

## APPENDIX A: THE SURFACES

In this section we describe all the surfaces listed in Table 1, 2 and 3

$K_S^2$	Sing( $X$ )	Sign.	$G^0$	$G$	$H_1(S, \mathbb{Z})$	$\pi_1(S)$	Label
1	$2C_{2,1}, 2D_{2,1}$	$2^3, 4$	$D_4 \times \mathbb{Z}_2$	$\mathbb{Z}_2^3 \rtimes \mathbb{Z}_4$	$\mathbb{Z}_4$	$\mathbb{Z}_4$	1.1
2	$6C_{2,1}$	$2^5$	$\mathbb{Z}_2^3$	$\mathbb{Z}_2^2 \rtimes \mathbb{Z}_4$	$\mathbb{Z}_2 \times \mathbb{Z}_4$	$\mathbb{Z}_2 \times \mathbb{Z}_4$	1.2
2	$6C_{2,1}$	$4^3$	$(\mathbb{Z}_2 \times \mathbb{Z}_4) \rtimes \mathbb{Z}_4$	$G(64, 82)$	$\mathbb{Z}_2^3$	$\mathbb{Z}_2^3$	1.3
2	$C_{2,1}, 2D_{2,1}$	$2^3, 4$	$\mathbb{Z}_2^4 \times \mathbb{Z}_2$	$\mathbb{Z}_2^4 \rtimes \mathbb{Z}_4$	$\mathbb{Z}_4$	$\mathbb{Z}_4$	1.4
2	$C_{2,1}, 2D_{2,1}$	$2^2, 3^2$	$\mathbb{Z}_3^2 \times \mathbb{Z}_2$	$\mathbb{Z}_3^2 \rtimes \mathbb{Z}_4$	$\mathbb{Z}_3$	$\mathbb{Z}_3$	1.5
2	$2C_{4,1}, 3C_{2,1}$	$2^3, 4$	$G(64, 73)$	$G(128, 1535)$	$\mathbb{Z}_2^3$	$\mathbb{Z}_2^3$	1.6
2	$2C_{3,1}, 2C_{3,2}$	$3^2, 4$	$G(384, 4)$	$G(768, 1083540)$	$\mathbb{Z}_4$	$\mathbb{Z}_4$	1.7
2	$2C_{3,1}, 2C_{3,2}$	$3^2, 4$	$G(384, 4)$	$G(768, 1083541)$	$\mathbb{Z}_2^2$	$\mathbb{Z}_2^2$	1.8
3	$C_{8,3}, C_{8,5}$	$2^3, 8$	$G(32, 39)$	$G(64, 42)$	$\mathbb{Z}_2 \times \mathbb{Z}_4$	$\mathbb{Z}_2 \times \mathbb{Z}_4$	1.9
4	$4C_{2,1}$	$2^5$	$D_4 \times \mathbb{Z}_2$	$D_{2,8,5} \rtimes \mathbb{Z}_2$	$\mathbb{Z}_2 \times \mathbb{Z}_8$	$\mathbb{Z}_2^2 \times \mathbb{Z}_8$	1.10
4	$4C_{2,1}$	$2^5$	$\mathbb{Z}_2^4$	$(\mathbb{Z}_2^2 \rtimes \mathbb{Z}_4) \times \mathbb{Z}_2$	$\mathbb{Z}_2^3 \times \mathbb{Z}_4$	$\infty$	1.11
4	$4C_{2,1}$	$4^3$	$G(64, 23)$	$G(128, 836)$	$\mathbb{Z}_2^3$	$\mathbb{Z}_4^2 \rtimes \mathbb{Z}_2$	1.12
8	$\emptyset$	$2^5$	$D_4 \times \mathbb{Z}_2^2$	$(D_{2,8,5} \rtimes \mathbb{Z}_2) \times \mathbb{Z}_2$	$\mathbb{Z}_2^3 \times \mathbb{Z}_8$	$\infty$	1.13
8	$\emptyset$	$4^3$	$G(128, 36)$	$G(256, 3678)$	$\mathbb{Z}_4^3$	$\infty$	1.14
8	$\emptyset$	$4^3$	$G(128, 36)$	$G(256, 3678)$	$\mathbb{Z}_2^4 \times \mathbb{Z}_4$	$\infty$	1.15
8	$\emptyset$	$4^3$	$G(128, 36)$	$G(256, 3678)$	$\mathbb{Z}_2^2 \times \mathbb{Z}_4^2$	$\infty$	1.16
8	$\emptyset$	$4^3$	$G(128, 36)$	$G(256, 3679)$	$\mathbb{Z}_2^2 \times \mathbb{Z}_4^2$	$\infty$	1.17

TABLE 1.  $p_g = q = 0$

$K_S^2$	$g_{alb}$	$\text{Sing}(X)$	Sign.	$G^0$	$G$	$H_1(S, \mathbb{Z})$	Label
2	2	$C_{2,1}, 2D_{2,1}$	$2^2$	$\mathbb{Z}_2$	$\mathbb{Z}_4$	$\mathbb{Z}^2$	2.1
2	2	$C_{2,1}, 2D_{2,1}$	2	$D_8$	$D_{2,8,3}$	$\mathbb{Z}^2$	2.2
2	2	$C_{2,1}, 2D_{2,1}$	2	$Q_8$	$BD_4$	$\mathbb{Z}^2$	2.3
4	3	$4C_{2,1}$	$2^2$	$\mathbb{Z}_4$	$\mathbb{Z}_8$	$\mathbb{Z}_2 \times \mathbb{Z}^2$	2.4
4	3	$4C_{2,1}$	$2^2$	$\mathbb{Z}_2 \times \mathbb{Z}_2$	$\mathbb{Z}_2 \times \mathbb{Z}_4$	$\mathbb{Z}_2 \times \mathbb{Z}^2$	2.5
4	2	$4C_{2,1}$	2	$\mathbb{Z}_2^2 \rtimes \mathbb{Z}_4$	$G(32,29)$	$\mathbb{Z}_2^2 \times \mathbb{Z}^2$	2.6
4	3	$4C_{2,1}$	2	$D_{4,4,3}$	$D_{4,8,3}$	$\mathbb{Z}_2 \times \mathbb{Z}^2$	2.7
4	3	$4C_{2,1}$	2	$D_{4,4,3}$	$D_{4,8,7}$	$\mathbb{Z}_2 \times \mathbb{Z}^2$	2.8
4	2	$4C_{2,1}$	2	$D_{4,4,3}$	$G(32,32)$	$\mathbb{Z}_2^2 \times \mathbb{Z}^2$	2.9
4	2	$4C_{2,1}$	2	$D_{4,4,3}$	$G(32,35)$	$\mathbb{Z}_2^2 \times \mathbb{Z}^2$	2.10
4	3	$4C_{2,1}$	2	$D_{2,8,5}$	$G(32,15)$	$\mathbb{Z}_2 \times \mathbb{Z}^2$	2.11
5	3	$C_{3,1}, C_{3,2}$	3	$BD_3$	$BD_6$	$\mathbb{Z}_2 \times \mathbb{Z}^2$	2.12
5	3	$C_{3,1}, C_{3,2}$	3	$D_6$	$D_{2,12,5}$	$\mathbb{Z}_2^2 \times \mathbb{Z}^2$	2.13
6	3	$2C_{2,1}$	2	$A_4 \times \mathbb{Z}_2$	$A_4 \rtimes \mathbb{Z}_4$	$\mathbb{Z}_2 \times \mathbb{Z}^2$	2.14
6	7	$2C_{2,1}$	2	$A_4 \times \mathbb{Z}_2$	$A_4 \times \mathbb{Z}_4$	$\mathbb{Z}_2 \times \mathbb{Z}^2$	2.15
6	5	$C_{5,3}$	5	$D_5$	$\mathbb{Z}_5 \rtimes \mathbb{Z}_4$	$\mathbb{Z}_2 \times \mathbb{Z}^2$	2.16
8	5	$\emptyset$	$2^2$	$\mathbb{Z}_2 \times \mathbb{Z}_4$	$D_{2,8,5}$	$\mathbb{Z}_4 \times \mathbb{Z}^2$	2.17
8	5	$\emptyset$	$2^2$	$D_4$	$D_{2,8,3}$	$\mathbb{Z}_4 \times \mathbb{Z}^2$	2.18
8	5	$\emptyset$	$2^2$	$\mathbb{Z}_2^3$	$\mathbb{Z}_2^2 \rtimes \mathbb{Z}_4$	$\mathbb{Z}_2^3 \times \mathbb{Z}^2$	2.19

TABLE 2.  $p_g = q = 1$ 

$K_S^2$	$\text{Sing}(X)$	Sign.	$G^0$	$G$	$H_1(S, \mathbb{Z})$	Label
8	$\emptyset$	-	$\mathbb{Z}_2$	$\mathbb{Z}_4$	$\mathbb{Z}_2 \times \mathbb{Z}^4$	3.1

TABLE 3.  $p_g = q = 2$ 

We will follow the scheme below:

$G$ : the Galois group.

$G^0$ : the index 2 subgroup of the elements that do not exchange the factors.

$T$ : the type of the generating vector  $(m_1, \dots, m_r)$ .

$L$ : here we list the set of elements of  $G$  that is a generating vector for  $G^0$  that gives the curve  $C$ :  $(a_1, b_1, \dots, a_q, b_q, c_1, \dots, c_r)$ .

$H_1$ : the first homology group of the surface.

$\pi_1$ : the fundamental group of the surface (only if  $q \neq 0$ ).

1.  $p_g = q = 0$

$K^2 = 1$ , **Basket**  $\{2 \times C_{2,1}, 2 \times D_{2,1}\}$ .

**1.1. Galois group  $G(32,6)$ :**  $(\mathbb{Z}_2)^3 \rtimes_{\varphi} \mathbb{Z}_4 : \varphi(1) = \begin{pmatrix} 1 & 1 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{pmatrix}$

$G$ :  $\langle (2, 5, 6, 8)(3, 7), (1, 2)(3, 5)(4, 6)(7, 8), (1, 3)(2, 5)(4, 7)(6, 8), (2, 6)(5, 8), (1, 4)(2, 6)(3, 7)(5, 8) \rangle < \mathfrak{S}_8$

$G^0$ :  $G(16,11)$ :  $D_4 \times \mathbb{Z}_2$

$T$ :  $(2, 2, 2, 4)$

L:  $(1, 8)(2, 7)(3, 6)(4, 5)$ ,  $(1, 7)(2, 8)(3, 4)(5, 6)$ ,  $(1, 3)(2, 8)(4, 7)(5, 6)$ ,  
 $(1, 5, 4, 8)(2, 7, 6, 3)$   
 $H_1: \mathbb{Z}_4$   
 $\pi_1: \mathbb{Z}_4$

$K^2 = 2$ , **Basket**  $\{6 \times C_{2,1}\}$ .

**1.2. Galois group  $\mathbf{G(16,3)}$ :**  $(\mathbb{Z}_2)^2 \rtimes_{\varphi} \mathbb{Z}_4$ :  $\varphi(1) = \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}$

G:  $\langle (1, 2, 4, 6)(3, 5, 7, 8), (2, 5)(6, 8), (1, 3)(2, 5)(4, 7)(6, 8),$   
 $(1, 4)(2, 6)(3, 7)(5, 8) \rangle < \mathfrak{S}_8$

$G^0: G(8,5): (\mathbb{Z}_2)^3$

T:  $(2, 2, 2, 2, 2)$

L:  $(1, 3)(4, 7)$ ,  $(1, 7)(2, 6)(3, 4)(5, 8)$ ,  $(1, 3)(2, 5)(4, 7)(6, 8)$ ,  $(2, 5)(6, 8)$ ,  
 $(1, 7)(2, 6)(3, 4)(5, 8)$

$H_1: \mathbb{Z}_2 \times \mathbb{Z}_4$

$\pi_1: \mathbb{Z}_2 \times \mathbb{Z}_4$

**1.3. Galois group  $\mathbf{G(64,82)}$ :** Sylow 2-subgroup of the Suzuki group  $Sz(8)$ ,

G:  $\langle g_1, g_2, g_3 \mid g_3^4, g_2^4, g_1^4, g_1 g_3 g_1^{-1} g_3 g_2^2, g_2^{-2} g_3^{-1} g_1^{-1} g_3^{-1} g_1,$   
 $g_2 g_3 g_1^2 g_2 g_3^{-1}, g_1^{-1} g_3^2 g_2 g_1 g_2^{-1}, g_2^{-1} g_3^2 g_2 g_3^2, g_1^{-2} g_3^{-1} g_2 g_3 g_2 \rangle$

$G^0: G(32,2): (\mathbb{Z}_2 \times \mathbb{Z}_4) \rtimes_{\varphi} \mathbb{Z}_4$  where  $\varphi(1) = \begin{pmatrix} 1 & 1 \\ 0 & 3 \end{pmatrix}$

T:  $(4, 4, 4)$

L:  $g_3^{-1}, g_1 g_3^{-2}, g_1 g_3 g_2^{-2} g_3^2 g_2^{-2} g_1^{-2}$

$H_1: (\mathbb{Z}_2)^3$

$\pi_1: (\mathbb{Z}_2)^3$

$K^2 = 2$ , **Basket**  $\{C_{2,1}, 2 \times D_{2,1}\}$ .

**1.4. Galois group  $\mathbf{G(64,32)}$ :**  $(\mathbb{Z}_2)^4 \rtimes_{\varphi} \mathbb{Z}_4$ :  $\varphi(1) = \begin{pmatrix} 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$

G:  $\langle (2, 6, 7, 12)(3, 9, 10, 16)(4, 11)(8, 14, 15, 13),$   
 $(1, 2)(3, 6)(4, 7)(5, 8)(9, 13)(10, 14)(11, 15)(12, 16),$   
 $(1, 3)(2, 6)(4, 9)(5, 10)(7, 13)(8, 14)(11, 16)(12, 15),$   
 $(2, 7)(3, 10)(6, 12)(8, 15)(9, 16)(13, 14),$   
 $(1, 4)(2, 7)(3, 9)(5, 11)(6, 13)(8, 15)(10, 16)(12, 14),$   
 $(1, 5)(2, 8)(3, 10)(4, 11)(6, 14)(7, 15)(9, 16)(12, 13) \rangle < \mathfrak{S}_{16}$

$G^0: G(32,27): (\mathbb{Z}_2)^4 \rtimes_{\psi} \mathbb{Z}_2$ ,  $\psi(1) = \varphi(2)$

T:  $(2, 2, 2, 4)$

L:  $(2, 7)(3, 10)(6, 12)(8, 15)(9, 16)(13, 14),$   
 $(1, 16)(2, 12)(3, 11)(4, 10)(5, 9)(6, 15)(7, 14)(8, 13),$   
 $(1, 14)(2, 10)(3, 8)(4, 12)(5, 6)(7, 16)(9, 15)(11, 13),$

(1, 2, 4, 7)(3, 14, 9, 12)(5, 8, 11, 15)(6, 16, 13, 10)  
 $H_1: \mathbb{Z}_4$   
 $\pi_1: \mathbb{Z}_4$

**1.5. Galois group G(36,9):**  $(\mathbb{Z}_3)^2 \rtimes_{\varphi} \mathbb{Z}_4: \varphi(1) = \begin{pmatrix} 0 & 1 \\ 2 & 0 \end{pmatrix}$

G:  $\langle (1, 2)(3, 4, 5, 6), (3, 5)(4, 6), (2, 4, 6), (1, 3, 5)(2, 4, 6) \rangle < \mathfrak{S}_6$   
 $G^0: G(18,4) : (\mathbb{Z}_3)^2 \rtimes_{\psi} \mathbb{Z}_2, \psi(1) = \varphi(2)$   
T: (2, 2, 3, 3)  
L: (3, 5)(4, 6), (2, 6)(3, 5), (1, 3, 5), (1, 5, 3)(2, 4, 6)  
 $H_1: \mathbb{Z}_3$   
 $\pi_1: \mathbb{Z}_3$

$K^2 = 2$ , Basket  $\{2 \times C_{4,1}, 3 \times C_{2,1}\}$ .

**1.6. Galois group G(128,1535):**

G:  $\langle g_1, g_2, g_3, g_4 \mid g_1^{-1}g_4g_1g_4, g_4^4, (g_2^{-1}g_3^{-1})^2, g_2^4, (g_3, g_4^{-1}), (g_3^{-1}g_2)^2, g_2^{-1}g_4g_2^{-1}g_4^{-1}, g_1^{-1}g_2^{-1}g_1g_2^{-1}, g_1^{-1}g_3^{-1}g_1^2g_3g_1^{-1}, g_3^{-2}g_1g_3^2g_1^{-1}, g_4^{-2}g_1g_3g_2^2g_1^{-1}g_3^{-1}, g_4^{-2}g_3^{-1}g_1g_3g_1^{-1}g_2^2, g_4^2g_1^{-2}g_3^{-1}g_2^2g_3^{-1}, g_4^{-1}g_1^{-1}g_2g_3g_2^{-1}g_4g_3^{-1}g_1^{-1}, g_4^{-2}g_1^3g_4^{-2}g_1 \rangle$   
 $G^0: G(64, 73): \langle h_1, h_2, h_3 \mid h_1^2, h_2^2, h_3^2, (h_1h_3)^4, (h_1h_2)^4, (h_2h_3)^4, (h_2h_3h_2h_1h_3)^2, (h_1h_2h_3h_1h_3)^2, (h_2h_1h_3)^4 \rangle$   
T: (2, 2, 2, 4)  
L:  $g_1g_3g_4^{-1}g_2^2, g_1g_3g_2^{-2}g_3^{-2}g_2^2, g_2g_3, g_2g_3g_4g_2^{-2}g_4^{-2}g_2^2g_3^{-2}g_2^2$   
 $H_1: (\mathbb{Z}_2)^3$   
 $\pi_1: (\mathbb{Z}_2)^3$

$K^2 = 2$ , Basket  $\{2 \times C_{3,1}, 2 \times C_{3,2}\}$ .

**1.7. Galois group G(768,1083540):**

G:  $\langle g_1, g_2, g_3, g_4, g_5, g_6, g_7, g_8, g_9 \mid g_1^3, g_2^2(g_5g_6g_7)^{-1}, g_3^2(g_5g_6), g_4^2(g_5)^{-1}, g_5^2, g_6^2, g_7^2, g_8^2, g_9^2, (g_2, g_1)(g_4g_6g_7g_9)^{-1}, (g_3, g_1)(g_3g_7g_9)^{-1}, (g_3, g_2)g_5^{-1}, (g_4, g_1)(g_8g_9)^{-1}, (g_4, g_2)g_6^{-1}, (g_4, g_3)g_7^{-1}, (g_5, g_1)(g_6g_7)^{-1}, (g_5, g_2)g_8^{-1}, (g_5, g_3)g_9^{-1}, (g_6, g_1)g_8^{-1}, (g_6, g_2) = g_8g_9, (g_6, g_3)g_9^{-1}, (g_6, g_4)g_8^{-1}, (g_7, g_1)g_9^{-1}, (g_7, g_2)g_9^{-1}, (g_7, g_3)g_8^{-1}, (g_7, g_4)g_9^{-1}, (g_8, g_1)g_9^{-1}, (g_9, g_1) \rangle$   
 $G^0: G(384, 4): \langle h_1, h_2 \mid h_1^3, h_2^4, (h_2^{-1}h_1)^3, (h_2^{-1}h_1^{-1})^6, (h_2, h_1)^4, h_1^{-1}h_2^{-2}h_1h_2^{-2}h_1^{-1}h_2^{-1}h_1^{-1}h_2h_1^{-1}h_2^{-1}, h_2^{-1}h_1h_2h_1h_2^{-1}h_1^{-1}h_2h_1h_2h_1^{-1}h_2^{-1}h_1^{-1}h_2h_1h_2^{-1}h_1^{-1} \rangle$   
T: (3, 3, 4)  
L:  $g_1^2g_4g_9, g_1g_6g_7g_9, g_2g_5g_8$   
 $H_1: \mathbb{Z}_4$   
 $\pi_1: \mathbb{Z}_4$

**1.8. Galois group G(768,1083541):**

G:  $\langle g_1, g_2, g_3 \mid g_1^3, g_3^4, g_2^4, g_2g_3g_1g_2^{-1}g_1^{-1}g_3, g_3^2g_2^2g_3^{-2}g_2^2, g_3g_2^{-1}g_3g_1^{-1}g_2g_3^{-2}g_1, g_1^{-1}g_3^{-2}g_1g_2g_3g_2^{-1}g_3^{-1} \rangle$

$$\begin{aligned}
& g_2 g_3 g_2^{-1} g_1 g_2 g_1^{-1} g_2^{-1} g_3, g_3 g_2^2 g_3 g_1^{-1} g_2 g_1 g_2, (g_3^{-1} g_2^{-1} g_3 g_2^{-1})^2, \\
& g_2^{-1} g_3^{-1} g_1^{-1} g_3^2 g_1 g_2 g_3, (g_3^{-1} g_2)^4, g_3 g_1 g_2^{-2} g_3^{-1} g_2^{-1} g_3^{-1} g_1^{-1} g_2^{-1} g_3, \\
& g_3 g_2^2 g_3^{-1} g_1^{-1} g_3 g_2^{-2} g_3^{-1} g_1, g_2 g_1^{-1} g_2 g_1 g_3^{-1} g_2 g_3 g_1 g_2 g_1^{-1}, \\
& g_3^{-1} g_2^2 g_1^{-1} g_3^{-1} g_1 g_3^{-1} g_1^{-1} g_3 g_1, g_3^{-1} g_2 g_3^2 g_2 g_1^{-1} g_2^2 g_1 g_3^{-1}, \\
& g_1^{-1} g_2 g_3^{-1} g_2 g_3^{-1} g_1 g_3^{-2} g_2^2, g_3 g_1^{-1} g_3 g_1 g_3^{-1} g_2^{-2} g_3 g_2^{-1} g_1 g_3 g_1^{-1}, \\
& g_2^{-1} g_1^{-1} g_2 g_3^{-1} g_1 g_3^{-1} g_1^{-1} g_3 g_2^{-2} g_1 g_3, \\
& g_3^{-1} g_1^{-1} g_2^{-1} g_3^{-1} g_2^{-1} g_1 g_3 g_2^{-1} g_3^{-1} g_1 g_3^{-2} g_2 g_3^{-1} g_1^{-1} g_2^{-1}
\end{aligned}$$

$G^0$ :  $G(384, 4)$ , as above.  
 $T$ :  $(3, 3, 4)$   
 $L$ :  $g_1^2 g_2 g_3 g_2^{-2} g_3 g_2^2 g_3 g_2^2 g_1 g_3^{-1} g_2^2 g_3 g_2^2 g_1^{-1}, g_1 g_2^3 g_3 g_2^2 g_1 g_3^{-1} g_2^2 g_3 g_2^2 g_1^{-1},$   
 $g_2 g_3^{-1} g_1^{-1} g_3 g_1 g_2^{-1} g_3^{-2} g_2^{-1} g_3^{-1} g_2^3 g_3 g_2^2$   
 $H_1$ :  $(\mathbb{Z}_2)^2$   
 $\pi_1$ :  $(\mathbb{Z}_2)^2$

$K^2 = 3$ , **Basket**  $\{C_{8,3}, C_{8,5}\}$ .

**1.9. Galois group  $G(64, 42)$ :**

$$\begin{aligned}
& G: \langle (1, 2, 3, 5, 8, 13, 6, 10)(4, 7, 11, 14, 15, 16, 9, 12), \\
& \quad (2, 4)(3, 6)(5, 9)(7, 12)(10, 11)(13, 15)(14, 16) \rangle < \mathfrak{S}_{16} \\
& G^0: G(32, 39): \langle (2, 4)(5, 7)(6, 8)(9, 11)(10, 12)(13, 15), \\
& \quad (1, 2)(3, 5)(4, 6)(7, 9)(8, 10)(11, 13)(12, 14)(15, 16), \\
& \quad (1, 3)(2, 5)(4, 7)(6, 9)(8, 11)(10, 13)(12, 15)(14, 16) \rangle < \mathfrak{S}_{16} \\
& T: (2, 2, 2, 8) \\
& L: (2, 13)(4, 15)(5, 10)(9, 11), \\
& \quad (1, 7)(2, 5)(3, 12)(4, 15)(6, 14)(8, 16)(10, 13), \\
& \quad (2, 15)(3, 6)(4, 13)(5, 11)(7, 12)(9, 10)(14, 16), \\
& \quad (1, 7, 3, 14, 8, 16, 6, 12)(2, 15, 10, 11, 13, 4, 5, 9) \\
& H_1: \mathbb{Z}_2 \times \mathbb{Z}_4 \\
& \pi_1: \mathbb{Z}_2 \times \mathbb{Z}_4
\end{aligned}$$

$K^2 = 4$ , **Basket**  $\{4 \times C_{2,1}\}$ .

**1.10. Galois group  $G(32, 7)$ :**  $D_{2,8,5} \rtimes_{\varphi} \mathbb{Z}_2$ ,  $\varphi(1) = \begin{cases} x \mapsto x \\ y \mapsto yxy^4 \end{cases}$

$$\begin{aligned}
& G: \langle (1, 2, 3, 6, 4, 5, 7, 8), (2, 5)(3, 7), (2, 5)(6, 8), (1, 3, 4, 7)(2, 6, 5, 8), \\
& \quad (1, 4)(2, 5)(3, 7)(6, 8) \rangle < \mathfrak{S}_8 \\
& G^0: G(16, 11):  $D_4 \times \mathbb{Z}_2$  \\
& T: (2, 2, 2, 2, 2) \\
& L: (2, 5)(6, 8), (1, 7)(2, 6)(3, 4)(5, 8), (1, 4)(2, 5), (1, 4)(2, 5), \\
& \quad (1, 7)(2, 8)(3, 4)(5, 6) \\
& H_1: \mathbb{Z}_2 \times \mathbb{Z}_8 \\
& \pi_1: G(32, 5):  $(\mathbb{Z}_2)^2 \rtimes_{\psi} \mathbb{Z}_8$ ,  $\psi(1) = \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}$ 
\end{aligned}$$

**1.11. Galois group  $\mathbf{G(32,22)}$ :**  $((\mathbb{Z}_2)^2 \rtimes_{\varphi} \mathbb{Z}_4) \times \mathbb{Z}_2$ ,  $\varphi(1) = \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}$

G:  $\langle (1, 2, 5, 8)(3, 7, 10, 14)(4, 6, 11, 13)(9, 12, 15, 16),$   
 $(2, 6)(7, 12)(8, 13)(14, 16),$   
 $(1, 3)(2, 7)(4, 9)(5, 10)(6, 12)(8, 14)(11, 15)(13, 16),$   
 $(1, 4)(2, 6)(3, 9)(5, 11)(7, 12)(8, 13)(10, 15)(14, 16),$   
 $(1, 5)(2, 8)(3, 10)(4, 11)(6, 13)(7, 14)(9, 15)(12, 16) \rangle < \mathfrak{S}_{16}$

$G^0: G(16,14): \mathbb{Z}_2^4$

T:  $(2, 2, 2, 2, 2)$

L:  $(1, 5)(2, 13)(3, 10)(4, 11)(6, 8)(7, 16)(9, 15)(12, 14),$   
 $(1, 3)(2, 12)(4, 9)(5, 10)(6, 7)(8, 16)(11, 15)(13, 14),$   
 $(1, 4)(3, 9)(5, 11)(10, 15),$   
 $(1, 10)(2, 16)(3, 5)(4, 15)(6, 14)(7, 13)(8, 12)(9, 11),$   
 $(1, 4)(2, 6)(3, 9)(5, 11)(7, 12)(8, 13)(10, 15)(14, 16)$

$H_1: (\mathbb{Z}_2)^3 \times \mathbb{Z}_4$

$\pi_1: \langle p_1, p_2, p_3, p_4 \mid p_1^2, p_3^2, (p_3 p_2)^2, (p_1 p_2^{-1})^2, p_4 p_2^{-1} p_4^{-1} p_2^{-1},$   
 $p_4 p_1 p_3 p_4^{-1} p_3 p_1, (p_1 p_4^2)^2, (p_4^{-2} p_3)^2 \rangle$

**1.12. Galois group  $\mathbf{G(128,836)}$ :** Sylow 2-subgroup of a double cover of the Suzuki group  $Sz(8)$

G:  $\langle (2, 4, 9, 13)(3, 7, 12, 15)(8, 10)(11, 16),$   
 $(1, 2, 5, 9)(3, 6)(4, 10, 13, 8)(7, 11)(12, 14)(15, 16),$   
 $(1, 3, 8, 7)(2, 6, 4, 11)(5, 12, 10, 15)(9, 14, 13, 16) \rangle < \mathfrak{S}_{16}$

$G^0: G(64,23): (\mathbb{Z}_2^2 \times \mathbb{Z}_4) \rtimes_{\varphi} \mathbb{Z}_4$ ,  $\varphi(1) = \begin{pmatrix} 1 & 0 & 1 \\ 1 & 1 & 1 \\ 0 & 0 & 3 \end{pmatrix}$

T:  $(4, 4, 4)$

L:  $(1, 12, 8, 15)(2, 14, 4, 16)(3, 10, 7, 5)(6, 13, 11, 9),$   
 $(1, 13, 5, 4)(2, 8, 9, 10)(3, 11)(6, 7)(12, 16)(14, 15),$   
 $(1, 14, 8, 16)(2, 3, 13, 15)(4, 7, 9, 12)(5, 6, 10, 11)$

$H_1: (\mathbb{Z}_2)^3$

$\pi_1: G(32,33): (\mathbb{Z}_4^2) \rtimes_{\psi} \mathbb{Z}_2$ ,  $\psi(1) = \begin{pmatrix} 3 & 2 \\ 2 & 1 \end{pmatrix}$

$K^2 = 8$ , Basket  $\emptyset$ .

**1.13. Galois group  $\mathbf{G(64,92)}$ :**  $(D_{2,8,5} \rtimes_{\varphi} \mathbb{Z}_2) \times \mathbb{Z}_2$ ,  $\varphi(1) = \begin{cases} x \mapsto x \\ y \mapsto yxy^4 \end{cases}$

G:  $\langle (1, 2, 4, 8, 5, 9, 12, 16)(3, 7, 10, 15, 11, 6, 13, 14),$   
 $(2, 6)(4, 12)(7, 9)(8, 15)(10, 13)(14, 16),$   
 $(1, 3)(2, 7)(4, 10)(5, 11)(6, 9)(8, 15)(12, 13)(14, 16),$   
 $(1, 3)(2, 6)(4, 10)(5, 11)(7, 9)(8, 14)(12, 13)(15, 16),$   
 $(1, 4, 5, 12)(2, 8, 9, 16)(3, 10, 11, 13)(6, 14, 7, 15),$   
 $(1, 5)(2, 9)(3, 11)(4, 12)(6, 7)(8, 16)(10, 13)(14, 15) \rangle < \mathfrak{S}_{16}$

$G^0: G(32,46): D_4 \times \mathbb{Z}_2 \times \mathbb{Z}_2$

T:  $(2, 2, 2, 2, 2)$

L:  $(1, 5)(2, 7)(3, 11)(6, 9)(8, 14)(15, 16),$

(2, 7)(4, 12)(6, 9)(8, 14)(10, 13)(15, 16),  
 (1, 13)(2, 8)(3, 12)(4, 11)(5, 10)(6, 14)(7, 15)(9, 16),  
 (1, 4)(2, 14)(3, 10)(5, 12)(6, 8)(7, 16)(9, 15)(11, 13),  
 (1, 3)(2, 6)(4, 10)(5, 11)(7, 9)(8, 14)(12, 13)(15, 16)

$H_1: (\mathbb{Z}_2)^3 \times \mathbb{Z}_8$

$\pi_1: 1 \rightarrow \Pi_{17} \times \Pi_{17} \rightarrow \pi_1 \rightarrow G \rightarrow 1$

**1.14. Galois group  $G(256, 3678)$ :**

$G: \langle g_1, g_2, g_3 \mid g_1^4, g_2^4, g_3^4, g_1 g_2 g_3^2 g_1^{-1} g_2^{-1},$   
 $g_2^{-1} g_1^2 g_3^{-1} g_2^{-1} g_3, g_3 g_1^{-1} g_2^{-1} g_3^{-1} g_1^{-1} g_2, g_1 g_2 g_3 g_2^{-1} g_1 g_3,$   
 $g_3 g_1^{-1} g_2^{-1} g_1 g_2 g_3, g_2^2 g_3 g_1^{-1} g_3 g_1, g_3 g_1 g_2^{-1} g_3^{-1} g_2^{-1} g_3^{-1} g_1 g_3,$   
 $g_2^{-1} g_1 g_2 g_1^2 g_3^{-2} g_1, g_1 g_2^2 g_1 g_3^{-1} g_1 g_3^{-1} g_1, g_2^{-2} g_1^{-1} g_3^{-1} g_1 g_3^3,$   
 $g_3^{-1} g_1 g_2^{-1} g_3^{-2} g_1^{-1} g_3^2 g_2 g_3^{-1} \rangle$

$G^0: G(128, 36): \langle h_1, h_2 \mid h_2^4, h_1^4, h_1 h_2^2 h_1^{-2} h_2^{-2} h_1, (h_2^{-1} h_1 h_2 h_1)^2, (h_1, h_2)^2,$   
 $(h_1^{-1} h_2^{-1} h_1 h_2^{-1})^2, (h_1^{-1} h_2^{-1} h_1^{-2} h_2 h_1^{-1})^2, (h_2^2 h_1^{-1} h_2^2 h_1)^2 \rangle$

$T: (4, 4, 4)$

$L: g_2 g_3, g_3 g_2^{-1} g_3^{-1} g_2 g_3 g_1^{-1} g_3 g_1 g_3 g_2 g_3^{-2} g_2 g_3^2, g_2^{-1} g_3^2 g_2^{-1} g_3^{-1} g_2 g_3$

$H_1: (\mathbb{Z}_4)^3$

$\pi_1: 1 \rightarrow \Pi_9 \times \Pi_9 \rightarrow \pi_1 \rightarrow G \rightarrow 1$

**1.15. Galois group  $G(256, 3678)$ :**

$G$ : as above

$G^0: G(128, 36)$ , as above

$T: (4, 4, 4)$

$L: g_1 g_3^{-2} g_1^{-1} g_3 g_1 g_3 g_2^2, g_3 g_2^{-2} g_3^2 g_2^{-1} g_3^{-1} g_2 g_3, g_1 g_3^{-1} g_2^{-2} g_3^2 g_1^{-1} g_3 g_1 g_3 g_2 g_3^{-2} g_2 g_3^2$

$H_1: (\mathbb{Z}_2)^4 \times \mathbb{Z}_4$

$\pi_1: 1 \rightarrow \Pi_9 \times \Pi_9 \rightarrow \pi_1 \rightarrow G \rightarrow 1$

**1.16. Galois group  $G(256, 3678)$ :**

$G$ : as above

$G^0: G(128, 36)$ , as above

$T: (4, 4, 4)$

$L: g_1 g_3 g_2^{-2} g_3^2 g_2^{-1} g_3^{-2} g_2 g_3^2, g_1 g_2 g_3 g_1^{-1} g_3 g_1 g_3 g_2^2, g_2 g_3^{-2} g_1^{-1} g_3 g_1 g_3 g_2^2$

$H_1: (\mathbb{Z}_2)^2 \times (\mathbb{Z}_4)^2$

$\pi_1: 1 \rightarrow \Pi_9 \times \Pi_9 \rightarrow \pi_1 \rightarrow G \rightarrow 1$

**1.17. Galois group  $G(256, 3679)$ :**

$G: \langle g_1, g_2, g_3 \mid g_3^4, g_1^4, g_2^4, g_2 g_3^2 g_1^{-1} g_2^{-1} g_1, g_3^{-1} g_2^{-1} g_3^{-1} g_1 g_2^{-1} g_1,$   
 $g_3^{-1} g_2 g_3 g_2 g_1^2, g_2^{-1} g_1 g_2^{-1} g_3^{-1} g_1^{-1} g_3, g_1^2 g_2^{-1} g_3^{-1} g_2^{-1} g_3, g_2^{-1} g_3 g_2 g_1 g_3 g_1,$   
 $g_1^{-1} g_2^{-1} g_1^2 g_3 g_1^{-1} g_3^{-1} g_2^{-1}, g_3^{-1} g_2 g_3 g_2^{-1} g_1^{-2} g_2^{-2}, (g_3^{-1} g_2)^4,$   
 $g_2^{-1} g_1 g_2^{-1} g_1 g_3^{-1} g_1 g_3 g_1 \rangle$

$G^0: G(128, 36)$ , as above

$T: (4, 4, 4)$

$L: g_2 g_3, g_3 g_2^{-1} g_3^{-1} g_2 g_3 g_1 g_3^2 g_1^{-1} g_3^{-2} g_2^{-1} g_3^{-2} g_2 g_3^{-2}, g_2^{-1} g_3^2 g_2^{-1} g_3^{-1} g_2 g_3$

$H_1: (\mathbb{Z}_2)^2 \times (\mathbb{Z}_4)^2$

$\pi_1: 1 \rightarrow \Pi_9 \times \Pi_9 \rightarrow \pi_1 \rightarrow G \rightarrow 1$

2.  $p_g = q = 1$

$K^2 = 2$ , Basket  $\{C_{2,1}, 2 \times D_{2,1}\}$ .

**2.1. Galois group  $G(4,1)$ :  $\mathbb{Z}_4$**

G:  $\mathbb{Z}_4$

$G^0$ :  $G(2,1)$ :  $\mathbb{Z}_2$

T: (2,2)

L: 2, 0, 2, 2

$H_1$ :  $\mathbb{Z}^2$

**2.2. Galois group  $G(16,8)$ :  $D_{2,8,3}$**

G:  $D_{2,8,3}$

$G^0$ :  $G(8,3)$ :  $D_4$

T: (2)

L:  $x, xy^2, y^4$

$H_1$ :  $\mathbb{Z}^2$

**2.3. Galois group  $G(16,9)$ :  $BD_4$**

G:  $BD_4$

$G^0$ :  $G(8,4)$ :  $Q_8$

T: (2)

L:  $y^6, yx, y^4$

$H_1$ :  $\mathbb{Z}^2$

$K^2 = 4$ , Basket  $\{4 \times C_{2,1}\}$ .

**2.4. Galois group  $G(8,1)$ :  $\mathbb{Z}_8$**

G:  $\mathbb{Z}_8$

$G^0$ :  $G(4,1)$ :  $\mathbb{Z}_4$

T: (2,2)

L: 2,0,4,4

$H_1$ :  $\mathbb{Z}_2 \times \mathbb{Z}^2$

**2.5. Galois group  $G(8,2)$ :  $\mathbb{Z}_2 \times \mathbb{Z}_4$**

G:  $\mathbb{Z}_2 \times \mathbb{Z}_4$

$G^0$ :  $G(4,2)$ :  $\mathbb{Z}_2 \times \mathbb{Z}_2$

T: (2,2)

L: (1,0), (1,2), (1,0), (1,0),

$H_1$ :  $\mathbb{Z}_2 \times \mathbb{Z}^2$

**2.6. Galois group  $G(32,29)$ :**

G:  $\langle x, y, z \mid z^2, y^4, x^4, x^{-1}y^2x^{-1}, x^{-1}y^{-1}xy^{-1}, xy^{-1}xy, y^{-1}zyz, zy^{-1}x^2zy^{-1}, (zxzx^{-1})^2 \rangle$

$G^0$ :  $G(16,3)$ :  $(\mathbb{Z}_2)^2 \rtimes_{\varphi} \mathbb{Z}_4$ :  $\varphi(1) = \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}$

T: (2)

L:  $z^2xzx^{-1}, x, zxzx^{-1}$

$H_1$ :  $\mathbb{Z}_2^2 \times \mathbb{Z}^2$

**2.7. Galois group  $G(32,13)$ :  $D_{4,8,3}$**

G:  $D_{4,8,3}$



$G^0: G(16,4): D_{4,4,3}$

T: (2)

L:  $y^6, x, y^4$

$H_1: \mathbb{Z}_2 \times \mathbb{Z}^2$

**2.8. Galois group  $G(32,14): D_{4,8,-1}$**

G:  $D_{4,8,-1}$

$G^0: G(16,4): D_{4,4,3}$

T: (2)

L:  $y^2, x, y^4$

$H_1: \mathbb{Z}_2 \times \mathbb{Z}^2$

**2.9. Galois group  $G(32,32):$**

G:  $\langle x, y, z \mid y^4, x^4, x^2z^2, x^{-1}yx^{-1}y^{-1}, (y, z^{-1}), y^{-1}xzx^{-1}z^{-1}y^{-1} \rangle$

$G^0: G(16,4): D_{4,4,3}$

T: (2)

L:  $yz^{-1}y^{-2}, xz, z^{-2}y^{-2}$

$H_1: \mathbb{Z}_2^2 \times \mathbb{Z}^2$

**2.10. Galois group  $G(32,35):$**

G:  $\langle x, y, z \mid x^2y^2, x^{-1}y^{-1}x^{-1}y, z^4, x^4, x^{-1}zxxz, (y, z^{-1}) \rangle$

$G^0: G(16,4): D_{4,4,3}$

T: (2)

L:  $xy^{-2}, z^{-1}, z^{-2}$

$H_1: \mathbb{Z}_2^2 \times \mathbb{Z}^2$

**2.11. Galois group  $G(32,15):$**

G:  $\langle x, y \mid x^{-1}y^3xy^{-1}, x^{-1}yx^{-1}yx^{-2}, xy^{-1}x^{-2}yx, xy^{-1}x^{-1}y^{-1}x^{-1}y^{-1}xy^{-1}, \rangle$

$G^0: G(16,8): D_{2,8,5}$

T: (2)

L:  $y^{-2}, x, y^{-4}$

$H_1: \mathbb{Z}_2 \times \mathbb{Z}^2$

$K^2 = 5$ , Basket  $\{C_{3,1}, C_{3,2}\}$ .

**2.12. Galois group  $G(24,4): BD_6$**

G:  $BD_6$

$G^0: G(12,1): BD_3$

T: (3)

L:  $y^8, xy^3, y^8,$

$H_1: \mathbb{Z}_2 \times \mathbb{Z}^2$

**2.13. Galois group  $G(24,5): D_{2,12,5}$**

G:  $D_{2,12,5}$

$G^0: G(12,4): D_6$

T: (3)

L:  $y^4x, y^4, y^2x$

$H_1: \mathbb{Z}_2^2 \times \mathbb{Z}^2$

$K^2 = 6$ , Basket  $\{2 \times C_{2,1}\}$ .

**2.14. Galois group  $\mathbf{G}(48,30)$ :**  $A_4 \rtimes_{\varphi} \mathbb{Z}_4$ ,

$$\mathbf{G}: A_4 \rtimes_{\varphi} \mathbb{Z}_4, \varphi_1(1) = \begin{cases} (1,2)(3,4) & \mapsto (1,2)(3,4) \\ (1,2,3) & \mapsto (2,3,4) \end{cases}$$

$$G^0: G(24,13): A_4 \times \mathbb{Z}_2$$

$$\mathbf{T}: (2)$$

$$\mathbf{L}: ((1,3)(2,4);0), ((2,4,3);0), ((1,2,3);2)$$

$$H_1: \mathbb{Z}_2^2 \times \mathbb{Z}^2$$

**2.15. Galois group  $\mathbf{G}(48,31)$ :**  $A_4 \times \mathbb{Z}_4$

$$\mathbf{G}: A_4 \times \mathbb{Z}_4$$

$$G^0: G(24,13): A_4 \times \mathbb{Z}_2$$

$$\mathbf{T}: (2)$$

$$\mathbf{L}: ((1,2)(3,4);0), ((1,3,4);0), ((1,4)(2,3);2)$$

$$H_1: \mathbb{Z}_2^2 \times \mathbb{Z}^2$$

$K^2 = 6$ , Basket  $\{C_{5,3}\}$ .

**2.16. Galois group  $\mathbf{G}(20,3)$ :**  $\mathbb{Z}_5 \rtimes_{\varphi} \mathbb{Z}_4$

$$\mathbf{G}: \mathbb{Z}_5 \rtimes_{\varphi} \mathbb{Z}_4, \varphi(1) = (1 \mapsto 3)$$

$$G^0: G(10,1): D_5$$

$$\mathbf{T}: (5)$$

$$\mathbf{L}: (3,0), (1,0), (2,2)$$

$$H_1: \mathbb{Z}_2^2 \times \mathbb{Z}^2$$

$K^2 = 8$ , Basket  $\emptyset$ .

**2.17. Galois group  $\mathbf{G}(16,6)$ :**  $D_{2,8,5}$

$$\mathbf{G}: D_{2,8,5}$$

$$G^0: G(8,2): \mathbb{Z}_2 \times \mathbb{Z}_4$$

$$\mathbf{T}: (2,2)$$

$$\mathbf{L}: y^2, xy^4, xy^4, xy^4$$

$$H_1: \mathbb{Z}_4 \times \mathbb{Z}^2$$

**2.18. Galois group  $\mathbf{G}(16,8)$ :**  $D_{2,8,3}$

$$\mathbf{G}: D_{2,8,3}$$

$$G^0: G(8,3): D_4$$

$$\mathbf{T}: (2,2)$$

$$\mathbf{L}: Id(G), xy^4, xy^2, xy^2$$

$$H_1: \mathbb{Z}_4 \times \mathbb{Z}^2$$

**2.19. Galois group  $\mathbf{G}(16,3)$ :**  $(\mathbb{Z}_2)^2 \rtimes_{\varphi} \mathbb{Z}_4: \varphi(1) = \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}$

$$\mathbf{G}: \langle (1,2,4,6)(3,5,7,8), (2,5)(6,8), (1,3)(2,5)(4,7)(6,8), (1,4)(2,6)(3,7)(5,8) \rangle < \mathfrak{S}_8$$

$$G^0: G(8,5): \mathbb{Z}_2^3$$

$$\mathbf{T}: (2,2)$$

L: (14)(28)(37)(56),(25)(68), (17)(26)(34)(58)  
 $H_1: \mathbb{Z}_2^3 \times \mathbb{Z}^2$

3.  $p_g = q = 2$

$K^2 = 8$ , **Basket  $\emptyset$ .**

**3.1. Galois group  $G(4,1): \mathbb{Z}_4$**

G:  $\mathbb{Z}_4$

$G^0: G(2,1): \mathbb{Z}_2$

T: –

L: 1, 1, 0, 1

$H_1: \mathbb{Z}_2 \times \mathbb{Z}^4$