Dynamic Phasor and Frequency Measurements by an Improved Taylor Weighted Least Squares Algorithm

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Abstract — *One of the most accurate phasor estimation procedures recently proposed in the literature is the so-called Taylor Weighted Least Squares (TWLS) algorithm, which relies on a dynamic phasor model of an electrical waveform at nominal frequency. In this paper an extension of the TWLS algorithm (called Generalized TWLS, or GTWLS, algorithm) to a generic (not only nominal) reference frequency is described and the accuracies of the returned estimates are analyzed through meaningful simulations, performed in different steady-state and dynamic testing conditions according to the Standard IEEE C37.118.1-2011 about synchrophasor measurement for power systems and its Amendment IEEE Standard C37.118.1a-2014. It is shown that the accuracy of the Total Vector Error (TVE), Frequency Error (FE), and Rate-of-change-of-Frequency Error (RFE) normally decreases as the deviation between the reference frequency and the true waveform frequency decreases. Furthermore, a two-step procedure for accurate estimation of the phasor parameters is proposed. In the first step the waveform frequency is estimated by a classical Interpolated Discrete Fourier Transform (IpDFT) algorithm. The second step then returns an estimate of the phasor parameters by applying the TWLS algorithm based on the frequency estimate returned by the first step. It is shown that the proposed procedure, called GTWLS-IpDFT algorithm, can comply with the P-class or the M-class of performances in all the considered testing conditions when an appropriate number of waveform cycles is considered and the most significant disturbances are removed from the analyzed* *waveform. Finally, uncertainties of the proposed estimators and the IpD^{2}FT algorithm recently presented in the literature are also compared.*

Index terms *— Phasor measurement, Error analysis, weighted least squares, Fourier transform, parameter estimation, power system monitoring*.

I. INTRODUCTION

The next-future extensive diffusion of nonlinear loads and distributed micro-generation, as well as the increasing energy demands due to new components (e.g., electrical vehicles) is progressively leading to new technologies and methodologies for electrical power grid monitoring and protection. Among the many research challenges that need to be tackled to optimize energy distribution and ensure reliable protection, high-accuracy, low-latency waveform parameters have to be measured at the same time in different geographically distributed points of the electrical network. This task can be performed by the so called Phasor Measurement Units (PMUs), i.e. smart devices able to measure various parameters of the waveforms at their input by processing waveform samples acquired in observation intervals synchronized to the Universal Coordinated Time (UTC) [1]. These devices constantly provide relevant data about crucial waveform quantities such as phasor amplitude and angle, waveform frequency, and Rate-Of-Change-Of-Frequency (*ROCOF*). In particular, frequency and *ROCOF* measurements are needed by the Phasor Data Concentrators (PDCs) to estimate at common reference time the waveform phasor values in distinct points of the grid. Indeed, different PMUs (because belonging to different performance classes or made by diverse manufacturers) may estimate waveform quantities using observations of different duration. As a consequence, the PDCs receive values at different reference time with different phase shifts that, hence, need to be properly estimated and compensated by integrating both frequency values (once) and *ROCOF* values (twice) over time.

The main PMU performance parameters are specified in the IEEE Standard C37.118.1-2011 for synchrophasor measurements [2] and in its recent Amendment IEEE Standard C37.118.1a-2014 [3]. In the following these documents are simply referred to as the Standard. According to the Standard, two PMU performance classes exist, i.e. the *P-class* (faster and generally less accurate, devoted to grid protection operations) and the *M-class* (slower and requiring better steady state accuracy, more appropriate for measurement applications). The main parameters suggested in the Standard to assess phasor measurement accuracy in both steady-state and dynamic conditions are the Total Vector Error (*TVE*), the Frequency Error (*FE*), and the *ROCOF* Error (*RFE*). Moreover, the maximum overshoot (or undershoot), the response time, and the delay time are proposed to assess the phasor measurement performances in transient conditions. *P-class* and *M-class* performance requirements are quite different and they are specified in the Standard in terms of the above listed parameters when measurement is performed in steady-state, dynamic, and transient conditions modeling situations commonly encountered in practice. It is worth noticing that the *M-class* of performances generally requires wider ranges of operating conditions and higher steady state accuracy than the *P-class* of performances.

Standard requirements are hard to meet because of two contrasting issues. On one hand, the need for tracking physiological phasor fluctuations demands observation intervals as short as few waveform cycles. On the other hand, accurate steady-state estimation of the status of the power network requires very small measurement uncertainties. Moreover, even stricter requirements are expected in next-generation power distribution networks. In this complex scenario, measuring waveform amplitude, phase, frequency, and *ROCOF* is still an open problem.

Most of the modern PMUs estimate the phasor amplitude and phase, the waveform frequency, and the *ROCOF* using digital signal processing techniques based on the Discrete Fourier Transform (DFT) of a record of waveform samples. Unfortunately, all these techniques are based on a static phasor model and so they are very sensitive to phasor amplitude and angle fluctuations often occurring in modern power grids [4], [5]. This is especially true when waveform frequency and *ROCOF* are of concern [6]. Therefore, research efforts are currently focused on the development of novel measurement algorithms based on dynamic phasor models [6] – [12]. Very accurate phasor measurements can be achieved by using the so-called Taylor Weighted Least Squares (TWLS) algorithm [8] even though observation intervals as short as two waveform nominal cycles are considered [13]. In the TWLS algorithm the analyzed waveform is *a-priori* weighted by a suitable window [7], [8] and the nominal frequency is used as reference frequency in the adopted waveform model [8]. Recently, it has been shown that more accurate phasor, frequency, and *ROCOF* estimates can be achieved when the reference frequency coincides with the true waveform frequency [14]. In particular, raw discretization of the allowed reference frequency range has been proposed in order to evaluate *a-priori* the matrix employed by the TWLS estimator, which considerably reduces the required processing effort. Nevertheless, a comprehensive analysis of the effect of the chosen reference frequency value on the estimation accuracy has not performed yet in the scientific literature. In this paper, the Generalized TWLS (GTWLS) algorithm is firstly analyzed as an extension of the classical TWLS algorithm obtained by modeling the acquired waveform as a sine-wave at a generic, but *a-priori* known, reference frequency. The accuracies of the related phasor, frequency, and *ROCOF* estimators are then analyzed as a function of the Frequency Deviation (*FD*) defined as the difference between the instantaneous waveform frequency and the adopted reference frequency. To this aim meaningful Monte Carlo simulations built upon steady-state or dynamic testing conditions specified in the Standard are considered.

A two-step procedure for phasor parameters estimation is then proposed. In the first step a classical Interpolated Discrete Fourier Transform (IpDFT) [15] – [19] algorithm is applied in order to achieve an initial estimate of both the waveform frequency and some other parameters useful to remove relevant disturbances from the acquired waveform, thus reducing their detrimental effect on measurement accuracy. In the second step, possible disturbances are effectively removed from the acquired waveform and the phasor parameters of interest are estimated by applying to the achieved waveform the GTWLS algorithm. In particular, the frequency estimated in the first step, or its value rounded to the closest integer, is employed as reference frequency. The performance of the proposed procedure, called GTWLS-IpDFT algorithm, and the GTWLS algorithm based on the true waveform frequency are then compared. Also, the performances of the GTWLS-IpDFT algorithm are compared with both *P-class* and *M-class* performance requirements and insights useful to select the best window and the optimal observation length are provided. Finally, the accuracies of the proposed estimators are compared with those returned by the IpD^{2}FT algorithm recently proposed in [12].

The paper is organized as follows. In Section II the dynamic phasor model and the GTWLS algorithm are shortly described. In Section III the accuracy of the phasor, frequency, and *ROCOF* estimators provided by the GTWLS algorithm is analyzed as a function of the frequency deviation in the case when the rectangular or the Hann windows weight the acquired data. In Section IV the GTWLS–IpDFT algorithm is described and an accuracy comparison with the GTWLS algorithm based on the true waveform frequency is performed. Moreover, two GTWLS-IpDFT estimators that comply with either the *P-class* or the *M-class* of performances in all the considered testing conditions are derived. Moreover, the accuracies of the proposed estimators and the IpD^{2}FT algorithm are compared. Finally, Section V concludes the paper.

II. WAVEFORM MODEL AND GENERALIZED TWLS ALGORITHM

A general dynamic and noisy electrical waveform $x(t)$ at frequency f can be represented by referring to a generic frequency f_0 by means of the following model:

$$x(t) = a(t)\cos(2\pi f t + \phi(t)) + \eta(t) = \mathrm{Re}\left\{a(t)\cdot e^{j[2\pi f_0 t + \varphi(t)]}\right\} + \eta(t) = \mathrm{Re}\left\{\sqrt{2}p(t)\cdot e^{j2\pi f_0 t}\right\} + \eta(t), \quad (1)$$

where $\mathrm{Re}\{\cdot\}$ is the operator returning the real part of its (complex valued) argument, $p(t)$ represents the so-called dynamic phasor related to the reference frequency f_0, $a(t)$ and $\varphi(t)$ are the phasor amplitude and angle, respectively, ϕ is the waveform phase, and $\eta(t)$ represents a disturbance signal which includes harmonics of the fundamental component, inter-harmonics, and additive wideband noise. It is worth observing that possible DC offset is not included in the waveform model (1) - even though it could be significant, especially for current phasor estimation - because it can be easily estimated and removed from the acquired data, even in off-nominal conditions, if suitable windowing is used.

From (1) it follows that:

$$\varphi(t) = \phi(t) + 2\pi(f - f_0) = \phi(t) + 2\pi FD, \tag{2}$$

where the Frequency Deviation FD is defined as the difference between the true (instantaneous) waveform frequency f and the considered reference frequency f_0, i.e. $FD = f - f_0$. Observe that, in principle, f_0 may assume any value. When f_0 is equal to the nominal frequency f_n (i.e., 50 or 60 Hz), the related synchronized phasor $p(t)$ is called the synchrophasor.

In order to track phasor variations over time, the phasor $p(t)$ defined at a time distance $\Delta t = t - t_r$ from the reference time t_r can be approximated by its complex Taylor's series expansion about t_r, truncated to the Kth order term, i.e.,

$$p(t) \cong p(t_r) + p'(t_r)\Delta t + \frac{p''(t_r)}{2!}\Delta^2 t + \ldots + \frac{p^{(K)}(t_r)}{K!}\Delta^K t, \qquad |\Delta t| \leq \tfrac{1}{2}T \tag{3}$$

where $p^{(k)}(t_r)$, $k = 1,\ldots,K$, is the k-th order derivative of $p(t)$ computed at the reference time, which is assumed at the center of an observation interval of duration T.

Let us assume that the waveform $x(t)$ is acquired at the sampling frequency f_s, which is chosen in such a way that an integer number J of nominal waveform cycles is observed, that is, $J = Tf_n = 2N_h f_n/f_s$, in which $2N_h = Tf_s$ is an even number and $M = 2N_h + 1$ is the overall number of acquired samples. Thus, if an integer number of samples per cycle $N = f_s/f_n$ is acquired, it follows that $2N_h = J{\cdot}N$.

If multiple acquisitions are considered, the coefficients of the phasor Taylor's polynomial (3) related to the r-th reference time can be estimated by the WLS approach as [8]:

$$\hat{P}_{rK} = 2\left(A_K^H W^H W A_K\right)^{-1} A_K^H W^H W\ x_r = \widetilde{A}_K x_r, \quad r = 0,1,2,\ldots \tag{4}$$

in which

$$\hat{P}_{rK} = \left[\hat{p}_{rK}^*\ \hat{p}_{r(K-1)}^* \cdots \hat{p}_{r1}^*\ \hat{p}_{r0}^*\ \hat{p}_{r0}\ \hat{p}_{r1} \cdots \hat{p}_{r(K-1)}\ \hat{p}_{rK}\right]^T, \quad r = 0,1,2,\ldots \tag{5}$$

represents the vector whose entries are the WLS estimates of

$$p_{r0} = p(t_r), \text{ and } p_{rk} = p^{(k)}(t_r)/\left(k!\, f_s^k\right), k = 1, 2, \ldots, K, \tag{6}$$

respectively. In (4) the vector x_r is expressed as

$$x_r = \left[x(-N_h + r)\, x(-N_h + 1 + r) \ldots x(r) \ldots x(N_h - 1 + r)\, x(N_h + r)\right]^T, \quad r = 0,1,2,\ldots \tag{7}$$

while the entries of the matrix A_K, of dimension Mx2(K + 1), are defined as in [8], except that in the exponential terms the normalized frequency $2\pi f_n / f_s$ is substituted by $2\pi f_0 / f_s$, and $\boldsymbol{W}$ is the diagonal matrix formed by the values of the chosen window $w(\cdot)$ [8]:

$$W = diag\left[w(-N_h)\, w(-N_h + 1) \ldots w(0) \ldots w(N_h - 1)\, w(N_h)\right]. \tag{8}$$

In (4) and (5), $(\cdot)^*$, $(\cdot)^H$ and $(\cdot)^T$ denote the conjugation, the Hermitian, and the transposition operators, respectively. Also it is worth noticing that the matrix W becomes the identity matrix when the rectangular window is used [7].

The Generalized TWLS algorithm described above allows to estimates phasor related parameters such as the frequency deviation and the $ROCOF$ by means of the following relationships [12]:

$$\hat{FD}(t_r) = \frac{1}{2\pi T_s} \frac{\mathrm{Im}\left\{\hat{p}_{r1} \hat{p}_{r0}^*\right\}}{\left|\hat{p}_{r0}\right|^2}, \tag{9}$$

and

$$\hat{ROCOF}(t_r) = \frac{1}{\pi T_s^2} \left[\frac{\mathrm{Im}\left\{\hat{p}_{r2} \hat{p}_{r0}^*\right\}}{\left|\hat{p}_{r0}\right|^2} - \frac{\mathrm{Re}\left\{\hat{p}_{r1} \hat{p}_{r0}^*\right\} \mathrm{Im}\left\{\hat{p}_{r1} \hat{p}_{r0}^*\right\}}{\left|\hat{p}_{r0}\right|^4} \right]. \tag{10}$$

Then, the algorithm total vector error TVE, frequency error FE, and ROCOF error RFE are given by:

$$TVE(t_r) = \frac{\left|\hat{p}_{r0} - p_{r0}\right|}{\left|p_{r0}\right|}, \qquad FE(t_r) = \hat{FD}(t_r) - FD(t_r), \qquad RFE(t_r) = \hat{ROCOF}(t_r) - ROCOF(t_r) \tag{11}$$

respectively.

III. ACCURACY OF THE GENERALIZED TWLS ALGORITHM

In this Section the accuracy of the GTWLS algorithm is analyzed as a function of the deviation *FD* between the instantaneous value of the waveform frequency f at the center of the analyzed data record and the reference frequency f_0. Specifically, the algorithm accuracy is assessed by evaluating the *TVE*, *FE*, and *RFE* parameters through extensive Monte Carlo simulations, which were performed assuming that:

- the reference time at which every phasor is estimated is located exactly at the center of the considered observation window;
- the rectangular window and the Hann (i.e. the two-term Maximum Sidelobe Decay (MSD)) window [20] are adopted;
- the waveform amplitude is 1 pu, the nominal frequency f_n is 50 Hz, the sampling frequency f_s is 1200 Hz (i.e. $N = 24$ samples/cycle);
- one-, two-, three-, and four-nominal cycle observation interval lengths are considered (i.e. $J = 1, 2, 3$ or 4); the related overall number of acquired samples is $M = 25, 49, 73$, and 97, respectively;
- the phasor Taylor's series is truncated to the order $K = 2$, that is only the terms of the Taylor expansion strictly needed to compute the estimates of *FD* and *ROCOF* in (9) and (10) are used;
- the magnitudes of the *TVE*, *FE*, and *RFE* parameters are evaluated repeatedly by shifting the observation window sample by sample, and considering an overall record of fixed length. However, the same results have been achieved when considering observation windows shifted by more than one sample or even not overlapped observation windows.

Simulations conditions are inspired to the worst-case values specified in the Standard for the *M-class* of performances [2], [3], but they are sometimes even worse, i.e.:

i) Fundamental tone only, with a maximum off-nominal frequency of ±5 Hz.

ii) Fundamental tone at nominal frequency affected by only a 2nd-order or a 3rd-order harmonic of amplitude equal to 10% of the fundamental, as required by *M-class* of performances. Higher harmonic orders are not considered because simulations showed that their effect on estimation accuracy is much smaller. The initial phases of the harmonics are chosen randomly in the range $[0, 2\pi)$ rad.

iii) Fundamental tone at nominal frequency affected by amplitude modulation (AM). The modulating signal is a sine-wave of amplitude equal to 10% of the fundamental and frequency 5 Hz.

iv) Fundamental tone at nominal frequency affected by phase modulation (PM). The modulating signal is a sine-wave of amplitude 0.1 rad and frequency 5 Hz.

v) Fundamental tone with frequency changing linearly from the nominal value f_n to $f_n \pm 5$ Hz, at a rate of ±1 Hz/s.

vi) Fundamental tone at frequency f affected by an out-of-band interference at frequency f_{ih} and amplitude equal to 10% of the fundamental. The frequency f_{ih} is varied in the ranges $[10, f_n - RR/2]$ and $[f_n + RR/2, 2f)$ Hz. Different values for the waveform frequency f are considered, i.e. $f = f_n = 50$ Hz, $f = f_n - RR/20$ and $f = f_n + RR/20$.

In steady-state and modulation testing the magnitudes of the *TVE*, *FE*, and *RFE* parameters were determined over 960 subsequent records shifted each other sample by sample, while a 6000 subsequent records were considered in the frequency ramp testing, in order to achieve an observation interval duration of 5 s as required by the Standard. The estimated magnitudes for the *TVE*, *FE*, and *RFE* were also compared with the related thresholds specified in the Standard.

In Tab. I the results returned by the GTWLS algorithm when $f_0 = 50$ Hz (i.e. when the classical TWLS algorithm is employed) are given. The related thresholds specified in the Standard are also reported for comparison (shadowed rows).

Table I. Maximum magnitude of the *TVE*, *FE*, and *RFE* values returned by the GTWLS algorithm based on the rectangular or the Hann windows when f_0 = 50 Hz (i.e. when it coincides with the classical TWLS algorithm) and J = 1, 2, 3, or 4 cycles. The simulation parameters are chosen according to the worst-case conditions specified in the Standard for the *P-class* or the *M-class* of performances, respectively.

		Rectangular window						Hann window					
		P-class			*M-class*			*P-class*			*M-class*		
Test type	*J*	*TVE* max (%)	|*FE*| max (mHz)	|*RFE*| max (Hz/s)	*TVE* max (%)	|*FE*| max (mHz)	|*RFE*| max (Hz/s)	*TVE* max (%)	|*FE*| max (mHz)	|*RFE*| max (Hz/s)	*TVE* max (%)	|*FE*| max (mHz)	|*RFE*| max (Hz/s)
off-nomin. frequency	1	0.01	5.1	0.52	0.15	75	7.1	0.01	5.9	0.45	0.19	85	5.5
	2	0.01	14.0	0.36	0.14	216	6.7	0.00	4.8	0.35	0.08	73	5.5
	3	0.03	30.2	0.45	0.59	455	8.2	0.00	10.0	0.00	0.04	152	0.2
	4	0.06	52.4	0.49	1.47	762	8.6	0.00	17.5	0.01	0.13	264	0.2
Std. thresh.	—	1	5	0.4	1	5	0.1	1	5	0.4	1	5	0.1
2nd harmonic	3	0.13	19.8	1.70	1.31	199	17.0	0.12	7.1	5.33	1.20	72.1	53.3
	4	0.08	11.5	0.63	0.84	116	6.3	0.01	0.9	0.31	0.13	8.8	3.1
3rd harmonic	1	0.22	93.6	28	2.2	948	280	7.7	$3\cdot10^3$	$1\cdot10^3$	77	$1\cdot10^5$	$8\cdot10^4$
	2	0.10	24.5	3.4	1.0	245	33.7	0.01	1.8	1.48	0.15	17.9	14.8
	3	0.06	11.2	0.94	0.62	112	9.4	0.00	0.1	0.04	0.01	1.3	0.4
	4	0.04	6.4	0.38	0.44	64	3.8	0.00	0.0	0.00	0.00	0.2	0.0
Std. thresh.	—	1	5	0.4	1	5(1)-25(2)	—	1	5	0.4	1	5(1)-25(2)	—
amplitude modulation	1	0.00	0.2	0.12	0.02	2.7	1.9	0.00	0.3	0.16	0.02	3.9	2.4
	2	0.00	0.2	0.03	0.01	2.3	0.5	0.00	0.0	0.04	0.01	0.7	0.6
	3	0.00	0.2	0.04	0.04	2.6	0.6	0.00	0.0	0.00	0.00	0.1	0.0
	4	0.00	0.2	0.04	0.11	2.9	0.6	0.00	0.0	0.00	0.01	0.0	0.0
phase modulation	1	0.00	0.5	0.06	0.02	8.4	0.9	0.00	0.6	0.06	0.02	9.9	0.9
	2	0.00	1.4	0.03	0.01	21.5	0.7	0.00	0.5	0.04	0.01	7.5	0.6
	3	0.00	3.0	0.05	0.04	45.4	1.1	0.00	1.0	0.01	0.00	15.4	0.4
	4	0.00	5.3	0.06	0.11	77.2	1.8	0.00	1.8	0.02	0.01	27.1	0.8
Std. thresh.	—	3	60	2.3	3	300	14	3	60	2.3	3	300	14

Note: Threshold values are related to $RR \leq 20$ (1) or $RR > 20$ (2).

The following conclusions can be derived from the achieved results:

- in off-nominal frequency, modulations, and ramp-frequency testing conditions, the accuracies of the *TVE*, *FE*, and *RFE* parameters increases as the |*FD*| decreases; however the GTWLS algorithm has a low sensitivity to *FD* as long as |*FD*| is less than about 1 Hz;

- in the off-nominal frequency and ramp-frequency testing conditions when the reference frequency is close to the waveform frequency the GTWLS algorithm provides much more accurate estimates than the classical TWLS algorithm; when $f_0 = f$, estimation errors are negligible, while only phasor estimates comply with both *P-class* and *M-class* performances when the classical TWLS algorithm is applied, i.e. $f_0 = 50$ Hz (see Tab. I);

- in the modulations testing conditions, the parameters returned by the GTWLS algorithm with $f_0 = f$ are almost equal to those returned by the classical TWLS algorithm since, according to the Standard, we have $f_0 = f_n$ and $f = f_n$ in the AM testing, while in the PM testing f varies in the range $[f_n - 0.1, f_n + 0.1]$ Hz.

- errors achieved in harmonic and in the out-out-band interference testing conditions are high because the model (1) does not explicitly include these disturbances;

- when considering the 2nd harmonic testing, all the phasor parameter estimates comply with the *P-class* and the *M-class* requirements (in this last case related to *RR* > 20 readings/s) only when $J = 4$ and the Hann window is used; conversely, when the 3rd harmonic testing is concerned, the Standard requirements are satisfied only when $J = 3$ and 4 and the Hann window is employed (see Tab. I);

- in the out-of-band interference testing, the achieved maximum *TVE* and *FE* values are much higher than the related Standard thresholds for all the considered observation interval lengths.

Moreover, simulation results show that in the off-nominal frequency, harmonics, modulations, and frequency ramp testing the GTWLS algorithm based on the Hann window is capable to return estimates compliant with both *P-class* and *M-class* of performances when observing $J = 1, \ldots, 4$ cycles, but an accurate *a-priori* waveform frequency estimate is needed and, when present, possible low order harmonics must be removed from the acquired data. Accurate waveform frequency estimates can be achieved by using the classical Interpolated Discrete Fourier Transform (IpDFT) algorithm [15]-[18], which is simple to apply and very fast to perform. This algorithm returns the parameters of each component of a multifrequency signal by compensating both spectral leakage and picket-fence effects by using windowing and evaluating the ratio between the two largest DFT samples of the corresponding spectrum peak, respectively. In the next Section this algorithm is used to provide an initial frequency estimate to the GTWLS algorithm. Accordingly, the whole related procedure is called the GTWLS-IpDFT algorithm.

IV. ACCURACY OF THE GTWLS-IpDFT ALGORITHM

In the IpDFT algorithm, the frequency of a spectral component is estimated as a function of the ratio between the two highest DFT spectral samples [15] – [17]. This function can be expressed using a simple analytical expression when the cosine class Maximum Sidelobe Decay (MSD) windows are adopted [17]. The two-term MSD window, or Hann window, is employed in the proposed algorithm. However, the frequency estimated in off-nominal frequency conditions can be significantly affected by the spectral interference from the image component or possible harmonics and interharmonics when the number of acquired waveform cycles is small. Windowing can effectively reduce this detrimental effect when the frequency distance between the fundamental and the interfering spectral tones is greater than the normalized width of the window spectrum mainlobe, which is equal to $H + 1$, where H is the number of cosine window terms ($H = 2$ for the Hann window) [16]. The worst case occurs in the presence of second harmonic, for which the above constraint is satisfied if the number of observed waveform cycle J is at least equal to $H + 1$. Thus, when the Hann window is adopted, at least $J = 3$ cycles should be acquired to achieve accurate IpDFT frequency estimates. However, in practice this constraint does not affect the estimator performances because of the algorithm robustness to frequency changes, which usually are slower than phasor variations.

In the following, the accuracies of the phasor, frequency, and *ROCOF* estimates returned by the GTWLS-IpDFT algorithm are analyzed through extensive Monte Carlo simulations and compared with both the *P-class* and the *M-class* requirements [2], [3]. The waveform frequency is firstly estimated by means of the IpDFT algorithm based on the Hann window applied to J cycles long observations, except when $J = 1$ or 2, for which three waveform cycles are considered in the IpDFT algorithm, while only the one or two cycles are used as input to the GTWLS algorithm. The pseudocode of the applied procedure for

$J \geq 3$ is given in Fig. 1. In steps 3 and 4 of the procedure the parameters of the 2nd harmonic are estimated and then this component is removed from the acquired data, respectively. However, if an out-of-band interference is detected, a similar approach can be used to remove it from the analyzed waveform.

Step 1: *Acquire M samples belonging to J cycles of the analyzed signal* $x(m)$, *$m = 0, 1, \ldots, M - 1$, where* $M = Jf_s/f_n + 1$.

Step 2: Determine the windowed signal $x_w(m) = x(m)w(m)$, $m = 0, 1, \ldots, M - 2$, *where w(m) is the Hann window.*

Step 3: Apply the IpDFT algorithm to the windowed signal $x_w(\cdot)$ *to obtain the estimated frequency* $\hat{f}$ *and the estimated amplitude* $\hat{A}_2$ *and phase* $\hat{\varphi}_2$ *of the 2nd-order harmonic of the analyzed waveform.*

Step 4: Remove the 2nd order harmonic and determine the new analyzed signal:

$$\tilde{x}(m) = x(m) - \hat{A}_2 \cos(4\pi\hat{f}m / f_s + \hat{\varphi}_2), \quad m = 0,1,\ldots,M-1.$$

Step 5: Determine the phasor amplitude and phase, FD, and ROCOF by applying the GTWLS algorithm (4), (9), (10), and (11) *to the signal* $\tilde{x}(\cdot)$ *and considering as reference frequency the estimated value* $\hat{f}$ *or its rounded value. If the rounded frequency value is used the matrix* $\tilde{A}_K$ *in (4) can be computed a-priori. The most suitable window can be the Hann or the Hamming window, depending on the observation length and the kind of disturbances applied to the analyzed waveform.*

Fig. 1. Pseudocode of the proposed GTWLS-IpDFT algorithm.

The maximum magnitude of the *TVE*, *FE*, and *RFE* values returned by the GTWLS-IpDFT algorithm are reported in Tab. II. The parameters considered in the performed simulations were chosen according to the worst-case conditions specified in the Standard for the *P-class* or the *M-class* of performances, respectively [2], [3]. The related thresholds specified in the Standard are also given in Tab. II for comparison. The results related to off-nominal frequency and ramp-frequency testing conditions are not reported in Tab. II since they assume negligible values. It is also worth noticing that results almost equal to those reported in Tab. II are achieved when the GTLWS algorithm is applied using the true value of the waveform frequency. This fact can be easily explained by considering that in most of the considered testing conditions the IpDFT algorithm provides enough accurate frequency estimates and the GTWLS algorithm exhibits low sensitivity to *FD* as long as its absolute value is less than about 1 Hz.

Table II. Maximum magnitude of the *TVE*, *FE*, and *RFE* values returned by GTWLS-IpDFT algorithm based on the rectangular or the Hann windows and *J* = 1, 2, 3 or 4 cycles. The waveform frequency was estimated by applying the IpDFT method based on the Hann window to three cycles long observations except when *J* = 4 when all the four cycles were considered. The simulation parameters are chosen according to the worst-case conditions specified in the Standard for the *P-class* or *M-class* of performances, respectively. Errors related to static off-nominal frequency and frequency ramp tests are negligible and not reported.

		Rectangular window						Hann window					
		P-class			*M-class*			*P-class*			*M-class*		
Test type	*J*	*TVE* max (%)	$\lvert FE\rvert$ max (mHz)	$\lvert RFE\rvert$ max (Hz/s)	*TVE* max (%)	$\lvert FE\rvert$ max (mHz)	$\lvert RFE\rvert$ max (Hz/s)	*TVE* max (%)	$\lvert FE\rvert$ max (mHz)	$\lvert RFE\rvert$ max (Hz/s)	*TVE* max (%)	$\lvert FE\rvert$ max (mHz)	$\lvert RFE\rvert$ max (Hz/s)
2nd harmonic	1(1)	0.00	0.00	0.00	0.00	0.0	0.0	0.00	0.0	0.00	0.00	0.0	0.0
	2(1)	0.00	0.00	0.00	0.00	0.0	0.0	0.00	0.0	0.00	0.00	0.0	0.0
	3(2)	0.13	19.8	1.70	1.31	198	16.9	0.12	7.1	5.33	1.21	72.1	53.3
	4(2)	0.08	11.5	0.63	0.84	115	6.3	0.01	0.9	0.31	0.13	8.8	3.1
3rd(3) harmonic	1	0.22	93.6	28.0	2.2	947	281	7.7	$3\cdot10^3$	$1\cdot10^3$	77	$1\cdot10^5$	$9\cdot10^4$
	2	0.10	24.5	3.4	1.0	245	33.6	0.01	1.8	1.48	0.15	17.8	14.8
	3	0.06	11.2	0.94	0.62	112	9.4	0.00	0.1	0.04	0.01	1.3	0.4
	4	0.04	6.4	0.38	0.44	64.1	3.8	0.00	0.0	0.00	0.00	0.2	0.0
Std. thresh.	-	1	5	0.4	1	5(4)-25(5)	—	1	5	0.4	1	5(4)-25(5)	—
amplitude modulation	1	0.00	0.2	0.12	0.02	2.8	1.8	0.00	0.3	0.16	0.02	4.0	2.4
	2	0.00	0.2	0.03	0.01	2.3	0.5	0.00	0.0	0.04	0.01	0.7	0.6
	3	0.00	0.2	0.04	0.04	2.6	0.6	0.00	0.0	0.00	0.00	0.1	0.0
	4	0.00	0.2	0.04	0.11	2.9	0.6	0.00	0.0	0.00	0.01	0.0	0.0
phase modulation	1	0.00	0.5	0.06	0.02	8.4	0.9	0.00	0.6	0.05	0.02	9.8	0.8
	2	0.00	1.4	0.03	0.01	21.6	0.7	0.00	0.5	0.04	0.01	7.5	0.6
	3	0.00	3.0	0.05	0.04	45.8	1.1	0.00	1.0	0.01	0.00	15.3	0.4
	4	0.00	5.3	0.06	0.11	78.0	1.9	0.00	1.8	0.02	0.01	26.9	0.8
Std. thresh.	-	3	60	2.3	3	300	14	3	60	2.3	3	300	14

Notes (1): Errors related to the 2nd order harmonic testing and *J* = 1 or 2 cycles are well above the related Standard thresholds, so errors of estimates achieved after harmonic removal are reported. Errors values compliant with the related *P-class* requirements (but not with the *M-class* requirements) are achieved for *J* = 2 cycles if the IpDFT algorithm is based on 2 waveform cycles.

(2): Errors related to the 2nd order harmonic testing and *J* = 3 or 4 cycles are negligible after harmonic removal, so errors of estimates achieved without harmonic removal are reported.

(3): Errors related to the 3rd order harmonic testing are negligible after harmonic removal, so errors of estimates achieved without harmonic removal are reported.

(4): Threshold values related to $RR \leq 20$.

(5): Threshold values related to $RR > 20$.

Tab. II also shows that more accurate estimates are obtained when the Hann window is adopted in the GTWLS algorithm. Thus, the rectangular window will not be more considered in the following. Conversely, in order to provide insights useful for the choice of the best two-term cosine window to be used in the GTWLS algorithm, results returned by either the Hann or the Hamming windows are discussed. These two windows have been selected since they exhibit very different spectral features: the former provides the maximum sidelobe decay rate, while the latter exhibits the minimum sidelobe level within the class of two-term cosine windows [20]. Anyway, the IpDFT frequency estimation is always based on the Hann window.

Compliance of the proposed algorithm to either *P-class* or *M-class* of performances is discussed in the next subsections. Notice that, due to the low sensitivity of the GTWLS-IpDFT algorithm to *FD* in most of the considered testing conditions, we can expect that almost the same estimation accuracy is achieved when the reference frequency employed in the GTWLS algorithm coincides with either the frequency estimate $\hat{f}$ returned by the IpDFT algorithm or its closest integer value, i.e. $round(\hat{f})$. This choice produces a significant impact on the required processing effort. Indeed, if the estimated frequency $\hat{f}$ is used, the matrix $\tilde{A}_K$ in (4) needs to be computed each time the GTWLS algorithm is implemented. Opposite, if the rounded frequency value is adopted as reference frequency, within the threshold specified in the Standard there are at most 11 different integer frequency values so the related matrices $\tilde{A}_K$ can be computed *a-priori*, stored in the system memory and read by the digital processor when needed. In this case the GTWLS algorithm requires about $6M - 6$ real products and $6M - 9$ real additions to return the phasor parameters. Moreover, the processing effort required by the IpDFT algorithm must be taken into account. Generally it includes the effort related to both the raw search of the spectrum peak and the estimation of the related tone parameters. However, since in practice a high number of samples M is considered, most of the processing effort related to the IpDFT algorithm is due to the calculation of the employed DFT samples, that is about $6M$ real products and additions by spectrum sample. Thus, most of the processing time required by the GTWLS-IpDFT based on rounded waveform frequency is due to the IpDFT algorithm.

A. *P-class of performances compliance*

When considering *P-class* of performances, harmonics are the most severe disturbances implied in the analyzed testing conditions. From Tab. II it follows that, in the considered situations, the GTWLS-IpDFT algorithm complies with the *P-class* requirements for J = 2-4 only when the low order harmonics are removed from the analyzed waveform. When J = 1 cycle, the number of harmonics to be removed from the analyzed waveform could be high. Thus, this situation is not considered in the following. Conversely, when J = 2 cycles, both the 2nd and the 3rd harmonics should be removed, while only the 2nd harmonic has a significant effect on estimation accuracy and should be removed when J = 3 or 4 cycles.

As for the estimation latency, since the reference time is exactly in the middle of the observation interval, then the estimated parameters are returned with a delay equal to half the observation interval length (i.e. $J/(2f_n)$ seconds) plus the processing delay. However, this last contribution can be often considered negligible with respect to the duration of the waveform cycle. According to the Standard, the maximum PMU reporting latency for *P-class* of performances is equal to 2/*RR* seconds. Hence, the GTWLS-IpDFT algorithm complies with the Standard requirements in all the considered situations and J = 1 - 4 (except when *RR* = 50 readings/s and J = 4) if the harmonics are removed. In the following, the performances achieved for J = 2-4 cycles are analyzed

The robustness of the proposed algorithm when multiple severe disturbances concurrently occur has also been analyzed. In Tab. III the results returned when the analyzed waveform is affected by 2nd harmonic and possibly by off-nominal frequency, amplitude or phase modulations are reported in the case when the Hann or the Hamming windows are employed in the GTWLS algorithm. Testing conditions are chosen according to the worst-case parameters specified in the Standard for the *P-class* of performances. In particular, the off-nominal frequency was set to 48 Hz. The thresholds specified in the Standard respectively for harmonic, off-nominal frequency, and modulation testing conditions are also given in Tab. III.

Table III. Maximum magnitude of the *TVE*, *FE*, and *RFE* values returned by GTWLS-IpDFT algorithm based on the Hann or the Hamming windows and 2nd-order harmonic removal. The GTWLS algorithm employs as reference frequency the estimate returned by the IpDFT algorithm (estimated frequency) or its rounded value (rounded frequency). The simulation parameters are chosen according to the worst-case conditions specified in the Standard for the *P-class* of performances.

		Hann window			Hamming window		
Test type	*J*	*TVE* max (%)	$\|FE\|$ max (mHz)	$\|RFE\|$ max (Hz/s)	*TVE* max (%)	$\|FE\|$ max (mHz)	$\|RFE\|$ max (Hz/s)
off-nominal + 2nd harm. (estimated frequency)	2	0.35	55.3	7.6	0.26	41.9	3.24
	3	0.08	5.3	0.16	0.04	3.4	0.08
	4	0.00	0.3	0.00	0.00	0.2	0.01
off-nominal + 2nd harm. (rounded frequency)	2	0.35	55.2	7.6	0.26	41.8	3.23
	3	0.08	5.4	0.16	0.04	3.4	0.08
	4	0.00	0.3	0.00	0.00	0.2	0.01
Std. thresh.	-	1	5	0.4	1	5	0.4
AM + 2nd harmonic (estimated freq.)	2	0.03	1.05	2.58	0.03	0.80	1.61
	3	0.01	0.13	0.28	0.00	0.09	0.13
	4	0.00	0.06	0.01	0.00	0.07	0.01
AM + 2nd harmonic (rounded freq.)	2	0.03	1.05	2.58	0.03	0.80	1.61
	3	0.01	0.11	0.28	0.00	0.07	0.13
	4	0.00	0.01	0.01	0.00	0.01	0.01
PM + 2nd harmonic (estimated freq.)	2	0.05	9.27	1.58	0.04	7.39	1.00
	3	0.01	1.42	0.31	0.01	1.49	0.17
	4	0.00	1.81	0.02	0.00	2.21	0.03
PM + 2nd harmonic (rounded freq.)	2	0.05	9.41	1.59	0.04	7.53	1.00
	3	0.01	1.42	0.31	0.01	1.50	0.17
	4	0.00	1.83	0.02	0.00	2.23	0.03
Std. thresh.	-	3	60	2.3	3	60	2.3

Tab. III shows that the results returned by using the two different windows are almost equal. In particular, in the off-nominal frequency testing when either the Hann or the Hamming windows are used and $J = 2$ cycles the achieved *FE* and *RFE* estimates are much higher than the related thresholds. Conversely, when the Hann window is used and $J = 3$ cycles are considered, the magnitude of the achieved *FE* is a bit higher than the related Standard threshold, while the magnitudes of the estimation errors are much smaller than the related Standard thresholds in all the remaining testing conditions. In addition, when the Hamming window is adopted with J = 3 cycles the returned estimates are smaller than the Standard thresholds in all the considered testing conditions. Finally, both the Hann and the Hamming windows allows to comply with the Standard requirements when J = 4 cycles.

Furthermore, the performances of the proposed algorithm have been assessed also in transient conditions by considering waveform amplitude or phase step changes accordingly to the Standard. The achieved worst-case phasor amplitude overshoot or undershoot (expressed as percent of the step magnitude) and phasor, frequency, and *ROCOF* response times are given in Tab. IV for both the Hann and the Hamming windows. The thresholds specified in the Standard are also reported for comparison. The initial phase of the analyzed waveform was linearly varied in the range [0, 2π) rad with a step of π/50 rad and the worst-case values of the transient performance parameters were determined when the step change is -10% or +10% for amplitude steps, and -10° or +10° when considering phase steps.

Table IV. Absolute value of the worst-case overshoot or undershoot, and phasor, frequency, and *ROCOF* response times returned by GTWLS-IpDFT algorithm based on the Hann or the Hamming windows when the analyzed waveform exhibits an amplitude or a phase step. The GTWLS algorithm employs as reference frequency the estimate returned by the IpDFT algorithm (estimated frequency) or its rounded value (rounded frequency).

		Hann window				*Hamming window*			
Test type	*J*	*Overshoot/ undershoot* (%)	*Phasor response time* (nominal cycles)	*Frequency response time* (nominal cycles)	*ROCOF response time* (nominal cycles)	*Overshoot/ undershoot* (%)	*Phasor response time* (nominal cycles)	*Frequency response time* (nominal cycles)	*ROCOF response time* (nominal cycles)
magnitude step (estimated freq.)	2	0.8	0.5	1.54	1.71	0.6	0.6	1.83	1.96
	3	0.7	0.8	2.29	2.46	0.6	0.8	2.63	2.92
	4	0.5	0.9	2.92	3.16	0.5	1.0	3.25	3.75
magnitude step (rounded freq.)	2	0.8	0.5	1.54	1.71	0.6	0.6	1.83	1.96
	3	0.7	0.8	2.33	2.46	0.6	0.8	2.75	2.96
	4	0.5	0.9	2.96	3.17	0.5	1.0	3.42	3.75
phase step (estimated freq.)	2	5.3	1.5	2.58	2.75	4.8	1.4	2.75	2.83
	3	5.7	1.4	2.46	2.58	4.8	1.0	2.96	2.96
	4	4.8	1.1	3.21	3.37	3.7	1.2	3.79	3.96
phase step (rounded freq.)	2	5.4	1.5	2.71	2.75	4.8	1.4	2.83	2.83
	3	5.8	1.4	2.46	2.58	4.9	1.0	2.96	2.96
	4	4.9	1.1	3.29	3.37	4.4	1.2	3.96	3.96
Std. thresh.	-	5	2	4.5	6	5	2	4.5	6

Tab. IV additionally shows that the results obtained when considering the estimated or the rounded frequencies are very close each other. Moreover, the proposed algorithm based on the Hann window complies with the *P-class* requirements in all the considered conditions, except for phase step change and $J = 2$ or 3. Conversely, by adopting the Hamming window, compliant estimates are achieved in all the above testing conditions, even though the frequency and *ROCOF* response times are a bit higher than those achieved using the Hann window.

The delay times related to the step changes have also been analyzed. When amplitude and phase steps are of concern, the achieved maximum delay time were 1.67 ms and 2.5 ms, respectively. Thus, all the achieved values are always smaller than the minimum threshold specified in the Standard, i.e. 5 ms, which is related to $RR = 50$ readings/s.

B. *M-class of performance compliance*

When considering *M-class* of performances, out-of-band interference is the most severe condition among the analyzed ones. *TVE* and *FE* are the parameters specified in the Standard to assess estimator performances in the related testing condition. In particular, the frequency of the out-of-band interference f_{ih} belongs to the ranges $[10, f_n - RR/2]$ and $[f_n + RR/2, 2f)$, while the fundamental frequency f is in the range $f_n \pm RR/20$. Thus, the minimum frequency distance between the two components is $\Delta f_{min} = \min| f_{ih} - f | = 9 \cdot RR/20$ Hz. In order to obtain accurate estimates of the interfering component parameters by the IpDFT algorithm (so allowing an effective removal of this disturbance from the acquired waveform as specified in the pseudocode reported in Fig. 1 for the 2nd-order harmonic) the absolute value $\Delta\lambda$ of the difference between the number of observed cycles of the interfering tone and the fundamental must be greater than about $\Delta\lambda_{min} = H + 1$ ($H = 2$ for both the Hann and the Hamming windows) [16]. Observing that $\Delta\lambda_{min} = J \cdot \Delta f_{min} / f_n$, where J is the number of observed nominal cycles, it follows that the minimum required value for J must be greater than about $20 \cdot (H + 1) \cdot f_n / (9 \cdot RR)$. Assuming $H = 2$ and the RR values specified in the Standard (i.e. 10, 25, and 50 readings/s), the minimum number of waveform cycles to be acquired is $J = 34$, 14, and 7, respectively. It is worth noticing that all these choices largely complies with the upper bound on latency specified in the Standard, which is equal to 7/*RR*. In the following the maximum *RR* value specified in the Standard is considered, that is *RR* = 50 readings/s, and the Hann window is used. In Tab. V the magnitudes of the *TVE* and *FE* values obtained using as reference frequency either the IpDFT estimate or its rounded value are reported for *J* = 7, 8, and 9 cycles in the case when *RR* = 50 reading/s and f_n= 50 Hz. Both the Hann and the Hamming windows are adopted and the worst case out-of-band interference is considered, i.e.: f = 47.5 Hz, f_{ih} = 25 Hz and f = 52.5 Hz, f_{ih} = 75 Hz [2].

Table V. Out-of-band interference testing: maximum magnitude of the *TVE* and *FE* values returned by GTWLS-IpDFT algorithm based on the Hann and Hamming windows when *J* = 7, 8, and 9 cycles. The simulation parameters are chosen according to the worst case specified in the Standard for the *M-class* of performances when *RR* = 50 readings/s and f_n = 50 Hz. The out-of-band component is estimated by the IpDFT algorithm based on the Hann window and then removed from the analyzed waveform.

				Hann window		Hamming window	
Test type	*J*	f (Hz)	f_{ih} (Hz)	*TVE* max (%)	$\lvert FE \rvert$ max (mHz)	*TVE* max (%)	$\lvert FE \rvert$ max (mHz)
out-of-band interference (estimated frequency)	7	47.5	25	5.80	203	5.11	145
		52.5	75	5.78	203	4.86	146
	8	47.5	25	0.39	7.6	0.21	5.2
		52.5	75	0.38	7.5	0.20	5.3
	9	47.5	25	0.01	1.2	0.01	0.8
		52.5	75	0.01	1.2	0.01	0.8
out-of-band interference (rounded frequency)	7	47.5	25	5.95	241	5.14	179
		52.5	75	5.93	241	5.11	178
	8	47.5	25	0.47	14.4	0.25	9.9
		52.5	75	0.46	14.4	0.25	9.9
	9	47.5	25	0.02	2.5	0.02	2.3
		52.5	75	0.02	2.5	0.01	2.3
Std. thresh.	-	-	-	1.3	10	1.3	10

Tab. V shows that the *TVE* and *FE* values achieved for $J = 7$ cycles are quite higher than the related thresholds specified in the Standard. When $J = 8$ cycles, the Standard compliance is achieved only by the Hamming window, while compliance with the *M*-class requirements is achieved by both considered windows when $J = 9$ cycles. Hence, only observation length equal to $J = 9$ cycles are considered in the following analysis.

The results achieved by the proposed algorithm when using as reference frequency either the IpDFT estimate or its rounded value, the Hann or the Hamming windows and $J = 9$ cycles are reported in Tab. VI for both the steady-state and the dynamic testing conditions specified in the previous Section, and in Tab. VII in the case of transient conditions.

Table VI. Maximum magnitude of the *TVE*, *FE*, and *RFE* values returned by GTWLS-IpDFT algorithm based on the Hann or the Hamming windows and $J = 9$ cycles. The reference frequency employed by the GTWLS algorithm is either the IpDFT estimate or its rounded value. The simulation parameters are chosen as in Tab. II.

	Hann window			Hamming window		
Test type	*TVE* max (%)	$\|FE\|$ max (mHz)	$\|RFE\|$ max (Hz/s)	*TVE* max (%)	$\|FE\|$ max (mHz)	$\|RFE\|$ max (Hz/s)
2nd harmonic (estimated/rounded freq.)	0.00	0.1	0.0	0.01	1.2	0.04
Std. thresh.	1	5(1)-25(2)	—	1	5(1)-25(2)	—
AM (estimated freq.)	0.33	12.6	0.1	0.52	14.7	0.2
AM (rounded freq.)	0.33	0.0	0.0	0.51	0.5	0.1
PM (estimated/rounded freq.)	0.30	124	3.5	0.46	146	4.4
Std. thresh.	3	300	14	3	300	14

Note: Threshold values are related to $RR \leq 20$ (1) or $RR > 20$ (2)

Table VII. Absolute value of the worst-case overshoot or undershoot, and phasor, frequency, and *ROCOF* response times returned by GTWLS-IpDFT algorithm based on the Hann or the Hamming windows and J = 9 cycles when the analyzed waveform exhibits an amplitude or a phase step. The GTWLS algorithm employs as reference frequency the estimate returned by the IpDFT algorithm (estimated frequency) or its rounded value (rounded frequency).

	Hann window				*Hamming window*			
Test type	*Overshoot (undershoot)* (%)	*Phasor response time* (nominal cycles)	*Frequency response time* (nominal cycles)	*ROCOF response time* (nominal cycles)	*Overshoot (undershoot)* (%)	*Phasor response time* (nominal cycles)	*Frequency response time* (nominal cycles)	*ROCOF response time* (nominal cycles)
amplitude step (estimated freq.)	0.5	2.0	5.7	6.8	0.5	2.2	6.0	7.7
amplitude step (rounded freq.)	0.5	2.0	5.0	6.7	0.5	2.1	5.1	7.2
phase step (estimated freq.)	4.5	2.3	6.8	7.5	4.0	2.5	7.8	8.9
phase step (rounded freq.)	4.5	2.3	6.7	7.5	4.0	2.5	7.7	8.9
Std. thresh.	10	7(1)	14(1)	14(1)	10	7(1)	14(1)	14(1)

Note: (1) Threshold values are related to *RR* = 50.

It is worth noticing that the errors achieved in the off-nominal frequency and the ramp frequency testing are negligible when either the estimate returned by the IpDFT algorithm or its rounded value are employed as reference frequency in the GTWLS algorithm.

Tabs. VI and VII show that the proposed GTWLS-IpDFT algorithm comply with *M-class* performance in all the considered situations. Also, the maximum delay times related to either amplitude or phase step changes are 1.67 and 2.50 ms, respectively. These values are smaller than 5 ms, that is the minimum threshold value specified in the Standard and related to *RR* = 50 readings/s.

The robustness of the proposed algorithm when multiple severe disturbances concurrently occur have also been analyzed. In Tab. VIII the results returned when the analyzed waveform is affected by out-of-band interference characterized by f = 47.5 Hz and f_{ih} = 25 Hz plus 2nd-order harmonic or amplitude or phase modulations are reported in the case when the Hann or the Hamming windows are employed in the GTWLS algorithm. Testing conditions are chosen according to the worst-case parameters specified in the

Standard for the *M-class* of performances. The thresholds specified in the Standard respectively for harmonic, and modulation testing conditions are also given in Tab. VIII.

Table VIII. Maximum magnitude of the *TVE*, *FE*, and *RFE* values returned by GTWLS-IpDFT algorithm based on the Hann or the Hamming windows when J = 9. Out-of-band interference at f = 47.5 Hz and f_{ih} = 25 Hz is removed by using the estimates returned by IpDFT algorithm. The GTWLS algorithm employs as reference frequency the estimate returned by the IpDFT algorithm (estimated frequency) or its rounded value (rounded frequency). The simulation parameters are chosen according to the worst-case conditions specified in the Standard for the *M-class* of performances.

	Hann window			Hamming window		
Test type	*TVE* max (%)	|*FE*| max (mHz)	|*RFE*| max (Hz/s)	*TVE* max (%)	|*FE*| max (mHz)	|*RFE*| max (Hz/s)
2nd harmonic + out-of-band (estimated frequency)	0.01	1.2	0.03	0.01	0.8	0.05
2nd harmonic + out-of-band (rounded frequency)	0.02	2.5	0.06	0.02	2.3	0.06
Std. thresh.	1	5	0.4	1	5	0.4
AM + out-of-band (estimated frequency)	0.47	5.24	0.7	0.59	5.78	0.4
AM + out-of-band (rounded frequency)	0.47	36.3	1.2	0.62	41.4	1.1
PM + out-of-band (estimated frequency)	0.43	128	4.12	0.54	150	4.73
PM + out-of-band (rounded frequency)	0.44	128	4.10	0.56	150	4.71
Std. thresh.	3	300	14	3	300	14

Tab. VIII shows that the achieved values for *TVE*, *FE*, and *RFE* estimates are well below the thresholds specified in the Standard for the related testing conditions without considering the out-of-band interference. This confirms the robustness of the proposed algorithm.

C. *Some remarks about computational complexity*

The GTWLS-IpDFT algorithm was implemented in a Matlab 7.0 environment running on a portable computer provided with a 2 GHz processor, 2046 MB RAM memory, and equipped with a Microsoft Windows Vista operating system. When the frequency estimate returned by the IpDFT algorithm is used as reference frequency in the GTWLS algorithm and so the matrix $\widetilde{A}_K$ in (4) needs to be evaluated at runtime, the average processing times needed to estimate the phasor parameters over 10,000 runs were 1.5, 2.1, and 5.9 ms when the GTWLS-IpDFT algorithm based on the Hamming window was implemented respectively over three, four, and nine waveform cycles, respectively and the 2nd harmonic or the out-of-band interference removal was used. Conversely, when the reference frequency used in the GTWLS algorithm is the rounded value of the estimate returned by the IpDFT algorithm, the related matrix $\widetilde{A}_K$ in (4) can be computed *a-priori* and stored in the system memory. In this case, considering the same testing conditions above, the average processing time over 10,000 runs was equal to 0.41, 0.48, and 0.72 ms, of which the IpDFT algorithm required 0.31, 0.37, and 0.57 ms, respectively. So, as expected, the processing effort is mainly due to the IpDFT algorithm. Thus, the GTWLS-IpDFT algorithm based on rounded waveform frequency and a pre-computed matrix can be easily implemented even using low-cost low performance digital hardware.

D. *Accuracy comparison with the IpD^{2}FT algorithm*

For completeness, the accuracy achieved with the GTWLS-IpDFT algorithm in steady-state and dynamic conditions is compared with that of the IpD^{2}FT algorithm [12] based on the Hann window, whose worst-case errors obtained in the simulation conditions considered above are reported in Tab. IX.

The IpD^{2}FT algorithm returns phasor estimates compliant with the *M-class* requirements in all the considered situations, except for waveforms affected by a 2nd harmonic and J = 2. The *FE* and the *RFE* values returned in the presence of harmonics and modulations are slight worse than the GTWLS-IpDFT algorithm with harmonic removal. The harmonic presence, instead, turns to be a challenging scenario for the IpD^{2}FT, where noncompliant *FE* values are returned in the case of 2nd harmonic when J = 2 and 3 for the *M-class* and when J = 2 for the *P-class*. As for *RFE*, the *P-class* Standard thresholds are satisfied except when the acquired waveform is affected by 2nd harmonic and J = 2 and 3. The main message here is that, on the whole, the GTWLS-IpDFT algorithm performs even better than an effective novel solution as the IpD^{2}FT algorithm in all the analyzed situations. However, for a fair comparison, it has to be noted that the GTWLS-IpDFT algorithm refines its estimates after harmonic removal, an idea that could be also applied to IpD^{2}FT algorithm.

Table IX. Maximum magnitude of the *TVE*, *FE*, and *RFE* values returned by IpD^{2}FT algorithm based on the Hann windows and J = 2, 3, or 4 cycles. The simulation parameters are chosen according to the worst-case conditions specified in the Standard for the *P-class* or *M-class* of performances, respectively.

		Hann window					
		P-class			*M-class*		
Test type	J	*TVE* max (%)	$\|FE\|$ max (mHz)	$\|RFE\|$ max (Hz/s)	*TVE* max (%)	$\|FE\|$ max (mHz)	$\|RFE\|$ max (Hz/s)
2nd harmonic	2	0.55	99.0	33.3	5.52	983	333
	3	0.05	4.38	1.69	0.49	42.4	17.0
	4	0.02	0.95	0.35	0.16	9.17	3.56
3rd harmonic	2	0.01	5.34	0.88	0.11	17.0	8.70
	3	0.00	0.41	0.14	0.03	2.51	1.38
	4	0.00	0.18	0.04	0.02	0.72	0.36
Std. thresh.	-	1	5	0.4	1	5$^{(1)}$-25$^{(2)}$	-
amplitude modulation	2	0.17	21.5	14.2	0.19	29.5	14.4
	3	0.05	3.60	2.13	0.07	5.27	2.48
	4	0.05	1.19	1.03	0.07	1.99	1.51
phase modulation	2	0.00	16.3	0.62	0.01	19.2	0.67
	3	0.01	22.3	0.76	0.02	22.9	0.77
	4	0.04	34.5	1.09	0.06	36.1	1.09
Std. thresh.	-	3	60	2.3	3	300	14

Note: Threshold values are related to $RR \leq 20$ $^{(1)}$ or $RR > 20$ $^{(2)}$.

V. CONCLUSIONS

In this paper an extension of the Taylor Weighted Least Squares (TWLS) algorithm for the estimation of the phasor, frequency, and *ROCOF* parameters of an electric waveform has been analyzed. This extension - called Generalized TWLS (GTWLS) algorithm – is based on *a-priori* known generic reference frequency, not only on the nominal waveform frequency as occurs in the classical TWLS algorithm. The GTWLS algorithm accuracy has been analyzed through meaningful Monte Carlo simulations, performed in different steady-state and dynamic testing conditions according to the requirements reported in the Standard for the *P-class* and the *M-class* of performances. As expected, in the off-nominal frequency, modulations, and ramp frequency testing conditions the best parameter estimation accuracy is achieved when the adopted reference frequency is equal to the waveform frequency. However, for frequency deviations smaller than about 1 Hz, its effect on the estimation accuracy is very low and can be further mitigated by windowing.

Furthermore, a two-step procedure for phasor parameters estimation - called GTWLS-IpDFT algorithm - has been proposed. In the first step the waveform frequency is estimated by a classical Interpolated Discrete Fourier Transform (IpDFT) algorithm based on the Hann window. The second step then returns an estimate of the parameters of interest by applying the GTWLS algorithm based on either the Hann or the Hamming windows. The reference frequency employed in the GTWLS algorithm can be either the estimate returned by the first step or its value rounded to the closest integer. This latter choice allows a significant reduction of the required processing effort since the matrix involved in the least squares estimation can be computed *a-priori* and stored in the system memory. It has been shown that the phasor parameter estimates returned by the proposed procedure comply with the *P-class* of performances in all the considered static or dynamic conditions when 2, 3 or 4 cycle long observations are considered. Either the Hann or the Hamming windows are used in the GTWLS algorithm, and both the 2nd and the 3rd harmonics (when J = 2) or the 2nd harmonic (when J = 3 or 4) are removed exploiting the estimates returned by the IpDFT algorithm based on at least 3 waveform cycles. However, when 4 cycle observations are employed, the algorithm cannot comply with the *P-class* requirements related to RR = 50 readings/s due to latency constraints. Moreover, the proposed algorithm based on either the Hann or the Hamming windows complies with the *M-class* requirements when RR = 50 readings/s and eight cycle long observations (or nine cycles if rounded values are used for the reference frequency) are considered, while the out-of-band interference is removed through the estimates returned by the IpDFT algorithm.

The GTWLS-IpDFT algorithm has been further compared with the IpD^{2}FT algorithm, recently presented in the literature. It has been shown that it performs better for both harmonic disturbance (assuming they are removed from the acquired data) and waveform modulations.

The achieved results show that the GTWLS-IpDFT algorithm can be advantageously employed in both *P-class* and *M-class* PMUs.

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