A theory of energy cost and speed of climbing

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(Received 27 June 2011; accepted 8 September 2011; published online 23 September 2011)

Climbing is an interesting form of quadrupedal locomotion on vertical substrates, and also a popular recreational activity. However, a theory of locomotor energetics of climbing has not been devised yet. Here we discuss an analytical model, based on simple physical principles, that gives the energy cost as a function of the vertical speed. We found that the energy cost monotonically decreases with speed, so that to minimize the energy spent to climb one should ascend at the highest possible speed. We propose that the actual climbing speed derives from the requirement of minimizing simultaneously the work per unit time as well as the work per unit length. Our predictions are in excellent agreement with measurements carried out on elite climbers. *Copyright 2011 Author(s). This article is distributed under a Creative Commons Attribution 3.0 Unported License*. [doi:10.1063/1.3646533]

Climbing is an interesting form of quadrupedal locomotion on vertical substrates, which has played a critical role in the hominoid locomotor evolution.[1] In addition, sports climbing is an increasingly popular recreational activity. However, differently from walking and running,[2] so far a theory of locomotor energetics for climbing is lacking. Here we derive a model starting from simple physical assumptions.

For climbing a vertical length L, a subject of mass m performs a work W that is composed of two terms: the increment in gravitational potential energy mgL, and the dissipative work E for generating the friction necessary for hanging and climbing. Thus we can write the energy cost per unit mass and lenght (EC) as:

$$EC = \frac{W}{mL} = g + \frac{E}{mL} \tag{1}$$

Substituting $L = vt$, where v is the vertical speed of the climber center of mass (CM), we get

$$EC = g + \frac{E}{mvt} \tag{2}$$

It is reasonable to assume that the work W linearly scales with m. This assumption was recently experimentally confirmed.[3] As a consequence, $E/(mt)$ in equation (2) must be independent on m. Setting $E/(mt) = k(v)$, we obtain

$$EC = g + \frac{k(v)}{v} \tag{3}$$

By expanding the dissipative power per unit mass $k(v)$ in series and retaining the first two terms we obtain

$$k(v) = k_0 + \acute{k}v \tag{4}$$

Substituting in equation (3), we finally get

$$EC = g + \acute{k} + \frac{k_0}{v} \tag{5}$$

The previous equation predicts that EC decreases by increasing the vertical speed. This characteristic behavior is due to the term k_0/v. The expansion of the dissipative power into two terms permits a straightforward physical interpretation: $g + \acute{K}$ is the force necessary to lift an unit mass sliding along a vertical surface with kinetic friction $\acute{K}$. The term $g + \acute{K}$ represents the minimum work at per unit length and mass to be done against gravity and friction. At finite velocities, the work is always greater than $g + \acute{K}$ due to the term $k_{0,}$ that represents the *isometric* (v=0) dissipative power per unit mass to hang on the holds.

Higher order terms are neglected in equation (4) because, in the first approximation, the force needed to lift the CM does not depend on the vertical speed. This statement can be justified by considering that in sports climbing the vertical motion is a sequence of cycles. In each cycle, starting from a stable stance, the climber moves only one limb at a time, keeping the other three at rest on the holds to produce the required friction. Climbing is a rhythmic motion from a static equilibrium stance to the next one and the CM moves anything but uniformly. An activity analysis on accomplished climbers showed that 38% of the route ascent time is spent in static positions.[4] Thus the average vertical speed is dominated by the duty cycle given by the ratio of the dynamic phase duration to the total duration of the cycle, rather than by the execution speed of the single movements. The force is then independent of the vertical speed.

By multiplying both terms of equation (5) by v, one gets the total power per unit mass required to climb:

$$\frac{P_{tot}}{m} = (g + \acute{k})v + k_0 \tag{6}$$

This equation predicts that the total power per unit mass is linear in v. This result can be directly compared with experimental data. Booth *et al*.[5] measured the oxygen uptake of seven subjects climbing at three different speeds (8, 10 and 12 m/min) by using a vertical climbing ergometer fitted with artificial holds. They observed that the mean oxygen uptake increased linearly with climbing speed. This finding has been confirmed by other authors.[4] Because of the well known linear dependence of the metabolic power on oxygen consumption (caloric equivalent of 20.9 MJ/m^3), a linear relationship also exists between the metabolic power and the speed, in perfect agreement with the prediction of our model.

The climbing efficiency can be obtained as the ratio between the mechanical gravitational work and the total work:

$$\eta = \frac{\mathrm{g}}{\mathrm{g} + \acute{\mathrm{k}} + \frac{\mathrm{k_0}}{\mathrm{v}}} \tag{7}$$

The efficiency η increases with v, from zero up to the maximum value $g/(g + \acute{K})$. For the data reported by Booth *et al*.,[4] the maximum efficiency is 0.2. Interestingly this asymptotic value is comparable with the typical efficiencies of running and walking. The actual efficiency is lower because a relevant fraction of the energy spent for climbing is used just to hang, an act that *per se* does not produce any useful work.

In human locomotion the choice of the gait is dictated by optimal energetic economy. Minimization of the energy cost discovers indeed the transition from walking at low speed to running at high speed.[6] Our results demonstrate that EC decreases with speed, so that optimal economy in climbing should be obtained at infinite speed. However, climbers do not ascend at their highest speed, which is related to early fatigue. This observation indicates that, differently from bipedal horizontal locomotion, the choice of the speed in vertical quadrupedal locomotion is not determined by a "least work criterion".

We then propose that the optimal speed is given by a tradeoff between the minimization of the work per unit time and the minimization of the work per unit length. In other words, climbing at the optimal speed might be the best tradeoff between the need to reduce the power necessary for hanging, which increases linearly with the speed, and the necessity to minimize EC, which decreases with the speed. Consequently, we propose that the cost of climbing (C) is given by a linear combination of P_{tot} and $m \cdot EC$:

$$C = \alpha P_{tot} + \beta mEC = \alpha m \left[\left(g = \acute{k} \right) v + k_0 \right] + \beta m \left[g + \acute{k} + \frac{k_0}{v} \right] \tag{8}$$

The minimization of C yields the optimal speed

$$v = \sqrt{\frac{\beta k_0}{\alpha \left(g + \acute{k} \right)}} \tag{9}$$

Interestingly, the optimal speed turns out to be proportional to the square root of the ratio between the isometric dissipative power per unit mass, and the minimum EC. The physical interpretation is straightforward: it is more convenient to move quickly for climbing a wall that requires a high isometric power, $k_{0,}$ to hang on; conversely, the higher the work required for moving up, the lower the optimal speed.

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