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Analytic Techniques for the Design of Non-Regular Arrays

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The cost, weight, power consumption, mutual coupling effects, HW and SW complexity of large arrays can be greatly reduced by suitable *non-regular array design* techniques, that is by considering array designs with a low number of elements with respect to that of a half-wavelength equispaced array [1]-[5]. Non-regular arrays, however, are known to exhibit higher peak sidelobe levels (*PSL*) if not suitably designed [1]-[5]. As a consequence, design techniques able to control and reduce the PSL of non-regular arrays have been subject of research since their introduction [1]-[5].

Deterministic techniques were the first methodologies applied to design of non-regular arrays [3]. However, they were soon shown to provide little or no improvement over random techniques, which were therefore preferred for their simplicity [3]. In the last years, random designs have been overcome by dynamic programming and stochastic optimization techniques [2][4][5]. Such techniques, however, have some drawbacks: in fact, on the one side it can be unfeasible to apply them to very large arrays, due to their computational costs; on the other side, it is not possible to *a-priori* estimate the performances which will be obtained for a given aperture size and thinning factor, due to their stochastic nature [1]. More recently, an innovative deterministic technique able to provide good and predictable performances with very low computational efforts has been proposed for the thinning of large arrays [1]. Such technique, which exploits the two-level autocorrelation function of binary sequences derived by *difference sets* (DSs), has been shown to provide predictable advantages in terms of PSL with respect to the corresponding random designs [1]. Unfortunately, the design of a DS-based thinned array has some limitation. As a matter of fact, it is known that DSs are not always available for the desired size of the array [1].

In order to overcome the limitations of DSs while retaining, at least partially, their advantages (in terms of performances and design speed), binary sequences with the autocorrelation properties most similar to DSs, called *Almost Difference Sets* [6]-[10], have been proposed in the design of non-regular arrays [11]. Despite their good performances, their application to planar arrangements has not been investigated yet. As a consequence, in the present paper their applicability and performances when dealing both with 1D and 2D arrangements is analyzed.

Towards this end, let us consider a planar uniform lattice of $N = P \times Q$ positions spaced by $s_x \times s_y$ wavelengths ($Q = 1$ corresponds to the linear case). A non-regular array with K active elements defined on such an aperture will exhibit a power pattern equal to $W_{AF}(u,v) = \left| \sum_{p=0}^{P-1} \sum_{q=0}^{Q-1} w(p,q) \exp\left[i2\pi (p s_x u + q s_y v) \right] \right|^2$, where $w(p,q) \in \{0,1\}$ ($p = 0,..,P-1$, $q = 0,..,Q-1$) and $\sum_{p=0}^{P-1} \sum_{q=0}^{Q-1} w(p,q) = K$. By exploiting and extending the ADS-technique outlined in [11], the design of a thinned array is carried out by selecting $w(p,q) = 1$ if $(p,q) \in \underline{A}$ ($w(p,q) = 0$ otherwise), where $\underline{A}$ is the (N, K, Λ, t)-ADS at hand (construction algorithms [6]-[9] and repositories [10] of ADSs are available). By considering the same approach outlined in [11], one can show that the associated PSL complies (both in the 1D and 2D cases) with the following inequalities

$$PSL^{MIN} \le PSL^{OPT} \le PSL^{MAX} \qquad (1)$$

where $PSL^{OPT} \equiv \min_{\tau_x, \tau_y} \left[PSL\left(\underline{A}^{(\tau_x, \tau_y)} \right) \right]$ ($\tau_x = 0,..,P-1$, $\tau_y = 0,..,Q-1$), $PSL(\underline{A}^{(\tau_x,\tau_y)}) \equiv \frac{\max_{(u,v) \notin R} W_{AF}(u,v)}{W_{AF}(0,0)}$, R is the main-lobe region [7], $\underline{A}^{(\tau_x,\tau_y)}$ is the cyclically shifted version of σ_x, σ_y positions of the original ADS, and

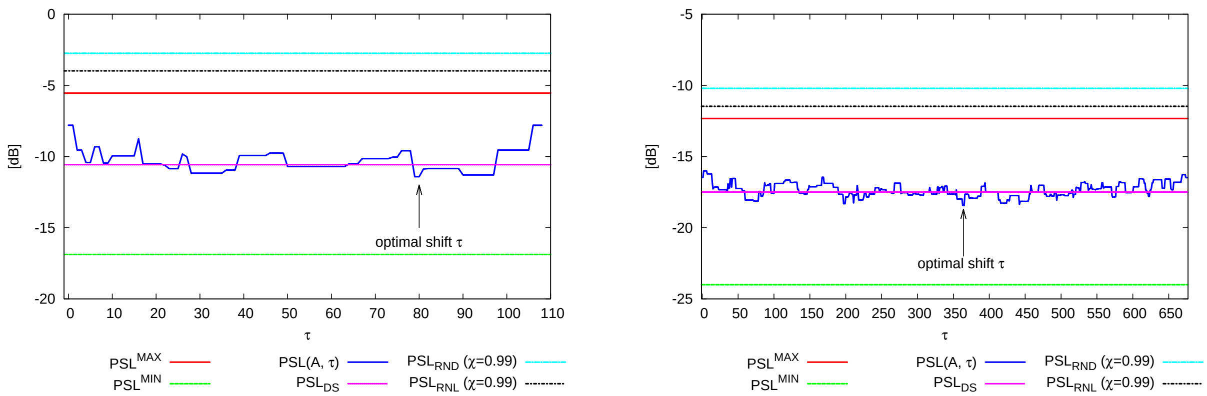
$$PSL^{MIN} = \frac{K - \Lambda - 1 - \sqrt{\frac{t(N-t)}{N-1}}}{(N-1)\Lambda + K - 1 + N - t}, \quad PSL^{MAX} = \left(0.8488 + 1.128 \log_{10} N \right) \frac{K - \Lambda - 1 + \sqrt{t(N-t)}}{(N-1)\Lambda + K - 1 + N - t}$$

($PSL^{MIN} = \frac{K-\Lambda-\sqrt{\frac{(t+1)(N-t-1)}{N-1}}}{K^2}$, $PSL^{MAX} = \left(-0.1+1.5\log_{10} N\right)\frac{K-\Lambda+\sqrt{(t+1)(N-t-1)}}{(N-1)\Lambda+K-1+N-t}$ in the planar case).

To assess the potentialities of such technique, the arrays deduced from three different ADSs (whose characteristics are resumed in Table I [10]) are analyzed, and their performances are compared to those of representative random, random lattice and DS arrays [1][3][9].

Table 1. Considered ADSs.

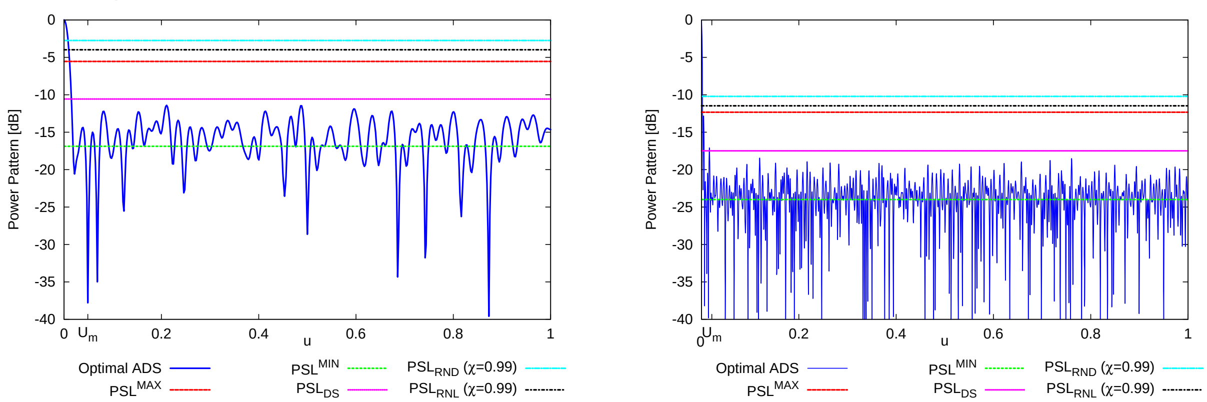
	ADS A_1	ADS A_2	ADS A_3
Geometry	1D	1D	2D
N	109	677	169
K	27	170	72
$\nu = K/N$	0.25	0.25	0.43
$\eta = t/(N-1)$	0.50	0.50	0.57



(*a*) (*b*)

Figure 1. Behavior of the PSL for the array derived from A_1 (*a*) and A_2 (*b*) and comparison with predicted performances for ADS, DS and random arrays.

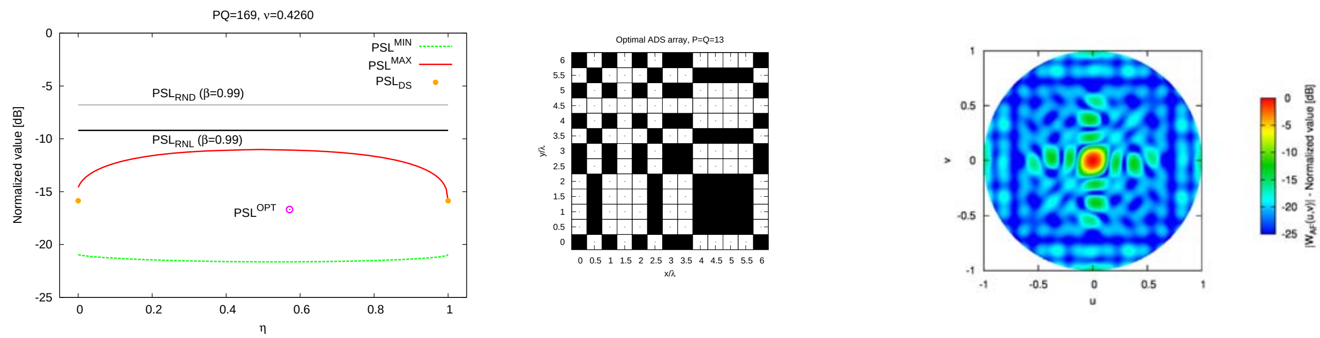
As a first numerical example, the PSL of the linear arrays derived from A_1 and A_2 is reported in Fig. 1 as a function of the applied cyclic shift τ. The obtained PSL, which complies with (1) for all τ, turns out to be always well below the upper bound PSL^{MAX} and the PSL for random and random lattice linear arrays (Fig. 1). Moreover, PSL^{OPT} is also below the expected PSL for DS arrays (Fig. 1), as it is also confirmed by the behavior of the associated power patterns (Fig. 2).



(*a*) (*b*)

Figure 2. Behavior of the power patterns for the optimal arrays derived from A_1 (*a*) and A_2 (*b*) and comparison with predicted performances for ADS, DS and random arrays.

It is worth remarking that, despite the very low number of active elements (only a quarter of the corresponding filled array), the array patterns do not exhibit any grating lobes in the visible range (Fig. 2).
As a further validation of the above conclusions, the optimal PSL obtained from the ADS A_3 is compared in Fig. 3(*a*) with the expected bounds from (1) and with the PSL values for DS and random/random lattice arrays. The reported results confirm the effectiveness of the considered approach for non-uniform array design also in the planar case [Fig 3(*a*)]. As a matter of fact, also in the 2D case the optimal arrangement resulting from the considered ADS [Fig. 3(*b*)] provides a beampattern with well controlled sidelobes [Fig. 3(*c*)], despite the low number of active elements. Moreover, it is worth remarking that the obtained PSL is well below corresponding random and DS-based designs [Fig. 3(*a*)].



(*a*) (*b*)

Figure 3. Behavior of the PSL for the array derived from A_3 and comparison with representative designs (*a*). Geometry of the optimal array resulting from the ADS A_3 (*a*) and associated power pattern (*b*).

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