One-loop Modified Gravity in de Sitter Universe, Quantum Corrected Inflation, and its Confrontation with the Planck Result

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Abstract

Motivated by issues on inflation, a generalized modified gravity model is investigated, where the model Lagrangian is described by a smooth function $f(R, K, \phi)$ of the Ricci scalar R, the kinetic term K of a scalar field ϕ. In particular, the one-loop effective action in the de Sitter background is examined on-shell as well as off-shell in the Landau gauge. In addition, the on-shell quantum equivalence of $f(R)$ gravity in the Jordan and Einstein frames is explicitly demonstrated. Furthermore, we present applications related to the stability of the de Sitter solutions and the one-loop quantum correction to inflation in quantum-corrected R^2 gravity. It is shown that for a certain range of parameters, the spectral index of the curvature perturbations can be consistent with the Planck analysis, but the tensor-to-scalar ratio is smaller than the minimum value within the $1\,\sigma$ error range of the BICEP2 result.

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1 Introduction

Relativistic theories of gravity have attracted a lot of interests in modern cosmology after the discovery of the current cosmic acceleration, namely, the dark energy problem, as well as after the confirmation of the early-time inflationary era. There exist several possible descriptions of the current accelerated expansion of the universe. The simplest one is the introduction of the small positive cosmological constant in the framework of General Relativity, so that we can deal with a perfect fluid whose equation of state (EoS) parameter $w = -1$. This fluid model is able to realize the current cosmic acceleration, but also other kinds of fluid (e.g., phantom, quintessence, inhomogeneous fluids...) with their suitable EoS have not been excluded yet (for a recent review, see [1]), because the observed small value of the cosmological constant leads to several conceptual problems, such as the vacuum energy and the coincidence problem. For this reason, several different approaches to the dark energy issue have been proposed. Among them, the modified theories of gravity represent an interesting extension of General Relativity (for example, see [2] and references therein).

On the other hand, very recently, after the release of the Planck mission results [3, 4], a lot of papers concerning the inflationary era have appeared. Among many models of inflation proposed in the past years, only a restricted class of models based on a single scalar field theory seems to be in agreement with the Planck data. In particular, the so-called Starobinsky inflation [5], where the action consists of the Einstein-Hilbert term plus a quadratic term in the Ricci curvature (i.e., R^2 gravity), seems to be quite successful. Note that the gravitational sector of the Starobinsky inflation model is equivalent to that of the scalar inflationary model proposed in Ref. [6]. Recently, a flow of works have been executed [7], in which such inflationary models related with R^2 gravity and its generalizations have been investigated.

R^2 gravity is one of the simplest modified gravity models. For this reason, it may be of interest to explore generalized modified gravitational models represented by the Lagrangian involving an arbitrary dependence on the Ricci scalar R, a scalar field ϕ and its kinetic energy K.

In this paper, we clearly show the on-shell quantum equivalence of $f(R)$ gravity in the Jordan and Einstein frames. It may also be interesting to compute the one-loop effective action of such a generalized model in a maximally symmetric space, namely, the de Sitter space, which is one of the most relevant ingredients due to its applications to inflation in quantum-corrected R^2 gravity. This kind of computation has been performed for $f(R)$ gravity in Ref. [8]. Here, our aim is to extend it in these generalized modified models. The results of such a study may directly be applied to the realization of the inflationary epoch by taking account of quantum gravity corrections. The one-loop effective action is calculated in the Euclidean sector, that is, the de Sitter space becomes the four dimensional compact sphere S_4 and the evaluation of the several functional determinants is made by making use of the zeta-function regularization (see, e.g., [9]), and making use of the quantum field theory (QFT) in curved space-time [10, 11]. Typically, the so-called on-shell one-loop effective action is relatively simple to compute, and hence it may be used in order to study the stability of the generalized background and the on-shell quantum equivalence between the Jordan and Einstein frames. Instead, the off-shell one-loop effective action suffers from gauge ambiguities, and therefore in order to avoid the problem, one has to evaluate it in the so-called Landau gauge [12], which is a somehow selected gauge owing to the relation with the gauge-fixing independent description. The off-shell one-loop effective action is useful for examining the relevance of the quantum corrections to inflation observables. We

note that in Refs. [13, 14], quantum corrections with completely different approaches from our method have been discussed.

In addition, we derive the spectral index n_s and the tensor-to-scalar ratio r so that we would examine whether those values can be consistent with not only the Planck data [3, 4] but also the BICEP2 result [15] on the B-mode polarization of the cosmic microwave background (CMB) radiation. One-scalar and quantum-corrected inflationary models consistent with the BICEP2 data have recently been discussed in Ref. [16]. As a proposal for inflation in R^2 gravity, the trace-anomaly driven inflation in $f(R)$ gravity [17] has been studied, and the n_s and r in it have been compared with the Planck and BICEP2 observations. We adopt the units $k_B = c = \hbar = 1$ and express the Newton's constant as $G = 1/M_P^2$ with $M_P = 2.43 \times 10^{18}$ GeV the reduced Planck mass.

The organization of the paper is the following. In Sec. II, the generalized modified gravity model is introduced and the classical equation of motion is derived. In Sec. III, the one-loop quantization of the generalized gravity model is presented in detail. In Sec. IV, the two one-loop effective actions, the on-shell and off-shell ones are written down in terms of functional determinants. In Sec. V, cosmological applications are presented. Particularly, the quantum corrections to the Starobinsky inflation are evaluated in the limit of large scalar curvature. Moreover, we examine the spectral index of scalar modes of the density perturbations and those tensor-to-scalar ratio, and investigate whether our model can explain the observational consequences found by the Planck satellite and the BICEP2 experiment. The paper ends with the Conclusion and two Appendices, where technical details are given.

2   Generalized Modified Gravity Models

The modified gravity model which we are interested in is described by the action

$$I = \frac{1}{2\kappa^2} \int d^4x \sqrt{-\tilde{g}} f(\tilde{R}, \tilde{K}, \tilde{\phi}) , \qquad \kappa^2 = 8\pi G . \tag{2.1}$$

The Lagrangian density f is a smooth function depending on the Ricci scalar $\tilde{R}$, the scalar field $\tilde{\phi}$ and the kinetic energy $\tilde{K} = (1/2) \tilde{g}^{ij} \tilde{\nabla}_i \tilde{\phi} \tilde{\nabla}_j \tilde{\phi}$, $\tilde{g}_{ij}$ is the metric tensor, $\tilde{g}$ its determinant, and $\tilde{\nabla}_i$ the covariant derivative, where $i, j, k, ...$ are tensorial indices that run over the range $0, ..., 3$. Here and in the following, we use the tilde for arbitrary quantities.

The field equations may be obtained by making the functional variation of the action in Eq. (2.1) with respect to the metric $\tilde{g}_{ij}$ and the scalar field $\tilde{\phi}$, given by

$$\begin{cases} f_{\tilde{R}} \tilde{R}_{ij} - \frac{1}{2} f \tilde{g}_{ij} + \left(\tilde{g}_{ij} \tilde{\Delta} - \tilde{\nabla}_i \tilde{\nabla}_j \right) f_{\tilde{R}} + \frac{1}{2} f_{\tilde{K}} \tilde{\nabla}_i \tilde{\phi} \tilde{\nabla}_j \tilde{\phi} = 0 , \\ \tilde{g}^{ij} \tilde{\nabla}_i \left(f_{\tilde{K}} \tilde{\phi} \tilde{\nabla}_j \phi \right) = f_{\tilde{\phi}} , \end{cases} \tag{2.2}$$

where $\tilde{\Delta}$ is the covariant Laplace operator in the metric $\tilde{g}_{ij}$, while $f_{\tilde{R}}$ represents the derivative of $f(\tilde{R}, \tilde{K}, \tilde{\phi})$ with respect to the variable $\tilde{R}$ ($f_{\tilde{K}}$ and $f_{\tilde{\phi}}$ have the similar meanings).

From equations (2.2), we see that there exists a constant curvature solution $\tilde{R} = R$. With a constant scalar field $\tilde{\phi} = \phi$, we find that the Lagrangian density satisfies the following conditions:

$$f_R R_{ij} - \frac{1}{2} f_0 g_{ij} = 0 , \qquad f_\phi = 0 , \tag{2.3}$$

where all the quantities in the latter equation are evaluated on the solution $\{R, \phi\}$. In particular, $f_0 = f(R, K, \phi)$. In such a case, the background fields (constant scalar field and curvature) are solutions of the equations

$$R f_R - 2f_0 = 0 \,, \qquad\qquad f_\phi = 0 \,. \tag{2.4}$$

Using these equations, a number of classical constant curvature solutions may be constructed.

3 One-loop quantization around a maximally symmetric solution

In this section, we discuss the one-loop quantization of the model on a maximally symmetric space. In these investigations, as quite usual, this should be regarded as only an effective approach (see, a book [10]). We may start from the Euclidean action

$$I_{\mathrm{E}}[\tilde{g}, \tilde{\phi}] = -\frac{1}{16\pi G} \int d^4 x \, \sqrt{-\tilde{g}} \, f(\tilde{R}, \tilde{K}, \tilde{\phi}) \,, \tag{3.1}$$

where the generic function f satisfies –on shell– the conditions in Eq. (2.3), that ensure the existence of the solutions of a constant curvature R and a constant scalar field ϕ. Here, we are interested in, particularly, S^4 (de Sitter) case, but also H^4 (anti de Sitter) or $\mathbb{R}^4$ (euclidean) cases are included in the general discussion. In all such cases, we have

$$R_{ijrs} = \frac{R}{12} \left(g_{ir} g_{js} - g_{is} g_{jr} \right) \,, \qquad R_{ij} = \frac{R}{4} \, g_{ij} \,, \qquad R = constant \,, \tag{3.2}$$

where g_{ij} is the metric of the maximally symmetric space. In the next step, we examine the small fluctuations around the constant curvature solution, that is,

$$\begin{cases} \tilde{g}_{ij} = g_{ij} + h_{ij} \,, & |h_{ij}| \ll 1 \,, \\ \tilde{g}^{ij} = g^{ij} - h^{ij} + h^{ik} h_k^j + \mathcal{O}(h^3) \,, & h = g^{ij} h_{ij} \,, \\ \tilde{\phi} = \phi + \varphi \,, & |\varphi| \ll 1 \,, \end{cases} \tag{3.3}$$

where indices are made lowered and raised by means of the background metric g_{ij}.

By performing the Taylor expansion of $\sqrt{-\tilde{g}} f(\tilde{R}, \tilde{K}, \tilde{\phi})$ around the background fields $\{g_{ij}, \phi\}$ up to the second order in the perturbations, we get

$$I_{\mathrm{E}}[g, \phi] \sim -\frac{1}{16\pi G} \int d^4 x \, \sqrt{-g} \, [f_0 + \mathcal{L}_1 + \mathcal{L}_2] \,, \tag{3.4}$$

where, up to total derivatives,

$$\mathcal{L}_1 = \frac{1}{4} X h + f_\phi \phi \,, \tag{3.5}$$

$$\begin{aligned} \mathcal{L}_2 =\ & -\frac{1}{2} f_R h_k^i \nabla_i \nabla_j h^{jk} + \frac{1}{4} f_R h_{ij} \Delta h^{ij} - \frac{1}{24} R f_R h_{ij} h^{ij} + \frac{1}{2} f_R h \nabla_i \nabla_j h^{ij} \\ & + f_{R\phi} \varphi \nabla_i \nabla_j h^{ij} + \frac{1}{2} f_{RR} \nabla_i \nabla_j h^{ij} \nabla_r \nabla_s h^{rs} - f_{RR} \Delta h \nabla_i \nabla_j h^{ij} - \frac{1}{4} R f_{RR} h \nabla_i \nabla_j h^{ij} \\ & - \frac{1}{48} R f_R h^2 - \frac{1}{2} f_K \varphi \Delta \varphi - \frac{1}{4} f_R h \Delta h - f_{R\phi} h \Delta \varphi + \frac{1}{4} R f_{RR} h \Delta h + \frac{1}{2} f_{\phi\phi} \varphi^2 \\ & - \frac{1}{4} R f_{R\phi} h \varphi + \frac{1}{32} R^2 f_{RR} h^2 + \frac{X}{16} \left(h^2 - 2 h_{ij} h^{ij} \right) + \frac{1}{2} f_\phi h \varphi \,. \end{aligned} \tag{3.6}$$

Here, we have set $X = 2f_0 - Rf_R$. In this way, the on-shell Lagrangian density can directly be obtained in the limit $X \to 0$ and $f_\phi \to 0$.

As is well known, it is convenient to carry out the standard expansion of the tensor field h_{ij} in irreducible components [12, 18, 19], namely,

$$h_{ij} = \hat{h}_{ij} + \nabla_i \xi_j + \nabla_j \xi_i + \nabla_i \nabla_j \sigma + \frac{1}{4} g_{ij}(h - \Delta_0 \sigma) , \tag{3.7}$$

where σ is the scalar component, while ξ_i and $\hat{h}_{ij}$ are the vector and tensor components with the properties

$$\nabla_i \xi^i = 0 , \qquad \nabla_i \hat{h}^i_j = 0 , \qquad \hat{h}^i_i = 0 . \tag{3.8}$$

In terms of the irreducible components of the h_{ij} field, the one-loop contribution to the Lagrangian density, again disregarding the total derivatives, becomes

$$
\begin{aligned}
\mathcal{L}_2 = {} & \frac{1}{32} \sigma \left(9f_{RR}\Delta\Delta\Delta - 3f_R\Delta\Delta\Delta + 6f_{RR}R\Delta\Delta \right. \\
& \qquad \left. - f_R R\Delta\Delta + f_{RR}R^2\Delta\Delta - 3X\Delta\Delta - RX\Delta \right) \sigma \\
& + \frac{1}{32} h \left(9f_{RR}\Delta\Delta - 3f_R\Delta + 6f_{RR}R\Delta - f_R{}^2 R + f_{RR}R^2 + X \right) h \\
& + \frac{1}{16} h \left(-9f_{RR}\Delta\Delta\Delta + 9f_R\Delta\Delta - 6f_R\Delta\Delta - 6f_{RR}R\Delta\Delta + f_R R\Delta - f_{RR}R^2\Delta \right) \sigma \\
& + \frac{1}{2} \varphi \left(-f_K\Delta + f_{\phi\phi} \right) \varphi + \frac{1}{4} h \left(-3f_{R\phi}\Delta + 2f_\phi - f_{R\phi}R \right) \varphi \\
& + \frac{1}{4} \sigma \left(+3f_{R\phi}\Delta\Delta + f_{R\phi}R\Delta \right) \varphi \\
& + \frac{1}{16} \xi_i \left(4X\Delta + 4RX \right) \xi^i + \frac{1}{24} \hat{h}_{ij} \left(6f_R\Delta - f_R R - 3X \right) \hat{h}^{ij} .
\end{aligned}
\tag{3.9}
$$

Since the invariance under the diffeomorphisms renders the operator in the (h, σ) sector not invertible, a gauge-fixing term and a corresponding ghost compensating term have to be added. We explore the class of gauge conditions, parameterized by the real parameter ρ as

$$\chi_k = \nabla_j h^j_k - \frac{1+\rho}{4} \nabla_k h .$$

This is the harmonic or the de Donder one corresponding to the choice $\rho = 1$. As the gauge fixing, we choose the quite general term [10]

$$\mathcal{L}_{gf} = \frac{1}{2} \chi^i G_{ij} \chi^j , \qquad G_{ij} = \gamma g_{ij} + \beta g_{ij}\Delta , \tag{3.10}$$

where the term proportional to γ on the right-hand side of the second equation is the one normally used in the Einstein gravity. The corresponding ghost Lagrangian reads [10]

$$\mathcal{L}_{gh} = B^i G_{ik} \frac{\delta \chi^k}{\delta \varepsilon^j} C^j , \tag{3.11}$$

where C_k and B_k are the ghost and anti-ghost vector fields, respectively, while $\delta \chi^k$ is the variation of the gauge condition due to an infinitesimal gauge transformation of the field. It reads

$$\delta h_{ij} = \nabla_i \varepsilon_j + \nabla_j \varepsilon_i \qquad \Longrightarrow \qquad \frac{\delta \chi^i}{\delta \varepsilon^j} = g_{ij}\Delta + R_{ij} + \frac{1-\rho}{2} \nabla_i \nabla_j . \tag{3.12}$$

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Neglecting total derivatives, we get

$$\mathcal{L}_{gh} = B^i \left(\gamma \, H_{ij} + \beta \, \Delta \, H_{ij} \right) C^j \,, \tag{3.13}$$

where we have set

$$H_{ij} = g_{ij} \left(\Delta + \frac{R_0}{4} \right) + \frac{1-\rho}{2} \, \nabla_i \nabla_j \,. \tag{3.14}$$

In irreducible components, we obtain

$$
\begin{aligned}
\mathcal{L}_{gf} \;=\; & \frac{\gamma}{2} \left[\xi^k \left(\Delta_1 + \frac{R_0}{4} \right)^2 \xi_k + \frac{3\rho}{8} \, h \left(\Delta_0 + \frac{R_0}{3} \right) \Delta_0 \, \sigma \right. \\
& \left. - \frac{\rho^2}{16} \, h \, \Delta_0 \, h - \frac{9}{16} \, \sigma \left(\Delta_0 + \frac{R_0}{3} \right)^2 \Delta_0 \, \sigma \right] \\
& + \frac{\beta}{2} \left[\xi^k \left(\Delta_1 + \frac{R_0}{4} \right)^2 \Delta_1 \xi_k + \frac{3\rho}{8} \, h \left(\Delta_0 + \frac{R}{4} \right) \left(\Delta_0 + \frac{R}{3} \right) \Delta_0 \sigma \right. \\
& \left. - \frac{\rho^2}{16} \, h \left(\Delta_0 + \frac{R_0}{4} \right) \Delta_0 h - \frac{9}{16} \, \sigma \left(\Delta_0 + \frac{R_0}{4} \right) \left(\Delta_0 + \frac{R_0}{3} \right)^2 \Delta_0 \sigma \right] \,, \tag{3.15}
\end{aligned}
$$

$$
\begin{aligned}
\mathcal{L}_{gh} \;=\; & \gamma \left\{ \hat{B}^i \left(\Delta_1 + \frac{R_0}{4} \right) \hat{C}^j + \frac{\rho-3}{2} \, b \left(\Delta_0 - \frac{R_0}{\rho-3} \right) \Delta_0 c \right\} \\
& + \beta \left\{ \hat{B}^i \left(\Delta_1 + \frac{R_0}{4} \right) \Delta_1 \hat{C}^j \right. \\
& \left. + \frac{\rho-3}{2} \, b \left(\Delta_0 + \frac{R_0}{4} \right) \left(\Delta_0 - \frac{R_0}{\rho-3} \right) \Delta_0 c \right\} \,, \tag{3.16}
\end{aligned}
$$

where the ghost irreducible components are defined by

$$
\begin{aligned}
C_k \;\equiv\;& \hat{C}_k + \nabla_k c \,, & \nabla_k \hat{C}^k = 0 \,, \\
B_k \;\equiv\;& \hat{B}_k + \nabla_k b \,, & \nabla_k \hat{B}^k = 0 \,, \tag{3.17}
\end{aligned}
$$

and for clarity, from now on we use the notation Δ_0, Δ_1, and Δ_2 for the Laplace-Beltrami operators acting on scalars, traceless-transverse vector fields, and traceless-transverse tensor ones, respectively.

In order to compute the one-loop contribution to the effective action, we have to analyze the path integral for the bilinear part of the total Lagrangian

$$\mathcal{L} = \mathcal{L}_2 + \mathcal{L}_{gf} + \mathcal{L}_{gh} \,, \tag{3.18}$$

and take into account the Jacobian due to the change of variables with respect to the original ones. In this way, we get [10, 12]

$$
\begin{aligned}
Z^{(1)} = e^{-\Gamma^{(1)}} \;=\;& (\det G_{ij})^{-1/2} \int D[h_{ij}] D[C_k] D[B^k] \exp \left(- \int d^4x \, \sqrt{g} \, \mathcal{L} \right) \\
\;=\;& (\det G_{ij})^{-1/2} \det J_1^{-1} \det J_2^{1/2} \\
& \times \int D[h] D[\hat{h}_{ij}] D[\xi^j] D[\sigma] D[\hat{C}_k] D[\hat{B}^k] D[c] D[b] \exp \left(- \int d^4x \, \sqrt{g} \, \mathcal{L} \right) \,. \tag{3.19}
\end{aligned}
$$

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Here, J_1 and J_2 are the Jacobians due to the change of the variables in the ghost and tensor sectors, respectively [12], described by

$$J_1 = \Delta_0, \qquad J_2 = \left(\Delta_1 + \frac{R_0}{4}\right)\left(\Delta_0 + \frac{R_0}{3}\right)\Delta_0, \qquad (3.20)$$

and the determinant of the operator G_{ij}, acting on vectors, can be written as

$$\det G_{ij} = \text{const } \det\left(\Delta_1 + \frac{\gamma}{\beta}\right)\det\left(\Delta_0 + \frac{R_0}{4} + \frac{\gamma}{\beta}\right), \qquad (3.21)$$

while it is trivial in the case $\beta = 0$.

4 One-loop effective action

Now, a straightforward computation leads to the approximate effective action with the one-loop quantum correction. This is a quite complicated gauge-dependent quantity. For simplicity, and since we are mainly interested in the Landau gauge, we restrict our investigations to the class of gauges with an arbitrary parameter γ and fixed ones as $\rho = 1$ and $\beta = 0$. In this way, we have the formal equations

$$\Gamma = I_{\text{E}}[g, \phi] + \Gamma^{(1)}, \qquad I_{\text{E}}[g, \phi] = \frac{24\pi f_0}{GR^2}, \qquad (4.1)$$

$$\begin{aligned}
\Gamma^{(1)} &= \frac{1}{2}\ln\det\left(b_4\Delta_0^4 + b_3\Delta_0^3 + b_2\Delta_0^2 + b_1\Delta_0 + b_0\right) - \ln\det\left(-\Delta_0 - \frac{R}{2}\right) \\
&\quad + \frac{1}{2}\ln\det\left(-\Delta_1 - \frac{R}{4} - \frac{X}{2\gamma}\right) - \ln\det\left(-\Delta_1 - \frac{R}{4}\right) \\
&\quad + \frac{1}{2}\ln\det\left(-\Delta_2 + \frac{R}{6} + \frac{X}{2f_R}\right), \qquad (4.2)
\end{aligned}$$

where the coefficients b_k are complicated expressions depending on the function $f(R, K, \phi)$ and its derivatives. These are explicitly written in Appendix A, where the expressions for the simpler case of $f(R)$ are also presented. The determinant of a differential operator can be well defined by means of zeta-functions [9], that on S^N can be expressed in terms of the Hurwitz zeta functions as is explained in Appendix C.

The "on-shell" contribution is obtained in the limit $X \to 0$ and $f_\phi \to 0$, and as is well known, it is gauge independent. For the one-loop contribution, we find

$$\begin{aligned}
\Gamma^{(1)}_{\text{on-shell}} &= \frac{1}{2}\ln\det\left[\frac{1}{\mu^4}\left(a_2\Delta_0^2 + a_1\Delta_0 + a_0\right)\right] \\
&\quad - \frac{1}{2}\ln\det\left[\frac{1}{\mu^2}\left(-\Delta_1 - \frac{R}{4}\right)\right] + \frac{1}{2}\ln\det\left[\frac{1}{\mu^2}\left(-\Delta_2 + \frac{R}{6}\right)\right]. \qquad (4.3)
\end{aligned}$$

Also the coefficients a_k depend on the function f and its derivatives and are written in Appendix A. Here, an arbitrary renormalization parameter μ^2 has been introduced for dimensional reasons. Furthermore, we should mention another delicate point. The Euclidean gravitational action is not bounded from below due to the presence of R, because arbitrary negative contributions can be induced on R by conformal rescaling of the metric. For this reason, we have also used the Hawking prescription to integrate over imaginary scalar fields. Finally, the problem of presence

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of additional zero modes introduced by the decomposition in Eq. (3.7) can be treated through the method proposed in Ref. [12].

For physical applications, the most appropriate one is the Landau gauge [10, 12, 20], that corresponds to the choice of gauge parameters $\rho = 1$, $\beta = 0$, and $\gamma = \infty$. It is known that such a gauge condition in one-loop approximation makes the convenient effective action to be equal to the gauge-fixing independent effective action (for reviews, see [12, 10]). In this case, we get

$$
\begin{aligned}
\Gamma^{(1)}_{\text{Landau}} &= \frac{1}{2} \ln \det \left[\frac{1}{\mu^8} \left(c_4 \Delta_0^4 + c_3 \Delta_0^3 + c_2 \Delta_0^2 + c_1 \Delta_0 + c_0 \right) \right] \\
&\quad - \frac{1}{2} \ln \det \left[\frac{1}{\mu^2} \left(-\Delta_1 - \frac{R}{4} \right) \right] - \ln \det \left[\frac{1}{\mu^2} \left(-\Delta_0 - \frac{R}{2} \right) \right] \\
&\quad + \frac{1}{2} \ln \det \left[\frac{1}{\mu^2} \left(-\Delta_2 - \frac{R}{3} + \frac{f_0}{f_R} \right) \right] .
\end{aligned}
\tag{4.4}
$$

The coefficients c_k are also represented in Appendix A.

5 Cosmological applications

5.1 Quantum equivalence between the Jordan and Einstein frames

Modified gravity models are usually formulated in the so-called Jordan frame, where the gravitational Lagrangian density only depends on geometric invariants. However, for some models like $f(R)$ gravity, it is possible to formulate the theory in the so-called Einstein frame, where, by means of a suitable conformal transformation involving geometric quantities only, the original Lagrangian is replaced by the Einstein-Hilbert one plus the Lagrangian of a scalar field that explicitly takes account of an additional degree of freedom which presents in the original theory. At the classical level, the equivalence of the two formulations has been studied in many works (see, for instance, reviews [2]). Here, we show that this is true also at the one-loop level for the on-shell effective actions (compare with corresponding one-loop equivalence of dilatonic gravity in different frames [21]). On the contrary, for the off-shell ones, the one-loop contributions are completely different.

To this aim, we investigate $f(R)$ gravity in the Jordan and Einstein frames. The corresponding classical actions are represented as

$$
I_{\text{Jord}} = \frac{1}{2\kappa^2} \int d^4x \sqrt{-g}\, f(R) , \qquad \text{for the Jordan frame,} \tag{5.1}
$$

$$
I_{\text{Eins}} = \frac{1}{2\kappa^2} \int d^4x \sqrt{-\tilde{g}}\, \tilde{f}(\tilde{R}, K, \sigma) , \qquad \text{for the Einstein frame,} \tag{5.2}
$$

where

$$
\tilde{f}(\tilde{R}, \tilde{K}, \sigma) = \tilde{R} - \frac{3}{2} \tilde{g}^{ij}\, \partial_i \sigma \partial_j \sigma - V(\sigma) . \tag{5.3}
$$

In this section, all the quantities with the tilde are related to the metric $\tilde{g}_{ij} = e^\sigma g_{ij}$ in the Einstein frame. Also, note that σ is not an arbitrary function, but it is related to R as follows

$$
e^\sigma = f'(R) , \qquad R = \Phi(e^\sigma) , \qquad \Phi \circ f' = 1 , \tag{5.4}
$$

where the prime denotes the derivative with respect to R. Moreover, the potential is implicitly defined by

$$V(\sigma) \equiv e^{-\sigma}\Phi(e^{\sigma}) - e^{-2\sigma}f(\Phi(e^{\sigma})). \tag{5.5}$$

It is clear from Eq. (4.3) that to verify the equivalence of the on-shell, one-loop effective actions corresponding to the classical actions above, it is sufficient to compare the corresponding scalar sectors.

In the first case, the scalar contribution to the effective action can directly be read off from Eq. (A.11), that is,

$$
\begin{aligned}
\Gamma^{\text{Jord}}_{\text{on-shell}} &= \frac{1}{2}\ln\det\left[\frac{1}{\mu^2}\left(-f_{RR}\left(\Delta_0 + \frac{R}{3}\right) + \frac{f_R}{3}\right)\right] \\
&\quad + \text{classical and higher spin contributions.}
\end{aligned}
\tag{5.6}
$$

In the second case, we have to examine Eq. (4.3) and compute the coefficients a_k by using the function $\tilde{f}(\tilde{R}, \tilde{K}, \sigma)$. We also take into consideration the fact that $\tilde{\Delta}_0$ is related to Δ_0 via a conformal transformation. Consequently, we obtain

$$
\begin{aligned}
\Gamma^{\text{Eins}}_{\text{on-shell}} &= \frac{1}{2}\ln\det\left[\frac{1}{\tilde{\mu}^2}\left(3\tilde{\Delta}_0 - V''(\sigma)\right)\right] \\
&\quad + \text{classical and higher spin contributions} \\
&= \frac{1}{2}\ln\det\left[\frac{1}{\tilde{\mu}^2}\left(\frac{3\Delta_0}{f_R} + \frac{R}{f_R} - \frac{1}{f_{RR}}\right)\right] \\
&\quad + \text{classical and higher spin contributions,}
\end{aligned}
\tag{5.7}
$$

where the prime means the derivative with respect to σ, and the latter equality equivalent to Eq. (5.6) with a trivial redefinition of $\tilde{\mu}$.

As an example, we now explore R^2 gravity. In the Jordan frame, we have

$$f(R) = R + \frac{R^2}{6M^2}, \tag{5.8}$$

with M^2 a mass parameter. For the Einstein gravity, there exists a new scalar degree of freedom, which is the so-called scalaron, as a consequence of the quadratic term in the classical action. It follows from Eq. (2.4) that there is the background solution of $R = 0$, and thus the related on-shell, one loop effective action reads

$$
\begin{aligned}
\Gamma^{\text{Jord}}_{\text{on-shell}} &= \frac{1}{2}\ln\det\left[\frac{1}{\mu^2}\left(-\Delta_0 + M^2\right)\right] \\
&\quad + \text{classical and higher spin contributions.}
\end{aligned}
\tag{5.9}
$$

The scalaron itself manifests in the scalar functional determinant.

In the Einstein frame, the new degree of freedom explicitly appears in the action as a scalar field with a suitable potential. In this particular case, we have

$$\tilde{f}(\tilde{R}, \tilde{K}, \sigma) = \tilde{R} - \frac{3}{2}\tilde{g}^{ij}\partial_i\sigma\partial_j\sigma - \frac{3}{2}M^2\left(1 - e^{-\sigma}\right)^2 = \tilde{R} - 3\tilde{K} - \frac{3}{2}M^2\left(1 - e^{-\sigma}\right)^2. \tag{5.10}$$

It can directly be verified that the corresponding one-loop effective action is equivalent to the one in Eq. (5.9), because $V''(0) = -3M^2$. It has to be noted that for such a simple model, the background solution corresponds to $\{R = 0, \sigma = 0\}$ as a consequence of Eq. (2.4).

Through the replacement $\sigma \to \sqrt{2/3}\phi/M_{\mathrm{P}}$, we find the Lagrangian density in the standard form (see Appendix B). In this case, the expression for the one-loop effective action is quite trivial, because the background geometry is flat ($R = 0$). Therefore, we get the exact well-known result

$$\Gamma^{(1)}_{\mathrm{on-shell}} = \frac{\mathcal{V}}{2}\, M^4 \left(\ln \frac{M^2}{\mu^2} - \frac{3}{2} \right) , \tag{5.11}$$

with $\mathcal{V}$ the (infinite) volume of the manifold.

5.2 One-loop quantum-corrected R^2 gravity

In order to study the role of the one-loop quantum corrections to R^2 gravity, the off-shell one-loop effective action has to be used. The idea is to work in the Jordan frame and take the off-shell effective Lagrangian in the Landau gauge. Making use of Appendix A, for the model described by Eq. (5.8) we get

$$
\begin{aligned}
\Gamma^{(1)}_{\mathrm{Landau}} &= \frac{1}{2}\, \mathcal{V}\, M_{\mathrm{P}}^2 \left(R + \frac{R^2}{6M^2} \right) + \frac{1}{2} \ln\det\left[\frac{1}{\mu^2} \left(-\frac{1}{2}\, \Delta_0 - \frac{R}{2} \right) \right] \\
&\quad + \frac{1}{2} \ln\det\left[\frac{1}{\mu^2} \left(-\Delta_1 - \frac{R}{4} \right) \right] - \ln\det\left[\frac{1}{\mu^2} \left(-\Delta_0 - \frac{R}{2} + M^2 \right) \right] \\
&\quad - \frac{1}{2} \ln\det\left[\frac{1}{\mu^2} \left(-\Delta_2 + \frac{R(R + 12M^2)}{6(R + 3M^2)} \right) \right] .
\end{aligned}
\tag{5.12}
$$

Here, $\mathcal{V} = \frac{384\pi^2}{R^2}$ is the volume of S^4.

For the classical description of inflation in the Jordan frame (see Appendix B), the inflationary stage consists of two regimes. The first one is $M^2/R \ll 1$, where the solution is a quasi de Sitter space-time, and the second one is $M^2/R \gg 1$, in which the solution oscillates and inflation becomes over.

In the first case (during inflation), by expanding Eq. (5.12) for large R, we obtain

$$L(R) = \frac{1}{2}\, M_{\mathrm{P}}^2 \left[R + \frac{R^2}{6M^2} + \frac{R^2}{384\pi^2 M_{\mathrm{P}}^2} \left(C_1 \ln \frac{R}{\mu^2} + C_2 \right) \right] + O\left(\frac{M^2}{R} \right) , \tag{5.13}$$

where C_1 and C_2 are pure numbers, given by

$$
\begin{aligned}
C_1 &= \zeta(0|-\hat{\Delta}_1 - 3) - \zeta(0|-\hat{\Delta}_2 + 2) = O(1) , \tag{5.14} \\
C_2 &= -\zeta'(0|-\hat{\Delta}_1 - 3) + \zeta'(0|-\hat{\Delta}_2 + 2) \sim 300 , \tag{5.15}
\end{aligned}
$$

with $\hat{\Delta}$ the Laplace-Beltrami operator acting on the unitary hypersphere S^4 (see Appendix C).

For the model described by the Lagrangian in Eq. (5.13), by performing the conformal transformation as in Eq (5.4), we obtain the following action in the Einstein frame:

$$I_{\mathrm{Eins}} = \frac{1}{\kappa^2} \int d^4x \sqrt{-\tilde{g}} \left(\tilde{R} - \frac{3}{2}\tilde{g}^{ij}\partial_i\sigma\partial_j\sigma - V(\sigma) \right) , \tag{5.16}$$

where

$$V(\sigma) = (1 - e^{-\sigma})^2\, \frac{a + 2b\left\{ 1 + \log|e^{\sigma} - 1| - \log\left[4|b\mu|W\left(\frac{|e^{\sigma}-1|e^{(a+b)/2b}}{4|b\mu|} \right) \right] \right\}}{\left[4bW\left(\frac{|e^{\sigma}-1|e^{(1+b)/2b}}{4|b\mu|} \right) \right]^2} , \tag{5.17}$$

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with W the Lambert function and

$$a = \frac{1}{6M^2} + \frac{C_2}{384\pi^2 M_P^2}, \qquad b = \frac{C_1}{384\pi^2 M_P^2}. \tag{5.18}$$

As we said above, with the replacement $\sigma \to \sqrt{2/3}\phi/M_P$ we get the action in the standard form.

The slow-roll parameters are pure numbers and so for their computation we can use units such that $1/2\kappa^2 = M_P^2/2 = 1$. We acquire

$$\varepsilon = \left(\frac{1}{V}\frac{dV}{d\phi}\right)^2 = \frac{1}{3}\left(\frac{V'(\sigma)}{V(\sigma)}\right)^2, \qquad \eta = \frac{2}{V}\frac{d^2V}{d\phi^2} = \frac{2}{3}\frac{V''(\sigma)}{V(\sigma)}. \tag{5.19}$$

Finally, the spectral index n_{s} and the tensor-to-scalar ratio are expressed as [22, 23]

$$n_{\mathrm{s}} = 1 - 6\epsilon + 2\eta, \qquad r = 16\epsilon. \tag{5.20}$$

In order to execute numerical calculations, now we choose $M \sim 0.1\,M_P$, $\mu \sim M$, and $\phi_k \equiv \phi(\tilde{t}_k) \sim 7.756 M_P$, where $\tilde{t}_k$ is the time when the density perturbation with a given scale k first crosses the horizon $k/\left(\tilde{a}\tilde{H}\right) = 1$. Here, $\tilde{t}$ is time in Einstein frame and $\tilde{a}$ the scale factor. With such values for the parameters, we find $n_s \sim 0.968$ and $r = 0.0028$, which are practically the same as those obtained for the classical Starobinsky model. In fact, such values are essentially determined by the huge value of ϕ_k independently of the other parameters. Thus, what we could observe here is that the quantum gravity corrections might be small in terms of the values of n_{s} and r. It is clearly seen that the value of n_{s} could be compatible with the Planck result of $n_{\mathrm{s}} = 0.9603 \pm 0.0073\,(68\%\,\mathrm{CL})$ [3, 4], whereas that of r is out of the $1\,\sigma$ error range of the BICEP2 consequence $r = 0.20^{+0.07}_{-0.05}\,(68\%\,\mathrm{CL})$ [15]. Note, however, that we have calculated quantum corrections in the de Sitter background. Furthermore, owing to very complicated form of quantum corrections expressed in terms of zeta-functions, the expansion of the one-loop effective action has been made to get the expression in Eq. (5.13). It could be considered that the calculation of even the one-loop quantum corrections in the most general background may bring new qualitative terms changing the above values of n_{s} and r.

In the second case (at the end of inflation), taking the opposite limit $M^2/R \gg 1$ in (5.12) and using equations in Appendix C, we have the one-loop effective Lagragian with the Coleman-Weinberg quantum correction, namely,

$$\begin{aligned} L(R) &= \frac{1}{2}M_P^2\left[R + \frac{R^2}{6M^2} - \frac{M^4}{32\pi^2 M_P^2}\left(\ln\frac{M^2}{\mu^2} - \frac{3}{2}\right) + O\left(R^2\ln\frac{R}{M_P^2\mu^2}\right)\right] \\ &\sim \frac{1}{2}M_P^2 F(R). \end{aligned} \tag{5.21}$$

We also note that here, there is two natural scales: the Planck scale M_P^2 and the mass M^2, while the effective Lagrangian explicitly depends on the scale parameter μ^2. Moreover, the effective cosmological constant

$$\Lambda(\mu) = \frac{M^4}{16\pi^2 M_P^2}\left(\ln\frac{M^2}{\mu^2} - \frac{3}{2}\right), \tag{5.22}$$

is positive or negative according to whether μ^2 is smaller or larger than $M^2 e^{-3/2}$. It vanishes for $\mu^2 = M^2 e^{-3/2}$, namely, the leading term of the quantum correction is absent at this scale.

The modified gravity $L(R)$ Lagrangian may be studied in the Einstein frame, where there are the Einstein gravity plus a scalar field ϕ (i.e., inflaton), that is induced by a conformal transformation related to the Ricci scalar by

$$R = 3M^2 \left(e^{\sqrt{\frac{2}{3}} \frac{\phi}{M_{\mathrm{P}}}} - 1 \right) . \tag{5.23}$$

We explore the quantum corrections only in the second regime, relevant for the end of inflation. In such a case, the inflaton potential reads

$$V(\phi) = \frac{1}{2} M_{\mathrm{P}}^2 \left[\frac{3M^2}{2} \left(1 - e^{-\sqrt{\frac{2}{3}} \frac{\phi}{M_{\mathrm{P}}}} \right)^2 + 2\Lambda(\mu) \, e^{-2\sqrt{\frac{2}{3}} \frac{\phi}{M_{\mathrm{P}}}} \right] . \tag{5.24}$$

This potential has a minimum at $\phi = \phi_*$ defined by

$$e^{-\sqrt{\frac{2}{3}} \frac{\phi_*}{M_{\mathrm{P}}}} = 1 + \frac{4\Lambda(\mu)}{3m^2} , \tag{5.25}$$

namely, its value is slightly different from zero and given by

$$V(\phi_*) = \frac{3M_{\mathrm{P}}^2 M^2 \Lambda(\mu)}{3m^2 + 4\Lambda(\mu)} . \tag{5.26}$$

Thus, for large ϕ, the potential does not differ from that for the Starobinsky inflation, but near the minimum, there is a very small difference. This type of potential may be examined along the standard approach. The quantum corrections give contributions of the order $(M/M_{\mathrm{P}})^2$.

5.3 Stability issues

Equation (4.3) is relatively simple also for quite general models, and it may be used to investigate the stability of de Sitter background with respect to arbitrary perturbations. For this reason, we require the coefficients a_0, a_1, and a_2 to satisfy some constraints so that all the eigenvalues of the operators in Eq. (4.3) can be non negative. The smallest eigenvalues of the Laplacian operators $-\Delta_0$, $-\Delta_1$, and $-\Delta_2$ acting on scalar, vector, and tensor fields, respectively, are 0, $R/4$, and $2R/3$. Thus, only the first term on the right-hand side of Eq. (4.3) could be relevant to the stability problem. We here explore several particular cases. As a first example, we study the Einstein gravity with a cosmological constant. In this case, by using $f = R - 2\Lambda$ and Eq. (A.11) in Appendix A, we obtain the well-known result [12, 18, 19]. For another approach to the same problem, see also Ref. [25].

As a second example, we investigate a generalized model of

$$f(R, K, \phi) = F(R) - \frac{1}{2} g^{ij} \, \partial_i \phi \partial_j \phi = F(R) - K , \tag{5.27}$$

where for convenience, we use units such that $M_P^2/2 = 1$. From Eqs. (A.1)–(A.3) in Appendix A, we find

$$f_K = -1 , \qquad a_0 = 0 , \qquad a_1 = -F_R(R F_{RR} - F_R) , \qquad a_2 = -3F_R F_{RR} . \tag{5.28}$$

In this case, what is related to the stability is the ratio a_1/a_2 that has to be non negative. The stability condition then becomes

$$\frac{F_R}{F_{RR}} - R \geq 0 . \tag{5.29}$$

Such a condition is exactly the same as the one obtained for the pure $F(R)$ case [8, 26].

Another phenomenologically interesting example is the following scalar tensor model

$$f(R, K, \phi) = R - \frac{1}{2} g^{ij} \partial_i \phi \partial_j \phi - \frac{1}{2} m^2 \phi^2 + \xi R \phi^2 \,. \tag{5.30}$$

Here,

$$f_R = 1 + \xi \phi^2 \,, \quad f_K = -1 \,, \quad f_\phi = (2\xi R - m^2)\phi \,, \quad f_{\phi\phi} = 2\xi R - m^2 \,, \quad f_{R\phi} = 2\xi\phi \,. \tag{5.31}$$

On the background solution, ϕ and R are constants, given by

$$R = \frac{m^2}{2\xi} \,, \quad \phi = \pm \frac{1}{\sqrt{\xi}} \,. \tag{5.32}$$

As a consequence of the latter equation, we get $f_{\phi\phi} = 0$. The coefficients a_k read

$$a_0 = 4m^2 \,, \quad a_1 = 4(1 + 6\xi) \,, \quad a_2 = 0 \,. \tag{5.33}$$

If $\xi = -1/6$ (conformally invariant case), we have $a_1 = 0$. Hence, the bosonic sector disappears from the on-shell effective action, and eventually it plays no role. On the contrary, if $\xi \neq -1/6$, the stability of de Sitter solution is assured by condition

$$-\frac{a_0}{a_1} = -\frac{m^2}{1 + 6\xi} \geq 0 \qquad \Longrightarrow \qquad \xi < -\frac{1}{6} \,. \tag{5.34}$$

This means that the stable constant solution has the negative scalar curvature (i.e., the anti-de Sitter).

In the general case, a_2 does not vanish, and the scalar contribution may be written in a factorized multiplicative form, neglecting the multiplicative anomaly, that can be "absorbed" in the μ^2 parameter [27, 28]. As a result, we acquire

$$\begin{aligned}
\Gamma_{\text{on-shell}} &= \frac{24\pi f_0}{GR^2} + \frac{1}{2} \ln\det \left[\frac{1}{\mu^2} \left(-\Delta_0 + X_1 \right) \right] + \frac{1}{2} \ln\det \left[\frac{1}{\mu^2} \left(-\Delta_0 + X_2 \right) \right] \\
&\quad - \frac{1}{2} \ln\det \left[\frac{1}{\mu^2} \left(-\Delta_1 - \frac{R}{4} \right) \right] + \frac{1}{2} \ln\det \left[\frac{1}{\mu^2} \left(-\Delta_2 + \frac{R}{6} \right) \right] \,,
\end{aligned} \tag{5.35}$$

where

$$X_{1,2} = \frac{1}{2} \left(-\frac{a_1}{a_2} \pm \sqrt{\frac{a_1^2}{a_2^2} - \frac{4a_0}{a_2}} \right) \,. \tag{5.36}$$

To have two positive roots, the following conditions have to be satisfied

$$\frac{a_1}{a_2} < 0 \,, \qquad \left(\frac{a_1}{a_2} \right)^2 \geq \frac{4a_0}{a_2} \geq 0 \,. \tag{5.37}$$

Thus, it has been performed that using the one-loop effective action, we can analyze the stability of the maximally-symmetric background under consideration.

6 Conclusions

In the present paper, to solve issues on inflation, we have studied a generalized modified gravity model whose action is written by a generic function $f(R, K, \phi)$ of R, K and ϕ. We have explored the one-loop effective action in the de Sitter background both on-shell and off-shell in the Landau gauge. Also, we have investigated the stability of the de Sitter solutions and the one-loop quantum correction to inflation in R^2 gravity.

Moreover, we have analyzed the spectral index $n_{\rm s}$ of scalar modes of the primordial density perturbations and those tensor-to-scalar ratio r, and make the comparison of the theoretical predictions with the observational data obtained by the Planck satellite as well as the BICEP2 experiments. Consequently, it has explicitly been shown that for sets of the wider ranges of the parameters, the value of $n_{\rm s}$ can explain the Planck analysis of $n_{\rm s} = 0.9603 \pm 0.0073\,(68\%\,{\rm CL})$, while the value of r is not within the $1\,\sigma$ error range of the BICEP2 result $r = 0.20^{+0.07}_{-0.05}\,(68\%\,{\rm CL})$. For instance, $n_s \sim 0.968$ and $r = 0.0028$ can be realized (when the values of model parameters are $M \sim 0.1\,M_P$, $\mu \sim M$, and $\phi_k(= \phi(\tilde{t}_k)) \sim 7.756 M_P$, as presented in Sec. 5.2). These resultant values mainly depend on the quite-large value of ϕ_k, and therefore they are basically independent of the other model parameters. Since these results are similar to those in the Starobinsky inflation model, it is considered that quantum gravity might present only small corrections to the values of $n_{\rm s}$ and r.

As a significant outlook, it should be emphasized that not only by the BICEP2 but also by other collaborations including B-Pol [29], LiteBIRD [30], POLARBEAR [31], and QUIET [32], non-zero r might be detected in the future. These data must present us some clues to understand high-energy physics describing the early universe.

A   Explicit calculations of the coefficients

In this Appendix, we explicitly write down the coefficients which appear in the one-loop effective action for the general case $f(R, K, \phi)$. Moreover, we study the simpler case of $f(R)$, that cannot be obtained as a trivial limit of the general one.

ßThe general case: $f(R, K, \phi)$

In such a case, we have

$$a_0 = f_R \left[R f_{R\phi}^2 + f_{\phi\phi}(f_R - R f_{RR}) \right], \tag{A.1}$$

$$a_1 = f_R \left[f_K (R f_{RR} - f_R) + f_{R\phi}^2 - 3 f_{\phi\phi} f_{RR} \right], \tag{A.2}$$

$$a_2 = 3 f_K f_R f_{RR}, \tag{A.3}$$

$$
\begin{aligned}
b_0 ={}& \frac{4 f_\phi{}^2 X}{\gamma} + 4 f_\phi{}^2 R - \frac{4 f_\phi f_{R\phi} R X}{\gamma} - 4 f_\phi f_{R\phi} R^2 + \frac{f_{\phi\phi} f_R R X}{\gamma} \\
&+ f_{\phi\phi} f_R R^2 - \frac{f_{\phi\phi} f_{RR} R^2 X}{\gamma} - f_{\phi\phi} f_{RR} R^3 - \frac{f_{\phi\phi} X^2}{\gamma} - f_{\phi\phi} R X
\end{aligned}
$$

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$$+ \frac{f_{R\phi}{}^2 R^2 X}{\gamma} + f_{R\phi}{}^2 R^3 \,,$$

$$
\begin{aligned}
b_1 &= -\frac{f_K f_R R X}{\gamma} - f_K f_R R^2 + \frac{f_K f_{RR} R^2 X}{\gamma} + f_K f_{RR} R^3 \\
&\quad + \frac{f_K X^2}{\gamma} + f_K R X + \frac{4 f_\phi{}^2 f_R}{\gamma} - \frac{4 f_\phi{}^2 f_{RR} R}{\gamma} \\
&\quad + 12 f_\phi{}^2 - \frac{12 f_\phi f_{R\phi} X}{\gamma} - 20 f_\phi f_{R\phi} R + \frac{2 f_{\phi\phi} f_R X}{\gamma} + 4 f_{\phi\phi} f_R R \\
&\quad - \frac{5 f_{\phi\phi} f_{RR} R X}{\gamma} - 7 f_{\phi\phi} f_{RR} R^2 - 2 f_{\phi\phi} X + \frac{5 f_{R\phi}{}^2 R X}{\gamma} + 7 f_{R\phi}{}^2 R^2 \,,
\end{aligned}
\tag{A.4}
$$

$$
\begin{aligned}
b_2 &= -\frac{2 f_K f_R X}{\gamma} - 4 f_K f_R R + \frac{5 f_K f_{RR} R X}{\gamma} + 7 f_K f_{RR} R^2 \\
&\quad + 2 f_K X - \frac{12 f_\phi{}^2 f_{RR}}{\gamma} - 24 f_\phi f_{R\phi} + 4 f_{\phi\phi} f_R - \frac{6 f_{\phi\phi} f_{RR} X}{\gamma} \\
&\quad - 16 f_{\phi\phi} f_{RR} R + \frac{6 f_{R\phi}{}^2 X}{\gamma} + 16 f_{R\phi}{}^2 R \,,
\end{aligned}
$$

$$
b_3 = -4 f_K f_R + \frac{6 f_K f_{RR} X}{\gamma} + 16 f_K f_{RR} R - 12 f_{\phi\phi} f_{RR} + 12 f_{R\phi}{}^2 \,,
$$

$$
b_4 = 12 f_K f_{RR} \,.
\tag{A.5}
$$

$$
c_0 = R \left[f_{\phi\phi}(2 R f_R - 2 f_0 - R^2 f_{RR}) + (2 f_\phi - R f_{R\phi})^2 \right] \,,
\tag{A.6}
$$

$$
\begin{aligned}
c_1 &= f_0 \left(2 f_K R - 4 f_{\phi\phi} \right) + 12 f_\phi{}^2 - 20 f_\phi f_{R\phi} R \,, \\
&\quad + R \left(-2 f_K f_R R + f_K f_{RR} R^2 + 6 f_{\phi\phi} f_R - 7 f_{\phi\phi} f_{RR} R + 7 f_{R\phi}{}^2 R \right) \,,
\end{aligned}
\tag{A.7}
$$

$$
\begin{aligned}
c_2 &= 4 f_0 f_K - 6 f_K f_R R + 7 f_K f_{RR} R^2 - 24 f_\phi f_{R\phi} \\
&\quad + 4 f_{\phi\phi} \left(f_R - 4 f_{RR} R \right) + 16 f_{R\phi}{}^2 R \,,
\end{aligned}
\tag{A.8}
$$

$$
c_3 = -4 \left(f_K \left(f_R - 4 f_{RR} R \right) + 3 f_{\phi\phi} f_{RR} - 3 f_{R\phi}{}^2 \right) \,,
\tag{A.9}
$$

$$
c_4 = 12 f_K f_{RR} \,.
\tag{A.10}
$$

ßThe simpler case: $f(R)$.

In this case, the one-loop effective action is quite simple as

$$
\begin{aligned}
\Gamma^{(1)}_{\text{on-shell}} &= \frac{1}{2} \ln \det \left[\frac{1}{\mu^2} \left(-f_{RR} \left(\Delta_0 + \frac{R}{3} \right) + \frac{f_R}{3} \right) \right] \\
&\quad - \frac{1}{2} \ln \det \left[\frac{1}{\mu^2} \left(-\Delta_1 - \frac{R}{4} \right) \right] + \frac{1}{2} \ln \det \left[\frac{1}{\mu^2} \left(-\Delta_2 + \frac{R}{6} \right) \right] \,.
\end{aligned}
\tag{A.11}
$$

$$
\begin{aligned}
\Gamma^{(1)}_{\text{Landau}} &= \frac{1}{2} \ln \det \left[\frac{1}{\mu^4} \left(f_{RR}(6 \Delta_0^2 + 5 R \Delta_0 - 2 f_R (\Delta_0 + R) + 2 f_0 \right) \right] \\
&\quad - \frac{1}{2} \ln \det \left[\frac{1}{\mu^2} \left(-\Delta_1 - \frac{R}{4} \right) \right] - \frac{1}{2} \ln \det \left[\frac{1}{\mu^2} \left(-\Delta_0 - \frac{R}{2} \right) \right] \\
&\quad + \frac{1}{2} \ln \det \left[\frac{1}{\mu^2} \left(-\Delta_2 - \frac{R}{3} + \frac{f_0}{f_R} \right) \right] \,.
\end{aligned}
\tag{A.12}
$$

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B R^2 gravity in the Jordan frame

The equation of motion for the classical R^2 gravity becomes

$$\ddot{H} + \frac{1}{2}\,M^2 H + 3H\,\dot{H} - \frac{1}{2H}\,\dot{H}^2 = 0\,. \tag{B.1}$$

This equation is a highly non-linear second order differential equation, that has no a constant curvature solution $H = H_0$. However, we can find a solution of the form

$$H(t) = H(t_0)(1 + v(t))\,, \tag{B.2}$$

where $v(t)$ is a small quantity. At the first order in $v(t)$, we have the differential equation

$$\ddot{v} + 3H(t_0)\dot{v} + \frac{1}{2}\,M^2 v = -\frac{1}{2}\,M^2\,. \tag{B.3}$$

There may be two regimes. The first one is the early stage of the inflationary period $t_0 = t_i$, when

$$\frac{M^2}{H_i^2} \ll 1\,, \qquad H_i = H(t_i)\,. \tag{B.4}$$

If this condition is met, a solution that satisfies the initial conditions $v(t_i) = \dot{v}(t_i) = 0$ is represented as

$$H(t) = H_i \left[1 + \frac{p_2(1 - e^{p_1(t - t_i)}) - p_1(1 - e^{p_2(t - t_i)})}{p_1 - p_2}\right]\,, \tag{B.5}$$

where

$$p_1 = -\frac{3H_i}{2}\left(1 - \sqrt{1 - \frac{2M^2}{H_i^2}}\right)\,, \qquad p_2 = -\frac{3H_i}{2}\left(1 + \sqrt{1 - \frac{2M^2}{H_i^2}}\right)\,. \tag{B.6}$$

For this solution, we see that $R(t) \sim 12H_i^2$.

The other interesting regime at the end of inflation ($t_0 = t_e$) is obtained for

$$\frac{M^2}{H_e^2} \gg 1\,, \qquad H_e = H(t_e)\,. \tag{B.7}$$

The solution is an oscillating one, described by

$$H(t) = H_e \left[1 + e^{-3/2 H_e t} \sin\left(\frac{M}{\sqrt{2}}\sqrt{1 - \frac{18H_e^2}{M^2}}(t_e - t)\right)\right]\,. \tag{B.8}$$

Within this regime, R is of the order of $12H_e^2$ which is much smaller than M^2.

C   Computation of the determinants

In this Appendix, we recall some known facts about the evaluation of functional determinant of a differential elliptic non-negative operator A defined on a compact N-dimensional manifold without boundary (see, for example, [9]). The starting point is the related zeta function

$$\zeta(s|A) = \sum_n \lambda_n^{-s}, \qquad \text{Re}\, s > \frac{N}{2}, \qquad (C.1)$$

with $\lambda_n > 0$ the non-vanishing eigenvalues of A. The analytic continuation of the zeta-function is, under general conditions, regular at $s = 0$. Thus, we may define

$$\ln \det \frac{A}{\mu^2} \equiv -\zeta'\left(0 \left| \frac{A}{\mu^2}\right.\right), \qquad (C.2)$$

where the prime indicates derivative with respect to s. Looking at Eq. (4.4), we see that the one-loop effective action can be written in terms of the derivative of the zeta-functions corresponding to the Laplace-like operators acting on scalar, vector, and tensor fields on the 4-dimensional de Sitter space. In all of such cases, the eigenvalues of the Laplace operator are explicitly known and the zeta-functions can directly be computed by using Eq. (C.2).

The exact evaluation of the functional determinant is a difficult task even though one is dealing with constant curvature spaces. In our case, the functional determinant of the Laplace-like operators can be written as a finite sum of the Hurwitz zeta-functions, but for physical applications, approximate methods present acceptable results. For this reason, now we explore a method, that gives an analytic approximate expression for the determinant of an elliptic non-negative operator defined on a compact manifold $\mathcal{M}$ without boundary.

In this work, we have to deal with the Laplace-like operators L acting on scalar, vector, and tensor fields on the hyper-sphere S^4. They have the form

$$L = -\Delta + E = \frac{R}{12}\, \hat{L}_u, \qquad \hat{L}_u = -\hat{\Delta} + \frac{12}{R}\, E, \qquad (C.3)$$

where E is a constant potential term, while $\hat{\Delta}$ and $\hat{L}_u$ are operators acting on the unitary hyper-sphere S^4. It is also convenient to introduce the Laplace-like operator $\hat{L}$, given by

$$\hat{L}_u = \hat{L} + \alpha, \qquad \alpha = \frac{12}{R}\, E - \rho, \qquad (C.4)$$

with ρ a pure number depending on the spin. The eigenvalues $\hat{\lambda}_n$ and their degeneration d_n of $\hat{L}$ are well known. They can be represented in the form

$$\lambda_n = (n + \nu)^n, \qquad d_n = c_1(n + \nu) + c_3(n + \nu)^3, \qquad (C.5)$$

so that the zeta-functions of $\hat{L}$ can trivially be expressed in term of the Hurwitz-zeta functions. In fact, we get

$$\zeta(s|\hat{L}) = c_1 \zeta_{\text{H}}(2s - 1, \nu) + c_3 \zeta_{\text{H}}(2s - 3, \nu), \qquad (C.6)$$

where ν, ρ, c_1, c_3 depends on the spin according to the following table:

$$
\begin{array}{lllll}
\hat{L}_0 & \implies & \nu = \dfrac{3}{2}, & \rho = \dfrac{9}{4}, & c_1 = -\dfrac{1}{12}, & c_3 = \dfrac{1}{3}, \\[2ex]
\hat{L}_1 & \implies & \nu = \dfrac{5}{2}, & \rho = \dfrac{13}{4}, & c_1 = -\dfrac{9}{4}, & c_3 = 1, \\[2ex]
\hat{L}_2 & \implies & \nu = \dfrac{7}{2}, & \rho = \dfrac{17}{4}, & c_1 = -\dfrac{125}{12}, & c_3 = \dfrac{5}{3},
\end{array} \qquad (C.7)
$$

As is well known, the zeta-function is related to the trace of the heat kernel $K(t|A) = \text{Tr}\, e^{-tA}$ via the Mellin transform. In particular,

$$\zeta(s|\hat{L}) = \frac{1}{\Gamma(s)} \int_0^\infty dt\, t^{s-1} K(t|\hat{L}) \qquad \Longrightarrow \qquad K(t|\hat{L}) = \frac{1}{2\pi i} \int ds\, t^{-s} \Gamma(s)\zeta(s|\hat{L}). \quad \text{(C.8)}$$

Integrating the last equation, we find the asymptotic expansion of $K(t|\hat{L})$, that is,

$$K(t|\hat{L}) \sim \sum_k \hat{A}_k t^{k-2}, \qquad \hat{A}_k = \text{Res}\left(\Gamma(s)\zeta(s|\hat{L}), s = 2-k\right). \quad \text{(C.9)}$$

Clearly, the coefficients $\hat{A}_k$ depend on the spin, and they become

$$\hat{A}_0 = c_3, \qquad \hat{A}_1 = c_1, \qquad \hat{A}_2 = c_1\zeta_H(-1,\nu) + c_3\zeta_H(-3,\nu),$$

$$\hat{A}_k = \frac{(-1)^k}{(k-2)!}\left[c_1\zeta_H(3-2k,\nu) + c_3\zeta_H(1-2k,\nu)\right], \qquad \text{for } k \geq 3. \quad \text{(C.10)}$$

Now, we have all the elements necessary to compute the zeta-function for the operator $\hat{L}_u$ in Eq. (C.4). Assuming $\alpha^2 \gg 1$, we find

$$\zeta(s|\hat{L}_u) = \frac{1}{\Gamma(s)} \int_0^\infty dt\, t^{s-1} e^{-t\alpha} K(t|\hat{L})$$

$$= \frac{1}{\Gamma(s)} \int_0^\infty dt\, t^{s-3} e^{-t\alpha}(\hat{A}_0 + \hat{A}_1 t + \hat{A}_2 t^2) + G(s,\alpha), \quad \text{(C.11)}$$

where $G(s,\alpha)$ is an entire function of s, vanishing for $s \to 0$ and for $\alpha \to \infty$. In this way, we get the asymptotic expression valid for large values of α

$$\zeta(s|\hat{L}_u) = \frac{\hat{A}_0}{(s-1)(s-2)}\alpha^{2-s} + \frac{\hat{A}_1}{(s-1)}\alpha^{1-s} + \hat{A}_2\alpha^{-s} + \frac{1}{\Gamma(s)}O(\alpha^{-(1+s)}). \quad \text{(C.12)}$$

As the first consequence, we have the results

$$\zeta(0|\hat{L}_u) = \frac{\hat{A}_0}{2}\alpha^2 + \hat{A}_1\alpha + \hat{A}_2, \quad \text{(C.13)}$$

$$\zeta'(0|\hat{L}_u) = -\frac{\hat{A}_0}{2}\alpha^2\left(\ln\alpha^2 - \frac{3}{2}\right) + \hat{A}_1\alpha\,(\ln\alpha - 1) - \hat{A}_2\ln\alpha + O(\frac{1}{\alpha}), \quad \text{(C.14)}$$

and within this approximation valid for $\frac{E}{R} \gg 1$, the regularized functional determinant for the original operator in Eq. (C.4) reads

$$\ln\det\frac{L}{\mu^2} = -\zeta'\left(0\left|\frac{L}{\mu^2}\right.\right) = -\zeta'(0|\hat{L}_u) + \zeta(0|\hat{L}_u)\ln\frac{R}{12\mu^2}$$

$$= \frac{A_0}{2}E^2\left(\ln\frac{E}{\mu^2} - \frac{3}{2}\right) - A_1 E\left(\ln\frac{E}{\mu^2} - 1\right) + A_2\ln\frac{E}{\mu^2} + O(\frac{1}{E}). \quad \text{(C.15)}$$

Here, the coefficients A_k are related to $\hat{A}_k$, given by

$$A_0 = \frac{72\hat{A}_0}{R^2}, \qquad A_1 = \frac{12(\hat{A}_1 + \varrho\hat{A}_0)}{R}, \qquad A_2 = \hat{A}_2 + \varrho\hat{A}_1 + \frac{1}{2}\varrho^2\hat{A}_0. \quad \text{(C.16)}$$

Using the above results, we can obtain the explicit expression of the one-loop effective action for the generalized gravity under consideration.

References

[1] K. Bamba, S. Capozziello, S. Nojiri and S. D. Odintsov, Astrophys. Space Sci. **342**, 155 (2012) [arXiv:1205.3421 [gr-qc]].

[2] S. Nojiri and S. D. Odintsov, Phys. Rept. **505**, 59 (2011) [arXiv:1011.0544 [gr-qc]]; eConf C **0602061**, 06 (2006) [Int. J. Geom. Meth. Mod. Phys. **4**, 115 (2007)] [hep-th/0601213]; Int. J. Geom. Meth. Mod. Phys. **11**, 1460006 (2014) [arXiv:1306.4426 [gr-qc]]; S. Capozziello and M. De Laurentis,Phys. Rept. **509**, 167 (2011) [arXiv:1108.6266 [gr-qc]]; V. Faraoni and S. Capozziello, *Beyond Einstein gravity : A Survey of gravitational theories for cosmology and astrophysics*, Fundamental Theories of Physics, Vol. 170 (Springer, Dordrecht, 2010); R. Myrzakulov, L. Sebastiani and S. Zerbini, Int. J. Mod. Phys. D **22**, 1330017 (2013) [arXiv:1302.4646 [gr-qc]]; A. de la Cruz-Dombriz and D. Saez-Gomez, Entropy **14**, 1717 (2012) [arXiv:1207.2663 [gr-qc]].

[3] P. A. R. Ade *et al.* [Planck Collaboration], arXiv:1303.5076 [astro-ph.CO].

[4] P. A. R. Ade *et al.* [Planck Collaboration], arXiv:1303.5082 [astro-ph.CO].

[5] A. A. Starobinsky, Phys. Lett. B **91**, 99 (1980).

[6] F. L. Bezrukov and M. Shaposhnikov, Phys. Lett. B **659**, 703 (2008) [arXiv:0710.3755 [hep-th]].

[7] R. Kallosh, A. Linde, B. Vercnocke and W. Chemissany, arXiv:1403.7189 [hep-th]; J. Amorós, J. de Haro and S. D. Odintsov, arXiv:1402.3071 [gr-qc]; G. F. Giudice and H. M. Lee, arXiv:1402.2129 [hep-ph]; K. Bamba, S. Nojiri and S. D. Odintsov, Phys. Lett. B **731**, 257 (2014) [arXiv:1401.7378 [gr-qc]]; J. Alexandre, N. Houston and N. E. Mavromatos, Phys. Rev. D **89**, 027703 (2014) [arXiv:1312.5197 [gr-qc]]; C. Pallis, arXiv:1312.3623 [hep-ph]; A. Kehagias, A. Moradinezhad Dizgah and A. Riotto, Phys. Rev. D **89**, 043527 (2014) [arXiv:1312.1155 [hep-th]]; E. J. Copeland, C. Rahmede and I. D. Saltas, arXiv:1311.0881 [gr-qc]; L. Sebastiani, G. Cognola, R. Myrzakulov, S. D. Odintsov and S. Zerbini, Phys. Rev. D **89**, 023518 (2014) [arXiv:1311.0744 [gr-qc]]; J. Ellis, D. V. Nanopoulos and K. A. Olive, JCAP **1310**, 009 (2013) [arXiv:1307.3537]; F. Briscese, A. Marcianò, L. Modesto and E. N. Saridakis, Phys. Rev. D **87**, 083507 (2013) [arXiv:1212.3611 [hep-th]]; F. Arroja and M. Sasaki, JCAP **1208**, 012 (2012) [arXiv:1204.6489 [astro-ph.CO]]; D. S. Gorbunov and A. G. Panin, Phys. Lett. B **718**, 15 (2012) [arXiv:1201.3539 [astro-ph.CO]]; F. L. Bezrukov and D. S. Gorbunov, Phys. Lett. B **713**, 365 (2012) [arXiv:1111.4397 [hep-ph]]; D. -i. Hwang, B. -H. Lee and D. -h. Yeom, JCAP **1112**, 006 (2011) [arXiv:1110.0928 [gr-qc]].

[8] G. Cognola, E. Elizalde, S. Nojiri, S. D. Odintsov and S. Zerbini, JCAP **0502**, 010 (2005) [hep-th/0501096].

[9] E. Elizalde, S. D. Odintsov, A. Romeo, A. A. Bytsenko and S. Zerbini, *Zeta regularization techniques with applications* (World Scientific, 1994); A. A. Bytsenko, G. Cognola, L. Vanzo and S. Zerbini, Phys. Rept. **269**, 1 (1996).

[10] I. L. Buchbinder, S. D. Odintsov and I. L. Shapiro, *Effective action in quantum gravity* (IOP Publishing, Bristol, 1992).

[11] N. D. Birrell and P. C. W. Davies, *Quantum fields in curved space* (Cambridge University Press, New York, 1982); V. F. Mukhanov and S. Winirzki, *Introduction to Quantum Effects in Gravity* (Cambridge University Press, New York, 2007).

[12] E. S. Fradkin and A. A. Tseytlin, Nucl. Phys. **B234**, 472 (1984).

[13] A. Y. Kamenshchik, A. Tronconi and G. Venturi, Phys. Lett. B **726**, 518 (2013) [arXiv:1305.6138 [gr-qc]].

[14] D. Bini and G. Esposito, Phys. Rev. D **89**, 084032 (2014) [arXiv:1402.3417 [gr-qc]].

[15] P. A. R. Ade *et al.* [BICEP2 Collaboration], arXiv:1403.3985 [astro-ph.CO].

[16] P. Channuie, arXiv:1312.7122 [gr-qc]; J. Joergensen, F. Sannino and O. Svendsen, arXiv:1403.3289 [hep-ph]; K. Nakayama and F. Takahashi, arXiv:1403.4132 [hep-ph]; K. Harigaya, M. Ibe, K. Schmitz and T. T. Yanagida, arXiv:1403.4536 [hep-ph]; L. A. Anchordoqui, V. Barger, H. Goldberg, X. Huang and D. Marfatia, arXiv:1403.4578 [hep-ph]; Y. -Z. Ma and Y. Wang, arXiv:1403.4585 [astro-ph.CO]; A. Kehagias and A. Riotto, arXiv:1403.4811 [astro-ph.CO]; Y. Hamada, H. Kawai, K. -y. Oda and S. C. Park, arXiv:1403.5043 [hep-ph]; T. Kobayashi and O. Seto, arXiv:1403.5055 [astro-ph.CO]; K. Freese and W. H. Kinney, arXiv:1403.5277 [astro-ph.CO]; C. Cheng and Q. -G. Huang, arXiv:1403.5463 [astro-ph.CO]; S. Choudhury and A. Mazumdar, arXiv:1403.5549 [hep-th]; H. M. Lee, arXiv:1403.5602 [hep-ph]; Y. Gong, arXiv:1403.5716 [gr-qc]; C. Germani, Y. Watanabe and N. Wintergerst, arXiv:1403.5766 [hep-ph]; A. Ashoorioon, K. Dimopoulos, M. M. Sheikh-Jabbari and G. Shiu, arXiv:1403.6099 [hep-th]; S. Dodelson, arXiv:1403.6310 [astro-ph.CO]; N. Okada, V. N. Senoğuz and Q. Shafi, arXiv:1403.6403 [hep-ph]; D. H. Lyth, arXiv:1403.7323 [hep-ph]; E. Palti and T. Weigand, arXiv:1403.7507 [hep-th]; K. Hamaguchi, T. Moroi and T. Terada, arXiv:1403.7521 [hep-ph]; D. K. Hazra, A. Shafieloo, G. F. Smoot and A. A. Starobinsky, arXiv:1403.7786 [astro-ph.CO]; P. Di Bari, S. F. King, C. Luhn, A. Merle and A. Schmidt-May, arXiv:1404.0009 [hep-ph]; C. Cheng and Q. -G. Huang, arXiv:1404.1230 [astro-ph.CO]; I. Oda and T. Tomoyose, arXiv:1404.1538 [hep-ph]; S. Antusch and D. Nolde, arXiv:1404.1821 [hep-ph]; Y. -F. Cai, J. -O. Gong and S. Pi, arXiv:1404.2560 [hep-th]; T. Kobayashi and O. Seto, arXiv:1404.3102 [hep-ph]; C. Cheng, Q. -G. Huang and W. Zhao, arXiv:1404.3467 [astro-ph.CO].

[17] K. Bamba, R. Myrzakulov, S. D. Odintsov and L. Sebastiani, arXiv:1403.6649 [hep-th].

[18] G. W. Gibbons and M. J. Perry, Nucl. Phys. **B146**, 90 (1978).

[19] S. M. Christensen and M. J. Duff, Nucl. Phys. **B170**, 480 (1980).

[20] S. D. Odintsov, Europhys. Lett. **10**, 287 (1989); Theor. Math. Phys. **82**, 66 (1990); T. R. Taylor and G. Veneziano, Nucl. Phys. B **345**, 210 (1990).

[21] S. Nojiri and S. D. Odintsov, Int. J. Mod. Phys. A **16**, 1015 (2001) [hep-th/0009202].

[22] V. F. Mukhanov and G. V. Chibisov, JETP Lett. **33**, 532 (1981) [Pisma Zh. Eksp. Teor. Fiz. **33**, 549 (1981)].

[23] A. R. Liddle and D. H. Lyth, *Cosmological inflation and large-scale structure* (Cambridge University Press, Cambridge, United Kingdom, 2000).

[24] G. Hinshaw *et al.* [WMAP Collaboration], Astrophys. J. Suppl. **208**, 19 (2013) [arXiv:1212.5226 [astro-ph.CO]].

[25] F. Finelli, G. Marozzi, G. P. Vacca and G. Venturi, "Adiabatic regularization of the graviton stress-energy tensor in de Sitter space-time, arXiv:gr-qc/0407101.

[26] G. Cognola and S. Zerbini, J. Phys. A **39**, 6245 (2006) [hep-th/0511233].

[27] E. Elizalde, L. Vanzo and S. Zerbini, Commun. Math. Phys. **194**, 613 (1998) [hep-th/9701060].

[28] E. Elizalde, A. Filippi, L. Vanzo and S. Zerbini, Phys. Rev. D **57**, 7430 (1998) [hep-th/9710171]; E. Elizalde, G. Cognola and S. Zerbini, Nucl. Phys. B **532**, 407 (1998) [hep-th/9804118].

[29] http://www.b-pol.org/index.php.

[30] http://cmbpol.kek.jp/litebird/.

[31] http://mountainpolarbear.blogspot.jp/.

[32] D. Samtleben [QUIET Collaboration], Nuovo Cim. B **122**, 1353 (2007) [arXiv:0802.2657 [astro-ph]]; D. Araujo *et al.* [QUIET Collaboration], Astrophys. J. **760**, 145 (2012) [arXiv:1207.5034 [astro-ph.CO]]; http://quiet.uchicago.edu/index.php.