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Original Article

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Do balanced-budget fiscal stimuli of investment increase its economic value?

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Abstract: We examine the timing of a business investment providing valuable external benefits to society. A surge in uncertainty about private returns, a typical feature if not a cause of recessions, delays capital outlays to an extent that may be detrimental to social welfare. Is there an efficiency-improving public policy directed at accelerating investment? By real option analysis, we try answering this question by comparing three fiscal policies: (i) a simple subsidy on investment, (ii) a balanced-budget fiscal stimulus where the subsidy is subsequently covered by profit taxation, and (iii) by taxing external benefits as well. We show that, under a balanced-budget stimulus, investment acceleration may come at the expense of a net economic loss, and the higher is uncertainty on private returns, the higher the likehood of a negative outcome. However, this risk strongly declines when government spending is balanced by taxing both private and public returns on investment.

Keywords: investment, Fiscal stimulus, balanced-budget constraints, Real options

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1 Introduction

It is a well-known empirical regularity brought to the forefront by Keynes in the General Theory (1936; 1937) and later confirmed by a large body of evidence over

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time and across countries (Fazzari et al. 1988; Hubbard 1990; Bond and Jenkinson 1996; Saltari and Ticchi 2007; Gennaioli et al. 2016) that investment is the most volatile component of GDP over the business cycle. It is highly sensitive to uncertainty, but less responsive to interest rates, making monetary policy insufficient to stimulate investment as much as needed during a slump.

Indeed, one of the most striking features of the Great Recession was the sharp decline of fixed capital expenditures compared to consumption (Hall 2010). The policy reaction saw central banks on the front line with deep cuts of policy interest rates that quickly reached the zero lower bound without major effects. To bypass this limit, monetary authorities switched to "unconventional policies", the bulk of which consists of Quantitative Easing (QE), i. e., large injections of liquidity by means of direct purchases of a variety of assets (Bernanke and Reinhart 2004; Borio and Zabai 2016; Driffil 2016). Yet, in spite of the long period of extremely easy monetary conditions, in many advanced economies recovery of investment remained slow and anemic (Banerjee et al. 2015; European Central Bank 2017). Thus, after decades of "monetary dominance", the Great Recession led to a "rehabilitation" of fiscal activism (Blanchard 2009; Blanchard et al. 2010).[1] For instance, the problem of persistent stagnation of investment and the limits of monetary stimuli have proven to be particularly challenging in the Euro Zone, where both the former President of the ECB (Draghi 2014a; 2014b) and the President of the European Commission (Juncker 2015) repeatedly called for fiscal policy to share responsibility in sustaining economic recovery.

To an even greater extent, the slowing of private investment figures prominently among the consequences of the Covid-19 pandemic (Baldwin and Weder di Mauro 2020a; Boone et al. 2020) and in several countries policy responses have been engineered with joint deployment of monetary and fiscal supports on an unprecedented scale (Baldwin and Weder di Mauro 2020b). With policy interest rates already stuck at the zero lower bound, central banks have intensified QE programmes, with State bond generally taking the largest share, so that monetary financing of fiscal stimuli is the practical result. For instance, the ECB has engaged into a major plan of asset purchases, the Pandemic Emergency Purchases Plan (Lane 2020; Schnabel 2020). However, the main novelty lies on the fiscal side. On the one hand, the suspension of the budgetary rules of the Stability and Growth Pact has allowed national governments to grant economic relief to households and to a large extent to businesses. On the other hand, at the Union level, an unprecedented amount of resources has been deployed to directly support public

1 See also Krugman (1998; 2005) for earlier reassessment of fiscal policy.

investments as well as to leverage private capital. Thus, public policy has taken centre stage in investment decisions.

Generally speaking, from a normative perspective, public intervention on private investment can be justified by the existence of (positive or negative) externalities, either in normal times or during economic downturns. Positive examples include R&D expenditures that generate significant intrasectoral or horizontal productivity gains, or investments reducing environmental harm through the deployment of less-polluting (e. g., less carbon-intensive) technologies. There are also "macroeconomic externalities" which manifest themselves in periods of low business confidence, when companies tend to delay capital outlays to reduce the chance of making a wrong decision (Bernanke 1983; Cooper and John 1988; Haltiwanger and Waldman 1989; Hargreaves Heap 1992). Since the social benefits (costs) of (not) having an additional unit of capital expenditure, in terms of aggregate demand and employment, are not internalised by individual decision-makers, policy measures are required to narrow the gap between the private and socially desirable size and timing of investment.

However, even though the existence of policy-relevant externalities provides a robust argument for interventions, the governments' ability to stimulate investment can be bound by budgetary constraints, either self-imposed or imposed from outside. For instance, during the Great Recession and the sovereign debt crisis in the Euro Zone, the potential conflict between expansionary fiscal policies and long-term sustainability of public finances proved to be one of the most controversial economic and political issues (CESifo 2019). While the exceptional gravity of the economic dislocation due to the pandemic has set aside budgetary concerns, the issue of budgetary sustainability retains importance in its own right in a long-run perspective. Another related emerging issue is the problem of so-called "zombie firms", i. e., whether subsidizing businesses with negative market value will actually leave a net positive impact on the society as a whole.

The developments of the real option theory of investment (Dixit 1992; Dixit and Pindyck 1994), on which we draw in this paper, have provided fruitful insights into the effects of uncertainty and the implications for public policy. One finding, directly relevant to the above mentioned recent policy strategies, is that a large upsurge of uncertainty, and hence high value of waiting, can impair stimulative monetary policy, even with the interest rate at its zero lower bound (see, e. g., Miyazaki et al. 2004; Belke and Göcke 2019).

As for the budgetary sustainability of fiscal stimuli, in parallel to the macroeconomic debate, a microeconomic literature, using real option models, has offered insights about the effects of public subsidies in accelerating business investments as well as the ultimate impact on public accounts (see, e. g., Danielova and Sarkar, 2011; Sarkar 2011; 2012; Barbosa et al., 2016). Much of this literature has been inspired by Pennings (2000), who examined the possibility of reconciling short-term incentives on investment with long-term sustainability of public finances, showing that a government could accelerate capital outlays, while keeping its long-term budget balanced, by subsidizing investment costs and by subsequently collecting a share in the generated profits. However, Maoz (2011) cast doubts about the seemingly free-lunch subsidy-tax scheme described by Pennings, by pointing out that investment acceleration might come at the expense of reducing the firm's market value. Thus, taken together, these findings suggest that government intervention must find a tighter justification than in Pennings' model, where it is implicitly taken for granted that investment acceleration will result into a social gain.

In this paper we address the above two interrelated issues. On the one hand, we show how an increase in uncertainty and thus an increase in the private option-value of waiting – a typical feature, if not a cause, of deep recessions – can lead to economically inefficient delays of business investments. On the other hand, we examine the economic impact of policies directed at accelerating investment. In our partial equilibrium framework, the focus is on the impact upon the firm's market value and the external benefits directly attributable to capital outlays, which jointly define what in the following will be referred to as the project's economic (private and public) value.

Our primary aim is to compare the economic value resulting from a fiscal stimulus with that obtained when the exercise of the option to invest is entirely left to the firm without any government interference. In particular, we compare three alternative fiscal policies: (i) a simple subsidy of investment costs, (ii) a balanced-budget stimulus where the subsidy is covered by taxing the private returns on investment, and (iii) a balanced-budget stimulus where the subsidy is covered by taxing both profits and the external returns of investment. Our main findings are the following.

First, we replicate the result that the subsidy is by itself an effective tool to accelerate investment in line with the social benefit pursued by the government. Why does a subsidy succeed whereas the interest rate cut may not? The reason must be found in the project's irreversibility. Unlike the interest rate, the subsidy, cutting directly the sunk cost of investment, reduces the expected loss to be incurred as a consequence of irreversibility and raises (reduces) the value of accelerating (postponing) investment.

Second, when the fiscal stimulus is balanced by a profit tax, and account is taken of both the firm's market value and the external benefits of investment, we find a kind of Laffer Curve, i. e., a combination of subsidy-and-tax rate with a critical value beyond which investment acceleration reduces the project's economic value. For investment acceleration, and thus higher public benefits, comes at the expense of a reduction of the firm's market value. Notably, the higher is uncertainty about the private returns on investment, the lower is the market value, and thus the higher the likehood of a net economic loss.

Third, the risk of such negative outcome strongly declines when the subsidy is subsequently balanced by taxing both private and external returns on investment. Since the subsidy is disbursed earlier and the tax revenue comes later, an analogy can be found between this kind of fiscal programme and the "golden rule of public finance", which justifies uncovered public expenditure to the extent that it gives rise to a broader tax base generating its own tax coverage.

The remainder is organised as follows. In Section 2 we present the model. In Section 3 we analyze the effects of alternative public policies. Section 4 concludes. The proofs are presented in the appendices.

2 The model

Consider a representative firm that holds an option to invest at any time $t \geq 0$ in a infinitely-lived project which requires a sunk implementation cost denoted by I.

The project is expected to generate a time-stream of profits x_t, defined as the difference between the operating cash flows (measured by the unit rate ρ) and the cost of capital to be paid out to funders (measured by the market unit rate r).[2]

Since we intend to focus on the effects of uncertainty, we assume that the state variable x_t follows a geometric Brownian motion:

$$dx_t = (\rho - r)x_t dt + \sigma x_t dz_t \qquad x_0 = x \tag{1}$$

where dz_t is the increment of a standard Wiener process and σ is the constant proportional volatility of x_t per unit time.

Equation (1) implies that future profits are lognormally distributed with a variance that grows with the time horizon. Thus, by varying σ, it is possible to analyze how different levels of uncertainty affect investment decisions dynamically.

Since $E_t(x_t \mid x) = xe^{(\rho-r)t}$, the present value of the expected profits at time t is simply given by: $V(x_t) = E_t(\int_t^{\infty} e^{-\rho(s-t)} x_s ds) = \frac{x_t}{r}$.[3]

2 For our purpose, it is immaterial whether r has to be reckoned as the cost of external funds or as the opportunity cost of internal funds.

3 As standard in the literature, if ρ is the rate of return for holding an asset whose price is V, then the price of this asset must satisfy the equation:

$$\underbrace{\frac{x}{V}}_{\text{dividend yield}} + \underbrace{\frac{dV}{V}}_{\text{Capital gain}} = \rho$$

Equation (1) and the sunkness of investment imply that there exists an option-value of waiting. Specifically, since at any time $t > 0$ all the information about the future evolution of profits is embodied in the current value x_t, there exists an optimal rule of the form: invest now if x_t is at or above a critical threshold, otherwise wait (Dixit and Pindyck 1994).

Hence the firm's problem consists in the choice, at $t = 0$, of the optimal time of investment, defined as $\tau^P = \inf(t > 0 \,/\, x_t = x_{\tau^P})$, i. e., the time that maximizes the expected net present value (NPV) given by:[4]

$$\begin{aligned} F(x, x_{\tau^P}) &\equiv E_0(e^{-\rho\tau^P})[V(x_{\tau^P}) - I] \\ &= \left(\frac{x}{x_{\tau^P}}\right)^{\beta} [V(x_{\tau^P}) - I] \end{aligned} \tag{2}$$

where $\beta = \frac{1}{2} - \frac{\rho-r}{\sigma^2} + \sqrt{\left(\frac{\rho-r}{\sigma^2} - \frac{1}{2}\right)^2 + \frac{2\rho}{\sigma^2}} > 1$ is the positive root of the characteristic equation $\Psi(\beta) = (\sigma^2/2)\beta(\beta - 1) + (\rho - r)\beta - \rho = 0$, and $E_0(e^{-\rho\tau^P}) = \left(\frac{x}{x_{\tau^P}}\right)^{\beta} < 1$ is the "expected discount factor".[5]

Equation (2) implies that there exists a particular value ρ^* (known in capital budgeting as the "internal rate of return") that makes the expected NPV equal to zero and which sets the highest payable cost of capital for the investment to remain feasible, i. e., $r \le \rho^*$.

Since the process (1) is time autonomous, the discount rate is constant and the option-to-invest perpetual, the optimal threshold of x_t for investing (the "entry trigger") is given by:

$$x_{\tau^P} = \frac{\beta}{\beta - 1} rI \tag{3}$$

where $\frac{\beta}{\beta-1} = 1 + \frac{1}{\beta-1} > 1$ is the option multiplier which captures the effect of uncertainty.

where x are the profits that the asset pays and dV is the market appreciation. If we assume a constant market appreciation per unit of asset value α then $\frac{x}{V} = \rho - \alpha = r$. Yet, if V moves stochastically as a Geometric Brownian motion, then also x moves with the same motion as in Eq. (1).

4 For the rest of the paper, we assume that the initial value $x_0 = x$ is always low enough to guarantee that immediate exercise does not happen.

5 The expected present value, which allows transforming tomorrow's a uncertain payoff into present value, can be determined by using dynamic programming (see, *e. g.*, Dixit and Pindyck 1994).

Equation (3) shows that, under uncertainty ($\sigma > 0$), the entry trigger is higher than the pure market return rI. Notice that Eq. (3) can be reformulated as follows:

$$z \equiv \frac{x_{\tau^P}}{I} = \frac{\beta}{\beta - 1} r$$

where z (the "hurdle rate", in the language of capital budgeting) represents the minimum rate that the firm is expected to earn while undertaking the project. While under certainty ($\sigma = 0$) the hurdle rate is just the market interest rate, under uncertainty $z > r$.

Since $\frac{d\beta}{d\sigma} < 0$, with $\lim_{\sigma\to\infty} \beta = 1$ and $\lim_{\sigma\to 0} \beta = \frac{\rho}{\rho - r}$, it follows that $\lim_{\sigma\to\infty} x_{\tau^P} = +\infty$ and $\lim_{\sigma\to 0} x_{\tau^P} = \rho I$, i. e., the entry trigger x_{τ^P} is increasing in σ and so is the hurdle rate.

By substituting (3) into (2), we get the firm's market value when investing at the optimal threshold:

$$F(x, x_{\tau^P}) = \left(\frac{x}{x_{\tau^P}}\right)^{\beta} \frac{1}{\beta - 1} I$$

However, while waiting until the process (1) hits the threshold (3) is optimal for the firm, it may be wasteful from a social perspective. This notably occurs when the investment generates positive externalities.

In order to capture the essence of the argument, let's suppose that the project's implementation will lead to aggregate external benefits denoted by B, with $0 < B < I$.[6,7]

Hence, for a benevolent government, i. e., a planner attaching the same value to both the firm's market value and the public benefits, the objective function to be maximized is:

$$W(x, x_{\tau^W}) \equiv E_0(e^{-r\tau^W})[B + V(x_{\tau^W}) - I]$$

leading to the following entry trigger:

$$x_{\tau^W} = \frac{\beta}{\beta - 1} r(I - B) < x_{\tau^P} \tag{4}$$

6 As shown by Eq. (4), $B > I$ would imply that $W > 0$, i. e., a positive project's economic value even if expected private profits could be negative ($V < 0$). Consequently, the government ought to take direct responsibility over the project, while we only focus on government's intervention directed towards accelerating private investment.

7 For the sake of simplicity, we assume that the public benefits B are deterministic. However, in Appendix A we extend our setup by modelling external benefits as a "mark-up" on the private return on investment, showing that our main results are qualitatively unaffected.

Comparison between Eq. (3) and (4) shows the difference between the private and socially optimal threshold for investing:

$$x_{\tau^P} - x_{\tau^W} = \frac{\beta}{\beta - 1} rB > 0 \tag{5}$$

From an efficiency perspective, this gap calls for policy action. In the next section we compare alternative policies, by taking both the uncertainty parameter σ and the market interest rate r as exogeneously given. Moreover, since we intend to focus on the effects of uncertainty shocks, we shall hold the marginal investment ($\rho = r$) as a benchmark.[8]

3 Accelerating investment by fiscal policy

3.1 When monetary policy is ineffective

In principle, the gap between the private and socially optimal threshold for investing could be narrowed by lowering the interest rate r. However, as shown by Miyazaki et al. (2004) and Belke and Göcke (2019) in a setup similar to ours, monetary policy alone can prove ineffective to spur investment. Here we consider the role that could be played by conventional monetary policy, namely, central bank's operations aimed at lowering the interest rate relevant to investment decisions. Leaving aside the details of the transmission mechanism from the policy rate to the relevant rate, we simply assume that the central bank has full leverage on the market cost of capital r as defined above.

In order to examine the effects of monetary policy, it is convenient to reformulate the entry trigger (3) as the hurdle rate $z = \frac{x_{\tau^P}}{I}$. To resume our previous results, under certainty the hurdle rate is just the market rate r, whereas uncertainty and irreversibility raise z above r.

These notions can usefully be portrayed in Figure 1, which plots the hurdle rate z as a concave function of r. The lowest straight line ($z = r$) represents the case

8 Instead of a trendless (i. e., $\rho - r = 0$) Brownian motion, profits could be modelled using a mean-reverting process. However, this would not allow us to get a closed-form solution for the firm's market value, and thus, the project's total value. Notice, however, that the comparative statistics analysis with respect to standard deviation σ, carried on later in the paper, is ended equivalent to a mean-preserving spread. In fact, if we added to Eq. (1) another Wiener process, uncorrelated with dz_t, i. e., $\frac{dx_t}{x_t} = \sigma dz_t + \Delta\sigma dw_t$, and $E(dz_t dw_t) = 0$, the expected value $E(dx_t)$ is still nil, while the variance is $E(dx_t^2) = (\sigma^2 + \Delta\sigma^2){x_t}^2 dt$. For further details on this point see Abel (1985).

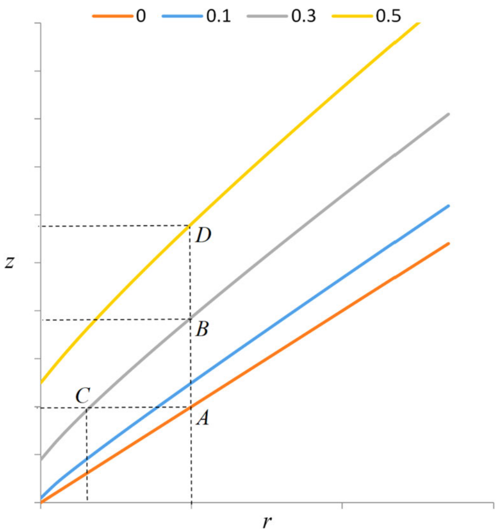

Figure 1: Relation between the hurdle rate z and the market interest rate z for different values of σ.

of certainty ($\sigma = 0$). The functions determined by increasing levels of uncertainty lie above the certainty line.

As a hypothetical starting point, let's consider the certainty case at point A. A surge of uncertainty (*e. g.*, $\sigma = 0.3$) shifts the hurdle rate to point B. The consequence for the marginal firm is that a project that was immediately feasible at point A is now delayed until point B is observed.[9] This effect may be offset by the central bank cutting the interest rate up to point C. However, if uncertainty is larger (e. g., $\sigma = 0.5$), then z rises up to point D, making the monetary policy impotent even when the zero bound of r is reached.[10]

Two further considerations are in order. First, the hurdle rate of inframarginal investment projects (with $\rho > r$) does fall to zero as the interest rate falls to zero, so that monetary policy may retain some efficacy as a means of accelerating investment. However, this amounts to assuming quasi-rents which require a motivated relaxation of the standard conditions of perfect competition and risk neutrality. Second, at the zero lower bound, the central bank may switch to "uncon-

9 More precisely, the aggregate effect on investment is that all inframarginal projects with $z \in [A, B]$ are delayed.

10 While stemming from a different conceptual and modeling framework, this result is in line with one of the key implications of Keynes's General Theory.

ventional" tools that in some way or another directly inject liquidity into the economy, as largely practised by major central banks in the last decade. Yet, the evidence of the stimulus effect on investment remains controversial. To our knowledge, a detailed analysis of this *modus operandi* of monetary policy in the investment theory under consideration is not well developed, and it falls outside the scope of this paper. However, it may be noted that the option value of waiting does not depend on firms being liquidity constrained. Indeed, the hurdle rate that affects the firm's investment decision does not necessarily represent the cost of funds to be borrowed, it may well represents the opportunity cost of alternative uses of funds that the firm owns. Hence, liquidity injections do not seem to be an effective policy for solving the underlying problem of the value of waiting.

3.2 A subsidy to the cost of investing

Suppose that the government decides to subsidize the investment cost at a rate $0 < \xi \le 1$. Consequently, the firm's optimal entry trigger, denoted by $\tau^S = \inf(t > 0 \,/\, x_t = x_{\tau^S})$, can be derived by maximizing:

$$F^S(x, x_{\tau^S}) \equiv E_0(e^{-r\tau^S})[V(x_{\tau^S}) - (1-\xi)I] \tag{6}$$

leading to the following threshold:

$$x_{\tau^S} = (1-\xi)x_{\tau^P} \tag{7}$$

Equation (7) shows that the government could, in principle, reshape the entry trigger by simply subsidizing the investment cost. For instance, the gap between the private and socially optimal threshold for investing could be fully bridged by setting $\xi = B/I$, i. e., by fully rewarding the firm for the external benefits.

3.3 A balanced-budget fiscal stimulus with a profit tax

Now suppose that the government, while willing to accelerate investment, must comply with budgetary constraints. Specifically, as in Pennings (2000), suppose that the government is allowed to subsidize private investments only on the condition of subsequently rebalancing the budget by taxing profits at the rate $0 < \gamma \le 1$, so as to render the NPV of the tax-subsidy program equal to zero. In the following, this program will be referred to as the "balanced-budget fiscal stimulus" (BBFS).[11]

Let's first derive the new optimal timing of investment $\tau^{TS} = \inf(t > 0 / x_t = x_{\tau^{TS}})$ that maximizes

$$F^{TS}(x, x_{\tau^{TS}}) \equiv E_0(e^{-r\tau^{TS}})[(1-\gamma)V(x_{\tau^{TS}}) - (1-\xi)I] \tag{8}$$

and thus the firm's optimal threshold for investing:

$$x_{\tau^{TS}} = \frac{1-\xi}{1-\gamma} x_{\tau^P} \tag{9}$$

Equation (9) shows that, as long as $\xi > \gamma$, the government enjoys a whole range of subsidy rates whereby it can reduce $x_{\tau^{TS}}$ relative to x_{τ^P} up to the first-best (i. e., $x_{\tau^{TS}} = x_{\tau^W}$) which now requires setting $\xi = \frac{B}{I} + \gamma(1 - \frac{B}{I})$.

However, the budget constraint requires that:

$$\xi I = \gamma \frac{x_{\tau^{TS}}}{r} \rightarrow \xi I = \frac{\beta\gamma}{\beta - (1-\gamma)} I \tag{10}$$

By substituting (10) into (9), we get the firm's entry trigger under BBFS:

$$x_{\tau^{BB}} = \frac{\beta}{\beta - (1-\gamma)} rI \tag{11}$$

Comparison between (3) and (11) shows two things.[12] First, as pointed out by Pennings (2000), the BBFS still induces a downward revision of the entry trigger. Second, the higher is the subsidy rate (and, consequently, the tax rate required to balance the budget), the lower will be the entry trigger. For instance, in the limit case where $\xi = 1$ and $\gamma = 1$ (i. e., where the government subsidizes entirely the investment ahead of 100 % taxation of future profits, or indeed the investment project becomes public), the BBFS would completely offset the option-value of waiting, by driving the hurdle rate to its zero-uncertainty value r.[13] Stated differently, under BBFS, the greater is the government interference, i. e., the higher are ξ and γ, the more effective is fiscal policy in terms of investment acceleration.

11 This program, too, entails that the government finances the initial subsidy by borrowing. Yet now, by Ricardain equivalence, there is no effect at all on the rate of interest because the amount borrowed is fully matched by the subsequent tax revenue.

12 By simple algebra (11) can be written as the project's NPV at the moment of investment:

$$\left[\frac{x_{\tau^{BB}}}{r} - I\right] - \frac{1-\gamma}{\beta - (1-\gamma)}$$

where the last term represents the effect of the tax-subsidy program.

13 From this point of view, fiscal policy is the right complement to monetary policy.

However, the question is whether government intervention actually increases the project's economic value relative to "laissez faire" (i. e., relative to the case where $\xi = 0$ and $\gamma = 0$).

The answer requires looking, first, at the impact upon the firm's market value. By substituting (10) and (11) into (8) and denoting with $F^{BB}(x, x_{\tau^{BB}})$ the market value under BBFS, we get (see Appendix B):

$$F^{BB}(x, x_{\tau^{BB}}) = \mu(\gamma, \beta) F(x, x_{\tau^P}) \tag{12}$$

where $\mu(\gamma, \beta) = (1-\gamma)\left(\frac{\beta-(1-\gamma)}{\beta-1}\right)^{\beta-1} \in [0, 1)$.

As in Maoz (2011), Eq. (12) shows that, under BBFS, investment acceleration comes at the expense of reducing the firm's market value. Specifically, within our framework, the term $\mu(\gamma, \beta)$, which summarizes the relevant parameters, can be interpreted as the measure of the "distortion" due to government intervention. Since $\frac{\partial \mu}{\partial \gamma} < 0$, the greater is the fiscal interference, the higher is the value loss to the firm.

Let's now look at the overall economic impact of BBFS. Denoting with $W^{BB}(x, x_{\tau^{BB}})$ and $W^P(x, x_{\tau^P})$ the project's economic value under BBFS and laisser-faire respectively, we get:

$$W^P(x, x_{\tau^P}) = \left(\frac{x}{x_{\tau^P}}\right)^{\beta} B + F(x, x_{\tau^P}) \tag{13a}$$

$$W^{BB}(x, x_{\tau^{BB}}) = \left(\frac{x}{x_{\tau^{BB}}}\right)^{\beta} B + \mu(\gamma, \beta) F(x, x_{\tau^P}) \tag{13b}$$

Rearranging, we get:

$$W^{BB}(x, x_{\tau^{BB}}) - W^P(x, x_{\tau^P}) = [\phi(\gamma, \beta) - 1]\left(\frac{x}{x_{\tau^P}}\right)^{\beta} B + [\mu(\gamma, \beta) - 1]\, F(x, x_{\tau^P}) \tag{14}$$

where $\phi(\gamma, \beta) = \left(\frac{\beta-(1-\gamma)}{\beta-1}\right)^{\beta} > 1$.

As already pointed out, the second term on the RHS of (14) is negative. However, the first term is always positive because investment acceleration increases the present value of public benefits. Thus, the net effect is ambiguous.

However, working on (14), we get that the sign of the difference between $W^{BB}(x, x_{\tau^{BB}})$ and $W^P(x, x_{\tau^P})$ depends on the sign of:

$$\Omega(\gamma, \sigma) \equiv B + \frac{(1-\gamma)}{\beta-(1-\gamma)} I - \left(\frac{\beta-1}{\beta-(1-\gamma)}\right)^{\beta}\left(B + \frac{I}{\beta-1}\right)$$

with $\Omega(0, \sigma) = 0$ and $\Omega(1, \sigma) = B - \left(\frac{\beta-1}{\beta}\right)^{\beta}\left(B + \frac{I}{\beta-1}\right) \gtrless 0$ (see Appendix C).

Thus, looking more in detail at the effect of uncertainty, we can show the following proposition.

Proposition 1. *For any given $B < I$, there exists a value of $\hat{\sigma}$ such that: for $\sigma < \hat{\sigma}$, $W^{BB}(x, x_{\tau^{BB}}) > W^{P}(x, x_{\tau^{P}})$ for all $\gamma \in (0, 1]$. Otherwise, for $\sigma \geq \hat{\sigma}$, there exists a tax rate $\hat{\gamma}(\sigma)$ such that $W^{BB}(x, x_{\tau^{BB}}) < W^{P}(x, x_{\tau^{P}})$ for all $\gamma > \hat{\gamma}(\sigma)$.*

Proof. See Appendix C. □

The proposition says two things. First, for any given subsidy-and-tax rate ensuring a balanced budget, the higher is the uncertainty about private earnings, the higher is the likelihood that a BBFS will not increase the project's economic value. Second, given the level of uncertainty, the greater is the fiscal interference, the greater the likehood that investment acceleration will come at the expense of a lower economic value.

This ambiguity of results sets limits to, but does not kill altogether, the viability of a BBFS as is clarified by the following numerical example.

Let's assume the following values for the relevant parameters: the investment cost is normalized to $I = 1$, the external benefits $B = 0.5$ and the cost of capital $r = 1\,\%$.[14] Given these parameters, in Figure 2 we plot $\Omega(\gamma, \sigma)$ as a function of γ for different values of σ: $\beta(\sigma = 10\,\%) = 2.0$ (Solid-Thin), $\beta(\sigma = 30\,\%) = 1.2$ (Solid-Dots), $\beta(\sigma = 40\,\%) = 1.1$ (Solid-Medium).

Figure 2 highlights the kind of "Laffer Curve" implied by Proposition 1. For any given level of uncertainty, one may spot a subsidy-and-tax rate such that the economic gains of BBFS are maximized. Beyond that point the project's economic value declines and eventually becomes negative.

Taking another viewpoint, an analogy can be drawn between the fiscal program considered here and the "golden rule of public finance", which, simply stated, posits that public deficits over the business cycle are justified, indeed they can be beneficial, if they are used to fund productive expenditures. However, our findings suggest that the range of viability of BBFS shrinks as uncertainty rises, i. e., exactly when investment delays are likely to be more severe and, thus, a government response is more needed. As the solid-medium line exemplifies, with

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14 From a macroeconomic point of view, since the subsidised share of I is public spending, B may be regarded as the induced increase in national income, and hence one may look for reference values at the empirical research on so-called "fiscal multipliers". Results are far from conclusive. However, the consensus estimates before the Great Recession may be located around 0.5, whereas post-crisis studies have unveiled that fiscal stimuli (contractions) in recessions are more powerful, with estimates pointing to higher values, around 1 or more (see, *e. g.*, IMF 2010; Gechert et al. 2015). The same conclusion is reached by the specific study of the impact of public expenditure via private investment by Carillo and Poilly (2013). Hence $B = 0.5$ can be considered as a conservative hypothesis.

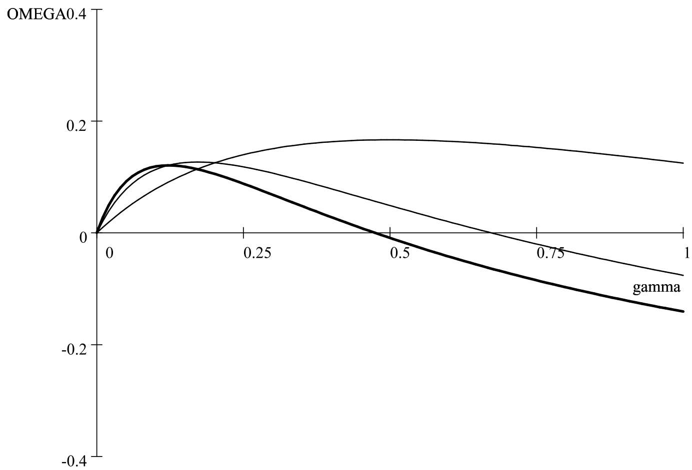

Figure 2: Difference between the economic value under BBFS and laissez-faire.

high uncertainty the maximal project's economic value is reached at a very low subsidy-and-tax rate, which generates a negligible acceleration of investment.

3.4 Taxing external benefits

The taxation arm of BBFS is one of the factors determining the rate of reduction of the project's economic value. This is largely attributable to the assumption that the initial increase of government expenditure will be entirely balanced by the revenues collected by taxing the profits generated by the project. However, since we are considering a situation where the investment generates economic benefits – such as revitalization of economy from recession – it may be thought that they can contribute to further increase the tax base and, thus, allow the government to reduce the tax burden to the firm required to achieve a balanced budget.

Clearly, the extent to which the tax-base actually increases will depend, *inter alia*, on the economic nature, and thus, the taxability of external benefits. Here, we simplify by assuming that all B can and will be taxed at the same rate γ as profits. In the following, this program will be denoted as BBT.

Thus, the government's budget constraint becomes:

$$\xi I = \gamma \frac{x_{\tau^{TS}}}{r} + \gamma B \rightarrow \xi I = \frac{\beta\gamma}{\beta - (1-\gamma)} I + \frac{(\beta - 1)\gamma(1-\gamma)}{\beta - (1-\gamma)} B \tag{15}$$

By substituting (15) into (9), we get the firm's entry trigger:

$$x_{\tau^{BBT}} = \frac{\beta}{\beta - (1-\gamma)} r(I - \gamma B) < x_{\tau^{BB}} < x_{\tau^{P}} \tag{16}$$

which, as can be expected, is lower than (11).

By substituting (15) and (16) into (8) and denoting with $F^{BBT}(x, x_{\tau^{BB}})$ the firm's market value a BBFS with external benefits taxed, we get:

$$F^{BBT}(x, x_{\tau^{BB}}) = \mu^{T}(\gamma, \beta) F(x, x_{\tau^{P}}) \tag{17}$$

where $\mu^{T}(\gamma,\beta) \equiv \mu(\gamma,\beta)\left(\frac{I}{I-\gamma B}\right)^{\beta-1} \in [0,1)$ (See Appendix D).

Again, we can compare the project's economic value with and without government intervention:

$$W^{BBT}(x, x_{\tau^{BB}}) - W^{P}(x, x_{\tau^{P}}) = \left[\phi^{T}(\gamma,\beta) - 1\right]\left(\frac{x}{x_{\tau^{P}}}\right)^{\beta} B + \left[\mu^{T}(\gamma,\beta) - 1\right] F(x, x_{\tau^{P}}) \tag{18}$$

where $\phi^{T}(\gamma,\beta) = \phi(\gamma,\beta)\left(\frac{I}{I-\gamma B}\right)^{\beta} > 1$.

The sign of the difference is given by:

$$\Omega^{T}(\gamma,\sigma) \equiv B + \frac{(1-\gamma)}{\beta-(1-\gamma)}(I-\gamma B) - \left(\frac{\beta-1}{\beta-(1-\gamma)}\right)^{\beta}\left(\frac{I-\gamma B}{I}\right)^{\beta}\left(B + \frac{I}{\beta-1}\right)$$

with $\Omega^{T}(0,\sigma) = 0$ and $\Omega^{T}(1,\sigma) = B - \left(\frac{\beta-1}{\beta}\right)^{\beta}\left(\frac{I-B}{I}\right)^{\beta}\left(B + \frac{I}{\beta-1}\right) > \Omega(1,\sigma)$ (see Appendix D).

Although the sign is still ambiguous we can prove the following proposition.

Proposition 2. *While an increase in uncertainty still reduces the benefits of a balanced-budget fiscal stimulus, the taxation of external benefits broadens the range of tax rates γ (for any given value of σ) and the range of σ (for a given value of γ) where a balanced-budget stimulus will provide a net economic gain.*

Proof. See Appendix D. □

A numerical example helps to illustrate these results. Using the same parameters used for generating Figure 2, in Figure 3 we plot $\Omega^{T}(\gamma,\sigma)$ as a function of γ.

Comparison between Figure 2 and 3 shows that the broadening of the tax-base, and thus, the reduction of the profit tax rate, modifies substantially the economic effects of BBFS. In fact, the contribution of external benefits to the tax revenues allows a faster acceleration of investment while ensuring a net economic gain. This effect can be seen in the convexity of the iso-uncertainty curves. Moreover, contrary to the previous case, there is now a subsidy-and-tax rate beyond which the project's economic value increases. This favourable combination can also be obtained with high uncertainty, though at a lower scale.

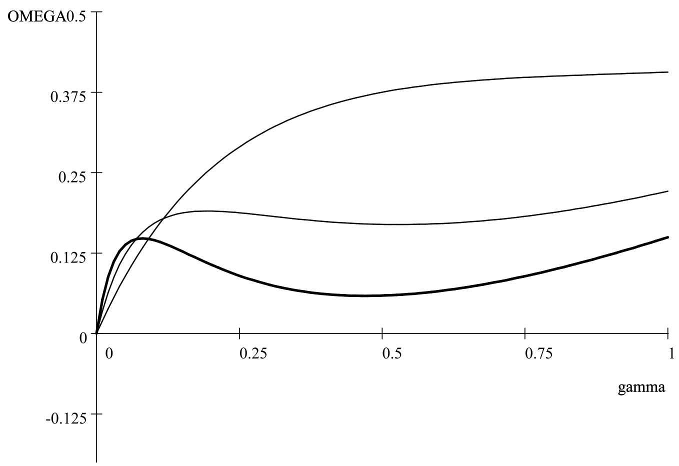

Figure 3: Difference between the economic value under BBT and laissez-faire.

4 Final remarks

A surge of uncertainty, a typical feature of deep recessions, has the effect of exacerbating the gap between the private and socially desirable timing of investment, to an extent that may not be offset by monetary policy (conventional or not).

Taking stock of other real option models, we have framed the external benefits of accelerating investment within the economic assessment of alternative fiscal policies: (i) a simple subsidy to the private cost of investment, (ii) a balanced-budget stimulus where the up-front subsidy is covered by subsequently taxing the profits generated by the project, and (iii) by taxing external benefits as well. The policy implications of our model can be summarised as follows.

First, a subsidy is a powerful tool that the government can use to spur investment so as to increase its total economic value.

Second, introducing a balanced-budget constraint, satisfied by future taxation of profits, has a twofold effect. On the one hand, the government can still gear the subsidy-tax scheme so as to accelerate investment. On the other, a balanced-budget fiscal stimulus has a negative impact on the firm's market value. Hence the net economic impact is ambiguous. However, we have shown that the net effect is more likely to turn negative the higher is uncertainty, i. e., when the public interest in spurring investment is stronger.

Third, the government can enlarge the scope of the economic gains of a balanced-budget fiscal stimulus by broadening the tax-base, so as to include the external benefits generated by the private investment. In fact, the enlargement of the tax-base allows the government to balance its budget at a lower profit tax rate and, in so doing, to further accelerate investment, while keeping the project's economic value positive even for higher levels of uncertainty.

In essence, we find support for the so-called "golden rule of public finance", which justifies deficits aimed at fostering investment (public or private as in our case) covered by future fiscal revenues, provided that these arise from an appropriately broad tax-base.

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Appendix A

In our setup, while private returns of investment are assumed to evolve stochastically, with a variance which grows with time, the external benefits (B) are described as deterministic. Here we extend the model, by modelling the instantaneous social returns, denoted by $b(.)$, as a stochastic variable. For instance, suppose that $b(x_t) = bx_t$, such that $E[db(x_t)] = b(\rho - r)x_t dt$, i. e., the rate of change of social return is modelled as a "mark-up" on the private return $\rho - r$. This leads to the following present value of the expected external benefits:

$$B(x) = E_0\left(\int_0^{\infty} e^{-\rho t} bx_t dt\right) = \frac{bx_0}{r}.$$

Hence, for a benevolent government, the optimal entry trigger becomes:

$$x_{\tau^W} = \frac{\beta}{\beta - 1} r \frac{I}{1 + b} < x_{\tau^P} \tag{A.1}$$

while, the difference between the private and the socially optimal trigger becomes:

$$x_{\tau^P} - x_{\tau^W} = \frac{\beta}{\beta - 1} rI - \frac{\beta}{\beta - 1} r \frac{I}{1 + b} = \frac{\beta}{\beta - 1} r \frac{b}{1 + b} I \tag{A.2}$$

Comparison between Eq. (A.2) and Eq. (5) shows two things. First, the difference happens to be identical if:

$$(I - B) = \frac{b}{1 + b} I$$

Second, an increase of external benefits, namely, an increase of the mark-up rate b, would lead to an increase of the gap between the private and socially optimal threshold for investing, i. e.:

$$\frac{\partial(x_{\tau^P} - x_{\tau^W})}{\partial b} = \frac{1}{(1+b)^2} > 0$$

Appendix B

First, substituting (3) in (2) we get:

$$F(x, x_{\tau^P}) = \left(\frac{x}{x_{\tau^P}}\right)^{\beta} \frac{x_{\tau^P}}{\beta\delta} \tag{B.1}$$

Second, substituting (10) and (11) in (8), we obtain:

$$\begin{aligned} F^{BB}(x, x_{\tau^{BB}}) &= \left(\frac{x}{x_{\tau^{BB}}}\right)^{\beta} (1-\gamma)\frac{x_{\tau^{BB}}}{\beta\delta} = (1-\gamma)F(x, x_{\tau^{BB}}) \\ &= (1-\gamma)\left(\frac{\beta-(1-\gamma)}{\beta-1}\right)^{\beta-1} F(x, x_{\tau^P}) \\ &= \mu(\gamma,\beta)F(x, x_{\tau^P}) \end{aligned} \tag{B.2}$$

where $\mu(\gamma,\beta) \equiv (1-\gamma)\left(\frac{\beta-(1-\gamma)}{\beta-1}\right)^{\beta-1}$. Since $\mu(1,\beta) = 0$, $\mu(0,\beta) = 1$, and:

$$\frac{\partial\mu}{\partial\gamma} = \left(\frac{\beta-(1-\gamma)}{\beta-1}\right)^{\beta-1}\left[-\frac{\gamma\beta}{\beta-(1-\gamma)}\right] < 0 \tag{B.3}$$

we may conclude that $\mu(\gamma,\beta) \in [0,1]$.

Finally, comparing (A.1) and (A.2), the difference between $F(x, x_{\tau^P})$ and $F^{BB}(x, x_{\tau^{BB}})$ becomes:

$$\begin{aligned} F(x, x_{\tau^P}) - F^{BB}(x, x_{\tau^{BB}}) &= F(x, x_{\tau^P}) - (1-\gamma)F(x, x_{\tau^{BB}}) \\ &= [1-\mu(\gamma,\beta)]F(x, x_{\tau^P}) > 0 \end{aligned} \tag{B.4}$$

Appendix C

The project's economic value is defined as the sum of the firm's market value and the external benefits. When the firm invests at (3), the total value is:

$$W^P(x, x_{\tau^P}) = \left[\left(\frac{x}{x_{\tau^P}}\right)^{\beta}\left(B + \frac{x_{\tau^P}}{\delta} - \pi(\tau^P)\right) + \left(\frac{x}{x_{\tau^P}}\right)^{\beta}\left(\frac{x_{\tau^P}}{\delta} - I + \pi(\tau^P)\right)\right] \tag{C.1}$$

$$= \left(\frac{x}{x_{\tau^P}}\right)^{\beta}\left(B + \frac{x_{\tau^P}}{\beta\delta}\right) = \left(\frac{x}{x_{\tau^P}}\right)^{\beta} B + F(x, x_{\tau^P})$$

When the firm invests at (11), the economic value is:

$$W^{BB}(x, x_{\tau^{BB}}) = \left[\left(\frac{x}{x_{\tau^{BB}}}\right)^{\beta}\left(B + \gamma\frac{x_{\tau^{BB}}}{\delta} - \pi(\tau^{BB})\right)\right. \tag{C.2}$$
$$\left. + \left(\frac{x}{x_{\tau^{BB}}}\right)^{\beta}\left((1-\gamma)\frac{x_{\tau^{BB}}}{\delta} - I + \pi(\tau^{BB})\right)\right]$$
$$= \left(\frac{x}{x_{\tau^{BB}}}\right)^{\beta}\left(B + \frac{(1-\gamma)x_{\tau^{BB}}}{\beta\delta}\right) = \left(\frac{x}{x_{\tau^{FBB}}}\right)^{\beta} B + (1-\gamma)F(x, x_{\tau^{BB}})$$

The difference between (C.2) and (C.1) becomes:

$$W^{BB}(x, x_{\tau^{BB}}) - W^{P}(x, x_{\tau^P}) = \left(\frac{x}{x_{\tau^{BB}}}\right)^{\beta}\left(B + \frac{(1-\gamma)x_{\tau^{BB}}}{\beta\delta}\right) - \left(\frac{x}{x_{\tau^P}}\right)^{\beta}\left(B + \frac{x_{\tau^P}}{\beta\delta}\right) \tag{C.3}$$
$$= \left(\frac{x}{x_{\tau^P}}\right)^{\beta}\left(\frac{\beta-(1-\gamma)}{\beta-1}\right)^{\beta}\left[B + I\frac{(1-\gamma)}{\beta-(1-\gamma)} - \left(\frac{\beta-1}{\beta-(1-\gamma)}\right)^{\beta}\left(B + \frac{I}{\beta-1}\right)\right]$$

Let's define $\Omega(\gamma, \sigma) \equiv \left[B + I\frac{(1-\gamma)}{\beta-(1-\gamma)} - \left(\frac{\beta-1}{\beta-(1-\gamma)}\right)^{\beta}\left(B + \frac{I}{\beta-1}\right)\right]$. We first prove that, for a given σ, there may be a value of $\gamma \in (0,1)$ such that $\Omega(\gamma,\sigma) = 0$. Then we show how this value varies with σ.

Since $\Omega(\gamma, \sigma)$ is continuous in γ, by fixing σ, it is easy to show that:

$$\Omega(0,\sigma) = 0 \text{ and } \Omega(1,\sigma) = B - \left(\frac{\beta-1}{\beta}\right)^{\beta}\left(B + \frac{I}{\beta-1}\right) < 0 \rightarrow \text{if } B < \frac{1}{\left[\left(\frac{\beta}{\beta-1}\right)^{\beta} - 1\right]}\frac{I}{\beta-1} \tag{C.4}$$

where $\left(\frac{\beta}{\beta-1}\right)^{\beta} > 1$. Further $\Omega(\gamma, \sigma)$ is a concave function on γ. Taking the first and second derivatives with respect to γ we get:

$$\frac{\partial\Omega}{\partial\gamma} = \frac{\beta}{(\beta-(1-\gamma))^2}\left[-I + (\beta-1)\left(\frac{\beta-1}{\beta-(1-\gamma)}\right)^{\beta-1}\left(B + \frac{I}{\beta-1}\right)\right] \tag{C.5}$$

$$\frac{\partial^2\Omega}{\partial\gamma^2} = \frac{\beta}{(\beta-(1-\gamma))^2}\left[-(\beta-1)^2\left(\frac{\beta-1}{\beta-(1-\gamma)}\right)^{\beta-2}\frac{\beta-1}{\beta-(1-\gamma)}\left(B + \frac{I}{\beta-1}\right)\right] < 0 \tag{C.6}$$

and the value of γ such that $\frac{\partial\Omega}{\partial\gamma} = 0$ is:

$$\gamma^{\max} = (\beta-1)\left[\left(1 + (\beta-1)\frac{B}{I}\right)^{\frac{1}{\beta-1}} - 1\right] \tag{C.7}$$

Since $1 + (\beta - 1)\frac{B}{I} > 1$ we get that $\gamma^{\max} > 0$ while it is less than 1 if:

$$B < \left[\left(\frac{\beta}{\beta - 1} \right)^{\beta - 1} - 1 \right] \frac{I}{\beta - 1} \tag{C.8}$$

Finally, comparing (C.4) and (C.8), it is easy to show that if (C.4) holds then (C.8) is always satisfied. This implies that there exists a value of $\hat{\gamma}(\sigma) \in (0, 1)$ such that for $\gamma \geq \hat{\gamma}(\sigma)$, $\Omega(\gamma, \sigma) < 0$ and positive otherwise.

Let's now consider the effect of σ. Recalling that $\frac{d\beta}{d\sigma} < 0$, with $\lim_{\sigma \to 0} \beta = +\infty$ and $\lim_{\sigma \to \infty} \beta = 1$, we get:

$$\lim_{\beta \to \infty} \Omega(\gamma, \sigma) = \lim_{\beta \to \infty} \left[B + I \frac{(1 - \gamma)}{\beta - (1 - \gamma)} - \left(\frac{\beta - 1}{\beta - (1 - \gamma)} \right)^{\beta} \left(B + \frac{I}{\beta - 1} \right) \right] = 0 \tag{C.9}$$

and:

$$\begin{aligned} \lim_{\beta \to 1} \Omega(\gamma, \sigma) &= \lim_{\beta \to 1} \left[B + I \frac{(1 - \gamma)}{\beta - (1 - \gamma)} - \left(\frac{\beta - 1}{\beta - (1 - \gamma)} \right)^{\beta} \left(B + \frac{I}{\beta - 1} \right) \right] \\ &= \left[B + I \frac{(1 - \gamma)}{\gamma} - \frac{I}{\gamma} \right] \\ &= B - I < 0 \end{aligned} \tag{C.10}$$

Note that (C.10) is negative. Thus, by (C.4), there exists a value of $\hat{\sigma}$ such that for $\sigma < \hat{\sigma}$, $\Omega(\gamma, \sigma) > 0$ for all $\gamma \in [0, 1]$. On the contrary, for $\sigma \geq \hat{\sigma}$, as proved above, there may exist a value $\hat{\gamma}(\sigma) > \gamma^{\max}$ such that for $\gamma < \hat{\gamma}(\sigma)$ we get $\Omega(\gamma, \sigma) > 0$, and $\Omega(\gamma, \sigma) < 0$ for $\gamma > \hat{\gamma}(\sigma)$.

Appendix D

Let's compare $F^{BBT}(x, x_{\tau^{BBT}})$ with the first-best, i. e.:

$$\begin{aligned} F^{BBT}(x, x_{\tau^{BB1}}) &= \left(\frac{x}{x_{\tau^{BBT}}} \right)^{\beta} \frac{(1 - \gamma)}{\beta - (1 - \gamma)} (I - \gamma B) \\ &= \left(\frac{x_{\tau^P}}{x_{\tau^{BBT}}} \right)^{\beta} \left(\frac{x}{x_{\tau^P}} \right)^{\beta} \frac{I}{\beta - 1} \frac{(I - \gamma B)}{I} \frac{(1 - \gamma)(\beta - 1)}{\beta - (1 - \gamma)} \\ &= (1 - \gamma) \left(\frac{\beta - (1 - \gamma)}{\beta - 1} \right)^{\beta - 1} \left(\frac{I}{I - \gamma B} \right)^{\beta - 1} F(x, x_{\tau^P}) \\ &= \mu^T(\gamma, \beta) F(x, x_{\tau^P}) \end{aligned} \tag{D.1}$$

where $\mu^T(\gamma, \beta) \equiv (1 - \gamma) \left(\frac{\beta - (1 - \gamma)}{\beta - 1} \right)^{\beta - 1} \left(\frac{I}{I - \gamma B} \right)^{\beta - 1}$. Since the term $\mu^T(\gamma, \beta)$ is monotone in γ with $\mu^T(1, \beta) = 0$ and $\mu^T(0, \beta) = 1$, we may conclude that $\mu^T(\gamma, \beta) \in [0, 1]$.

Let's now consider the project's economic value, by taking into account the external benefits associated with project acceleration. Denoting with $W^{BBT}(x, x_{\tau^{BB}})$ the value, the difference is:

$$W^{BBT}(x, x_{\tau^{BB1}}) - W^{P}(x, x_{\tau^{P}}) = \left[\phi^{T}(\gamma,\beta) - 1\right]\left(\frac{x}{x_{\tau^{P}}}\right)^{\beta} B + \left[\mu^{T}(\gamma,\beta) - 1\right] F(x, x_{\tau^{P}}) \quad (D.2)$$

where $\phi^{T}(\gamma,\beta) = \left(\frac{\beta-(1-\gamma)}{\beta-1}\right)^{\beta}\left(\frac{I}{I-\gamma B}\right)^{\beta} > 1$. By simple algebra we get:

$$W^{BB1}(x, x_{\tau^{BBT}}) - W^{P}(x, x_{\tau^{P}}) = \left(\frac{x}{x_{\tau^{P}}}\right)^{\beta}\left(\frac{x_{\tau^{p}}}{x_{\tau^{BBT}}}\right)^{\beta} \Omega^{T}(\gamma,\sigma) \quad (D.3)$$

where:

$$\Omega^{T}(\gamma,\sigma) \equiv \left[B + \frac{(1-\gamma)}{\beta-(1-\gamma)}(I-\gamma B) - \left(\frac{\beta-1}{\beta-(1-\gamma)}\right)^{\beta}\left(\frac{I-\gamma B}{I}\right)^{\beta}\left(B + \frac{I}{\beta-1}\right)\right] \quad (D.4)$$

$$= \Omega(\gamma,\sigma) - \frac{(1-\gamma)}{\beta-(1-\gamma)}\gamma B + \left(\frac{\beta-1}{\beta-(1-\gamma)}\right)^{\beta}\left(B + \frac{I}{\beta-1}\right)\left[1 - \left(\frac{I-\gamma B}{I}\right)^{\beta}\right]$$

Since $\Omega^{T}(\gamma,\sigma)$ is continuous in γ, it is easy to show that for any given σ:

$$\Omega^{T}(0,\sigma) = 0 \text{ and } \Omega^{T}(1,\sigma) = B - \left(\frac{\beta-1}{\beta}\right)^{\beta}\left(\frac{I-B}{I}\right)^{\beta}\left(B + \frac{I}{\beta-1}\right) \quad (D.5)$$

As $\Omega^{T}(1,\sigma) > \Omega(1,\sigma)$, and $\frac{\partial\Omega^{T}}{\partial\sigma} > 0$, this confirms the result in Proposition 2.

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