A spectral-based stochastic ground motion model with a non-parametric time-modulating function

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Abstract: This study presents a novel fully non-stationary stochastic model for simulation of synthetic arrays of ground motion. The model employs a modulated and filtered white-noise process defined via spectral representation. This study is an extension of and improvement upon the previous work of Rezaeian and Der Kiureghian. The proposed model preserves the key advantage of separating the temporal and spectral non-stationary characteristics of the process, thus simplifying the modeling and parameter estimation. Moreover, the spectral representation used here allows added flexibility in the choice of the filter. Additionally, we propose a novel non-parametric time-modulating function based on a monotonic cubic interpolation and that has the following advantages: (i) it is directly defined by physically meaningful parameters related to time intervals of the buildup of Arias intensity of the recorded ground motion, thus simplifying the fitting procedure; (ii) it allows for a more flexible range of shapes, thereby accommodating multiple peaks. To ensure zero residual velocity and displacement, a high-pass filter is applied according to the evolutionary theory of Priestley. This, together with an energy correction factor, eliminates a bias on the cumulative energy of the post-processed simulated motions, which is implicitly present in most stochastic simulations of ground motion.

1 Introduction

There has been growing interest in modeling earthquake ground motions and in developing methods for generation of synthetic ground motions, which can be used in performance-based earthquake engineering in addition to or in place of recorded motions. Obviously, it is crucial that such synthetic motions be realistic and have characteristics that are consistent with recorded ground motions. In this paper, we propose a site-based stochastic ground motion model which directly describes the ground motion time-series recorded at a site. Such models are attractive to design engineers because they are computationally-efficient, have simple formulations, and only require readily available knowledge of the earthquake source and site characteristics such as earthquake magnitude, source-to-site distance, shear-wave velocity of the site [8, 9]. Recent examples of site-based stochastic ground motion models include the non-stationary filtered white-noise model for far-field ground motions [8, 9], a wavelet-based model [13], and a multimodal non-stationary spectral model [12]. Model parameters in all three cases are estimated by fitting to a database of recorded motions. These three models account for both temporal and spectral non-stationarity, which is an important feature of earthquake ground motions. In this paper, we develop a spectral-based parameterized stochastic model of broadband ground motion. The model employs a modulated and filtered white-noise process defined via Evolutionary Power Spectral Density (EPSD). For the time-modulating function, we propose a novel non-parametric function based on a monotonic cubic spline interpolation. The spectral non-stationarity is described by a time-frequency modulating function defined by four meaningful engineering parameters, namely main frequency and bandwidth of the motion in the strong phase, and their rates of change with respect to time. This study is an extension of and improvement upon the previous work of Rezaeian and Der Kiureghian [8, 9]. The proposed model preserves the key advantage of separating the temporal and spectral non-stationary characteristics of the process, thus simplifying the modeling and parameter estimation. Moreover, the spectral representation used here allows added flexibility in the choice of the filter. To ensure zero residual velocity and displacement, a high-pass filter is applied according to the evolutionary theory of Priestley [7]. This in conjunction with an energy correction factor eliminates a bias on the cumulative energy of the post-processed simulated motions, which is implicitly present in stochastic models of ground motion. The proposed model is fitted to recorded far-field ground motions. Example synthetic motions are then generated using the fitted model parameters, and are compared with the corresponding recorded motions.

2 Formulation of the ground motion model

Earthquakes are non-stationary in both the intensity and frequency content. The temporal non-stationarity arises from the evolution of the ground acceleration intensity over time. Typically for far-field motions (which are the focus of this study), the intensity increases from zero to a nearly constant value during the "strong motion" phase, followed by a gradual decrease back to zero. The spectral non-stationarity arises from the different arrival times of the seismic waves, which depend on the wavelength and propagation speed. Typically, P waves dominate the initial phase of the motion, and they are characterized by high-frequency content. These are followed by S waves, which usually dominate the strong phase of the motion. Finally, surface waves dominate the last phase of the motion, and they are characterized by low-frequency content. Consequently, a typical earthquake time history is a mix of these waves, with predominant frequency content that tends towards lower values with time.

In this study, we propose a site-based stochastic ground motion model which aims to represent, separately, both the temporal and spectral non-stationarity. The model builds on a filtered white noise process in the frequency domain; it can be seen as the frequency domain counterpart of the time domain formulation proposed by [8]. As was done in [8], the proposed synthetic ground motion model is defined in terms of a limited number of parameters. These parameters describe important physical characteristics of the ground motion that are of interest for engineering applications. The selected physical characteristics are related to duration, frequency content characteristics, and cumulative energy of the acceleration ground motion. In this section, we present the time and frequency modulated white noise model that is used to represent the ground acceleration process. We start by formulating and discretizing the process in the frequency domain. Then, we introduce: (i) the novel time-modulating function that describes the temporal non-stationarity of the ground motion; and (ii) the evolutionary power spectral density that describes the spectral non-stationarity of the ground motion.

2.1 Formulation and discretization in the frequency domain

A spectral representation of a zero-mean stationary stochastic process can be written as

$$\bar{X}(t) = \int_{-\infty}^{\infty} e^{-i\omega t} dZ(\omega), \tag{1}$$

where $dZ(\omega)$ is a zero-mean complex valued incremental process, i.e. $E[dZ(\omega)] = 0$, $E[dZ(\omega)dZ^H(\gamma)] = 0$ for $\omega \neq \gamma$, and $E[|dZ(\omega)|^2] = S_{\bar{X}\bar{X}}(\omega)d\omega$, where $S_{\bar{X}\bar{X}}(\omega)$ is the two-sided Power Spectral Density (PSD) of the process. If $S_{\bar{X}\bar{X}}(\omega)$ is discretized at equally spaced frequencies, i.e. $\hat{S}_{\bar{X}\bar{X}}(\omega) = \sum_k S_{\bar{X}\bar{X}}(\omega)\delta[\omega - \omega_k]$, where $\omega_k = k\Delta\omega$, and $\delta[x] = 1$ for $x = 0$ and 0 otherwise, equation (1) can be approximately rewritten as [10, 11]

$$\bar{X}(t) = \sum_{k=0}^{K} \sigma_k [u_k \sin(\omega_k t) + u_{K+k} \cos(\omega_k t)], \tag{2}$$

where u_k and u_{K+k} are statistically independent standard normal random variables, and $\sigma_k = \sqrt{2\hat{S}_{\bar{X}\bar{X}}(\omega_k)\Delta\omega}$. The process in (2) is periodic with period $T = 2\pi/\Delta\omega$, and it has a cut off frequency $\Omega = (K+1)\Delta\omega$. Moreover, it is a simple matter to show that $E[\bar{X}(t)] = 0$ and $E[\bar{X}^2(t)] = \sigma_{\bar{X}}^2 = \sum_{k=0}^{K} \sigma_k^2$. Equation (2) can be used to simulate both stationary band-limited white-noise processes by selecting $S_{\bar{X}\bar{X}}(\omega) = S_0$, or stationary band-limited colored-noise processes by selecting $S_{\bar{X}\bar{X}}(\omega) = A(\omega)S_0$, where $A(\omega)$ is a frequency modulating function. This formulation can be extended to simulate weakly non-stationary excitations via Priestley's evolutionary theory of oscillating processes [7]. Specifically, Priestley defines a modulated oscillatory process as

$$X(t) = \int_{-\infty}^{\infty} A(t,\omega) e^{-i\omega t} dZ(\omega), \tag{3}$$

where $A(t,\omega)$ is a time-frequency modulating function, and $S_{XX}(t,\omega) = |A(t,\omega)|^2 S_{\bar{X}\bar{X}}(\omega)$ is defined as the two-sided EPSD of the process. Selecting for convenience $S_{\bar{X}\bar{X}}(\omega) = 1$, equation (2) can be rewritten as

$$X(t) = \sum_{k=0}^{K} \sigma_k(t) [u_k \sin(\omega_k t) + u_{K+k} \cos(\omega_k t)], \tag{4}$$

where $\sigma_k(t) = \sqrt{2|\hat{A}(t,\omega_k)|^2\Delta\omega}$, $\hat{A}(t,\omega) = \sum_k A(t,\omega)\delta[\omega - \omega_k]$, $\hat{S}_{XX}(t,\omega) = |\hat{A}(t,\omega)|^2$, and $E[X^2(t)] = \sigma_X^2(t) = \sum_{k=0}^{K} \sigma_k^2(t)$. Observe that a unit variance process with spectral non-stationary is obtained by imposing

$$|\hat{A}(t,\omega)|^2 = \frac{|\hat{\phi}(t,\omega)|^2}{2\sum_{k=0}^{K} |\hat{\phi}(t,\omega_k)|^2 \Delta\omega}, \tag{5}$$

where $\phi(t,\omega)$ is a generic time-frequency modulating function, and $\hat{\phi}(t,\omega) = \sum_k \phi(t,\omega)\delta[\omega - \omega_k]$. A fully non-stationary process with separable time and frequency non-stationarity can be obtained by selecting

$$|\hat{A}(t,\omega)|^2 = q^2(t) \frac{|\hat{\phi}(t,\omega)|^2}{\sum_{k=0}^{K} 2|\hat{\phi}(t,\omega_k)|^2 \Delta\omega}, \tag{6}$$

where $q(t)$ is a time modulating function. Equation (4) with $|\hat{A}(t,\omega)|$ defined as (6) completely separates the spectral non-stationarity, which is completely defined by the modulating function $\phi(t,\omega)$, from the temporal non-stationarity which is completely defined by $q(t)$. Given this, it is a simple matter to show that $E[X^2(t)] = q^2(t)$.

2.2 Formulation of the time-modulating function

In this study, the temporal non-stationarity of the process is defined by a non-parametric time modulating function $q(t)$. Several models for the parametric modulating function have been proposed in the past (e.g. [2, 8, 9, 5, 12]). All these models are parameterized; the shape of the evolution of the ground motion amplitude is constrained by the function selected. These functions usually consider a single strong-motion phase preceded by a buildup phase and followed by a decay phase. To account for multiple strong motion phases, more complex formulations can be used but at the expense of a larger number of parameters. The parameters defining existing modulating functions do not all have straightforward physical interpretations (e.g., the rates of buildup and decay of strong ground motion in [5, 8], or the parameters of the "gamma" modulating function in [2, 9]). Moreover, fitting them to recorded ground motions requires solving an optimization problem. For example in [5, 8, 9], fitting is performed numerically by matching the expected cumulative energy of the process ($E\left[\frac{\pi}{2g}\int_0^t X^2(\tau)d\tau\right]=\frac{\pi}{2g}\int_0^t q^2(\tau)d\tau$) with the cumulative energy contained in the motion ($\frac{\pi}{2g}\int_0^t \ddot{u}^2_{g,rec}(\tau)d\tau$ also known as its Husid plot) over the duration of the ground motion. In [8], the integrated squared difference between the Husid plot of the modulating function and that of the recorded motion is minimized. In [5, 9], the two Husid plots are matched at a number of selected points equal to the number of modulating function parameters. The fitted modulating function parameters are calculated numerically by relating them to physically-based variables that describe the Husid plot.

In this study, the time-modulating function is directly defined by carefully selected physically meaningful parameters that describe the buildup of cumulative Arias intensity of the ground motion time series. Such a formulation simplifies the fitting procedure and allows for a more flexible range of shapes including multiple peaks. Instead of modeling the modulating function directly, we model the expected cumulative energy (or Husid plot) of the process, which is equal to the cumulative energy of the modulating function. The cumulative energy of a ground motion is a smooth, continuous, and monotonically increasing function. Therefore, we describe the cumulative energy of the modulating function using a smooth, continuous and monotone piecewise cubic interpolator or cubic Hermite spline [6]. This function can easily be fitted to pass by any number of discrete points on the cumulative energy function of a recorded ground motion. The first derivative of the function is continuous and corresponds to $\frac{\pi}{2g}q^2(t)$.

2.3 Formulation of the time-varying frequency modulating function

In this study, the spectral non-stationarity of the process is defined by a parametric time-frequency modulating function $\phi(t,\omega;\theta_\phi)$. Within the class of parametric functions, different choices can be made for selecting $\phi(t,\omega;\theta_\phi)$. For instance in [2], Broccardo and Der Kiureghian proposed a second order filter with time varying parameters, while Vlachos et al. [12] proposed a mixture of Kanai-Tajimi filters with time varying parameters. In this short study, we build on the study in[2] by selecting a second order filter, which essentially is the frequency-domain counterpart of the time-domain filter used in [8]. In particular, we can write

$$|\phi(t,\omega;\theta_\phi(t))|^2 = \frac{\omega_g^4(t)}{(\omega_g^2(t)-\omega^2)^2+4\zeta_g(t)^2\omega_g(t)^2\omega^2}, \tag{7}$$

where $\theta_\phi(t) = [\omega_g(t), \zeta_g(t)]$, and $\omega_g(t)$ and $\zeta(t)$ are the main frequency and the bandwidth of the filter, respectively. We choose a simple linear model to describe the evolution of the model parameters with time

$$\omega_g(t) = \omega_{mid} + \omega'(t-t_{mid}), \tag{8}$$

$$\zeta_g(t) = \zeta_{mid} + \zeta'(t-t_{mid}). \tag{9}$$

It follows that the complete set of model parameters describing the spectral evolution is $\theta_\phi = [\omega_{mid}, \omega', \zeta_{mid}, \zeta']$. Observe that the stationary version of (7) (i.e. when the filter parameters are time invariant) is a differentiable (only once), but not integrable process. In fact, the

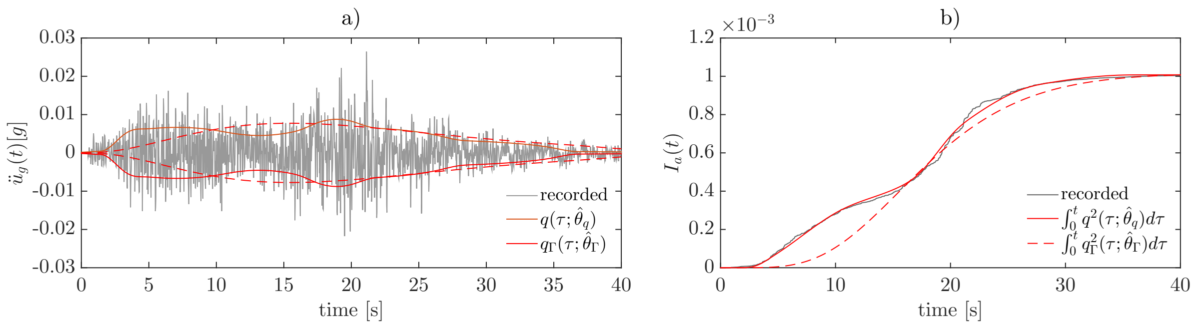

Figure 1: a) Intermediate component of 1999 Hector Mine earthquake acceleration time series and fitted non-parametric and gamma function. Station Anza - Pinyon Flat. NGA RSN 1763, b) Corresponding cumulative energy

Power Spectral Density (PSD) has non-zero value at $\omega = 0$. Non-integrability is also a property of the more classical Kanai-Tajimi filter. To overcome this caveat in the stationary case, Clough and Penzien [3] introduced a high-pass filter to enforce zero value of the PSD at $\omega = 0$. However, because (7) describes a non-stationary process (like any filter with time varying parameters), we apply the high-pass filter via Priestley's evolutionary theory [7]; see Section 4. This is shown to be convenient because the high-pass filter can also be used to ensure zero velocity and displacement residuals.

3 Parameter identification

In this study, we focus only on fitting the proposed parametric stochastic model to a given target ground motion. In this section, we describe the fitting procedure for estimating the model parameters. We choose as target ground motion the intermediate component of the 1999 Hector Mine earthquake recorded at station Station Anza - Pinyon Flat.; see Figure 1a). Since the time non-stationarity is completely decoupled from the spectral non-stationarity, we first introduce the fitting procedure for θ_q, followed by the fitting procedure for θ_ϕ.

3.1 Identification of the parameters of the time modulating function

Let $I_{a,rec}(t) = \frac{\pi}{2g}\int_0^t \ddot{u}^2_{g,rec}(\tau)d\tau$ be the cumulative energy contained in the recorded ground motion, and let $t_{z\%}$ be the time at which $z\%$ of the total Arias intensity of the component in question is reached. To overcome the arbitrariness in the start and end times of a ground motion record, we define the start time $t_s = t_{0.01\%}$ and the end time $t_f = t_{99.99\%}$. We match the Husid plots of the modulating function and recorded motion at seven discrete times namely $[t_s,\ t_{5\%}, t_{30\%},\ t_{45\%},\ t_{75\%},\ t_{95\%},\ t_f]$. Within each interval $[D_{s-5\%}, D_{5\%-30\%}, D_{30\%-45\%}, D_{45\%-75\%}, D_{75\%-95\%}, D_{95\%-f}]$, where $D_{z'-z\%} = t_{z\%} - t_{z'\%}$, a third-degree polynomial specified in terms of Hermite basis functions is fitted and constitutes a piece of the cubic Hermite spline. The coefficients of these basis functions are the values and first derivatives of the piecewise cubic function at the end points of the corresponding domain interval. The values of the function at the end points, $I_{a,rec}(t_z)$ and $I_{a,rec}(t_{z'})$, are directly specified. The derivatives are calculated using an algorithm proposed by [6] that yields a monotone piecewise cubic interpolant. For more details, see [6]. The fitted cubic Hermite spline is smoothed by a normalized Hann function with a time lag 3[s] to guarantee a higher differentiability and to be consistent with Priestley's evolutionary theory. Given the start time t_s of a recorded motion, the fitted and smoothed cumulative Arias function, $I_a(t)$, is completely defined by $\theta_q = [I_{a,rec}(t_f), D_{s-5\%}, D_{5\%-30\%}, D_{30\%-45\%}, D_{45\%-75\%}, D_{75\%-95\%}, D_{95\%-f}]$. The derivative of the fitted cumulative Arias intensity gives $\frac{\pi}{2g}q^2(t;\theta_q)$. Observe that θ_q is not a set of parameters to be estimated, but rather a set of coefficients directly available from the Husid plot of the target recorded ground motion. Figure 1a) shows the smoothed cubic Hermite spline (solid line) that is fitted to the selected target motion. Observe that the proposed modulating function is able to capture the two distinct strong motion phases, which is not possible when a "gamma" modulating function [2, 9] is used (dashed line). The Husid plots of the recorded motion and of the two fitted modulating functions are shown in Figure 1b).

3.2 Identification of the parameters of the time-variant frequency modulating function

The set of parameters $\hat{\theta}_\phi$ is estimated so that $\phi(t,\omega;\hat{\theta}_\phi)$ "best fits" an empirical EPSD, $\tilde{S}_{XX}(t_n,\omega_k)$ with $n \in [0,...N]$ and $t_n = n\Delta t$, estimated with a variation of the classical short-time Fourier transform of the target ground motion. As suggested by [12], we use the short-time Thomson's multiple-window (STTMW) spectrum estimation technique introduced by [4]. Then, $\tilde{S}_{XX}(t_n,\omega_k)$ is normalized at each instant of time to obtain

$$|\tilde{\phi}(t_n,\omega_k)|^2 = \frac{\tilde{S}_{XX}(t_n,\omega_k)}{2\sum_{k=0}^{K}\tilde{S}_{XX}(t_n,\omega_k)\Delta\omega}. \tag{10}$$

Equation (10) represents the EPSD estimation of a unit variance process, but with the same spectral evolution of the target record. Moreover, equation (10) can be interpreted as an evolutionary probability density function (since at each instant of time the area of the instantaneous EPSD is one). At each t_n, the set of parameters $\tilde{\theta}_\phi(t) = [\tilde{\omega}_g(t), \tilde{\zeta}_g(t)]$ is estimated as follow

$$\hat{\tilde{\theta}}_\phi(t_n) = \underset{\tilde{\theta}_\phi(t_n),\phi_0(t_n)}{\arg\min} \sum_{k=0}^{K}\left[\hat{\phi}(t_n,\omega_k;\tilde{\theta}(t_n))\phi_0(t_n) - \left(\sum_{m=-M/2}^{M/2}\pi(m)\tilde{\phi}(t_{n+m},\omega_k)\right)\right]^2, \tag{11}$$

where $\hat{\phi}(t,\omega;\tilde{\theta}_\phi(t)) = \sum_k \phi(t,\omega;\tilde{\theta}_\phi(t))\delta[\omega-\omega_k]$, $\phi_0(t_n)$ is a scaling factor introduced only for the optimization, $\pi(m)$ is a normalized ($\sum_m \pi(m) = 1$) Hann window function, and M is an even number. In this study, we select M such that $(M+1)\Delta t = 3[s]$. We choose to perform the optimization (11) with a weighted sum of neighboring spectra in order to guarantee a smooth transition of the filter parameters. Finally, given the set of $\hat{\tilde{\theta}}_\phi(t_n)$ we fit the parametric model described in (8) and (9) using weighted least squares optimization,

$$[\hat{\omega}_{mid},\ \hat{\omega}'] = \underset{[\omega_{mid},\ \omega']}{\arg\min} \sum_{l=0}^{L} w(t_l)\left[\hat{\tilde{\omega}}_g(t_l) - \left(\omega_{mid} + \omega'(t_l - t_{mid})\right)\right]^2, \tag{12}$$

$$[\hat{\zeta}_{mid},\ \hat{\zeta}'] = \underset{[\zeta_{mid},\ \zeta']}{\arg\min} \sum_{l=0}^{L} w(t_l)\left[\hat{\tilde{\zeta}}_g(t_l) - \left(\zeta_{mid} + \zeta'(t_l - t_{mid})\right)\right]^2, \tag{13}$$

where $t_0 = t_{5\%}$ and $t_L = t_{95\%}$, and $w(t_l)$ is given by

$$w(t_l) = \frac{q(t_l;\hat{\theta}_q)}{\sum_{l=1}^{L} q(t_l;\hat{\theta}_q)}. \tag{14}$$

The optimization is performed over the interval $D_{5\%-95\%}$ to improve the accuracy of the fitting at the strong shaking phase of the motion. Moreover, we handle the boundary intervals $D_{s-5\%}$ and $D_{95\%-f}$ by imposing $\hat{\omega}_g(t) = \hat{\omega}_g(t_{5\%})$ and $\hat{\zeta}_g(t) = \hat{\zeta}_g(t_{5\%})$ for $D_{s-5\%}$, and $\hat{\omega}_g(t) = \hat{\omega}_g(t_{95\%})$ and $\hat{\zeta}_g(t) = \hat{\zeta}_g(t_{95\%})$ for $D_{95\%-f}$. This eliminates undesired and unphysical estimation of the

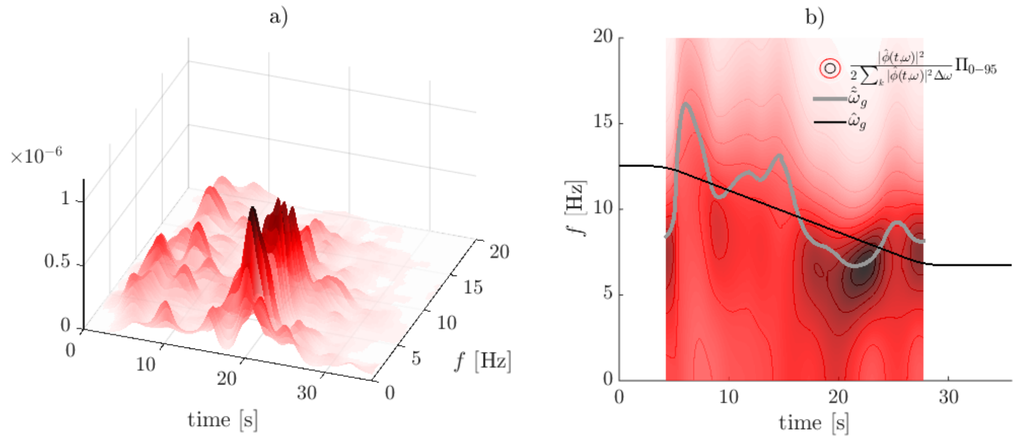

Figure 2: a) Empirical EPSD estimated via STTMW, b) Fitting procedure for $\hat{\omega}_g(t)$

filter parameters outside the $D_{5\%-95\%}$ interval. Figure 2 shows the estimated filter parameters for the target ground motion. Observe that we smooth the final version of $\hat{\omega}_g(t)$ and $\hat{\zeta}_g(t)$ to avoid abrupt changes in the spectrum. Finally, we combine $\hat{\phi}(t,\omega;\hat{\theta}_\phi)$ with $q(t,\hat{\theta}_q)$ to obtain the EPSD $\hat{S}_{XX}(t,\omega) = |\hat{A}(\omega,t)|^2$ from equation (6). Figure 3 a) shows $S_{XX}(t,\omega)$ for the given target motion.

4 Definition of the complete Evolutionary Power Spectral Density

Using the process $X(t)$ defined in equation (4) with the EPSD of equation (6) to represent a ground acceleration process has two drawbacks. First, the process is not integrable because $S_{XX}(t,0) \neq 0$; second, realizations of this process have non-zero residual velocity and displacement. Generally, to overcome the first problem a high-pass filter is applied to $\phi(t,\omega)$, i.e. $|H(\omega)|^2\phi(t,\omega)$ [2, 12], as suggested by Clough and Penzien [3] for stationary processes. The second problem is solved either by applying a second high-pass filter to each of the simulated realizations of the process, or by base-line correction [1]. In this study, we apply a single high-pass filter directly to the EPSD of the process, $S_{XX}(t,\omega)$, using the Priestley evolutionary theory. In particular, we apply a critically damped filter as used in [8]. Let $x(t)$ be a realization of $X(t)$ and let $h(t;\omega_f) = \frac{1}{\omega_f}\sin(\omega_f t)$ be the impulse response function of a critically damped oscillator, where ω_f is the cut off frequency. Then, the ground displacement time series is obtained as $u_g(t) = h(t;\omega_f) * x(t)$ (where $*$ denotes convolution for $t \in [0,+\infty)$), the velocity time series as $\dot{u}_g(t)$, and the acceleration time series as $\ddot{u}_g(t)$. In this study, instead of applying the filter to each time series $x(t)$, we derive the EPSD of the displacement process as $S_{U_gU_g}(t,\omega) = |M(t,\omega)|^2$, where

$$M(t,\omega) = \int_0^t |A(t-\tau,\omega)|h(\tau;\omega_f)e^{i\omega t}d\tau. \tag{15}$$

The EPSD $S_{U_gU_g}(t,\omega)$ can be used directly to simulate $u_g(t)$, by using equation (4) with $\sigma_k(t) = \sqrt{2S_{U_gU_g}(t,\omega_k)\Delta\omega}$, and consequently $\dot{u}_g(t)$ and $\ddot{u}_g(t)$ by differentiation. Observe that this procedure yields an acceleration process that is twice integrable and with zero residual velocity and displacement. Moreover, applying the filter as in (15) is computationally more efficient because the filter is applied once, and the complete set of time series $[\ddot{u}_g(t), \dot{u}_g(t), u_g(t)]$ is given by differentiation rather than integration. It can be shown that the EPSDs of the velocity

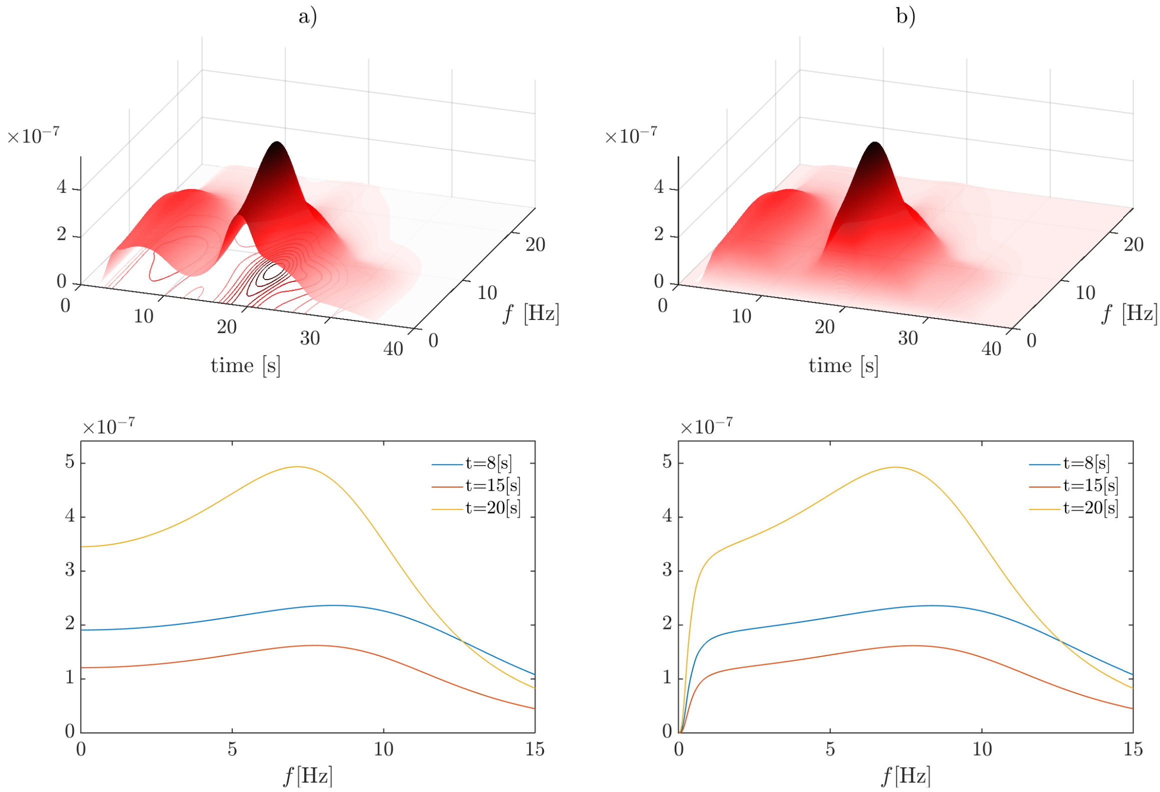

Figure 3: a) $S_{XX}(t,\omega)$, and b) $S_{\ddot{U}_g\ddot{U}_g}(t,\omega)$

and acceleration processes can be written as

$$S_{\dot{U}_g\dot{U}_g}(t,\omega) = |\dot{M}(t,\omega) + i\omega M(t,\omega)|^2, \tag{16}$$

$$S_{\ddot{U}_g\ddot{U}_g}(t,\omega) = |\ddot{M}(t,\omega) + 2i\omega M(t,\omega) - \omega^2 M(t,\omega)|^2. \tag{17}$$

It is clear that although similar, $S_{\ddot{U}_g\ddot{U}_g}(t,\omega) \neq S_{XX}(t,\omega)$. In fact, the high pass filter eliminates part of the low frequency content of the process. As shown in Figure 3, this does not affect the predominant frequency content of the process; however, it does affect the energy content. In fact, the exclusion of the low-frequencies decreases the total energy of the process. This effect can be seen in Figure 4a), where the mean cumulative energy of the simulated process is lower that the target motion. This energy bias is inherently present in all synthetic ground motions which are either filtered or base-line corrected. To account for this effect, we introduce an energy correction factor, K, defined as follow

$$K = \frac{I_a(t_f)}{\int_0^{t_f} \sum_{k=0}^{K} 2S_{\ddot{U}_g\ddot{U}_g}(t,\omega_k)\Delta\omega}. \tag{18}$$

Then, we can define the energy consistent EPSDs as $S^c_{U_gU_g}(t,\omega) = KS_{U_gU_g}(t,\omega)$, $S^c_{\dot{U}_g\dot{U}_g}(t,\omega) = KS_{\dot{U}_g\dot{U}_g}(t,\omega)$, and $S^c_{\ddot{U}_g\ddot{U}_g}(t,\omega) = KS_{\ddot{U}_g\ddot{U}_g}(t,\omega)$. Figure 4b) shows the energy consistent simulations, while Figure 4c) shows a simulated ground motion versus the target motion.

Conclusion

A novel fully non-stationary stochastic model for simulation of synthetic arrays of ground motions has been developed. The model is formulated in the frequency domain and in terms of

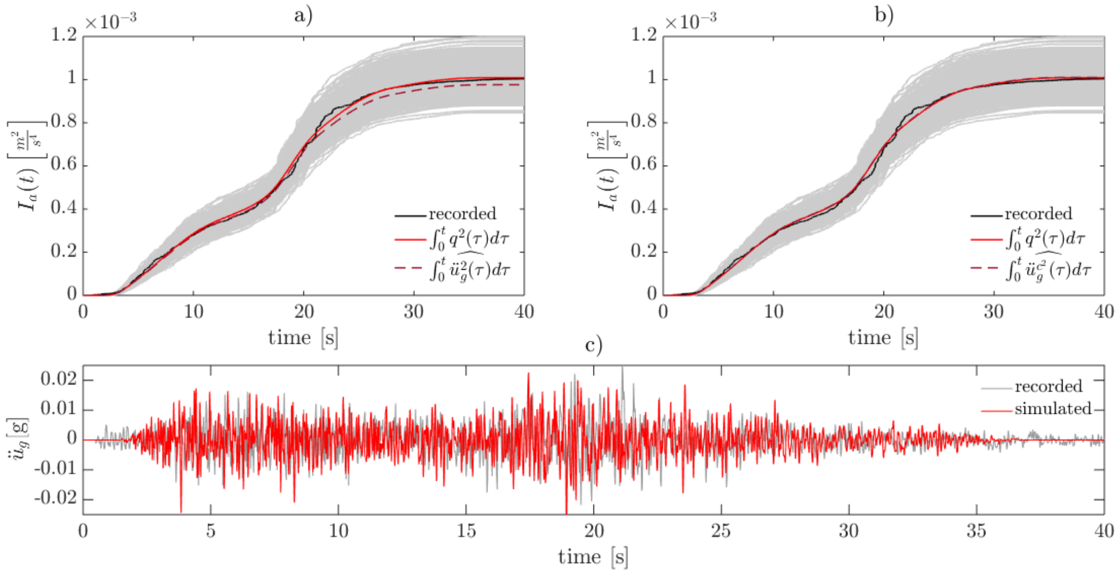

Figure 4: Cumulative Arias intensity of the recorded motions, fitted modulating function, and simulated motions with a) $\hat{S}_{\ddot{U}_g\ddot{U}_g}(t,\omega)$, and b) $\hat{S}^c_{\ddot{U}_g\ddot{U}_g}(t,\omega)$. Gray lines represent 1,000 simulations. c) Recorded vs simulated motions with $\hat{S}^c_{\ddot{U}_g\ddot{U}_g}(t,\omega)$

seven time modulating parameters and four frequency parameters. As its earlier time-domain counterparts [8, 9], the proposed model has separate temporal and spectral non-stationarity characteristics. In the current model, we propose a novel time-modulating function based on a monotonic cubic interpolator. This modulating function is completely defined by the Arias intensity intervals, which are of great importance for engineering applications. The new model accommodates a broader range of shapes and it is completely defined without any optimization procedure. The spectral non-stationarity is obtained by a second-order filter with time-varying parameters. The model is fitted to a normalized empirical evolutionary power spectral density, estimated via short-time Thomson's multiple-window spectrum estimation. A high-pass filter is applied via Priestley's evolutionary theory. It is shown that this procedure guarantees both integrability of the acceleration process and zero residual velocity and displacement. Finally, an energy correction factor is introduced to eliminate a bias in the cumulative Arias intensity. Such bias is inherently present in all stochastic models for generating ground motions that rely on high-passed filtering.

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References

[1] D. M. Boore and J. J. Bommer. "Processing of strong-motion accelerograms: needs, options and consequences". In: *Soil Dyn. Earthq. Eng.* 25.2 (2005), pp. 93–115.

[2] M. Broccardo and A. Der Kiureghian. "Simulation of near-fault ground motions using frequency-domain discretization". In: *Proceedings of the 10th National Conference on Earthquake Engineering. Anchorage, Alaska*. 2014.

[3] R. Clough and J Penzien. *Dynamics of structures*. ISBN 0-07-011392. Machw Hill, 1975, pp. 614–615.

[4] J. P. Conte and B. F. Peng. “Fully nonstationary analytical earthquake ground-motion model”. In: *J. Eng. Mech.-ASCE* 123.1 (1997), pp. 15–24.

[5] M. Dabaghi and A. Der Kiureghian. “Stochastic model for simulation of near-fault ground motions”. In: *Earthq. Eng. & Str. Dyn.* (2016), n/a–n/a.

[6] F. N. Fritsch and R. E. Carlson. “Monotone piecewise cubic interpolation”. In: *SIAM Journal on Numerical Analysis* 17.2 (1980), pp. 238–246.

[7] M. B. Priestley. “Evolutionary spectra and non-stationary processes”. In: *Journal of the Royal Statistical Society. Series B (Methodological)* (1965), pp. 204–237.

[8] S. Rezaeian and A. Der Kiureghian. “A stochastic ground motion model with separable temporal and spectral nonstationarities”. In: *Earthq. Eng. & Str. Dyn.* 37.13 (2008), pp. 1565–1584.

[9] S. Rezaeian and A. Der Kiureghian. “Simulation of synthetic ground motions for specified earthquake and site characteristics”. In: *Earthq. Eng. & Str. Dyn.* 39.10 (2010), pp. 1155–1180.

[10] S. O. Rice. “Mathematical analysis of random noise”. In: *Bell Syst. Techn. J.* 23 (1944), pp. 282–332.

[11] D. G. Shinozuka M. “Simulation of Stochastic Processes by Spectral Representation.” In: *ASME. Appl. Mech. Rev.* 44.4 (1991), pp. 191–204.

[12] C. Vlachos, K. G. Papakonstantinou, and G. Deodatis. “A multi-modal analytical nonstationary spectral model for characterization and stochastic simulation of earthquake ground motions”. In: *Soil Dyn. Earthq. Eng.* 80 (2016), 177–191.

[13] Y. Yamamoto and J. W. Baker. “Stochastic Model for Earthquake Ground Motion Using Wavelet Packets”. In: *Bull. Seismol. Soc. Amer.* 103.6 (2013), pp. 3044–3056.