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A numerical study on Rank Reversal in AHP: the role of some influencing factors

Fedrizzi M.
Department of Industrial Engineering, University of Trento, TRENTO, Italy
michele.fedrizzi@unitn.it

Predella N.
University of Trento, TRENTO, Italy
nicolas.predella@unitn.it

Abstract. We study, by numerical simulations, how rank reversal phenomenon in the Analytic Hierarchy Process is influenced by two different factors in the case of consistent judgements. We consider a three level hierarchy, where the alternatives constitute the lowest layer and the (k) criteria the layer immediately above. First, we focus on the distribution of the normalized criteria weights $(v_1, \dots, v_k)$. It is known that no rank reversal occur if all the criteria weights but one are zero, for instance, if $(v_1, \dots, v_k) = (1,0, \dots,0)$. In fact, this case corresponds to the single criterion case, which is rank reversal free if the judgements are consistent. By drifting away from this polarized case, rank reversal can arise. We assume that the entropy is a suitable quantity to describe how far the distribution of criteria weights is from the extreme case described above. Indeed, the maximum entropy case is the uniform case $(v_1, \dots, v_k) = (1/k\,, 1/k\,, \dots, 1/k)$. We obtained an interesting monotone behavior, that is, the probability of rank reversal increases when the entropy of $(v_1, \dots, v_k)$ increases. We obtained a similar result by substituting the entropy with the standard deviation. Clearly, in this latter case the estimated probability of rank reversal decreases as the standard deviation increases. The second part of our study focuses on the method used for the aggregation of the local weight vectors. It is known that, if the weighted geometric mean is used, then rank reversal cannot occur. Conversely, by using the weighted arithmetic mean, as AHP suggests, rank reversal can occur. Therefore, we used the more general aggregation method of the weighted power mean. The weighted arithmetic mean and the weighted geometric mean are particular cases of the weighted power mean. More precisely, they are obtained by setting the power parameter p to the values $p = 1$ and $p \to 0$ respectively. We studied the estimated probability of rank reversal as a function of p. In this case too, we obtain a regular monotone behavior. As expected, the estimated probability of rank reversal is zero for $p \to 0$ and it increases in the interval $[0,1]$. For each study, we randomly generated 500.000 consistent pairwise comparison matrices.

Keywords: Rank Reversal, AHP, numerical simulations.

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