Analysis of digital maps based on simple feature quantities

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The digital map (Dmap) is digital dynamical system defined on a set of lattice point [1]. Since the number of the lattice point is finite, the steady state is a periodic orbit (PEO). Although analog one-dimensional maps (e.g., the logistic map) can generate chaos, the Dmap can generate various PEOs. The Dmaps are related to various systems such as cellular automata, digital spiking neurons, dynamic binary neural networks, and logical/sequential circuits ([1], [2], and references therein). Such digital systems have been applied to various engineering systems. Analysis of the Dmaps is important not only as a basic study but also for engineering applications. The Dmap is described by $\theta_{n+1} = F_D(\theta_n)$, $\theta_n \in L_N \equiv \{l_1, l_2, ..., l_N\}$, where L_N is a set of N lattice points l_1 to l_N. Figure 1 illustrates the Dmap where several PEOs co-exist. Depending on initial condition and parameters, the Dmap exhibits either PEO. In general, the Dmap can have a variety of PEOs and transient phenomena. In order to analyze of Dmap, we introduce two feature quantities. The first quantity α represents plentifulness of the steady states. The second quantity β represents deviation of transient phenomena to steady states. One Dmap corresponds to one point on the α vs β plane on which we can classify/consider the dynamics. There exist many examples of the Dmaps and their general consideration is extremely hard. For simplicity, we consider one example: the digital tent map given by discretizing the tent map on the set of lattice points. Using the feature quantities, we have given basic classifications.

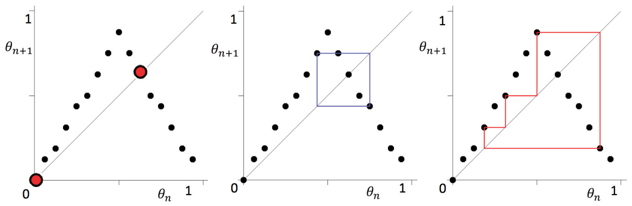

Fig. 1: Digital tent map. Left to right: Fixed points; PEO with period 2; PEO with period 4.

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New tools to determine the optimal embedding of a time series

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According to the Takens-Mane embedding theorem, embedding makes up a basic tool to analyze a time series and thus investigate the dynamical properties of the underlying chaotic system. Unfortunately the embedding theorem provides no clues on how to choose the embedding dimension m and the embedding lag L. Although in the last two decades several methods have been devised to tackle this crucial issue, a conclusive criterion to make the most appropriate choice is still lacking. We present a new approach that relies on the analysis of the statistical properties of the uncertainty of the maximum Lyapunov exponent (MLE), calculated via the divergence rate method on finite-time sequences [1]. Using chaotic systems with explicit analytic representation that are widely used as references in the scientific literature we show that 'good' embedding pairs (m, L) tend to form Gaussian-like clusterings of the uncertainty. This property can be used also in the reverse way: embedding pairs that form quasi-normally distributed clusterings of the MLE uncertainty are to correspond to optimal embedding choices. The method complies with the theory of the statistical properties of the finite-time MLE first discussed by Grassberger, Badii and Politi [2]. A second aspect regards the investigation of how the MLE uncertainty scales with the sampling frequency: the theory [2] predicts a power law scaling with typical exponents (for example, -0.5 in the case of continuous systems with short correlation times). We discuss how this scaling can be exploited to provide an additional tool to identify optimal embedding parameters, and in particular the lag L. Finally, we present preliminary results concerning the application of these methods to the analysis of time series generated by synthetic chaotic and noisy systems and to electroencephalografic recordings.

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Hyperchaos and synchronization in paralleled power converters

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The paralleled systems of switching power converters have been studied from fundamental and application viewpoints. In the fundamental study, the paralleled systems are interesting examples of switched dynamical systems that can exhibit a variety of nonlinear phenomena [1,2]. In the applications, the paralleled systems can realize current sharing and ripple reduction which are effective in robust and reliable power management [2,3]. In these studies, analysis of nonlinear