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CS-based computational methods for inverse problems arising in arrays processing and design

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Abstract. This work presents a review of the recent Compressive Sensing (*CS*)-based computational methodologies when applied to the design and processing of antenna arrays. The arising inverse problems have been properly formulated in order to deals with the *CS* theoretical requirements. Two instances of array processing and design, namely the diagnosis of failed elements and the synthesis of clustered arrays, are proposed and validated by a simple numerical example.

1. Introduction

In the last years the Compressive Sensing (*CS*) paradigm has been proposed as a promising and effective strategy for solving inverse problems in different research fields, from video/image processing to electromagnetics [1]-[3]. Starting from a reduced set of measurements, fewer than required from standard Nyquist based strategies, *CS* allows fast and reliable inversions, assuming the sparsity of the unknowns [1]. Anyway the use of *CS* in generic inverse problems is not trivial: the choice of the basis function used for the sparse representation of the unknowns is of non-negligible importance [3], and the problem matrix, linking the measurements to the unknowns, must satisfy the restrict isometry property (*RIP*) [3][4]. Unfortunately checking the *RIP* condition is not a simple task and in most of the cases it is computationally unfeasible. Indeed the problem must be properly formulated in order to comply with the *CS* theoretical requirements [2][3]. As instance, *CS* has been recently applied to free-space inverse scattering problems [5]-[8] by mans of Bayesian formulations, for the imaging of intrinsically sparse (i.e. pixel sparse) targets [5], and of scatterers that are sparse in transformed domains [6]-[8]. The effectiveness of *CS* methodologies in solving other electromagnetics related problems [2] has been effectively showed when applied to the synthesis of sparse arrays of antennas [9]-[17], considering linear [10], planar [11]-[14] and conformal geometries [16][17], and properly handling the complex nature of the unknown variables [10]. *CS* has been also successfully exploited for the estimation of direction-of-arrivals (*DoA*) signals [18]-[21], thanks to the intrinsic sparsity of the *DoA*s in the angular domain, considering single [18][19] and multiple snapshot techniques [20][21], and a multi-resolution scheme for real elements arrays affected by mutual-coupling [19].

In this work the recent advancements of *CS* as applied to inverse problems arising in array processing and synthesis are reported. More in detail, the diagnosis problem of failures in planar

phased arrays [22]-[25] and the design of contiguously clustered linear arrays [26] are formulated and validated with a set of numerical examples. Finally the limitations and the future trends of *CS* applied to array processing related inverse problems are discussed as well.

2. Mathematical Formulation

Let us consider an array of P elements having barycenters $\mathbf{r}_p = (x_p, y_p)$, $p = 1, ..., P$ weighted by a set of complex coefficients $\mathbf{w} = \{w_p; p = 1, ..., P\}$ related to the corresponding radiated pattern as:

$$\mathbf{F} = \mathbf{A}\mathbf{w} \tag{1}$$

where in (1) $\mathbf{A} = \left\{a_{np} = e^{jk(x_p u_n + y_p v_n)}; n = 1, ..., N, p = 1, ..., P\right\}$ and $\mathbf{F} = \{F(u_n, v_n); n = 1, ..., N\}$, are the measurement matrix and the pattern samples vector, respectively, $k = 2\pi/\lambda$ is the wave number, λ being the wavelength, $u_n = \sin\theta_n \cos\phi_n$ and $v_n = \sin\theta_n \sin\phi_n$, $n = 1, ..., N$, are the direction cosines, N being the number of pattern samples. In the following, according to the above formulation, two different inverse problems related to array processing, namely the diagnosis of array faulty elements and the design of unconventional clustered array architectures, are described and formulated in order to allow the use of *CS* in solve the arising inverse problems.

2.1. CS-based Diagnostic of Planar Phased Arrays

Let us assume that N_f elements of the array under test (*AUT*) are failed and the goal is to precisely estimate such failures starting from the knowledge of N measured samples of the respective far-field pattern. The vector of the *AUT* excitations can be mathematically described as

$$\mathbf{w}^{AUT} = diag(\mathbf{D})\mathbf{w} \tag{2}$$

where $\mathbf{D}$ is an $P \times P$ diagonal matrix whose diagonal values are equal to $d_{pp} = \alpha_p$, $p = 1, ..., P$, where if $\alpha_p = 1$ the p-th element is correctly working, while if $\alpha_p < 1$ the p-th element is failed ($\alpha_p = 0$ if the p-th element is totally failed). The radiated *AUT* far-field patter is then acquired in N sampling points $\widetilde{\mathbf{F}} = \{\widetilde{F}(u_n, v_n); n = 1, ..., N\}$ as:

$$\widetilde{\mathbf{F}} = \mathbf{A}\mathbf{w}^{AUT} + \mathbf{e} \tag{3}$$

where $\mathbf{e} = \{e(u_n, v_n); n = 1, ..., N\}$ is the vector of the measurement zero-mean Gaussian noise. Assuming the knowledge of the reference, fault-free array pattern $\mathbf{F}$, the difference between the *AUT* array pattern and the one radiated by the reference array , $\Delta\mathbf{F} = \mathbf{F} - \widetilde{\mathbf{F}}$, can be obtained as the pattern radiated by an array fed by the differential weights $\Delta\mathbf{w} = \mathbf{w} - \mathbf{w}^{AUT}$. According to the above formulation, the addressed failure diagnosis problem is then reduced to the estimation of $\Delta\mathbf{w}$ starting from a set of N field measurements $\widetilde{\mathbf{F}}$ satisfying the following relation

$$\Delta\mathbf{F} = \mathbf{A}\Delta\mathbf{w} + \mathbf{e} \tag{4}$$

It is worth noting here that if $N_f \ll P$, thanks to the above formulation, the vector $\Delta\mathbf{w}$ turns out to be sparse. Consequently, thanks to the linear relationship between the measured far-field samples, $\widetilde{\mathbf{F}}$

and the differential weights vector, $\Delta \mathbf{w}$, the inverse problem (4) can be solved using the single-task Bayesian *CS* (*ST-BCS*) propsed in [22] for linear layouts, after properly transforming the complex matrix equation into a real-one [2][3].
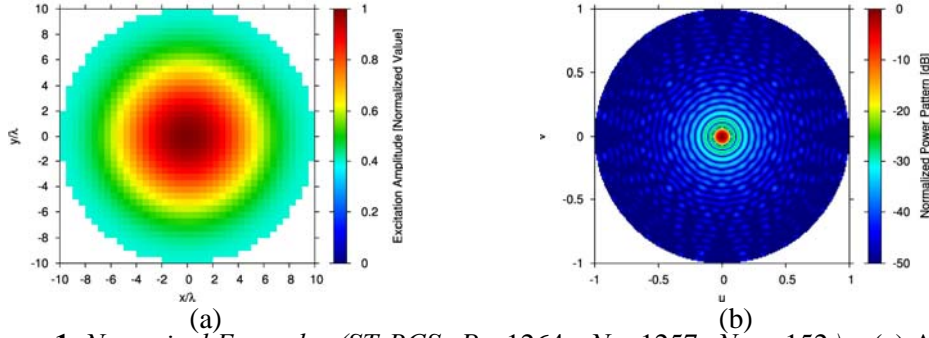

Figure 1. *Numerical Examples (ST-BCS, $P = 1264$, $N = 1257$ $N_f = 152$)* – (a) Array excitation amplitude and (b) the array normalized power pattern.

2.2. Design of Clustered Linear Arrays by means of a TV-CS Strategy

Dealing with the design of clustered linear arrays by means of a CS-based scheme, let us formulate the synthesis problem as a pattern matching with respect to an available reference fully-populated (i.e. non-clustered) array. Let us consider a linear array of P elements placed along the x-axis, contiguously partitioned into Q sub-arrays. The vector of the equivalent array weights $\overline{\mathbf{w}} = \left\{ \overline{w}_p; p = 1,...,P \right\}$ contains the P equivalent excitations coefficients expressed as:

$$\overline{w}_p = \sum_{q=1}^{Q} w_q^{SUB} \delta_{c_p q} \quad p = 1,...,P \tag{5}$$

where in eq. (5) w_q^{SUB} $q = 1,...,Q$ are the complex excitation weights of the Q sub-arrays, $\mathbf{c} = \left\{ c_p \in [1, Q], p = 1,...,P \right\}$, is the clustering vector that univocally defines the sub-array membership of the *p-th* element to the *q-th* sub-arrays by means of Q integer numbers, and $\delta_{c_p q}$ is the Kronecker delta function ($\delta_{c_p q} = 1$ if $c_p = q$, otherwise $\delta_{c_p q} = 0$). Let us assume that the reference array far-field pattern $\mathbf{F}$ is known, the goal is to estimate the equivalent array excitations $\overline{\mathbf{w}}$ satisfying the pattern matching condition

$$\sum_{n=1}^{N} \left| F(u_n) - \sum_{p=1}^{P} \overline{w}_p e^{jkx_p u_n} \right|^2 \le \varepsilon \tag{6}$$

with ε reasonably small. It is worth noting here that if the array is partitioned into $Q << P$ contiguous clusters, the vector $\overline{\mathbf{w}}$ turns out to be a constant function with few discontinuities. Consequently the gradient of the vector $\overline{\mathbf{w}}$ defined as:

$$\nabla \overline{\mathbf{w}} = \left\{ \nabla_p \overline{\mathbf{w}} = w_p - w_{p-1} \right\} \tag{7}$$

turns out to be a sparse vectors with all zero entries, except for the p indexes corresponding to the elements that are located at the border of the clusters. Accordingly, the pattern matching problem (6) can be solved using a *CS*-based approach. More in detail the Total Variation *CS* (*TV-CS*) method [6] can be directly applied in order to retrieve TV-sparse solutions effectively fitting the proposed formulation. With respect to the state-of-art *CS*-based technique [26] the proposed method does not

need to define a-priori the size of the clusters, allowing a complete exploration of the existing solutions.
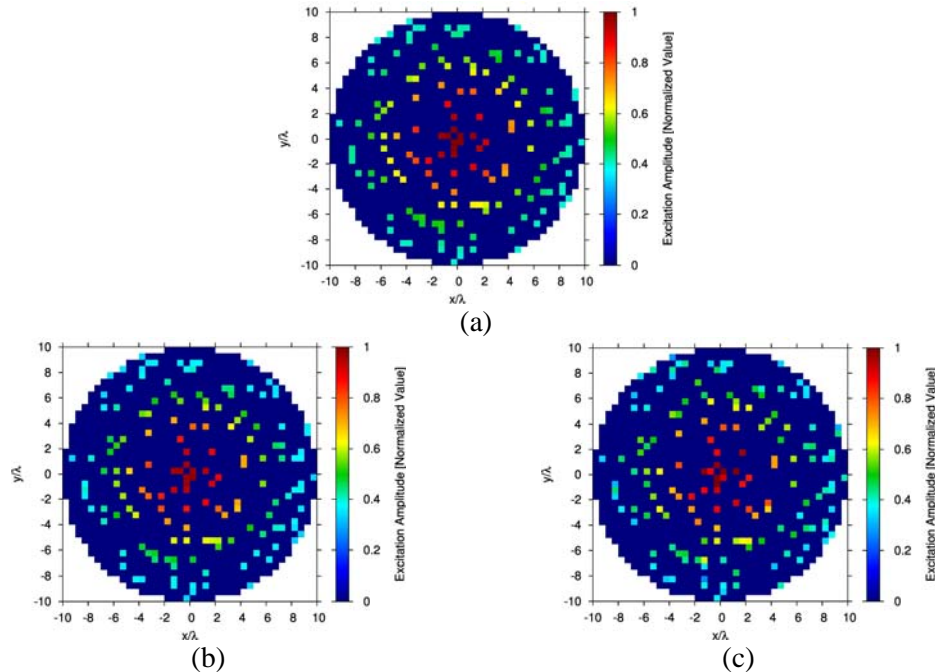

(a)

(b)

(c)

Figure 2. *Numerical Examples (ST-BCS,* $P = 1264$, $N = 1257$ $N_f = 152$ *) –* (a) The actual failures and (b)(c) the *CS* estimated failures considering noisy field samples with (b) $SNR = 40$ [dB] and (c) $SNR = 20$ [dB].

3. Numerical Examples

As a first numerical example of array diagnosis using a *CS* approach, a planar circular array of $P = 1264$ isotropic sources equally spaced by $d_x = d_y = 0.5\lambda$ is considered. The excitations amplitude have been obtained according to a Taylor distribution ($\bar{n} = 3$, $SLL = -25$ [dB]) as shown in Fig. 1(a), radiating the far-field power pattern reported in Fig. 1(b). The test case considers an *AUT* in which a subset of elements is totally failed while all the other elements are correctly working (i.e. $\alpha_p = 0$ or $\alpha_p = 1$, $p = 1,...,P$). The number of failed elements is equal to $N_f = 152$ (i.e. a percentage of failure equal to $\chi = 12\%$) as shown in Fig. 2(a), reporting the differential weights $\Delta\mathbf{w}$. Starting from the knowledge of $N = 1257$ noisy samples of the far field radiated by the *AUT*, the failure configuration has been estimated using the *ST-BCS* solver [22], and reported in Fig. 2(b)-(c). As can be seen, the *CS* method allows to estimate all the $N_f = 152$ failures with small deviations in terms of the amplitude for both the considered noisy levels [i.e. *SNR*=20 [dB], Fig. 2(b) and SNR=40 [dB], Fig. 2(c)]. Of course the percentage of failures that can be reliably estimated is limited, due to the sparseness of the solution enforced in solving eq. (4). A maximum number of failures recovered by the proposed ST-BCS based method, has been estimated in the interval $\chi_{max} = [16\%, 20\%]$ if reliable reconstruction must be guaranteed. In order to improve such limitation, the use of the *MT-BCS* approach [10] will be considered as a possible extension of the methodology, exploiting as instance, the correlation of the real and imaginary parts of the measured pattern values. It is worth noting that the proposed approach, according to the formulation reported in Sec. 2, can handle only pixel-like failures, consequently, future extensions will consider non-pixel like failures, such as clusters of failed elements complying with unconventional tiled architectures [27].

As a second numerical example a linear array of $P = 20$ ideal isotropic elements, laying in the x-axis equally spaced by $d_x = 0.5\lambda$ is considered. The goal is to find a clustered solution that minimize the matching with the reference array reported in Fig. 3(c) (Taylor, $\bar{n} = 6$, $SLL = -20$ [dB]). Figure 3(a) shows the clustering configuration corresponding to the solution provided by the *TV-CS* solver [Fig. 3(b)], which allows to reduce the number of control points to $Q = 13$. Figure 3(c) compares the power pattern of the reference array with the *TV-CS* synthesized pattern. As can be seen the *TV-CS* pattern is very close to the reference with a non negligible simplification of the feeding network (i.e. a reduction of $\chi = 35\%$ of amplifiers). Higher reduction percentages can be achieved when considering larger arrays. As instance, Fig. 4 shows the case of a $P = 100$ elements array. Two different solutions, selected according to two different sampling rates, are shown in Fig. 4(a) ($N = 90$) and Fig. 4(b) ($N = 200$), characterized by $Q = 25$ and $Q = 33$ clusters, respectively. The clustered amplitude coefficients are compared in Fig. 4(c) while the respective power patterns are reported in Fig. 4(d). As can be notice, the power pattern radiated by the $Q = 33$ sub-arrays solution shows negligible deviations from the optimal pattern, with a final reduction of $\chi = 65\%$ of control points. Instead, the $Q = 25$ solution allows a further simplification of the feeding network ($\chi = 75\%$) accepting the presence of slightly higher lobes in the side-lobe region [Fig. 4(d)]. Future extensions of the proposed approach include the possibility to handle planar array structures and, by exploiting suitable basis functions, to optimize the partitioning of tile-based modular arrays [27].
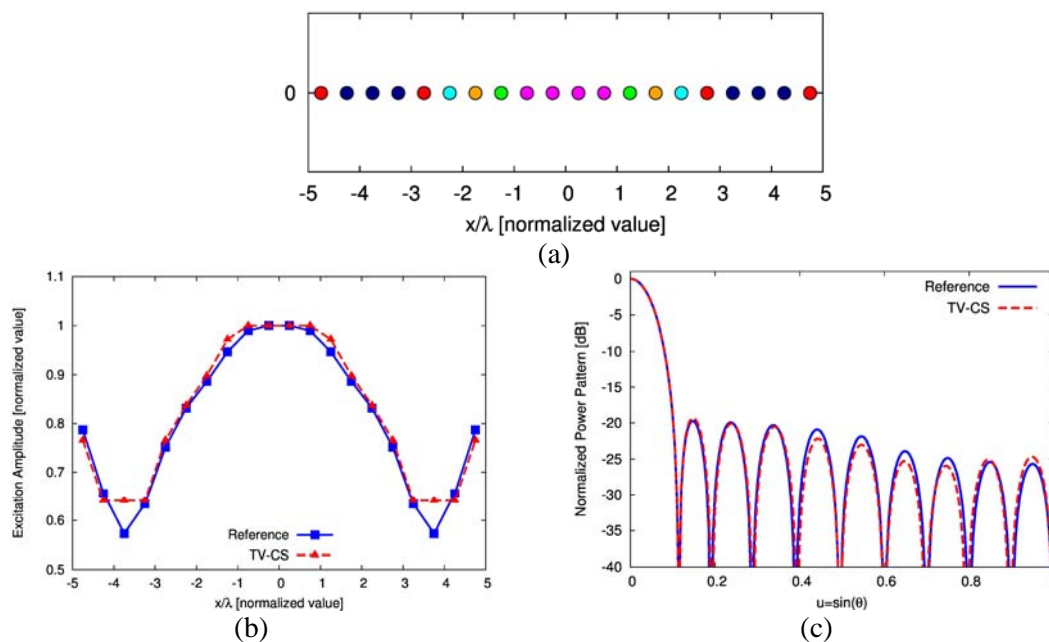

Figure 3. *Numerical Examples (TV-CS, $P = 20$, $N = 90$, $Q = 13$) – (a) The CS clustering configuration, (b) the reference and TV-CS excitations amplitudes and (c) the normalized reference and TV-CS synthesized power patterns.*

4. Conclusions

In this work, a review of the main *CS*-based methodologies applied to inverse problems arising in the design and processing of antenna arrays, is reported. Two instances of inverse problems have been formulated in order to comply with the *CS* theoretical requirements, namely the diagnosis of failures in planar arrays and the design of contiguously clustered arrays, discussing the limitations and the future

trends of both techniques. Finally two illustrative examples are reported, validating the effectiveness of the proposed method.
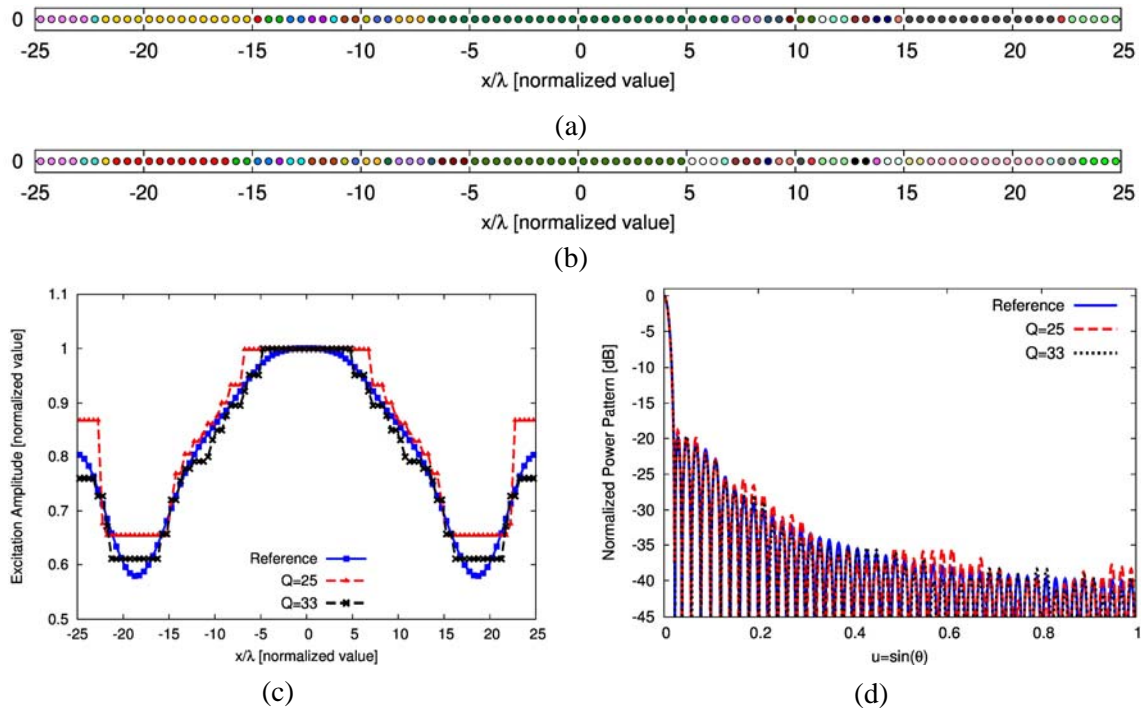

Figure 4. *Numerical Examples (TV-CS, $P = 100$) – (a)(b) The CS clustering configurations of two TV-CS solutions with (a) $Q = 25$ and (b) $Q = 33$ clusters. (c) the reference and TV-CS excitations amplitudes and (d) the normalized reference and TV-CS synthesized power patterns.*

Acknowledgments

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