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Ab initio calculations for non-strange and strange few-baryon systems

Winfried Leidemann

Dipartimento di Fisica, Università di Trento, I-38123 Trento, Italy
and INFN, TIFPA, I-38123 Trento, Italy

E-mail: `winfried.leidemann@unitn.it`

Abstract. Concerning the non-strange particle systems the low-energy excitation spectra of the three- and four-body helium isotopes are studied. Objects of the study are the astrophysical S-factor S_{12} of the radiative proton deuteron capture $d(p,\gamma)^3$He and the width of the ^{4}He isoscalar monopole resonance. Both observables are calculated using the Lorentz integral transform (LIT) method. The LIT equations are solved via expansions of the LIT states on a specifically modified hyperspherical harmonics (HH) basis. It is illustrated that at low energies such a modification allows to work with much higher LIT resolutions than with an unmodified HH basis. It is discussed that this opens up the possibility to determine astrophysical S-factors as well as the width of low-lying resonances with the LIT method. In the sector of strange baryon systems binding energies of the hypernucleus $^3_\Lambda$H are calculated using a nonsymmetrized HH basis. The results are compared with those calculated by various other groups with different methods. For all the considered non-strange and strange baryon systems it is shown that high-precision results are obtained.

1. Introduction

Non-strange and strange few-baryon systems are particularly interesting particle systems in the hadronic sector. On the one hand they serve for parametrization and test of potential models for nucleon-nucleon (NN), nucleon-hyperon (NY), hyperon-hyperon (YY) interactions and the various analogous three-body interactions. On the other hand they play an important role in testing the quality of ab initio methods, for example in benchmark calculations. In the present work the second of these two aspects is of relevance. In fact our aim is to test the quality of two different ab initio techniques for specific physical questions as is explained in greater detail in the following.

One of the tested ab initio methods is the LIT. The LIT approach is well-established [1] and allows to determine observables involving the many-body continuum without the necessity to calculate continuum wave functions. In the present work it is investigated to what extent specific features in the low-energy electromagnetic response of nuclei can be determined with the LIT method. We consider two examples: (i) the width of the ^{4}He isoscalar monopole resonance 0^+ and (ii) the threshold cross section in ^{3}He photodisintegration. The inverse reaction of the latter, the radiative proton-deuteron capture, is of relevance for the nucleosynthesis and usually parametrized via the astrophysical S-factor S_{12}. As explained in section 2 the crucial point for an exact description of the observables mentioned above lies in the question whether a sufficiently high density of LIT states can be obtained in the low-energy region. In a rather recent LIT

calculation based on HH expansions [2], where the ^{4}He inelastic isoscalar monopole response function was computed with realistic nuclear forces, this aim could not be achieved even though the HH basis was quite large. Therefore it was not possible to determine the width of the 0^+ resonance in this calculation. In [3] it was then shown that the problem is due to the employed HH basis and that a somewhat modified many-body basis solves the problem of a too low low-energy density of LIT states. The modification consists in using for an A-body system, instead of an A-body HH basis, an $(A-1)$-body HH basis times a basis set for the relative motion of the A-th particle with respect to the center of mass of the $(A-1)$-body system.

For the strange particle systems responses to external probes have not yet been determined in experiment. In fact the knowledge of such systems is still rather scarce. One source of experimental information are binding energies of hypernuclei. One of the future aims of our group are ab initio calculations of such binding energies with realistic forces. However, first we want to test the reliability and the precision of the ab initio approach chosen by us. Different from the basis systems, which is used for the above mentioned LIT calculations, where the various HH basis states have a well-defined permutational symmetry, we take for the bound-state calculations of strange few-body baryon systems a nonsymmetrized HH (NSHH) basis. The results are then compared with results coming from other ab initio approaches. We switch from a symmetrized to a nonsymmetrized HH basis because we are confident that calculations for A-body baryon systems with $A \geq 6$ can be carried out with less computational effort.

Since in the present work it is the aim to test the precision of various theoretical ab initio approaches we do not employ realistic interaction models, but use instead simpler potential models, which will be defined in the following sections.

This work is organized as follows. In section 2 the LIT method and the used many-body basis systems are briefly described. Furthermore, the LIT results for the above mentioned S-factor S_{12} as well as for the 0^+ resonance of ^{4}He are discussed. In both cases results with the HH and the new basis are compared. In section 3 it is described how nonsymmetrized basis systems can nonetheless be used to determine ground states of systems which obey a specific permutational symmetry. Subsequently the results for binding and Λ separation energies of $^3_\Lambda$H are illustrated in comparison to results from other authors with different ab initio few-body techniques. Finally, in section 4 a summary is given.

2. The LIT method

Nuclear cross sections of inclusive reactions with electromagnetic probes are expressed in terms of inclusive response functions, which contain the information about the dynamics of the nucleus under investigation. Inclusive response functions are in general of the following form

$$R(\omega) = \int df |\langle f|\hat{O}|0\rangle|^2 \delta(E_f - E_0 - \omega)\,, \qquad (1)$$

where $|0\rangle$ and $|f\rangle$ are nuclear ground and final states, E_0 and E_f are the corresponding eigenenergies and ω is the energy of the exchanged real (photoabsorption) or virtual photon (electron scattering). Finally, $\hat{O}$ denotes the operator inducing the reaction.

A calculation of $R(\omega)$ can be become very difficult or even impossible for cases where $|f\rangle$ is a many-body continuum state. However an explicit calculation of $|f\rangle$ can be avoided by the use of the LIT, which is an integral transform defined as follows

$$L(\sigma) = \int d\omega \, \frac{R(\omega)}{(\omega - \sigma_R)^2 + \sigma_I^2} \qquad (2)$$

with $\sigma = \sigma_R + i\sigma_I$. Due to the variable width of $2\sigma_I$ of the Lorentzian kernel the LIT is an integral transform with a controlled resolution. But it is important to realize that in a given

calculation one cannot simply increase the resolution by choosing smaller and smaller σ_I values. In fact one has to make sure that the precision of the LIT calculation allows the choice of a smaller σ_I value. How this can be achieved becomes clearer in the discussion that follows next.

Since the aim is to determine the response function without the knowledge of the continuum wave function it is useless to calculate the LIT via its definition of eq. (1). Fortunately, the LIT can be determined in an alternative way, namely by solving an equation, the LIT equation, given by

$$(\hat{H} - E_0 - \sigma)|\tilde{\Psi}(\sigma)\rangle = \hat{O}|0\rangle \,, \tag{3}$$

where $\hat{H}$ is the nuclear Hamiltonian. The important feature of the solution $\tilde{\Psi}(\sigma)$ is that it is a localized function. Therefore one can compute $\tilde{\Psi}(\sigma)$ using bound-state methods.

After having determined $\tilde{\Psi}(\sigma)$ one calculates the LIT from the following expression

$$L(\sigma) = \langle \tilde{\Psi}(\sigma)|\tilde{\Psi}(\sigma)\rangle \,. \tag{4}$$

In order to obtain the response function $R(\omega)$ one has to invert the LIT. Details about inversion methods are described in [1, 4].

As already mentioned in the introduction an expansion on a complete many-body basis is used for the solution of the LIT equation (3). First, the Hamiltonian matrix for such a basis is determined, then, in a subsequent diagonalization of this matrix, N eigenvalues E_n and eigenstates ϕ_n (LIT states) ($n = 1, 2, ..., N$) are obtained, where N is the dimension of the basis. The LIT can then be expressed in terms of the energy eigenvalues and the LIT states. One obtains

$$L(\sigma) = \sum_{n=1}^{N} \frac{S_n}{(\sigma_R - (E_n - E_0))^2 + \sigma_I^2} \,. \tag{5}$$

with

$$S_n = |\langle \phi_n|\hat{O}|0\rangle|^2 \,. \tag{6}$$

Coming back to the question which resolution or in other words which value of σ_I can be sustained in a given LIT calculation it is already pointed out in the introduction that the density of LIT states plays a crucial rule. In fact the higher the density of LIT states the higher is also the resolution. This point will be better illustrated in section 2.2.

2.1. Many-body basis systems
In most of the LIT applications the HH basis has been used for the expansions of ground-state wave functions and LIT states of the considered A-body system. This is mainly attributed to its property of being a complete A-body basis for localized states.

An HH basis has the following form

$$HH_{[K]n}(\Omega_A, \rho_A) = \mathcal{Y}_{[K]}(\Omega_A)R_n(\rho_A) \,. \tag{7}$$

It consists of an hyperangular part $\mathcal{Y}_{[K]}(\Omega_A)$ and a hyperradial part $R_n(\rho_A)$, where Ω_A is a set of $3A - 4$ hyperangles, ρ_A denotes the hyperradius and $[K]$ stands for a set of hyperspherical quantum numbers. For the hyperradial basis functions Laguerre polynomials $L_n^{(\beta)}(\rho_A)$ times an exponential factor $\exp(-\rho_A/2b)$ are used, where β and b are a free parameters. In addition one can also introduce as a multiplicative factor NN short-range correlation functions of Jastrow type, which can be purely central or can also become spin and/or isospin dependent (see e.g. [5]).

Usually the hyperangular states $\mathcal{Y}_{[K]}$ are constructed with a well-defined permutational symmetry. With a complementary permutational symmetry of the spin-isospin part of the nuclear wave function one then obtains an antisymmetric basis. In case of $A = 2$ the

hyperangular basis functions reduce to the well-known spherical harmonics Y_{lm}. A detailed description of the HH expansion technique is given e.g. in [1].

In the LIT applications of the present work a new basis $\Phi_{[K]nn'l}$ is employed in addition. It consists of a separation of the A-body basis in a $(A-1)$-part with HH basis functions $R_n(\rho_{A-1})\mathcal{Y}_{[K]}(\Omega_{A-1})$ and a single-particle part with basis functions $R_{n'}^{(2)}(r'_A)Y_{lm}(\Omega_{r'_A})$:

$$\Phi_{[K]nn'l} = \mathcal{Y}_{[K]}(\Omega_{A-1})\, R_n(\rho_{A-1})\, R_{n'}^{(2)}(r'_A)Y_{l(m)}(\Omega_{r'_A}) \tag{8}$$

with $\mathbf{r}'_A = \mathbf{r}_A - \mathbf{R}_{cm}^{(A-1)}$, where $\mathbf{r}_A$ and $\mathbf{R}_{cm}^{(A-1)}$ is the position of the A-th particle and the center of mass of the $(A-1)$-particle system, respectively. For $R_{n'}^{(2)}(r'_A)$ a similar expansion as for the hyperradial part is taken, namely a Laguerre polynomial $L_{n'}^{(2)}(r'_A)$ times an exponential factor $\exp(-r'_A/2b_A)$. Also in this case one may use the NN correlation functions, discussed above, in addition.

Of course, as for the HH basis given in eq. (7), one has to multiply the basis functions of eq. (8) with appropriate A-body spin-isospin wave functions and then one has to care for an antisymmetric state by making a proper antisymmetrization of the basis states.

2.2. Photodisintegration of ^3He and astrophysical S-factor S_{12}
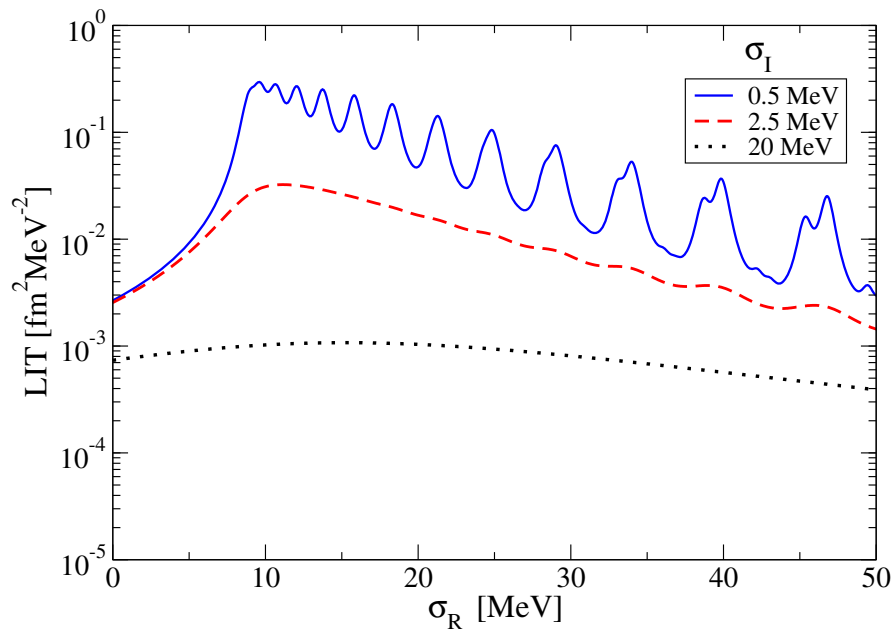
Figure 1. LIT of the ^{3}He response function $R_{\mathrm{E1}}(\omega)$ using an HH basis with 31 hyperradial ($b = 0.3$ fm) and 30 hyperangular basis states.

As pointed out in the introduction, in this work the S-factor of the reaction $d(p,\gamma)^3$He is determined via the inverse reaction, the ^{3}He photodisintegration, then time reversal invariance is applied to obtain S_{12}.

We take the unretarded dipole approximation for the calculation of the ^{3}He photodisintegration cross section, which is given by

$$\sigma_{\mathrm{E1}}(\omega) = 4\pi^2\alpha\omega R_{\mathrm{E1}}(\omega)\,, \tag{9}$$

where α is the fine structure constant and $R_{\text{E1}}(\omega)$ denotes the dipole response function. In this case the components $\hat{O}$ and $|0\rangle$ of eq. (1) become equal to D_z, the third component of the nuclear dipole operator $\mathbf{D}$, and the ^{3}He ground-state wave function, respectively.

In order to determine the S-factor S_{12} one only needs to take into account the low-energy ^{3}He photoabsorption cross section, which is exclusively due to the two-body breakup channel ^{3}He$+\gamma \to p + d$. Since the pd channel has isospin $T = 1/2$, only the $T = 1/2$ channel is considered for the LIT equation (3). As NN potential the MT-I/III potential [6] is employed. To speed up the convergence of the expansions, both with HH basis and new basis, the already mentioned central NN short-range correlation functions are taken into account in addition.

First LIT results with a three-body HH basis are considered. As mentioned in section 2.1 the hyperradial basis functions contain Laguerre polynomials $L_n^{(\beta)}$, here $\beta = 5$ is taken. In fig. 1 the LIT for the case with 30 hyperangular and 31 hyperradial states ($b = 0.3$ fm) is shown. One sees that a smooth LIT is obtained with $\sigma_I = 20$ MeV, while with $\sigma_I = 2.5$ MeV the contributions of single LIT states becomes visible at higher energies. Such contributions due to single LIT states become even the dominant feature for $\sigma_I = 0.5$ MeV. This is a clear sign that the LIT-state density is too low to support a resolution with a σ_I value of 0.5 MeV. In fact the resolution of strength encoded in the LIT depends on the relative distance ΔE of two neighbouring LIT states. Structures with a width smaller than ΔE can hardly be resolved by an inversion of the LIT. In other words the higher the density of LIT states the finer the details that can be resolved inverting the LIT.
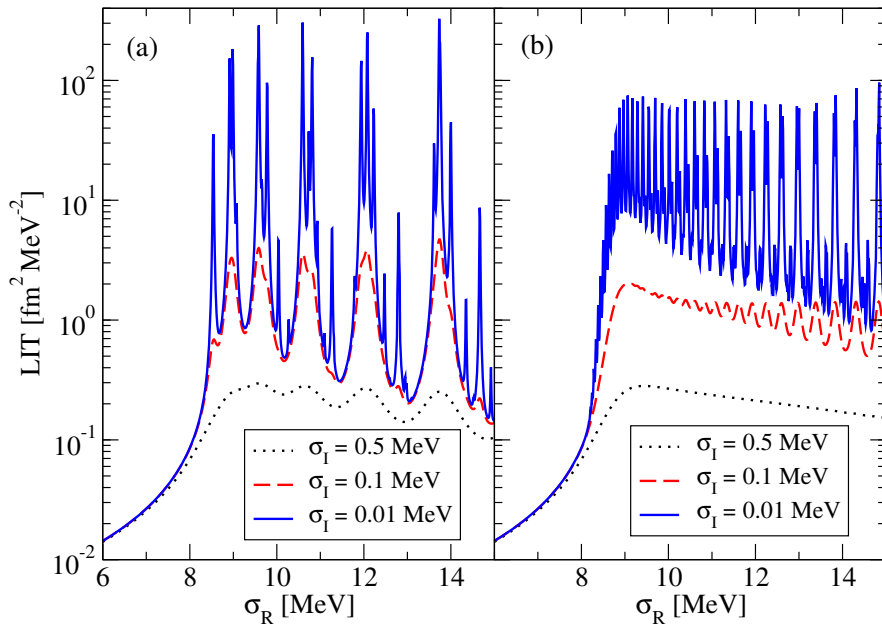
Figure 2. LIT of the ^{3}He response function $R_{\text{E1}}(\omega)$ using an HH basis as in fig. 1 (a) and with 76 hyperradial ($b = 1$ fm) and 40 hyperangular basis states (b).

In order to enhance the LIT-state density one can increase the number of HH basis states taking more hyperangular and/or hyperradial states. In addition one can use a larger spatial extension of the basis by taking a greater value for the hyperradial parameter b, which then leads to a shift of LIT states towards lower energies. The resulting effects on the LIT are discussed in

greater detail in [7]. Here, in fig. 2, we only compare the LITs of fig. 1 at low energies with those obtained with an HH basis of 40 hyperangular and 76 hyperradial states with $b = 1$ fm. From the LIT results with $\sigma_I = 0.01$ MeV it is readily seen that the density of LIT states becomes much larger with the increased HH basis. Accordingly one finds for the lower resolutions of σ_I equal to 0.1 and 0.5 MeV much smoother LIT results with the increased HH basis. However, one also notes a very important point: even in fig. 2b there is not a single LIT state below the three-body breakup threshold at about 8 MeV (^{3}He binding energy with MT potential). Thus the information about the response function is only rather scarce in the energy range between the two-body breakup threshold at about 5.8 MeV and the three-body breakup threshold. Also with regard to the results following in section 2.3 and to those of [2] one may conclude that for an HH basis a systematic increase of the LIT-state density cannot easily be achieved in an energy region where only two-body breakup channels are open. Therefore the HH basis is not very suitable to obtain precise results for such cases, as for example the present one of low-energy ^{3}He photodisintegration.
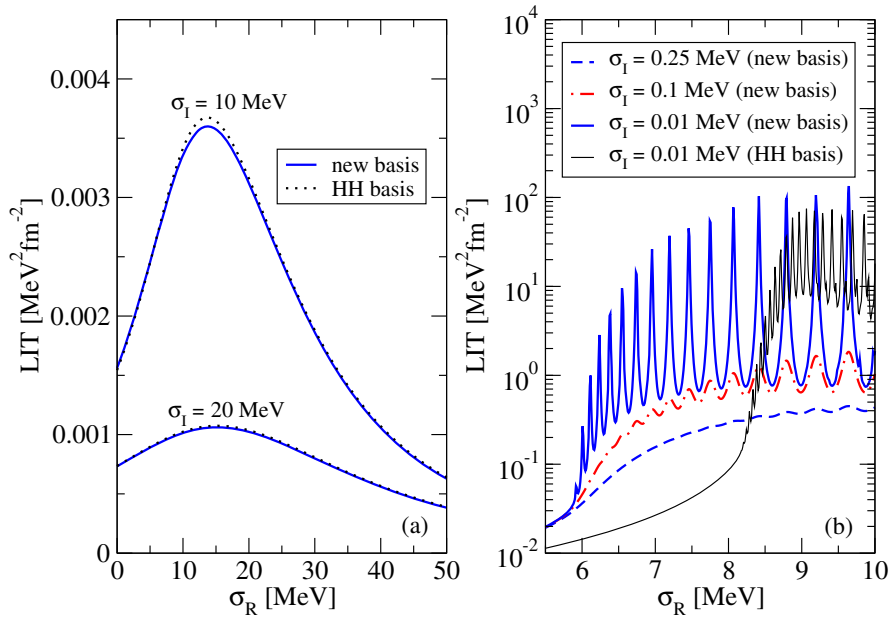
Figure 3. LIT of the ^{3}He response function $R_{\mathrm{E1}}(\omega)$ using an HH basis as in fig. 2b (a) and new basis as described in the text (b).

Now we turn to the results with the new basis described in section 2.1. The $(A - 1)$-basis corresponds in the present three-body case to a two-body basis with basis states $Y_{lm}(\Omega_r) L_{n'}^{(2)}(r) \exp(-r/b_2)$ with $\mathbf{r} = \mathbf{r}_2 - \mathbf{r}_1$, where $\mathbf{r}_i$ is the position of the i-th particle. A basis is used with 25 and 80 radial states for the two-body and single-particle basis, respectively ($b_2 = 0.75$ fm, $b_3 = 0.5$ fm). Since in the present case only the low-energy part of the response function is relevant, it is sufficient to take into account only s-states for the two-body basis.

In fig. 3 LITs resulting from the increased HH basis of fig. 2b and those obtained with the new basis are shown. Figure 3a illustrates that both results are very similar for $\sigma_I = 20$ MeV, while with $\sigma_I = 10$ MeV one finds some differences in the region of the maximum. For a much smaller σ_I of 0.01 MeV, shown in fig. 3b, strong differences become evident. In fact, only with

the new basis LIT states are present right above the two-body breakup threshold at about 5.8 MeV. Moreover these states have a rather high density. More details of the LIT calculation with the new basis, like for example the convergence behaviour with respect to the two-body and single-particle basis systems, are discussed in [7].

The results presented in fig. 3 show that the use of a proper many-body basis can become important for specific questions. There are two conditions which should be considered: (i) is the density of LIT states sufficiently high in order to extract specific structures in the response function and (ii) is the LIT-state density sufficiently regular in order to work with a single σ_I value. If the second condition is not fulfilled one should take in a region of lower LIT-state density a different, more suitable, value for σ_I, otherwise one risks to misplace strength in the inversion. In fact in [7] quite a number of different σ_I values were used in order to take into account a lower LIT-state density with growing energy.

For the LIT with the present HH basis one can conclude that the completely missing LIT states in the two-body breakup region do not only prevent to resolve the correct threshold behaviour of $R_{E1}(\omega)$, but that one would also obtain an overestimation of the peak height of the response function if for the inversion one uses a LIT with a σ_I value much smaller than 20 MeV. As illustrated in [7] (see fig. 7 therein) one can still obtain a rather reasonable inversion result with $\sigma_I = 20$ MeV using the standard inversion method, described in [1], where the correct threshold behaviour of the response function is implemented.

Concerning the LIT results with the new basis one can certainly say that the low-energy density of LIT states is quite high and that the pattern is very regular. Thus one may expect that the low-energy response, and thus the astrophysical S-factor S_{12}, can be determined very precisely by the inversion. In fig. 4 we show the result for S_{12} obtained with the new basis in comparison to a calculation of S_{12} with explicit wave functions for the proton-deuteron continuum states (for details of the continuum state calculation see [7]). As anticipated one observes an excellent agreement between both results. Note that the figure also contains an error estimate due to the LIT inversion, also here we refer to [7] for a more detailed description of the error estimate.

2.3. The ^4He isoscalar monopole resonance

The isoscalar monopole resonance 0^+ of the α-particle leaves a strong signal in inclusive inelastic electron scattering experiments [8, 9, 10]. The corresponding transition form factor was studied in a LIT calculation, where an HH basis and modern realistic forces were used [2]. A rather strong potential model dependence was found, but the experimental data were overestimated quite a bit. The present work, however, is not devoted to determine the strength of the transition form factor, but rather to a different aspect of the resonance, namely its rather small width of 270(50) keV as determined in the ^{4}He(e, e') experiments mentioned above. In [2] this question could not be addressed because the density of LIT states was not sufficiently high in the region of the 0^+ resonance, which is located closely above the lowest ^{4}He two-body breakup threshold.

The isoscalar monopole response function $R_{C0}(q, \omega)$ depends on energy transfer ω and momentum transfer q mediated in electron scattering by the exchanged virtual photon. Thus the corresponding transition operator $\hat{O}$ of eq. (1) becomes q-dependent:

$$\hat{O}(q) = \frac{G_E^s(q^2)}{2} \sum_{i=1}^A j_0(qr_i).$$
(10)

In the equation above $G_E^s(q^2)$ is the nucleon isoscalar electric form factor, $\mathbf{r}_i$ is the position of nucleon i, and j_0 is the spherical Bessel function of 0th order.
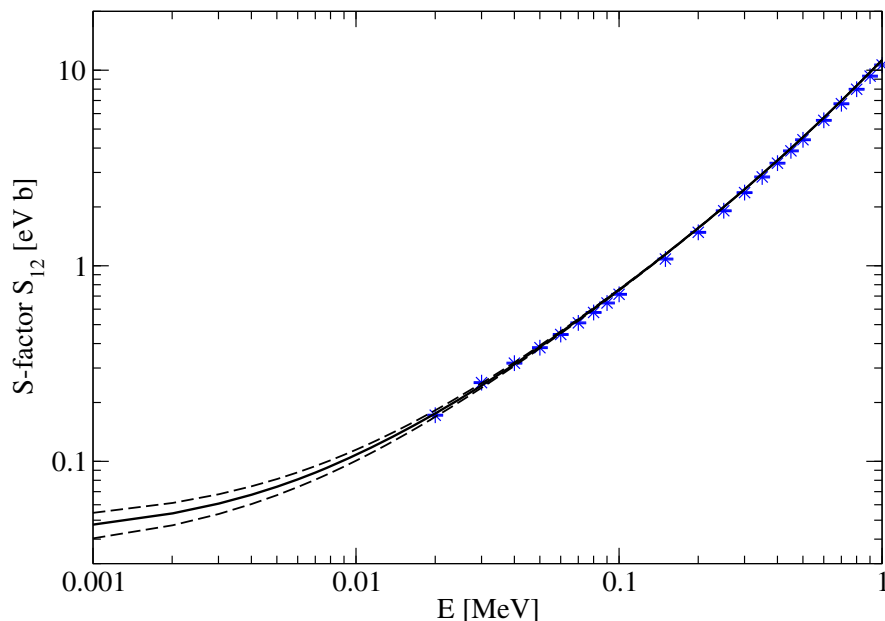
Figure 4. Astrophysical S-factor S_{12} calculated with the LIT method (full curve) with additional error estimate due to inversion (dashed curves); in addition results of a calculation with explicit *pd* continuum wave functions (stars).

For the present study the LIT of $R_{C0}(q, \omega)$ is taken at $q = 300$ MeV/c, a q value, which lies in the momentum transfer range of maximal strength of the 0^+ transition form factor. Here we consider results with an HH basis for the four-body system and in addition the new basis as described in section 2.1 (three-body HH basis plus single-particle basis). For details of the used basis states we refer to [3]. As NN potential model the central TN potential is taken, it has been used in the very first LIT applications for the α-particle (see for example [1]). Like in the previous case in section 2.2 central NN correlation functions are used in order to accelerate the convergence of HH and new basis.

In fig. 5 the LIT results for the response function are shown. For the HH basis in fig. 5a one sees a very similar picture as in fig. 2a. There is only one essential difference, in fig. 2a there are no LIT states below the many-body breakup threshold, whereas in fig. 5a one finds just one LIT state below the many-body breakup threshold at about 30 MeV (note ^{4}He binding energy with TN potential is 31.4 MeV). The isolated low-energy LIT state for the ^{4}He case is due to the 0^+ resonance. It is evident that with a single LIT state it is impossible to determine a resonance width. In the already mentioned LIT calculation for $R_{C0}(q, \omega)$ with realistic nuclear forces of [2] the situation was somewhat better, but the LIT-state density could not be systematically improved in order to have sufficient information to compute the 0^+ resonance width.

As illustrated in fig. 5b the situation is much better in case of the new basis. It is interesting to study the results with the various σ_I values. With a low resolution of $\sigma_I = 5$ MeV one does not realize that there is a resonance (note the logarithmic scale). If one increases the resolution using smaller σ_I values the resonance becomes more and more distinct from the background. For the highest resolution of $\sigma_I = 0.01$ MeV one sees that there are various LIT states in the region of the resonance at about 26 MeV. In fact in [3] it was possible to determine the width using besides the results of fig. 5b also LIT results with a different basis size (for details see [3]). The obtained width of 180(70) keV agrees quite well with the experimental value of 270(50) keV.
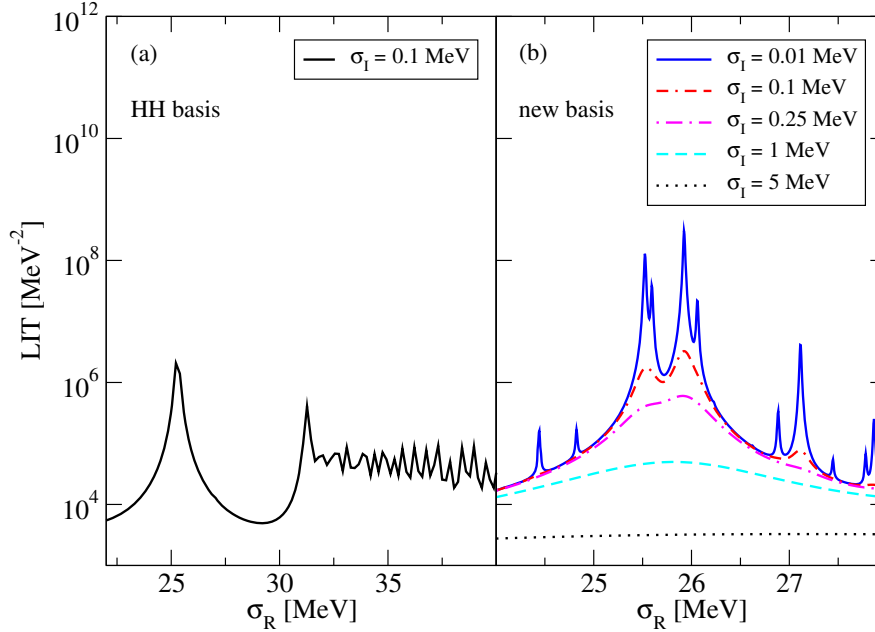
Figure 5. LIT of the ^{4}He response function $R_{C0}(q, \omega)$ at $q = 300$ MeV/c using a four-body HH basis (a) and the new basis $\Phi_{[K]nn'l}$ of eq. (8) (b) (a detailed description of the considered basis states is given in [3]).

3. Strange baryon systems

In the benchmark calculation of [11] bound baryon systems from three up to five particles are considered, where one of the baryons is the Λ hyperon which has strangeness $S = -1$. As already mentioned in the introduction the experimental information about the YN interaction is still rather scarce. On the other hand, our present aim is not yet a realistic calculation of the binding energy of hypernuclei, but rather a check of the precision of the ab initio method used by us. Therefore in [11] calculations with non-fully realistic interactions models are made. Before coming to some of these results in section 3.2, first, a short description of the ab initio method of our choice is given in the following section.

3.1. The nonsymmetrized hyperspherical harmonics (NSHH) expansion

The NSHH expansion relies on the HH expansion given in eq. (7), but the hyperspherical functions $\mathcal{Y}_{[K]}(\Omega_A)$ are not constructed with any permutational symmetry. Also the spin-isospin part of a hypernuclear basis state is taken without imposing a permutational symmetry. On the other hand it is clear that a hypernuclear wave function has to be antisymmetric under the exchange of two identical fermions. At this point it is helpful to consider the Casimir operator $\hat{C}$ of the particle system. Taking an A-body baryon system with N nucleons ($n = 1, 2, ..., N$) and L Λ hyperons ($n = N + 1, N + 2, ..., N + L = A$) one has

$$\hat{C} = \hat{C}_N + \hat{C}_\Lambda = \sum_{j>i=1}^{N} \hat{P}_{ij} + \sum_{j>i=N+1}^{A} \hat{P}_{ij}, \tag{11}$$

where the operator $\hat{P}_{ij}$ exchanges particles i and j. The eigenvalues $\lambda_{[I]}$ of the Casimir operator depend on the specific permutational symmetry of the eigen functions.

Table 1. Binding energy BE and Λ separation energy S_Λ for the hypernucleus $^3_\Lambda$H.

Potential model	Energy	NSHH	AFDMC	FY	GEM
AV4'+Bodmer-Usmani	BE	2.530(3)	2.42(6)	2.537(1)	-
	S_Λ	0.290(3)	0.18(6)	0.292(1)	-
AV8'+NSC97f	BE	2.41(2)	-	2.415(1)	-
	S_Λ	0.17(2)	-	0.189(1)	0.19(1)

In our case with just one Λ particle only the permutational symmetry of the nucleons is relevant. One has the lowest eigenvalue for the antisymmetric case ($\lambda_{[A]} = -N(N-1)/2$) and the highest for the symmetric case ($\lambda_{[S]} = +N(N-1)/2$). Thus diagonalizing the Hamiltonian matrix for an NSHH basis, one can find out the symmetry of a given eigenstate by calculating the corresponding eigenvalue $\lambda_{[I]}$ applying the Casimir operator. This is the strategy which has been put forward in [12]. If, however, the NSHH basis is very large and one wants to find the lowest state being antisymmetric for the nucleonic part, which in general is not the lowest energy state, it is more convenient concerning the computational resources to apply the strategy of [13]. In fact in case of large basis systems it is not advisable to perform a complete diagonalization of H. It is much better to use the Lanczos technique, which saves computational resources and leads to a fast determination of the lowest energy state. In order to bring the lowest antisymmetric state for the nucleonic part to the lowest overall state the following fictitious Hamiltonian $\hat{H}'$ has been introduced in [13]:

$$\hat{H}' = \hat{H} + \gamma \hat{C}_N \,. \tag{12}$$

Thus, for a sufficiently large γ, such a lowest antisymmetric state will become the absolute ground state of the particle system with ground-state energy E_0. Therefore, taking $\hat{H}'$ instead of $\hat{H}$, one can apply the Lanczos technique to find the proper ground state for an A-baryon system with $A-1$ nucleons and one Λ hyperon. In order to have the correct bound-state energy one needs to correct E_0 only by $\gamma N(N-1)/2$.

3.2. The hypernucleus $^3_\Lambda H$
Here the benchmark results for one of the strange baryon systems discussed in [11] are illustrated, namely those for $^3_\Lambda$H. Two different potential sets were used in these calculations: (i) the AV4' NN potential [14] together with the Bodmer-Usmani YN potential [15] and (ii) the AV8' NN potential [14] together with a parametrization [16] of the meson-theoretical NSC97f YN potential [17]. In order to accelerate the NSHH expansion an effective interaction is used as described in [18]. Besides the NSHH results new results with the auxiliary field diffusion Monte Carlo (AFDMC) technique [19] have been obtained. Also results due to the Faddeev approach (FY) and due to the Gaussian expansion method [16] (GEM) are included in [11].

In table 1 the binding energy and the Λ separation energy of $^3_\Lambda$H are listed for the two different potential models defined above. One observes a rather good agreement between the various methods. Only the AFDMC results are a bit different, but this is not a real surprise since the AFDMC is an ab initio method more suitable for systems with more than three particles and with some preference for closed shell nuclei. In fact further calculations discussed in [11] show that the comparison of AFDMC results with those of the other ab initio methods become decisively better for $^4_\Lambda$H and $^5_\Lambda$He.

With the results of table 1 and the further ones given in [11] one can conclude that the NSHH method is very well suited to give precise results for observables of hypernuclei. A

further benchmark with the AFDMC method, where also three-body interactions are taken into account, will be published in the near future. Thus, more ambitious calculations with more realistic interaction models can be tackled with the NSHH method in future.

4. Summary

The purpose of this work is twofold. Firstly, it is a check of the applicability of the LIT method for a precise determination of specific details in nuclear low-energy cross sections that are induced by external electromagnetic probes. To this end the reactions ^{3}He(γ) and ^{4}He(e, e') have been considered. The ^{3}He photodisintegration has been calculated in order to obtain the astrophysical S-factor S_{12} of its inverse reaction, i.e. $d(p, \gamma)^3$He, by applying time reversal invariance. Thus the actual aim has been a precise determination of S_{12} via the LIT method. Comparing to results of a calculation with explicit proton-deuteron continuum wave functions it has been shown that the LIT leads to excellent results for S_{12}. The calculation has been carried out with a simple central NN potential, but the real importance of the calculation does not lie in a realistic calculation of S_{12}, more important is the fact that the LIT method could serve to calculate astrophysical S-factors of reactions involving more than three nucleons. Also the inclusive electrodisintegration of ^{4}He has been computed here with a central NN interaction. Again, the essential aim has not been to obtain realistic results, but to test the possibility to determine the width of a narrow resonance with the LIT method. In fact the ^{4}He continuum exhibits a rather narrow resonance, the so-called isoscalar monopole resonance 0^+. Therefore the LIT calculation has been performed for the ^{4}He isoscalar monopole response function $R_{C0}(q, \omega)$. It has been shown that the resonance width can be determined with the LIT method and that a value of 180(70) keV is obtained. This agrees quite well with the experimental result of 270(50) keV. Thus, there is cause for hope that the 0^+ resonance width can be determined with the LIT method also for the case of a realistic nuclear force.

It has been pointed out that the decisive point for precise determinations of S-factor S_{12} and of ^{4}He 0^+ resonance width is a sufficiently high density of LIT states. Unfortunately, this seems to be very difficult to achieve if one uses as A-body basis an HH A-body basis. It is much better to take an HH hybrid basis consisting of an $(A - 1)$-body HH basis and an additional single-particle basis for the $A - th$ nucleon. It has been illustrated that with such a basis it is possible to systematically increase the density of LIT states in the two-body breakup region at low-energies.

The second purpose of the present work has been a benchmark calculation for $^3_\Lambda$H. The ab initio method of our choice, expansion of the hypernuclear ground state on a nonsymmetrized HH basis, has been discussed, in particular, how such a basis can serve to obtain a ground state with a proper permutational symmetry. Two different potential model sets have been employed for NN and NΛ interactions. In comparison to results from other ab initio approaches it has been found that the present calculation leads to reliable results for the $^3_\Lambda$H binding energy and the corresponding Λ separation energy.

Finally, it should be mentioned that more details of the various calculations are given in [3, 7, 11].

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