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Double parton scattering: A study of the effective cross section within a Light-Front quark model

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ABSTRACT

We present a calculation of the effective cross section σ_{eff}, an important ingredient in the description of double parton scattering in proton–proton collisions. Our theoretical approach makes use of a Light-Front quark model as a framework to calculate the double parton distribution functions at low-resolution scale. QCD evolution is implemented to reach the experimental scale. The obtained values of σ_{eff} in the valence region are consistent with the present experimental scenario, in particular with the sets of data which include the same kinematical range. However the result of the complete calculation shows a dependence of σ_{eff} on x_i, a feature not easily seen in the available data, probably because of their low accuracy. Measurements of σ_{eff} in restricted x_i regions are addressed to obtain indications on double parton correlations, a novel and interesting aspect of the three dimensional structure of the nucleon.

1. Introduction

Multi Parton Interactions (MPI), occurring when more than one parton scattering takes place in the same hadron–hadron collision, have been discussed in the literature since long time ago [1] and are presently attracting considerable attention, thanks to the possibilities offered by the Large Hadron Collider (LHC) (see Refs. [2–6] for recent reports). In particular, the cross section for double parton scattering (DPS), the simplest MPI process, depends on specific non-perturbative quantities, the double parton distribution functions (dPDFs), describing the number density of two partons with given longitudinal momentum fractions and located at a given transverse separation in coordinate space. dPDFs are naturally related to parton correlations and to the three-dimensional (3D) nucleon structure, as discussed also in the past [7].

No data are available for dPDFs and their calculation using non perturbative methods is cumbersome. A few model calculations have been performed, to grasp the most relevant features of dPDFs [8–10]. In particular, in Ref. [10] a Light-Front (LF) Poincaré covariant approach, naturally reproducing the essential sum rules of dPDFs, has been described. Although it has not yet been possible to extract dPDFs from data, a signature of DPS has been observed and measured in several experiments [11–16]: the so called "effective cross section", σ_{eff}. Despite of large errorbars, the present experimental scenario is consistent with the idea that σ_{eff} is constant w.r.t. the center-of-mass energy of the collision.

In this letter we present a predictive study of σ_{eff} which makes use of the LF quark model approach to dPDFs developed in Ref. [10].

The definition of σ_{eff} is reviewed in the next section, where an operative expression, suitable for microscopic studies and model calculations, is derived and the present experimental situation is summarized. Then the results of our approach are presented critically, discussing the dynamical dependence of σ_{eff} in view of future experiments. Conclusions are drawn in the last section.

2. The effective cross section

The effective cross section, σ_{eff}, is defined through the so called "pocket formula", which reads, if final states A and B are produced in a DPS process (see, e.g., [5]):

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$$\sigma_{eff} = \frac{m}{2} \frac{\sigma_A^{pp'} \sigma_B^{pp'}}{\sigma_{double}^{pp}}. \quad (1)$$

m is a process-dependent combinatorial factor: $m = 1$ if A and B are identical and $m = 2$ if they are different. $\sigma_{A(B)}^{pp'}$ is the differential cross section for the inclusive process $pp' \to A(B) + X$, naturally defined as:

$$\sigma_A^{pp'}(x_1, x_1', \mu_1) = \sum_{i,k} F_i^p(x_1, \mu_1) F_k^{p'}(x_1', \mu_1)\, \hat{\sigma}_{ik}^A(x_1, x_1', \mu_1), \quad (2)$$

$$\sigma_B^{pp'}(x_2, x_2', \mu_2) = \sum_{j,l} F_j^p(x_2, \mu_2) F_l^{p'}(x_2', \mu_2)\, \hat{\sigma}_{jl}^B(x_2, x_2', \mu_2), \quad (3)$$

where $F_{i(j)}^p$ is a one-body parton distribution function (PDF) with $i, j, k, l = \{q, \bar{q}, g\}$, $\mu_{1(2)}$ is the factorization scale for the process $A(B)$, σ_{double}^{pp}, the remaining ingredient in Eq. (1), appears in the natural definition of the cross section for double parton scattering:

$$\sigma_d = \int \sigma_{double}^{pp}(x_1, x_1', x_2, x_2', \mu_1, \mu_2)\, dx_1 dx_1' dx_2 dx_2', \quad (4)$$

and reads:

$$\sigma_{double}^{pp}(x_1, x_1', x_2, x_2', \mu_1, \mu_2) = \frac{m}{2} \sum_{i,j,k,l} \int D_{ij}(x_1, x_2; \mathbf{k}_\perp, \mu_1, \mu_2)\, \hat{\sigma}_{ik}^A(x_1, x_1', \mu_1) \times D_{kl}(x_1', x_2'; -\mathbf{k}_\perp, \mu_1, \mu_2) \hat{\sigma}_{jl}^B(x_2, x_2', \mu_2)\, \frac{d\mathbf{k}_\perp}{(2\pi)^2}. \quad (5)$$

In the above equation, $\mathbf{k}_\perp$ ($-\mathbf{k}_\perp$) is the transverse momentum unbalance of the parton 1 (2), conjugated to the relative distance $\mathbf{r}_\perp$ (the reader should not confuse $\mathbf{k}_\perp$ with the intrinsic momentum of the parton, argument of transverse momentum dependent parton distributions). The quantity $D_{ij}(x_1, x_2; \mathbf{k}_\perp)$, called sometimes "double generalized parton distributions" ($_2GPDS$) [17,18], is therefore the Fourier transform of the so called double distribution function, $D_{ij}(x_1, x_2; \mathbf{r}_\perp)$, which represents the number density of partons pairs i, j with longitudinal momentum fractions x_1, x_2, respectively, at a transverse separation $\mathbf{r}_\perp$ in coordinate space. dPDFs, describing soft Physics, are nonperturbative quantities.

Two main assumptions are usually made for the evaluation of dPDFs:

a) factorization of the transverse separation and the momentum fraction dependence:

$$D_{ij}(x_1, x_2; \mathbf{k}_\perp, \mu) = D_{ij}(x_1, x_2, \mu)\, T(\mathbf{k}_\perp, \mu); \quad (6)$$

b) factorized form also for the x_1, x_2 dependence:

$$D_{ij}(x_1, x_2, \mu) = F_i(x_1, \mu)\, F_j(x_2, \mu)\, \theta(1 - x_1 - x_2)(1 - x_1 - x_2)^n. \quad (7)$$

The expression $\theta(1 - x_1 - x_2)(1 - x_1 - x_2)^n$, where $n > 0$ is a parameter to be fixed phenomenologically, introduces the natural kinematical constraint $x_1 + x_2 \leq 1$ (in Eqs. (6) and (7) the same scale $\mu = \mu_1, \mu_2$ is assumed, for brevity).

One comment about the physical meaning of σ_{eff} is in order. In Eq. (1), if the occurrence of the process B were not biased somehow by that of the process A, instead of the ratio σ_B/σ_{eff} one would read σ_B/σ_{inel}, representing the probability to have the process B once A has taken place assuming rare hard multiple collisions. The difference between σ_{eff} and σ_{inel} measures therefore correlations between the interacting partons in the colliding proton.

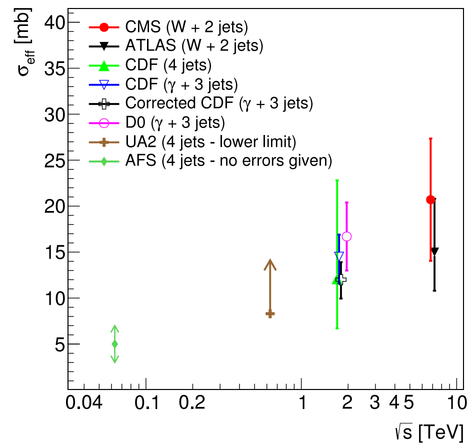

Fig. 1. Center-of-mass energy dependence of σ_{eff} measured by different experiments using different processes [11–16]. The figure is taken from [16].

Let us discuss now the dynamical dependence of σ_{eff} on the fractional momenta x_1, x_1', x_2, x_2'. By inserting Eqs. (2)–(5) in Eq. (1), and omitting the dependence on the factorization scales for simplicity, one gets the following expression for σ_{eff}:

$$\sigma_{eff}(x_1, x_1', x_2, x_2') = \frac{\left\{\sum_{i,k} F_i^p(x_1) F_k^{p'}(x_{1'})\, \hat{\sigma}_{ik}^A(x_1, x_{1'})\right\} \left\{\sum_{j,l} F_j^p(x_2) F_l^{p'}(x_{2'})\, \hat{\sigma}_{jl}^B(x_2, x_{2'})\right\}}{\sum_{i,j,k,l} \hat{\sigma}_{ik}^A(x_1, x_1')\, \hat{\sigma}_{jl}^B(x_2, x_2') \int D_{ij}(x_1, x_2; \mathbf{k}_\perp)\, D_{kl}(x_1', x_2'; -\mathbf{k}_\perp) \frac{d\mathbf{k}_\perp}{(2\pi)^2}}. \quad (8)$$

Eq. (8) clearly shows the dynamical origin of the dependence of σ_{eff} on the fractional momenta x_1, x_1', x_2, x_2'. Even within the "zero rapidity region", ($y = 0$), where $x_1 = x_1'$, $x_2 = x_2'$, such a dependence, although simplified, is still effective.

Assuming that heavy flavors are not relevant in the process, the dependence on the "parton type", $i = q, \bar{q}, g$, of the elementary cross section is basically [19]:

$$\hat{\sigma}_{ij}(x, x') = C_{ij} \bar{\sigma}(x, x'), \quad (9)$$

where $\bar{\sigma}(x, x')$ is a universal function, and C_{ij} are color factors which stay in the ratio:

$$C_{gg} : C_{qg} : C_{qq} = 1 : (4/9) : (4/9)^2. \quad (10)$$

Using Eq. (9), Eq. (8) simplifies considerably:

$$\sigma_{eff}(x_1, x_1', x_2, x_2') = \frac{\sum_{i,k,j,l} F_i(x_1) F_k(x_1') F_j(x_2) F_l(x_2') C_{ik} C_{jl}}{\sum_{i,j,k,l} C_{ik} C_{jl} \int D_{ij}(x_1, x_2; \mathbf{k}_\perp) D_{kl}(x_1', x_2'; -\mathbf{k}_\perp) \frac{d\mathbf{k}_\perp}{(2\pi)^2}}. \quad (11)$$

The present experimental scenario is illustrated in Fig. 1. The experiments [11–16], at different values of the center-of-mass energy, $\sqrt{s}$, and with different final states, explore different regions of x_i. Experiments at high $\sqrt{s}$ access low x_i regions, in general. The old AFS data [11] are in the valence region ($0.2 \leq x_i \leq 0.3$), the Tevatron data [13,14] are in the range $0.01 \leq x_i \leq 0.4$ while the recent LHC data [15,16] cover a lower average x_i range and are dominated by the glue distribution.

Remarkably the experimental evidences are compatible with a constant value of σ_{eff} in Eq. (1), the x_i-dependence being probably hidden within the experimental uncertainties. In fact one should stress that the knowledge of the x_i-dependence of σ_{eff} would open the access to information on the x_i-dependence of the dPDFs

$D_{ij}(x_1, x_2; \mathbf{r}_\perp)$, entering the definition of σ_{eff}: a direct way to access the 3D nucleon structure [7]. Nowadays, the aspects of the 3D nucleon structure related to the transverse position of partons are investigated through hard-exclusive electromagnetic processes, such as deeply virtual Compton scattering (DVCS), extracting the Generalized Parton Distributions (GPDs) (see Ref. [20] for recent results). The information encoded in DPS, dPDFs and in σ_{eff}, in its full x_i dependence, are anyway different and complementary to those provided by GPDs in impact parameter space. While the latter quantities are one-body densities, depending on the distance of the interacting parton with given x from the transverse center of the target, in DPS one is sensitive to the *relative* distance between two partons with given longitudinal momentum fractions. In other words, the investigation of dPDFs from DPS, is relevant to know, for example, the average transverse distance of two fast partons or two slow partons: a very interesting dynamical feature, not accessible through GPDs.

3. Light-Front quark model calculation of the effective cross section

dPDFs have a non-perturbative nature, and, at present, cannot be calculated in QCD. However they can be explicitly calculated, at a low resolution scale, $Q_0 \sim \Lambda_{QCD}$, using quark models, as extensively done for the usual PDFs. The results of these calculations should be then evolved using perturbative QCD (pQCD) in order to match data taken at a momentum scale $Q > Q_0$. The procedure is nowadays well established (see, e.g., Ref. [21] and the references therein). The QCD evolution of the x_i-dependences of dPDFs (from Q_0 to $Q > Q_0$) is known [22,23], and currently implemented in a systematic way (see Ref. [3,24] and the references therein).

The first model calculations of dPDFs in the valence region, at the hadronic scale Q_0, have been presented in a bag model framework [8], and in a constituent quark model (CQM), [9]. Of course CQM have the specific advantage of including correlations in a way consistent with the quark dynamics, from the very beginning, a property that the bag model cannot fulfill.

In particular the fully Poincaré covariant Light-Front model approach we developed in Ref. [10] respects relevant symmetries, broken in the descriptions of Refs. [8,9], allowing for a correct evaluation of the Mellin moments of the distributions and, consequently, for a precise pQCD evolution to high momentum transfer. In this way our model calculations can be relevant for the analysis of high-energy data. The model, extensively applied to the evaluation of different parton distributions, (see, e.g., Refs. [25–27] and the references therein), is a good candidate to grasp the most relevant features of dPDFs. For the present study it is enough to recall that the proton state is given by a spatial wave function and an SU(6) symmetric spin-isospin part (see Ref. [25] for details). The spatial part is numerical solution of a relativistic Mass equation, dynamically responsible for the presence of correlations between the two quarks in the CQM wave function (a non-relativistic version of the model was introduced in Ref. [28]). The Light-Front calculations of $D_{ij}(x_1, x_2; \mathbf{k}_\perp, \mu)$, in Ref. [10], shows that the factorization of Eq. (6) is basically valid, but the common assumption of Eq. (7) is strongly violated. Besides, the strong correlation effects present at the scale of the model are still sizable, in the valence region, at the experimental scale, i.e. after QCD evolution. At the low values of x, presently studied at the LHC, the correlations become less relevant, although their effects are still important for the spin-dependent contributions to unpolarized proton scattering.

We have explicitly calculated single and double parton distributions entering Eq. (11), and then σ_{eff} relying on the natural assumption Eq. (10) only. We adhere, in addition, to the simplifying choice of a single factorization scale $\mu_1 = \mu_2 = \mu_0$, used in, e.g., Refs. [2,4,29]. μ_0 has to be interpreted, in the present approach, as the hadronic scale, where only valence quarks u and d are present. Considering the symmetries of our model, one has $u(x, \mu_0) = 2d(x, \mu_0)$, $D_{uu}(x_1, x_2, k_\perp, \mu_0) = 2D_{ud}(x_1, x_2, k_\perp, \mu_0)$ and Eq. (11) simplifies to

$$\sigma_{eff}(x_1, x_1', x_2, x_2', \mu_0) = \frac{81 u(x_1, \mu_0)\, u(x_1', \mu_0)\, u(x_2, \mu_0)\, u(x_2', \mu_0)}{64 \int D_{uu}(x_1, x_2; \mathbf{k}_\perp, \mu_0)\, D_{uu}(x_1', x_2'; -\mathbf{k}_\perp, \mu_0)\, \frac{d\mathbf{k}_\perp}{(2\pi)^2}}. \quad (12)$$

In principle, σ_{eff} depends therefore on four momentum fractions. For the seek of convenience, in order to clearly show the main features of our results, σ_{eff} has been evaluated at zero rapidity ($y = 0$), where $x_i = x_i'$, so that one remains with

$$\sigma_{eff}(x_1, x_2, \mu_0) = \frac{81 u(x_1, \mu_0)^2 u(x_2, \mu_0)^2}{64 \int D_{uu}(x_1, x_2; \mathbf{k}_\perp, \mu_0)^2 \frac{d\mathbf{k}_\perp}{(2\pi)^2}}. \quad (13)$$

In order to illustrate our results we will concentrate on the valence region where the present model is more predictive. In particular we concentrate on the kinematics of the old AFS data [11], which means $y \simeq 0$ ($x_1 \simeq x_1'$, $x_2 \simeq x_2'$) and $0.2 \le x_{1,2} \le 0.3$. The average momentum scale, again assumed to be the same for the processes initiated by the two different collisions, turns out to be $Q^2 \simeq 250$ GeV^2. The results of the calculations are shown in Fig. 2, at the scale of the model, $\mu_0^2 \simeq 0.1$ GeV^2, and after non-singlet evolution to Q^2 (details on the fixing of the hadronic scale and on the calculation of the QCD evolution can be found in Ref. [10]).

What is immediately seen is an $x_{1,2}$ dependence of the results, which change up to 100% even in this narrow kinematical range. Such a dependence is found at both the experimental and the model scale. The slope of the surface in the right panel of Fig. 2 is inverted w.r.t. that in the left panel. It is not a surprising feature, due to the different evolution properties of the numerator and denominator in Eq. (12) and consistent with the evolution calculated in Ref. [10].

It is worth to notice that the three old experimental extractions of σ_{eff} from data [11–13], which involve the valence region (cf. Fig. 1), lie in the obtained range of values of σ_{eff}, shown in Fig. 2. It is important to remark that, since the dPDF evolution on $k_\perp$ is still an open challenge, only the $x_{1,2}$ evolution at fixed $k_\perp$ has been taken into account in evolving the model effective cross section to $Q^2 \simeq 250$ GeV^2. Moreover, we show our results, for the moment being, only in a region dominated by valence quarks, where the non-singlet sector of the evolution equations plays a major role. We have solved therefore only this sector, where the inhomogeneous part of the evolution equations, describing the possibility that the quarks belonging to the same proton and participating to the interaction are originated from the same parton, has no effect. At low x, in processes initiated by gluons, this contribution, sometimes called in the literature, for brevity, "1v2", has been found to give an important contribution [18,33], although some doubts on its actual impact have been raised recently in Ref. [34].

This x dependence, found at the hadronic scale as well as at high Q^2, can be attributed to different dynamical and kinematical properties:

1. both the numerator and the denominator vanish quickly with x_i through the valence region, but the latter vanishes faster, mainly due to the kinematic constraint $x_1 + x_2 \le 1$ of the dPDF, a quantity appearing only in the denominator. The LF model correctly reproduce such a kinematical constraint;
2. the correlations introduced by the LF dynamics and effective both in the x_i and $k_\perp$ dependence;

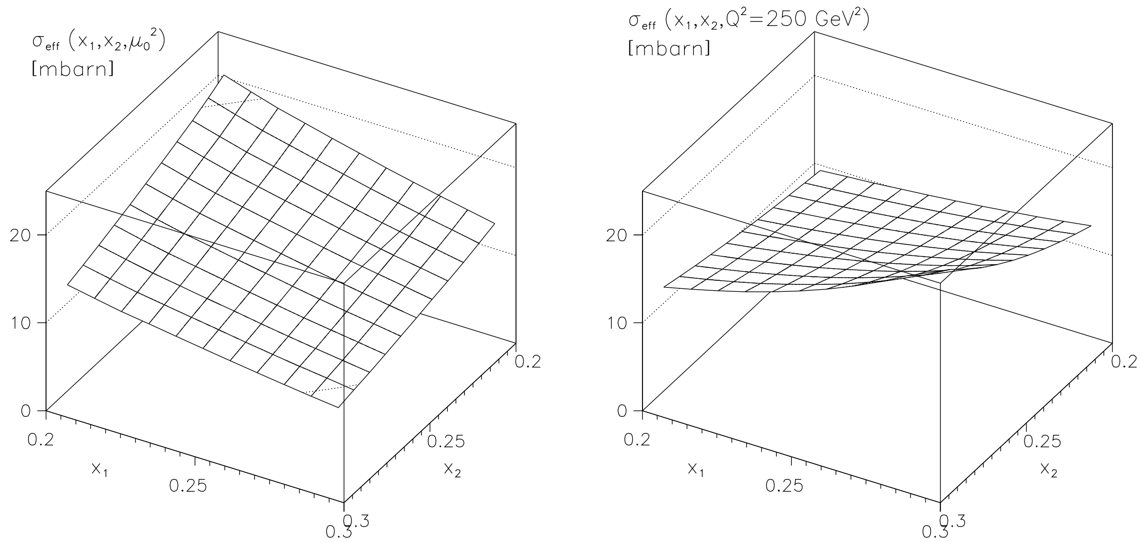

Fig. 2. $\sigma_{eff}(x_1, x_2, Q^2)$ for the values of x_1, x_2 measured in Ref. [11]. Left panel: hadronic scale; right panel: $Q^2 = 250$ GeV^2.

3. the correlations induced by the pQCD evolution in the valence region.

Let us compare our predictions for σ_{eff} to other estimates, mostly performed in a very far kinematical regime. At very low-x gluons are strongly dominating (this is the hypothesis in [17], partially corrected in [18]), so that it is enough to consider $i, j, k, l = g$ in Eq. (11). Assuming, in addition, a fully factorized approach, $D_{gg}(x, x'; \mathbf{k}_\perp) = F_g(x)F_g(x')g(\mathbf{k}_\perp)$, σ_{eff} becomes:

$$\sigma_{eff}(x_1, x_1', x_2, x_2') \to \sigma_{eff} = \frac{1}{\int g^2(\mathbf{k}_\perp)\frac{d\mathbf{k}_\perp}{(2\pi)^2}}. \tag{14}$$

An equivalent scheme is used in Ref. [17], relying on an uncorrelated ansatz in coordinate space, leading, in momentum space, to a dPDF given by the product of two gluon GPDs at zero "plus" momentum transfer.[1] Writing in turn each GPD in an x, $t = -k_\perp^2$ factorized form, σ_{eff} is obtained to be about twice the experimental value. Later, the same authors have corrected this prediction taking into account a possible contribution coming from interacting partons originated by the same parton. This "$1v2$" mechanism was found to be able to reconcile to a large extent the prediction with the measured values [18]. Obviously, the validity of Eq. (14) is spoiled by correlation effects and restricted to very low-x_i. The problems related to the uncorrelated ansatz are discussed in a number of papers (see, e.g., Ref. [2,30–32]). In particular, in the valence region we are discussing here, this assumption is not supported by model calculations [8–10] and it is certainly untrue in pQCD, being also spoiled by QCD evolution. In other words, several arguments lead to the conclusion that, in general, σ_{eff} should be x_i dependent, namely: breaking of the factorization ansatz; the QCD evolution; contribution of more than one parton type (not only gluons as at very low x_i) to the DPS cross section.

None of the assumptions leading to Eq. (14), valid at very low x, is reliable in our calculation. Nevertheless, to obtain a single number to be qualitatively compared with the present x independent values which have been reported by the experimental collaborations so far, one can try to reduce the results of our calculation to a fully factorized approach to dPDFs, following the hypothesis often assumed (cf. Eqs. (6), (7)):

$$D_{uu}(x_1, x_2; \mathbf{k}_\perp, \mu_0) = u(x_1, \mu_0)u(x_2, \mu_0)f_{uu}(\mathbf{k}_\perp), \tag{15}$$

where a natural definition for the "effective form factor", $f_{uu}(\mathbf{k}_\perp)$, in our approach, is

$$f_{uu}(\mathbf{k}_\perp) = \frac{1}{4}\int dx_1 dx_2 D_{uu}(x_1, x_2; \mathbf{k}_\perp, \mu_0), \tag{16}$$

a quantity which turns out to be scale independent. Within this approximation, Eq. (12) yields:

$$\sigma_{eff}(x_1, x_1', x_2, x_2', \mu_0) \to \sigma_{eff} = \frac{81}{64\int f_{uu}^2(\mathbf{k}_\perp)\frac{d\mathbf{k}_\perp}{(2\pi)^2}} \simeq 10.9\,mb, \tag{17}$$

a value which turns out to be independent on the momentum scale Q and on the longitudinal momentum fractions x_i, x_i', and compares reasonably well with the sets of data shown in Fig. 1, in particular with those of the three oldest experiments, at lower values of $\sqrt{s}$, strongly involving the valence region. There is no reason why the LHC experimental scenario should be described by our model, which is, for the moment being, a valence quark one. This simplified result, restricted to the valence region, is connected to the ability of the model to capture (in its wave function) the correct average distance between the valence quarks in transverse space, related to the "effective form factor", $f_{uu}(\mathbf{k}_\perp)$, Eq. (16), which can be written also as follows:

$$\begin{aligned} f_{uu}(\mathbf{k}_\perp) &= 3\,\langle\Psi(\mathbf{k_1}, \sigma_1, \tau_1; \mathbf{k_2} + \mathbf{k}_\perp, \sigma_2, \tau_2; \mathbf{k_3}, \sigma_3, \tau_3)|\,\delta(\mathbf{k_1} + \mathbf{k_2} + \mathbf{k_3}) \\ &\quad \times P_u(1)P_u(2)\,|\Psi(\mathbf{k_1} + \mathbf{k}_\perp, \sigma_1, \tau_1; \mathbf{k_2}, \sigma_2, \tau_2; \mathbf{k_3}, \sigma_3, \tau_3)\rangle, \end{aligned} \tag{18}$$

using directly the proton state $|\Psi\rangle$ and the u-flavor projector for the particle i, $P_u(i) = (1 + \tau_z(i))/2$. Looking at Eq. (18), one can notice that, in the ansatz given by Eqs. (15) and (16), the model correlations described by the dPDF are still in part effectively present through $f_{uu}(\mathbf{k}_\perp)$.

The x-dependence we are discussing does not emerge from the present data, probably not accurate enough. Our study points out therefore to an experimental scenario where more precise mea-

[1] For a generic four momentum a^μ, we use the following notation for the "plus" ("minus") light cone component: $a^+ = (a^0 + a^3)/\sqrt{2}$ ($a^- = (a^0 - a^3)/\sqrt{2}$).

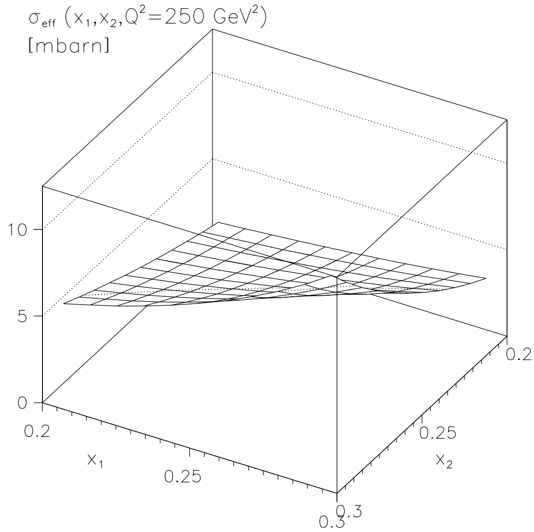

Fig. 3. $\sigma_{eff}(x_1, x_2, Q^2)$ for the values of x_1, x_2 measured in Ref. [11], at $Q^2 = 250$ GeV^2, considering the contribution of gluons perturbatively generated.

surements in narrow x_i regions could shed new light on the structure of the proton and on the nature of hard proton–proton collisions. If the x-dependence is seen, one will gain, through σ_{eff}, a first indication of double parton correlations and a fresh look at the 3D proton structure.

Our treatment so far has been developed considering only valence quarks, and one could wonder whether, in the valence region, the inclusion of other degrees of freedom (sea quarks, gluons), possibly relevant in specific channels, could change this important x_i dependence. For the moment being, we are able to show, in Fig. 3, in the same valence window of Fig. 2 and for an illustrative purpose only, the effective cross section as result of a singlet evolution which takes into account the contribution of perturbatively generated gluons, at $Q^2 = 250$ GeV^2. This amounts at calculating, according to Eq. (11), the following expression:

$$\sigma_{eff}(x_1, x_2, \mu) = N(x_1, x_2, \mu)/D(x_1, x_2, \mu), \tag{19}$$

where

$$\begin{aligned} N(x_1, x_2, \mu) = {} & C_{gg}^2 g_1^2 g_2^2 + 3C_{gg}C_{qg}[u_1 g_1 g_2^2 + u_2 g_2 g_1^2] \\ & + \frac{9}{4} C_{gg} C_{qq}[u_1^2 g_2^2 + u_2^2 g_1^2] \\ & + \frac{27}{4} C_{gq} C_{qq}[u_1^2 u_2 g_2 + u_2^2 u_1 g_1] \\ & + 9C_{gq}^2 u_1 u_2 g_1 g_2 + \frac{81}{16} C_{qq}^2 u_1^2 u_2^2, \end{aligned} \tag{20}$$

and

$$\begin{aligned} D(x_1, x_2, \mu) = \int \frac{d\mathbf{k}_\perp}{(2\pi)^2} & [C_{gg}^2 D_{gg}^2 + 8C_{gg}C_{gq}D_{gg}D_{gu} + 8C_{gq}^2 D_{gu}^2 \\ & + 4C_{gq}^2 D_{gg} D_{uu} + 8C_{gg}C_{qq}D_{gu}^2 \\ & + 16C_{gq}C_{qq}D_{gu}D_{uu} + 4C_{qq}^2 D_{uu}^2]. \end{aligned} \tag{21}$$

In Eqs. (20) and (21) we have used, for brevity, the notation $u_i = u(x_i, \mu)$ and $D_{ij} = D_{ij}(x_i, x_j; k_\perp, \mu)$. Looking at these equations, one should notice that: *i*) if the gluon contributions are not inserted, Eq. (13) is recovered from Eq. (19); *ii*) use has been made of the symmetries of the model, given above Eq. (12), for the quark sector; *iii*) the possible contribution of the sea quarks in this kinematical window in specific channels has not been included. Indeed, with respect to the gluon one, the latter will be suppressed by the color factor (cf. Eq. (10)) and it has been neglected for the sake of clarity. As it is easily seen in Fig. 3, the new result does not differ dramatically from that shown in Fig. 2, right panel. In particular, the x_i dependence, the most important feature of the results of the present investigation, is found to be very similar, while a significant change in size is seen, up to a factor of two in some regions. However, we observe that the "$1v2$" contribution is not included, neither at the perturbative nor at the non perturbative level, and could have, according to some studies, an important impact, of the same order of the calculated effective cross section [18,33]. The study of its possible role, as well as the analysis of the low x region, are beyond the scope of the present paper and will be treated elsewhere.

In closing this section, let us qualitatively address the experimental scenarios where our prediction, valid for the moment only in the valence region, can be tested. The present run of the LHC is very promising for the detection of DPS but only the low x region will be accessed. It appears therefore difficult to detect a DPS event generated by four valence quarks, as the one we are addressing here. Nonetheless, valence quarks contributing to a DPS process together with sea quarks or gluons have a chance to be observed. These contributions could be selected looking at jets of different rapidity, so that the x_1, x'_1 and/or x_2, x'_2 values lie in different regions, namely, one in the valence region and the other at lower longitudinal momentum. The study of the dPDFs associated to these processes would require the inclusion of perturbative and non-perturbative gluon and sea contributions in our analysis, and the check of a possible role of the "$1v2$" processes. This will be treated elsewhere. Concerning the possibility to observe our predicted x dependence of the effective cross section for valence quarks, hardly accessible at the LHC, some information could be got, in principle, inspecting carefully the old Tevatron data. The latter have indeed been taken at typical values of $\sqrt{s}$ and looking at observed final states which enter deeply the valence region.

4. Conclusions

An operative expression of σ_{eff} has been obtained in terms of dPDFs and standard PDFs. Thanks to this new result, direct microscopic calculations of σ_{eff} can be performed in different scenarios and kinematical frames. In particular, in the present analysis, we have calculated the effective cross section σ_{eff} within a relativistic Poincaré covariant quark model. Extracted from proton–proton scattering data by several experimental collaborations in the last 30 years, σ_{eff} represents a tool to understand double parton scattering in a p–p collision. Our investigation predicts a behavior of σ_{eff} which, when averaged over the longitudinal momentum fractions x_i, is consistent with the present experimental scenario, in particular with the sets of data which include the valence region. However, at the same time, an x_i dependence of σ_{eff} is found, a feature not easily read in the available data. This dependence is found when a non singlet evolution of the valence distributions is performed, as well as when perturbatively generated gluons are included into the scheme.

We conclude that the measurement of σ_{eff} in restricted x_i ranges would lead to a first indication of double parton correlations in the proton, addressing a novel and interesting aspect of the 3D structure of the nucleon. The analysis of particular processes where these effects could be most easily seen, as well as the extension of the model to obtain a better description of the low-x region, presently studied at LHC, are in progress.

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