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Synthesis of Arbitrary Sidelobes Sum and Difference Patterns with Common Excitation Weights

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Introduction

The synthesis of sum and difference patterns is a canonical problem widely dealt with by researchers working on antenna array synthesis. As a matter of fact, they are used as transmitting/receiving devices for search-and-track systems (e.g., monopulse radars [1]). In this framework, several procedures have been proposed to reduce the complexity of the beam forming network aimed at generating at least a couple of radiation patterns. Among them, the generation of an optimal sum pattern and a difference one has been carried out by means of a sub-arraying strategy [2,3]. The simplification of the hardware complexity has also been addressed by sharing some excitations for the sum and difference channels [4].

Recently, the synthesis of low-sidelobe sum and difference patterns with a common aperture has been carried out by perturbing the roots of the Bayliss distribution to match as much as possible a given Taylor distribution [5]. The discrete linear arrays have been successively obtained by sampling the resulting continuous apertures.

In this work, the same array synthesis problem dealt with in [5] is addressed, and an innovative approach based on a deterministic optimization strategy is presented wherein the problem is formulated as the minimization of a linear function over a convex set. Taking advantage from the approaches proposed in [6] and [7] for the optimal synthesis of sum and difference patterns respectively, the proposed method allows one to synthesize patterns with arbitrary sidelobes (unlike [5]).

Mathematical Formulation

Let us consider a linear array of N elements equally spaced of d along the z-axis and symmetric with respect antenna centre. Accordingly, the array factor of the sum pattern is given by

$$AF_s(\vartheta) = \sum_{\substack{n=-M \\ n\neq 0}}^{M} I_n^s e^{j\beta nd\cos(\vartheta)} \tag{1}$$

and that of the difference mode is expressed as

$$AF_d(\vartheta) = \sum_{\substack{n=-M \\ n\neq 0}}^{M} I_n^d e^{j\beta nd\cos(\vartheta)} \tag{2}$$

where I_n^s and I_n^d, $n=-M,...,1,1,...,M$ are sets of excitation weights for the sum and difference patterns characterized by an even (i.e., $I_n^s = I_{-n}^s$) and odd (i.e., $I_n^d = -I_{-n}^d$) distribution, respectively. Moreover, $\beta = 2\pi/\lambda$, λ being the free space wavelength, and ϑ is the direction angle.

The approach is aimed at determining the two sets I_n^s and I_n^d, $n=-M,...,1,1,...,M$ in such a way to provide in the target direction ϑ_0 both a maximization of the slope of the difference pattern and a sum pattern amplitude greater than a given threshold, while subjecting to different and arbitrary upper bounds the sidelobes of both the patterns and sharing a number of excitation amplitudes. Accordingly, the synthesis problem can be formulated as the optimization of the following cost function

$$\min_{\mathrm{Re}\left(I_1^d\right),...,\mathrm{Re}\left(I_M^d\right),\mathrm{Im}\left(I_1^d\right),...,\mathrm{Im}\left(I_M^d\right)} \left\{ -\mathrm{Im}\left(\frac{\partial AF_d}{\partial \vartheta} \right) \right\} \Bigg|_{\vartheta=\vartheta_0} \tag{3}$$

With

$$\left| AF_d\left(\vartheta_k\right) \right|^2 \le UB_d\left(\vartheta_k\right), k=1,...,K \tag{4.a}$$

$$\mathrm{Re}\left\{ AF_s\left(\vartheta_0\right) \right\} \ge T \tag{4.b}$$

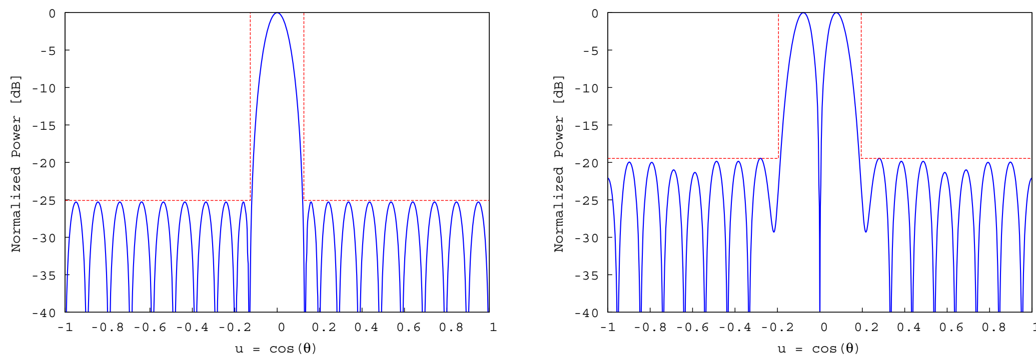
$$\left| AF_s\left(\vartheta_k\right) \right|^2 \le UB_s\left(\vartheta_k\right), k=1,...,K \tag{4.c}$$

and subject to the following constraints on the excitations

$$\left| I_n^s \right| = \left| I_n^d \right| \text{ for } |n| \ge \eta \tag{5}$$

where T is a user-defined value imposed according to the problem requirements and η is an integer threshold governing the complexity of the array structure.

In particular, since the optimal excitations for the two modes are generally very different in the central part of the array, while tending both to decay away from the center (as shown in [5]), we look herein for common amplitude weights in the border of the antenna. Moreover, UB_s and UB_d are non-negative upper bounds for the sum and difference patterns respectively, defined through a set of K samples taken on the antenna visible range. Finally, $\mathrm{Re}(\cdot)$ and $\mathrm{Im}(\cdot)$ respectively denote the real and imaginary part of the complex number at hand.



(a) (b)

Fig. 1 – Plot of the normalized power (a) sum and (b) difference patterns.

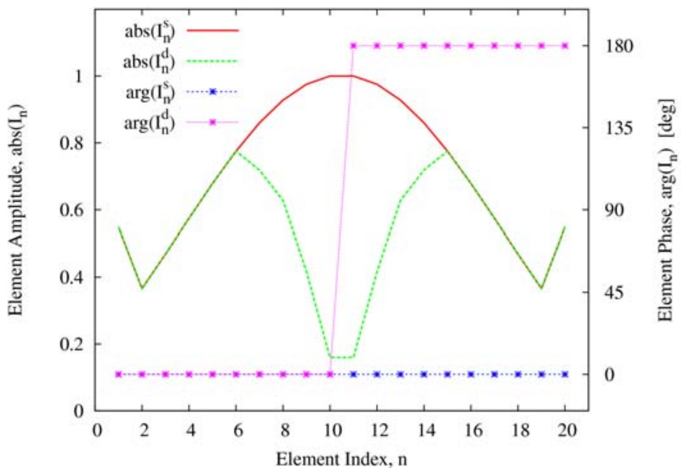


Fig. 2 – Excitation weights of the sum and difference patterns.

It is worth pointing out that, since $\left|AF_s(\vartheta_k)\right|^2$ and $\left|AF_d(\vartheta_k)\right|^2$ ($k=1,\ldots,K$) are positive semi-definite quadratic forms in terms of the excitation weights, the constraints (4.a) and (4.c) define a convex set in the space of the unknowns. Moreover, by virtue of the fact that constraints (4.b) and (5) are linear in terms of the excitations, they also define a convex set in the space of the unknowns. Since the intersection of convex sets is still convex, constraints (4)-(5) define a convex set. Finally, being (3) a linear function expressed in terms of the unknowns, the synthesis problem has been in this way formulated as the minimization of a linear function in a convex set and so can be solved in a globally optimal fashion without recurring to global optimization techniques.

Numerical Results

As a representative example, let us consider a uniform linear array with $N=20$ elements equally spaced of $d=\lambda/2$. The value η has been set to 5 such that 12 element among 20 (the 60%) are shared between the two patterns. By imposing the upper masks as shown in Fig. 1 (dashed line), the sum pattern and difference pattern reported in Fig. 1(a) and Fig. 1(b), respectively, have been achieved at the end of the optimization process completed after less than 5 seconds using a 3 GHz PC with 2 GB of RAM. The sum power pattern is characterized by a sidelobe level (SLL) equal to $SLL_s = -25.2\,dB$ and a $-3\,dB$ main lobe beamwidth (BW) of $BW_s = 5.9[\deg]$. The difference pattern has $SLL_d = -19.4\,dB$ and $BW_d = 4.9[\deg]$. The (real) excitation values synthesized by means of the Convex Programming procedure are given in Fig. 2. The two set of excitations provide a sum and a difference pattern with maximum directivity values equal to $D_s^{\max} = 12.6\,dB$ and $D_d^{\max} = 10.4\,dB$, respectively. Moreover, two difference pattern is characterized by a slope in the boresight direction of $1.15[rad^{-1}]$.

Conclusions

In this work, the synthesis of sum and difference patterns generated through linear arrays having common excitation weights has been addressed by means of an innovative approach. By virtue of the fact that the problem has been formulated as the optimization of a linear function over a convex set in the space of the unknowns, the array excitations have been obtained in a globally optimal fashion by means of a Convex Programming procedure. The main advantage of the present formulation lies in the possibility to synthesize in a fast and effective fashion whatever sum and difference beam with arbitrary upper bound masks.

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