2-D Model for friction of complex anisotropic surfaces

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The friction force observed at macroscale is the result of interactions at various lower length scales, which are difficult to model in a combined manner. For this reason, simplified approaches are required, depending on the specific aspect to be investigated. In particular, the dimensionality of the system is often reduced, especially in models designed to provide a qualitative description of friction properties of elastic materials, e.g. the spring-block model. In this paper, we implement a two dimensional extension of the spring-block model, aiming to investigate by means of numerical simulations the frictional behaviour of a surface in the presence of surface features like cavities, pillars or complex anisotropic structures. We show how friction can be effectively reduced or controlled by appropriate surface features design.

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I. INTRODUCTION

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II. MODEL

m, N_x x- N_y y- l_x l_y.

$N_b \equiv N_x N_y$.

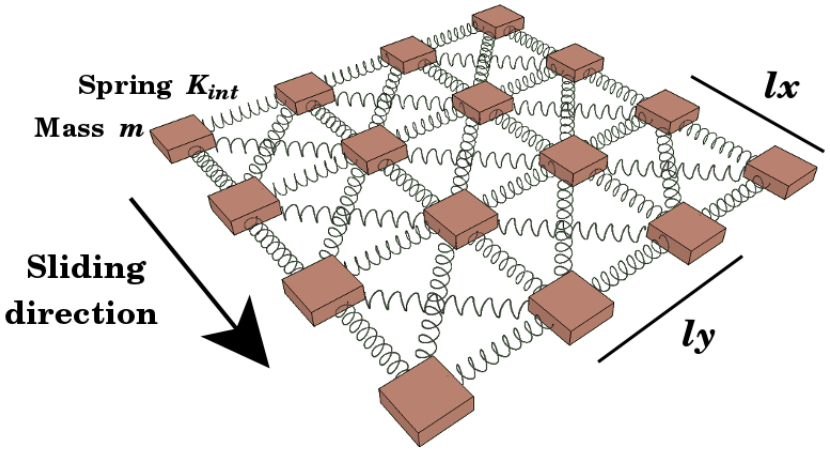

Figure 1. Discretization of a square surface into a 2-D spring-bock model, showing the mesh of the internal springs. The shear springs K_s attached above the blocks are not shown.

fl2-Ipldhneg [15][19] eldlgplwd

ldldldFfe(dsdsdsl0).

Idddfigsgsdlgs

hfg3Yh E, lRldfdo $\nu = 1/3$ [19] lw

pdpds $l_x = l_y \equiv l$ nl $K_{int} = 3/8El_z$, lw l_z isdh

fl2-lgl K_{int} isffsfigfdg

fdie lldiFfsfigdo

lfsdie lgdsdo $K_{int}/2$. ldh

hfdo i sdlg j is $\mathbf{F}_{int}^{(ij)} = k_{ij}(r_{ij}-l_{ij})(\mathbf{r}_j-\mathbf{r}_i)/r_{ij}$,

lw $\mathbf{r}_i$, $\mathbf{r}_j$ dsfdo r_{ij} sdldfl

l_{ij} sdfdsdl k_{ij} sdsfigdm

lddsdlgffs K_s, dsdsgm

tdy v in x idie ldsds $\mathbf{v} = (v, 0)$. fG

dsdsp $\mathbf{r}_i^0$ fo i, dsfs $\mathbf{F}_s^{(i)} = K_s(\mathbf{v}t + \mathbf{r}_i^0 - \mathbf{r}_i)$. dWn

ldfm i s $\mathbf{F}_{mot}^{(i)} = \sum_j \mathbf{F}_{int}^{(ij)} + \mathbf{F}_s^{(i)}$. ffs K_s dsdh

pdsn $G = 3/8E$, sdsgdp

sdydsdn $K_s = K_{int}l^2/l_z^2$.

ddddgdldldw

idpdgdfo] dddpeg

[15–18] is a widely used framework for
is e.g. [11–32] with applications in [19] and
the (A)ff The is to
g the p y
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for $\mathbf{F}_{fr}^{(i)}$ p ie $\mathbf{F}_{fr}^{(i)} = -\mathbf{F}_{mot}^{(i)}$, p
$F_{fr}^{(i)} = \mu_{si}\, F_n^{(i)}$, w μ_{si} i $F_n^{(i)}$ i
i. M a ie
$\mathbf{F}_{fr}^{(i)} = -\mu_{di}\, F_n^{(i)} \widehat{\mathbf{r}}_i$, w μ_{di} i $\widehat{\mathbf{r}}_i$ i
f s,d y
p
f ie $p(\mu_i) = (\sqrt{2\pi}\sigma)^{-1} \exp\left[-(\mu_i - (\mu)_m)^2/(2\sigma^2)\right]$
w $(\mu)_m$ σ i
$(\mu_s)_m = 1.0(1)$ a $(\mu_d)_m = 0.50(5)$, p
$\mu)_M$.
is
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[10]

III. RESULTS

A. Non-patterned surface

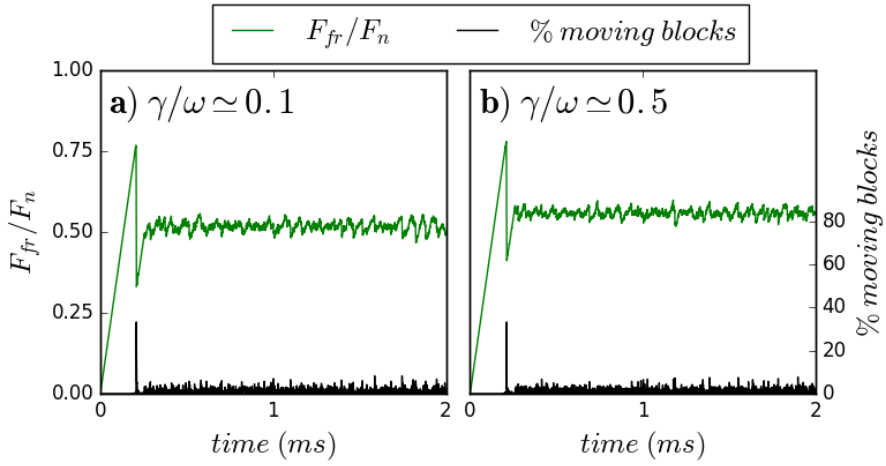


Figure 2. Time evolution for the total friction force and percentage of moving blocks for $N = 20$, pressure $P = 0.1$ MPa, velocity $v = 0.1$ cm/s, $\gamma/\omega = 0.1$ (a) or $\gamma/\omega = 0.5$ (b), where ω is the internal frequency $\omega \equiv \sqrt{K_{int}/m}$. The other parameters are set to the default values. Greater damping enhances the dynamic friction coefficient and reduces stick-slip oscillations.

$N \gtrsim 100$

P

v

$v = 0.05$

$P = 50$

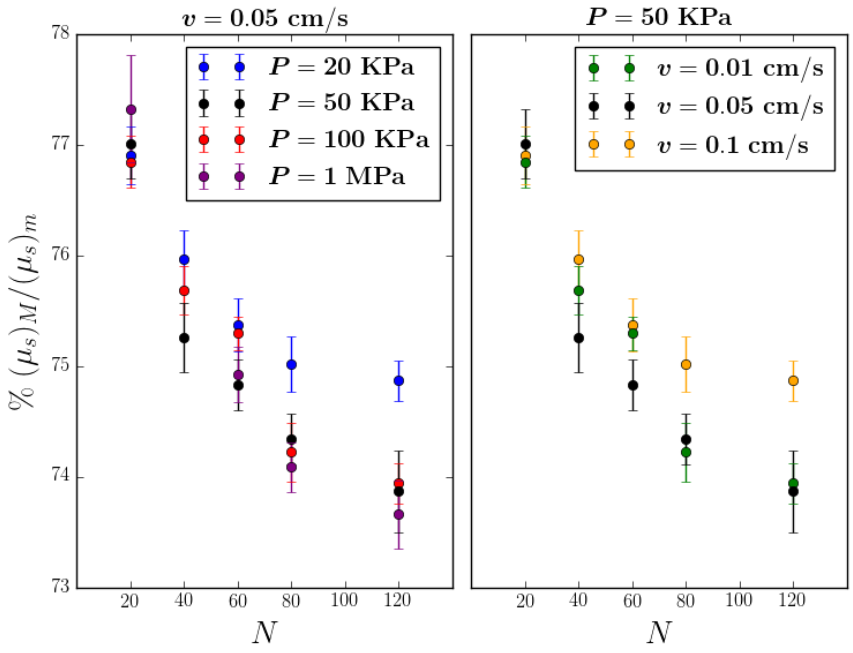


Figure 3. Static friction coefficient as a function of the number of blocks N by varying the applied pressure P with the default velocity $v = 0.05$ cm/s (a), and by varying the velocity v with the default pressure $P = 50$ KPa (b). Thus, the black dots on both sides show the curve for the default set of parameters. Variations with respect to this are limited to few percent in the typical ranges of these parameters.

1. *Role of damping*

$\mathbf{F}_d^{(i)} = -\gamma m \dot{\mathbf{r}}_i$,

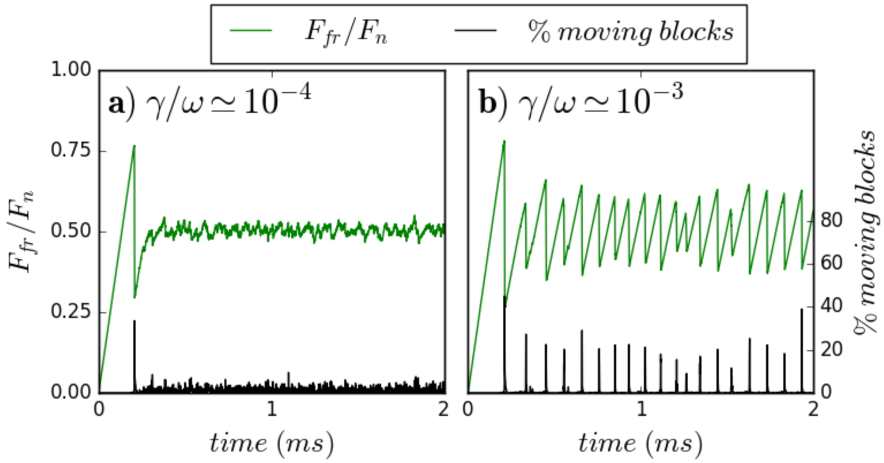


Figure 4. Time evolution for the total friction force with the same parameters of figure 2, except that the damping is imposed on the relative velocity between neighbouring blocks, i.e. using a viscoelastic material model, respectively with $\gamma/\omega = 10^{-3}$ (a) or $\gamma/\omega = 10^{-4}$ (b), where ω is the internal frequency ($\omega \equiv \sqrt{K_{int}/m}$). The static friction coefficient remains unchanged, but the kinetic phase is totally different, in particular for higher damping values there are greater stick-slip oscillations.

[illegible] [40]. [illegible] γ [illegible] $\gamma < \omega \equiv \sqrt{K_{int}/m}$.

[illegible] $\mathbf{F}_d^{(ij)} = -m\gamma\,(\dot{\mathbf{r}}_i - \dot{\mathbf{r}}_j)$, [illegible] (fig. 4). [illegible]

[illegible] N: [illegible] $N = 80$, [illegible]

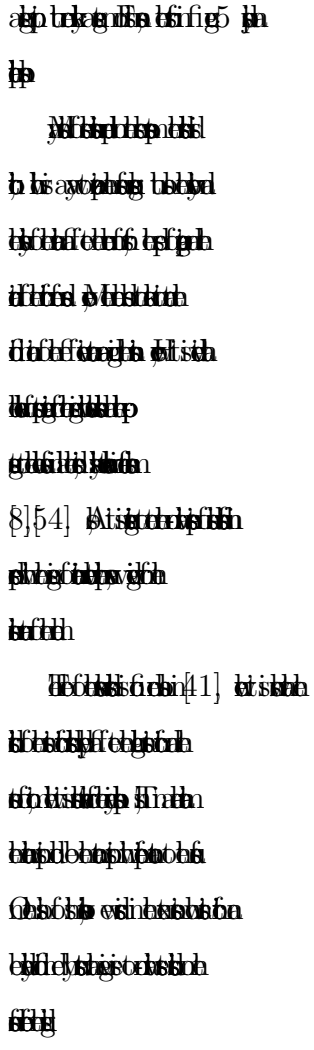
[illegible] ie a [illegible] $\gamma = 500$ s m $^{-1}$ $(\gamma/\omega \simeq 0.1)$.

2. Detachment fronts

[illegible] fig. 2. [illegible] [8]–[11] [illegible] [7],[50]–[52]. [illegible] [53]–[58] [illegible] 2-D [illegible]

In fig. 5, [illegible] fig. 5 [illegible] ie [illegible] 1-D [32] [illegible] [7], [illegible] [37]

In 2-D, [illegible]



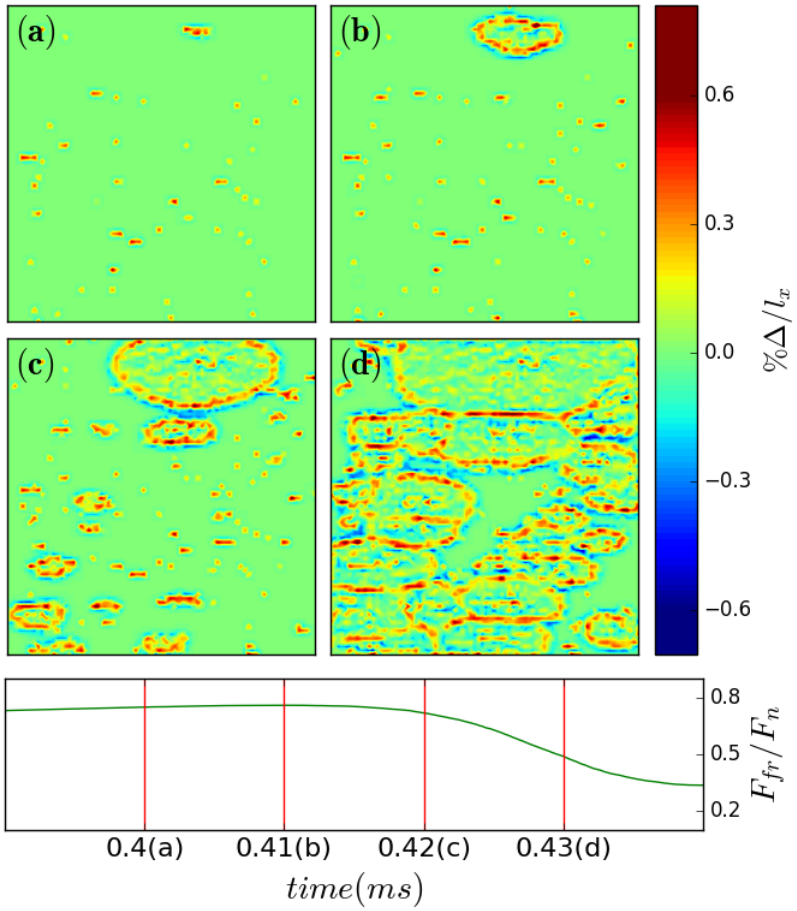


Figure 5. Time snapshots of the deformation Δ along the longitudinal direction on the surface divided by the block distance l_x, so that positive values (red) indicate compression and negative values (blue) tension. Before the maximum of the friction force is reached, some blocks with weak static friction thresholds detach (a), then a rupture front nucleates from the weakest point, corresponding to the instant of the maximum force before the drop (b); the front propagates while other fronts nucleate elsewhere (c) finally, the whole surface slides leaving a non-uniform distribution of regions under tension/compression (d).

B. Patterned structures

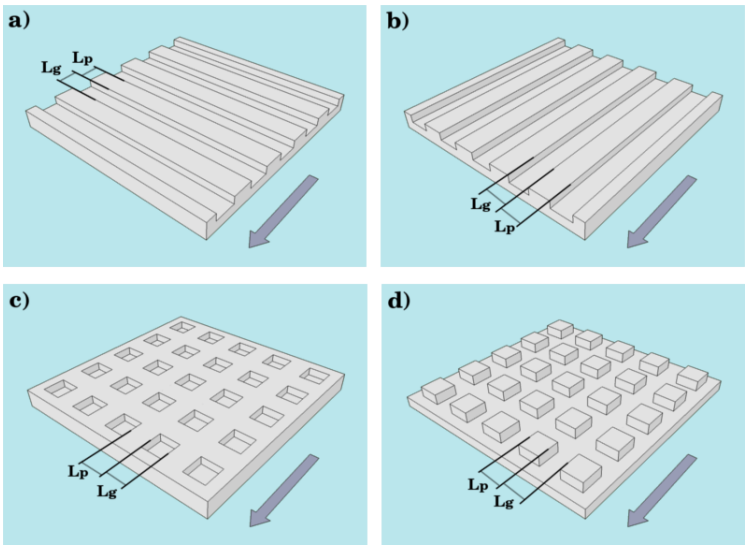


Figure 6. Single-level surface structures considered in the simulations: patterning with grooves in direction perpendicular (a) or parallel (b) to the motion. Square cavities (c) and pillars (d). The number $l_g \equiv L_g/l_x$ is the ratio between the size of the structure and the elementary block distance. The arrow denotes the sliding direction.

L_g

L_p

$n_g \equiv L_g/l_x$

$n_p \equiv L_p/l_x$

n_g

$n_g = n_p$.

$N_x = N_y = 120.$

1. *Static Friction*

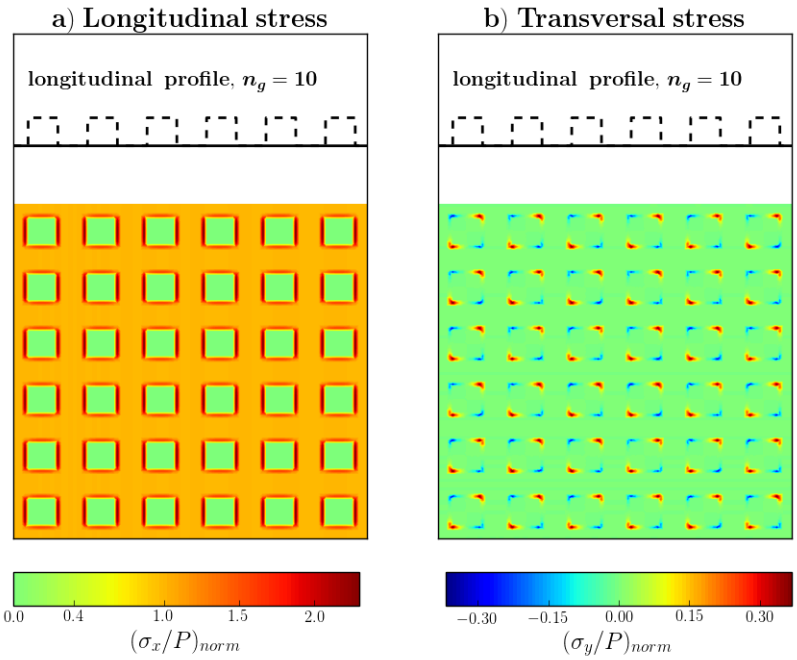
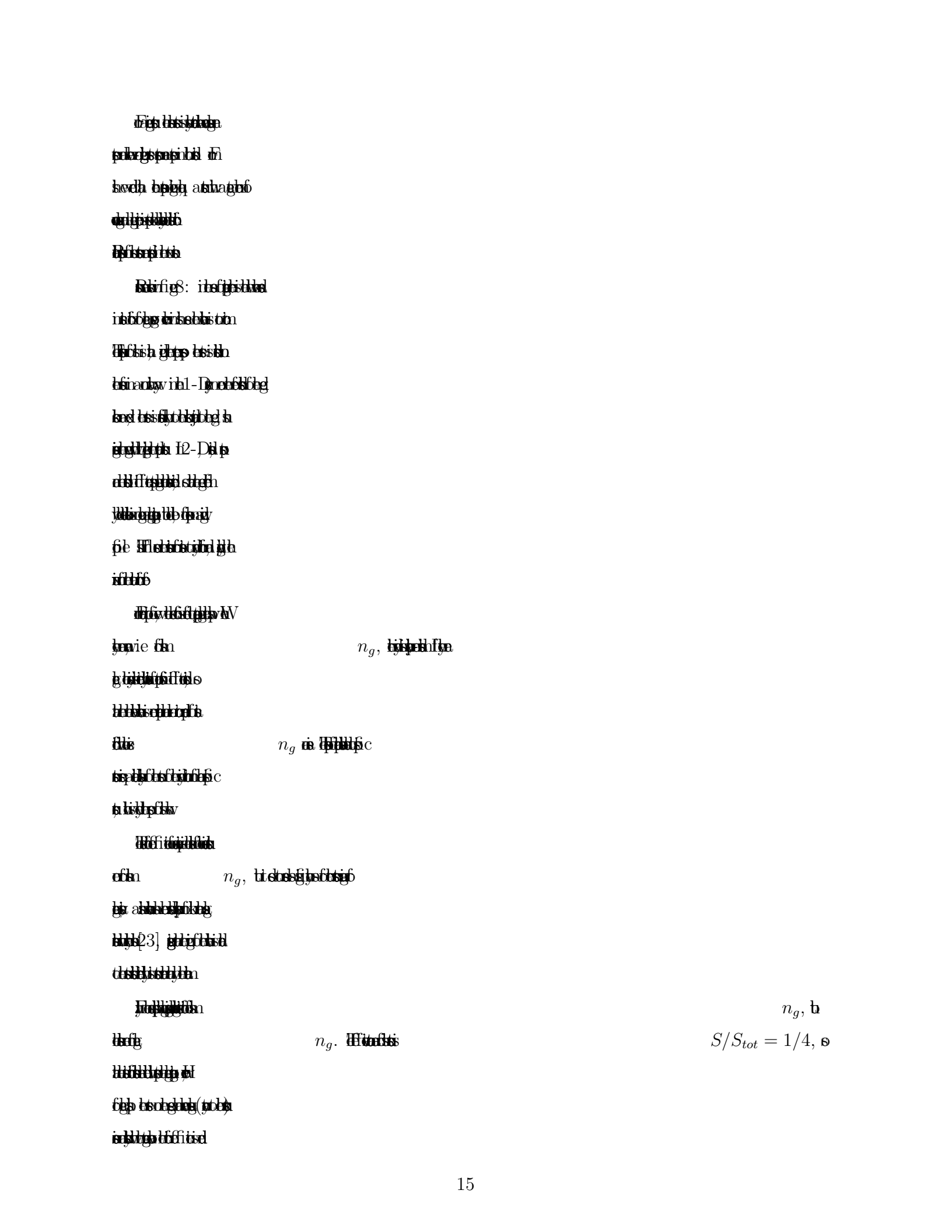


Figure 7. Longitudinal (a) and transversal stress (b), divided by the pressure on the 2-D surface, before the blocks motion, for a structure with square cavities as in figure 6c and $n_g = 10$ (the dotted lines above shows the surface profile). The stress-pressure ratio is also normalized with the value obtained for a smooth surface, so that, for example, the normalized value is fixed to one for non-edging blocks. The stress is accumulated at the edge of the cavities with a non-zero component in the transversal direction.



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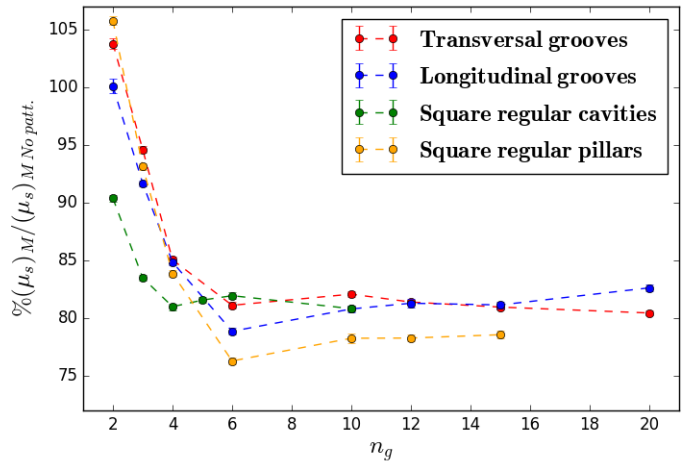

Figure 8. Normalized static friction coefficients for the four single-level structures of figure 6. Results are normalized with respect to the static friction coefficients for a smooth surface (non-patterned case) and are displayed as a function of the structure characteristic width $n_g = n_p$. Notice the decrease of static friction for $n_g \simeq 2$ and the non monotonic behavior for larger sizes ($n_g > 6$).

2. *Dynamic friction*

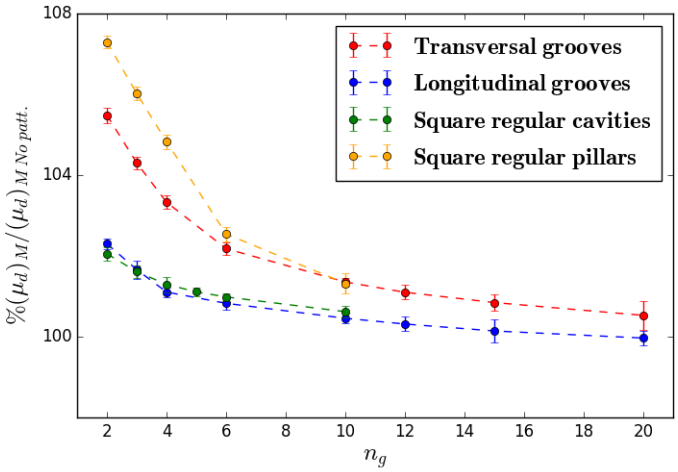

Figure 9. Normalized dynamic friction coefficients for the four single-level structures of figure 6. Results are normalized with respect to the dynamic friction coefficients for a smooth surface (non-patterned case) and are displayed as a function of the structure typical width $n_g = n_p$. The decreasing trend with the size of the structures is limited to few percent with respect the non-patterned case.

C. Winding tread patterns

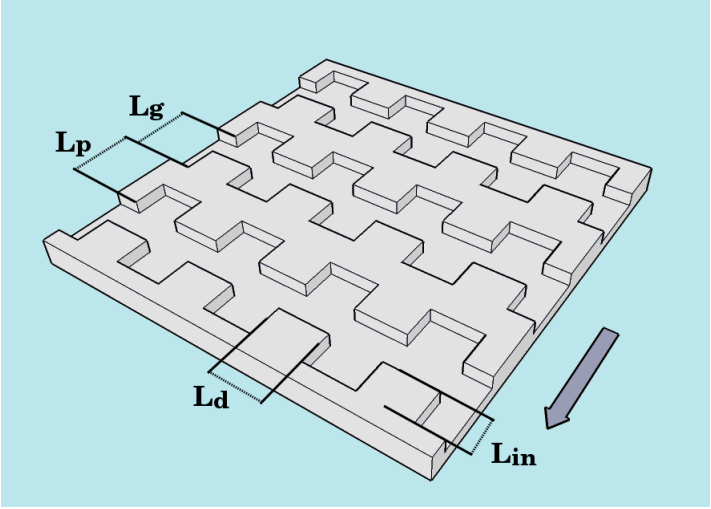

Figure 10. Structure derived from that in figure 6a, in which the straight edge of the grooves has been modified to a winding profile with ratchets of width L_d and depth L_{in}. The effective contact area is halved, like in the case of regular patterning with grooves and pawls of the same size.

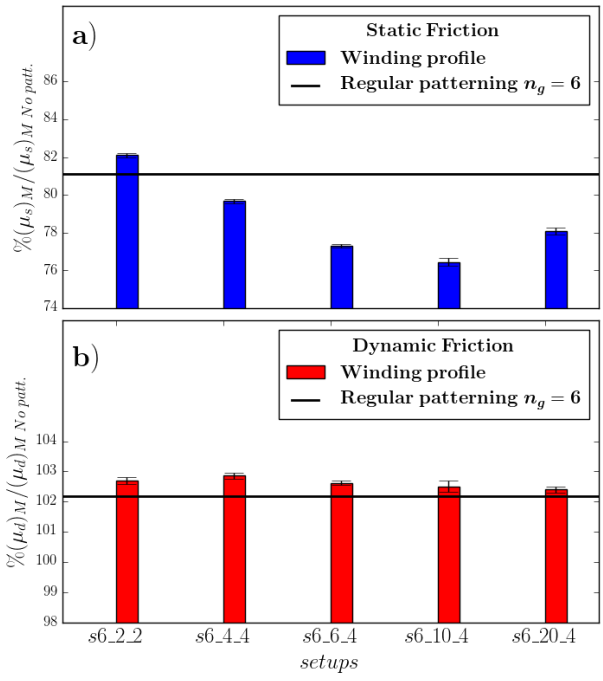

Figure 11. Normalized static (a) and dynamic (b) friction coefficients for the different tread patterns in table I compared to those for a regular patterning with $n_g = n_p = 6$ (black line). The static coefficient can be further reduced with respect to the case of periodic regular patterning.

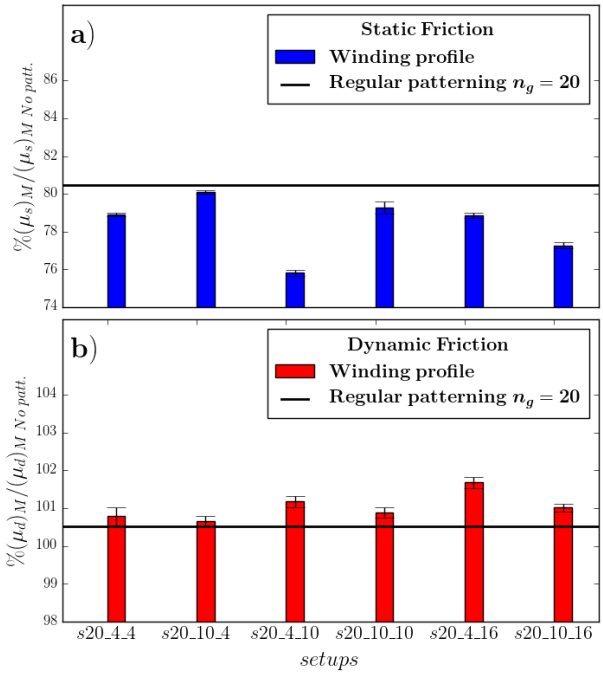

Figure 12. Normalized static (a) and dynamic (b) friction coefficients for the different tread patterns in table I compared to those for a regular patterning with $n_g = n_p = 20$ (black line). While the dynamic friction coefficient displays little variation, the static friction coefficient can be remarkably reduced with an optimal combinations of parameters.

tread pattern	grooves n_g	width n_d	depth n_{in}	tread pattern	grooves n_g	width n_d	depth n_{in}
$s6\_2\_2$	6	2	2	$s20\_4\_4$	20	4	4
$s6\_4\_4$	6	4	4	$s20\_10\_4$	20	10	4
$s6\_6\_4$	6	6	4	$s20\_4\_10$	20	4	10
$s6\_10\_4$	6	10	4	$s20\_10\_10$	20	10	10
$s6\_20\_4$	6	20	4	$s20\_4\_16$	20	4	16
				$s20\_10\_16$	20	10	16

Table I. Table reporting the setups of the structure of figure 10 corresponding to the results presented in figures 11 and 12. For all the setups only n_g is reported since $n_p = n_g$.

IV. CONCLUSIONS

ACKNOWLEDGMENTS

[illegible] 2015 [illegible] N693670,
[illegible] 2020 [illegible] N 696656
[illegible] N 732344.
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