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On the Effects of Phase Information on the Reconstruction Capabilities of the Iterative Multi-Scaling Strategy

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***Abstract* – This paper focuses on the dielectric reconstruction from amplitude-only data by means of the Iterative Multi-Scaling Strategy. Such a methodology is considered to fully exploit the reduced information content achievable from phaseless measurements allowing a satisfactory spatial resolution in the investigation domain. In order to preliminary evaluate the performance of the proposed approach, a representative test case is analyzed and the results are compared with those of the full-data approach.**

1 INTRODUCTION

The problem of imaging cylindrical structures with electromagnetic waves has been addressed in several works (see for example [1] and the references cited therein) by exploiting inverse-scattering techniques.

However, it is well known that inverse-scattering problems arising from the mathematical model of electromagnetic interactions are ill-posed and nonlinear [2]. Such drawbacks are caused by the lack of information on the scenario under test. As a matter of fact, the collection of information through the measurement of the scattered electromagnetic field in an observation domain outside the investigation area is not enough for providing a sufficient amount of information. The collectable information is limited [3] and it cannot be extended at will even though multi-illumination experiments [4] enlarge the dimension of the data space. Therefore, a great care must be exercised in defining the problem unknowns as well as the functional to be minimized, since the original problem is generally recast as an optimization one.

In addition, the need of measuring complex quantities (i.e., the amplitude and the phase of the scattered field) requires a reference channel and a synchronous detection. Although the measurement of the phase is not impracticable, it generally represents an expensive task at microwaves and lower frequencies. Thus, the use of phaseless data can notably simplify the design of the acquisition system and some useful attempts towards this end have been recently investigated [5] [6].

Within this framework, since the efficient exploitation of the reduced information content of amplitude-only data and a reduced ratio between problem unknowns and data are key issues, the use of multi-resolution strategies could be very appealing. As a matter of fact, such approaches are aimed at providing an optimal representation of the unknowns in order to minimize their number. As far as complete data are concerned, the iterative multi-scaling strategy (IMSS) [7]–[9] has been successfully employed in reconstructing dielectric objects.

In this paper, a suitable version of the IMSS is applied to amplitude-only data (Sect. 2). The approach is assessed by considering a selected test case (Sect. 3). Moreover, for comparison purposes, the obtained results are compared with those obtained starting from complete data.

2 MATHEMATICAL FORMULATION

The inversion procedure will be described referring to a two-dimensional geometry. An investigation domain D_I embeds an unknown scatterer, described by the contrast function

$$\tau(x,y)=\varepsilon_r(x,y)-1-j\frac{\sigma(x,y)}{\omega\varepsilon_0} \qquad (1)$$

where $\varepsilon_r(x,y)$ and $\sigma(x,y)$ are the dielectric permittivity and the electric conductivity, respectively; ω is the working angular frequency.

The scenario under test is illuminated by a set of V TM-polarized monochromatic incident fields, $E_{inc}^{v}(x,y)$ $v=1,..,V$. The reconstruction is performed starting from the measurements of the amplitudes of the fields, $\left|E_{tot}^{v,meas}(x_m,y_m)\right|$ $v=1,..,V$, collected at M positions in a circular observation domain D_M external to D_I. Moreover, the knowledge of the field without the object in D_M, $E_{inc}^{v,meas}(x_m,y_m)$, and its amplitude in D_I, $\left|E_{inc}^{v,meas}(x,y)\right|$, is assumed.

In this scenario, the inverse scattering equations [10] describing the relations between object and fields are:

$$\left|E_{tot}^{v,meas}(x_m,y_m)\right| = \left|E_{inc}^{v,meas}(x_m,y_m) + k_0^2 \int_{D_I} G(x_m,y_m/x',y')\tau(x',y')E_{tot}^{v}(x',y')dx'dy'\right| \quad (x_m,y_m)\in D_M \qquad (2)$$

$$\left|E_{inc}^{v,meas}(x,y)\right| = \left|E_{tot}^{v}(x,y) - k_0^2 \int_{D_I} G(x,y/x',y')\tau(x',y')E_{tot}^{v}(x',y')dx'dy'\right| \quad (x,y)\in D_I \quad (3)$$

where $G(x,y/x',y')$ and $G(x_m,y_m/x',y')$ are the internal and external Green's functions [10] for the homogeneous background, respectively. $k_0=\omega\sqrt{\varepsilon_0\mu_0}$ is the background wave number.

In order to determine the unknown quantities $\left[\tau(x,y),\ E_{tot}^{v}(x,y)\quad (x,y)\in D_I\right]$, the inverse scattering problem in hand is recast as an optimization one, by defining a suitable cost function, which estimates the fitting between measured and reconstructed data:

$$F_{PL}\left\{\tau(x,y),E_{tot}^{v}(x,y)\right\} = \frac{\sum_{v=1}^{V}\sum_{m=1}^{M}\left\|E_{tot}^{v,meas}(x_m,y_m)\right| - \left|E_{tot}^{v,rec}(x_m,y_m)\right\|^2}{\sum_{v=1}^{V}\sum_{m=1}^{M}\left|E_{tot}^{v,meas}(x_m,y_m)\right|^2} + \frac{\sum_{v=1}^{V}\sum_{n=1}^{N}\left\|E_{inc}^{v,meas}(x_n,y_n)\right| - \left|E_{inc}^{v,rec}(x_n,y_n)\right\|^2}{\sum_{v=1}^{V}\sum_{n=1}^{N}\left|E_{inc}^{v,meas}(x_n,y_n)\right|^2} \qquad (4)$$

where the reconstructed data, computed from an estimate of the unknowns, are denoted by the superscript "*rec*". Moreover, $(x_n,y_n)\in D_I$ indicates the centre of the n-th subdomain of the discretized investigation domain. Since the dimension of the discretized cell defines the resolution accuracy of the reconstruction, N should be very large. However, the limited amount of information collectable from amplitude-only data does not allow this choice. Consequently, an inhomogeneous discretization is needed. Towards this end, the retrieval process is carried out by iteratively minimizing the amplitude-only cost function (4), thorough the Iterative Multi-Scaling Strategy [8].

Starting from a homogeneous low-resolution reconstruction of the area under test, an higher spatial accuracy is looked for in a limited number of regions-of-interest (RoIs) defined at each step of the procedure by means of a post-processing of the previous estimate of the dielectric distribution [8].

Consequently, a sort of synthetic zoom is performed at each step in limited portions of D_I until the stationariness condition holds true [8].

3 NUMERICAL RESULTS

In this section, the results of a representative set of numerical experiments will be analyzed in order to evaluate the effectiveness of the IMSS in imaging unknown scatterer from amplitude-only data. Comparisons with the full-data approach will be presented, as well.

Let us consider the dielectric distribution shown in Figure 1 concerned with a homogeneous square scatterer ($\tau_{ref} = 0.5$) centered at $x_{ref}^{c} = y_{ref}^{c} = -0.3\lambda_0$ in an investigation domain $L_{D_I} = 2.4\lambda_0$ -sided. Such a scenario is illuminated by a set of plane waves impinging from $V = 8$ different directions. The electric field scattered by the object is collected for each view with $M = 26$ point-like sensors.

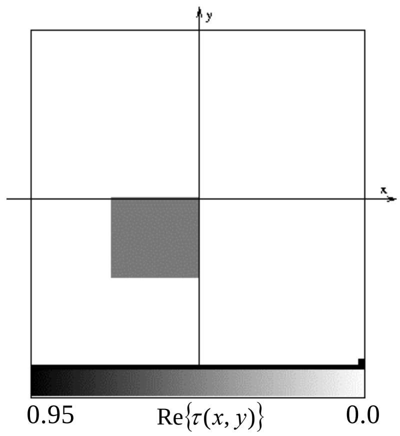

Figure 1: Reference distribution.

As far as the multi-scaling process is concerned, the first experiment has been carried out by considering the full-data approach (IMSS-FD), i.e. where the phase information is available. Figure 2 shows the evolution of the retrieved profile during the iterative process starting from the initial step ($s = 1$) [Figure 2(a)] up to the convergence step [Figure 3(b)] $(s = S_{opt} = 3)$. Noiseless conditions have been assumed. For a successive comparative analysis, the values of the error figures related to the reconstruction accuracy in the whole investigation domain, χ_{tot} , in the area belonging to the actual scatterer, χ_{int} , and in the surrounding background, χ_{ext} , are reported in Table 1.

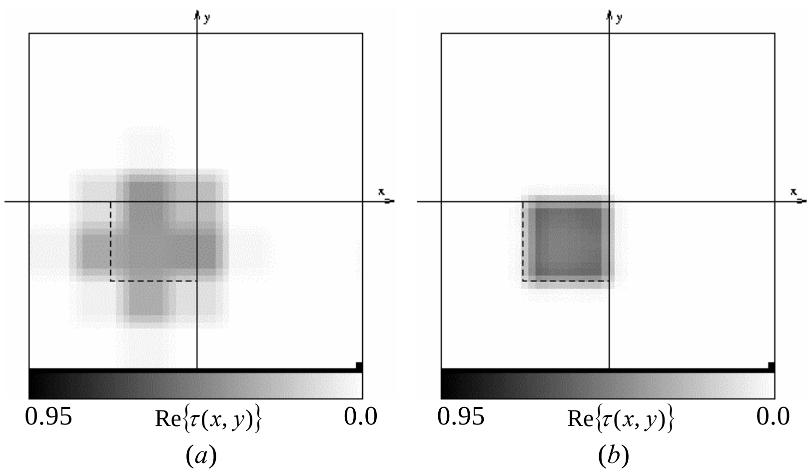

Figure 2: Reconstruction of a homogeneous scatter with the IMSS-FD – Noiseless conditions. Profiles retrieved at: (a) $s = 1$ and (b) $s = S_{opt} = 3$.

The grey-level images of the reconstructed objects as well as the reconstruction errors indicate that the IMSS-FD approach guarantees reliable results. Both the shape and the homogeneity of the actual scatterer are faithfully retrieved.

Error Figures	$s=1$	$S_{opt}=3$
χ_{tot}	3.8	0.47
χ_{int}	11.1	5.1
χ_{ext}	3.4	0.16

Table 1: Reconstruction of a homogeneous scatter with the IMSS-FD – Noiseless Conditions. Error figures.

The same experiment has been also carried out by processing amplitude-only data as detailed in Sect. 2.

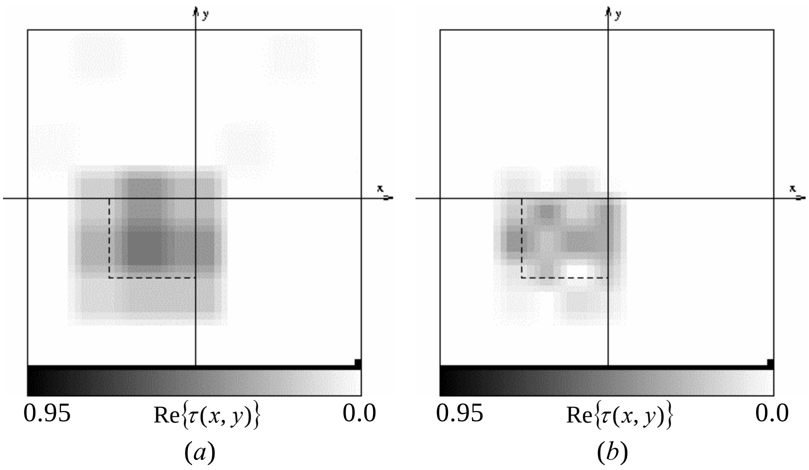

Figure 3: Reconstruction of a homogeneous scatterer with the IMSS-AD – Noiseless Conditions. Profiles retrieved at: (a) $s=1$ and (b) $s=S_{opt}=2$.

Starting from the amplitude-only data, the distribution of the object function reconstructed with the IMSS-AD at different steps of the multi-step process are shown in Figure 3. Moreover, the values of the reconstruction error figures are reported in Table 2.

As can be noticed by comparing Figure 2(a) and Figure 3(a), the reconstructions obtained with the two approaches are almost comparable at the first step. As a matter of fact, during such a phase, the iterative process is aimed at imaging the investigation domain with a low resolution in order to localize the RoIs where the scatterers are supposed to be located. Such an operation does not require a large amount of information, therefore information achievable from amplitude-only data turns out to be enough.

Successively ($s>1$), the level of the spatial resolution is enhanced in the RoIs to better detail the quantitative distribution of the object function. Even though, whatever the approach, the error values decrease (Tables 1,2), the performance of the IMSS-FD overcome those obtained with IMSS-AD. This is due to the larger information available when full-data are processed. However, although the quantitative reconstruction of the actual scatterer is worsened ($\chi_{\mathrm{int}}^{IMSA-FD}=5.2$ vs. $\chi_{\mathrm{int}}^{IMSA-AD}=17.1$), the estimated profile still represents an acceptable approximation of the object.

Error Figures	$s=1$	$S_{opt}=2$
χ_{tot}	5.4	2.2
χ_{int}	20.1	17.1
χ_{ext}	4.3	1.2

Table 2: Reconstruction of a homogeneous scatter with the IMSS-AD – Noiseless Conditions. Error figures.

4 CONCLUSIONS

In this paper, the problem of reconstructing unknown dielectric profiles starting form amplitude-only field data has been addressed. The performance of the IMSS-AD and those of the IMMS-FD have been compared by considering a preliminary test case. As expected, the reconstructions have shown a reduction of the resolution accuracy. However, the multi-resolution strategy has still been able to accurately localize the unknown target giving a coarse approximation of the dielectric distribution of the actual scatterer, as well.

Certainly, further synthetic and experimental benchmarks are needed to asses the approach defining potentialities and current limitations.

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