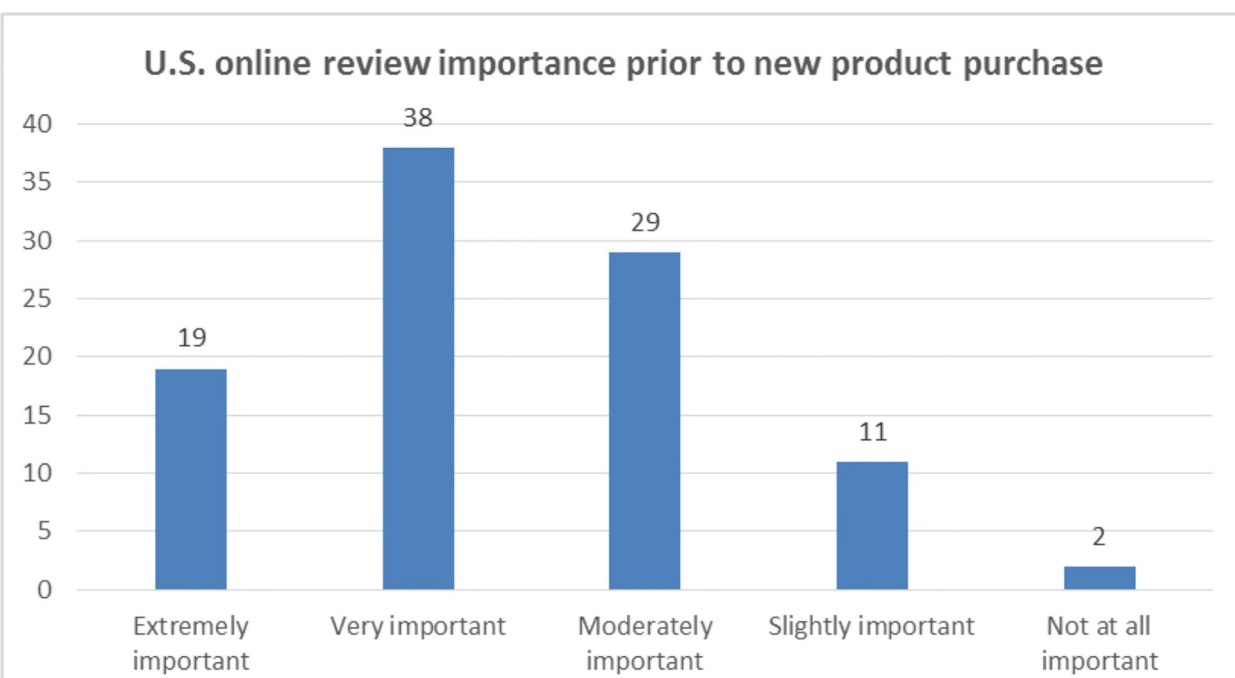


Fig. 8. When looking for a new product, how important are the online reviews before deciding to purchase the product? Source: Worldpay; Socratic Technologies. Survey conducted online in the United States on April 2017. The sample consisted of 501 respondents 25 years and older.

Fig. 9. Whose recommendations are you most likely to trust when you have to choose between different products? Source: Statista Survey conducted online in the United States from April 12 to 14, 2017. The sample consisted of 1052 respondents 18 years and older. Shop online at least once per year. Multiple answers were possible.

$$EV^{I}_{\substack{regret\\prevent}}(x_1) = \begin{cases} \int_{P^+(x_1)} \eta(y)(u(x_1) + v(y) - E_X - E_Y)\,dy, & \text{if } x_1 < c_X \\ \int_{P^-(x_1)} \eta(y)(E_X + E_Y - u(x_1) - v(y))\,dy, & \text{if } x_1 \geq c_X \end{cases} \tag{18}$$

$$EV^{I}_{\substack{choice\\confirm}}(x_1) = \begin{cases} \int_{P^-(x_1)} \eta(y)(E_X + E_Y - u(x_1) - v(y))\,dy, & \text{if } x_1 < c_X \\ \int_{P^+(x_1)} \eta(y)(u(x_1) + v(y) - E_X - E_Y)\,dy, & \text{if } x_1 \geq c_X \end{cases} \tag{19}$$

7.2. Expected information value of option (II)

To compute the expected value of the information corresponding to Option (II), that is, the expected value of checking the first characteristic x_2 of the alternative A_2, we reason as follows.

Case (II.a): Suppose that $x_1 < c_X$.

If the first characteristic x_2 of A_2 is such that $u(x_2) > E_X$, then A_2 would be a better choice than both $\bar{A}$ and A_1. Hence, rejecting A_2 would be regrettable.

Thus, the regret-preventing value that D expects to derive from selecting Option (II) can be computed as follows:

$$EV^{II}_{\substack{regret\\prevent}}(x_1) = \int_{Q^+(x_1)} \mu(x)(u(x) - E_X)\,dx. \tag{20}$$

If the first characteristic x_2 of A_2 is such that $u(x_2) < E_X$, then A_2 would be a suboptimal choice confirming $\bar{A}$ as the optimal one.

This means that the choice-confirming value that D expects to derive from selecting Option (II) is as follows:

$$EV^{II}_{\substack{choice\\confirm}}(x_1) = \int_{Q^-(x_1)} \mu(x)(E_X - u(x))\,dx. \tag{21}$$

Case (II.b): Suppose that $x_1 \geq c_X$.

If the first characteristic x_2 of A_2 is such that $u(x_2) > u(x_1)$, then A_2 is a better choice than A_1 and $\bar{A}$. Hence, A_1, corresponding to D's choice absent the new information, would be a regrettable choice.

Thus, we can compute the regret-preventing value that D expects to derive from selecting Option (II) as follows:

$$EV^{II}_{\substack{regret\\prevent}}(x_1) = \int_{Q^+(x_1)} \mu(x)(u(x) - u(x_1))\,dx. \tag{22}$$

The fact that the first characteristic x_2 of A_2 is such that $u(x_2) < u(x_1)$ does not constitute valuable information regarding the complete verification of any of the products. Indeed, the fact that $u(x_2) < u(x_1)$ does not suffice to state that A_1 is a non-regrettable choice, since the second characteristic of A_1 could still transform A_1 in a regrettable option with respect to $\bar{A}$. Similarly, A_2 could be an optimal product for a sufficiently high realization of the second characteristic. The immediate value of the information follows from the partial confirmation of A_1 as the best potential alternative.

This leads to the following expression for the choice-confirming value that D expects to derive from selecting Option (II):

$$EV^{II}_{\substack{choice\\confirm}}(x_1) = \int_{Q^-(x_1)} \mu(x)(u(x_1) - u(x))\,dx. \tag{23}$$

Together with Definitions 2 and 3, Cases (II.a) and (II.b) above yield the following proposition.

Proposition 4. *Let x_1 be the value observed by D for the first characteristic of A_1. Acquiring the information described by Option (II) is valuable from both a regret-preventing and choice-confirming viewpoint. Furthermore, given a regret level $\alpha \in [0, 1]$, the expected information value that D associates to Option (II) is the weighted sum of the corresponding regret-preventing and choice-confirming values, that is:*

$$EV^{II}_{total}(x_1, \alpha) = \alpha \cdot EV^{II}_{\substack{regret\\prevent}}(x_1) + (1 - \alpha) \cdot EV^{II}_{\substack{choice\\confirm}}(x_1) \tag{24}$$

where:

$$EV^{II}_{\substack{regret\\prevent}}(x_1) = \begin{cases} \int_{Q^+(x_1)} \mu(x)(u(x) - E_X)\,dx, & \text{if } x_1 < c_X \\ \int_{Q^+(x_1)} \mu(x)(u(x) - u(x_1))\,dx, & \text{if } x_1 \geq c_X \end{cases} \tag{25}$$

$$EV^{II}_{\substack{choice\\confirm}}(x_1) = \begin{cases} \int_{Q^-(x_1)} \mu(x)(E_X - u(x))\,dx, & \text{if } x_1 < c_X \\ \int_{Q^-(x_1)} \mu(x)(u(x_1) - u(x))\,dx, & \text{if } x_1 \geq c_X \end{cases} \tag{26}$$

7.3. Deciding between option (I) and option (II)

Letting the value x_1 vary in X, $P^+(x_1)$, $P^-(x_1)$, $Q^+(x_1)$ and $Q^-(x_1)$ can be interpreted as set-valued functions of the observed value x_1. Similarly, given a regret level $\alpha \in [0, 1]$, $EV^I_{total}(x_1, \alpha)$ and $EV^{II}_{total}(x_1, \alpha)$ define two expected information value functions that we will refer to as "continuation" and "starting" functions subject to the regret level α.

By Propositions 3 and 4, it follows that the crossing points between the graphs of the continuation and starting functions work as thresholds defining the "optimal information gathering behaviour" for D. More precisely, having fixed $\alpha \in [0, 1]$, if $EV^I_{total}(x_1, \alpha) \neq EV^{II}_{total}(x_1, \alpha)$, then D proceeds with the option delivering the highest expected value for the information to acquire. If $EV^I_{total}(x_1, \alpha) = EV^{II}_{total}(x_1, \alpha)$, then D is actually indifferent between Option (I) and Option (II); however, without loss of generality, we can assume that D prefers Option (I) to Option (II). In other words, D implements the following selection criterion.

Proposition 5. (Selection Criterion) *Let $\alpha \in [0, 1]$ be the regret level fixed by D. If x_1 is the value observed by D for the first characteristic of A_1, then:*

(a) $EV^I_{total}(x_1, \alpha) \geq EV^{II}_{total}(x_1, \alpha) \quad \rightarrow \quad D$ selects Option (I)
(b) $EV^I_{total}(x_1, \alpha) < EV^{II}_{total}(x_1, \alpha) \quad \rightarrow \quad D$ selects Option (II)

Remark 3. In the case when $\alpha = 1$, $EV^I_{total}(x_1, \alpha)$ and $EV^{II}_{total}(x_1, \alpha)$ coincide with the expected information value functions defined by Santos-Arteaga et al. [51]. Thus, the selection criterion presented by Santos-Arteaga et al. [51] constitutes a particular case of Proposition 5, that is, the case when D gives no importance to the choice-confirming value of the information. ■

8. Numerical examples

Consider, as the basic reference case, the behavior of a risk neutral DM when facing complete initial uncertainty, i.e. maximum information entropy [58]. We will therefore assume throughout the numerical examples presented in this section that D defines linear utility functions and uniform probability densities both on X and Y. That is,

$$\forall\, x \in X,\ u(x) = x \quad \text{and} \quad \mu(x) = \frac{1}{x_M - x_m};$$
$$\forall\, y \in Y\ v(y) = y \quad \text{and} \quad \eta(y) = \frac{1}{y_M - y_m}.$$

Recall that the regret value $\alpha \in [0, 1]$ is fixed by D and expresses the relative importance that D assigns to the choice-confirming and regret-preventing values that he expects from acquiring a new piece of information. Moreover, we should note that while risk neutrality constitutes a plausible assumption in the standard decision models dealing with regret, our formal framework allows for the inclusion of risk averse and risk seeking DMs, an extension that could be based on several of the many different factors affecting the behavior of D, ranging from the amount of money being spent on a product to the trust placed in the different sources of information.

Example 1. Fig. 10 shows the case where $X = [0, 10]$ and $Y = [0, 5]$, which yield $c_X = 5$ and $c_Y = 2.5$, respectively. In particular, Fig. 10(a) provides a general perspective of the functions $EV^I_{total}(x_1, \alpha)$ and $EV^{II}_{total}(x_1, \alpha)$ for different values of α and x_1. Fig. 10(b) shows the graphs of the continuation and starting functions when $\alpha = 0$ whose intersection point is given by $x_1 = 3.9645$. Fig. 10(c) shows the graphs in the case when $\alpha = 0.5$ with an intersection point at $x_1 = 2.5$. Fig. 10(d) illustrates the graphs in the case when $\alpha = 1$: in this case, there is no intersection point. ■

Note that as the value of α increases, i.e. as the subjective importance assigned to the regret-preventing capacity of the information increases, the continuation area vanishes. That is, observing the second characteristic of an alternative does not provide as much value as starting observing a new one when D assigns a relatively higher value to prevent a regrettable choice. If, on the other hand, confirming a choice gains relative importance for D, continuation becomes a reliable option, particularly for lower realizations of the first characteristic so as to verify whether the alternative considered should be discarded.

Additional intuition justifying this result can be obtained when considering the spread of the interval on which the second characteristic is defined relative to that of the first one. That is, in addition to the value of α, the relative spread of the domains on which the characteristics are defined determines the relative value of the information, as we will illustrate through the following example.

Example 2. Fig. 11 shows the case where $X = [0, 10]$ and $Y = [0, 10]$, which yield $c_X = 5$ and $c_Y = 5$, respectively. In particular, Fig. 11(a) provides a general perspective of the functions $EV^I_{total}(x_1, \alpha)$ and $EV^{II}_{total}(x_1, \alpha)$ for different values of α and x_1. Fig. 11(b), (c) and (d) show the cases where $\alpha = 0$, $\alpha = 0.5$ and $\alpha = 1$, respectively. Note that in all the three cases, the functions intersect at $x_1 = 5$, that is, the CE reference value of the first characteristic. For realizations of the first characteristic above c_X both functions overlap. ■

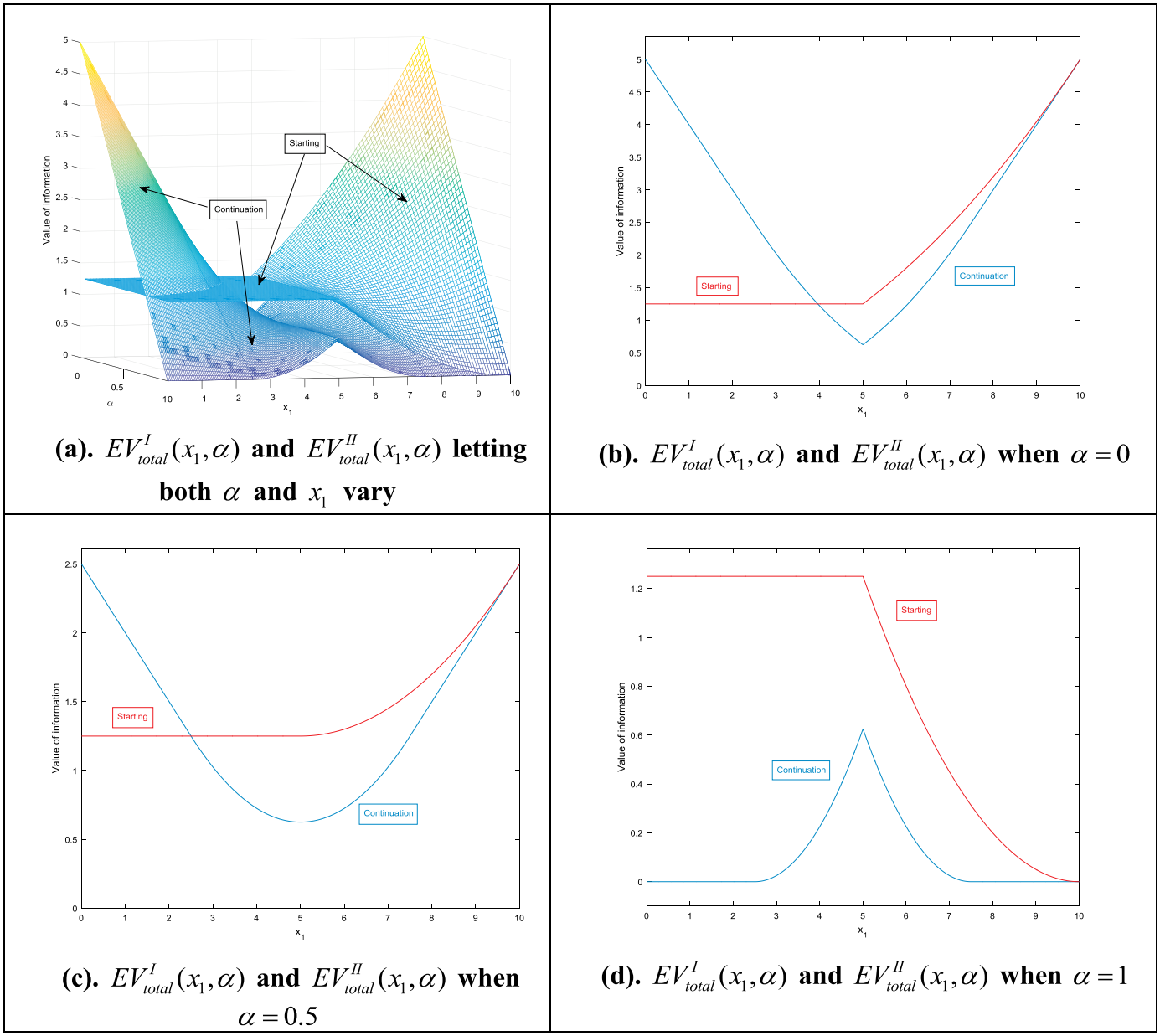
The difference between the settings considered in the two examples above is the increase in the spread of the second characteristic, leading to a higher potential value of the continuation function. Note that the starting function is identical in both settings, since the domain of X remains unchanged. Thus, the value of information derived from starting is the same in both cases. However, the higher spread defined in the latter case leads to a larger continuation area for low values of α and to indecision when the realization of x_1 is above the CE value.

More in detail, the increase in the domain leads to a higher spread of the continuation realizations through Y relative to the CE alternative. In Example 1, the CE value equals $c_X + c_Y = 7.5$ and the lowest and highest potential realizations of an alternative are given by $x_m + y_m = 0$ and $x_M + y_M = 15$, respectively. This spread leads to a potential value gain of 7.5 for the continuation function ($x_M + y_M - (c_X + c_Y) = 7.5$ and $c_X + c_Y - (x_m + y_m) = 7.5$). On the other hand, the value gain for the starting function is equal to 5 ($x_M - c_X = 5$ and $c_X - x_m = 5$).

Example 2 shows that an increase in the domain of the second characteristic leads to an increase in the potential information value obtained from continuing. That is, the new spread of Y leads to a potential value gain of 10 for the continuation function, i.e. $x_M + y_M - (c_X + c_Y) = 20 - 10 = 10$ and $c_X + c_Y - (x_m + y_m) = 10 - 0 = 10$. On the other hand, the domain of the first characteristic defining the potential value gain derived from the starting function remains unchanged. Thus, the increment in the relative value gain derived from continuing determines the shift of the corresponding function in Fig. 11.

Example 3. Fig. 12 validates the previous results by describing the case where $X = [0, 10]$ and $Y = [0, 20]$, which yield $c_X = 5$ and $c_Y = 10$, respectively. In this case, the spread of Y leads to a potential value gain of 15 for the continuation function, i.e. $x_M + y_M - (c_X + c_Y) = 30 - 15 = 15$ and $c_X + c_Y - (x_m + y_m) = 15 - 0 = 15$. Note how continuation dominates starting throughout the whole set of figures, with Fig. 12(d) illustrating the intersection point between both functions at $x_1 = 2.0711$ when $\alpha = 1$. ■

All in all, the incentives of the DMs in terms of value of information have been shown to be directly determined by the subjective importance assigned to the choice-confirming and regret-preventing capacity of the information together with the spread of the potential realizations of the characteristics defining the alternatives. Note also that, Example 2, where $X = [0, 10]$ and $Y = [0, 10]$, displays a



(a). $EV^{I}_{total}(x_1,\alpha)$ **and** $EV^{II}_{total}(x_1,\alpha)$ **letting both** α **and** x_1 **vary**

(b). $EV^{I}_{total}(x_1,\alpha)$ **and** $EV^{II}_{total}(x_1,\alpha)$ **when** $\alpha=0$

(c). $EV^{I}_{total}(x_1,\alpha)$ **and** $EV^{II}_{total}(x_1,\alpha)$ **when** $\alpha=0.5$

(d). $EV^{I}_{total}(x_1,\alpha)$ **and** $EV^{II}_{total}(x_1,\alpha)$ **when** $\alpha=1$

Fig. 10. The expected information value functions $EV^{I}_{total}(x_1, \alpha)$ and $EV^{II}_{total}(x_1, \alpha)$ when $X = [0, 10]$ and $Y = [0, 5]$.

considerable area of indecision arising from the selection criterion. This result can be used to explain the incapacity of DMs to make actual real life choices when the realizations observed are above the reference values [59].

Finally, inaction, namely, the decision of not acquiring any information and selecting the best alternative available, will be explicitly considered among the potential options arising when evaluating a given decision. In this case, the anticipated emotions associated with a given alternative do not define a comparison between continuing acquiring information on a product and starting acquiring information on a new one, but between acquiring and not acquiring information. In all these scenarios, the expected value derived from either acquiring or not acquiring information is determined by the realization of x_1 observed.

Proposition 6. *Let x_1 be the value observed by D for the first characteristic of A_1. Deciding whether to purchase the alternative observed or randomly select a new one without acquiring any additional information delivers either a positive gain from a correct decision or a negative loss from a regrettable one. Furthermore, given a regret level $\alpha \in [0, 1]$, the expected value that D associates to the inaction alternative is the weighted value from both potential outcomes:*

$$EV^{Inact}_{total}(x_1, \alpha) = (1-\alpha)\cdot EV^{choice}_{utility}(x_1) - \alpha\cdot EV^{regret}_{loss}(x_1) \tag{27}$$

where:

$$EV^{regret}_{loss}(x_1) = \begin{cases} u(x_1) + \int_{P^+(x_1)} \eta(P^+(x_1))v(y)\,dy - E_X - E_Y, & \text{if } x_1 < c_X \\ E_X + E_Y - u(x_1) - \int_{P^-(x_1)} \eta(P^-(x_1))v(y)\,dy, & \text{if } x_1 \geq c_X \end{cases} \tag{28}$$

$$EV^{choice}_{utility}(x_1) = \begin{cases} E_X + E_Y - u(x_1) - \int_{P^-(x_1)} \eta(P^-(x_1))v(y)\,dy, & \text{if } x_1 < c_X \\ u(x_1) + \int_{P^+(x_1)} \eta(P^+(x_1))v(y)\,dy - E_X - E_Y, & \text{if } x_1 \geq c_X \end{cases} \tag{29}$$

with $\eta(P^+(x_1)) = \frac{1}{x_1}$, and $\eta(P^-(x_1)) = \frac{1}{x_M - x_1}$, defined over the domains of the $P^+(x_1)$ and $P^-(x_1)$ sets, respectively.

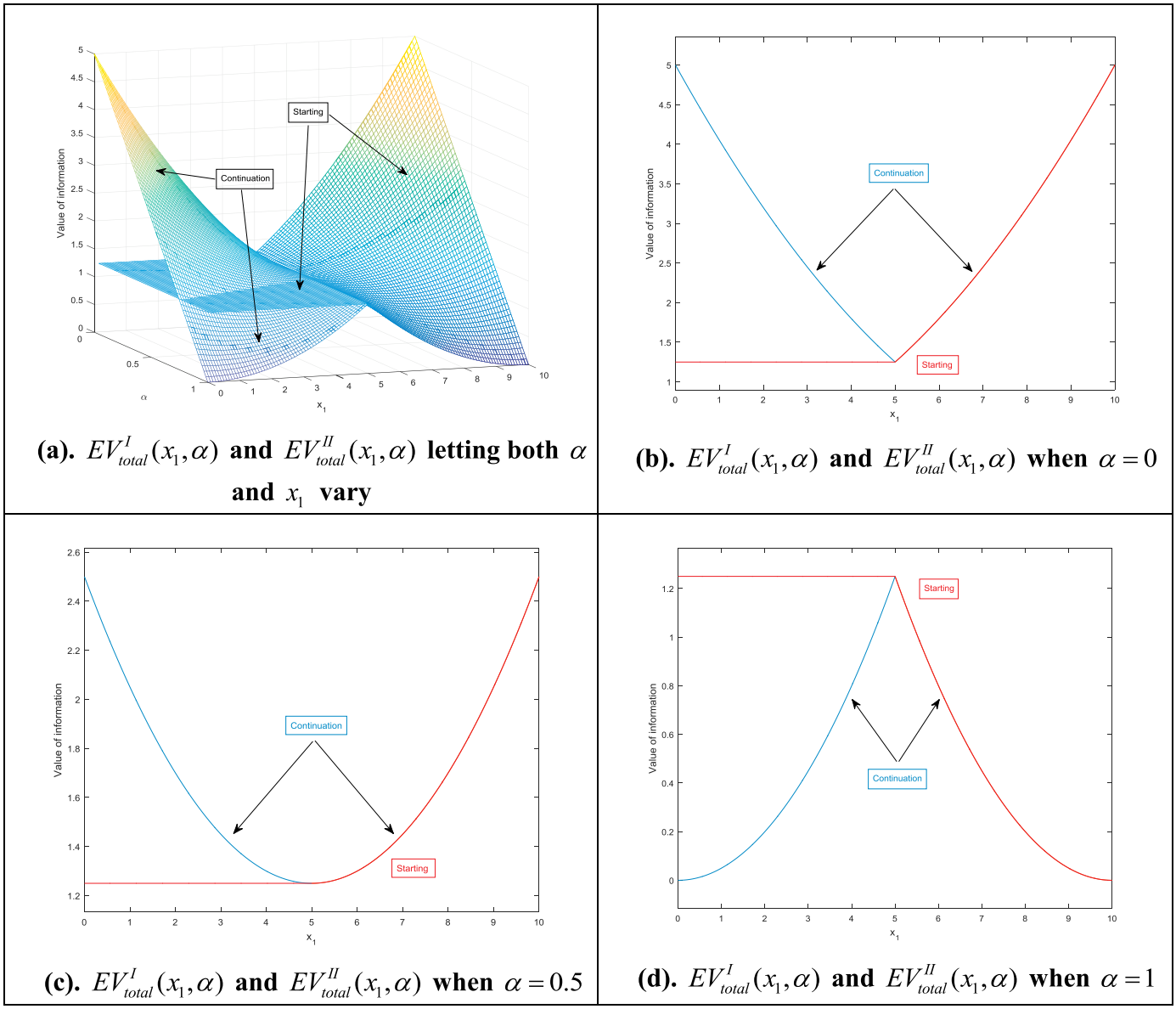


Fig. 11. The expected information value functions $EV^{I}_{total}(x_1, \alpha)$ and $EV^{II}_{total}(x_1, \alpha)$ when $X = [0, 10]$ and $Y = [0, 10]$.

The intuitive interpretation of Proposition 6 is straightforward. In the case where $x_1 < c_X$, D selects a random alternative since it delivers a higher expected utility than the one observed. As a result, the expected value from inaction consists of the potential gain derived from confirming the avoidance of an inferior alternative minus the regret from having foregone a superior one. On the other hand, if $x_1 \geq c_X$, then D selects the alternative observed since it delivers a higher expected utility than a randomly chosen one. As in the $x_1 < c_X$ case, the expected value from inaction consists of the potential gain from having selected a superior alternative minus the regret from having selected an inferior one. Note that, in both cases, the utility assigned to the partially observed alternative is defined in expected terms, since inaction implies that D does not acquire any additional information.

Fig. 13 presents the consequences from introducing the inaction alternative within the $X = [0, 10]$ and $Y = [0, 10]$ framework. Fig. 13(a) illustrates how the expected value of inaction decreases as the subjective importance of regret increases, a result that follows intuitively from Eq. (27). More importantly, Fig. 13(b) shows that inaction constitutes a preferred alternative whenever $\alpha = 0$, i.e. when D does not regard the prevention of regret as an important determinant of his decisions. As described in Fig. 13(c) and (d), inaction is not considered by D whenever $\alpha > 0.5$. Thus, D may decide not to acquire any information when assigning a sufficiently low importance to its regret-preventing value. In other words, if D trusts the source of information, he may decide not to acquire any additional information when selecting (or ignoring) a given alternative.

9. Conclusion

We have defined a novel information acquisition model that incorporates the influence of positive and negative anticipated emotions in the evaluation and selection incentives of DMs. The model has been designed to account for the subjective relative importance assigned by the DMs to the verification and regret value of information. We have also illustrated how the incentives defining the sequential information retrieval process of DMs are affected by the relative width of the domains on which the different characteristics describing the alternatives are defined.

The strategic side of the model becomes evident when considering the fact that the attitude of DMs towards receiving advice, the trust

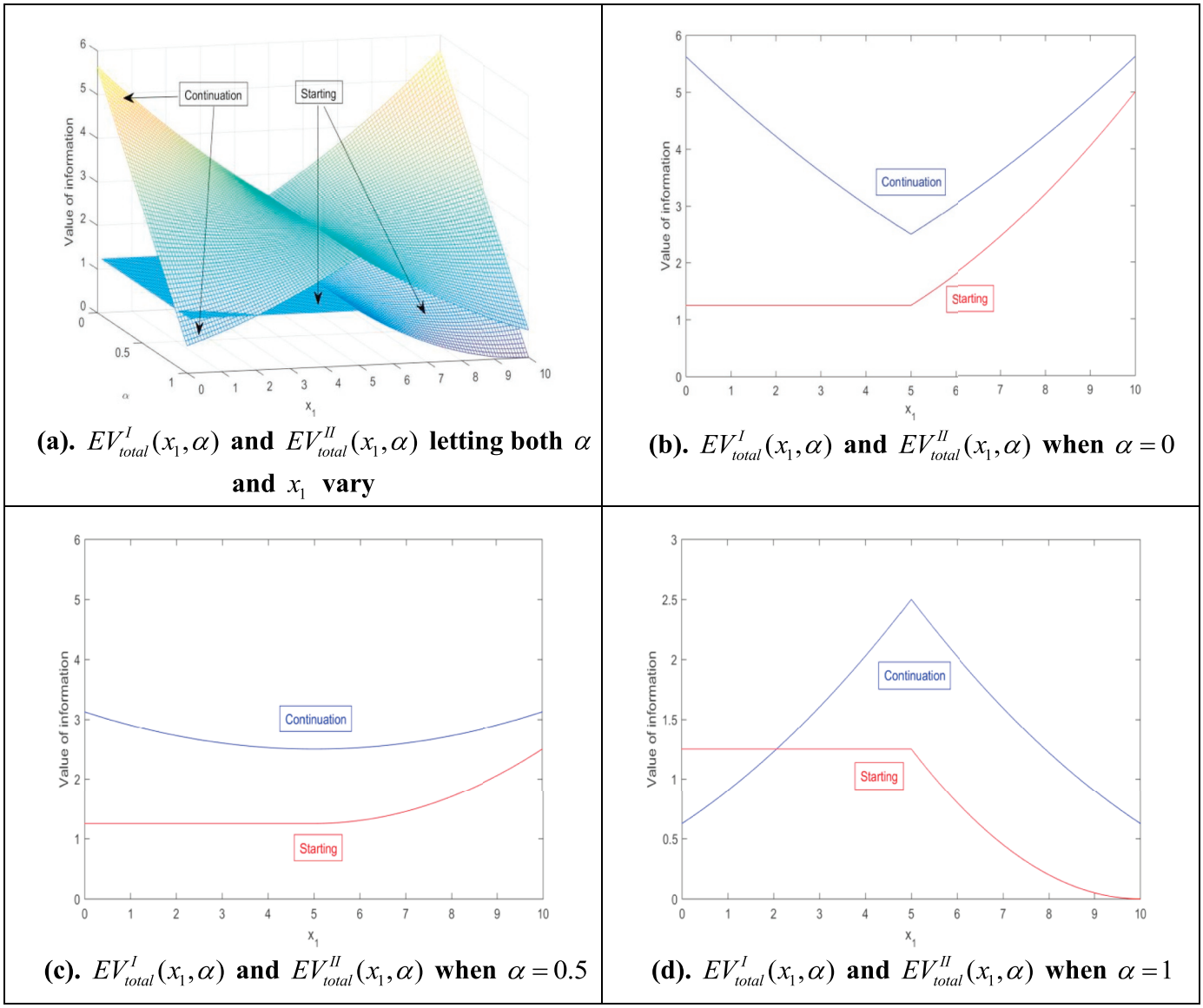


Fig. 12. The expected information value functions $EV^{I}_{total}(x_1, \alpha)$ and $EV^{II}_{total}(x_1, \alpha)$ when $X = [0, 10]$ and $Y = [0, 20]$.

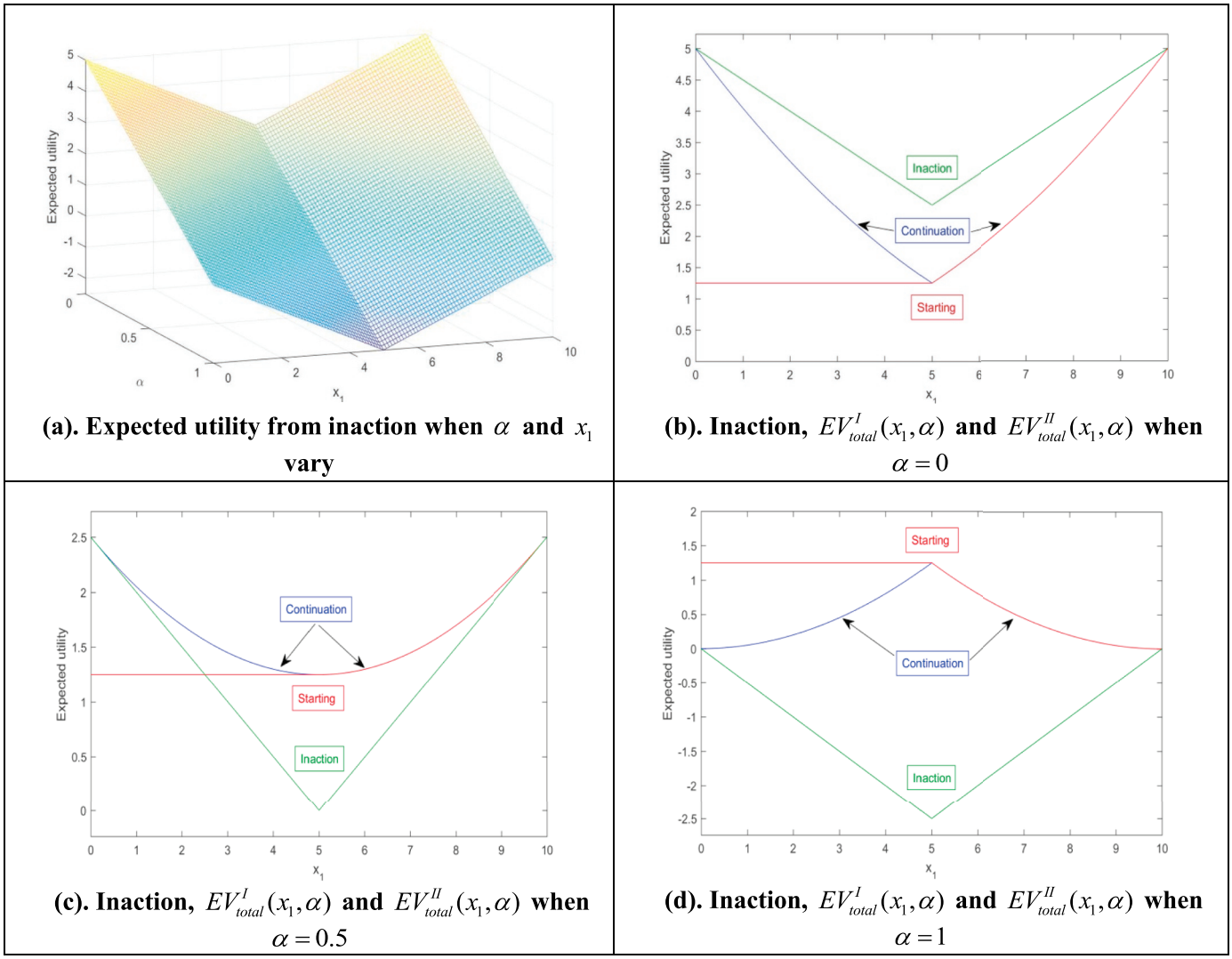
placed in the information source and its perceived usefulness all determine the intention of DMs to follow a given advice [8]. From a purely applied perspective, it is known that a strategic component is inherent to most rating sites and the methods they implemented to build trust [27]. The use of biased reviews allows for the manipulation of DMs, with the resulting consequences being substantial and measurable [43].

Consider, for instance, the results obtained in the current paper regarding the variability of the second characteristic relative to the first one and its potential effects on the information acquisition incentives of DMs. While not guaranteeing the final purchase of a product, exploiting the variability of the second characteristic strategically would allow information providers to capture the process of information acquisition within a given subset of products.

Moreover, instead of a precise account of all the combinations existing within the domains of the different characteristics, heuristic approximations to the computations that must be performed by the DMs could be introduced in future extensions of the current paper. This would help highlighting the cognitive limitations of DMs and the potential differences in search behavior that may arise across normative settings.

It should be noted that, given the overload of free information faced by DMs, we have not explicitly defined any information acquisition cost. However, the limited capacity of DMs to assimilate information could constrain the total amount of information expected to be acquired. This constraint would modify the resulting information retrieval incentives of DMs. A similar setting would result from endowing the DMs with basic memory capacities through the sequential information retrieval process. Note also that our model can account for different attitudes towards risk on the side of the DMs and modifications in their subjective evaluations and beliefs as information is acquired, providing several potential scenarios to define dynamic extensions of the model.

For instance, a scenario that could be analyzed in future research is the case where anticipated regret increases as the DM observes an alternative delivering a similar level of utility to a previously observed one taken as a reference. In this case, the value of α could be defined in terms of the first characteristic of the (partially or fully observed) alternative delivering the highest utility to the DM, x_1^*. Such alternative would be initially given by the certainty equivalent one, i.e. $\bar{A}$. This will be the case until the DM observes an alternative satisfying either $u(x_1^*) + v(y_1^*) > E_X + E_Y$ or $u(x_1^*) + E_Y > E_X + E_Y$. We could therefore define $\alpha(x_1^*) = \frac{x_1^M - x_1}{x_1^M - x_1^*}$ for $x_1 \geq x_1^*$, and $\alpha(x_1^*) = \frac{x_1 - x_1^m}{x_1^* - x_1^m}$ for $x_1 < x_1^*$, with



(a). Expected utility from inaction when α and x_1 vary

(b). Inaction, $EV^{I}_{total}(x_1,\alpha)$ and $EV^{II}_{total}(x_1,\alpha)$ when $\alpha = 0$

(c). Inaction, $EV^{I}_{total}(x_1,\alpha)$ and $EV^{II}_{total}(x_1,\alpha)$ when $\alpha = 0.5$

(d). Inaction, $EV^{I}_{total}(x_1,\alpha)$ and $EV^{II}_{total}(x_1,\alpha)$ when $\alpha = 1$

Fig. 13. The expected value from inaction relative to $EV^{I}_{total}(x_1, \alpha)$ and $EV^{II}_{total}(x_1, \alpha)$ when $X = [0, 10]$ and $Y = [0, 10]$.

x_1 denoting the first characteristic of the alternative being observed. Note that incorporating this modification requires defining a dynamic evaluation process for the DM based on the set of previously observed alternatives. More importantly, the incorporation of this type of extensions to the model would allow us to generate potential classifications of the DMs that could be implemented in real-life environments so as to cluster them and extrapolate their behavior based, for example, on their attitudes towards risk, learning capacities, and trust on the information sources.

Acknowledgement

This work was supported by the Open Access Publishing Fund of the Free University of Bozen-Bolzano.

Supplementary materials

Supplementary material associated with this article can be found, in the online version, at doi:10.1016/j.orp.2019.100106.

Conflict of interest

The authors have no conflict of interest.

References

[1] Al-Samarraie H, Eldenfria A, Dawoud H. The impact of personality traits on users' information-seeking behavior. Inf Process Manag 2017;53(1):237–47.
[2] Anderson EW, Sullivan MW. The antecedents and consequences of customer satisfaction for firms. Market Sci 1993;12(2):125–43.
[3] Bae S, Lee T. Product type and consumers' perception of online consumer reviews. Electron Markets 2011;21(4):255–66.
[4] Bagozzi RP, Belanche D, Casaló LV, Flavián C. The role of anticipated emotions in purchase intentions. Psychol Market 2016;33(8):629–45.
[5] Bartels DM, Johnson EJ. Connecting cognition and consumer choice. Cognition 2015;135:47–51.
[6] Bettman JR. An information processing theory of consumer choice. Addison-Wesley; 1979.
[7] Carr N. The shallows: what the internet is doing to our brains. W. W. Norton & Company; 2011.
[8] Casaló LV, Guinalíu CFM. Understanding the intention to follow the advice obtained in an online travel community. Comput Hum Behav 2011;27(2):622–33.
[9] Cegarra-Navarro JG, Soto-Acosta P, Wensley AKP. Structured knowledge processes and firm performance: the role of organizational agility. J Bus Res 2016;69(5):1544–9.
[10] Chater N. Can cognitive science create a cognitive economics? Cognition 2015;135:52–5.
[11] Chen YC, Shang RA, Kao CY. The effects of information overload on consumers' subjective state towards buying decision in the internet shopping environment. Electron Commerce Res Appl 2009;8(1):48–58.
[12] Cleveland S, Ellis TJ. Rethinking knowledge sharing barriers: a content analysis of 103 studies. Int J Knowl Manag 2015;11(1):28–51.
[13] Debreu G. Representation of a preference ordering by a numerical function. In: Debreu G, editor. Mathematical economics: twenty papers of gerard debreu. Cambridge University Press; 1983.

[14] Di Caprio D, Santos-Arteaga FJ. Cardinal versus ordinal criteria in choice under risk with disconnected utility ranges. J Math Econ 2011;47(4–5):588–94.
[15] Di Caprio D, Santos-Arteaga FJ, Tavana M. The optimal sequential information acquisition structure: a rational utility-maximizing perspective. Appl Math Modell 2014;38:3419–35.
[16] Dimoka A, Hong Y, Pavlou PA. On product uncertainty in online markets: theory and evidence. MIS Q 2012;36(2):395–426.
[17] Dou W, Lim KH, Su C, Zhou N, Cui N. Brand positioning strategy using search engine marketing. MIS Q 2010;34(2):261–79.
[18] Eweje J, Turner R, Müller R. Maximizing strategic value from megaprojects: the influence of information-feed on decision-making by the project manager. Int J Project Manag 2012;30(6):639–51.
[19] Gigerenzer G, Garcia-Retamero R. Cassandra's regret: the psychology of not wanting to know. Psychol Rev 2017;124(2):179–96.
[20] Gigerenzer G, Selten R. Bounded rationality: the adaptive toolbox. The MIT Press; 2002.
[21] Gilboa I. Theory of decision under uncertainty. Cambridge University Press; 2009.
[22] Goodwin P, Wright G. Decision analysis for management judgment. third ed. Chichester, UK: Wiley; 2004.
[23] Hennig-Thurau T, Walsh G. Electronic word-of-mouth: motives for and consequences of reading customer articulations on the Internet. Int J Electron Commerce 2003;8(2):51–74.
[24] Hsu M-H, Chang C-M, Chuang Ll-W. Understanding the determinants of online repeat purchase intention and moderating role of habit: the case of online group-buying in Taiwan. Int J Inf Manag 2015;35(1):45–56.
[25] Ingwersen P. Psychological aspects of information retrieval. Social Sci Inf Stud 1984;4(2–3):83–95.
[26] Janssen M, van der Voort H, Wahyudi A. Factors influencing big data decision-making quality. J Bus Res 2017;70:338–45.
[27] Jeacle I, Carter C. In TripAdvisor we trust: rankings, calculative regimes and abstract systems. Account Organ Soc 2011;36(4–5):293–309.
[28] Joho H, Jatowt A, Blanco R. Temporal information searching behaviour and strategies. Inf Process Manag 2015;51(6):834–50.
[29] Kirkebøen G, Teigen KH. Pre-outcome regret: widespread and overlooked. J Behav Decis Making 2011;24(3):267–92.
[30] Koppius OR, Van Heck E, Wolters M. The importance of product representation online: empirical results and implications for electronic markets. Decis Supp Syst 2004;38(2):161–9.
[31] Kreye ME, Goh YM, Newnes LB, Goodwin P. Approaches to displaying information to assist decisions under uncertainty. Omega 2012;40:682–92.
[32] Lanzetta JT. Information acquisition in decision-making. In: Harvey OJ, editor. Motivation and social interaction - cognitive determinants. New York: Ronald Press; 1963. p. 239–65.
[33] Lanzetta JT, Driscoll JM. Effects of uncertainty and importance on information search in decision making. J Personal Social Psychol 1968;10(4):479–86.
[34] Lauraéus T, Saarinen T, Öörni A. Factors affecting consumer satisfaction of online purchase. 48th Hawaii international conference on system sciences 2015. p. 3364–73. https://doi.org/10.1109/HICSS.2015.406.
[35] Lerner JS, Li Y, Valdesolo P, Kassam KS. Emotion and decision making. Ann Rev Psychol 2015;66:799–823.
[36] Lin Y, Cole C, Dalkir K. The relationship between perceived value and information source use during KM strategic decision-making: a study of 17 Chinese business managers. Inf Process Manag 2014;50(1):156–74.
[37] Litvin SW, Goldsmith RE, Pan B. Electronic word-of-mouth in hospitality and tourism management. Tourism Manag 2008;29(3):458–68.
[38] Lo LYS, Lin S-W, Hsu Ll-Y. Motivation for online impulse buying: a two-factor theory perspective. Int J Inf Manag 2016;36(5):759–72.
[39] Loomes G, Sugden R. Regret theory: an alternative theory of rational choice under uncertainty. Econ J 1982;92:805–24.
[40] March JG. Bounded rationality, ambiguity, and the engineering of choice. Bell J Econ 1978;9(2):587–608.
[41] Mas-Colell A, Whinston MD, Green JR. Microeconomic theory. New York: Oxford University Press; 1995.
[42] Massey C, Robinson D, Kaniel R. Can't wait to look in the mirror: the impact of experience on better-than-average effect. Proceedings of the annual INFORM meeting. 2006.
[43] Mayzlin D, Dover Y, Chevalier J. Promotional reviews: an empirical investigation of online review manipulation. Am Econ Rev 2014;104(8):2421–55.
[44] Mellers BA. Choice and the relative pleasure of consequences. Psychol Bull 2000;126:910–24.
[45] Mintzberg M. Patterns in strategy formulation. Manag Sci 1978;24(9):934–48.
[46] O'Reilly CA. Variations in decision makers' use of information sources: the impact of quality and accessibility of information. Acad Manag J 1982;25(4):756–71.
[47] Park J, Hill WT, Bonds-Raacke J. Exploring the relationship between cognitive effort exertion and regret in online vs. offline shopping. Comput Hum Behav 2015;49:444–50.
[48] Park DH, Lee J. eWOM overload and its effect on consumer behavior al intention depending on consumer involvement. Electron Commerce Res Appl 2009;7(4):386–98.
[49] Payne WJ, Bettman JR, Johnson EJ. The adaptive decision maker. Cambridge University Press; 1993.
[50] Samset K, Christensen T. Ex ante project evaluation and the complexity of early decision-making. Public Organ Rev 2017;17(1):1–17.
[51] Santos-Arteaga FJ, Di Caprio D, Tavana M. A self-regulating information acquisition algorithm for preventing choice regret in multi-perspective decision making. Bus Inf Syst Eng 2014;6(3):165–75.
[52] Sarangee K, Schmidt JB, Wallman JP. Clinging to slim chances: the dynamics of anticipating regret when developing new products. J Product Innovat Manag 2013;30(5):980–93.
[53] Schulz M, Winter P, Choi SKT. On the relevance of reports – Integrating an automated archiving component into a business intelligence system. Int J Inf Manag 2015;35(6):662–71.
[54] Schwartz B. The paradox of choice: why more is less. New York: Ecco Press; 2004.
[55] Simon HA. Administrative behavior. New York: The Free Press; 1997.
[56] Simon HA. A behavioral model of rational choice. Q J Econ 1955;79:99–118.
[57] Stewart N, Chater N, Brown GD. Decision by sampling. Cognit Psychol 2006;53:1–26.
[58] Tavana M. A subjective assessment of alternative mission architectures for the human exploration of Mars at NASA using multicriteria decision making. Comput Oper Res 2004;31:1147–64.
[59] Tavana M, Di Caprio D, Santos-Arteaga FJ. A multi-criteria perception-based strict-ordering algorithm for identifying the most-preferred choice among equally-evaluated alternatives. Inf Sci 2017;381:322–40.
[60] Tavana M, Di Caprio D, Santos-Arteaga FJ. Modeling sequential information acquisition behavior in rational decision making. Decis Sci 2016;47(4):720–61.
[61] Tavana M, Di Caprio D, Santos-Arteaga FJ. An ordinal ranking criterion for the subjective evaluation of alternatives and exchange reliability. Inf Sci 2015;317:295–314.
[62] Tzini K, Jain K. The role of anticipated regret in advice taking. J Behav Decis Making 2018;31(1):74–86.
[63] Wilde LL. On the formal theory of inspection and evaluation in product markets. Econometrica 1980;48(5):1265–79.
[64] Williams TM, Samset K, Sunnevåg KJ. Making essential choices with scant information. Palgrave Macmillan; 2009.
[65] Woudstra L, van den Hooff B, Schouten A. The quality versus accessibility debate revisited: a contingency perspective on human information source selection. J Assoc Inf Sci Technol 2016;67(9):2060–71.
[66] Wright WF. Cognitive information processing biases: implications for producers and users of financial information. Decis Sci 1980;11:284–98.
[67] Xianzhong D, Xu M, Kaye GR. Knowledge workers for information support: executives' perceptions and problems. Inf Syst Manag 2002;19(1):81–8.
[68] Zeelenberg M. Anticipated regret, expected feedback and behavioral decision making. J Behav Decis Making 1999;12(2):93–106.
[69] Zeelenberg M, Nelissen RMA, Breugelmans SM, Pieters R. On emotion specificity in decision making: why feeling is for doing. Judgment Decis Making 2008;3(1):18–27.
[70] Zeelenberg M, Van Dijk WW, Van der Pligt J, Manstead ASR, Van Empelen P, Reinderman D. Emotional reactions to the outcomes of decisions: the role of counterfactual thought in the experience of regret and disappointment. Org Behav Human Decis Process 1998;75:117–41.
[71] Zimmer JC, Henry RM. The role of social capital in selecting interpersonal information sources. J Assoc Inf Sci Technol 2017;68(1):5–21.