

Running into tension: Primordial black holes from ultra-slow-roll inflation, spectral running, and the Hubble tension

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Single-field ultra-slow-roll (USR) inflation is among the most studied mechanisms for primordial black hole (PBH) formation. When a nonstationary inflection point close to cosmic microwave background (CMB) scales is present, these models predict a negative spectral running ($\alpha_s < 0$), whose magnitude increases with the PBH mass. This is in tension with recent hints for positive running from Atacama Cosmology Telescope (ACT) CMB data. However, inflationary parameters inferred from CMB data are sensitive to the assumed prerecombination expansion history, which is precisely where new physics motivated by the Hubble tension should operate. Focusing on axionlike early dark energy (EDE) as a benchmark, we investigate the effect of such prerecombination new physics on α_s , and hence on the viability of USR PBH models, in light of state-of-the-art CMB data from *Planck*, ACT, and the South Pole Telescope, together with baryon acoustic oscillation data from the Dark Energy Spectroscopic Instrument Data Release 2. Our analysis, therefore, provides an updated set of constraints on α_s and the running of the running β_s . For most dataset combinations, moving from Λ cold dark matter to EDE increases the inferred α_s : once the acoustic angular scale θ_s is fixed, EDE increases the diffusion-to-acoustic angular scale ratio θ_d/θ_s , and the shift in α_s compensates this extra damping by increasing small-scale power. In this sense, tension calls for tension: taking the Hubble tension seriously as an indication for new physics strengthens the challenges faced by USR PBH models. More broadly, our analysis stresses that inflationary model selection using CMB-inferred inflationary parameters such as n_s and α_s may be premature, especially until the Hubble tension, and more generally the prerecombination expansion history, is understood.

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I. INTRODUCTION

Primordial black holes (PBHs) [1–4] have emerged as a remarkably well-motivated target, both from a theoretical and observational standpoint (see, e.g., Refs. [5–9] for reviews). On the theoretical side, PBHs offer a compelling dark matter candidate, particularly in the asteroid-mass window $M_{\text{PBH}} \sim 10^{-17} - 10^{-12} M_{\odot}$, where they can constitute the entirety of the observed dark matter abundance [10–13]. On the observational side, several of the compact binary mergers detected by LIGO-Virgo-KAGRA (LVK)

collaboration [14–18] feature subsolar or low-mass components [19–23] whose astrophysical origin remains uncertain, and for which PBHs are a natural candidate [24–33]. In addition, a plausible interpretation for the nHz gravitational wave (GW) signal recently detected by pulsar timing array (PTA) experiments [34–37] is that of a scalar-induced GW (SIGW) background sourced by the same enhanced curvature perturbations that are responsible for PBH formation [38–65]. In most of the standard formation scenarios, PBHs form from the gravitational collapse of rare overdense regions seeded by large-amplitude peaks in the (dimensionless) primordial curvature power spectrum, which must be enhanced by several orders of magnitude above the nearly scale-invariant cosmic microwave background (CMB) value, $\mathcal{P}_{\zeta}(k_{\text{CMB}}) \sim 10^{-9}$, up to $\mathcal{P}_{\zeta}(k_{\text{pk}}) \sim 10^{-2}$ at the relevant small scales. Among the simplest and most studied single-field inflationary mechanisms capable of generating such an enhancement is a transient phase of ultra-slow-roll (USR) inflation, during which the inflaton traverses a flat feature in its potential, typically an inflection

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point, causing a dramatic amplification of curvature perturbations [66–80].¹

A key feature of USR models for PBH formation is that the dynamics that generate the enhanced peak in the curvature power spectrum cannot be decoupled from those that control the CMB-scale part of the inflationary trajectory. Such a coupling has important consequences. USR inflation is generically divided into an initial slow-roll (SR) phase that lasts for N_1 e -folds, during which the CMB modes exit the horizon and the large-scale power spectrum is imprinted, followed by the USR phase, which ultimately seeds PBH formation. The position of the resulting enhanced peak is set by N_1 : the shorter the initial SR phase, the closer the USR feature lies to the CMB window, and the heavier the PBHs produced. The fact that the PBH mass controls the location of the USR feature relative to the CMB scales leads in turn to a mass-dependent imprint on the inflationary observables constrained by CMB experiments: the scalar spectral index n_s , the tensor-to-scalar ratio r , and the spectral running α_s . The closer the feature, the stronger the impact [70,74,84]. It is, therefore, CMB-scale observables, and not just the small-scale power spectrum amplitude, that carry direct information about these models. As a result, precision CMB measurements can play a key role when testing the viability of USR models for PBH formation.

A central prediction of these scenarios is that of negative running ($\alpha_s < 0$) [84]. This follows from the necessity of an initial SR phase of reduced duration, followed by a successful PBH-producing USR phase: together, these force the inflaton to cross a nonstationary inflection point close to the CMB scales and push the primordial power spectrum away from scale invariance [84]. This expectation, however, is in tension with recent primordial power spectrum constraints from the Atacama Cosmology Telescope (ACT) CMB data. The latest ACT Data Release 6 (DR6) measurements [85], once combined with large-to-intermediate scale CMB data from the *Planck* satellite, shift n_s toward larger values, closer to the scale-invariant value of $n_s = 1$, which has itself led to a renewed interest in inflationary model-building activity (see, e.g., Refs. [86–163]). The same combination, with the further addition of CMB lensing and baryon acoustic oscillation (BAO) data, also leads to $\alpha_s = 0.0062 \pm 0.0052$ [85]. This mild preference for positive running points precisely in the opposite direction to that required in USR models for PBH formation: as pointed out in Ref. [84], the resulting tension for these models is progressively more pronounced for heavier PBHs (given that the relevant feature lies closer to the CMB scales), especially for those in the solar mass range whose mergers are relevant for ground-based GW experiments.

The previous conclusions rest upon an essential caveat. Constraints on inflationary parameters are often treated as

direct, model-independent measurements of the primordial power spectrum and, therefore, ultimately on the parameters of the inflationary Lagrangian. In truth, they are not. As is the case with any cosmological parameter, the inferred inflationary parameters depend on the assumed cosmological model [typically the Λ cold dark matter (Λ CDM) model], and are particularly sensitive to the assumed prerecombination expansion history, or more generally the postinflationary history [164–192]. These considerations open a potential loophole to the “USR-ACT tension,” in case the preference for positive running could be removed or at least weakened by introducing new prerecombination physics. We, therefore, believe that the tension should not be overinterpreted, at least not until the stability of constraints on α_s under the assumed cosmological model, and in particular under physically motivated modifications of the (prerecombination) expansion history, is better understood.

The above is not merely an academic question, and is unavoidable if one seriously takes the Hubble tension (see, e.g., Refs. [193–207] for reviews) as an indication for new physics [208–282], as opposed to unaccounted systematics [283–287]. In this case, modifications to the prerecombination expansion history are precisely those required to solve the tension, for two reasons [288–297]: firstly, consistency with BAO measurements requires a reduction of the sound horizon, which necessarily requires new physics operating prior to recombination. At the same time, postrecombination attempts to solve the tension are severely constrained by BAO and unanchored Type Ia Supernovae (SNe Ia) measurements. Prerecombination solutions to the Hubble tension, such as early dark energy (EDE), are indeed known to strongly affect the inference of inflationary parameters such as n_s , which typically increases. The reason is that these models affect the CMB damping tail in such a way as to increase the amount of damping, which in turn can be compensated by an enhanced small-scale power spectrum. It is then natural to ask whether prerecombination new physics called to solve the Hubble tension can affect α_s in a similar way. With these premises in mind, in this work we address two closely related questions:

- (1) From the PBH point of view: can Hubble tension-motivated prerecombination new physics shift current constraints on α_s toward the negative values preferred by USR models for PBH formation, thereby “saving” these models?
- (2) From the cosmological point of view: how robust are current constraints on α_s , with respect to both the assumed prerecombination expansion history and adopted CMB dataset?

Answering these two interconnected questions will allow us to determine the extent to which the challenges faced by USR models for PBH formation depend on the assumption of Λ CDM in the prerecombination history, as well as the adoption of the ACT CMB measurements.

¹The level of fine-tuning required to achieve the correct PBH abundance in these models has been recently discussed in Refs. [81–83].

To address these questions, we use axionlike EDE as a benchmark model of prerecombination new physics. The reason is that it provides a well-studied model of new physics, which is relatively successful at alleviating the Hubble tension, while exhibiting the shift toward larger values of n_s due to its impact on the damping tail: it is, therefore, a useful benchmark for testing the extent to which the “USR-ACT tension” is affected by Hubble tension-motivated modifications of the prerecombination expansion history. To assess the dependence on the adopted CMB dataset, we consider a large set of dataset combinations, involving CMB measurements from *Planck*, ACT, and the South Pole Telescope (SPT), alone or in combination with each other, and/or in combination with BAO measurements from the Dark Energy Spectroscopic Instrument (DESI) Data Release 2 (DR2) release. Overall, our analysis includes 32 runs, obtained by combining different cosmological models (Λ CDM and EDE, allowing for nonzero spectral running α_s and potentially for the running of the running β_s) and dataset combinations: since several of these dataset combinations had not been previously used to constrain α_s , even within Λ CDM, a useful byproduct of our work is an updated set of limits on α_s in light of state-of-the-art cosmological data (see also Ref. [298]). In short, we find that for most models/dataset combinations, moving from Λ CDM to EDE increases the inferred α_s (as one could *a priori* expect, given the impact on n_s): in fact, for our most constraining combination involving all three CMB datasets, α_s is always positive and larger with respect to the value inferred assuming Λ CDM. While our analysis focuses on EDE, we argue that this is expected to apply more broadly to any prerecombination new physics model aimed at solving the Hubble tension, such as models involving extrarelativistic components. In this sense, tension calls for tension: because of the model dependence of the inferred inflationary parameters, taking the Hubble tension seriously strengthens, rather than weakens, the tension faced by USR models for PBH formation.

The rest of this work is then organized as follows. In Sec. II we briefly review various theoretical aspects that are central to our study: production of PBHs from USR inflation (Sec. II A), the reason why the Hubble tension generically affects the inference of cosmological parameters (Sec. II B), and the axionlike EDE model (Sec. II C). In Sec. III we discuss the datasets and methodology adopted to obtain our results. These are presented in Sec. IV, and critically discussed with an eye to their broader implications in Sec. V. Finally, in Sec. VI we close by drawing concluding remarks.

II. THEORY

We now review some background material that is useful to understand the premises of our work. We begin by reviewing the formation of PBHs from USR inflation, and why this generally requires $\alpha_s < 0$. We then explain why inflationary

parameters inferred from cosmological observations are generally model dependent, and should, therefore, not be treated as direct, model-independent measurements of the primordial power spectrum. Finally, we review the axionlike EDE model we adopt as benchmark prerecombination modification aimed at solving the Hubble tension.

A. Primordial black holes from ultra-slow-roll inflation

The simplest single-field realization of inflation that can produce a sizeable abundance of PBHs relies on a transient USR phase, during which the inflaton crosses a quasistationary feature of the potential, typically an inflection point or a shallow local extremum, that strongly amplifies the curvature power spectrum at small scales [66,67]. The required hierarchy between the curvature power spectrum at CMB scales, $\mathcal{P}_\zeta(k_{\text{CMB}}) \sim 10^{-9}$, and at PBH scales, $\mathcal{P}_\zeta(k_{\text{pk}}) \sim 10^{-2}$, must be generated within a finite number of e -folds, since heavy nonevaporating PBHs require the USR feature to occur relatively close in e -fold time to the CMB window. Schematically, denoting by N_1 the number of e -folds elapsed between horizon exit of CMB scales and the onset of the USR phase, the typical horizon mass at the spectral peak scales as follows [84]:

$$M_{\text{PBH}} \simeq \mathcal{O}(1)M_\odot \times e^{-2(N_1-18)}. \quad (1)$$

Hence, solar-mass PBHs (relevant for LVK [14–18,25–29,31]) are produced when the USR feature is close to the scales relevant for CMB spectral distortion constraints from FIRAS [299,300] (small N_1), while asteroid-mass PBHs (relevant for the dark matter window) require larger $N_1 \sim [30 - 40]$. PTA signals associated with USR scenarios, through the SIGW background sourced by the same enhanced curvature spectrum, correspond to a peak scale around $k_{\text{pk}} \sim 10^7 \text{ Mpc}^{-1}$, again pointing to relatively small N_1 and to PBHs of solar or subsolar mass [34–37,60].

The reduced duration of the initial SR phase, combined with the necessity of a sizeable spectral peak, push USR models systematically away from scale invariance [70,74]. As shown in detail in Ref. [84], the requirement of a successful PBH-producing USR phase forces the inflaton to cross a nonstationary inflection point near CMB scales, where the inflationary observables admit a controlled expansion in two parameters: the offset $\delta_\star$ from the inflection point and the parameter β measuring the deviation from a stationary one.² The resulting structure is most transparent when expressed at the level of the

²The fact that ACT data are in tension with USR models of PBH formation was first pointed out by Frolovsky and Ketov [301], who showed, working exclusively within a specific modified Starobinsky-like realization, that the new ACT preference for positive α_s is in tension with the values typically required to obtain a sizeable PBH peak. Their analysis, unlike that of Ref. [84], was confined to a single benchmark model.

running of the spectral index, which to leading order takes the schematic form

$$\alpha_s \simeq -\frac{12\beta}{V_0^2} - \frac{36\delta_\star^2}{V_0^2}, \quad (2)$$

where V_0 denotes the height of the potential at the inflection point. The crucial point is that, in order for the inflaton to actually cross the inflection point and reach the small-field region where the USR feature is located, β must be strictly positive: a negative β would generate a local maximum that traps the field. Together with the manifestly negative $\delta_\star^2$ contribution, this leads to the robust prediction that $\alpha_s < 0$ in this entire class of polynomial USR models. The same negative-running tendency is shared by other commonly studied USR realizations, including the well-known Higgs- and fiber-inflation-inspired potentials [68,69,73].

The meaning of the three quantities entering Eq. (2) is illustrated in Fig. 1, for the representative polynomial potential $V(\phi) = m^2\phi^2/2 - \mu\phi^3/3 + \lambda\phi^4/4$, with the quartic coupling parametrized as $\lambda = \mu^2(1 + \beta)/(4m^2)$. This parametrization is convenient because $\beta = 0$ corresponds to an exactly stationary inflection point at $\phi_0 = \mu/(2\lambda)$, where $V'(\phi_0) = V''(\phi_0) = 0$ and $V_0 \equiv V(\phi_0)$, whereas for $\beta \neq 0$ one finds exactly $V'(\phi_0) = 2\beta$ and $V''(\phi_0) = 3\beta$ (in units $m = \mu = 1$). The parameter β , therefore, has a transparent geometrical meaning: it is a direct measure of the residual slope of the potential at the feature, i.e., of its departure from stationarity. Its sign is not

a matter of choice. For $\beta > 0$ the potential remains monotonic and the inflaton traverses the feature, experiencing the transient USR phase, which enhances the curvature perturbations. For $\beta < 0$, on the other hand, a local minimum and a local maximum are generated on either side of ϕ_0 , and the inflaton becomes trapped before it can reach the small-field region, so that no PBH-producing background exists in the first place. This is the sense in which $\beta > 0$ is required, rather than assumed. The remaining quantity, $\delta_\star$, is the displacement of the inflaton from the feature at the time the CMB modes leave the horizon, and enters Eq. (2) squared, hence with a definite negative sign irrespective of the side from which the feature is approached. The combination of these two facts is what makes the negative sign of α_s robust within the given class of models.

This sign of α_s is in direct tension with recent CMB data from ACT, which, when combined with *Planck* data, indicates a positive running, $\alpha_s = 0.0062 \pm 0.0052$ [85,302], alongside a higher value of the spectral index, $n_s = 0.9743 \pm 0.0034$. The tension is not uniform across the PBH mass range: the magnitude of $|\alpha_s|$ predicted by USR models grows with the produced mass of PBHs, and hence becomes more pronounced for solar-ish mass PBHs. As our work will show, this conclusion is sensitive to the assumed CMB dataset.

B. Inflationary parameters and the Hubble tension

We now review why the type of new physics required to solve the Hubble tension affects the determination of

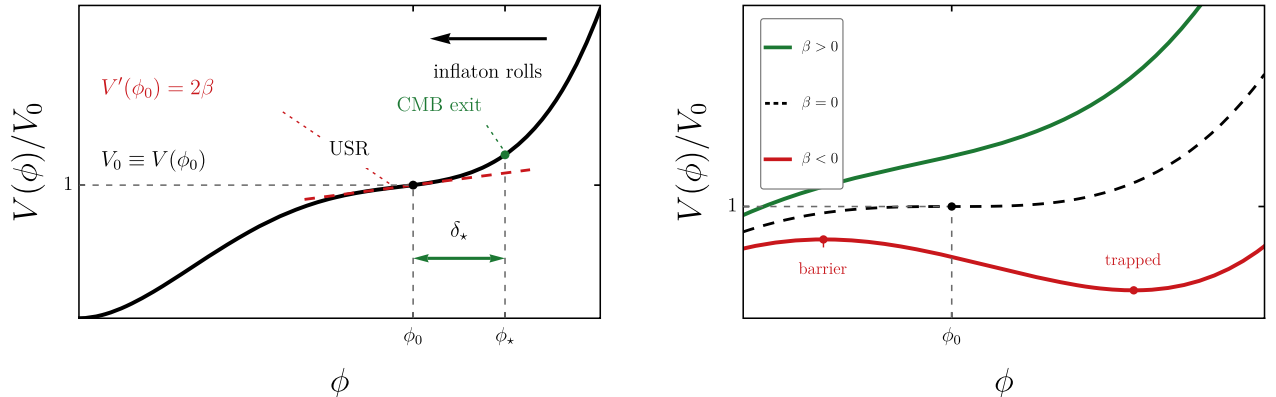


FIG. 1. Illustration of the quantities entering the right-hand side of Eq. (2), for the representative polynomial potential $V(\phi) = m^2\phi^2/2 - \mu\phi^3/3 + \lambda\phi^4/4$ with $\lambda = \mu^2(1 + \beta)/(4m^2)$, shown in units $m = \mu = 1$. Left panel: the inflaton rolls from large- to small-field values (black arrow) and traverses the near-inflection point at ϕ_0 , whose height defines V_0 . The CMB modes exit the horizon earlier, at $\phi_\star$, and the offset $\delta_\star \equiv \phi_\star - \phi_0$ (green double arrow) measures how far the inflaton still is from the feature at that moment: heavier PBHs correspond to smaller N_1 , hence smaller $\delta_\star$ and a larger imprint on CMB-scale observables. The red dashed line is the tangent to the potential at ϕ_0 : for this parametrization one has exactly $V'(\phi_0) = 2\beta$ and $V''(\phi_0) = 3\beta$, so that β measures precisely the deviation of the feature from a stationary inflection point. Right panel: zoom on the feature, showing why $\beta > 0$ is required. For $\beta > 0$ (green) the potential is monotonic and the inflaton crosses the feature, undergoing the USR phase, which seeds PBH formation. $\beta = 0$ (black dashed) corresponds to an exactly stationary inflection point, and for $\beta < 0$ (red) a local minimum/maximum pair is generated and the inflaton is trapped before reaching the small-field region, so that no viable PBH-producing background is obtained. The requirement $\beta > 0$, combined with the manifestly negative $\delta_\star^2$ term, is what makes $\alpha_s < 0$ a robust prediction of this class of models.

inflationary parameters, typically increasing n_s . Our discussion also clarifies why constraints on inflationary parameters cannot be treated as direct, model-independent measurements of the primordial power spectrum.

New physics aiming to address the Hubble tension must do so while keeping the acoustic angular scale θ_s fixed, as this is measured by the CMB to exquisite precision, with very little model dependence. This quantity is given by the following:

$$\theta_s \equiv \frac{r_s^*}{D_A^*}, \quad (3)$$

where r_s^* is the comoving sound horizon at recombination, and D_A^* is the comoving angular diameter distance to the surface of last-scattering. In principle one could raise H_0 while keeping θ_s fixed in (at least) two ways:

- (1) lowering r_s^* , with the corresponding increase in H_0 decreasing D_A^* by the same amount: since the integral determining r_s^* runs over the prerecombination epoch, this requires prerecombination (early-time) new physics;
- (2) keeping r_s^* fixed, while altering the shape of the postrecombination expansion rate, i.e., $E(z) \equiv H(z)/H_0$, in such a way so as to reach a higher H_0 while keeping D_A^* fixed: this is the route followed by postrecombination (late-time) new physics attempts to solve the Hubble tension.

At the geometrical level, CMB measurements cannot distinguish between the above two options toward raising the inferred value of H_0 .

BAO measurements play a key role because they break this geometrical degeneracy in favor of the first option, i.e., the prerecombination class of solutions reducing r_s^* , as extensively discussed in the literature (see, for instance, Refs. [288–297]). BAO essentially measures the angular size of the comoving sound horizon at baryon drag, r_{drag} (itself closely related to $r_s^* \simeq 1.018 r_{\text{drag}}$), along and transverse to the line-of-sight, as well as a volume-averaged combination of the two. It is easy to show (see, e.g., Eqs. (3)–(5) of Ref. [303]) that BAO measurements are directly sensitive to the combination $r_{\text{drag}} H_0$. Loosely speaking, current BAO measurements are consistent with a value $r_{\text{drag}} H_0 \approx 10000$ km/s, which in Λ CDM corresponds to a “high” sound horizon $r_{\text{drag}} \sim 147$ Mpc and a “low” $H_0 \approx 67$ km/s/Mpc. Keeping the product $r_{\text{drag}} H_0$ constant requires that an increase in H_0 be accompanied by a proportional decrease in r_{drag} (see, e.g., Fig. 1 of Ref. [293]), i.e., $\delta r_{\text{drag}}/r_{\text{drag}} \simeq -\delta H_0/H_0$. At present, solving the Hubble tension requires a decrease in the sound horizon at the $\simeq 7\%$ level, $\delta r_{\text{drag}}/r_{\text{drag}} \simeq -0.07$. As stressed earlier, this requires new physics operating prior

to recombination, as the integral determining r_{drag} runs across redshifts $z > z_{\text{drag}} \approx 1060$.³

If r_{drag} decreases by the same relative amount by which H_0 increases ($\delta r_{\text{drag}}/r_{\text{drag}} \simeq -\delta H_0/H_0$), while the shape of the postrecombination expansion rate remains unchanged, the CMB constraint on θ_s is automatically satisfied. In fact, the relation between the two sound horizons implies $\delta r_s^*/r_s^* \simeq \delta r_{\text{drag}}/r_{\text{drag}}$, whereas $\delta D_A^*/D_A^* = -\delta H_0/H_0$, implying that $\delta r_s^*/r_s^* \simeq \delta D_A^*/D_A^*$. Since the numerator and denominator of Eq. (3) decrease by the same relative amount, θ_s remains fixed, as required by CMB observations. However, the CMB damping tail also tightly constrains the angular size of the photon diffusion scale at recombination θ_d , defined as follows:

$$\theta_d \equiv \frac{r_d^*}{D_A^*}, \quad (4)$$

where r_d^* is the comoving photon diffusion scale at recombination and, crucially, the denominator is the same as in Eq. (3). Whereas r_s^* is sensitive to the expansion history in the entire decade of scale factor prior to recombination, the sensitivity of r_d^* to the expansion history is much more localized around recombination (compare the blue and orange kernels in Fig. 2 of Ref. [293]). As a result, models that increase the prerecombination expansion rate in order to reduce r_s^* are only able to reduce r_d^* by a smaller relative amount: $|\delta r_d^*/r_d^*| < |\delta r_s^*/r_s^*|$.⁴ However, it is the same quantity that appears at the denominators of Eqs. (3) and (4). Therefore, a model that can keep θ_s fixed will in general not be able to keep θ_d fixed, as $|\delta r_d^*/r_d^*| < |\delta r_s^*/r_s^*| \simeq |\delta D_A^*/D_A^*|$.⁵ Since in Eq. (4) the numerator decreases more slowly than the denominator, keeping θ_s fixed will result in θ_d increasing.

An increase in θ_d implies that diffusion damping becomes relevant at smaller multipoles, since $\ell \propto 1/\theta$. The damping tail in the CMB temperature power spectrum

³Aside from the amplitude of BAO measurements and their sensitivity to the product $r_{\text{drag}} H_0$, another important obstruction to late-time solutions to the Hubble tension comes from unanchored SNe Ia, which place tight constraints on the shape of the expansion history $E(z)$ [304–310].

⁴As discussed in Refs. [293,311], for many models that attempt to solve the Hubble tension by increasing the prerecombination rate close to matter-radiation equality, the fractional decrease in r_d^* is about half that of r_s^* : $\delta r_d^*/r_d^* \simeq 1/2 \delta r_s^*/r_s^*$. The factor of 1/2 is related to the random walk nature of diffusion damping, because of which r_d^* depends on the integral of $\sqrt{1/H(z)}$, as opposed to $1/H(z)$ in the case of r_s^* .

⁵Stated otherwise, the different sensitivities of r_s^* and r_d^* to the prerecombination expansion history make it very difficult to keep the diffusion-to-acoustic angular scale ratio θ_d/θ_s fixed. As observed in Ref. [312], this can potentially be achieved by considering modifications to the expansion history that are much more extended compared to the ones considered so far, mostly concentrated around matter-radiation equality.

will, therefore, be more strongly suppressed, at a given multipole, relative to the reference best-fit Λ CDM model. To maintain consistency with the measured CMB power spectrum, this extra damping needs to be compensated, and the most economical way to do so is to increase n_s , thereby providing additional power to smaller scales. This is the reason why most early-time modifications aimed at solving the Hubble tension do so at the expense of an increase in n_s (alongside shifts in other cosmological parameters, including but not limited to ω_c and ω_b [313–317]). This behavior has been extensively discussed, for instance, in the case of EDE [170,182], where the inferred value of n_s can move significantly closer to the scale-invariant value of $n_s = 1$, motivating extensive theoretical follow-up.

Summing up, early-time modifications to the expansion rate typically shift the inferred inflationary parameters. These affect the primordial power spectrum so as to compensate the extra damping associated to the increase in θ_d , itself tied to the different responses of r_d^* and r_s^* to modifications to the prerecombination expansion rate. This discussion clarifies a very important point: inflationary parameters inferred from cosmological observations depend on the assumed underlying model, and *cannot* be treated as direct, model-independent measurements of the primordial power spectrum. Given that attempts to resolve the Hubble tension are generally accompanied by an increase in n_s , it is natural to ask whether similar considerations hold for α_s as well. Indeed, increasing α_s increases the amount of small-scale power (at fixed n_s and above the pivot scale), which counteracts the extra damping following from a larger value of θ_d . As we shall see, this expectation is confirmed within our EDE benchmark: α_s does indeed increase, although the increase is much milder compared to that in n_s .

C. Axionlike early dark energy

We now turn to EDE as a benchmark model of early-time new physics aimed at addressing the Hubble tension (see, e.g., Refs. [318–320] for reviews). In general, the term EDE refers to a component that was dynamically relevant at $z \gg 1$, and whose equation of state was $w \simeq -1$ at some point during its evolution. It is often (although not always) the case that EDE takes the form of a scalar field initially frozen in its potential by Hubble friction. In our case, we specialize to the so-called axionlike EDE model, first introduced in the context of the Hubble tension in Refs. [321–323]. Our motivation is that this is a concrete and widely studied prerecombination new physics model that exhibits the features relevant for our analysis, i.e., a decrease in the sound horizon accompanied by an increase in n_s to compensate the impact on the damping tail. For this reason, we adopt it as a useful benchmark for testing the impact of early-time new physics on the inferred value of α_s . Nevertheless, we stress that several other EDE models beyond axionlike EDE are being developed and studied,

such as (cold and hot) new EDE, anti-de Sitter EDE, acoustic EDE, and rock 'n roll EDE, among others: we refer the reader to Refs. [324–398] for examples in this direction, as well as various studies on EDE.

The axionlike EDE model consists of an ultralight scalar field ϕ (with mass typically $m \lesssim 10^{-27}$ eV), governed by the following potential [322,323]:

$$V(\phi) = m^2 f^2 \left[1 - \cos\left(\frac{\phi}{f}\right) \right]^n, \quad (5)$$

where m and f are the field's mass and decay constant, respectively, and the potential index n takes integer values.⁶ Initially, EDE is displaced from its minimum and frozen by Hubble friction, resembling a cosmological constant. When the Hubble rate is of order the field's mass, the field becomes dynamical and starts rolling down its potential. Eventually, it oscillates around the minimum of its potential and dilutes, redshifting as an effective fluid with equation of state (after averaging over oscillations) $\langle w \rangle = (n-1)/(n+1)$.

In cosmological analyses, it is common practice to trade the fundamental parameters m and f for two phenomenological parameters: the critical redshift z_c at which EDE becomes dynamical, and f_{ede} , the fractional energy density contributed by EDE at redshift z_c . Aside from these parameters, EDE is characterized by the initial misalignment angle $\theta_i = \phi_i/f$, where ϕ_i is the initial field value, and the potential index n . Without loss of generality, θ_i can be taken in the range $0 \leq \theta_i \leq \pi$. Since cosmological data are not particularly sensitive to n [399], in what follows we will set $n = 3$: with this choice, once the field starts oscillating around the minimum its effective equation of state is $\langle w \rangle = 1/2$, allowing EDE to decay more quickly than the radiation component. As commonly done in such studies, the behavior of EDE perturbations is followed at the scalar field level (rather than treating it as an effective fluid), with linear fluctuations in ϕ evolved consistently with the metric and matter perturbations in the perturbed Einstein-Boltzmann-Klein-Gordon system. We refer the reader to Refs. [319,323] for further details on the equations governing the perturbations of the axionlike EDE model.

On the model side, constructing well-motivated theoretical realizations of EDE (e.g., within string theory [400,401]) has proven challenging since, taken at face value, the potential in Eq. (5) requires a delicate

⁶In the context of cosmological studies one often refers to the field as an “axion” and, therefore, to the model as “axionlike” EDE, despite the field being actually a scalar rather than a pseudoscalar. The reason is that, for $n = 1$, the above potential naturally appears in axionlike/pseudo-Nambu-Goldstone boson settings. Nevertheless, we stress that in the context of our study, the potential should be interpreted as a toy potential, and not as one tied to a fundamental theory.

cancellation between various higher-order instanton corrections. Moreover, the required energy injection has to occur within a relatively narrow redshift window, raising questions about the naturalness of the mechanism. We stress that, while important, these challenges are not unique to EDE, but shared to varying degrees with other cosmological models based on scalar fields, such as inflation, dark energy, and fuzzy dark matter. Most importantly, they do not affect the role of axionlike EDE in our work: that of providing a concrete benchmark for assessing how a phenomenological cosmological model, which has so far proven relatively successful at alleviating the Hubble tension, affects the inferred value of α_s .

III. DATASETS AND METHODOLOGY

To determine constraints on α_s and assess the extent to which the “USR-ACT tension” depends on the assumed prerecombination expansion history and adopted CMB dataset, we make use of the following measurements:

- (i) *Planck*—CMB temperature, polarization, and lensing measurements from the *Planck* NPIPE data release (PR4) [402]. Specifically, we make use of the CamSpec high- ℓ TTTEEE likelihood [403], the Commander low- ℓ TT likelihood [404], the *Scroll2* low- ℓ EE likelihood [405], and the PR4 lensing likelihood [406]. We refer to the resulting combination as *Planck*.
- (ii) ACT—CMB temperature, polarization, and lensing measurements from ACT DR6. Specifically, we adopt the ACT lite (compressed) DR6 TTTEEE likelihood and the ACT DR6 lensing likelihood [302]. Following the ACT convention, we include the *Planck Scroll2* low- ℓ EE likelihood to provide the large-scale polarization information required to constrain the optical depth to reionization τ . We refer to this combination as ACT.
- (iii) SPT—CMB temperature, polarization, and lensing measurements from the SPT-3G D1 observations. Specifically, we include the SPT-3G D1 TTTEEE likelihood [407] and the MUSE lensing likelihood [408]. Following the same logic as with the ACT dataset combination, we add the *Planck Scroll2* low- ℓ EE likelihood (note that our approach differs from the SPT convention of simply including a Gaussian prior on τ , as it retains the full E-mode polarization information rather than compressing it into a value of τ). We refer to the above combination as SPT.
- (iv) DESI—BAO measurements from the DESI DR2 release. This includes measurements of D_V/r_{drag} , D_M/r_{drag} , and D_H/r_{drag} across seven redshift bins in the range $0.295 < z < 2.330$ [409], where D_V , D_M , and D_H are the volume-averaged distance, transverse comoving distance, and Hubble distance, respectively. We refer to the combinations of these measurements as DESI.

Following recent works, we also combine data from all three CMB experiments to fully take advantage of their complementary scale coverage, and derive the tightest CMB-based constraints on the relevant cosmological parameters. When doing so, following the prescriptions of ACT and SPT collaborations, we combine the *Planck* likelihoods with multipole cuts of $\ell < 1000$ in TT and $\ell < 600$ in EE, the ACT DR6 TTTEEE likelihood, the SPT-3G D1 TTTEEE likelihood, the *Planck Scroll2* likelihood, the joint *Planck*-ACT lensing likelihood, and the MUSE lensing likelihood. The specific choice of scale cuts is designed to minimize the correlations between the *Planck* and ACT datasets, themselves assumed to be uncorrelated with the SPT dataset, an assumption that is well justified by the relatively small overlap in sky regions. We refer to this combination of CMB datasets as South Pole–Planck–Atacama (CMB-SPA).

In our analysis, we consider eight combinations of the above datasets, i.e., the three CMB datasets and their combination, with and without BAO data. Explicitly, we consider these datasets/dataset combinations:

- (i) *Planck*
- (ii) ACT
- (iii) SPT
- (iv) CMB-SPA
- (v) *Planck* + DESI
- (vi) ACT + DESI
- (vii) SPT + DESI
- (viii) CMB-SPA + DESI

This choice of combinations allows us to systematically study the robustness of the inferred values of α_s . While the most constraining combinations are those involving CMB-SPA, considering also the three CMB experiments individually allows us to determine the extent to which shifts in α_s are driven by a specific CMB dataset. In addition, although BAO data are not directly sensitive to inflationary parameters, the addition of the DESI dataset is extremely important. In fact, constraints on n_s are driven in part by a degeneracy between n_s and ω_m , which manifests in CMB data, as well as the mild tension between CMB and BAO data [410]. Comparing results from the CMB-only and CMB + DESI dataset combinations, therefore, allows us to assess whether the inferred value of α_s is affected by the CMB-BAO tension. We note that the choice of dataset combinations resembles that used in Ref. [410] (which focused solely on n_s), although it differs in the use of *Planck* PR4 data instead of PR3 data, as well as the use of the *Scroll2* low- ℓ EE likelihood when using SPT data. En passant, we also note that most of these dataset combinations had not been previously used to constrain α_s . For instance, the constraints on α_s explicitly reported by ACT collaboration always make use of *Planck* data as well [85]. Our analysis thus provides, as a useful byproduct, a set of updated constraints on α_s (as well as n_s) in light of various combinations of recent cosmological datasets.

In our analysis we consider four different cosmological models. Our baseline model is the seven-parameter Λ CDM + α_s model, whose free parameters are the standard six Λ CDM cosmological parameters (the acoustic angular scale θ_s , the physical baryon and cold dark matter densities ω_b and ω_c , the amplitude and spectral index of the primordial power spectrum of curvature perturbations A_s and n_s , and the optical depth to reionization τ), as well as the running of the spectral index α_s .⁷ We also consider the eight-parameter Λ CDM + $\alpha_s + \beta_s$, where the running of the spectral index β_s is an additional free parameter. In what follows, when no ambiguity arises, we will refer to both the Λ CDM + α_s and Λ CDM + $\alpha_s + \beta_s$ models as simply Λ CDM, as a shorthand to avoid an unnecessarily heavy notation.

The other two cosmological models we consider are extensions of the nine-parameter axionlike EDE model. Beyond the six Λ CDM ones, its three additional free parameters are the axion misalignment angle θ_i , as well as the two phenomenological parameters z_c and f_{EDE} . Specifically, we consider the 10-parameter EDE + α_s model, and the 11-parameter EDE + $\alpha_s + \beta_s$ model. We fix the potential index to $n = 3$, noting that cosmological data are largely insensitive to the value of n . As previously, when no ambiguity arises, we will refer to both these models as simply EDE.

To obtain theoretical predictions for the relevant cosmological observables, we use the CLASS Boltzmann solver for the two Λ CDM-based models, and the CLASS_EDE Boltzmann solver [416] for the two EDE-based models, assuming adiabatic initial conditions in both cases.⁸ CLASS_EDE is an extension of the Boltzmann solver CLASS [417], which explicitly solves the perturbed Klein-Gordon equation for the axionlike EDE field.

We sample the posterior distributions for the parameters of the four cosmological models using Monte Carlo Markov chain (MCMC) methods, using the cosmological MCMC sampler Cobaya [418].⁹ We set wide, flat priors on all cosmological parameters (except for z_c , for which we instead sample $\log_{10} z_c$, consistent with earlier analyses), and verify that our posteriors are not affected by the choice of lower and upper prior boundaries. We assess the convergence of our MCMCs using the Gelman-Rubin $R - 1$ parameter [419], requiring $R - 1 < 0.03$ for our chains to be considered converged. Our chains are then analyzed using the GetDist package [420].¹⁰

⁷We fix the sum of the neutrino masses to $\sum m_\nu = 0.06$ eV, given the current upper limits on $\sum m_\nu$ [411–415].

⁸The CLASS and CLASS_EDE Boltzmann solvers are both publicly available at https://github.com/lesgourg/class_public and https://github.com/mwt5345/class_edc, respectively.

⁹The Cobaya MCMC sampler is publicly available at <https://github.com/CobayaSampler/cobaya>.

¹⁰The GetDist package for analyzing MCMCs is publicly available at <https://github.com/cmbant/getdist>.

IV. RESULTS

Our main results are summarized in four tables and three plots. In particular, constraints on cosmological parameters within the Λ CDM + α_s and EDE + α_s models are presented in Table I for the four CMB-only dataset combinations, and in Table II when DESI data are included. The counterparts of these tables for the cases where β_s is varied as well are Tables III and IV, respectively. Corner plots showing constraints on selected EDE parameters within the EDE + α_s model, for all eight dataset combinations, are provided in Fig. 2. Comparisons between the 1D α_s posteriors within the Λ CDM + α_s and EDE + α_s models are shown in Fig. 3. Finally, corner plots showing constraints on selected EDE parameters within the EDE + $\alpha_s + \beta_s$ model can be found in Fig. 4.

We begin by discussing the results obtained when, in addition to the standard Λ CDM and EDE parameters, only α_s is allowed to vary. For the four CMB-only dataset combinations, we find the results reported in Table I. Within Λ CDM, and considering the three individual CMB experiments, we see that only ACT reports the well-known $\simeq 2.2\sigma$ indication for positive running, $\alpha_s = 0.033 \pm 0.015$. On the other hand, neither *Planck* nor SPT show a preference for nonzero running, with both central values being negative, and both inferences being consistent with $\alpha_s = 0$ well within the 1σ level: $\alpha_s = -0.0042 \pm 0.0068$ and $\alpha_s = -0.004 \pm 0.023$, respectively (for recent discussions on this mild disagreement regarding the sign of α_s between different CMB probes, see, for instance, Ref. [143]). Our most robust constraint on the running is obtained by combining all three CMB experiments within the CMB-SPA combination, from which we infer $\alpha_s = 0.0054 \pm 0.0050$, i.e., a factor of 3 improvement in sensitivity compared to ACT alone. While the central value is positive, not only is it almost an order of magnitude smaller than the central value inferred from ACT alone but, most importantly, this inference is now consistent with $\alpha_s = 0$ at the 1.1σ level. To the best of our knowledge, this is among the first constraints on the running of the spectral index obtained by combining data from all three CMB experiments.

A comment on the ACT results within Λ CDM is in order before moving on. From Table I one sees that, within the Λ CDM + α_s model, $n_s = 0.946 \pm 0.012$ is inferred. This value may appear somewhat surprising, especially in light of the well-known ACT preference for a bluer spectrum. Indeed, combining ACT data with *Planck* and BAO data leads to $n_s = 0.974 \pm 0.003$ [302]. We stress, however, that this usually quoted “high n_s ” value is obtained *within the baseline Λ CDM model, where $\alpha_s = 0$* . Once varied, α_s is able to partially absorb the preference for larger values of n_s , as manifested by a clear negative correlation between n_s and α_s (see, e.g., Fig. 2 within EDE). This highlights how the relevant quantity is not simply the value of n_s at ACT’s pivot scale, but rather the *convex* shape of the power

TABLE I. 68% credible intervals on the scalar spectral index and its running n_s and α_s , the physical cold dark matter density ω_c , and the EDE fraction f_{ede} , obtained within the $\Lambda\text{CDM} + \alpha_s$ and EDE + α_s models, in light of the four CMB-only dataset combinations. When only upper limits are reported (as is the case for f_{ede}), these should be understood as 95% upper limits.

Parameter	<i>Planck</i>		ACT		SPT		CMB-SPA	
	ΛCDM	EDE	ΛCDM	EDE	ΛCDM	EDE	ΛCDM	EDE
n_s	0.9616 ± 0.0045	$0.9670^{+0.0061}_{-0.0090}$	0.946 ± 0.012	$0.945^{+0.016}_{-0.014}$	0.955 ± 0.018	$0.955^{+0.019}_{-0.022}$	0.9690 ± 0.0033	$0.9746^{+0.0047}_{-0.0068}$
α_s	-0.0042 ± 0.0068	-0.0031 ± 0.0076	0.033 ± 0.015	0.033 ± 0.017	-0.004 ± 0.023	0.011 ± 0.027	0.0054 ± 0.0050	0.0063 ± 0.0057
ω_c	0.1199 ± 0.0011	$0.1228^{+0.0017}_{-0.0031}$	0.1236 ± 0.0024	$0.1262^{+0.0028}_{-0.0037}$	0.1209 ± 0.0016	$0.1252^{+0.0022}_{-0.0047}$	0.11997 ± 0.00098	$0.1230^{+0.0016}_{-0.0032}$
f_{ede}	...	< 0.082	...	< 0.087	...	< 0.144	...	< 0.082
H_0 [km/s/Mpc]	67.19 ± 0.48	$68.14^{+0.66}_{-1.2}$	66.16 ± 0.87	$67.34^{+0.91}_{-1.7}$	66.89 ± 0.61	$68.44^{+0.68}_{-1.9}$	67.29 ± 0.40	$68.31^{+0.60}_{-1.1}$
Ω_m	0.316 ± 0.007	0.314 ± 0.007	0.335 ± 0.014	0.330 ± 0.016	0.321 ± 0.009	$0.317^{+0.013}_{-0.0097}$	0.316 ± 0.006	0.313 ± 0.006

TABLE II. As in Table I, but adding DESI data to each of the four CMB-only dataset combinations.

Parameter	+ DESI							
	<i>Planck</i>		ACT		SPT		CMB-SPA	
	ΛCDM	EDE	ΛCDM	EDE	ΛCDM	EDE	ΛCDM	EDE
n_s	0.9677 ± 0.0038	$0.9772^{+0.0067}_{-0.010}$	0.960 ± 0.011	0.951 ± 0.018	0.959 ± 0.017	0.962 ± 0.024	0.9739 ± 0.0029	$0.9825^{+0.0049}_{-0.0082}$
α_s	-0.0026 ± 0.0066	$-0.0002^{+0.0075}_{-0.0087}$	0.031 ± 0.016	$0.033^{+0.019}_{-0.017}$	-0.009 ± 0.025	0.004 ± 0.027	0.0050 ± 0.0049	$0.0064^{+0.0058}_{-0.0067}$
ω_c	0.11760 ± 0.00063	$0.1229^{+0.0025}_{-0.0047}$	0.11731 ± 0.00080	$0.1252^{+0.0034}_{-0.0069}$	0.11748 ± 0.00080	$0.1261^{+0.0040}_{-0.0063}$	0.11779 ± 0.00061	$0.1234^{+0.0026}_{-0.0046}$
f_{ede}	...	< 0.113	...	$0.066^{+0.024}_{-0.057}$	...	$0.078^{+0.033}_{-0.055}$	...	< 0.115
H_0 [km/s/Mpc]	68.22 ± 0.28	$69.71^{+0.80}_{-1.3}$	68.50 ± 0.31	$70.7^{+1.0}_{-1.9}$	68.22 ± 0.32	$70.7^{+1.2}_{-1.8}$	68.19 ± 0.25	$69.82^{+0.86}_{-1.3}$
Ω_m	0.302 ± 0.004	0.301 ± 0.004	0.299 ± 0.004	0.297 ± 0.004	0.301 ± 0.004	0.298 ± 0.004	0.303 ± 0.003	0.301 ± 0.004

TABLE III. As in Table I, but also including the running of the running β_s as a free parameter.

Parameter	<i>Planck</i>		ACT		SPT		CMB-SPA	
	ΛCDM	EDE	ΛCDM	EDE	ΛCDM	EDE	ΛCDM	EDE
n_s	0.9598 ± 0.0048	$0.9649^{+0.0056}_{-0.0085}$	0.948 ± 0.014	0.948 ± 0.018	0.955 ± 0.017	0.960 ± 0.022	0.9648 ± 0.0042	$0.9694^{+0.0054}_{-0.0071}$
α_s	$0.0044^{+0.011}_{-0.0092}$	0.004 ± 0.011	0.033 ± 0.015	0.034 ± 0.017	$0.003^{+0.024}_{-0.021}$	0.008 ± 0.026	0.00996 ± 0.0055	$0.0109^{+0.0063}_{-0.0072}$
β_s	$0.013^{+0.013}_{-0.011}$	$0.011^{+0.014}_{-0.010}$	-0.006 ± 0.022	$-0.005^{+0.026}_{-0.018}$	$-0.026^{+0.024}_{-0.027}$	-0.026 ± 0.026	0.0145 ± 0.0084	0.0138 ± 0.0087
ω_c	0.1200 ± 0.0011	$0.1223^{+0.0014}_{-0.0027}$	0.1235 ± 0.0024	$0.1261^{+0.0027}_{-0.0038}$	0.1212 ± 0.0016	$0.1253^{+0.0021}_{-0.0042}$	0.12019 ± 0.00098	$0.1227^{+0.0013}_{-0.0029}$
f_{ede}	...	< 0.068	...	< 0.092	...	< 0.123	...	< 0.082
H_0 [km/s/Mpc]	$67.11^{+0.47}_{-0.52}$	$67.93^{+0.58}_{-1.1}$	66.20 ± 0.87	$67.47^{+0.98}_{-1.8}$	66.73 ± 0.61	$68.15^{+0.73}_{-1.8}$	67.19 ± 0.41	$68.06^{+0.44}_{-1.1}$
Ω_m	0.317 ± 0.007	0.315 ± 0.007	0.335 ± 0.014	0.328 ± 0.016	0.324 ± 0.009	$0.319^{+0.013}_{-0.011}$	$0.317^{+0.005}_{-0.006}$	$0.315^{+0.007}_{-0.006}$

TABLE IV. As in Table II, but also including the running of the running β_s as a free parameter.

Parameter	DESI +							
	<i>Planck</i>		ACT		SPT		CMB-SPA	
	ΛCDM	EDE	ΛCDM	EDE	ΛCDM	EDE	ΛCDM	EDE
n_s	0.9667 ± 0.0038	$0.9756^{+0.0072}_{-0.010}$	0.960 ± 0.013	0.950 ± 0.020	0.961 ± 0.018	0.967 ± 0.025	0.9704 ± 0.0038	$0.9774^{+0.0058}_{-0.0080}$
α_s	0.0039 ± 0.0096	0.004 ± 0.013	0.031 ± 0.016	$0.031^{+0.018}_{-0.015}$	-0.011 ± 0.025	$0.006^{+0.025}_{-0.028}$	0.0088 ± 0.0057	$0.0097^{+0.0063}_{-0.0077}$
β_s	0.011 ± 0.012	0.007 ± 0.013	$0.002^{+0.025}_{-0.018}$	$0.004^{+0.023}_{-0.014}$	-0.008 ± 0.025	-0.014 ± 0.025	0.0119 ± 0.0086	0.0098 ± 0.0089
ω_c	0.11761 ± 0.00063	$0.1226^{+0.0024}_{-0.0049}$	0.11729 ± 0.00074	$0.1247^{+0.0033}_{-0.0065}$	0.11747 ± 0.00082	$0.1274^{+0.0046}_{-0.0065}$	0.11788 ± 0.00062	$0.1220^{+0.0020}_{-0.0040}$
f_{ede}	...	< 0.121	...	$0.062^{+0.022}_{-0.055}$	...	$0.089^{+0.036}_{-0.058}$	...	< 0.101
H_0 [km/s/Mpc]	68.20 ± 0.29	$69.63^{+0.81}_{-1.4}$	68.52 ± 0.30	$70.5^{+1.0}_{-1.8}$	68.21 ± 0.31	$71.0^{+1.3}_{-1.8}$	$68.15^{+0.25}_{-0.28}$	$69.38^{+0.63}_{-1.1}$
Ω_m	0.302 ± 0.004	0.301 ± 0.004	0.299 ± 0.004	0.297 ± 0.004	0.302 ± 0.004	0.298 ± 0.004	0.304 ± 0.004	0.302 ± 0.004

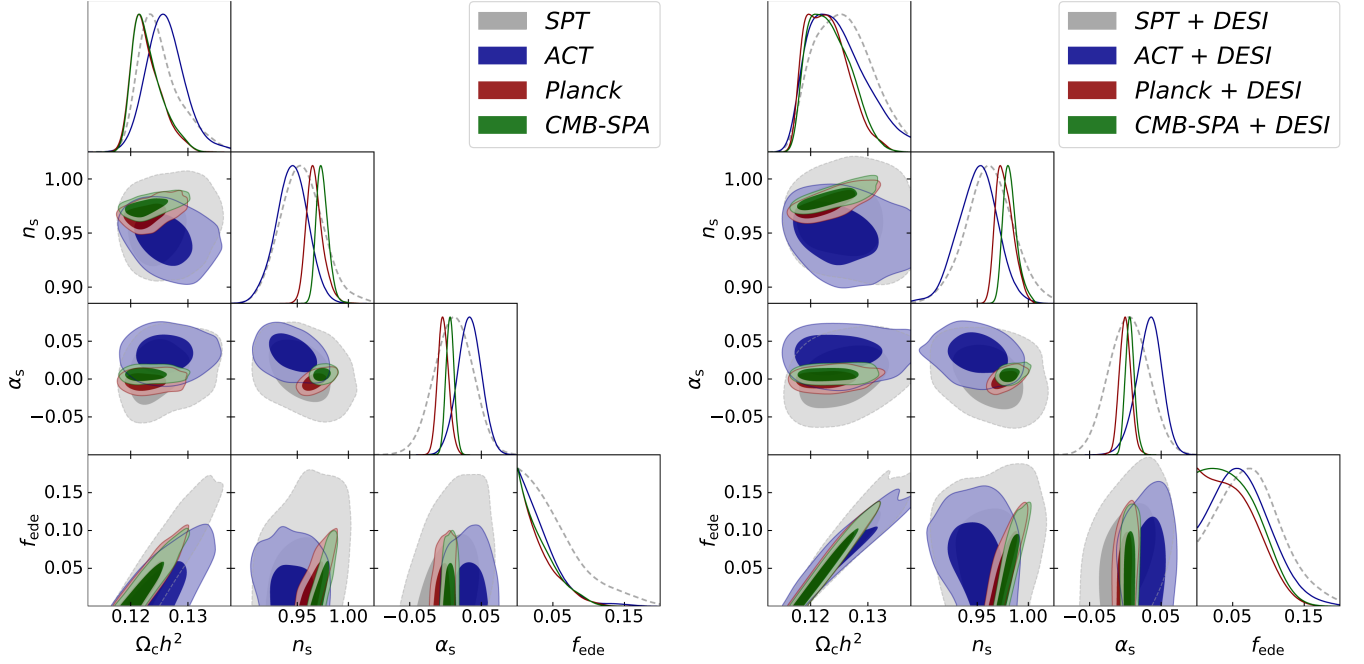


FIG. 2. Triangular plot showing 2D joint and 1D marginalized posterior probability distributions for the physical cold dark matter density ω_c , the scalar spectral index n_s and its running α_s , and the EDE fraction f_{ede} , obtained within the EDE + α_s model in light of the four CMB-only dataset combinations (left panel), and the same combinations supplemented by DESI BAO data (right panel). The 2D contours show 68% and 95% credible regions. In the α_s - n_s panel, a clear negative correlation between the two parameters is visible, reflecting the fact that a positive running can partially compensate the ACT preference for a bluer spectrum. The addition of DESI data does not qualitatively alter our conclusions.

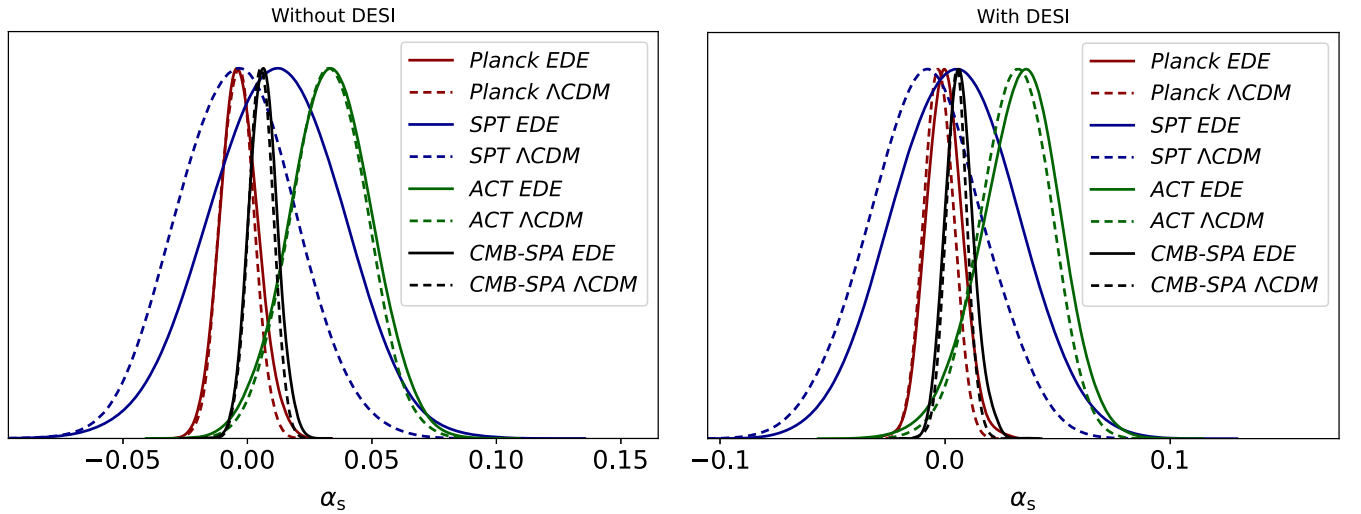


FIG. 3. 1D marginalized posterior distributions for the running of the spectral index α_s , obtained within the $\Lambda\text{CDM} + \alpha_s$ (dashed curves) and EDE + α_s (solid curves) models, in light of all eight dataset combinations considered in this work: the four CMB-only dataset combinations (left panel), and the same dataset combinations supplemented by DESI BAO data (right panel). For most dataset combinations, the central value of α_s increases when moving from ΛCDM to EDE, consistent with the theoretical expectation discussed in Sec. II B.

spectrum preferred by ACT, which gets increasingly blue toward the scales that are well probed by ACT (or, with a slight abuse of language, the overall scale-dependent effective tilt). Another important point to note is that

ACT alone lacks measurements of the first acoustic peak and, therefore, the lever arm required to robustly constrain n_s and α_s , unless it is combined with experiments (such as *Planck*) that do measure large angular scales, as explicitly

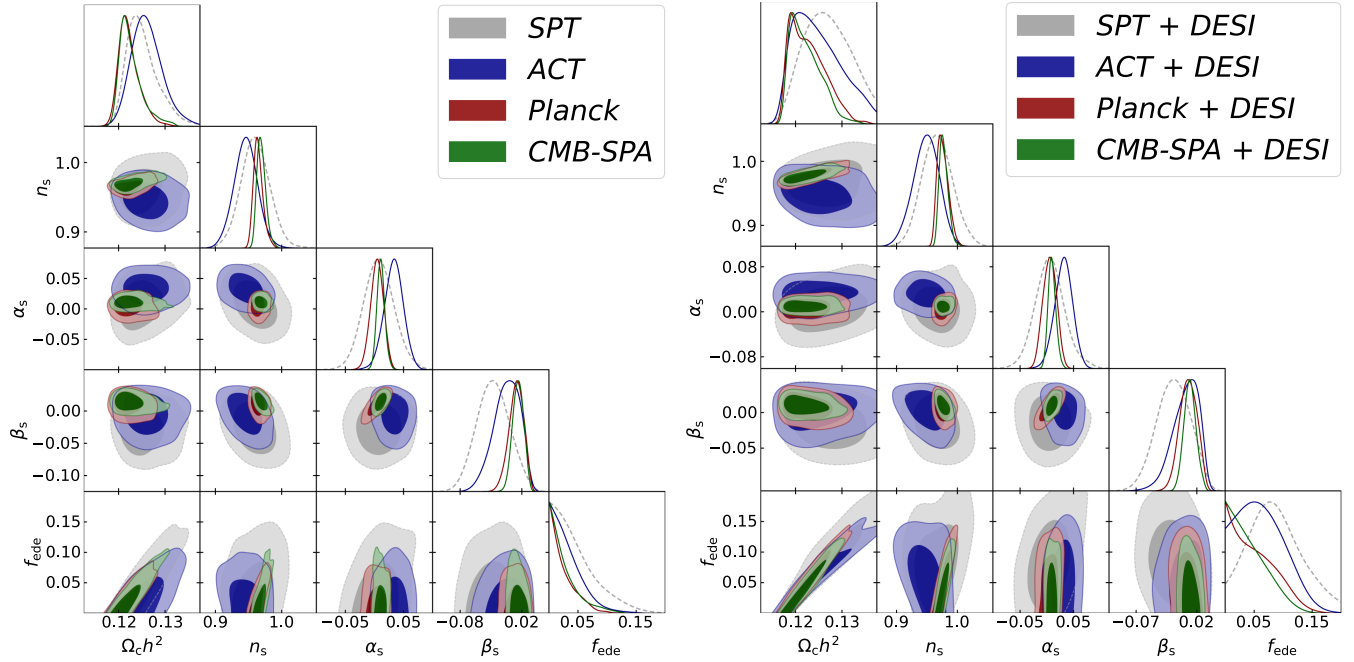


FIG. 4. Same as in Fig. 2, but also including the running of the running β_s as a free parameter. Doing so slightly broadens all posteriors (including those of α_s) and redistributes the increase in small-scale power between α_s and β_s , but does not qualitatively alter our conclusions.

discussed in Sec. IV.1 of the ACT DR6 extended models paper [85]. The ACT-only primordial power spectra shown in Fig. 5 of the same paper further support this interpretation of ACT data alone preferring a redder and more convex spectrum. As a sanity check, we have explicitly verified that we can reproduce the results of the ACT public chains for exactly the same dataset combinations considered there, and that our treatment of low-multipole polarization (S₀112 likelihood versus Gaussian prior on τ) has a negligible impact on these conclusions. We conclude that the inference of a substantially lower value of n_s within the Λ CDM + α_s model is a consequence of ACT's scale coverage and intrinsic preference for a somewhat convex spectrum, alongside the $\alpha_s - n_s$ degeneracy, rather than an issue with our chains. This discussion highlights the importance of combining all three CMB datasets within the CMB-SPA combination, to ensure wide scale coverage and the lever arm required to robustly measure parameters governing the shape of the primordial power spectrum: this is the reason why we regard the results obtained from the CMB-SPA and CMB-SPA + DESI dataset combinations as being our most trustworthy ones.

When moving to EDE (see Fig. 2 for a corner plot showing our constraints on selected cosmological parameters) we observe that for the *Planck*, SPT, and CMB-SPA combinations, α_s always increases (whereas for the ACT dataset, the central value of α_s remains stable at the positive and relatively large value of 0.033), although the increase is always very mild. Under the assumption of Gaussianity, which is well motivated (see Fig. 3 for 1D α_s posteriors in

Λ CDM and EDE for all dataset combinations considered), the increase is by 0.1σ , 0.5σ , and 0.1σ , respectively. For reference, while remaining stable within ACT and SPT, for the *Planck* and CMB-SPA dataset combinations, n_s increases by $\approx 0.7\sigma$ and $\approx 1.0\sigma$, respectively. The increases in α_s are also accompanied by a 10–20% broadening of the uncertainties, due to the marginalization over the three additional EDE parameters (f_{ede} , z_c , and θ_i). Our most robust constraint on the running within EDE is obtained within the CMB-SPA dataset combination, from which we infer $\alpha_s = 0.0063 \pm 0.0057$.

Once DESI data are included, we find the results reported in Table II. Up to a shift in n_s toward larger values, which is consistent with the findings of Ref. [410] and is inevitably intertwined with DESI's hints for dynamical dark energy, the results we find are qualitatively consistent with those from the DESI-free dataset combinations. In particular, for all four dataset combinations we observe a mild increase in α_s (at most at the 0.4σ level for the SPT + DESI dataset combination) accompanied by a 10–20% broadening of the associated uncertainties. From the CMB-SPA + DESI dataset combination, we infer $\alpha_s = 0.0050 \pm 0.0049$ and $\alpha_s = 0.0064^{+0.0058}_{-0.0067}$ within Λ CDM and EDE, respectively. These two figures, which to the best of our knowledge represent the first constraints on α_s from a combination of all available state-of-the-art cosmological data, are essentially consistent with no running within 1σ . We note (see both Table I and Table II) that for all eight dataset combinations considered, no detection of EDE at $\gtrsim 1.5\sigma$ is observed, with our tightest

2σ upper limit being $f_{\text{ede}} < 0.082$ from the CMB-SPA dataset combination. For the CMB-SPA + DESI dataset combination, the value of $H_0 = 69.82^{+0.86}_{-1.30}$ km/s/Mpc we find is in $\approx 3.1\sigma$ Gaussian tension with the latest local distance network (H0DN) determination of $H_0 = (73.50 \pm 0.81)$ km/s/Mpc [421].

Allowing β_s to vary does not qualitatively change our conclusions: see Tables III and IV for the obtained parameter constraints, and the corner plots of Fig. 4. We note that once β_s is freed, the inferred values of α_s become positive for most dataset combinations, even within Λ CDM. The trend of α_s increasing when moving from Λ CDM to EDE overall persists, but is even milder than previously. This is unsurprising, since part of the role previously played by α_s in providing more small-scale power to counteract the extra damping due to the larger θ_d is now “redistributed” between α_s and β_s . We also note that the n_s trends are altered to a more significant extent, with n_s in some cases decreasing when moving from Λ CDM to EDE, although not in the case where all three CMB datasets are combined. For what concerns β_s itself, we observe no obvious trend when moving from Λ CDM to EDE, with the shifts depending on the specific dataset adopted. From the CMB-SPA dataset combination, we infer $\beta_s = 0.0145 \pm 0.0084$ and $\beta_s = 0.0138 \pm 0.0087$ within Λ CDM and EDE, respectively, in both cases consistent with $\beta_s = 0$ within $\approx 1.5\sigma$. Once DESI data are added, the constraints we obtain within Λ CDM and EDE are $\beta_s = 0.0119 \pm 0.0086$ and $\beta_s = 0.0098 \pm 0.0089$, respectively, again consistent with no running of the running within $\approx 1.5\sigma$. We, therefore, conclude that our overall findings are only marginally affected by the choice of varying β_s , and remain robust against this parameter extension.

Summing up, the overall increase observed in α_s is consistent with the theoretical expectation we laid out previously in Sec. II B. In particular, the larger value of the running increases the amount of small-scale power, which compensates the extra damping due to the larger value of θ_d . For our most robust dataset combinations involving all three CMB datasets, despite the positive central values, α_s remains consistent with no running, albeit slightly disfavoring negative values. Compared to the inference of n_s , the impact of the CMB-BAO tension, and correspondingly DESI’s hints for dynamical dark energy (see Ref. [410]), are much milder for what concerns α_s , as one could have expected. If we were to summarize our results in one simple sentence, it would be as follows: *there is certainly no evidence for nonzero running from state-of-the-art cosmological data—there may be extremely mild hints, if at all, for a positive running.*

V. DISCUSSION

We now discuss the implications of our results for the viability of USR models of PBH formation, in light of the constraints on α_s obtained across all dataset combinations

and cosmological models considered in this work. The central question is whether the tension between the negative running generically predicted by USR inflation and the constraints from CMB data can be alleviated by invoking prerecombination new physics motivated by the Hubble tension.

Although the precise relation between α_s and the mass of the PBHs produced in a given USR model depends on the specific inflationary potential, Ref. [84] has shown that for a broad class of polynomial single-field models this relation can be cast in the following remarkably simple power-law form:

$$\alpha_s \simeq a_1 \left(\frac{M_{\text{PBH}}}{M_\odot} \right)^{a_2}, \quad (6)$$

where the coefficients (a_1, a_2) depend on the specific model considered. Two particularly well-studied cases are minimally coupled (MC) and nonminimally coupled (NMC) polynomial inflation [70,72,74,78,84,422], for which one finds $(a_1, a_2) \sim (-0.035, -0.046)$ and $(a_1, a_2) \sim (-0.018, -0.035)$, respectively. In both cases the coefficients $a_1 < 0$ and $a_2 < 0$, reflecting two important physical facts: first, USR models predict a negative running at all masses ($\alpha_s < 0$ for all M_{PBH}), and second, the magnitude $|\alpha_s|$ increases as M_{PBH} increases [84]. This is a direct consequence of the fact that heavier PBHs require the USR feature to sit closer to the CMB window (smaller N_1), therefore, leaving a more significant imprint on the large-scale power spectrum.

It is important to stress that the relation shown in Eq. (6) carries a degree of model dependence that goes beyond the choice of (a_1, a_2) . In particular, the coefficients reported above were derived under specific assumptions on the amplitude of the curvature power spectrum peak, i.e., the requirement that $\mathcal{P}_\zeta(k_{\text{pk}}) \sim 10^{-2}$ in order to produce a sizeable PBH abundance. Since the amplitude of the peak is not a free parameter but is rather constrained by the requirement of a given f_{PBH} , variations in the assumed PBH dark matter fraction will translate into mild shifts of the coefficients (a_1, a_2) . Nevertheless, the qualitative picture, i.e., negative running increasing in magnitude toward heavier PBH masses, is robust and model independent within the class of single-field USR models based on inflection-point potentials.

In Fig. 5, we translate our constraints on α_s into implications for PBH formation, by overplotting the theoretical predictions of Eq. (6) for the NMC (black solid curves) and MC (black dash-dotted curves) models as a function of $M_{\text{H}}/M_\odot$ (where M_{H} is the PBH horizon mass) against the 2σ lower limits on α_s inferred from the different dataset combinations, shown as horizontal bands. The arrows at the top of the figure indicate the approximate PBH mass ranges to which different classes of experiments are sensitive: asteroid-mass PBHs

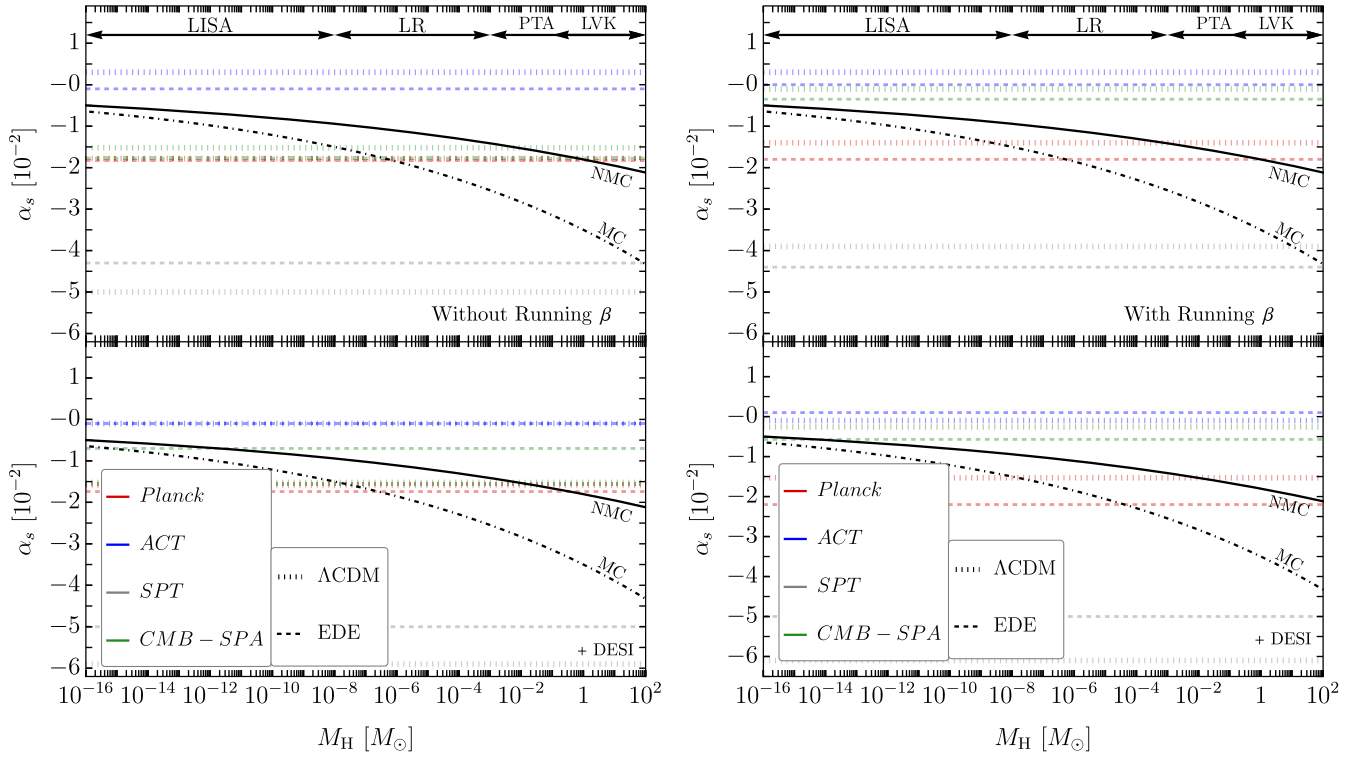


FIG. 5. Theoretical predictions for the running of the spectral index α_s as a function of the PBH horizon mass M_H , for the MC (black dash-dotted curves) and NMC (black solid curves) polynomial USR inflation models. The horizontal bands show the 2σ lower limits on α_s inferred from each dataset combination, both within Λ CDM (dotted horizontal lines) and EDE (dashed horizontal lines), with the legend color coding indicating the CMB dataset adopted. The lower (upper) subpanels correspond to the cases that include (do not include) DESI BAO data, whereas the right (left) subpanels correspond to the cases that include (do not include) the running of β_s as a free parameter. The arrows at the top are indicative of the PBH mass ranges probed by different experimental targets: LISA [423–428] (asteroid masses, $M_H \sim 10^{-18} - 10^{-8} M_\odot$), LR experiments [429,430] ($M_H \sim 10^{-8} - 10^{-3} M_\odot$), PTAs [62] ($M_H \sim 10^{-3} - 10^2 M_\odot$), and LVK [14,16–18] (solar masses, $M_H \sim 1 - 10^2 M_\odot$). USR PBH models are viable, in light of a given dataset combination and within a given cosmological model, only within the M_H region, where the theoretical curves lie *above* the corresponding 2σ lower limit on α_s . We, therefore, recommend reading the plot from right to left: starting from the largest masses, a model becomes viable once the theoretical curve crosses the horizontal line of interest, with the viable M_H region lying to the left of this crossing.

($M_{\text{PBH}} \sim 10^{-18} - 10^{-8} M_\odot$) are the target of space-based interferometers such as LISA [423–428], laser ranging (LR) experiments [429,430] probe the mass range $M_{\text{PBH}} \sim 10^{-8} - 10^{-3} M_\odot$, while microlensing surveys [431–438] are sensitive across the broader window $M_{\text{PBH}} \sim 10^{-12} - 1 M_\odot$, thus overlapping with the LISA, LR, and PTA ranges. Subsolar- to solar-mass PBHs ($M_{\text{PBH}} \sim 10^{-3} - 10^2 M_\odot$) source a SIGW signal in the nHz band observable by PTAs, such as NANOGrav [62]. Finally, solar-mass PBHs ($M_{\text{PBH}} \sim 1 - 10^2 M_\odot$) are the primary target of ground-based GW interferometers, such as those of LVK [14,16–18].

A particularly instructive way to read Fig. 5 is to trace the fate of USR models as a function of PBH mass, moving from right to left along the horizontal axis, and to ask, for each dataset combination and cosmological model, which portion of the PBH mass spectrum remains within the 2σ lower limits allowed region of α_s . USR PBH models are

viable, in light of a given dataset combination and within a given cosmological model, only within the M_H region, where the theoretical curves lie *above* the corresponding 2σ lower limit on α_s . Starting from the largest masses to the right, a model, therefore, becomes viable once the theoretical curve crosses the horizontal line of interest, with the viable M_H region lying to the left of this crossing.

We begin with the *Planck* dataset, which provides the most conservative constraints. Within Λ CDM, the 2σ lower limit on α_s is consistent with the NMC prediction across nearly the entire mass range shown, excluding only PBHs heavier than the sun, and with the MC prediction up to planetary masses. The inclusion of EDE shifts α_s downward by a small amount, consistent with the mild 0.1σ shift reported in Sec. IV, leaving the allowed mass windows largely unchanged. The *Planck* dataset, therefore, allows asteroid-mass USR PBH models as viable dark matter candidates, and leaves a meaningful allowed window in the

planetary mass range, irrespective of whether Λ CDM or EDE is assumed. Moreover, when the running of the running β_s is also freed, the situation does not change appreciably for both models.

The SPT dataset presents a qualitatively different situation. The uncertainties on α_s from SPT alone are sufficiently large, with the 2σ limits spanning a range of order ~ 0.05 , such that both the MC and NMC theoretical curves lie within the allowed region across essentially the entire mass range of interest, from asteroid masses up to solar masses. This holds both within Λ CDM and EDE, and even when β_s is freed. The SPT dataset is, therefore, currently not constraining enough to discriminate between viable and excluded USR PBH models across the mass range of interest, and no meaningful exclusion can be derived from it in isolation.

Moving to the ACT dataset, the constraints become significantly more stringent. The strong preference for positive running, $\alpha_s = 0.033 \pm 0.015$ [85], places the 2σ lower limit well above zero, in severe tension with the negative values predicted by both the MC and NMC models across all masses. Notably, the inclusion of EDE leaves the central value of α_s essentially unchanged within ACT (as discussed in Sec. IV), so the tension persists regardless of the assumed prerecombination expansion history. Within ACT, USR models for PBH formation are disfavored at more than 2σ across the entire mass range shown, from asteroid to solar masses, with no meaningful allowed window remaining.

Considering the CMB-SPA dataset, within Λ CDM, the 2σ lower limit on α_s already excludes both the MC and NMC models in the solar-mass and subsolar-mass ranges, while leaving the asteroid-mass window marginally open. When EDE is assumed, however, α_s shifts upward with the inclusion of DESI, unlike the downward shift observed for most other dataset combinations, by approximately 0.1σ , enough to close even the asteroid-mass window for both models. Allowing β_s to vary within the CMB-SPA dataset combination further modifies the picture: the redistribution of small-scale power between α_s and β_s shifts the inferred constraints in a way that completely closes the asteroid-mass window within both Λ CDM and EDE.

Taken together, these results paint a consistent but nuanced picture. The allowed mass window for USR PBH models depends sensitively on the adopted CMB dataset and, to a lesser extent, on the assumed prerecombination expansion history. The *Planck* dataset is the most permissive, allowing asteroid and subsolar masses robustly, with the main caveat being the MC model when β_s is freed. The SPT dataset is too uncertain to be constraining. The ACT dataset is the most restrictive, excluding the entire range of PBH masses regardless of the prerecombination expansion history. In the case of CMB-SPA the precise prediction depends on the assumed

prerecombination expansion history, with the overall picture that subsolar mass PBHs, or heavier ones, are excluded in both models.

Before closing, three important caveats of our analysis and a broader methodological point are worthy of further discussions. The first caveat is that, while we have considered a relatively wide range of dataset combinations, our conclusions are still contingent on the set of (CMB and BAO) datasets adopted. Constraints on the running have also been obtained from other datasets, e.g., the Lyman- α forest, with varying conclusions [439]. While perhaps stating the obvious, we stress that our findings could of course change should one choose to adopt different datasets. At the same time, the breadth of CMB combinations considered in the present work makes our analysis a useful baseline against which constraints on α_s from complementary probes can be compared.

The second caveat is that we have focused on a specific model of Hubble tension-motivated prerecombination new physics, i.e., axionlike EDE, which we have shown generically leads to an increase in α_s . The skeptical reader may object that this conclusion is specifically valid within axionlike EDE, and strictly speaking they would not be wrong. However, the argument according to which prerecombination new physics motivated to address the Hubble tension leads to an increase in θ_d , and with it the amount of damping, is quite generic (see, e.g., the discussion in Sec. V of the seminal Ref. [293]). This is true at least for the classes of new physics models considered so far, and can be avoided only by invoking a quite specific shape for the modified expansion history (see Fig. 1 of Ref. [312]), which, however, does not match any model constructed so far. For this reason, we actually have good reason to expect that our results should apply more broadly beyond axionlike EDE, although checking this model-by-model would be too computationally expensive, and is beyond the scope of our paper.

Nonetheless, as further support to this qualitative argument, it is worth considering the results of Ref. [190], which studied two related dark radiation extensions (with $\Delta N_{\text{eff}} > 0$) injecting extra energy in the prerecombination era in order to address the Hubble tension. These include a class of models where the dark radiation fluid is initially coupled to dark matter, and undergoes dark radiation-matter decoupling close to matter-radiation equality, as well as the self-interacting dark radiation subset of this class. While the scope of the work was rather different (and arguably complementary) to ours, i.e., to show that allowing for α_s and β_s to vary substantially loosens the otherwise very tight constraints on ΔN_{eff} from ACT, their results indeed support the trend we observed in axionlike EDE of α_s increasing in the presence of prerecombination new physics. In particular, Ref. [190] finds a positive correlation between α_s and ΔN_{eff} (see their Fig. 1), which

unsurprisingly maps onto a positive correlation between α_s and H_0 (see their Fig. 3). This is in line with the physical picture we discussed previously: extra prerecombination energy injection increases the amount of damping, which can be partially compensated by positive running. These results provide independent, although not conclusive, support for the idea that the upward shift in α_s may not be specific to axionlike EDE, but rather a broader feature of prerecombination solutions that increase the photon diffusion angular scale: a definitive statement would require a model-by-model analysis, which is beyond the scope of this work.

The third caveat, and arguably the most important one for what concerns the PBH side of our analysis, concerns the class of inflationary models to which our conclusions apply. As anticipated in Sec. I, the prediction $\alpha_s < 0$ is not a statement about PBH formation in general. It rests on two assumptions: that a single field is dynamically relevant during inflation, and that the enhancement of the curvature power spectrum originates from a (near-) inflection point of the potential that the inflaton must traverse, rather than from a feature engineered directly at small scales. It is precisely these assumptions that tie the small-scale peak to the CMB-scale trajectory: the same Taylor coefficients of the potential around the feature govern both, and cannot be adjusted independently. We stress that this is not a peculiarity of polynomial potentials, which we adopted merely as a concrete and calculable representative: the expansion leading to Eq. (2) only requires that the feature be a nonstationary inflection point, and applies equally to the Higgs- and fiber-inflation-inspired realizations [68,69,73], as well as to the modified Starobinsky construction of Ref. [301]. Conversely, our conclusions can be evaded, and we make no claim about models that break the link between small and large scales. Two obvious examples are multifield scenarios, most notably the curvaton [440–444], where the curvature perturbation on CMB scales and the small-scale peak originate from different fields, and single-field constructions in which a localized bump or dip is added by hand to an otherwise featureless potential. In the latter case the feature can in principle be placed arbitrarily far from the CMB window without affecting large-scale observables, although the required shape is typically postulated at the level of the effective potential rather than derived, and its ultraviolet origin is usually left unspecified. We, therefore, regard the class of models we consider as the physically best-motivated and most widely studied one, but not as exhaustive, and we caution the reader against extrapolating our conclusions beyond it.

This brings us to a broader methodological point. Constraints on inflationary parameters such as n_s and α_s should *not* be interpreted as direct measurements of the primordial power spectrum independently of the assumed

cosmological model and primordial power spectrum parametrization, and adopted dataset. Nevertheless, it is useful to distinguish between three effects, which are sometimes mixed up in the literature¹¹:

- (1) the dataset dependence of constraints on inflationary parameters, e.g., the ACT preference for larger n_s (more evident in ACT DR4, but still evident in ACT DR6);
- (2) the model dependence, or more precisely early-time physics dependence, of constraints on inflationary parameters;
- (3) the more general dependence of inflationary constraints on the assumed primordial power spectrum, for instance, the choice of whether α_s and β_s are allowed to vary.

As our work explicitly shows, constraints on inflationary parameters are particularly sensitive to the prerecombination expansion history, which is itself a major open issue in cosmology at the moment, in light of the Hubble tension. Similar considerations hold for the CMB-BAO tension, whose impact on n_s constraints has been extensively discussed in Ref. [410]. There it has been shown that current constraints on n_s obtained from combinations including DESI data are inevitably entangled with DESI's preference for dynamical DE, and more generally the tension between CMB and BAO data, which manifests also, though not only, in differences among the preferred values of Ω_m [445–447]. Although we have stressed these points several times throughout our work, they bear repeating, as they are particularly relevant for the use of CMB constraints in inflationary model building. To be as blunt as possible, we believe that *inflationary model selection based on n_s , α_s , and β_s is premature until the cause of the Hubble tension (as well as the CMB-BAO tension) is understood*, a point that has also been stressed in other works in the cosmology literature [164–192,448]. Such model selection is, at best, conditional on the cosmological model assumed in obtaining these constraints, and this is especially important in the inflationary model-building literature. While n_s , α_s , and β_s are without doubt extremely powerful probes of inflation, their interpretation as setting constraints on the inflationary Lagrangian requires specifying the assumed postinflationary cosmological model.

¹¹In the literature, there has been some confusion particularly for what concerns the first two points. This is perhaps understandable, since the strong ACT DR4 preference for larger values of n_s and the possible preference for EDE in the same dataset appeared almost simultaneously. These two effects are not necessarily different aspects of the same physical trend, and are in principle logically distinct. The distinction has become clearer with ACT DR6 data, where the preference for nonzero f_{ede} has disappeared, while the preference for larger values of n_s somewhat survived, albeit less strongly than previously.

VI. CONCLUSIONS

Single-field ultra-slow-roll inflation is one of the most studied PBH formation mechanisms. This class of models has been argued to predict a negative running of the spectral index ($\alpha_s < 0$), in potential tension with recent ACT data showing a $\approx 2.2\sigma$ preference for $\alpha_s > 0$ [85]. However, inflationary parameters (such as n_s and α_s) inferred from cosmological observations are especially sensitive to the assumed prerecombination expansion history, which is exactly where modifications are required to address the Hubble tension, if this is indeed to be taken as an indication of new physics [288–297]. With these premises in mind, the goal of our work has been a simple one: to study the impact of the Hubble tension, and in particular of prerecombination new physics, on the tension between single-field USR models of PBH formation and current CMB constraints on α_s . On the model side, we have focused on axionlike EDE, as a concrete and widely studied benchmark model for Hubble tension-motivated prerecombination new physics. On the data side, we have considered CMB data from *Planck*, ACT, SPT, and the joint CMB-SPA combination, with and without DESI DR2 BAO data. Overall, our analysis includes 32 MCMC runs (eight dataset combinations, within Λ CDM and axionlike EDE, with half of the runs treating the running of the running β_s as a free parameter), so a byproduct of our work is a set of up-to-date constraints on α_s from recent cosmological data.

Our main findings can be summarized as follows. Within Λ CDM, only ACT shows the well-known $\approx 2.2\sigma$ preference for positive running, while *Planck* and SPT are individually consistent with $\alpha_s = 0$ well within 1σ . Combining all three CMB experiments, we find $\alpha_s = 0.0054 \pm 0.0050$, consistent with no running at the 1.1σ level. Adding DESI data or freeing up β_s does not qualitatively alter the picture, and from our most complete dataset combination (CMB-SPA + DESI) we infer $\alpha_s = 0.0050 \pm 0.0049$. We stress that these are among the first constraints on α_s obtained by combining data from all three state-of-the-art CMB experiments. When moving from Λ CDM to EDE, the overall trend we observe is one where α_s increases mildly, thereby increasing the tension with USR models for PBH formation. This is consistent with our theoretical expectations: raising the prerecombination expansion rate increases the angular size of the photon diffusion scale at recombination θ_d , and the additional damping in the CMB can be compensated by enhancing the small-scale primordial power spectrum, explaining the increase in α_s , alongside the well-known and widely discussed increase in n_s .

When we translate our α_s constraints into implications for PBH formation, the picture we find is a consistent but nuanced one. SPT is the most permissive dataset, allowing the entire PBH mass range regardless of whether Λ CDM or EDE is assumed, while *Planck* allows only asteroid- to subsolar-mass PBHs from USR, and ACT excludes the

entire USR PBH mass spectrum at $> 2\sigma$ regardless of the prerecombination expansion history. Assuming instead the complete CMB-SPA combination within Λ CDM excludes subsolar mass and heavier masses, while leaving the asteroid-mass window marginally open, crucially, even this last remaining window is closed when moving to EDE, given the upward shift in α_s . Consistent with our theoretical expectation, we find that in no dataset combination does EDE widen the allowed mass window with respect to Λ CDM. The same qualitative conclusions persist overall when including DESI BAO data, or when β_s is freed.

Our results illustrate a broader principle: inflationary parameters inferred from CMB data are model dependent, and are especially sensitive to the assumed prerecombination expansion history. Given the different responses of the sound horizon and diffusion scale to changes in the prerecombination expansion history, any successful prerecombination solution to the Hubble tension that reduces the sound horizon while keeping the acoustic angular scale θ_s fixed will almost inevitably increase the angular size of the photon diffusion scale at recombination θ_d —this can be compensated by enhanced small-scale power, generically shifting n_s and α_s upwards. This argument applies, with varying degree, to most prerecombination new physics models aiming to address the Hubble tension, including those based on extrarelativistic species. Taking the Hubble tension seriously as evidence for new physics, therefore, strengthens, rather than weakens, the challenges faced by USR models of PBH formation [84]. The solar- and subsolar-mass PBH candidates directly relevant to current and upcoming GW phenomenology, including LVK mergers, the SIGW interpretation of PTA signals, and future Einstein Telescope observations, are thus disfavored by ACT—this conclusion only becomes more robust, not less, when the prerecombination model uncertainty due to the Hubble tension is taken into account. Finally, an important message that our analysis stresses once more is that inflationary parameters such as n_s and α_s should *not* be interpreted as direct measurements of the primordial power spectrum independently of the assumed cosmological model and adopted dataset, and inflationary model selection based on these parameters is premature until the causes of the Hubble tension and the CMB-BAO tension are well understood.

We can identify several interesting directions for future studies. First, the dataset dependence of our results highlights the importance of resolving the mild discrepancies currently observed between CMB experiments, and future CMB data [449,450] will be especially crucial in this respect. Second, while axionlike EDE is a useful and widely studied benchmark for our study, it is worth exploring other prerecombination solutions to the Hubble tension, even within the wide EDE class—we expect our qualitative conclusions to be robust against this

choice, but quantitative differences are to be expected. Third, the mass-running relation of Eq. (6) carries a degree of model dependence tied to the assumed PBH abundance, and the specific USR model under consideration. However, the take-home message is clear: *future analyses of USR models for PBH formation must account for the joint model dependence of both n_s and α_s when assessing their viability in light of CMB data.*

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DATA AVAILABILITY

There are no publicly available research data or software supporting this manuscript. Requests for further information or data should be sent to the authors.

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