Synergistic Control and Cooperative Operation of Distributed Harmonic and Reactive Compensators

Elisabetta Tedeschi[**], Paolo Tenti[*], and Paolo Mattavelli[**]

*Department of Information Engineering, University of Padova, Italy

[**]Department of Technologies and Management of Industrial Systems, University of Padova, Italy

Abstract— **The paper describes an approach to cooperative operation of a system of distributed harmonic and reactive compensators acting in the same network. The goal is to optimize the network operation at a given port by fully exploiting the capability of the various compensation units. The background for cooperative operation lays on the conservation of instantaneous complex power, a quantity which envelopes power and energy information of electrical networks and keeps validity even in presence of non-sinusoidal voltage supply and distorting loads. Based on this principle, a control approach is developed which applies to static reactive compensators (*SVC*) as well as to active power filters (*APF*). In particular, the high-power capability of *SVC* is combined with the fast response of *APF* to achieve full compensation of reactive power and harmonic currents. Single-phase and three-phase simulations demonstrate the capability of the proposed compensation approach.**

I. INTRODUCTION

The physical meaning of reactive and distortion power under non-sinusoidal conditions is a matter of discussion since the first decades of 20th century [1]. The subject is relevant not only for theoretical reasons, but because it is closely related to reactive and harmonic compensation, which is a fairly practical issue with severe technical and economic implications.

Several theories have been developed in the frequency domain [2-6] to define suitable power and current terms and to approach reactive and harmonic compensation issues in the non-sinusoidal regime [7-9].

More recently, approaches in the time domain have been devoted to the definition of instantaneous power terms, providing a background for dynamic compensation of reactive, distorting and unbalanced loads [10-16]. For this purpose, fast and accurate response is required, thus active power filters are employed [17-18], which may also help in damping harmonic oscillations [19].

In practical applications, the most popular solutions are the cheapest ones, i.e., passive filters for selective harmonic elimination or low-pass filters [20]. Thyristor-controlled static VAR compensators (*SVC*) are also popular in the high power range, because they feature high power density and good reliability at a cost which is not far from passive solutions. In spite of their fast and accurate compensation capability, active power filters (*APF*) are less common, especially in the mid/high power range, mainly because of cost.

If one approaches the compensation problem in large plants or local area networks, he will probably find a situation where some compensation devices are already installed in proximity of critical loads. Such distributed compensation units are normally designed and driven independently and perform their duty locally. They can however interact through the distribution lines, with effects which are difficult to predict, especially in case of non-sinusoidal voltage supply. Sometimes the interaction may help compensation, sometimes may result in detrimental resonances or control instabilities.

In general, coordinated selection and cooperative operation of the various units would be needed for effective distributed compensation. This is a challenging issue, analyzed in the literature in recent years [21-26] but not fully solved yet.

This paper presents the theoretical framework for the development of cooperative operation of distributed compensators, which results in desired performance at a selected network port. In particular, a control technique of *SVC* and *APF* is devised, which is based on complex power tracking principle and allows full and synergistic utilization of the available distributed compensation capability, even in presence of non-sinusoidal voltage supply and distorting loads.

II. OPERATORS

A. Basic average operators

Consider periodic quantities of *period T*, fundamental frequency $f = 1/T$ and angular frequency $\omega = 2\pi f$. The *average value* of periodic quantity *x(t)* is:

$$\bar{x} = \frac{1}{T}\int_0^T x(t)\,dt \tag{1.a}$$

The *internal product* of periodic quantities *x(t)* and *y(t)* is defined by:

$$\langle x, y\rangle = \langle y, x\rangle = \frac{1}{T}\int_0^T x(t)\,y(t)\,dt \tag{1.b}$$

Correspondingly the *norm* is:

$$\|x\| = \sqrt{\langle x, x\rangle} = \sqrt{\frac{1}{T}\int_0^T x^2 dt} = X \tag{1.c}$$

where *X* is the *rms* value of variable *x(t)*.

B. Homo-integral operator

For any given periodic quantity *x(t)* we define its *homo-integral* as:

$$\hat{x} = \omega\left(x_\int - \overline{x_\int}\right) \tag{2.a}$$

where $x_\int$ is time integral of *x*:

$$x_\int = x_\int(t) = \int_0^t x(\tau)\,d\tau \tag{2.b}$$

and $\overline{x_\int}$ is its average value. Note that homo-integral $\hat{x}$ is dimensionally homogeneous to basic quantity *x*.

C. Properties of homo-integrals

It is easy to verify that:

$$\langle x, \hat{x}\rangle = 0 \tag{3.a}$$

$$\langle \hat{x}, y\rangle = -\langle x, \hat{y}\rangle \tag{3.b}$$

978-1-4244-1668-4/08/$25.00 ©2008 IEEE

Authorized licensed use limited to: UNIVERSITA TRENTO. Downloaded on September 28,2026 at 07:59:16 UTC from IEEE Xplore. Restrictions apply.

Moreover, if x and y are *sinusoidal quantities* with *rms* values respectively equal to X and Y and phase difference equal to φ, the following properties hold:

$$\|x\| = \|\hat{x}\| = X \tag{4.a}$$

$$x^2 + \hat{x}^2 = 2X^2 \quad \forall t \in [0,T] \tag{4.b}$$

$$\langle x, y \rangle = XY\cos\varphi \qquad \langle \hat{x}, y \rangle = XY\sin\varphi \tag{4.c}$$

III. Power Terms Definition

Given electrical network Π, let u and i be the instantaneous voltage and current absorbed at network port π. Under the assumption of periodic behaviour we define the following quantities, which are valid for both sinusoidal and distorted operation.

A. Instantaneous Power Terms

The *instantaneous complex power* is defined as:

$$\dot{s} = p + jq = ui + j\hat{u}i \tag{5}$$

where the real term is *instantaneous (active) power* and the imaginary term is *instantaneous reactive power*.

Instantaneous *power terms p,q, $\dot{s}$ are conservative in every real network*. In fact, voltage terms *u and* $\hat{u}$ across every network branch comply with *KLV* (Kirchhoff's Law for Voltages), while the branch currents comply with *KLC* (Kirchhoff's Law for Currents). Therefore, the conservation (*Tellegen*) theorem applies to both real and imaginary power and, consequently, to complex power too.

B. Average Power Terms

Averaging (5) we get:

$$\dot{S} = \bar{\dot{s}} = \bar{p} + j\bar{q} = P + jQ \tag{6.a}$$

where $\dot{S}$ is the *complex power*, real term P is the *active power* and imaginary term Q is the *reactive power*. P, Q and S (absolute value of $\dot{S}$) are given by:

$$P = \langle u, i \rangle \qquad Q = \langle \hat{u}, i \rangle \qquad S = \sqrt{P^2 + Q^2} \tag{6.b}$$

While active power P represents the average power absorption at the given network port, reactive power Q is related to the average energy stored in the network (see Appendix I for a description of properties and physical meaning of Q). Observe that under sinusoidal operation, owing to (4.c) and (4.d), definitions (6.a) and (6.b) coincide with the usual ones.

C. Apparent and Distortion Power

The *apparent power* is defined by:

$$A = \|u\|\|i\| \tag{7.a}$$

It can be expressed as:

$$A = \sqrt{P^2 + Q^2 + D^2} = \sqrt{S^2 + D^2} \tag{7.b}$$

where D is *distortion power*. The origin of distortion power is discussed in Appendix II.

IV. Current Terms Definition

A. Active current i_a

It is defined as the minimum current (i.e., with minimum norm) conveying active power P absorbed by the network at port π. It is given by:

$$i_a = \frac{\langle u, i \rangle}{\|u\|^2} u = \frac{P}{\|u\|^2} u = G_e u \tag{8.a}$$

where G_e is the *equivalent port conductance.* Note that the power terms associated to current i_a are:

$$\begin{aligned} P_a &= \langle u, i_a \rangle = \|u\|\|i_a\| = \langle u, i \rangle = P \\ Q_a &= \langle \hat{u}, i_a \rangle = G_e \langle u, \hat{u} \rangle = 0 \end{aligned} \tag{8.b}$$

B. Reactive current i_r

It is defined as the minimum current conveying reactive power Q absorbed by the network at port π. It is given by:

$$i_r = \frac{\langle \hat{u}, i \rangle}{\|\hat{u}\|^2} \hat{u} = \frac{Q}{\|\hat{u}\|^2} \hat{u} = B_e \hat{u} \tag{9.a}$$

where B_e is the *equivalent port susceptance*. Note that the power terms associated to current i_r are:

$$\begin{aligned} P_r &= \langle u, i_r \rangle = B_e \langle u, \hat{u} \rangle = 0 \\ Q_r &= \langle \hat{u}, i_r \rangle = \|\hat{u}\|\|i_r\| = \langle \hat{u}, i \rangle = Q \end{aligned} \tag{9.b}$$

C. Void Current i_v

It is the remaining current term and is given by:

$$i_v = i - i_a - i_r \tag{10.a}$$

It is not associated to active or reactive power. In fact:

$$\begin{aligned} P_v &= \langle u, i_v \rangle = \langle u, i - i_a - i_r \rangle = P - P_a - P_r = 0 \\ Q_v &= \langle \hat{u}, i_v \rangle = \langle \hat{u}, i - i_a - i_r \rangle = Q - Q_a - Q_r = 0 \end{aligned} \tag{10.b}$$

The physical meaning of the void current can be explained in the frequency domain [9]. It includes *scattered currents*, due those harmonic conductances and susceptances which do not coincide with G_e and B_e, respectively, and *generated currents*, i.e., those harmonic current terms which do not exist in the voltage spectrum. It easy to check that void currents may exist only in presence of current distortion.

D. Orthogonality of Current Terms

By definition, all current terms are orthogonal, i.e:

$$\langle i_a, i_r \rangle = \langle i_r, i_v \rangle = \langle i_v, i_a \rangle = 0 \tag{11.a}$$

thus:

$$\|i\|^2 = \|i_a\|^2 + \|i_r\|^2 + \|i_v\|^2 \tag{11.b}$$

We can therefore express the apparent power as:

$$A = \|u\| \sqrt{\|i_a\|^2 + \|i_r\|^2 + \|i_v\|^2} \tag{12}$$

V. Distributed Compensation

A. Principle of Distributed Compensation

Consider Fig.1, where Π is a network including linear, non-linear and time-varying loads. A set of compensators are connected at various network ports.

Effective distributed compensation strategies aims at:

- *Cost effectiveness*: the distributed compensation system makes use of cheap units whenever possible, minimizing application of expensive compensators; redundancy and overrating are avoided and existing compensation equipment are fully exploited.
- *Cooperative operation*: compensators operation is coordinated to get full exploitation of the installed compensation capability while avoiding detrimental

Authorized licensed use limited to: UNIVERSITA TRENTO. Downloaded on September 28,2026 at 07:59:16 UTC from IEEE Xplore. Restrictions apply.

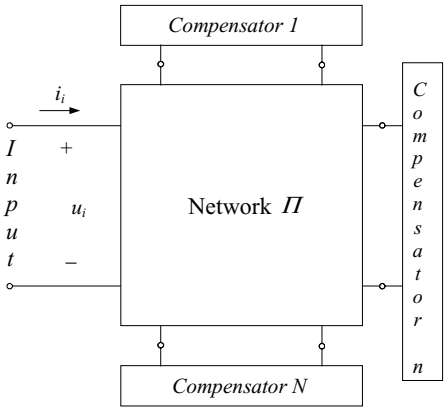


Fig. 1 – Representation of distributed compensation system

interactions of the various units (oscillations due to resonance, negative impedances generated by control, etc.).

- *Synergistic control*: The compensation units are controlled so as to keep, as a whole, the network behaviour at the input port within given specifications (power factor, total harmonic distortion, phase displacement at fundamental frequency, etc.).

B. Control Strategy of Distributed Compensators

If the compensators are off, the input current is due only to network *Π*. It can be generally split into active, reactive and void components. The function of the compensators is to reduce the unwanted components of such current, i.e., reactive and void terms, so that a suitable power factor is measured at the input port. Note that *SVC* and *APF* absorb only a marginal amount of active power, thus they do not influence active current.

The compensation strategy follows a two-level hierarchical approach.

- First, *SVC* are driven to absorb a reactive power Q^{SVC} which totally or partially compensates reactive power Q^{π} absorbed by the network, thus reducing reactive current absorption. As mentioned before, however, *SVC* may inject current harmonics, thus worsening void current absorption.
- Second, *APF* are driven to compensate - totally or partially - the remaining reactive and void current, including void current component caused by *SVC*.

C. Remote Control of Compensators by Power Command

In general, the compensators are connected to network ports different from the input port and are therefore fed by voltages different from the input voltage. In fact, line impedances may cause voltage drops, which result in phase shift and increased voltage distortion, while transformers may cause level shift and phase rotation. However, if *the compensation command is given in power terms* it is marginally affected by line impedances and transformers, which contribute to total power absorption only with their parasitic parameters. In addition, a power command naturally allows sharing of the compensation effort among the units according to their compensation capability, which is related to power ratings.

According to this approach *SVC* are driven by a reactive power command Q^{SVC}, which sets the inductive or capacitive kVAR demand. Instead, *APF* are driven by instantaneous power command $\dot{s}^{APF}$, which is then transformed into suitable current reference. The general *APF* control scheme is shown in Fig.2.

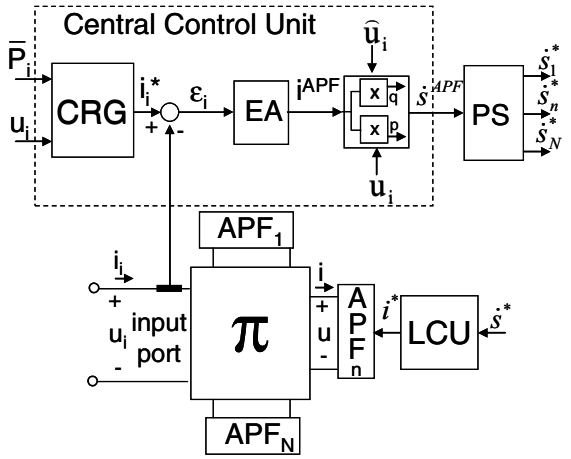


Fig. 2 – Structure of control system

The *Central Control Unit* (*CCU*) performs different functions, represented by three blocks:

- *CRG* (Current Reference Generator): it determines input current reference i_i^* as a function of required performances (*PF*, *THD*), voltage u_i and measured active power absorption $\bar{p}_i$ (moving average filter);
- *EA* (Error Amplifier): it processes input current error signal ε_i to produce *APF* current reference i^{APF}, which is then transformed into complex power reference:

$$\dot{s}^{APF} = p^{APF} + jq^{APF} = u_i i^{APF} + j\hat{u}_i i^{APF} \qquad (13)$$

For three phase applications, the complex power reference is obtained by adding the contributions of the individual phases.

In order to mitigate the effects of control delays, error amplifier *EA* is based on harmonic reference frames [18], which ensures precise current tracking even in presence of control delays, provided that load current and supply voltage are periodic and/or slowly varying.

- *PS* (Power Sharing unit): it distributes the complex power reference among the various *APF* according to their power ratings and utilization factor.

The *Local Control Unit* (*LCU*) transforms the complex power command received by each *APF* unit into a suitable current reference, according to the approach discussed in the following section.

VI. APF Control by Complex Power Command

The generation of a current reference from the power command follows different approaches depending on the

Authorized licensed use limited to: UNIVERSITA TRENTO. Downloaded on September 28,2026 at 07:59:16 UTC from IEEE Xplore. Restrictions apply.

kind of *APF*: three-phase three-wires, single-phase, three-phase four-wires.

A. Three-phase Three-wire Units

In three-phase three-wire systems there is a univocal correspondence between the complex power absorbed at a given port and the corresponding currents. In fact, let u_n be the port voltages and i_n the port currents, the instantaneous complex power is:

$$\dot{s} = p + jq = \sum_{n=1}^{3} u_n i_n + j\sum_{n=1}^{3} \hat{u}_n i_n \qquad (14.a)$$

On the other side, given the complex power we have:

$$\begin{cases} \sum_{n=1}^{3} u_n i_n = p \\ \sum_{n=1}^{3} \hat{u}_n i_n = q \\ \sum_{n=1}^{3} i_n = 0 \end{cases} \Rightarrow \begin{vmatrix} u_1 & u_2 & u_3 \\ \hat{u}_1 & \hat{u}_2 & \hat{u}_3 \\ 1 & 1 & 1 \end{vmatrix} \begin{vmatrix} i_1 \\ i_2 \\ i_3 \end{vmatrix} = \begin{vmatrix} p \\ q \\ 0 \end{vmatrix} \qquad (14.b)$$

Since the matrix can be inverted, the currents can be univocally determined from the complex power.

Thus, when the *LCU* receives complex power command $\dot{s}^*$, it sets current references $\underline{i}^*$ by solving:

$$\begin{vmatrix} i_1^* \\ i_2^* \\ i_3^* \end{vmatrix} = \begin{vmatrix} u_1 & u_2 & u_3 \\ \hat{u}_1 & \hat{u}_2 & \hat{u}_3 \\ 1 & 1 & 1 \end{vmatrix}^{-1} \begin{vmatrix} p^* \\ q^* \\ 0 \end{vmatrix} \qquad (14.c)$$

B. Single-phase Units

For a given *APF* supply voltage *u*, the task of the *LCU* is to determine current reference i^* which better suits complex power reference $\dot{s}^*$. In this case, the real and imaginary part of the equation $\dot{s}^* = u i^* + j\hat{u} i^*$ set two independent conditions, which cannot be met by a single value of i^*. The current reference is therefore selected to minimize the error function:

$$\psi = a\left(u i^* - p^*\right)^2 + b\left(\hat{u} i^* - q^*\right)^2 \qquad (15.a)$$

where *a* and *b* are proper weights. The solution is:

$$i^* = \frac{a u p^* + b\hat{u} q^*}{a u^2 + b\hat{u}^2} \qquad (15.b)$$

Coefficients *a* and *b* can be dynamically adjusted to provide good accuracy, in spite of the variation of power and voltage terms, while ensuring smooth operation even around the critical times when *u* and $\hat{u}$ vanish. It can be shown that an effective selection is:

$$a = \frac{u^2}{\hat{u}^2} \;\; if \;\; \left|\frac{u}{\hat{u}}\right| < 1 \qquad b = \frac{\hat{u}^2}{u^2} \;\; if \;\; \left|\frac{\hat{u}}{u}\right| < 1 \qquad (15.c)$$

while keeping the condition:

$$a u^2 + b\hat{u}^2 = u^2 + \hat{u}^2 \qquad (15.d)$$

C. Three-phase Four-wire Units

In this case the *APF* can control homopolar currents, which can be used to compensate for zero-sequence power absorption. Let u_0 be the voltage of the neutral wire and i_0 the neutral current, the instantaneous power terms absorbed by the *APF* are:

$$p = \sum_{n=0}^{3} u_n i_n \qquad q = \sum_{n=0}^{3} \hat{u}_n i_n \qquad (16.a)$$

Selecting the voltage reference so that $u_1 + u_2 + u_3 = 0$ and observing that $i_1 + i_2 + i_3 + i_0 = 0$, (16.a) gives:

$$\begin{aligned} p &= u_0 i_0 + \sum_{n=1}^{3} u_n \left(i_n + \frac{i_0}{3}\right) = p_0 + \sum_{n=1}^{3} u_n i_{no} \\ q &= \hat{u}_0 i_0 + \sum_{n=1}^{3} \hat{u}_n \left(i_n + \frac{i_0}{3}\right) = q_0 + \sum_{n=1}^{3} \hat{u}_n i_{no} \end{aligned} \qquad (16.b)$$

where $i_{1o} + i_{2o} + i_{3o} = 0$ Equation (16.b) shows that zero-sequence components can be treated separately from the other ones. Given total power reference $\dot{s}^* = p^* + jq^*$ and, separately, zero-sequence reference $\dot{s}_o^* = p_o^* + jq_o^*$ we obtain:

$$\begin{vmatrix} i_{1o}^* \\ i_{2o}^* \\ i_{3o}^* \end{vmatrix} = \begin{vmatrix} u_1 & u_2 & u_3 \\ \hat{u}_1 & \hat{u}_2 & \hat{u}_3 \\ 1 & 1 & 1 \end{vmatrix}^{-1} \begin{vmatrix} p^* - p_o \\ q^* - q_o \\ 0 \end{vmatrix} \qquad (16.c)$$

which is the solution for non-zero-sequence current references i_{1o}^*, i_{2o}^*, i_{3o}^*. As concerns zero-sequence current reference i_0^*, it can be obtained from power reference $\dot{s}_o^*$ according to the minimum error approach followed for single-phase units.

VII. Application Examples

Let's consider first the three-phase network of Fig.3. It includes: three-phase wye/delta transformer feeding ohmic-inductive load R_L-L_L (Z_L = 1 pu, $cos\phi$ = 0.9) through an inductive connection line L_S (Z_S =0.05 pu at fundamental frequency); a *SVC*, which includes a resonant filter tuned on the fifth harmonic and a *TCR*; an *APF* connected in parallel to the load, acting as controlled current source with high-frequency filter capacitors C_F.

The goal of the compensation is to make the network appearing purely resistive at the input terminals by proper control of *SVC* and *APF*.

The SVC is designed and driven to compensate for the reactive power absorbed by the load while injecting limited current harmonics. The *APF* performs the rest of the job by compensating the remaining reactive and void current components.

Fig.4.a shows the simulated waveforms of the input voltage, which is slightly distorted, and current. Initially ($t < 60$ *ms*) all compensators are off and only the load current appears at the input port. At $t = 60$ *ms*, the *SVC* is switched on and compensates for the reactive power of the load at the expense of increased input current distortion. At $t = 100$ *ms*, the *APF* is switched on and, after a ramp-up, brings the system to a full compensation.

Fig. 4.b shows the corresponding real and imaginary power absorption of load, *SVC, APF* and total network.

The active power is due to R-L load, while *SVC* and *APF* do not contribute to such power term. Instead, they absorb fluctuating components of real power, which are associated to harmonics and void current.

Authorized licensed use limited to: UNIVERSITA TRENTO. Downloaded on September 28,2026 at 07:59:16 UTC from IEEE Xplore. Restrictions apply.

The load reactive power is compensated by the *SVC*. The *APF* absorbs an imaginary power fluctuation term, which relates to harmonic and void current compensation.

As a whole, the network behaves as a resistive load and the *SVC* and *APF* perform cooperatively although connected to different network ports.

As a second case of application, let's consider the single-phase replica of the network of Fig.3 with the *SVC* substituted by a capacitor bank (Z_C = 2,3 pu). The *APF* takes charge of the entire compensation effort.

In order to show the capability of the proposed control approach, it is assumed that the line inductance is very

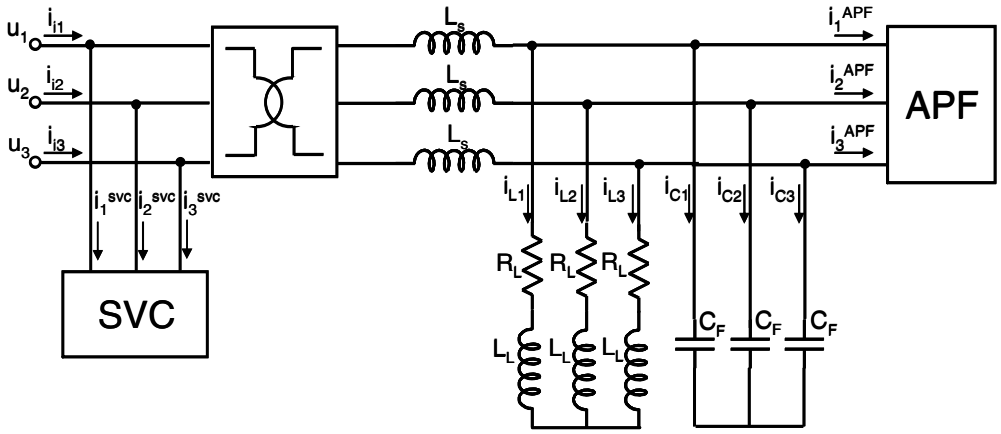


Fig. 3 – Application example: three phase network

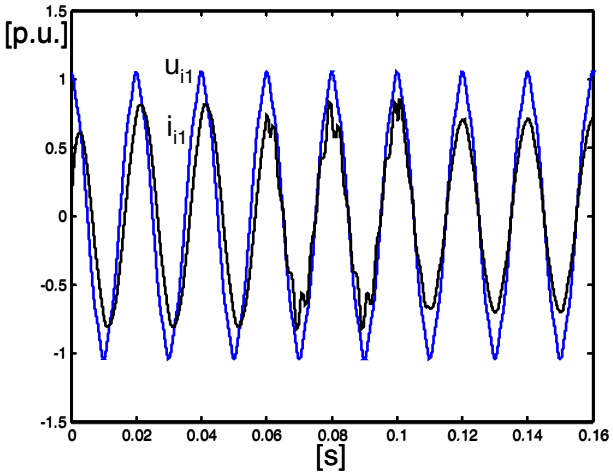


Fig. 4A – Three-phase application: Phase 1 input current (plotted with 0.8 scale factor) and voltage

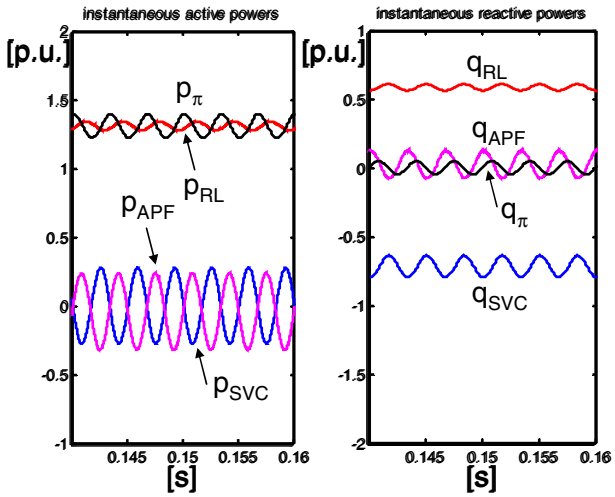


Fig. 4B – Three-phase application: Instantaneous active and reactive power in steady state-conditions

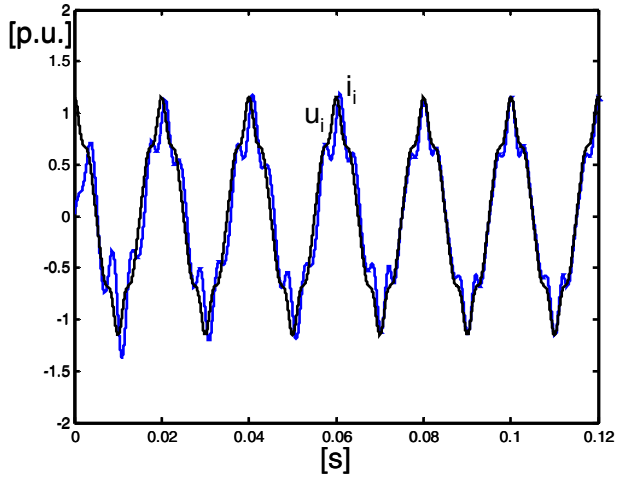


Fig. 5A – Single phase application: Input current and voltage

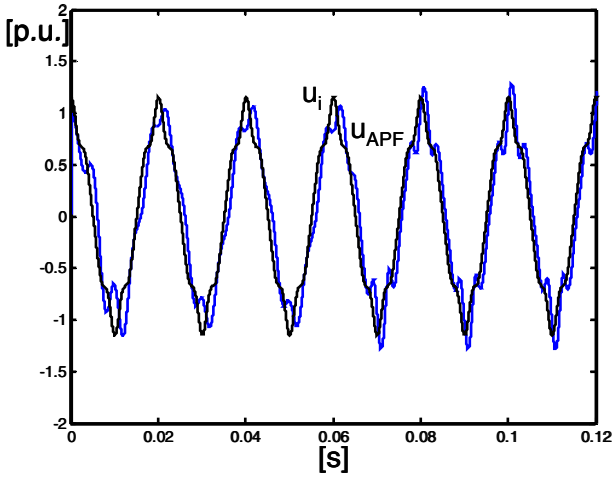


Fig. 5B – Single-phase application: Input voltage and *APF* voltage

Authorized licensed use limited to: UNIVERSITA TRENTO. Downloaded on September 28,2026 at 07:59:16 UTC from IEEE Xplore. Restrictions apply.

high (Z_s = 0.2 pu), causing a significant voltage difference between the input and *APF* terminals (Fig.5.b). Moreover, a high distortion of the input voltage is considered (10% of 5th harmonic, 5% of 7th harmonic).

Fig. 5.a shows the input current absorption before and after insertion of the *APF*, while Fig.5.b shows the *APF* voltage. In both figures the input voltage is plotted too.

With the *APF* on, the input current tracks the input voltage with good accuracy, in spite of the inherent error introduced by the computation algorithm (15.a).

Also in this case the *APF* performs well, in spite of the heavy effect of the line impedance.

CONCLUSIONS

A general approach to the problem of cooperative operation of various kinds of compensators distributed in the same network was presented. The approach lays on a general theory related to the definition of power and current terms under non-sinusoidal conditions.

In particular, reference was made to the instantaneous complex power, a conservative quantity which extends the properties of the complex power to the non-sinusoidal domain.

Control of static reactive compensators and active power filters based on instantaneous complex power tracking was then discussed for three-phase three-wire, single-phase and three-phase four-wire systems. In all cases, the power absorption at the input port can be effectively controlled by acting on the distributed compensator system.

Some application examples were finally shown, which demonstrate the capabilities of the proposed approach even in case of high voltage distortion and nonlinear loads.

APPENDIX I

Physical meaning of reactive power

For *linear passive networks*, reactive power Q is related to the total average energy stored in the network.

Let's compute reactive power Q for elementary linear components.

- for a *resistor* we have:

$$u = Ri \Rightarrow Q_R = \langle \hat{u}, i \rangle = {}^{1}\!/_{R} \langle \hat{u}, u \rangle = 0 \qquad \text{(I.1)}$$

- for an *inductor* we have:

$$\hat{u} = \omega L i \Rightarrow Q_L = \langle \hat{u}, i \rangle = \omega L \|i\|^2 = 2\omega W_L \qquad \text{(I.2)}$$

where W_L is the average energy in the inductor:

$$W_L = \frac{1}{T}\int_0^T \frac{1}{2} L i^2 dt = \frac{1}{2} L \|i\|^2 \qquad \text{(I.3)}$$

- for a *capacitor* we have:

$$\hat{i} = \omega C u \Rightarrow Q_C = -\omega C \langle u, \hat{i} \rangle = \\ = -\omega C \|u\|^2 = -2\omega W_C \qquad \text{(I.4)}$$

where W_C is the average energy in the capacitor:

$$W_C = \frac{1}{T}\int_0^T \frac{1}{2} C u^2 dt = \frac{1}{2} C \|u\|^2 \qquad \text{(I.5)}$$

Considering that Q is a conservative quantity, we can say that *under periodic operation*, either sinusoidal or distorted, *the reactive power Q measured at the input port of a passive linear network* is:

$$Q = 2\omega (W_{Lt} - W_{Ct}) \qquad \text{(I.6)}$$

where W_{Lt} and W_{Ct} are respectively *total average inductive energy* and *total average capacitive energy* in the network, given by:

$$W_{Lt} = \sum_{k=1}^{N_L} W_{Lk} = \sum_{k=1}^{N_L} \frac{1}{2} L_k I_{Lk}^2 \quad W_{Ct} = \sum_{k=1}^{N_C} W_{Ck} = \sum_{k=1}^{N_C} \frac{1}{2} C_k U_{Ck}^2 \qquad \text{(I.7)}$$

In (I.7) L_k are all network inductors and I_{Lk} their *rms* currents. Similarly, C_k are all network capacitors and U_{Ck} their *rms* voltages.

We can conclude that while *active power P represents average power absorption*, *reactive power Q represents* (in linear passive networks) *average energy storage*, inductive energy being counted with positive sign and capacitive energy with negative sign.

Moreover, *whichever is the origin of reactive power, it can be compensated by reactive elements*.

APPENDIX II

Origin of distortion power

The apparent power can be expressed as:

$$A^2 = \|u\|^2 \|i_a\|^2 + \|u\|^2 \|i_r\|^2 + \|u\|^2 \|i_v\|^2 = P^2 + \frac{\|u\|^2}{\|\hat{u}\|^2} Q^2 + \|u\|^2 \|i_v\|^2$$

Thus:

$$D^2 = A^2 - P^2 - Q^2 = \frac{\|u\|^2 - \|\hat{u}\|^2}{\|\hat{u}\|^2} Q^2 + \|u\|^2 \|i_v\|^2 = D_u^2 + D_i^2$$

where: (II.1)

$$D_u = Q \sqrt{\frac{\|u\|^2 - \|\hat{u}\|^2}{\|\hat{u}\|^2}} = Q\sigma_u \qquad \text{(II.2)}$$

$$D_i = \|u\| \|i_v\| \qquad \text{(II.3)}$$

- D_u is called *voltage distortion power*, and exists only in presence of reactive power absorption and voltage distortion at the measuring port. In fact, under sinusoidal operation $\|u\| = \|\hat{u}\|$ and D_u vanishes.
- D_i is called *current distortion power* and exists only in presence of void current absorption, i.e., current distortion at the measuring port.

Contrary to active and reactive power, *distortion power is a non-conservative quantity*.

APPENDIX III

Distortion factors and power factor

The *voltage distortion factor* is commonly defined as the ratio between total *rms* harmonic voltage and fundamental voltage component u_f. Considering that homo-integral voltage $\hat{u}$ is a low-pass filtered replica of voltage u, we may assume that $\hat{u} \approx u_f$, thus:

$$\sigma_u = \frac{\sqrt{\|u\|^2 - \|\hat{u}\|^2}}{\|\hat{u}\|} = \frac{D_u}{Q} \qquad \text{(III.1)}$$

The *current distortion factor* is defined as the ratio between the rms values of void current and total current:

Authorized licensed use limited to: UNIVERSITA TRENTO. Downloaded on September 28,2026 at 07:59:16 UTC from IEEE Xplore. Restrictions apply.

$$\sigma_i = \frac{\|i_v\|}{\|i\|} = \frac{D_i}{A} \qquad \text{(III.2)}$$

The apparent power can therefore be expressed as:

$$A^2 = P^2 + Q^2 + D^2 = \frac{P^2 + Q^2\left(1+\sigma_u^2\right)}{1-\sigma_i^2} \qquad \text{(III.3)}$$

Assuming a limited voltage distortion, we have:

$$A \approx \frac{S}{\sqrt{1-\sigma_i^2}} \qquad \text{(III.4)}$$

The *power factor* is:

$$\lambda = \frac{P}{A} \approx \frac{P}{S}\sqrt{1-\sigma_i^2} \qquad \text{(III.5)}$$

If the network absorbs a limited void current (i.e., in presence of compensating units), the current distortion approaches zero and the power factor becomes $\lambda \approx P/S$.

REFERENCES

[1] C.I.Budeanu: Puissances reactives et fictives, Institute Romain de l.Energie, Bucharest, 1927.

[2] N.L.Kuster, W.J.M.Moore: On the Definition of Reactive Power under Non-Sinusoidal Condition. IEEE Trans. on Power Apparatus and Systems, PAS-99 (1980), pp.1845-1854.

[3] W.Shepherd, P.Zakikhani: Suggested definition of reactive power for nonsinusoidal systems. Proc. Inst. Elec. Eng., vol. 119, pp.1361-1362, Sept. 1972.

[4] L.S.Czarnecki.: Orthogonal Decomposition of the Currents in a 3-Phase Nonlinear Asymmetrical Circuit with a Nonsinusoidal Voltage Source. IEEE Trans. on Instrumentation and Measurements, vol.IM-37, n.1, pp.30-34, March 1988.

[5] C.H.Page: Reactive Power in Non-Sinusoidal Situations. IEEE Trans. of Instrumentation and Measurement, Vol IM-29, No.4, Dec 1990, pp 420-423.

[6] L.S. Czarnecki, "Currents' Physical Components (CPC) in circuits with nonsinusoidal voltages and currents. Part 1: Single-phase linear circuits," Electrical Power Quality and Utilization Journal, Vol. XI, No. 2, 2005, pp. 3-14.

[7] Willems, J.L.: Power factor correction for distorted bus voltages. "Electr. Mach. a. Power Syst", 13 (1987), pp.207-218.

[8] L.S.Czarnecki, L.S.: Power factor improvement of three-phase unbalanced loads with nonsinusoidal supply voltages, European Trans. on Electrical Power Engineering (ETEP), Vol. 3, No. 1, Jan./Feb. 1993, pp. 67-72.

[9] P.Tenti, E.Tedeschi, P.Mattavelli, "Compensation Techniques based on Reactive Power Conservation", Seventh International Workshop on Power Definition and Measurements under Nonsinusoidal Conditions, Cagliari (Italy), July 2006.

[10] H.Akagi, Y.Kanazawa, A.Nabae: Generalized theory of the instantaneous reactive power in three-phase circuits, Proc. of the Int. Power Electron. Conf., (JIEE IPEC) Tokyo/Japan, 1983, pp. 1375-1386.

[11] H.Akagi, Y.Kanazawa, A.Nabae: Instantaneous reactive power compensators comprising switching devices without energy storage components, IEEE Trans. Ind. Appl., IA-20, 1984, No. 3, pp. 625-630.

[12] H. Akagi, A. Nabae: „The p-q Theory in Three-Phase Systems under Non-Sinusoidal Conditions", European Trans. on Electrical Power Engineering (ETEP), Vol. 3, No. 1, Jan./Feb. 1993, pp. 27-31.

[13] M. Depenbrock, "The FBD-method, A Generally Applicable Tool For Analyzing Power Relations", IEEE Transactions on Power Systems, Vol. 8 (1993), No. 2, pp. 381–387.

[14] M.Depenbrock, D.A.Marshall, J.D.van Wyk: "Formulating Requirements for a Universally Applicable Power Theory as Control Algorithm in Power Compensators", European Trans. on Electrical Power Engineering (ETEP), Vol. 4, No. 6, Nov/Dec 1994, pp. 445-455.

[15] J.L.Willems: Mathematical foundations of the instantaneous power concept: a geometrical approach, European Trans. on Electrical Power, ETEP, Vol. 6, No. 5, Sept./Oct.1996, pp. 299-304.

[16] L.Cristaldi, A.Ferrero: Mathematical foundations of the instantaneous power concept: an algebraic approach, European Trans. on Electrical Power, ETEP, Vol. 6, No. 5, Sept./Oct. 1996, pp. 305-309.

[17] F.Z.Peng, H.Akagi, A.Nabae "A New Approach to harmonic Compensation in Power Systems - A Combined system of shunt passive and series active filters", IEEE Trans on Industry Applications, vol.26, no.6, 1990, pp. 983-990.

[18] P. Mattavelli, "A closed-loop selective harmonic compensation for active filters", IEEE Trans. Industry Applications, vol. 37, no. 1, Jan./Feb. 2001, pp. 81-89.

[19] H.Akagi, "Control strategy and site selection of a shunt active filter for damping of harmonic propagation in power distribution systems," IEEE Trans. Power Delivery, vol. 12, pp. 354–363, Jan. 1997.

[20] S.J.Merhej, W.H.Nichols, "Harmonic Filtering for the Offshore Industry", IEEE Trans. on Industry Applications, Vol. 30, No. 3, May/June,1994, pp. 533-542.

[21] P.Jintakosonwit, H.Fujita, H.Akagi, S.Ogasawara: "Implementation and Performance of Cooperative Control of Shunt Active Filters for Harmonic Damping Throughout a Power Distribution System" IEEE Trans on Industry Applications., vol. 39,no.2, pp. 556–563, March/April 2003.

[22] T. L. Lee, P.T. Cheng, "Design of a new cooperative harmonic filtering strategy for the distributed generation systems", IEEE IAS 40th Annual Meeting 2005, pp. 549-556.

[23] P.T. Cheng, T. L. Lee, "Analysis of harmonic damping effect of the distributed active filter system", IEEJ Trans. Ind. Applicat., vol. 126, no. 5, pp.605-614, May 2006.

[24] P.T. Cheng, T. L. Lee, "Distributed active filter systems (DAFS): A new approach to power systems harmonics", IEEE Trans. Ind. Applicat., vol 42, no. 5, pp. 1301-1309, Sept/Oct. 2006.

[25] P.T. Cheng, T. L. Lee, H. Akagi, H. Fujita, "A Dynamic Tuning Method for Distributed Active Filter Systems", IEEE IAS 41th Annual Meeting 2006, pp. 175 - 182.

[26] E.Tedeschi, P.Tenti, P.Mattavelli: "Cooperative Operation of Active Power Filters by Instantaneous Complex Power Control", Proc. of the 7th Int. Conf .on Power Electronics and Drive Systems (PEDS 07), Bangkok, Nov 2007.

Authorized licensed use limited to: UNIVERSITA TRENTO. Downloaded on September 28,2026 at 07:59:16 UTC from IEEE Xplore. Restrictions apply.