CHOICE WITH INDEXED ALTERNATIVES
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Outline of the Thesis

In the last forty years, a considerable amount of experimental research in both psychology and economics has reported various violations of the axioms of classical choice theory, but only recently has axiomatic theory started to take into account this empirical evidence. In particular, the recent evidence collected by the new approach of neuroeconomics and the rapid growth of new theories have triggered a methodological debate about whether and how these sources of new empirical data and psychological insights should be used in economics (Caplin and Schotter, 2008). While some authors suggest to dismiss classical revealed preference analysis arguing that the presence of systematic biases between what people like and what people choose impair the possibility to reveal something by simply observing choices (Köszegi and Rabin, 2008), others remain skeptical about extending the classical model to include additional components that cannot be inferred from choice data (Gul and Pesendorfer, 2008). A third group of authors proposes to use the new evidence combined with standard choice–theoretic tools to build economic models that are both more realistic and choice–based (Caplin, 2008; Rubinstein and Salant, 2008). The present work is in this spirit: on one hand it builds on empirical evidence and psychological literature on salience, bandwagon and snob effects, and heuristic behaviour; on the other hand, it adopts a choice–theoretic approach to embed these phenomena into axiomatic models.

The first chapter of the thesis covers the recent methodological debate concerning classical decision theory. It briefly points out how economic theory has developed a coherent and organic framework that links together choices, utility, and preference by means of important formal results, and how some of the implicit difficulties regarding the psychological aspects have been neglected. In particular, the chapter discusses the critical assumption that choices depend only on the set of available alternatives and presents the relevant psychological literature and experimental evidence about the effect of alleged irrelevant aspects on choices, i.e., ancillary conditions (Bernheim and Rangel, 2009, p. 55).

The second chapter proposes an axiomatic model where choice behaviour of the decision maker is influenced by ancillary conditions. Specifically, the present thesis extends the concept of choices with frames proposed by Bernheim and Rangel (2008, 2009) and by Salant and Rubinstein (2008) according to which choices do not simply depend upon the set of available alternatives, but also upon additional components called *frames*. The present work defines the abstract concept of frame as a vector of indexes representing a psychological measure that agents attach to each alternative. Choices are then conditioned to the indexes attached to the alternatives. This chapter shows that, if the conditional choice behaviour satisfies two intuitively appealing properties—namely Monotonicity and Conditional IIA—, then the observable part of choice behaviour, i.e., the unconditional choices, can be interpreted as resulting from the maximisation of a preference relation. The chapter discusses also some welfare considerations regarding the choice model and proposes some interpretations of the indexes.

The third chapter considers a narrower interpretation of the indexes—each index represents the number of people in a group that choose each alternative—and discusses the properties an extended choice function should satisfy in order to capture the behavioural implications of the "Do What The Majority Do" heuristic

(Gigerenzer, 2004). This heuristic prescribes that, whenever the choice task is too difficult, the consequences of the alternatives are too complex to evaluate, or the subject is unsure about what to choose, he simply looks at what the majority of his peers does and then engages in the same behaviour. The chapter axiomatises the contents of the "Do What The Majority Do" heuristic by using the monotonicity axiom introduced in chapter 2 together with a maximality axiom, and then shows that an equilibrium is reached if choices satisfy maximality and monotonicity.

The fourth chapter presents an experimental test of the two axioms proposed in the third chapter. The test of the axioms is performed using sets of lotteries and groups of 7 people. Participants are asked to choose repeatedly from the same set of lotteries and, after each choice, they are informed about the number of people choosing each of the alternatives. The reaction to changes in the indexes—i.e., to the choices of the members of the group—is thus recorded and the robustness of the axioms is tested. Even though the results support the idea that choices are affected by what the others members of the group do, there is mixed evidence regarding the empirical validity of the two axioms. While strong support in favour of monotonicity is found, there is no clear cut evidence in support of maximality.

Chapter 1

Choices, Preferences, and Utility: Psychological Models of Economics

1.1 Introduction

The "act of choice" lies at the core of microeconomic theory. The most basic action that an agent has to make is indeed the choice of an alternative from a menu of available alternatives. At the most abstract level, the choice situation is usually modelled by defining the set of items from which the individual can choose and the behaviour of the individual is described as a function that assigns one or more chosen alternatives for each situation. All the information about the choice that is irrelevant for the assessment of the alternatives—i.e., ancillary information[1]—and the psychological states of the agent are assumed to be unimportant in the decision process and therefore not to influence choices. However, contrary to this assumption, evidence from psychology and behavioural economics suggests that these additional aspects have important effects on choice behaviour. For instance, the presence of framing effects (Tversky and Kahneman,

[1]The term ancillary information is inspired to the concept of ancillary conditions proposed by Bernheim and Rangel (2008)

1981) and other context effects—e.g., order effects, the endowment effect, the status quo bias, etc.—has been reported in many experimental studies challenging the assumptions of economic theory.

Combining the instruments of choice theory and the mounting evidence from experimental and behavioural economics, some authors have recently attempted to give a formal content to these effects by incorporating an additional parameter into the choice function (Salant and Rubinstein, 2008; Bernheim and Rangel, 2008, 2009). This chapter is organised as follows: section two critically discusses the classical approach used in decision theory and the recent methodological issues raised by some authors; section three focuses on the evidence about the effect of ancillary information on choices.

1.2 Decision Theory: classical results and new issues

Microeconomics models the agent at two different conceptual levels that are related to two groups of different primitives of human behaviour: choices on one side and utility or preferences on the other. Despite the two levels are formally related, the historical evolution of these primitives started from utility and, by progressively reducing the psychological component of the decision maker's model, ended with choices. In what follows we briefly explore the historical evolution of utility, preferences, and choices by underlining the links and the differences among these approaches. In addition we discuss three main weaknesses of the traditional methodology.

In the models where utility is taken as the primitive, the economic agent attaches a subjective value to each available option. This value, called utility, is considered as a measure of the pleasure the option provides to the agent. With this description of economic agent, the obvious natural tendency is to consider choices that maximise utility. That is, the option chosen by the agent is the one that gives him the highest level of pleasure.

This description of the economic decision maker originated from the so called Marginalist Revolution (See, e.g., Stigler, 1950a,b). Marginalists assumed that each commodity provides some level of pleasure when consumed and this pleasure becomes less intense for each additional unit of good consumed. The agent is assumed to compare the marginal utility of additional goods and to increase the consumption of the good that yields the highest pleasure per unit of income, i.e., the highest marginal utility per unit of money. The comparison of marginal utilities given prices has then become the psychological pillar on which economic theory based the internal functioning of its decision maker, and has been the starting point for the hypothesis of maximising behaviour. The idea of human beings as maximisers has been very successful in economics, it has permitted to consider a more general theory of values and prices as compared to the classical theories, hence providing empirically testable results concerning demand functions,[2] and an ideal starting point for welfare decisions.

However, the definition of utility given by the Marginalists presents some shortcoming that have been neglected since its introduction. The most important difficulty with utility is related to its measurability. Even though measurability of utility was not explicitly considered by the Marginalists, they implicitly assumed utility being a cardinal measure capturing the level of pleasure derived from the good by the individual. Some attempts to measure utility, either with money or by setting a unit of measure in terms of one good, has been made (See, e.g., Stigler, 1950b).

Despite the fact that measurability was problematic in several respects, the

[2] e.g., Decreasing marginal utility with additively separable utility functions is a sufficient condition for downward sloping demand functions.

assumption that utility is an objectively measurable entity was maintained by theorists, mainly because this would have implied more difficult interpersonal comparisons (Stigler, 1950a,b). The problem of measurability of utility was pointed out by Pareto and then discussed by Slutsky and Hicks (Hicks and Allen, 1934) that noticed how the level of utility cannot be inferred by observable data. Indeed, the mere knowledge of sets of bundles that yield the same level of utility does not permit to infer, i.e., "integrate", the overall level of pleasure perceived by the individual. The reason is very simple, even if an utility function is given, any strictly increasing transformation of the utility function leaves the set of bundles with the same utility unaltered (Hicks and Allen, 1934). Hence, it is impossible to univocally identify the utility level by the mere observation of sets of indifferent bundles. Therefore, utility has been interpreted as an ordinal, rather than cardinal, measure. That is, if a given bundle of commodities yields an utility higher than another bundle, this only means that the first allocation is preferred to the second, and the utility numbers attached to these bundles are interpreted as indexes that permit to "order" the various alternatives. This new interpretation had no particular consequences on the results derived by using the concept of marginal utility. Indeed, as pointed out by Hicks and Allen (1934), considering only the ratio of the marginal utilities and not their overall level—i.e. indifference curves or better marginal rates of substitution and not marginal utilities—it is possible to derive analogous conclusions regarding the most important economic concepts, e.g., demand functions.

These results have been the first step toward the exclusion of unobservable entities from the economic analysis. According to Pareto, the idea to move away from a cardinal concept of utility has been also a liberation from the necessity to refer to introspection and psychological components: "The entire theory . . . rests only on a fact of experience, that is to say, on the determination of the quantities of goods which constitute combinations which are equivalent for the individual.

The theory of economic science thus acquires the rigour of rational mechanics; it deduces its results from experience, without the intervention of any metaphysical entity" (Pareto (1897), in Stigler, 1950b, p. 381).

Thus, in the models where preferences are taken as primitives, the economic agent does not attach any psychological value to the options, but is able to perform binary comparisons between options; as a result, the necessity to have a measure of the pleasure given by each alternative vanishes. The agent only needs able to decide which option to reject when confronted with two alternatives. Similarly to the previous approach, choices are determined by a maximisation. Having a preference ranking of the alternatives, it seems natural to choose either the alternatives that "beat" all the other alternatives—i.e., the alternatives that are weakly preferred to all the alternatives in the set—or the alternatives that are unbeaten —i.e. alternatives that do not have another option that is strictly preferred to them.[3]

Note that both approaches, namely preferences and utilities, consider choices as the outcome of a sensible process that picks the best option according to a procedure, and indeed there is a close relationship between the two approaches from a formal standpoint. Many authors have discussed the conditions under which the formal equivalence of the two approaches is guaranteed, i.e. the representability of preferences with real valued functions. A standard result is that, if the preference relation $\succsim$ is complete and transitive, then there is a real valued function u such that $x \succsim y \Leftrightarrow u(x) \geq u(y)$.[4]

Whereas the two approaches may be formally equivalent, the underpinning

[3]The two methods not always give the same set of chosen elements; it depends upon the ordering properties of the preference relation. Suzumura (1976) and Sen (1997) present a discussion about the two types of optimisation and their interpretation.

[4]There are many representability results involving different assumptions about either the domain or the properties of preferences. See Richter (1971), Fishburn (1985), and Mas-Colell et al. (1995), for results about representability of Weak Orders; see Fishburn (1985) for results about representability of Interval Orders; see Luce (1956) and Fishburn (1985) for results about representability of Semiorders.

psychological motives differ greatly. As already pointed out, the first approach assumes the existence of a cardinal measure of pleasure/pain that is necessarily complete and transitive, while the second approach considers ordinal ranking of the alternatives that can have properties weaker than transitivity and completeness (e.g., Interval Order or Semiorder properties).

A further step toward the exclusion of unobservable psychological elements concerning the decision process has been done by Samuelson (1938). The author was critical about the success of the ordinal approach in excluding psychological explanations of consumer's behaviour and, indeed, he "tried to developed the theory of consumer's behaviour freed from any vestigial traces of the utility concept"(Samuelson, 1938, p. 71) by linking directly choices and demand theory. The basic intuition of the author has been to build the "selected over" relation: given a level of prices and income, if two bundles of goods x and y are affordable and the bundle x is chosen, then the bundle x is selected over the bundle y. The author proposed a property this relation should satisfy in order to have consistent behaviour (i.e., the Weak Axiom of Revealed Preferences), that is a property of choices not referring to psychological element.

This modelling strategy considers choices as the primitives of the economic man. In this case, the economic agent is described by using either a correspondence or a function $C()$ that, given a set of options A, selects a subset or an element $C(A)$ from the set of options. This case is the most general description of a decision maker one can think of and it is profoundly different from the previous conceptualisations in many respects. First, it refers only to the observable component of human behaviour—i.e., choices—and does not assume anything about the psychology of the decision maker. Second, it does not imply or require any maximisation process. Notwithstanding, the main difference between the aforementioned approaches is that, while the first two build on unobservable components, i.e., preferences and utility, the third approach starts from observable choices. The first two models can be considered as *psychological models*, in the sense that they speculate about how choices are determined, while the third is based on the observable and tangible output of the decision process and it should hence be considered as a *behavioural model*.

However, without imposing any restriction on the shape of the choice function, the behavioural model based on choices is vacuous; indeed, with no restrictions, this approach only provides a list of the chosen elements for each set of available alternatives. The real objective of this approach is to identify consistency properties of choices in order to link data with some choice generating process.[5] The most standard consistency conditions considered by classical decision theory is the Weak Axiom of Revealed Preferences (WARP), which assumes that, given two sets of alternatives A and B with $x, y \in A \cap B$, if $x \in C(A)$ and $y \in C(B)$, then $x \in C(B)$.

As the name suggests, the WARP is strictly connected with the idea of revealing behavioural preferences directly from choice data, and, indeed, it is possible to construct a binary relation from choices in several ways. For instance, one can define the "at least as good as" relation R as follows: an alternative x is revealed "at least as good as" another alternative y, i.e. xRy, if there is a set of alternatives A containing both x and y such that x is chosen over y. That is, $xRy \Leftrightarrow x \in C(A)$ and $y \in A$ (Sen, 1971). A classic result concerning WARP and the relation R identifies WARP as a sufficient and necessary condition for the revealed preference relation R to be a transitive and complete relation whose maximisation coincides with choices, i.e., $C(A) = \{x | x \in A$ and xRy for all $y \in A\}$.[6]

[5] Note that this was not the original aim of Samuelson (1938) who proposed his consistency condition with the purpose of getting rid of psychological comparisons and linking directly choices and demand theory. Ironically, it turned out that the consistency condition he proposed has become the pillar of the axiomatic foundation of preference maximisation.

[6] Given the definition of WARP and of R provided here, additional assumptions are needed

The aforementioned result is of great importance since it provides a link between the unobservable domain of preferences and the observable domain of choices, and, taken together with representability results, it allows to link properties of choices with the concept of utility. All these formal connections, from choices to preferences throughout rationalisations and from preferences to utility throughout representability, have made decision theory an harmonic picture (Caplin, 2008). If axioms about choices hold, one can indeed go from choices to utilities and vice versa.[7]

Hence, the exclusion of psychological elements from the economic model of decision making was completed by the behavioural approach first proposed by Samuelson. Various results concerning both rationalizability of choices—i.e., the properties choices should satisfy in order to be interpreted as the result of the maximisation of a preference relation—and representability of preferences—the properties that preferences must possess in order to be representable by an utility function—relegated the concept of utility maximisation to the role of synonymous for preference maximisation and choices consistency.[8]

The historical development of microeconomics, led economists to place maximisation and rational behaviour at the core of their theories, hence moving towards a more and more abstract description of the psychological and environmental elements characterising economics decisions. In the last forty years, however, a considerable amount of experimental research in both psychology and economics reported violations of various assumptions, either implicit or explicit,

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for this result being true. In particular one needs to assume that the domain of the choice function consists of the set all the non–empty subsets of a finite collection of elements. More general result are presented in, e.g Sen (1971) and Richter (1971).

[7] After the seminal paper of Samuelson (1938) many authors worked on the revealed preference concept, generalising the work of Samuleson and providing different consistency conditions either equivalent or weaker than WARP. Among others Houthakker (1950), Richter (1966), Sen (1971), Suzumura (1976).

[8] Notice that this is not the case for non deterministic economic models, where the concept of cardinal utility is central to the development of all the classical results of Expected Utility Theory.

made by economic theory. For instance, the presence of context effects (like framing and anchoring) (Kuhberger, 1998), of status quo bias and endowment effect (Kahneman et al., 1991), of anomalies concerning choices under risk, such as Gambler's Fallacy and the Law of Small Numbers (Tversky and Kahneman, 1974), and of preference reversals (Lichtenstein and Slovic, 1971) have been largely documented. Nonetheless, the aforementioned evidence have long been neglected, and very few attempts to model these phenomena by means of a choice–theoretic approach have been made until recently.

While behavioural economists provide models that take into account evidence of anomalous behaviours, they approach behavioural anomalies by using a perspective different from the choice–theoretic one. The methodological approach of behavioural economists is to observe empirical behaviour, by means of both experiments and empirical data, and to explain behavioural regularities using standard maximisation of enriched utility functions taking into account some psychological variable (See, for a survey, Camerer and Lowenstein, 2003). However, if one starts from behaviour it seems more natural to model directly axioms regarding choices, like in the revealed preference tradition, rather than explaining behaviour proposing particular functional forms for the utility function.[9]

Recently, axiomatic theory has started to take into account empirical evidence about behavioural anomalies, that resulted in a growing number of new theories that triggered a methodological debate about whether and how these sources of new empirical data and psychological insights should be used in economics (Caplin and Schotter, 2008). While there is some agreement about the necessity to take into account these aspects, opinions diverge regarding how it should be done. Some authors suggest to abandon classical revealed preference analysis by arguing that the presence of systematic biases between what people like and

[9]See, for instance: Laibson (1997) for an intertemporal behavioural model and Fehr and Schmidt (1999) for a model of other regarding behaviour.

what people choose impair the possibility to reveal preferences without having a model embedding both "true" preferences and mistakes (Kőszegi and Rabin, 2008). Other authors support the use of the revealed preference approach underlining its flexibility (Caplin, 2008; Gul and Pesendorfer, 2008); however, some of them remain skeptical about extending the classical model to include additional components that cannot be inferred from choice data (Gul and Pesendorfer, 2008), while others admit the use of different sources of data and retain the conceptual tools of revealed preferences (Caplin, 2008; Rubinstein and Salant, 2008). This second group of authors propose to use evidence from psychology and economics combined with standard choice–theoretic tools to build economic models that both provide a more realistic psychological picture of the decision maker, and can be identified by precise implication on data.

In what follows, we discuss some flaws related to both the formalisation and the interpretation of the framework of classical choice theory and some of the choice–theoretic models addressing these issues. The first issue we will discuss is the observation that the psychology of maximisation is just one of the possible models that produce choices compatible with the standard axioms of choice; the second issue we will report on concerns the implicit assumption that the set of available alternative is given to the decision maker; and the third issue we consider addresses the assumption that subjects exhibit stable choices when facing a given set of options.

1.2.1 Revealed preferences and the psychology of maximisation

The psychology of maximisation is just one of the possible models that produces choices compatible with the standard axioms of choice. This feature is usually addressed with the "as if" term. That is, when choices satisfy the WARP, it is "as if" these choices are produced by the maximisation of a complete and transitive preference relation that can be recovered from choices; nevertheless, nothing implies that the actual choice process used by an agent is a different one. Rubinstein and Salant (2008) provide an extreme example where choices can be rationalised. They consider an agent that, for some reasons, minimises his true preference relation instead of maximising it. In this case, the revealed preference relation R rationalises the choice behaviour but does not represent the decisional process adopted by the agent. Another interesting example is provided by Mandler et al. (2009), where choices are determined by a list of desirable properties. According to the procedure proposed by the authors, the agent goes through the list considering sequentially the properties one at a time and dropping those alternatives not possessing the current property. The alternatives that survive until the end of the procedure are the alternatives chosen by decision maker. The authors show that this choice procedure produces rationalizable choices but has a different psychological content. That is, the agent is adopting a procedure that is psychologically far from the maximisation of a preference relation.

Caplin (2008) clearly points out that axioms about choice behaviour are capable to identify only *classes* of psychological models. That is, all the models producing the same pattern of choices are indistinguishable and thus formally equivalent. This has two implications: first, if two models are equivalent they should deserve the same consideration, however, classical economics usually considers preferences maximisation preferable to other forms of explanations; second, a radical interpretation of the principle of revealed preferences can overcome this drawback by disregarding the psychological components leading to choices and fully identifying theories with their behavioural implications.

However, the unobservability of the psychological level does not mean that one should model only properties of choices. As Sen (1993) and Rubinstein and Salant

(2008) argued, a psychological component of the model is needed for two reasons. First, considering only consistency conditions—i.e., the axioms about choice behaviour—is meaningless: indeed, the consistency condition consider choices not possessing some property as contradictory but, in Sen's words, "statements A and not-A are contradictory in a way that choosing x from $\{x, y\}$ and y from $\{x, y, z\}$ cannot be. If the latter pair of choices were to entail respectively the statements (1) x is a better alternativethan y, and (2) y is a better alternative than x, then there would indeed be a contradiction here (assuming that the content of "being better than" requires asymmetry). But those choices do not, in themselves, entail any such statements." (Sen, 1993, p. 499). This statement clearly points out that it is impossible to provide consistency condition with a neutral interpretation: whenever a consistency condition is provided, it implicitly assumes a psychological model. Second, different psychological models entailing the same behavioural pattern may have different normative implications (Bernheim, 2009). Consider, for instance, the Checklist model presented above (Mandler et al., 2009); if a policy maker only observes choices, since WARP is satisfied he may infer that, an alternative x always chosen over y is preferred, but in the previous model there are no assumptions about what the agents likes, indeed, he may stick on the procedure even if delivering suboptimal alternatives.

Thus, both levels are needed: the psychological motivations and the axioms constraining choices. The former is needed to inform the construction of axioms, while the latter is needed to precisely identify the implications of the psychological model on the observables, i.e. on the choice data. There is an ongoing debate regarding how to use the psychological evidence to inform axiomatic models. For instance, psychological data can be incorporated by either directly modelling the psychological components—e.g. what Caplin (2008) calls "predicting mouse clicks"—or simply inspiring the psychological part of the model, and hence not entering into the axioms (Gul and Pesendorfer, 2008).

1.2.2 The set of alternatives is not given

A second issue of classic decision theory is that the set of alternatives is usually considered as given. This assumption possibly stems from the original definition of the sets of available options used in consumer's theory; indeed, the set of alternatives was first defined as the set of bundles of goods one can afford to buy. With this formulation, taking the available alternatives as given seems a sensible and harmless assumption. However, this assumption has been challenged since the 1950s by Simon's work about bounded rationality (Simon, 1955). Knowing all the available options requires cognitive abilities that human agents do not possess and hence, when observing the choice of an alternative x from an hypothetical set of alternatives A, one cannot sensibly assume that the agent is considering all the alternatives contained in A. For example, when one observes that a person has purchased a DVD of a movie in a big DVD shop, the assumption would be that the buyer had a clear picture of all the available options, i.e., of all the DVDs present in the shop. This assumption seems quite unrealistic at best. Human agents must search for the relevant options, and even if one assumes that all the relevant options are available to the subject (e.g., they are listed in a catalogue), one cannot assume that the subject is able to take into consideration all the alternatives if the number of options is big.

Some theorists have tackled these issues by modelling the selection of the alternatives that the agent is able to take into consideration in several ways. One solution is to model a search process that leads to the sequential definition of the considered alternatives. For instance, Masatlioglu and Nakajima (2008) consider a choice process where a set of alternatives S and an initial provisional choice x_0 are given, and, at each step k, the chooser is able to consider only a subset $\Omega(S, x_{k-1})$ of S containing those alternatives that are "similar" to the provisional choice x_k. If the provisional choice is the best alternative in the subset according to the agent's preference ordering, then the algorithm stops, otherwise the best alternative of the subset becomes the new provisional choice and the process is iterated.

A second solution proposed in the literature is to simply consider a fixed selection of the relevant alternatives. Masatlioglu et al. (2009) assume that the decision maker is able to consider only a subset of the available alternatives, and they propose to apply consistency conditions to the set of alternatives that are able to attract the attention of the decision maker. They define an *attention filter* as a selection of alternatives possessing the following property: if an alternative which does not attract attention is dropped from the set of available alternatives, the subset of those alternatives attracting the attention remains unchanged. A closely related model is the one by Manzini and Mariotti (2007) that considers choices rationalised by the sequential maximisation of two asymmetric binary relations. That is, the decision maker first eliminates the dominated alternatives according to some criterion (i.e., using the first binary relation) and then chooses the most preferred elements among the remaining ones (i.e., using the second binary relation). In this case the first preference relation can be seen as generating a selection of alternatives that possess stronger properties than the attention filter.

Notice that this issue poses a very serious question about the idea of revealing preferences by observing the choice x and the set A of available alternatives. Indeed, the basic underlying assumption is the ability to observe the couple (x, A) and revealed preference concludes that if y is in A then x is revealed preferred to y. However, if one does not know which is the A considered by the subject the task of revealing preferences is difficult to pursue. In the model by Masatlioglu and Nakajima (2008), for instance, preferences can be only partially deducted from choices: e.g., the extreme case in which $\Omega(S, x_0) = \{x_0\}$ for all S and x_0 does not permit to reveal anything about the underlying preferences.

1.2.3 Choices are affected by external features

The third issue relates to the fact that in classical decision theory it is implicitly assumed that the agent always makes the same choice when presented with the same set of alternatives, i.e., choices depend only upon the alternatives and not upon additional features. However, there is evidence suggesting that choices may vary according to some environmental or psychological variables that do not belong to the description of the alternatives (see the next section). However, it is not always clear which features pertain to the identification of the alternatives and which do not. The "boundaries" between alternatives and environment are not easy to be defined and observed. Consider for instance the choice between coffee and tea in a restaurant: should or should not the gender of the waiter be considered as a feature of the alternative? More precisely, should coffee brought by a waiter be considered different from coffee brought by a waitress?

Indeed, while there are features that one can undoubtedly consider part of the alternative—e.g. when choosing a mug is hard to consider its colour an environmental or psychological variable—,other features can be sensibly considered as environmental or psychological variables—e.g., the position of the available mugs on the shelf.[10] Classical decision theory builds on the implicit assumption that only the first type of information, i.e., the information needed to physically identify the objects, is important for the choice process, while the environmental information, i.e.,the information unnecessary to the identification of the set of alternatives, is considered irrelevant to the choice.[11]

Concerning this issue, Rubinstein and Salant (2008) have pointed out that

[10] The formalisation of a theory of object identification and conceptualisation goes beyond the scope of economic theory. In cognitive science there are some attempts to formalise the psychological space of concepts (Gärdenfors, 2000)

[11] Notice that one can always include all the environmental features into the alternatives, but this would have the extreme consequences that one would never face the same alternative twice and hence all the consistency conditions proposed by the theorists would be completely vacuous.

there should be "room for models in which the observable information about a choice situation is richer than just the set of available alternatives and the alternative chosen" (Rubinstein and Salant, 2008, p. 122). An example of such model is given by Rubinstein and Salant (2006a): where the choice environment is a list of elements and not simply a set, and choices may vary according to the position of the alternatives in the list. Notice that also the classical intertemporal choice model can be considered as a particular case of the dependence of choices upon additional features. In the model, time can be seen as the environmental description, and indeed, given a set of alternatives, choices vary with the dates at which the alternatives are delivered.

Note that there are other features influencing choices that pertain to the psychological domain rather than to the environmental one. For instance, the "salience" of an alternative (i.e., the psychological relevance or the ability to attract attention), is an unobservable feature of the alternative depending upon the interaction between the environment and the alternatives that can influence choices (Romaniuk and Sharp, 2004).[12] The integration of unobservable features into a choice model is more problematic than the integration of observable environmental features. In their suggestion to open the model to richer descriptions of the choice situation, Rubinstein and Salant (2008) explicitly refer only to observable environmental features. This is indeed a logical assumption, especially if one wants to empirically test the model. However, there are also good reasons for theoretical explorations of unobservable, but sensible, psychological effects: a better understanding of the behavioural implications of psychological processes may help to provide a more realistic description of theoretical agents, and, if one considers the possibility of discrepancies between what the agent wants and what he chooses, the support for modelling only the observable elements loses part of

[12]Tyson (2008) provides a choice–theoretic model where both cognitive limits and salience are considered.

its strength. On the basis of these considerations, the discussion of models where choices are affected by unobservables should not be ruled out a priori, also because it is in the tradition of economic theory to model unobservable psychological elements, such as utility and preferences that determine the choice.

The overall picture emerging from the above discussion demonstrates a tension between the harmony of classical decision theory and the necessity to improve the theoretical framework by taking into consideration the issues reported above. In the last decade, theorists have started to deal with these drawbacks of the classical theory proposing new models that incorporate empirical evidence and psychological phenomena. This thesis, like the models presented above, is an attempt to take into account some of these issues. It focuses mainly on the third issue outlined before: that is, the effect of the environmental and psychological aspects producing choice reversals. Specifically, it builds on empirical evidence on salience, bandwagon and snob effects, and heuristic behaviour to inform a choice–theoretic model characterising these elements. The next section examines more deeply the relevant evidence about choices depending on "ancillary information".[13]

1.3 Ancillary information: evidence and models

The standard model of decision theory formalises choices as a correspondence $C()$ that, for each set of available alternatives A, attaches a non–empty subset $C(A)$ of chosen alternatives—$C(A) \subseteq A\ \forall A \in D$, where D is the class of all possible choice problems. In this model, choices are independent of ancillary information and the psychological state of the agent. Thus, choices are assumed to be unaffected by, for example, the order of the alternatives or the level of salience that in different situations are attached to the alternatives. What matters

[13] Hereafter ancillary information is used for both observable environmental features and unobservable psychological features that leads to changes in the chosen alternative.

are the elements contained in A: whenever the decision maker faces the set of alternatives A, he is assumed to select the same subset of elements $C(A)$ without any possibility to change his mind.

Although the standard treatment neglects any influence of the description of the situation on choices, there is substantial evidence that the way the situation is framed influences choices. The "framing effect" was originally introduced by Kahneman and Tversky, who demonstrated important violations of the invariance principle using pairs of gambles (Tversky and Kahneman, 1981; Kahneman and Tversky, 1984). Following the approach of these authors, many experiments have reported that describing risky choices in terms of losses produces a more risk seeking behaviour as compared to describing the same risky choices in terms of gains (Levin et al., 1998; Kuhberger, 1998).

Different effects of ancillary information involving riskless situations are the so–called Status Quo Bias and Default Alternative Bias (Samuelson and Zeckhauser, 1988).[14] For both these biases, it has been shown that the presence of an option as the status quo (or default) inflates its attractiveness. For example, Johnson et al. (2002) have shown that proposing a default alternative in on-line forms almost twice as many people choose an option when the question is posed with an opt-out format than with an opt-in format. Another well–known example of status quo bias is the so called Endowment Effect (Thaler, 1980; Knetsch, 1989), for which a person that is endowed with an object attaches to the object a higher value than when he has to buy it. These types of effect have also been studied outside the laboratory: Johnson et al. (1993) have examined the effect of different default alternatives on car insurance decisions and Hartman et al. (1991) have studied the presence of a Status Quo Bias on the reliability of residential electricity service among Pacific Gas & Electric Company customers. However, it

[14]The distinction between the Status Quo Bias and the Default Option Bias is that in the former one considers the current state of affairs, while in the latter the default options can be different from the current state of affairs.

should be noted that there is also countervailing evidence about the presence of these effects. For example, Plott and Zeiler (2007) have shown that under some experimental conditions—e.g., changing the position of the endowed good and the words used in the instructions—the Endowment Effect disappears, hence suggesting that this effect may be due to experimental artifacts. The presence of mixed evidence strengthens the need for a better understanding of the conditions—i.e., frames—under which the effect is present and under which is not.

Additional effects that have attracted little attention in economics are about the order in which alternatives are presented to the subject and the number of times an alternative is present in the choice set. Examples of the former type are the "primacy" and "recency" effects or, more generally, order effects. Primacy and recency gives advantages to the first and the last alternatives in a list respectively. Indeed, since people examine initial alternatives more attentively and they recall more easily the alternatives that appear at the end of a list (see, e.g., Ward, 2002), the position in which an alternative is presented may affect the choice behaviour. The determinants of order effects have been extensively explored in psychology and marketing with regard to issues such as opinion formation (Wilson and Insko, 1968), advertising persuasion (Haugtvedt and Wegener, 1994; Brunel and Nelson, 2003), products taste evaluation (Dean, 1980), and belief revision (Hogarth and Einhorn, 1992); nevertheless, they have received little attention in economic theory. The lack of economic models including the aforementioned effects is surprising especially if we consider the fact that ordering is also an important methodological concern for the methods of experimental economics.[15] Effects involving the number of times an alternative is present in the choice set has been studied by Mittone et al. (2005) and Mittone and Savadori (2009), who have shown the presence of a Scarcity Bias. The authors produced

[15]Only few experimental contributions addressed the effect of ordering of experimental tasks, see, e.g., the debate between Holt and Laury (2002, 2005) and Harrison et al. (2005).

evidence that the likelihood of choice of an object is positively related to its relative scarcity. Studies that have examined how the perceived scarcity affects the attractiveness of an alternative has also been conducted by psychologists (Szybillo, 1975; Lynn, 1989; Lynn and Bogert, 1996) that, using assessments of attractiveness in hypothetical situations, have shown how declared attractiveness increases with the availability of the good.

Concerning the effect of unobservable psychological phenomena affecting choices, the idea that some alternatives can be less or more salient have been largely studied in marketing literature. The concept of salience is very broad and the term "salient" has often been used as a synonymous of noticeable, prominent, but it has also been interpreted differently in different fields. In marketing literature the concept of brand salience has been often equated with "the prominence or level of activation of a brand in memory" (Alba and Chattopadhyay, 1986) or with the idea of "brand awareness"(Romaniuk and Sharp, 2004), while in cognitive science—in particular in vision research—the concept of salience is closely related to a measure of the ability of different stimuli to attract the visual attention of the decision maker (Huang and Pashler, 2005; Itti, 2006, 2007).

Experiments measuring salience usually consider the likelihood that a given brand is recalled as first when subjects are asked to list the brands in a given product category (i.e., "Top of mind" brand) (Alba and Chattopadhyay, 1986). More complex measures of brand salience have also been developed; van der Lans et al. (2008), for instance, measure brand salience by using the eye–tracking techniques defining it as the probability to fixate a target product in pictures of supermarket shelves. Other studies relate salience and (hypothetical) choices showing that awareness of the product, i.e., having heard about the product before the experiment, increases the likelihood of the product to be chosen and may reduce the likelihood to choose better quality products (Hoyer and Brown, 1990; Macdonald and Sharp, 2000).

Finally, there are effects for which is more difficult to understand how ancillary information affecting choices can be formalised. Among them there is the so called Elicitation Effect (Rabin, 1998), for which people show a lack of stability of preferences when the elicitation method changes: e.g., preference reversal phenomena have been empirically observed using gambles and asking subjects both to choose and to price a gamble (Grether and Plott, 1979). As for the Endowment Effect, the evidence about preference reversal due to elicitation effect is not conclusive, there are studies (Chu and Chu, 1990; Cox and Grether, 1996) that show how this effect is reduced under particular experimental conditions, e.g., after market–like experiences.

A common feature of many of these works is that, given their empirical nature, they fail to give a formal model of how ancillary information affects behaviour. Only few empirical studies have included an attempt to provide a formal model of behaviour, and many economic models that embrace framing effects are based on the model of Prospect Theory proposed by Kahneman and Tversky (1979). These authors developed a model of choices under risk that accounts for the empirical regularities they have found in their experiments. The most interesting innovation of their model is the use of a value function that is able to capture the behavioural differences observed when the same problem is described in terms of gains or losses with respect to a reference point. Other models incorporate the reference dependence of choices: an extension of Prospect Theory to riskless choices was made by Tversky and Kahneman (1991), and Koszegi and Rabin (2006) also built on the value function proposed by Khaneman and Tversky to develop a model of choice that treats the reference point as an endogenous expectation. Another model by Masatlioglu and Ok (2006) treats the Status Quo Bias from a revealed preference point of view. Even if these models introduce a framing effect into the choice procedure, they focus mainly on the effect of a default alternative or a status quo on choices, while they do not take into account other types of effects that ancillary information has on choices—e.g., the order in which the alternatives are presented, the number of times the alternatives are repeated, etc. An attempt to embed menu dependence and chooser dependence of the choices has been made by Sen (1997), who was among the first scholars that pointed out the necessity to shift the level of the analysis from preferences over "culmination outcomes" to preferences influenced by other aspects of the "act of choice". Finally, the model of choice from lists proposed by Rubinstein and Salant (2006a) embeds order effects on choices and the most general framework of choices with frames captures the more general idea of framing effect (Salant and Rubinstein, 2008).

The intuition of Salant and Rubinstein (2008) is to consider choice functions $c()$ defined not only upon the sets of available alternatives A but also upon additional variables f they call $frames$.[16] According to the authors, a *frame* is an abstract object that can reflect both observable ancillary information, such as the position of alternatives, and unobservable internal manipulation used by an agent in the choice process. Formally, the authors define an extended choice problem as a pair (A, f) where A is the set of available alternatives and f is the *frame.* Then they define an extended choice function c^* as a function that assigns an element chosen from A for every extended choice problem (A, f). In this way the choice of an element from a given set of alternatives is conditioned to the frame: if the frame changes, the chosen element may change too. Some examples of ancillary information that can be incorporated into this framework are presented by the authors in an early version of the paper (Rubinstein and Salant, 2006b):

1. Default alternative: when an alternative in the choice problem is designed to be the default option for the agent, like in many internet pages when some boxes are already checked. In this case the extended choice problem can be formalised as (A, x), where $x \in A$ is an alternative that is designed

[16] Bernheim and Rangel (2008) independently proposed the same modelling strategy. They use the term "ancillary conditions" to refer to the additional component added to the choice function.

as the default.

2. Choice from lists: where the elements of the choice problem are sorted as a list, such as in menus or lists of political candidates. In this case the extended choice problem can be formalised as $(A, >)$ where $>$ is an ordering over the alternatives in A.

3. Number of appearances: when alternatives can appear more than once in the alternatives' menu. In this case an extended choice problem is a pair (A, i), where $i()$ is a function that assigns a natural number to each element of A that represents the number of times the alternative appears in the menu.

The idea of frame introduced by Salant and Rubinstein facilitates the introduction of procedural and psychological aspects of choices into economic models and allows for the characterisation of the effect of ancillary information on choices in a formal way. Indeed, once the frame has been formally defined, one can characterise the extended choice function according to some hypotheses about the way the frame is supposed to affect choices (e.g., in the default alternative model one can impose a default tendency, that is if a is chosen from A when x is the default then a is also chosen from A when a is the default option).

This section outlined the existence of various environmental and psychological variables affecting choices that classical decision theory ignores. What emerges is that, despite the body of empirical knowledge produced by experimental economists and psychologists, only part of these aspects have been seriously taken into account by theoretic models, e.g., Status Quo Bias and Endowment Effect; some aspects have not been considered yet, e.g., the Scarcity Bias; and others have been examined only recently, e.g., Salience and Order effects. Thus, there is room for new formal models that will help to have both a better understanding and a more precise framework about the economic implications of these effects.

1.4 Conclusions

This chapter has briefly discussed the issues that both empirical evidence and recent theoretical models pose to classical decision theory. In particular, the second part of the chapter presented a review of the effects that ancillary information, such as the position or the perceived availability of the alternatives, has on choices and introduced the flexible modelling tool developed by Salant and Rubinstein (2008) and Bernheim and Rangel (2008, 2009). Both the evidence and the tool are used to inspire the analysis presented in the next chapter of this thesis. The evidence is used to inform the psychological component of the model, while the tool is used as the starting point for the formalisation of the model.

Chapter 2

Revealed Preferences, Choices, and Psychological Indexes

2.1 Introduction

The standard model of choice (see Richter, 1966; Sen, 1971) considers a universal collection of alternatives X that are the possible objects of choice and defines a choice problem A as a non–empty subset of the universal collection X. In this model, both the environmental information that is not–necessary for describing and identifying the elements of the choice problem—i.e., ancillary information—and the psychological aspects involving the alternatives— e.g., salience and emotive states—are assumed to be irrelevant for choices made by economic agents. Choices are formalised by means of a correspondence $C()$ that attaches to each choice problem a non–empty subset of alternatives $C(A)$—$C(A) \subseteq A \ \forall A \in D$, where D is the class of all choice problems—and choices are independent of ancillary information and the psychological state of the agent. Therefore, the choice is assumed to be unaffected by, for example, the order of the alternatives, the description of the situation, the number of times an alternative is repeated in the choice set, or various levels of salience that in different situations are attached to the alternatives. The only thing that matters is the composition of the set A, and once the elements of A have been defined there are no ways to change the elements chosen from A.

This chapter borrows from the idea developed by Bernheim and Rangel (2008, 2009) and Salant and Rubinstein (2008) that additional components should be attached to the choice problem in order to capture the effect on choices of unobservable information and psychological states. That is, the authors allow choices to be conditioned by abstract entities they call "frames", and consequently, the decision maker can choose different elements from the same set of alternatives based on the frame that is attached to the set. Building on the aforementioned approach, this chapter proposes a more specialised conceptualisation of frame. We link each alternative in the choice problem with an index that represents an unobservable physical or psychological measure that the decision maker attaches to each alternative in the set. This index can be interpreted in various ways. For instance, the index can be interpreted as a measure of perceived availability of the alternative, or as the level of salience of the alternative in the specific situation, or as the level of perceived popularity of that alternative in a reference group. All these interpretations will be examined in detail in the present chapter.

In the next section, theories and evidence motivating the adoption of indexed alternatives are presented. We consider how psychological features affect choices; specifically, we constrain our analysis to psychological features that can be captured by means of indexes attached to the alternatives. In this chapter we propose two intuitively appealing properties of choices from sets of indexed alternatives that are inspired by the stylized facts we reported on and we show that, if choices from these sets of indexed alternatives satisfy these properties, then observing the unconditional behaviour, one cannot distinguish between these choices and choices produced by the maximisation of a preference relation that is complete and quasi–transitive[1]. Afterwards the connection between the proposed model and the

[1] A quasi–transitive preference relation is a preference relation for which the strict preference

more general model of "choice with frames" developed by Salant and Rubinstein (2008) are examined and finally some welfare considerations are discussed.

2.2 Psychological indexes

As mentioned in the previous sections the model considers choices from sets of alternatives that have an index attached to them. In the following, we propose evidence and theories supporting the idea to employ indexed alternatives in a choice theoretic model in order to capture psychological features influencing choices. Among the possible interpretations of the indexes, the more interesting ones are the following:

- **Perceived Availability.** This interpretation of the indexes is suggested by the psychological studies of the so called Commodity Theory developed by Brock (1968) and others (Brock and Brannon, 1992). According to this theory the more an alternative is perceived difficult to obtain, i.e., it is perceived "scarce", the more the alternative become attractive for a subject. Indeed, there are experiments, mainly in experimental psychology, which tested the effect of the perceived availability of goods on preferences (Verhallen, 1982; Lynn, 1989, 1991; Lynn and Bogert, 1996; Mittone et al., 2005; Mittone and Savadori, 2009). Manipulating the information about the easiness of obtaining the goods or the numerosity of each good available for the choice, these experiments have provided evidence supporting the sensitivity of preferences to the perceived availability of the good. In particular, it has been shown that a reduction (increment) of the manipulated availability (scarcity) of the goods increases (decreases) the likelihood of the good to be chosen.

  In light of this evidence, one can interpret the indexes of our model like a

part is transitive and the indifference part is not necessarily transitive.

measure of the perceived availability of the alternative. More precisely, the index attached to an alternative can be thought as a psychological measure of the easiness of obtaining the alternative. Using this interpretation of the indexes and considering the fact that increasing the scarcity of the good increases its attractiveness, it seems sensible to assume that, if an alternative is chosen when it has a given level of perceived availability, by reducing its availability the decision maker will continue to choose the same alternative. Similarly, if a non chosen alternative is perceived as more easy to obtain, i.e. an increase in the index, this will not affect choices since the relative scarcity of the chosen alternative will increase.

- **Snob and Bandwagon effects.** The second interpretation of the indexes builds on the work of Leibenstein (1950), in which the author pointed out that the individual demand of some good can be influenced by the overall level of its market demand. Considering the direction of the relation between individual and market demand, Leibenstein (1950) defines two types of effects: if the individual demand increases with the market demand we have the so called "bandwagon effect"; while if the individual demand decreases with the market demand of the good we have the so called "snob effect". Therefore, reinterpreting the two effects from a choice perspective, we can look at the index attached to the alternative like the level of popularity (diffusion) of the alternative in a reference group, e.g., we can see the indexes like the number of decision maker's friends that already possess the alternative. Notice that this interpretation of the indexes can be also motivated by the psychological phenomena regarding the "need for uniqueness" and the "need for conformity" (Hornsey and Jetten, 2004). According to these theories people compare themselves to others to assess their similarity to and distinctiveness from the others, this because of the opposing needs to be included in social groups and to be distinctive from others.

Interpreting the indexes as the level of popularity of the alternatives, we can model the snob and bandwagon effect from a choice perspective. Our model allows subjects to show snob effect for some alternatives and bandwagon effect for some other alternatives. In particular, we have “snob” alternatives when a reduction of the popularity of the chosen alternative does not alter the choice—if one chooses an alternative when it has a given index and one is snob regarding that type of alternative, one continue to choose it when the index of that alternative is reduced—while we have “bandwagon” alternatives when it is an increase of the popularity of the chosen alternatives that does not alter the choice—if one chooses an alternative when it has a given index and one is a bandwagoner regarding that type of alternative, one continue to choose it when the index of that alternative is increased. Obviously if there are alternative that does not trigger any effect, the index level of that alternative does not have any impact on the choice.

- **Salience.** A third interpretation of the indexes is to assume that they represent the salience of the alternatives for the decision maker in that particular choice task. The concept of salience is very broad and the term “salient” has often been used as a synonymous of noticeable and prominent; the interpretation of salience, however, varies greatly from field to field. For instance, in marketing literature the concept of brand salience has been often equated with “the prominence or level of activation of a brand in memory” (Alba and Chattopadhyay, 1986) while in cognitive science—in particular in vision research—the concept of salience is closely related to a measure of the ability of different stimuli to attract the visual attention of the decision maker in different situations (Huang and Pashler, 2005; Itti, 2006, 2007).

For our purposes, we consider salience as the level of importance or relevance that the decision maker attaches to the alternative with respect to the particular choice task he is facing.

Given this definition of salience, it is sensible to assume that if an alternative is chosen when it has a given level of salience, then it has to be chosen when the salience of that alternative becomes higher. In addition, if an alternative is non–chosen when it has a given level of salience, then it continues to be non–chosen when the salience of that alternative becomes lower. Hence, also in this case, we restrict choice behaviour in a way similar to the previous interpretations.

- **Reason–Based Choices.** The fourth interpretation of the indexes is based on the psychological research about reason based choices. As Shafir et al. (1993) pointed out, the making of a decision is often difficult because of uncertainty and conflict. People usually consider reasons for and against each option in order to choose and the decisions depends on the weights attached to the option's pros and cons (e.g., the famous list of pros and cons written by Charles Darwin to decide whether to get married or not). Considering a simplified version of this approach one can interpret the index of an alternative like the number of reasons supporting the choice of that alternative, or like the net number of pros and cons of the alternative or also like the net sum of the weighted pros and cons[2].

  When interpreting the indexes like the number of supporting reasons for choosing each alternative, it seems to be sensible to assume the following: on the one hand, if I choose an alternative when I have a given number of pros, by incrementing the pros of the chosen alternative I should continue

[2]See Bettman et al. (1998) for procedures that attach to each alternative a weighted sum of psychological evaluations of the alternative's attributes and see Alba and Marmorstein (1987) for evidence of the use of frequencies of pros like a choice heuristic.

to choose that alternative; and, on the other hand, if I reject an alternative when I have a given number of pros I still reject it when the number of pros decreases.

After having introduced some possible interpretation of the indexes, a couple of important aspects deserve to be mentioned. First of all, it is not necessarily true that the chosen alternative is the scarcest one, the most salient one, or the one that has the higher number of pros. Indeed, the choice depends not just on the indexes but also on the different alternatives. The choice is the product of the interplay between the indexes and the alternatives to which the indexes are attached. Even if this feature may sound a little bit odd in the first place, we want to underline the fact that both perceived availability and salience do not require the scarcer or more salient alternative in a set to be chosen. Indeed, if we consider salience as prominence the decision maker can be immediately attracted by the most prominent alternative but then he can become aware of the presence of less salient alternatives and he may finally choose one of these alternatives. However, notice that also the procedure that selects the alternative with the highest (lowest) index is compatible with the properties of our model as the proof of Lemma 4 will show.

A second related aspect is that, for almost all the interpretations, the choice depends on the the relative level of the index. That is, if something is chosen when it has a given level of salience (or scarcity), it seems natural to maintain the choice of that alternative when either its salience (scarcity) increases or the salience (scarcity) of a non–chosen alternative decreases. This is because, in both cases, the relative level of salience (scarcity) of the chosen alternative is either increased or unchanged, and hence, there are no obvious reasons to change the choice. The same reasoning apply to other interpretations of the indexes such as reason–based choices.

A final remark about the interpretations of the indexes is that they can be always considered also as a measure of distance between the alternatives and a reference point in a psychological space in which the decision maker may encode the alternatives. An example may be a manager that has to hire a new secretary. In this case, the alternatives are the candidates for the job, and the indexes may represent the unobservable information about the social distance between the manager and the candidates.

2.3 A model of choice with psychological indexes

As mentioned above, the main idea of this chapter is to attach to each alternative in a finite universal collection X an index that can reflect some unobservable physical or psychological feature that the alternatives possess. Hence, instead of defining a choice problem simply as a subset A of the collection X, we consider an *indexed choice problem* (A, f) that is a non–empty subset A of X along with a function f from X into $\mathbb{R}$, that we will call *index function*. That is, an index function f attaches a real number to each alternative present in X. As we will see, the indexes attached to the alternatives can be interpreted in various ways. For instance, these indexes can be interpreted as perceived availability of the alternative by the decision maker, as an index of salience of the alternative, or as the number of people already possessing that alternative. In this model a set of alternatives A is part of many choice problems. Indeed, each set A coupled with distinct index functions defines distinct choice problems and hence the set of all the indexed choice problems D^* becomes $D^* = (P(X) - \emptyset) \times \mathbb{R}^X$.

Having defined the concept of choice problem used here, we will use standard definitions concerning choice functions and choice correspondences. We use the term *choice function* for a function $c()$ that attaches to each choice problem

$A \in D$ a single element in A, while we use the term *choice correspondence* for a correspondence $C()$ that attaches to each choice problem $A \in D$ a non–empty subset of A. Therefore, given a choice problem, a choice function selects only one chosen element among the available alternatives while a choice correspondence can select many chosen elements among the available alternatives.

In what follows, we assume that a decision maker has a choice function $c()$—called *indexed choice function*—that is defined over the set of all the indexed choice problems D^* and selects a single chosen item from the set of available alternatives. It has to be noted that in this framework one can allow for changes in the chosen alternatives according to the index function attached to the choice set. Indeed, a decision maker facing a set of alternatives A can choose an alternative x from A when the choice problem is (A, f) while he can choose an alternative y when the choice problem is (A, g). Moreover, an indexed choice function $c()$ defined over D^* induces a choice correspondence $C()$ defined over $D = P(X) - \emptyset$ such that $C(A) = \bigcup_{f \in \mathbb{R}^X} c(A, f)$. The induced choice correspondence just defined includes all the elements that are chosen from a set A for some function f. That is, given a set A, x belongs to the induced choice correspondence $C(A)$ if and only if there is a function $f \in \mathbb{R}^X$ such that $c(A, f) = x$.

The reason motivating the introduction of an induced choice correspondence is the fact that in many cases the index that is attached to each alternative is unobservable by an external observer while it can be perfectly known by the decision maker—e.g., interpreting the indexes as levels of perceived availability of the alternative or as levels of salience. In such cases an observer of the choice behaviour of the decision maker will be unable to distinguish the circumstances under which the decision maker chooses the alternative x over the alternative y and under which he chooses x over y. Hence the observer can simply record that the individual have chosen both. There are also cases in which the indexes attached to the alternatives can be known—e.g., interpreting the value as the number of people that already possess the alternative or as the supply of that alternative in the market. However, in such cases it may be difficult to observe the indexes and interesting to disregard the information about the values attached to each alternative by focusing on the unconditional behaviour of the decision maker.

So far we have imposed no restriction on the indexed choice function, that is there are no limitations in what the decision maker can choose from a set A under different indexes attached to the alternatives. Obviously without limitations one can obtain every type of choice behaviour and hence we should introduce some restrictions on the behaviour of the indexed choice function $c()$ under a given index function. This is done by introducing two properties: the first constrains the behaviour of the indexed choice function when the indexes of the alternatives are fixed and the choice problem can vary; while the second constrains the behaviour of the indexed choice function when the choice problem is fixed and the indexes attached to the alternatives can vary.

The first property of the choices over the set of all the indexed choice problems is called *Conditional IIA*.

Def (Conditional IIA)**.** Given $f \in \mathbb{R}^X$, if $x \in B \subseteq A$ and $x = c(A, f)$ then $x = c(B, f)$.

This property says that having fixed an index function i, the indexed choice function $c()$ satisfies the standard Independence from Irrelevant Alternatives property (IIA). Conditional IIA captures the idea that if the index function has not been changed, the elimination of a non–chosen alternative from the set has no effect on the chosen alternative that must remain the same as chosen before the elimination occurred. In other words, this property says that once the decision maker has a complete psychological picture of the alternatives in X—i.e., he/she has attached to each alternative in the universal collection a psychological value—and has decided to choose one alternative from a choice problem A according to this picture, then removing a non-chosen alternative and holding fixed the psychological situation does not change his/her mind.

It has to be noted that Conditional IIA taken together with the fact that the indexed choice function is single valued implies that there exist a linear ordering $\succ_f$ such that its maximisation describes the choices of $c()$ over D whenever the index function is f, i.e., $\{c(A, f)\} = \{x \in A \mid \forall y \in A, x \succ_f y\}$ for all $(A, f) \in D^*$. Hence, the decision maker can be considered as completely rational when the index function is kept constant and moreover he cannot be indifferent between two alternatives. This means that indifference emerges only because of the unobservability of the indexes attached to the alternatives and that all the irrationality in the decision maker's behaviour is captured by indexes' movements. In other words, the decision maker is never indifferent between two alternatives, but he/she can reverse the preferences according to the particular index function attached to the choice set. Indifference is only in the eyes of the external observer, but we will discuss afterward the interpretation of indifference in this model.

The second property of choices over the set of all the indexed choice problems is the *Monotonicity* property which is composed of two parts.

Def (Monotonicity)**.** For each $x \in X$ either

$x \uparrow$**.** for all $(B, f) \in D^*$: $x = c(B, f) \Rightarrow \forall f'$ such that $f'(x) \geq f(x)$ and $f'(a) = f(a)$ $\forall a \in X - \{x\}$, $x = c(B, f')$ and; $x \neq c(B, f) = y \Rightarrow \forall f'$ such that $f'(x) \leq f(x)$ and $f'(a) = f(a)$ $\forall a \in X - \{x\}$, $y = c(B, f')$ or,

$x \downarrow$**.** for all $(B, f) \in D^*$: $x = c(B, f) \Rightarrow \forall f'$ such that $f'(x) \leq f(x)$ and $f'(a) = f(a)$ $\forall a \in X - \{x\}$, $x = c(B, f')$ and; $x \neq c(B, f) = y \Rightarrow \forall f'$ such that $f'(x) \geq f(x)$ and $f'(a) = f(a)$ $\forall a \in X - \{x\}$, $y = c(B, f')$.

Monotonicity means that, if an item is chosen under some circumstances, it has to be chosen when its index is altered in a given direction. More precisely, in case of alternatives of type '$\uparrow$', whenever the alternative is chosen, an increase in its index cannot affect choices. While in case of alternatives of type '$\downarrow$', whenever the alternative is chosen, a decrease in its index cannot affect choices. Obviously there can be alternatives for which both '$\uparrow$' and '$\downarrow$' hold true. In this case we use the notation '$\updownarrow$' and we have that, whenever those alternatives are chosen, both decreasing and increasing the index attached to them cannot alter the choice. Monotonicity has also implications for movements of the indexes of non–chosen alternatives. In particular, when an alternative of type '$\uparrow$' ('$\downarrow$') is non–chosen a reduction (increment) of the alternative's index does not alter the chosen alternative.

This property well captures the behavioural implications of the different psychological indexes and their interpretations discussed in the previous section of the chapter. For instance, if one interpret the indexes as the level of salience of an alternative, the '$\uparrow$' part of the Monotonicity property captures all the effect that salience have on choices. The same applies for the Perceived Availability and Reason Based interpretations. Concerning the Bandwagon and snob interpretation for which the indexes captures the level of popularity (i.e., the diffusion of the alternative in a reference group), the possibility to have simultaneously bandwagon and snob effects for different alternatives requires the simultaneous presence of both types of alternatives, i.e., the '$\uparrow$' type for bandwagon effect alternatives and the '$\downarrow$' type for the snob effect alternatives.

It has to be pointed out that Monotonicity is defined for all the alternatives in X, hence it applies also to the alternatives that do not belong to the current choice problem. This means that, for instance, if $x \notin A$ and x is of type '$\uparrow$', then reducing the index attached to x the alternative chosen from A does not change. We are aware that conditioning the behaviour of the indexed choice function to the index of an alternative that is not under consideration may appear quite counterintuitive. However, since an alternative that is not in the current choice problem cannot be chosen, it turns out that Monotonicity implies that the indexes of alternatives outside the current choice problem do not have any effect on choices. In order to see this we introduce the following lemma.

Lemma 1. *If $c()$ is an indexed choice function satisfying Monotonicity, $x \neq c(A, f)$ for all $f \in \mathbb{R}^X$, and $y = c(A, g)$ for an index function $g \in \mathbb{R}^X$ then for all h such that $h(x) \neq g(x)$ and $h(a) = g(a)$ $\forall a \in X - \{x\}$ we have that $y = c(A, h)$*[3].

Proof. Let $c()$ be an indexed choice function satisfying Monotonicity, $x \neq c(A, f)$ for all $f \in \mathbb{R}^X$, and $y = c(A, g)$ for an index function $g \in \mathbb{R}^X$. Suppose now that there is an index function h such that $h(x) \neq g(x)$ and $h(a) = g(a)$ $\forall a \in X - \{x\}$ and $y \neq c(A, h) = z$. Notice that since x is never chosen then $z \neq x$. So suppose w.l.o.g. that $h(x) > g(x)$. In this case if $x \uparrow$ we have $y = c(A, h)$ contradicting $z = c(A, h)$, and if $x \downarrow$ we have $z = c(A, g)$ contradicting $y = c(A, g)$. $\square$

Notice that Lemma 1 holds not only for those alternatives that do not belong to the current choice problem, but also for those alternatives that are in the current choice problem but are non–chosen for all the index functions. This means that the indexes of the alternatives that are non–chosen from a set A have no effect on the choice of the decision maker. In other words, only the indexes attached to alternatives that are chosen for some index function i matter for determining the choice of the decision maker. Finally, Lemma 1 also implies that alternatives that are never chosen in all the choice problems can be only of type '$\updownarrow$'.

In order to have a better understanding of how the indexes of the alternatives may affect choices it is worth underlining the implications of Monotonicity. As already shown with Lemma 1, the only indexes that may have effect on choices

[3]Notice that in the light of this lemma, the Monotonicity property can be restated using two separate properties: a Monotonicity property restricted to the elements belonging to the current choice problem plus an Invariance property that excludes the effect of a change in the indexes of alternatives outside the choice problem.

from a set A are the ones that belong to those alternatives that are chosen from A at some point. However, there is another interesting observation regarding the effect of a change of the indexes of such alternatives. Indeed, if we have that $x \updownarrow$ is a chosen alternative from A, this implies that the index of x does not affect the choice of the decision maker in any way. Both when x is chosen and x is non–chosen from A, an alteration of the index of x cannot lead to changes in the choice.

If instead we have an alternative x such that $x \uparrow$ and not $x \downarrow$ then the index of x has an effect on the choice of the decision maker. $x \uparrow$ and not $x \downarrow$ implies that: either there is a situation (A, f) in which x is chosen and there is a $h \in \mathbb{R}^X$ such that $h(x) < f(x)$ and x is not chosen from A, or there is a situation (B, g) in which $y \neq x$ is chosen and there is a $q \in \mathbb{R}^X$ such that $q(x) > g(x)$ and y is not chosen from B. Hence there is at least one set in which a movement in the index of x produces a change in the choice. An interesting observation is that, in the former case, a reduction in the index of x can change the choice from x to an element y, but, $x \uparrow$ implies that further reductions of the index do not affect the choice anymore—i.e., once the choice has switched from x to y a further reduction of the index of x has no consequence for the choice. In the latter case we have that increasing the index of x up to $q(x)$ an element different from y is chosen, but the element chosen cannot be different from x. Indeed, if we suppose that z different from x is chosen, we have that z is chosen from (B, g) contradicting $y = c(B, g)$. Therefore, even in case x is not chosen there can be only a single change due to the effect of the index of x. Combining the two cases, we have that the situation $x \uparrow$ and not $x \downarrow$ implies that there is at least a choice set A and a combination of indexes for the alternatives different from x for which there exists a value k such that x is chosen when its index is above and $y \neq x$ is chosen when the index is below that threshold.[4]

[4] Suppose the case $x \uparrow$ and not $x \downarrow$. Then not $x \downarrow$ implies either that: (1) $\exists (A, f)$ s.t.

A similar situation arises when considering alternatives of type $x\downarrow$ and not $x\uparrow$. That is, there is at least one choice set A and a combination of indexes for the alternatives different from x for which there exists a threshold k such that x is chosen when its index is below and $y \neq x$ is chosen when the index is above that threshold.

After having introduced and explained the two properties that constrain the behaviour of the indexed choice function, we define the concept of *constrained indexed choice function.*

Def (Constrained indexed choice function)**.** A *constrained indexed choice function* is a choice function $c()$ on D^* that satisfies Conditional IIA and Monotonicity.

The main result of the chapter is that the choice correspondence induced by a constrained indexed choice function can be rationalised by a quasi–transitive preference relation—i.e., there is a preference relation whose maximisation produces the same choices produced by the induced choice correspondence—and, moreover, the choices produced by the maximisation of a quasi–transitive preference relation can be produced by a choice correspondence induced by a constrained indexed choice function.

Theorem 1. *A choice correspondence is induced by a constrained indexed choice*

$x = c(A, f)$ and $\exists k < f(x)$ s.t. $y = c(A, f')$ where $f'(a) = f(a) \forall a \in X$ and $f'(x) = k$; or (2) $\exists (B, g)$ s.t. $z = c(B, g)$ and $\exists k' > g(x)$ s.t. $x = c(B, g')$ where $g'(a) = g(a) \forall a \in X$ and $g'(x) = k'$. Notice that (1) if and only if (2). Hence take (1). First we show that no elements different from x and y can be chosen from A for all the index function h such that $h(a) = f(a) \forall a \in X - \{x\}$. Indeed if $z = c(A, h)$ for some h such that $h(a) = f(a) \forall a \in X - \{x\}$ we have that: $h(x) \geq f(x)$ implies that $z = c(A, f)$ contradicting $x = c(A, f)$, and $h(a) < f(x)$ this implies either $z = c(A, f')$ (if $h(x) \geq f'(x)$) or $y = c(A, h)$ (if $h(x) < f'(x)$) producing a contradiction in both cases. Hence no elements different from x and y can be chosen from A for all the index function h such that $h(a) = f(a) \forall a \in X - \{x\}$. Now we want to show that there exists a number r for which $f(x) > r$ implies that x is chosen and $f(x) < r$ implies that y is chosen. In order to do this suppose not, suppose that there are two numbers a and b such that $a > b$ and y is chosen from A when $f(x) = a$ and x is chosen from A when $f(x) = b$. Using again the fact that $x\uparrow$ we have that x is chosen from A when $f(x) = b$ with $a > b$ implies that x is chosen from A when $f(x) = a$ a contradiction. Hence there has to be a number r for which $f(x) > r$ implies that x is chosen from A and $f(x) < r$ implies that y is chosen from A. Notice that we cannot say anything about the behaviour of the choice function when $f(x) = r$. We just know that one between x and y must be chosen in r but we do not know which one.

function if and only if it is rationalizable by a complete and quasi–transitive preference relation $\succsim$.

In order to prove the main statement we need to prove some preliminary result, but first we need to introduce the definition of three properties that will be used in the proof. Namely, Sen's properties α, γ, and δ (Sen, 1971).

Def (Sen's Property α)**.** A choice correspondence $C()$ satisfies Sen's Property α if $x \in C(A)$ and $x \in B \subseteq A$ implies that $x \in C(B)$.

Def (Sen's Property γ)**.** A choice correspondence $C()$ satisfies Sen's Property γ if $x \in C(A)$ and $x \in C(B)$ implies that $x \in C(A \cup B)$.

Def (Sen's Property δ)**.** A choice correspondence $C()$ satisfies Sen's Property δ if for any pair of sets $A, B \in D$ such that $A \subseteq B$ and $x, y \in C(A)$ then $C(B) \neq \{x\}$.

In what follows, we show that a choice correspondence $C()$ induced by an indexed choice function $c()$ satisfies Sen's properties α, γ, and δ (Sen, 1971), and hence the revealed preference relation R defined as xRy if and only if $\exists A \subseteq X$ such that $x \in C(A)$ and $y \in A$ is a complete and quasi–transitive binary relation whose maximisation produces the same choices as $C()$.

For the proof that a Choice Correspondence $C()$ defined over $D = P(X) - \emptyset$ satisfies Sen's properties α, γ, and δ if and only if there exists a quasi–transitive and complete preference relation R such that $C(A) = \{x \in A \mid \forall y \in A, xRy\}$ for all $A \in D$ see Sen (1971).

We start proving that the induced choice correspondence $C()$ satisfies properties α and γ.

Lemma 2. *The choice correspondence* $C()$ *induced by a constrained indexed choice function* $c()$ *satisfies Sen's properties* α *and* γ*.*

Proof. Concerning property α suppose that $x \in C(A)$, $x \in B$, and $B \subseteq A$, then for the definition of induced choice correspondence, i.e. $C(A) = \bigcup_{f \in \mathbb{R}^X} c(A, f)$, there is an index function f such that $x = c(A, f)$. Then, if $x = c(A, f)$, for

Conditional IIA $x = c(B, f)$ and by definition of induced choice correspondence we have that $x \in C(B)$. Thus property α is satisfied.

Moving to property γ suppose that $x \in C(A)$ and $x \in C(B)$ then, for the definition of induced choice correspondence, there exists an index function f such that $x = c(A, f)$ and an index function g such that $x = c(B, g)$. Now what is needed in order to have property γ satisfied is the existence of an index function h for which $x = c(A \cup B, h)$. The proof is by construction of h and consists of 2 steps.

Step 1: We start proving that, given $x = c(A, f)$ and $x = c(B, g)$, there is an index function h such that $x = c(A, h)$ and $x = c(B, h)$. Consider the index function h on X defined as follows:

$$h(z) = \begin{cases} \min\left(f(z), g(z)\right), & \text{if } z \uparrow \wedge \neg z \downarrow \text{ and } z \neq x; \\ \max\left(f(z), g(z)\right), & \text{if } z \downarrow \wedge \neg z \uparrow \text{ and } z \neq x; \\ \max\left(f(z), g(z)\right), & \text{if } z \uparrow \wedge \neg z \downarrow \text{ and } z = x; \\ \min\left(f(z), g(z)\right), & \text{if } z \downarrow \wedge \neg z \uparrow \text{ and } z = x; \\ f(z), & \text{otherwise.} \end{cases}$$

First we prove that $x = c(A, h)$. Notice that, by construction of h, for all the non–chosen alternatives y in $X - \{x\}$, we have that: if $y \uparrow$, $h(y) \leq f(y)$; if $y \downarrow$, $h(y) \geq f(y)$; and if $y \updownarrow$, $h(y) = f(y)$. Hence by Monotonicity the indexes of those alternatives cannot affect the choice. Moreover, concerning $h(x)$ we have that: if $x \uparrow$, $h(x) \geq f(x)$; if $x \downarrow$, $h(x) \leq f(x)$; and if $x \updownarrow$, $h(x) = f(x)$. Thus, also in this case Monotonicity prevents the index of x from affecting choices and hence $x = c(A, h)$ by Monotonicity.

Now we prove that $x = c(B, h)$. Applying the same reasoning as before we have that, for all the alternatives in $y \in X$ such that $y \uparrow \wedge \neg y \downarrow$ or $y \downarrow \wedge \neg y \uparrow$, the change of the index from g to h does not affect the choice. Considering the alternatives y such that $y \updownarrow$ notice that $h(y) = f(y)$ that can be different from $g(y)$. But we already know that Monotonicity implies that a change in the index of an alternative of type '$y \updownarrow$' does not have effect on the chosen alternative.

Hence Monotonicity implies that $x = c(B, h)$.

Step 2: Now we show that $x = c(A, h) = c(B, h)$ implies $c(A \cup B, h) = x$. Suppose $c(A \cup B, h) = z \neq x$, then we have two cases either: $z \in A$ or $z \in B - A$. *CASE 1:* Suppose $z \in A$. In this case $z = c(A, h)$ by Conditional IIA and this contradicts Step 1. *CASE 2:* Suppose $z \in B - A$. In this case $z = c(B, h)$ by Conditional IIA, and again a contradiction with Step 1. Therefore, for non-emptiness of $c()$, we conclude that $c(A \cup B, h) = x$.

In order to complete the proof one needs to note that, by definition of induced choice correspondence, $x = c(A \cup B, h)$ belongs to $\bigcup_{f \in \mathbb{R}^X} c(A \cup B, f)$ and therefore $x \in C(A \cup B)$. Thus, the induced choice correspondence satisfies Sen's Property γ. □

Since the induced choice correspondence satisfies both Sen's property α and γ it is Normal (Sen, 1971). This means that $C(A) = R$–$\mathrm{gr}(A)$ where the binary relation R is the revealed preference relation—i.e., xRy if and only if $\exists A \subseteq X$ such that $x \in C(A)$ and $y \in A$—and R–$\mathrm{gr}(A)$ is the set of the greatest elements in A according to R—i.e., $R\text{–}\mathrm{gr}(A) = \{x \in A \mid xRy \ \forall y \in A\}$.

The fact that the revealed preference relation R rationalises the choice correspondence induced by a constrained indexed choice function implies that the behaviour of the agent can be interpreted as a maximising one; that is, one can retain the classical assumption about the rationality of the agent: he/she chooses what is the best for him/her according to a complete and acyclic preference relation. In the light of this result, an interesting interpretation of the effect of psychological indexes can be given. If the agent behaves according to a constrained indexed choice function his/her behaviour can be interpreted "as if" he/she maximises a weak preference relation R obtaining a set of preferred items, and then he/she uses a tie breaking rule based on the "indexes" he/she attaches to the alternatives to choose one item among the preferred items. Notice that the decision maker's tie–breaking rule is based on regions of the space of the indexes and that these regions are convex. Indeed, if you choose the element x from the set A both in the situation g and in the situation f, a convex combination of the indexes is such that $\min(f(a), g(a)) \leq \sigma g(a) + (1 - \sigma) f(a) \leq \max(f(a), g(a))$ for all the a in A and for all the $\sigma \in [0, 1]$, hence Monotonicity guarantees that x is chosen for all the index function that are convex combinations of g and f. Thus the decision maker breaks the ties according to the region to which the current index function belongs to.

What is left to prove is the fact that the revealed preference relation R is indeed a quasi–transitive preference relation. This is assured by the following lemma.

Lemma 3. *The choice correspondence $C()$ induced by a constrained indexed choice function $c()$ satisfies Sen's property δ.*

Proof. Suppose $A \subset B$, $x \neq y$, $x, y \in C(A)$ and $\{x\} = C(B)$. $\{x\} = C(B)$ implies that $x = c(B, f)\ \forall f \in \mathbb{R}^X$. Therefore by Conditional IIA one gets that $x = c(A, f)\ \forall f \in \mathbb{R}^X$ that contradicts $y \in C(A)$. So the induced choice correspondence satisfies Sen's property δ. $\square$

Since the induced choice correspondence satisfies Sen's properties α, γ and δ, it is Normal and the revealed preference relation R is quasi–transitive (Sen, 1971). That is, the strict preference relation P—i.e., the asymmetric part of R—is transitive but the indifference relation I—i.e., the symmetric part of R—is not necessarily transitive.[5]

A specification making more clear the role of indifference in the models is due. As already mentioned, indifference is in the eyes of the external observer. When two alternatives x and y are revealed indifferent this means that there is an index function in which x is chosen over y and another in which y is chosen over x, and that the decision maker is not willing to change his choice. That is in the first case he truly prefers x over y and in the second case he truly prefers y over x. This implies that the interpretation of indifference in this model is slightly different from the standard one: indifference should be thought as the absence of an unambiguous strict preference under all the possible psychological situations.

[5] The non transitivity of indifference implies that we can have situations in which $xIy \wedge yIz$ but zPx. Notice that this situation is not so unnatural. Indeed, there are no compelling philosophical reasons for requiring transitivity of indifference. See Luce (1956) for a discussion about this issue.

So far we have shown that the induced choice correspondence satisfies Sen's properties α, γ and δ, and hence it is quasi–transitive. Now we show that it may not be a Weak-order, i.e., it may not satisfy Sen's property β.

Def (Sen's Property β)**.** A choice correspondence $C()$ satisfies Sen's Property β if for any pair of sets $A, B \in D$ such that $A \subseteq B$, $x, y \in C(A)$ and $x \in C(B)$ then $y \in C(B)$.

In other to see that the induced choice correspondence does not satisfy Sen's property β consider the following situation, suppose $X = \{x, y, z\}$ and suppose that:

1. $x = c(\{x, y, z\}, f)$, $x = c(\{x, z\}, f)$, $x = c(\{x, y\}, f)$, and $z = c(\{z, y\}, f)$ for all the index functions where $f(x) \geq k \in \mathbb{R}$;

2. $z = c(\{x, y, z\}, f)$, $z = c(\{x, z\}, f)$, $y = c(\{x, y\}, f)$, and $z = c(\{z, y\}, f)$ for all the index functions where $f(x) < k$

This indexed choice function satisfies the Monotonicity and Conditional IIA properties but the induced choice correspondence does not satisfy Sen's property β. In fact we have that $\{x, y\} = C(\{x, y\})$ and that $y \notin C(\{x, y, z\})$.

The implications of Lemmas 2 and 3 are that, given a choice procedure that satisfies Monotonicity and Conditional IIA, one can interpret the observable pattern of choices of the agent "as if" he/she is maximising a quasi–transitive preference relation. But given an agent that maximises a quasi–transitive preference relation is it possible to interpret his behaviour "as if" he is adopting a choice procedure that satisfies Monotonicity and Conditional IIA? We show this with lemma 4.

Lemma 4. *If choices are determined by the maximisation of a complete and quasi–transitive preference relation $\succsim$ on the finite set X, then there exists a constrained indexed choice function $c()$ defined over $D^* = (P(X) - \emptyset) \times \mathbb{R}^X$ that induces a choice correspondence $C()$ defined over $D = (P(X) - \emptyset)$ such that*

$C(A) = \succsim$ –gr(A) for all the non–empty subsets A of X[6].

Proof. We define the indexed choice function $c()$ explicitly, and then we prove that $c()$ satisfies Conditional IIA and Monotonicity. Let $<_O$ be an arbitrary linear order on X—i.e., $<_O$ is a complete, transitive and antisymmetric binary relation on X. Define $c(A, f) = x \; \forall f \in \mathbb{R}^X$ such that: $x \in \succsim$ –gr(A) and, $\forall y \in \succsim$ –gr(A), either $f(x) < f(y)$ or $f(x) = f(y) \wedge x <_O y$. First we show that the indexed choice function $c()$ just defined is indeed a choice function—i.e., that is non–empty and single–valued for all the sets $(A, F) \in D^*$. Considering that the set $\succsim$ –gr(A) is finite and not–empty for each non–empty subset A of X, the non-emptiness and the single-valuedness of $c()$ for all $f \in \mathbb{R}$ follow directly by the fact that a finite set of numbers in $\mathbb{R}$ has always a minimal element and that $<_O$ is a linear order—i.e., the set $<_O$ –gr(A) is a singleton for each subset A of X.

Now we show that $c()$ is a constrained indexed choice function, that is, $c()$ satisfies Conditional IIA and Monotonicity.

Conditional IIA: suppose $x = c(A, f)$. Then consider the set $B \subseteq A$ such that $x \in B$. We show that $x = c(B, f)$. The first consideration is that for each $z \in B$ if $z \in \succsim$ –gr(A) then it belongs also to $\succsim$ –gr(B). Indeed if z is a $\succsim$–greatest element of A it means that $z \succsim k$ for all $k \in A$ and hence $z \succsim k$ for all $k \in B$ that in turn implies $z \in \succsim$ –gr(B). The second consideration is that, by definition of $c()$, $x = c(A, f)$ implies that $x \in \succsim$ –gr(A) and $\forall z \in \succsim$ –gr(A), $f(x) < f(z)$ or $f(x) = f(z) \wedge x <_O z$. Hence combining the two considerations we have that for all $z \in \succsim -gr(B)$, $f(x) < f(z)$ or $f(x) = f(z) \wedge x <_O z$. That implies that $x = c(B, f)$.

Monotonicity: The proof that $c()$ satisfies Monotonicity is based on the fact that increasing the index of a non–chosen alternative y or reducing the index of the chosen one x does not alter the fact that $f(x) \leq f(y) \; \forall y \in \succsim$ –gr(A) and hence does not alter the chosen alternative. Suppose $x = c(A, f)$, then $\forall y \in \succsim$ –gr(A), $f(x) < f(y)$ or $f(x) = f(y) \wedge x <_O y$. Take an index function

[6] The relation $\succsim$ is complete and the asymmetric part $\succ$ is transitive, thus the set $\succsim$ –gr(A) is non–empty for each non–empty subset A of X because of finiteness of X. Suppose not, then for all x in A there exists y such that $\neg x \succsim y$ that implies $y \succ x$ and since the set A is finite, this lead to a contradiction with transitivity of $\succ$.

g, $g(z) = f(z)\ \forall z \in X - \{x\}$ and $g(x) < f(x)$. In this case $g(x) < g(y)\ \forall y \in \succsim$ $-\text{gr}(A) - \{x\}$ and hence, since $g(x) = g(x) \wedge x <_O x$, we have $x = c(A, g)$. Take now and index function g', $g'(z) = f(z) \forall z \in X - \{y\}$, $y \neq x$, and $g'(y) > f(y)$. In this case $g(x) \leq g(y)\ \forall y \in \succsim -\text{gr}(A)$ and, since $x <_O z \forall z \in \succsim -\text{gr}(A)$ such that $f(z) = f(x)$, then $x <_O z \forall z \in \succsim -\text{gr}(A)$ such that $g(z) = g(x)$. Thus we have $x = c(A, g')$. Therefore each element x in X satisfies the condition $x \downarrow$ and hence Monotonicity is satisfied.

Therefore the choice function $c()$ on D^* is a constrained indexed choice function. What we have left to prove is that the induced choice correspondence $C()$ is such that $C(A) = \succsim -\text{gr}(A)$ for all the non–empty subsets A of X. If $x \in C(A) = \bigcup_{h \in \mathbb{R}^X} c(A, h)$ then it exists an index function f such that $x = c(A, f)$ and hence $x \in \succsim -\text{gr}(A)$ by construction of $c()$. Suppose instead that $x \in \succsim -\text{gr}(A)$ and consider an index function f in which $f(x) < f(y)$ for all y in $\succsim -\text{gr}(A)$. In this case $x = c(A, f)$ and hence $x \in C(A)$. Therefore the constrained indexed choice function $c()$ is such that $C(A) = \succsim -\text{gr}(A)$ for all the non–empty subsets A of X. □

In the proof of Lemma 4 we build a constrained indexed choice function for which all the alternatives are of type '$\downarrow$' and hence, either the index of an alternative has no effect on choices, or the effect is in one particular direction. But this is just one of the possible choice functions that one can construct in order to prove Lemma 4. For instance another possibility is to build $c()$ such that it satisfies condition $x \uparrow$ for all the alternatives in X—i.e. defining $c(A, f) = x\ \forall f \in \mathbb{R}^X$ such that: $x \in \succsim -\text{gr}(A)$; and $\forall y \in \succsim -\text{gr}(A)$, $f(x) > f(y)$ or $f(x) = f(y) \wedge x <_O y$[7]. The interpretation of Lemma 4 is that, if an agent chooses by maximising a complete and quasi–transitive preference relation, then his/her behaviour can be seen "as if" he chooses according to a constrained indexed choice function.

After having obtained the results in Lemmas 2, 3, and 4, it is straightforward to prove the main proposition of the chapter. In fact, Lemmas 2 and 3 prove that

[7] A third different possibility is to choose a number $k \in \mathbb{R}$ and to define $c(A, f) = x$ for all the $f \in \mathbb{R}^X$ such that: $x \in \succsim -\text{gr}(A)$; and either (1) $\forall y \in \succsim -\text{gr}(A)$ such that $\neg x <_O y$, $f(y) < k \wedge f(x) \geq k$; or (2) $\forall y \in \succsim -\text{gr}(A)$, $f(y) < k \wedge y <_O x$.

a choice correspondence $C()$ induced by a constrained indexed choice function $c()$ is rationalised by the revealed preference relation R that is complete and quasi–transitive, while Lemma 4 shows that if there is a complete and quasi–transitive preference relation $\succsim$ whose maximisation determines the choice correspondence $C()$, then there exists a constrained indexed choice function $c()$ that induces $C()$.

What is left to prove is the independence of the axioms and in order to do this, we provide two examples. In the first example we discuss a case in which Monotonicity is satisfied and Conditional IIA is not, while in the second example Conditional IIA is satisfied and Monotonicity is not.

EXAMPLE 1. Consider the following situation, suppose $X = \{x, y, z\}$ and suppose that, for all the index function $f \in \mathbb{R}^X$, $x = c(\{x, y, z\}, f)$, $x = c(\{x, z\}, f)$, $y = c(\{x, y\}, f)$, and $y = c(\{z, y\}, f)$. This indexed choice function satisfies the Monotonicity property but not the Conditional IIA property. Indeed if Conditional IIA had been satisfied we would have that $x = c(\{x, y, z\}, f)$ implying $x = c(\{x, y\}, f)$ while we have that $y = c(\{x, y\}, f)$.

EXAMPLE 2. Consider the following situation, suppose $X = \{x, y, z\}$ and suppose that: $x = c(\{x, y, z\}, f)$ for all $f \in \mathbb{R}^X$ such that $f(x) \neq 0$ and $y = c(\{x, y, z\}, f)$ for all $f \in \mathbb{R}^X$ such that $f(x) = 0$; $x = c(\{x, z\}, f)$ for all $f \in \mathbb{R}^X$; $x = c(\{x, y\}, f)$ for all $f \in \mathbb{R}^X$ such that $f(x) \neq 0$ and $y = c(\{x, y\}, f)$ for all $i \in \mathbb{R}^X$ such that $f(x) = 0$; and $y = c(\{z, y\}, f)$ for all $f \in \mathbb{R}^X$. In this case it is easy to verify that Conditional IIA is satisfied while Monotonicity is not. Indeed taking an index function such that $f(x) > 0$ we have not $x \downarrow$ since $y = c(\{x, y, z\}, f)$ for all $f \in \mathbb{R}^X$ such that $f(x) = 0$, and taking an index function such that $f(x) < 0$ we have not $x \uparrow$ for the same reason. Hence this indexed choice function does not satisfy Monotonicity.

2.3.1 An alternative axiomatisation

Concerning the main result of the section, it may be interesting to explore the implications of a property of the indexed choice function other than Monotonicity. As a sensible alternative to Monotonicity one may consider a maximality property that, given an alternative x chosen from A for some index function f, imposes the choice of x whenever it possesses the globally maximal index.

Def (Maximality)**.** If $x = c(A, f)$ for some (A, f), then $x = c(A, f')$ for all f' such that $f'(x) > f'(z)\ \forall z \in X - \{x\}$.

This property, that will be used in chapter 3, captures the idea that if an alternative is chosen when it has a given index level, e.g., a given salience, then it must be chosen when it has the highest index level among all the alternatives, i.e., it is the most salient one. Notice that, while this feature may be sensible for some of the interpretation, it is not always supported by theory and evidence. In the case of salience, for instance, an agent may be attracted by the most salient alternative but then he may recognise that there is a less salient alternative that is better than the previous one, and hence he may finally choose the less salient alternative.

An interesting observation concerns the relationship between Maximality and Monotonicity. Indeed, while Maximality and Monotonicity show an high degree of similarity for the upward part of Monotonicity, the same is not true for the downward part of Monotonicity. More precisely, Maximality produces a clear contradiction when combined with the downward part of Monotonicity. Suppose that x and y are chosen from a set A and some index function, i.e., $x = c(A, f)$ and $y = c(A, g)$ and let $y \downarrow$. Then consider the following index function: f' where $f'(a) = f(a)$ for all $a \in X - \{y\}$ and $f'(y) > f(a)$ for all $a \in X - \{y\}$. We have that Maximality implies $y \in c(A, f')$ while $y \downarrow$ implies $x \in c(A, f')$.

However, also Maximality and the upward part of Monotonicity only appear to be formally related. Indeed, the upward part of Monotonicity implies that if x is chosen, then it must be chosen in some of the situations where x have the maximal index this does not imply that x is chosen every time it has the maximal index. In the next chapter we will show that the two properties are independent.

A second important aspect of Maximality is the possibility to substitute Monotonicity with Maximality in the proof of Theorem 1. Indeed, while the proofs that the induced choice correspondence satisfies Sen's properties α and δ make only use of Conditional IIA, the proof that it satisfies property γ (lemma 2) uses both Monotonicity and Conditional IIA. In lemma 2, Monotonicity and the fact that $x = c(A, f)$ and $x = (B, g)$ are both used to build an index function h such that $x = c(A, h) = c(B, h)$. It is immediate to see that choosing an index function h such that $h(x) > h(z)$ for all $z \in X - \{x\}$, Maximality and the assumptions that $x = c(A, f)$ and $x = (B, g)$ imply that $x = c(A, h) = c(B, h)$. Thus, Conditional IIA and Maximality are sufficient conditions for the induced choice correspondence being rationalizable by a complete and quasi–transitive preference relation.

Concerning the other direction, if one consider a choice correspondence $C()$ generated by a quasi–transitive preference relations, then it is easy to check that the indexed choice function defined as follows: $c(A, f) = x \; \forall f \in \mathbb{R}^X$ such that: $x \in \succsim$ –gr(A); and $\forall y \in \succsim$ –gr(A), $f(x) > f(y)$ or $f(x) = f(y) \wedge x <_O y$ is a costrained indexed choice function that satisfies Maximality and induces the choice correspondence $C()$.

Thus Maximality can substitute Monotonicity in the proof of Theorem 1. Moreover, Maximality does not have stronger implications than Monotonicity concerning the properties of the induced choice correspondence. That is, substituting Maximality for Monotonicity, the induced choice correspondence does not necessarily satisfy Sen's property β as the following example shows.

Consider the following situation, suppose $X = \{x, y, z\}$ and suppose that:

1. $x = c(\{x, y, z\}, f)$, $x = c(\{x, z\}, f)$, $x = c(\{x, y\}, f)$, and $z = c(\{z, y\}, f)$ for all the index functions where $f(x) \geq f(z)$ for all $z \neq x$;

2. $z = c(\{x, y, z\}, f)$, $z = c(\{x, z\}, f)$, $y = c(\{x, y\}, f)$, and $z = c(\{z, y\}, f)$ for all the index functions where $f(x) < f(z)$ for all $z \neq x$;

This indexed choice function satisfies the Monotonicity, Maximality, and Conditional IIA properties but the induced choice correspondence does not satisfy Sen's property β. In fact, we have that $\{x, y\} = C(\{x, y\})$ and that $y \notin C(\{x, y, z\})$.

2.4 Connections with other models

In this section we review two works closely related to our model. The first model was developed by Salant and Rubinstein (2008) and proposes a more abstract framework of choices conditioned to unobservable information. The second model is a model of social choice developed by Sen (1969) in which the author shows that, under some conditions, the aggregation of individual preference relations produces a quasi–transitive social preference relation, which is a result similar to the aggregation of constrained indexed choice functions.

The idea to attach an unobservable component to the choice set is not new. Indeed, Bernheim and Rangel (2008, 2009) and Salant and Rubinstein (2008) have developed a framework of choices with frames. The main intuition of Salant and Rubinstein is to attach to the class of choice problems (D) a class of frames called F. According to Salant and Rubinstein (2008), a *frame* is an abstract object that can reflect both observable ancillary information, such as the position of alternatives, and unobservable internal manipulation used by an agent in the choice process. Formally, an extended choice problem is defined as a pair (A, f) where $A \in D$ and $f \in F$ is the abstract object called *frame*. Accordingly, the extended choice function c^* is a function that assigns an element of A to every extended choice problem (A, f) and a standard choice correspondence induced by the extended choice function C_{c^*} where $C_{c^*}(A)$ is the set of elements chosen from the set A for some frame f (Salant and Rubinstein, 2008).

Salant and Rubinstein (2008) show that, if an extended choice function is a *Salient Consideration* function and it satisfies property γ–extended, then the standard choice correspondence induced by the extended choice function is indistinguishable from the choice produced by the maximisation of an asymmetric and transitive binary relation. In their model, an extended choice function is *Salient Consideration* if for every frame $f \in F$, there exists a corresponding ordering $\succ_f$ such that $c(A, f)$ is the $\succ_f$ –max(A). Moreover the the authors say that an extended choice function satisfies property γ–extended if $c(A, f) = x$ and $c(B, g) = x$, implies that there exists a frame h such that $c(A \cup B, h) = x$. Notice that there is a manifest relationship between the definitions proposed by Salant and Rubinstein (2008) and our definition and indeed, as we point out in the next paragraph, the two models are equivalent from a formal point of view.

In order to better understand the close relationship between the two models, note that a Salient Consideration function also satisfies Independence of Irrelevant Alternatives and a choice function that satisfies IIA is a Salient Consideration function. Hence, since a constrained indexed choice function satisfies Conditional IIA, then it is a Salient Consideration Function. Moreover, a constrained indexed choice function also satisfies property γ–extended. This can be easily seen by looking at the proof of Lemma 2, where we show that the choice correspondence induced by an indexed choice function satisfies Sen's property γ. Therefore, our model of choice with psychological index can be seen as a member of the bigger family of models defined by Salant and Rubinstein (2008). Indeed, the index function attached to the choice set can be thought as a *frame* in the model of choice with frames.

Although our model can be seen like a specification of the Salant and Rubinstein's model, it is worth considering that, if it is impossible to distinguish choices produced by the maximisation of an asymmetric and transitive relation—i.e., the optimisation of a complete and quasi–transitive relation—and choices induced by a Salient Consideration function satisfying property γ–extended, then it is also impossible to distinguish them from choices induced by a constrained indexed choice function. In this situation, whenever the frame is unobservable, one can interpret choices “as if” there was a measure attached to each alternative in the choice set that can affect choices according to the Monotonicity property. That is, if choices can be interpreted by using Salant and Rubinstein (2008)'s model you can always interpret the same choices in light of our model. Notice, however, that this is just an interpretation of the choice behaviour; the decision maker does not necessarily choose according to a measure attached to the alternatives, but he chooses "as if" that was the case. Moreover, even if a constrained indexed choice function which induces a choice correspondence compatible with the observed behaviour may exist, it may be the case that it is impossible to provide a meaningful interpretation to the indexes attached to the alternatives. On the other hand, there are psychological models in the literature that indeed use unobservable psychological measures in order to explain behaviour, e.g., availability or salience, and hence if it is supposed that these measures have a monotonic effect on choices, then the model of choices with psychological indexes is well-suited for showing that such a behaviour is not far from rationality.

The model about social choices by Sen (1969) is closely linked with both our and Salant and Rubinstein (2008) models. The author shows that it is possible to find an aggregation of complete and transitive individual preference relations that results into a quasi–transitive social preference relation. In his model, Sen considers a complete and transitive relation R_i for each subject i in the community and then he proposes the following aggregation rule in order to build the social preference relation S:

$$xSy \Leftrightarrow \neg\left[(\forall i\ yR_ix) \wedge (\exists i \mid yP_ix)\right]$$

where R_i is the weak preference relation of agent i, P_i is the strict preference of agent i, and S is the social preference relation. Notice that the meaning of this aggregation procedures is that x is socially excluded from being chosen by y—i.e., x is not socially weakly preferred to y—only if all the members of the community weakly prefers y to x and there is someone in the community that strictly prefers y to x. Using this aggregation procedure, the author shows that the social preference relation S is a complete and quasi–transitive relation whose maximisation produces a social choice satisfying Arrow's conditions (see Sen, 1969, p. 386).

Concerning our model, the most interesting aspect of Sen (1969) is that there is an aggregation of complete and transitive individual preference that produces a complete and quasi–transitive social choice relation. Indeed, since the Conditional IIA property implies that fixing the index function f one has a linear ordering $\succ_f$ representing preferences, one can interpret the final choice as the aggregation of preferences of a community of "multiple-selves" composed by the index functions. However, our aggregation procedure is different from the one used by Sen. Our procedure is motivated by the unobservability of the indexes—i.e., the unobservability of the voter that is in charge of taking the decision—while the aggregation used by Sen is a social decision function—i.e., a voting rule—used to produce a collective choice.

Notwithstanding, it may be interesting to compare the two aggregation procedures. Conditional IIA implies that for each index function f (i.e., for each voter) we have a linear ordering $\succ_f$, that is a complete, transitive, and antisymmetric preference relation. Antisymmetry implies that given $x, y \in X$ such that $x \neq y$, $x \succ_f y$ implies $\neg y \succ_f x$, thus $x \succ_f y$ implies yP_fx. Hence, by using the aggregation rule proposed by Sen we have that if $x \succ_f y$ for some index function f, then xSy where S is the social preference defined as $xSy \Leftrightarrow \neg\left[(\forall f\ yR_fx) \wedge (\exists f \mid yP_fx)\right]$. Consider now the revealed preference relation R constructed by using a choice correspondence $C()$ induced by an indexed choice function $c()$ that satisfies Conditional IIA, i.e., $xRy \Leftrightarrow x \in C(A) = \bigcup_{f\in\mathbb{R}^X} c(A, f) \wedge y \in A$. On the one hand, we have that if xRy then there exists a set $A \in D$ such that $x \in C(A) \wedge y \in A$ that means that there is an index function i such that $x \succ_i y$ and hence xSy. On the other hand, suppose that xSy, then there is an index function f such that $x \succ_f y$ and hence $x = c(\{x, y\}, f)$. Thus $x \in C(\{x, y\})$ that implies xRy. Hence, if the indexed choice function satisfies the Conditional IIA property—that is equivalent to have a population of linear orderings—it implies that the revealed preference relation R constructed using the induced choice correspondence is equivalent to the Sen's social preference relation S, and hence it is complete and quasi–transitive (see Sen, 1969, p. 287, Theorem V).

Thus it seems that the Conditional IIA is sufficient in order to get the quasi–transitivity of our revealed preference relation R, but notice that without the Monotonicity property we cannot show that $R-gr(A) = C(A) = \bigcup_{f \in \mathbb{R}^X} c(A, f)$. For this purpose, consider the following example satisfying Conditional IIA but not Monotonicity.

EXAMPLE 3. Consider the following situation, suppose $X = \{x, y, z\}$ and suppose that: (1) for all the index function $f \in \mathbb{R}^X$ such that $f(x) > 0$: $z = c(\{x, y, z\}, f)$, $z = c(\{x, z\}, f)$, $x = c(\{x, y\}, f)$, and $z = c(\{z, y\}, f)$; while (2) for all the index function $f \in \mathbb{R}^X$ such that $f(x) \leq 0$: $y = c(\{x, y, z\}, f)$, $x = c(\{x, z\}, f)$, $y = c(\{x, y\}, f)$, and $y = c(\{z, y\}, f)$. This indexed choice function satisfies the Conditional IIA property but not the Monotonicity property and it is easy to verify that in case (1) choices are produced by the maximisation of the following linear order $z \succ_f y$, $y \succ_f x$, $z \succ_f x$, $x \succ_f x$, $y \succ_f y$, and $z \succ_f z$; while in case (2) choices are produced by the maximisation of this linear order $y \succ_f x$, $x \succ_f z$, $y \succ_f z$, $x \succ_f x$, $y \succ_f y$, and $z \succ_f z$. Thus if we consider the induced choice correspondence $C()$ we have that $C(X) = \{z, y\}$ while the set $R-gr(X)$ is equal to $\{x, y, z\}$.

Example 3 shows that the two models are different in the sense that, even if we can interpret our choice procedure like an aggregation of Multiple Selves preferences, we require that the alternatives chosen by the maximisation of the social preference relation are equal to the union of the alternatives chosen by each "self" present in the decision maker. Moreover, notice that is not true that all the alternatives chosen by the Social Preference $S = R$ in a situation A are chosen by the decision maker. Indeed, we have a rule that decides which self is responsible for the decision according to the indexes attached to the alternatives. We have that, for each psychological situation, there is a "self" i that is the dictator of the decision maker, and only the unconditional behaviour can be interpreted as if it was an aggregation of preferences satisfying Arrow's conditions.[8]

[8] Notice that with Monotonicity property we indeed constrain the possible combination of individual orderings, hence not all the Arrow's conditions are satisfied. In particular condition

A final remark about the relationship between our model and models of social choices has to be made. Even if the multiple self interpretation looks appealing, it might be useful to point out that it is not the main interpretation of our model. Indeed, if we follow this interpretation, then the Monotonicity property will impose restrictions on the behaviour of the selves in the population. For instance, an implication of the Monotonicity property is that, if there are two selves i and j that choose the same alternative x from respectively the sets A and B, then there must exist a self u that chooses the alternative x from the set $A \cup B$. Clearly, this feature of the model is at odds with the idea of having a population of multiple selves. Indeed the idea of multiple selves implicitly assumes that the behaviour of one self is independent from the behaviour of another self.

2.5 Some welfare consideration

This section of the chapter discusses the issue of welfare analysis using behavioural models. Indeed, one of the main weaknesses of behavioural models is that they are well suited for a positive description of choice behaviour while they usually fail to provide normative guidance for welfare decisions.

One of the main difficulties of behavioural models is that they usually allow for preference reversal, i.e., the decision maker is willing to choose an alternative x from a set A under some circumstances while he is willing to choose y from A under some others—e.g., endowment effect, framing effect, status quo bias, etc.— These circumstances are usually not observable or are at least difficult to observe externally. Hence, the external observer cannot unambiguously determine if the individual will be better–off or not by switching from the alternative x to the alternative y. For instance, a social planner that has to decide between x and y for the individual cannot determine which alternative is preferred. The problem is even more severe in cases where the change in choices depends upon

of Unrestricted Domain of the aggregation function (see condition "U" Sen, 1969, p. 386) turns out to be violated. The Monotonicity property implies that if there are two individuals i and j that choose the same alternative x from respectively the sets A and B then there exists an individual u that chooses the alternative x from the set $A \cup B$.

the intervention of the social planner. Indeed, also by assuming that the social planner knows the subject is willing to choose x over y, the social planner probably has no basis for determining how the taste of the decision maker is going to be altered by an intervention. It could be the case that the decision maker has changed his mind and is now willing to choose y over x.

These considerations cast doubts on the possibility of developing normative welfare analysis in behavioural models. However, some authors (Bernheim and Rangel, 2008, 2009; Green and Hojman, 2007) have recently tried to overcome these shortcomings by providing a generalisation of welfare concepts that would allow welfare policies to be developed on the basis of behavioural models. Starting from the consideration that also standard welfare analysis is based on choices and not on utility, Bernheim and Rangel (2008, 2009) propose a revealed preference framework for welfare analysis which, in theory, is able to include all behavioural models. These authors define the notion of ancillary condition as "a feature of the choice environment that may affect behaviour, but is not taken as relevant to a social planner's evaluation" (Bernheim and Rangel, 2009, p. 55). Then they model a generalised choice situation as a subset of the universal collection of alternative X coupled with an ancillary condition—i.e., a pair (A, d) where A is a subset of X and d is an ancillary condition—and they let choices be dependent upon different ancillary conditions. Using this approach, Bernheim and Rangel (2009) are able to build individual welfare relations and to define the concept of individual welfare optima, thus showing that is possible to make basic welfare comparisons also without well–behaved preference relations or utility functions[9].

The model proposed in this chapter is similar to that of Bernheim and Rangel (2009) in interpreting the index function as an ancillary condition. Hence, following the same line of reasoning of Bernheim and Rangel (2009), it can be used to determine whether the decision maker is better–off by switching from one alternative to another. The model assumes that the external observer cannot know the psychological indexes the decision maker attaches to the alternatives and that

[9]In the article Bernheim and Rangel (2009) go further than this. They provide a generalisation for the concepts of equivalent and compensating variation and they also suggest a generalisation of the first welfare theorem. But this it is outside the scope of this section.

choices are conditioned upon the psychological situation the decision maker perceives. As shown above, the observable part of the decision maker's behaviour is given by the induced choice correspondence, which simply records all the alternatives chosen by the agent from a set A for all psychological situations. This implies that, for instance, when the external observer knows that x and y are chosen from $\{x, y\}$, he is unable to say whether the subject is currently willing to choose x over y or y over x. The external observer only knows that there are situations—i.e., some index function f—in which the subject is willing to choose x and other situations—another index function g—in which the subject is willing to choose y.

Despite this difficulty, the revealed preference relation R provides insights into the welfare of the decision maker. The relation R—defined as xRy iff $\exists A \subseteq X$ such that $x \in C(A)$ and $y \in A$, where $C(A) = \bigcup_{f \in \mathbb{R}^X} c(A, f)$—is based on the observable part of choices and thus it can be inferred by the external observer. In section 2.3, it was shown that the revealed preference relation has the property of rationalising the observable part of the choices, but it can also be used for deriving welfare implications. If we consider the asymmetric part P of the relation, i.e., xPy iff $xRy \wedge \neg yRx$, having xPy for some alternative x and y expresses the fact that the agent is always willing to choose x over y, independently from the psychological situation he faces. Thus, whenever the external observer can record that xPy, he can unambiguously conclude that the decision maker is better–off when he is given x instead of y. Hence, P can be used as an individual welfare relation, and it is also possible to define the concept of individual welfare optimum. Borrowing the definition of individual welfare optima by Bernheim and Rangel (2009), we can say that an alternative $x \in A$ is improvable when there is an alternative $y \in A$ such that yPx, and whenever this is not the case we can say that x is an individual welfare optimum in A[10].

[10]Note that the asymmetric part of our revealed preference relation R is equivalent to the welfare relation P^*—xP^*y iff, for all (A, d) such that $x, y \in A$, $y \in C(A, d)$ implies $x \in C(X, d)$—proposed by Bernheim and Rangel (2009). Note also that while the authors define the relation R'—$xR'y$ iff, for all (A, d) such that $x, y \in A$, $y \in C(A, d)$ implies $x \in C(X, d)$—and they use its asymmetric part when defining the concept of weak improvement, such relation is based on the observability of the ancillary conditions, which is ruled out in our setting.

Under these circumstances some basic welfare consideration can be drawn, but it is necessary to underline that, whenever two alternatives x and y are not ranked by the asymmetric part P of R, it is impossible for the external observer to decide whether the decision maker is willing to choose x over y or y over x. More precisely, the observer knows that there are psychological situations in which the decision maker is willing to choose x over y and other situations in which the subject is willing to choose y over x.

This consideration also helps in interpreting the symmetric part of the revealed preference relation R. Indeed, the symmetric part I of the revealed preference relation R is usually thought as revealing indifference between two alternatives, but in our model I can be given a different and more precise interpretation. Recalling the definition of I—i.e., xIy iff $xRy \wedge yRx$—xIy implies that there are situations in which the agent chooses x over y and situations in which he chooses y over x, but given a specific situation he is never indifferent between the two. Thus we can say that instead of revealing indifference, the relation I reveals the absence of an observable unambiguous preference ordering between the two alternatives. That is, I can be thought not to capture the indifference of the decision maker but the impossibility for the external observer to observe some relevant part of information.

Further insights can be gained by interpreting the model as a multiple–self model. This interpretation clarifies the concept of individual welfare optimum. Considering each index function as a distinct self we find that if two alternatives $x, y \in A$ belong to the items chosen from A—i.e., $x, y \in C(A)$—then they are not ranked by P and hence are both individual welfare optima in A. Nevertheless, these alternatives are also two Pareto equilibria for the society of multiple selves. Indeed, if we fix x, we have xIy for all the $y \in C(A)$, and is impossible to find an alternative in A that is preferred by all the selves; hence, it is also impossible to improve the well–being of one self without reducing the well-being of another self by moving from x. Consequently, without additional information about the indexes, the social planner cannot do better than randomly assigning one of the

Consequently we cannot speak about weak and strict welfare optima.

individual Pareto optima to the agent[11].

2.6 Concluding remarks

This chapter has developed a model in which choices are affected by some psychological elements assumed to be an unobservable measure that the decision maker attaches to each alternative in a universal collection. It has been shown that if choices conditioned by these measures satisfy two intuitively appealing properties, namely Monotonicity and Conditional IIA, then the observable part of the choice behaviour, i.e., the unconditional choices, can be interpreted as the product of the maximisation of a preference relation. Two related models were then examined and four interpretations of the measure attached to the alternatives were provided.

Before discussing some model's implications, a couple of remarks regarding the nature of the indexes deserve to be mentioned. A first issue concerning the indexes is related to their domain. In the discussion of the model the indexes are assumed to be real numbers. However, for some of the interpretations that have been provided, it seems reasonable to restrict the domain of the indexes. For instance, when interpreted as salience, the indexes can be sensibly restricted to the positive real. While, when interpreted as the number of decision maker's friends possessing the alternative, the indexes can be restricted to be natural numbers. These restrictions of the domain of the indexes do not alter the results of the model. The proofs of the propositions are still valid considering indexes that belong to the set of the natural numbers, or to convex subsets of the set of reals, or also to closed intervals of natural numbers.

Indeed, the propositions presented in the present chapter do not require to assume any cardinal measure attached to the alternatives. All the proofs are based only on ordinal comparisons of the indexes according to the greater or

[11] Note that this interpretation has to be taken with some reservations, we have already pointed out the difficulties of this interpretation at the end of the previous section and we want to stress the fact that this interpretation has to be considered a good tool of analysis but is not the main interpretation of the model.

equal relation. In the light of this consideration one can simplify the theory considering the index of each alternative as an element of a more general totally–ordered set $\langle S, \leq\rangle$ where $\leq$ is a linear order on S. That is, the index function f goes from X into S instead of the set of real numbers. However, despite the appeal of the more general ordinal approach, we decided to use real valued indexes in order to keep a closer relation between the formal development of the model and the interpretations of the indexes motivating the model. The case of Reason–Based choices and other interpretation based on aggregations of values (See, e.g., Bettman et al. (1998)), for instance, can be described and analysed more easily by means of a cardinal rather than an ordinal index.

Relatedly, there is the relationship between our model and the concept of fuzzy sets (Zadeh, 1965). Indeed, if the indexes are restricted to the unit interval, they can be interpreted as a fuzzy membership function. According to this interpretation, an index function f identifies a fuzzy set defined on the collection X of alternatives. Each index can be interpreted as the degree to which the associated alternative possesses a given property (i.e., the degree with which the alternative belongs to the set of alternatives defined by that property). Thus, in this case choices are defined on two sets: first, a standard set of available alternatives $A \subseteq X$ and, second, the fuzzy set of the alternatives f. This interpretation of the indexes, even though really appealing, presents some difficulties. In particular, it is unclear how to interpret changes in the indexes. Indeed, changes in the indexes can be interpreted in at least two ways: (a) as a change in the property identifying the set of alternatives; or (b) as the possibility to have changes in the degree to which the alternative belong to the set. Both interpretations, however, present important drawbacks: in case interpretation (a) is adopted, the meaning of Monotonicity becomes very difficult to interpret; in case interpretation (b) is considered, a fuzzy set is not univocally identified by its membership function.

To clarify this point, let consider the following example. Consider an universal collection X composed by apples and suppose that the fuzzy set of red apples is identified by the index function f. Consider then a change of the indexes from f to f'. If one adopts interpretation (a) this means that while f captures the property of being red, f' captures some other property, e.g., the property of being sweet. According to this interpretation, Monotonicity implies that, for instance, if x is chosen considering the fuzzy set of red apples, it has to be chosen also in all the fuzzy sets g where $f(x) \geq g(x)$ and $f(a) \leq g(a)$ for $a \neq x$ that is an absurd implication. If one adopts interpretation (b) one assumes that the degree to which an alternative belong to a fuzzy set can vary. That is, for some reasons the alternative x may be more or less red according to some other feature. If this solution solves the problems of interpreting Monotonicity, it produces other difficulties concerning the logic of functioning of fuzzy sets. Indeed, it would imply that the fuzzy set of red alternatives is not univocally identified by a membership function, and hence we have different sets identifying the same property.

Nonetheless, despite the difficulties of interpretation of Monotonicity, we think that the idea to consider the index function as a fuzzy membership function is worth to be further explored. For instance, one could explore the implications on the indexed choice function of the expansion and contraction properties that are used in classical choice theory when adapted to the union and intersection of fuzzy sets.

The last comment concerning the indexes is about the Monotonicity property. As one can observe from the proof of Lemma 4, the model could have been developed using only one of the two parts of Monotonicity, e.g., one could have considered only alternatives of type '$\uparrow$'. Indeed, the alternatives of type '$\downarrow$' can be transformed in alternatives of type '$\uparrow$' by multiplying their index by -1. Thus, for all the interpretations for which only one direction of monotonicity is required, such as, salience or perceived availability, the bidirectional formulation of the Monotonicity property is a complication of the model. However, when the interpretation of the indexes requires both the directions, e.g., Bandwagon and Snob effects, the adoption of both the directions simplifies the interpretation of the indexes. This because, if on the one hand the simpler formulation of Monotonicity reduces the complexity of the proof of lemma 2, on the other hand it implies different interpretations of the indexes for different alternatives: for the alternatives where "Bandwagon Effects" are present the index represents the popularity, while for those where the Snob effects are present the index represents the unpopularity of the alternative.

In conclusion two remarks about the implications of the model are due. The first remark is related to the rationality issue, while the second one regards the issue of the experimental testability of the model.

Starting with the first issue, the main result of the present chapter supports the idea that the decision maker's unconditional behaviour can be interpreted "as if" it was rational. However, the spirit of the chapter is not to pursue the idea that all the psychological phenomena can or should be rationalised, but to show that some of these phenomena can be treated by means of standard economic tools. The chapter is aimed to show that, whenever it is possible to have a sensible explanation of the indexes, e.g., in case of salience or perceived availability, it is possible to give to the choice behaviour an economic interpretation in terms of underlying preferences and to use the revealed preference relation to derive some basic welfare considerations.

Testability of the theory remains a more problematic issue; indeed, the unobservability of the indexes raises a major concern about testability. Choice axioms provide simple statements about choices that can be tested experimentally, but in our case choice behaviour is conditional to an unobservable component that cannot be controlled in an experimental setting. To make more clear the issue consider the following example: suppose an experimenter wants to test Conditional IIA. This person should first let the decision maker choose from a set of available alternatives and then let him choose from a smaller set in order to check if the subject sticks to the same choice or not. Suppose then that the experimenter observes a change in the chosen alternative, in this case he is unable to determine whether the change is due to a change in the psychological situation or to a violation of the Conditional IIA. The only way the experimenter can test Conditional IIA is to be sure to observe the choices from the smaller set for all the psychological situations. The previous example raises important concerns about testability. In order to test the model one have to make a strong assumption about the independence of indexes and sets of alternatives.

Another issue concerns the testability of Monotonicity, in this case the set of alternatives is kept fixed but one have to assume that the experimenter has some form of control over the indexes. Consider the perceived availability interpretation of the indexes. In this case one has to assume that manipulating the experimental situation it is possible to, e.g., reduce the perceived availability of the chosen alternative in order to check whether the decision maker sticks to that alternative. Thus also in this case we have to assume that the experimental manipulations change the psychological situation in the desired direction.

Notice, however, that the need of additional assumptions—although not strong as our assumptions—is not a peculiarity of our model, but is shared by a variety of other models. Testing the standard theory of intertemporal choices, for instance, requires some additional assumptions about the stability of preferences and income level in time.

Chapter 3

“Do What The Majority Do” Heuristic: A Choice Theoretic Analysis

3.1 Introduction

The idea of bounded rationality (Simon, 1955) suggests that human beings cannot produce optimal choices in many real life situations. The reason motivating this impossibility is double. On one hand, the limited computational power and memory of the human agent impede the evaluation of the consequences of all the possible alternatives one can choose, and on the other hand the complexity of the environment provides too many decision variables to take into account. Thus, according to Simon, the outcome of the decision process must be a satisficing one, that is the choice is the outcome of some decision heuristic that is shaped by both the complexity of the environment and the limits of the decision maker and that is aimed to produce satisficing but fast decisions (Gigerenzer and Todd, 1999).

In this chapter we focus on a very specific decision rule called “Do What the Majority Do” (DWMD hereafter) heuristic (Gigerenzer, 2004). The DWMD heuristic prescribes that, whenever the choice task is too difficult, the consequences of the alternatives are too complex to evaluate, or the subject is simply unsure of what to choose, he/she just looks at what the majority of his/her peers does and then he/she engages in the same behaviour. There are many cases of complex decisions in which people tend to imitate the behaviour of the majority of their peers. For example, when in a restaurant we do not know what to choose we often use a indecisiveness breaking rule by imitating the choice of the other people at the table. The same is true when we have to choose under some lack of information or lack of expertise. When buying a product we have no great experience with (e.g., a new laptop), we often survey our friends asking what you consider the best. Imitative behaviour has been already studied as a social learning tool among various animal species (see e.g., Laland (2001)) and, as an economic phenomena, by the early theoretical work on Bandwagon effects (Leibenstein, 1950) and the work on herding in finance (Devenow and Welch, 1996), but it has not received a formal choice theoretic treatment. Thus the aim of this chapter is, first, to give a formal content to the procedure described by the DWMD heuristic by formalising a choice function capturing the properties of the heuristic, and second, to determine what happens when a group of people is adopting this heuristic by looking at the other members' choices.

More precisely, by using the same formal modelling of Chapter 3, this chapter provides the axioms the choice function should satisfy in order to capture the idea of the DWMD heuristic and discusses the conditions of existence of a choice equilibrium for a multiagent environment in which the heuristic is adopted.

3.2 Choices and DWMD heuristic: a simple choice model

Let $N = \{0, 1, ..., n\}$ be a finite set of agents, and let X be a finite collection of alternatives. Let then $A_i \subseteq X$ be a subcollection of alternatives available to an agent $i \in N$. In order to formalise the dependency of agent's i choice by the choices of the set N of agents we use the same formulation used in the previous chapter which is based on the idea of choices with frames developed by Bernheim and Rangel (2008, 2009) and Salant and Rubinstein (2008). The basic idea of these models is to condition the choice of each agent not only to the set of available alternatives, but also to an additional component. In the terminology of Salant and Rubinstein (2008) this additional component of the choice function is called *frame* and in our specific case the frame is the number of agent that choose each specific alternative in X. Thus the choice function $c_i()$ of agent i has two components: the set of available alternatives A_i, and the frame f_i that is a function that attaches a number $f_i(x) \in N$ to each alternative x in the universal collection X. More formally, we say that $c_i()$ is the *indexed choice function* of agent i and it is a function from $(P(X) - \emptyset) \times N^X$ to X such that $c_i(A_i, f_i) \in A_i$ for all $(A_i, f_i) \in (P(X) - \emptyset) \times N^X$.[1]

In our interpretation of the frames, the numbers $f_i(x)$ attached to the alternatives represent the numbers of agents choosing each alternative. For instance, if $f_i(x) = k$, then agent i is counting k agents in the group that are choosing the alternative x. The fact that choice behaviour is conditioned by the frame implies that, given two different allocations f_i and f_i', agent i can choose different alternatives from the same set of alternatives: $c_i(A_i, f_i) \neq c_i(A_i, f_i')$. Thus agent i can change his/her choice according to the number of people choosing each alternative.

So far, there are no restrictions on how choices depend upon the frame f_i. That is, given a set of alternatives A_i, agent i can choose different alternatives according to different indexes, so we need to impose some restrictions on the behaviour of the choice function in order to model the "Do What the Majority Do" heuristic behaviour. In order to do this, we start defining more precisely the contents of the heuristic and to characterise the heuristic's behavioural implications in terms of properties of the indexed choice function. Loosely speaking, the prescription of the "Do What the Majority Do" procedure is the following one: whenever there

[1]Notice that the set of frames is bigger than needed, in the sense that choice behaviour is defined also for situations that are impossible, e.g., the case in which each alternative is chosen by n individuals.

is a subset of alternatives the decision maker is unable to discriminate, he will break the indecision by choosing one alternative among those that are chosen by the highest number of people. The heuristic can be thought as the second stage of a two–stage procedure in which, at the first stage, the agent is identifying a subset of alternatives that are potentially good, and at the second stage, the agent is using the number of people choosing those alternatives to break the indecisiveness.

At this point we introduce and comment about two restrictions of the indexed choice function: the *Maximality property* and the *Monotonicity property*, that are aimed to capture the salient features of the heuristic.

3.2.1 Maximality property

The first restriction we introduce is the *Maximality Property*. This property captures the idea that if something is chosen, then it has to be chosen in all those situations where the number of people choosing it is strictly maximal.

Def (Maximality)**.** If $x = c_i(A_i, f_i)$ for some (A_i, f_i), then $x = c_i(A_i, f_i')$ for all f_i' such that $f_i'(x) > f_i'(z)\ \forall z \in X - \{x\}$.[2]

The rationale of this property is very simple. According to the heuristic rule, the number of people choosing each alternative is used as an indecisiveness breaking rule; thus, the heuristic is used only when there is a subset of alternatives that the individual is unable to rank, i.e., there are two or more alternatives that are choosable and the decision maker is unable to decide which one is better. The underlying idea is that, if an alternative x is chosen from a set A_i for some f_i this implies that this alternative is among those alternatives the decision maker deems choosable, and hence it has to be selected by the application of the heuristic when competing with other choosable alternatives possessing a smaller number

[2]Notice that this definition requires the strict inequality sign. Adopting the weak inequality, one obtains a contradiction with single–valuedness of the indexed choice function. Indeed, if $x = c_i(A_i, f_i)$ and $y = c_i(A_i, f_i')$ then Maximality with weak inequality implies both $x = c_i(A_i, g_i)$ and $y = c_i(A_i, g_i)$ for all g_i such that $g_i(x) = g_i(y) \geq g_i(z)\ \forall z \in X$. A contradiction.

of choosers.[3] Some other observations about this property are proposed below.

- The first observation is that, keeping the set of available alternatives A_i fixed, if something is not chosen when its index is maximal, then it is never chosen. This is simply the converse of the property, but it helps understand how this property is consistent with the idea of the DWMD heuristic. Indeed, if the agent does not choose an alternative x when it is chosen by the majority of people, he/she does not need to break the indecision. He/she has deemed the x as unwanted and hence there are no reasons why he/she has to choose x when the majority of people is choosing something else.

- The second observation is that, keeping the set of available alternatives A_i fixed, and having that x or y are both chosen for some index function, then the property implies that they have to be chosen just when their index are strictly bigger than the indexes of all the other alternatives. Thus, in case $f_i(x) = f_i(y) > f_i(z)$ for all z distinct from x and y, the property tells nothing about what alternative has to be chosen. In this case, an alternative different from both y and x can be chosen. Thus, this situation is at odd with the behavioural implications of the DWMD heuristic.

- A third observation comes from the scope of the property. The property tells us that, if something is chosen, it has to be chosen when its index is strictly bigger than the indexes of all the alternatives in the collection. Thus, the property has no implications for those situations in which an unavailable alternative z has an index higher than that of x. That is, the index of x can be maximal in A_i but there can be an alternative z in $X - A_i$ possessing an higher index. In this case, the property is silent concerning the behaviour of the indexed choice function.

[3] Notice that the implicit assumption is that the selection process at the first stage is independent by the number of people choosing each alternative, i.e., the number of people choosing each alternative matters only at the second stage.

According to these observations, the maximality property is implied by the definition of the "Do what the majority do" heuristic, but the converse is not true. We need to strengthen the constraints on the behaviour of the indexed choice function to fully capture the implications of the heuristic. We do this by introducing the next property.

3.2.2 Monotonicity property

The second property we introduce is called monotonicity and is inspired by the Monotonicity property presented in chapter 3. The property assumes that, if the number of people choosing a chosen alternative increases or the number of people choosing a non-chosen alternative decreases, then the individual has no reasons to change his choice. More formally:

Def (Monotonicity)**.** If $x = c_i(B_i, f_i) \Rightarrow \forall f_i'$ such that $f_i'(x) \geq f_i(x)$ and $f_i'(z) = f_i(z)$ $\forall z \in X - \{x\}$, $x = c_i(B_i, f_i')$; and

If $x \neq c_i(B_i, f_i) = y \Rightarrow \forall f_i'$ such that $f_i'(x) \leq f_i(x)$ and $f_i'(z) = f_i(z)$ $\forall z \in X - \{x\}$, $y = c_i(B_i, f_i')$.

The rationale of this property is that, if something is chosen (in particular when it is not maximal), then there are no reasons to switch from the current alternative to another one when the relative popularity of the current alternative is increasing. That is, if some people in the group change alternative and pick the alternative the agent is currently choosing, there are no reasons for the agent to choose something else. Suppose that the agent chooses x from A given the frame f. Then x is deemed choosable and possesses the highest index among the alternatives that are deemed choosable (else something else would have been chosen). Thus, an increase in the number of people choosing x and/or a reduction in the number of people choosing another alternative should not affect the decision of the agent. The alternative x is still the one with the highest index among the choosable ones.

Maximality and Monotonicity together have an important implication that is summarised by the following lemma.

Lemma 5. *If* $x = c_i(A_i, f_i)$ *and* $f_i(y) > f_i(x)$ *then* $y \neq c_i(A_i, f_i')$ *for all* f_i'.

Proof. Monotonicity (plus Maximality) implies that if x is chosen when (A_i, f_i) and $f_i(y) > f_i(x)$, then y cannot be chosen from A_i. This is a stronger result than the one obtained with maximality alone. Suppose not and let $y = c_i(A_i, f_i')$, then consider the following frame f_i'': $f_i''(y) = f_i(y)$ and $f_i''(z) = min(f_i(x), f_i'(z))\ \forall z \neq y$. By maximality, we have that $y = c_i(A_i, f_i'')$ and for monotonicity we have $x = c_i(A_i, f_i'')$, a contradiction. The idea is that, if starting from f_i, the indexes of all the alternatives $z \neq y$ such that $f_i(z) > f_i(x)$ are lowered to $f_i(x)$, then y is maximal, thus it has to be chosen. But this violates monotonicity.[4] $\square$

Lemma 5 implies that, whenever something is chosen and it is not maximal, then all the alternatives possessing an higher index are never chosen. This is perfectly in line with the spirit of the heuristic. Indeed, the chosen alternative x must have the highest index among the choosable ones. Thus, if someting having an index strictly higher than the one attached to x is choosable, it should be chosen instead of x. Hence, nothing having an index strictly higher than x can be choosable.

Notice that the addition of the Monotonicity property solves also the problems of ties. Suppose x and y are chosen from A and suppose f_i such that $f_i(x) = f_i(y) > f_i(z)\ \forall z \neq y \neq x$. In this case the previous implications lead to the conclusion that neither x nor y can be chosen from A. Thus, if two alternatives x and y are chosen, then for all the situations in which $f_i(x) = f_i(y)$ is maximal, one of the two must be chosen.

Notice also that Monotonicity implies that the number of people choosing non–available alternatives, i.e., alternatives that are not in the choice set, does not affect the choice of the agent.[5]

So far, we introduced two properties that capture all the behavioural implications of the heuristic. With lemma 5 we point out that the choice of one of

[4] Because of the definition of Monotonicity, in order to make the argument more formal, one should have defined a sequence of frames $\{f_i^k\}_k$ starting from f_i and reaching f_i'' such that at each step k Monotonicity implies that $x = c_i(A_i, f_i^k)$.

[5] See Lemma 1 in Chapter 3.

the choosable alternatives having the highest index is completely captured by Maximality and Monotonicity. Now we focus on the independence of the two properties and then we move to the main question of the chapter: what happens if a group of people uses the "Do What the Majority Do" heuristic?.

3.2.3 Independence of Monotonicity and Maximality

The independence of the two properties is not intuitive, but we provide two examples showing that this is the case.

Maximality and not Monotonicity. This first case exploits the fact that, in case of ties, maximality is silent about the behaviour of the choice function. Suppose $x = c_i(A_i, f_i)$, where $f_i(z) = k \ \forall z \in X - \{x\}$ and suppose $y = c_i(A_i, f_i')$ where $f_i'(y) < f_i(y) \wedge f_i(z) = k \ \forall z \neq y$. This case satisfies Maximality but not Monotonicity.

Monotonicity and not Maximality. This case exploits the fact that monotonicity is silent about the effect of an increase in the index of a non–chosen alternative. Suppose $x = c_i(A_i, f_i)$, where $f_i(x) > f_i(z) + 1 \ \forall z \in X$ and suppose $y = c_i(A_i, f_i')$ where $f_i'(z) = f_i(z) \forall z \neq y \wedge f_i'(y) = f_i(y) + 1$. This case satisfies Monotonicity but not Maximality. The alternative x is still maximal under f_i', but is not chosen.

3.3 Interacting when using the DWMD heuristic

So far we have proposed two properties, Maximality and Monotonicity, that capture the behavioural content of the DWMD heuristic. Now we turn to the main question of the chapter and explore what happens when each person in a group of people behaves according to the heuristic. Indeed, in this situation, the choice of each agent depends upon the choices of the other agents in the group, and hence, if choices of the other members change, the agent may react by changing his chosen option, that, in turn, may trigger a subsequent counter–reactions by the other members. In this situation, nothing guarantees that at some point all the agents will be willing to stop changing their chosen alternative. In what follows we want to determine if there are conditions under which the choices of the agents stop changing. More precisely, we want to establish whether choices may reach an equilibrium or not and, in case they do, which are the conditions under which the equilibrium situation will be reached.

We start setting the structure of the multiagent model and defining formally what an equilibrium is. We consider an *interactive situation* as a triplet $\langle N, \{c_i\}_{i\in N}, \{A_i\}_{i\in N}\rangle$ where: $N = \{0, 1, ..., n\}$ is a finite set of agents, $c_i()$ is the indexed choice function of agent i and is assumed to satisfy both Monotonicity and Maximality, and A_i is a non-empty subset of the finite collection of alternatives X available to agent i. We define also the set $A_i^* = \{x \in A_i | \exists f \text{ s.t. } x = c_i(A_i, f)\}$, the subset of A_i of those alternatives that are chosen for some index function, as the set of the choosable alternatives of agent i.[6]

Before providing the definition of equilibrium in terms of choices it is necessary to discuss whether an agent should count or should not count his previously chosen alternative when determining the number of people choosing each alternative. That is, suppose $x = c_i(A_i, f_i)$ and suppose there are other k people in the group choosing x, then the issue is whether the "new" $f_i(x)$ should be k or $k + 1$. As a first step we study the implications of the former solution, i.e., not-counting one's own choice when calculating the number of people choosing each alternative.

Def (Equilibrium (Not–counting one's choice))**.** Let $\langle N, \{c_i\}_{i\in N}, \{A_i\}_{i\in N}\rangle$ be an interactive situation, then we say that the vector of alternatives $\{x_i^*\}_{i\in N}$ is a *choice equilibrium* in which the agents do not count their own choices iff for each $i \in N$: $x_i^* = c_i(A_i, f_i)$ where $\forall x \in X : f_i(x) = \#(\{j \in N - \{i\} | x = x_j^*\})$.

This first definition of equilibrium says that an equilibrium occurs when each

[6]Notice that, in the terminology of chapter 2, the set of alternatives A_i^* is simply the choice correspondence $C_i(A_i)$ induced by the indexed choice function $c_i(A_i, f)$, but since we are not interested in exploring the effect of changes in the set of available alternatives A_i here we adopt this alternative formulation.

agent, after considering the alternatives possessed by the other people in the group, maintains the alternative he was given. The definition implies that the agent's choice does not directly depend on his own previous choice.

Now we present some results concerning the existence of an equilibrium. In particular we present an example showing that the existence of an equilibrium is not generally guaranteed.

Example 1. Consider a group of 3 people, $N = \{1, 2, 3\}$, an universal collection of three alternatives $X = \{x, y, z\}$ and let the sets of available alternatives be X for all the three agents, i.e., $A_1 = A_2 = A_3 = X$. Finally, let the choice functions of the three agents be the ones presented in Table 3.1. In this situation, for each possible combination of alternatives the three agents may be endowed with, there is always one agent that is willing to change the alternative he was endowed with. For instance, in the first row of Table 3.1, Agent 1 and 3 possess x, and Agent 2 possesses y. Notice that while Agents 1 and 3 are keeping their alternatives—$c(A_1, f_1') = x$ and $c(A_3, f_3') = x$—Agent 2 is willing to change his alternative from y to z. The same happens for each possible combination of initial choices of the three agents. Thus, in this illustrative situation there are no equilibria.

Example 1 shows that, with the equilibrium definition given above, the existence of an equilibrium of choices is not a general result. But still there are some special cases in which an equilibrium arises. The following cases provide such an example:

- If $\bigcap_{i \in N} A_i^* \neq \emptyset$, then there exists an equilibrium in which all the people choose an alternative $x \in \bigcap_{i \in N} A_i^*$. If all the agents are assigned x, then the index of x is obviously maximal and since x is chosen for some f, then for Max it is chosen also in this case. Notice that, if $\bigcap_{i \in N} A_i^*$ is not a singleton, then there are more than one equilibrium.

- If $A_i^* \cap A_j^* = \emptyset \; \forall i, j \in N, i \neq j$, then there exists an equilibrium in which all the players choose something different. This case is equivalent to consider all players choosing with the frame $f(x) = 0 \; \forall x \in X$. Indeed, Monotonicity,

x_1^*	x_2^*	x_3^*	$(f_1(x), f_1(y), f_1(z))$	$c(A_1, f_1)$	$(f_2(x), f_2(y), f_2(z))$	$c(A_2, f_2)$	$(f_3(x), f_3(y), f_3(z))$	$c(A_3, f_3)$
x	y	x	$(1, 1, 0)$	x	$(2, 0, 0)$	z	$(1, 1, 0)$	x
x	y	z	$(0, 1, 1)$	y	$(1, 0, 1)$	z	$(1, 1, 0)$	x
x	z	x	$(1, 0, 1)$	x	$(2, 0, 0)$	z	$(1, 0, 1)$	z
x	z	z	$(0, 0, 2)$	y	$(1, 0, 1)$	z	$(1, 0, 1)$	z
y	y	x	$(1, 1, 0)$	x	$(1, 1, 0)$	y	$(0, 2, 0)$	x
y	y	z	$(0, 1, 1)$	y	$(0, 1, 1)$	y	$(0, 2, 0)$	x
y	z	x	$(1, 0, 1)$	x	$(1, 1, 0)$	y	$(0, 1, 1)$	z
y	z	z	$(0, 0, 2)$	y	$(0, 1, 1)$	y	$(0, 1, 1)$	z

Table 3.1: An Example in which there are no equilibria.

In this example $N = \{1, 2, 3\}$, $X = \{x, y, z\}$, $A_1 = A_2 = A_3 = X$. The first three columns of the table list all the possible combinations of players' choices. Columns 4, 6, and 8 present the frame f_i', respectively, for Agent 1, Agent 2, and Agent 3 given the choices in columns 1–3. Columns 5, 7, and 9 present the outcome of choice function $c(A_i, f_i)$ respectively for agent 1, agent 2, and agent 3. Notice that these choice functions satisfy Monotonicity and Maximality, and, for each initial combination (Cols 1–3), there is always an agent that is willing to alter his/her choice.

allows for a reduction of the index of all the non–chosen alternatives without altering the choice of the agents.

So far, we have shown that, if the agents do not count their own choices, the existence of an equilibrium is not generally guaranteed. Now the next step is to explore the implications of counting one's choice. Thus, we introduce the following equilibrium definition.

Def (Equilibrium (counting one's choice))**.** Let $\langle N, \{c_i\}_{i\in N}, \{A_i\}_{i\in N}\rangle$ be an interactive situation, then we say that the vector of alternatives $\{x_i^*\}_{i\in N}$ is a *choice equilibrium* in which the agents count their own choices iff for each $i \in N$: $x_i^* = c_i(A_i, f_i)$ where $\forall x \in X : f_i(x) = \#(\{j \in N | x = x_j^*\})$.

The only difference between the two equilibrium concepts is that in the former the agents consider their previous choice while in the latter they don't. Some considerations about the implications of counting one's own choice are due before analysing the existence of equilibria using this second equilibrium definition.

Concerning the behavioural implication of counting one's own choice, these are very similar to a status quo maintenance strategy. In particular, counting one's own choice has effect only when facing ties in the maximal index of the alternatives: if an agent is currently choosing x, the only difference between counting and not counting his own choice is an increase by one in the index of x and this modifies the set of elements possessing the highest index only in two cases.

The first case is when there is a tie between x and another choosable alternative y when counting only the other people's choice. If the indexes of x and y are maximal in A_i^* then, in virtue of Lemma 5, one of the two alternatives should be chosen. In this case, counting one's choice makes the index of x strictly maximal and therefore Maximality implies that x should be chosen again. Thus counting one's choice implies a Status Quo Maintenance strategy in case of multiple alternatives possessing the highest index.

The second case is when the index of x and the index of y are both maximal after having considered one's own choice. In this case counting only the other people's choices would make the index of x strictly smaller than the index of y and hence x will never be chosen in that situation. Counting one's choice, instead, allows x to be added to the set of the possible choices. This does not mean that it has to be chosen, but that Maximality and Monotonicity do not exclude it from being a possible choice given that indexes. The second case shows that counting one's own choice has more consequences than a simple Status Quo Maintenance strategy.

The most important implication of counting one's own choice is the former. As we will see the first implication—i.e., the Status Quo Maintenance—is sufficient for proving the existence of an equilibrium, and possesses also a good behavioural rationale. As Mandler (2004, 2005) points out maintaining the status quo can be a very efficient behavioural strategy in case of incomplete preferences. The author shows that, under some general conditions, the Status Quo Maintenance prevents the agent to fall into money pump traps. In order to see this consider the following example.

Example 2. Consider a set of alternatives $A = \{x, y, z\}$ and let the set of choosable alternatives be determined by the maximisation of a Partial Order $\succ$ such that $x \succ y$ but x and z, and y and z are unranked. Then $\{x, z\}^* = \{x, z\}$ and $\{z, y\}^* = \{z, y\}$. Consider a situation in which the agent does not count his/her previous choice, suppose a frame f such that $f_i(x) = f_i(y) = f_i(z)$, and we let $z = c_i(\{x, z\}, f_i)$ and $y = c_i(\{z, y\}, f_i)$.

Give now to the agent the possibility to keep the previously chosen alternative and present him $\{x, z\}$ as a first set of options keeping fixed the frame f. Given the set and the frame he will choose z over x. Giving now to the agent the opportunity to keep the alternative z or to change for y he will pick y ending up with an alternative he considers inferior.

Notice that this is not possible when keeping the status quo: if z is chosen from $\{x, z\}$ and then the agent is faced with the set $\{y, z\}$, he must choose z over y. Indeed in that case the $f(z) > f(y)$. That is, the agent cannot incur in money pump traps.

Example 2 provides a strong reason supporting the Status Quo Maintenance under restrictive conditions: it is assumed that the choosable alternatives A_i^* are generated by a preference relation, the set A_i must be allowed to change, and the agent can keep the previously chosen alternative. However, the motivation for maintaining the status quo is less pressing in our case, where the set A_i^* does not need to be generated by a Partial Order and the set of alternatives is not allowed to change. But still, keeping an alternative that is already known can be a sensible rule of behaviour in case of boundedly rational agents. They may prefer not to change a known alternative for a new one considering that they are unable to rank the two.

Now we examine the general case and we provide a procedure for finding an equilibrium. The procedure is reported as a proof for the following theorem.

Theorem 2. *Let $\langle N, \{c_i\}_{i \in N}, \{A_i\}_{i \in N} \rangle$ be an interactive situation, then there exists an equilibrium $\{x_i^*\}_{i \in N}$ in which the agents count their own choice.*

Proof. In order to prove the theorem we need first to set an auxiliary definition. Let $\langle N', \{c_i\}_{i \in N}, \{A_i\}_{i \in N} \rangle$ be a subproblem of $\langle N, \{c_i\}_{i \in N}, \{A_i\}_{i \in N} \rangle$ iff $N' \subset N$.

Now we consider a procedure that assign to each agent i in N an alternative x'_i. Afterwards we will show that the allocation $\{x'_i\}_{i \in N}$ is indeed an equilibrium. We assign the alternatives by induction over t.

- Let $m^t(x) = \#(\{i \in N^t | x \in A^*_i\})$. That is $m^t(x)$ is the number of people in N^t that deem choosable the alternative x at step t.

- Let $x^t \in \{x \in X | \forall y \in X, m^t(x) \geq m^t(y)\}$. That is, x^t is one of the alternatives that are deemed choosable by the highest number of people at step t.

- Let $x'_i = x^t$ for all $i \in N^t$ such that $x^t \in A^*_i$.

- Let $\langle N^{t+1}, \{c_i\}_{i \in N^{t+1}}, \{A_i\}_{i \in N^{t+1}} \rangle$ be the subproblem of $\langle N^t, \{c_i\}_{i \in N^t}, \{A_i\}_{i \in N^t} \rangle$ such that $N^{t+1} = N^t - \{j \in N^t | x'_j = x^t\}$. This is simply the subproblem considering only the agents that did not receive an alternative so far.

- Finally, let $\langle N^0, \{c_i\}_{i \in N^0}, \{A_i\}_{i \in N^0} \rangle = \langle N, \{c_i\}_{i \in N}, \{A_i\}_{i \in N} \rangle$.

Notice that, by definion, if $x'_i \notin \{x^k\}_{k \leq t}$ then $\{x^k\}_{k \leq t} \cap A^*_i = \emptyset$. Thus if agent i has not received an alternative at step t, it means that he/she considers unchoosable the alternatives given so far. Notice that this has another implication, if $x \in \{x^k\}_{k \leq t}$ then $m^w(x) = 0$ for all $w > t$.

First of all we prove that this procedure assigns an alternative to every agent in N. Suppose not, and suppose the procedure does not assign any alternative to agent i. By non-emptiness of A^*_i, there is an alternative $x \in A^*_i$ and thus $m^t(x) \geq 1$. Moreover, because the procedure does not assign an alternative to agent i, at each step t there exists $y \in X$ such that $m^t(y) \geq m^t(x)$ and $y \notin A^*_i$. The contradiction follows from the fact that, for all $w > t$, if $y \in \{x^k\}_{k \leq t}$, then $m^w(y) = 0$, and from the fact that X is finite.

Now we prove that the allocation $\{x'_i\}_{i \in N}$ is an equilibrium. Let $f_i(x) = \#(\{j \in N | x'_j = x\})$ for all $x \in X$. We have to show that for each $i \in N$, $x'_i = c_i(A_i, f_i)$. Consider an arbitrary $i \in N$, then $x'_i = x^t$ for some t and, since both $x'_i \in A^*_i$ and $\{x^k\}_{k \leq t} \cap A^*_i = \emptyset$, we have $c_i(A_i, f_i) \notin \{x^k\}_{k \leq t}$. Thus, if $c_i(A_i, f_i) \neq x'_i$ there exists an alternative $y \in A^*_i$ such that $f_i(y) \geq f_i(x'_i)$ (because of Lemma 5). Alternative y must have been assigned to $f_i(y)$ agents at some step $t' > t$. Thus $m^{t'}(y) \geq m^t(x)$. Notice now that, for all the alternatives $x \in X$ if $t' > t$, then $m^t(x) \geq m^{t'}(x)$[7]. Thus $i \in N^t$, $i \notin N^{t'}$, and $y \in A^*_i$ imply $m^t(y) > m^{t'}(y)$. Consequently $m^t(y) > m^{t'}(y) \geq m^t(x)$ that implies agent i should have been assigned y at step t. A contradiction. This concludes the proof that $\{x'_i\}_{i \in N}$ is an equilibrium. □

The intuition of the proof is to assign the alternatives to the agents in a recursive way. At each step t, one chooses one alternative x that is deemed choosable by the highest number of people among those not already possessing an alternative—suppose k people. Then one assigns x to all of the people that deem x choosable and are without an alternative.

The agents receiving x do not want alternatives that have been assigned at previous steps—else they would already have an alternative. Moreover, they do not want to choose alternatives that are assigned at successive steps—in that case there would be an alternative y assigned to at least k people that must have been deemed choosable by some people possessing x. But in such a case at step t, y would have been distributed instead of x.

The recursive process obviously stops since the number of agents is finite, and at the final step all the agents will retain the assigned alternative.

So far we have shown that, if people are counting their own choices when determining the indexes of the alternatives, then an equilibrium in the choice of the group arises. The next step is to check whether, starting from an arbitrary alternative assigned to each individual, a dynamic process of reaction will converge to an equilibrium or not. That is, given an arbitrary initial situation, if people is continuously revising their chosen alternative according to what the members of the group do, will they ever reach an equilibrium.

Before doing this we want to comment on the relationship between our equilibrium concept and the standard definition of Nash Equilibrium. Specifically,

[7] Obviously the number of people that deems choosable x can only shrink after each step.

notice that the set of choice equilibria coincides with the set of Nash Equilibria of the following strategic game $\langle N, \{\succcurlyeq_i\}_{i\in N}, \{A_i\}_{i\in N}\rangle$ where for each $i \in N$, $\succcurlyeq_i$ is a preference relation over $A = \times_{j\in N} A_j$ defined as follows: $\succcurlyeq_i = (R_i^{-1})^c$ where $(a_i, a_{-i}) R_i (a_i', a_{-i})$ iff $a_i = c_i(A_i, f_i)$ for $f_i(x) = \#(\{j \in N | x = a_j\})$.[8] This obvious relationship comes from the fact that, fixing the set A_i and the choices of the other people, one can rationalise the choice of i from A_i by using the revealed preference relation. Although this result may suggest the appealing interpretation that choices are determined by a preference relation over the strategy profiles, this is not in the spirit of the chapter. Indeed, the DWMD heuristic builds on the idea that agents have difficulties when comparing alternatives and hence they use the DWMD rule, while the maximisation of preferences conditional to the behaviour of the group's members imply a very high ability to compare alternatives. In addition, if one allows for revealing different preferences for each subset of X and each frame f_i, everything is rationalizable by the maximisation of some preference relation (Sen, 1997; Kalai et al., 2002) and thus the interpretation looses a lot of appeal as an explanation of the agents' behaviour.

3.3.1 Choices always converge to an equilibrium

Now we turn to the question whether, starting from an arbitrarily assigned alternative, a process of choice revision will end up in an equilibrium.

Let $\langle N, \{c_i\}_{i\in N}, \{A_i\}_{i\in N}\rangle$ be an interactive situation. Let $\{c_i^0\}_{i\in N}$ be the initial situation at $t = 0$ in which each individual is given an alternative $c_i^0 \in A_i$ and let c_i^t be the alternative agent i chooses at step $t > 0$, i.e. $c_i^t = c_i(A_i, f_i^t)$. Then we show that by letting the agents revise their choices according to the frame $f_i^t(x) = \#\{j \in N | x = c_j^{t-1}\}$ the process will converge to an equilibrium, i.e, there exists $t \in \mathbb{N}$ such that, for all $i \in N$: $c_i(A_i, f_i^t) = c_i^{t-1}$.

Before going into the details of the proof we have to make an important consideration. Since each player considers his own choice at the previous step when calculating the frame at step t, from step $t = 1$ on, all the players share the

[8] That is $\succcurlyeq_i$ is the complement of the dual of R with respect to $A \times A$.

same frame f^t. Moreover starting with step $t = 1$ the choice c_i^1 of each player belongs to A_i^* and thus each player is possessing something he deems choosable. Notice that we would have obtained the same result if we started from an arbitrary frame $f_i^0 \in N^X$: the frame f_i^1 would have been the same for all i and c_i^1 will belong to A_i^* for all i.

The previous consideration helps to interpret the consequences of starting from an arbitrary initial endowment $\{c_i^0\}_{i \in N}$. That is, the situation obtained at step $t = 1$ by assigning an arbitrarily chosen initial alternative can be obtained both from endowing each player with that alternative—a sort of status quo or default alternative—or from supposing different initial beliefs about what the other players will choose—when starting from an arbitrary frame $\{f_i^0\}_{i \in N}$. Thus since at step $t = 1$ the two different interpretation converge to the same situation, the following analysis is valid for both the cases.

Given the previous consideration we can start directly with a frame f^0 common to all the agents without any loss of generality. The revision of the choices produces a sequence $\{f\}_t$ of frames, thus in what follows we will show that there is a $t^* \in \mathbb{N}$ such that for all the $t > t^*$: $f^t = f^{t^*}$. This is enough for proving that the revision process converges to an equilibrium. Obviously, if no changes are observed in the frame, no changes will be observed in the choice.

Theorem 3. *Let $\langle N, \{c_i\}_{i \in N}, \{A_i\}_{i \in N} \rangle$ be an interactive situation, let f^0 be the initial frame common to all agents, and let $\forall x \in X : f^{t+1}(x) = \#\{j \in N | x = c_j(A_j, f^t)\}$. Then there exists a $t^* \in \mathbb{N}$ such that the sequence $\{f\}_t$ is such that $f^t = f^{t^*}$ for all $t \geq t^*$.*

Proof. Consider the following sequences $\{s^k\}_t$ derived from the sequence of frames $\{f\}_t$ defined as follows:

- $\{s^k\}_t$ such that $s_t^k = \max\limits_{x \in X - \bigcup_{i=0}^{k-1} S_i} f^t(x)$;
- $M_k = \max\limits_{t > t_{k-1}} \{s^k\}_t$. That is M_k is the maximum of the sequence $\{s^k\}_t$;
- $t_k = \min\limits_{t > t_{k-1}} \{t \in \mathbb{N} | \{s^k\}_t = M_k\}$ is the smallest t such that the sequence $\{s^k\}_t$ reaches its maximum.

- $S_k = \{x \in X | f^{t_k} = M_{t_k}\}$;
- Finally let $S_0 = \emptyset$ and $t_0 = 1$

In order to make the notation less heavy let $c_i^t = c_i(A_i, f^t)$. Now we will show by induction that there exists t^* such that $\{f\}_t = \{f\}_{t^*}$ for all $t \geq t^*$.

[**k = 1**] We show that the sequence $\{s^1\}_t$ **is a non–decreasing sequence**. Suppose the sequence is not non–decreasing and let $s^1_{t+1} < s^1_t$ for some t. Consider then the set $S = \{x \in X | f^t(x) = s^1_t\}$. If $S = X$ we have an immediate contradiction with $s_{t+1} < s_t$. Thus $s_{t+1} < s_t$ implies that there is an agent i such that: $c_i^{t-1} \in S$, and $c_i^t \notin S$. But then, since c_i^t is chosen when it is not maximal, lemma 5 implies that $x = c_i^{t-1}$ should be never chosen, a contradiction. Thus the sequence $\{s^1\}_t$ is non–decreasing.

In order to show that the sequence $\{s^1\}_t$ **is bounded above** it is sufficient to recall that the number of agent is finite ad hence also the maximum number of agents choosing a particular alternative is finite, so the number $\max_{x \in X}(\#(\{i \in N | x \in A_i^*\}))$ bounds the sequence.

Finally $\{s^1\}_t$ **reaches a maximum** M_1 **in a finite numbers of steps** $t_1 \in \mathbb{N}$. This again because the number of frames is finite. Since the sequence is a non–decreasing sequence of natural numbers the only way for not reaching a maximum in a finite number of steps is to have an increase in the index after an infinite sequence of frames that is obviously impossible. Thus the sequence $\{s^1\}_t$ is non-decreasing and reaches a maximum M_1 at step $t_1 \in \mathbb{N}$.

Consider now the nonempty set $S_1 = \{x \in X | f^{t_1} = M_1\}$. We show that:

[**1**] if $c_i^{t_1-1} \notin S_1$ then $A_i^* \cap S_1 = \emptyset$.

[**2**] $\forall t \geq t_1$, $\forall i$ such that $c_i^{t_1-1} \in S_1$: $c_i^t \in S_1$.

[**3**] $\forall x \in S_1$, $\forall t > t_1$: $f^t(x) = f^{t_1}(x)$;

[**1**] Suppose $i \in N$ such that $x = c_i^{t_1-1} \notin S_1$ and $A_i^* \cap S_1 \neq \emptyset$. Obviously $x \notin S_1$ implies $f^{t_1}(x) < M_1$ and $x \in S_1$ implies $f^{t_1}(x) = M_1$. Thus $c_i^{t_1} \in S_1$ else $A_i^* \cap S_1 \neq$

$\emptyset$ and lemma 5 will be in contradiction—who is choosing something outside S_1 and deems choosable something in S_1 will move to it at t_1—Moreover, for all j such that $c_j^{t_1-1} \in S_1$: $c_j^{t_1} \in S_1$ else lemma 5 will imply a contradiction—who is choosing something in S_1 do not choose something outside S_1 at t_1. These two considerations imply that $f^{t_1+1}(x) > M_1$ for some $x \in S_1$, another contradiction. Thus if $c_i^{t_1-1} \notin S_1$ then $A_i^* \cap S_1 = \emptyset$.

[**2**] Let $T = \{t \geq t_1 | \exists i \in N$ such that $c_i^{t_1-1} \in S_1 \wedge c_i^t \notin S_1\}$. If T is empty then $f^t(x) = f^{t_1}(x)$ for all $x \in S_1$ and $t \geq t_1$. This because $T = \emptyset$ implies that the number of people choosing something in S_1 can only increase, but, if either this happens, or there is an alternative $x \in S_1$ such that $f^{t'}(x) < M_1$ for some $t' \geq t_1$, there is an alternative $y \in S_1$ such that $f^{t'}(y) > M_1$. A clear contradiction. It is important to note that T is empty when $S_1 = X$. If $T \neq \emptyset$ let t' be the minimal element in T. The non-emptiness of T implies that there exists $i \in N$ such that $c_i^{t'-1} \in S_1$ and $c_i^{t'} \notin S_1$. Moreover since at t' there is the first deviation from S_1 we have $\forall x \in S_1 : f^{t'}(x) = M_1$. Finally the set $S' = \{x \in X - S_1 | f^{t'} = M_1\}$ is non-empty—$c_i^{t'} \in S'$—else we would have a contradiction by lemma 5. Notice that, by [**1**] and because in t' there is the first deviation, for all j such that $c_j^{t'-1} \in S'$, we have $A_j^* \cap S_1 = \emptyset$. Hence $c_j^{t'} \in S'$. But this implies that there is an alternative $z \in S'$ such that $f^{t'+1} > M_1$ a contradiction. Thus, for all $t \geq t_1$, and all the agents i such that $c_i^{t_1-1} \in S_1$: $c_i^t \in S_1$.

[**3**] Obviously, the fact that for all $t \geq t_1$: $f^t(x) \leq M_1$ and $\forall i$ such that $c_i^{t_1-1} \in S_1$: $c_i^t \in S_1$ implies directly that $f^t(x) = f^{t_1}(x)$ for all $x \in S_1$.

[$\mathbf{k = q}$] Suppose now that for all $k \leq q$:

1. each subsequence $\{s^k\}_{t > t_{k-1}}$ of $\{s^k\}_t$ reaches a maximum M_k at step $t_k \in \mathbb{N}$;

2. if $c_i^{t_k-1} \notin \bigcup_{l=0}^{k} S_l$ then $A_i^* \cap S_k = \emptyset$;

3. $\forall t \geq t_k$, $\forall i$ such that $c_i^{t_k-1} \in S_k$: $c_i^t \in S_k$.

4. $\forall x \in S_k$, $\forall t > t_k$: $f^t(x) = f^{t_k}(x)$;

Notice that, by inductive assumption 2 and 3 we have that, for all $t \geq t_q$:

$$c_i^t \notin \bigcup_{l=0}^{q} S_l \Leftrightarrow A_i^* \cap \bigcup_{l=0}^{q} S_l = \emptyset \tag{3.1}$$

Obviously, if $A_i^* \cap \bigcup_{l=0}^{q} S_l = \emptyset$ then $c_i^t \notin \bigcup_{l=0}^{q} S_l$. For the other direction notice that $c_i^t \notin \bigcup_{l=0}^{q} S_l$ and inductive assumption 3 imply $\forall k \leq q$, $c_i^{t_k-1} \notin S_k$ and hence $\forall k \leq q : c_i^{t_k-1} \notin \bigcup_{l=0}^{k} S_l$ that implies $\forall k \leq q : A_i^* \cap S_k = \emptyset$ by inductive assumption 2.

First of all consider the case $\bigcup_{k=0}^{q} S_k = X$. In that case, by the inductive assumption 4 there exists $k \leq q$ such that $\forall x \in X$, $\forall t > t_k$: $f^t(x) = f^{t_k}(x)$ and this would have concluded the proof of the theorem. So suppose $X - \bigcup_{k=0}^{q} S_k \neq \emptyset$. Now we prove the inductive step for the four inductive assumptions.

[**Step 1**]. Fist we prove that the subsequence $\{s^{q+1}\}_{t>t_q}$ reaches a maximum M_{q+1} in the finite number of steps t_{q+1}. We show that the sequence is non-decreasing and bounded above. Suppose there is a $t > t_q$ such that $s_t^{q+1} > s_{t+1}^{q+1}$. In that case there exists $i \in N$ such that $c_i^{t-1} \in S = \{x \in X - \bigcup_{l=0}^{q} S_l | f^t(x) = s_t^{q+1}\}$, and $c_i^t \notin S$[9]. Notice also that $c_i^t \notin S \cup \bigcup_{l=0}^{q} S_l$ because of (3.1). If $S \cup \bigcup_{l=0}^{q} S_l = X$ we have a contradiction: there cannot be i such that $c_i^{t-1} \in S \wedge c_i^t \notin S$, thus let $X - S \cup \bigcup_{l=0}^{q} S_l \neq \emptyset$ and let $c_i^t \in X - (S \cup \bigcup_{l=0}^{q} S_l)$. Since $c_i^t \notin S$ then $f^t(c_i^t) < f^t(c_i^{t-1})$ and hence by lemma 5 c_i^{t-1} should never be chosen, a contradiction.

The subsequence $\{s^{q+1}\}_{t>t_q}$ is non decreasing and bounded above by M_q[10]. Finally it reaches a maximum M_{q+1} in the finite number of steps t_{q+1} because of

[9]Notice that: if $X - \bigcup_{k=0}^{q} S_k \neq \emptyset$ then $S = \{x \in X - \bigcup_{l=0}^{q} S_l | f^t(x) = s_t^{q+1}\}$ is non-empty by construction.

[10]Notice that: for all h such that $h > k$ and $X - \bigcup_{l=0}^{h-1} S_l \neq \emptyset$; and for all $t \geq t_k$: $s_t^k \geq s_t^h$. If not we would have a contradiction with M_k being the maximal attainable index in $X - \bigcup_{l=0}^{k-1} S_l$ for $t \geq t_k$.

the finiteness of the number the frames.

[**Step 2**]. Suppose now there exists $i \in N$ such that $c_i^{t_{q+1}-1} \notin \bigcup_{l=0}^{q+1} S_l$. We have to prove that $A_i^* \cap S_{q+1} = \emptyset$. So suppose not, suppose $A_i^* \cap S_{q+1} \neq \emptyset$. Then (3.1) and $c_i^{t_{q+1}-1} \notin \bigcup_{l=0}^{q+1} S_l$ imply that $A_i^* \cap \bigcup_{l=0}^{q} S_l = \emptyset$. This plus $A_i^* \cap S_{q+1} \neq \emptyset$ imply that, in t_{q+1}, the choosable alternative with the highest index is in S_{q+1}, so $c_i^{t_{q+1}} \in S_{q+1}$ by lemma 5. Moreover, always by (3.1), $\forall j$ such that $c_j^{t_{q+1}-1} \in S_{q+1}$: $A_i^* \cap \bigcup_{l=0}^{q} S_l = \emptyset$, thus $c_j^{t_{q+1}} \in S_{q+1}$ else lemma 5 implies a contradiction. These two considerations imply that $f^{t_{q+1}+1}(x) > M_{q+1}$ for some $x \in S_{q+1}$ that is a contradiction. Thus $c_i^{t_{q+1}-1} \notin \bigcup_{l=0}^{q+1} S_l$ then $A_i^* \cap S_{q+1} = \emptyset$.

[**Step 3**]. Now we have to prove that $\forall t \geq t_{q+1}$, $\forall i$ such that $c_i^{t_{q+1}-1} \in S_{q+1}$: $c_i^t \in S_{q+1}$. Let $T = \{t \geq t_{q+1} | \exists i \in N$ such that $c_i^{t_{q+1}-1} \in S_{q+1} \wedge c_i^t \notin S_{q+1}\}$. If T is empty then $f^t(x) = f^{t_{q+1}}(x)$ for all $x \in S_{q+1}$ and $t \geq t_{q+1}$. This because $T = \emptyset$ implies that the number of people choosing something in S_{q+1} can only increase, but, if either this happens, or there is an alternative $x \in S_{q+1}$ such that $f^{t'}(x) < M_{q+1}$ for some $t' \geq t_{q+1}$, there is an alternative $y \in S_{q+1}$ such that $f^{t'}(y) > M_{q+1}$. A clear contradiction. Notice that T is empty when $\bigcup_{l=0}^{q+1} S_l = X$. If $T \neq \emptyset$, let t' be the minimal element in T. The non-emptiness of T implies that there exists $i \in N$ such that $c_i^{t'-1} \in S_{q+1}$ and $c_i^{t'} \notin S_{q+1}$. Moreover since at t' there is the first deviation from S_{q+1} we have $\forall x \in S_{q+1} : f^{t'}(x) = M_{q+1}$. Finally the set $S' = \{x \in X - \bigcup_{l=0}^{q+1} S_l = X | f^{t'} = M_{q+1}\}$ is non-empty—$c_i^{t'} \in S'$—else we would have a contradiction either by lemma 5 or by (3.1). By (3.1), for all j such that $c_j^{t'-1} \in S'$ we have $A_j^* \cap \bigcup_{l=0}^{q} S_l = \emptyset$ and, because this is the first deviation $c_j^{t_{q+1}-1} \notin S_{q+1}$. Hence, $\forall j$ such that $c_j^{t'-1} \in S'$: $c_j^{t_{q+1}-1} \notin \bigcup_{l=0}^{q+1} S_l$ then $A_i^* \cap S_{q+1} = \emptyset$. The consequence is that $c_j^{t'} \in S'$. But this implies that there is an alternative $z \in S'$ such that $f^{t'+1} > M_{q+1}$ a contradiction. Thus for all $t \geq t_{q+1}$ and all the agents i such that $c_i^{t_{q+1}-1} \in S_{q+1}$: $c_i^t \in S_{q+1}$.

[**Step 4**]. Obviously, the fact that for all $t \geq t_{q+1}$: $f^t(x) \leq M_1$ and that $\forall i$ such that $c_i^{t_{q+1}-1} \in S_{q+1}$: $c_i^t \in S_{q+1}$ implies directly that $f^t(x) = f^{t_{q+1}}(x)$ for all

$x \in S_{q+1}$.

The very last step is to show that there exist a $k \in \mathbb{N}$ such that $\bigcup_{l=0}^{k} S_l = X$. The existence of such k is a consequence of the finiteness of X plus the fact that whenever $X - \bigcup_{l=0}^{k} S_l \neq \emptyset$ the set S_{k+1} is non-empty by construction, and hence at each step k the finite set of alternatives $X - \bigcup_{l=0}^{k} S_l \neq \emptyset$ is shrinking. Thus there cannot be an infinite sequence of non-empty sets. □

The previous theorem shows that, if the members of the group are arbitrarily endowed with an alternative or they choose starting from different beliefs about the number of people choosing each alternative, a process of choice revision leads to a situation of equilibrium in a finite number of steps.

Despite the length, the intuition of the proof is to exploit that, at a given a step t, who is choosing something inside the set of alternatives possessing the highest index cannot choose something outside the set, and who is choosing something outside the set and deems choosable something inside it, will jump in. Thus, at the next step, some of the alternatives in the set will have an index greater or equal than before. This argument plus the fact that the number of agents is finite implies that, at some step the sequence of the maximal indexes will reach a maximum. The rest of the proof shows that, from that point on, the indexes of the alternatives having the maximal index cannot change anymore. Hence, one can iterate the argument considering a new problem without the alternatives that have reached a fixed index.

3.4 Concluding remarks

In this chapter we explored the implications of a simple behavioural rule, the "Do What the Majority Do". This rule prescribes that, whenever the choice task is too difficult, the consequences of the alternatives are too complex to evaluate, or the subject is unsure about what to choose, he just looks at what the majority of his peers does and then he engages in the same behaviour. The first part of the chapter proposed a choice–theoretical implementation of the heuristic that has been modelled using the idea of indexed alternative proposed in chapter 3. Two properties that captures the behavioural implications of the DWMD heuristic have been discussed.

The second part of the chapter examined the implications of this rule when it is used by all the members of a group of individuals. The main result identify the conditions for the existence of an equilibrium of choices—i.e., of an allocation of alternatives such that the alternative assigned to each member of the group is also the alternative chosen by him given all the allocated alternatives. The chapter shows also that, under these conditions, if everybody reviews the choice according to the axioms, then an equilibrium is reached independently of the initial allocation of alternatives.

In conclusion of the chapter some comments about the properties of the equilibrium allocations deserve to be made. In particular, we want to comment about some interesting questions arising from the interpretation of the model as a voting situation. If one considers the interactive situation modelled in the chapter as a collective decision where people vote for the choice of alternative options, one may be interested in understanding whether the equilibrium allocations possess any consensus or majority reaching property. Regarding consensus reaching, an obvious condition for the existence of an equilibrium where unanimity is attained is the presence of an alternative x that all the agents deem choosable, i.e., $x \in \bigcap_{i \in N} A_i^*$. Notice that if this condition is not met, then there would be an agent i for which, either x is not considered choosable or x is not available to him ($x \notin A_i$) and, hence, it is impossible to reach unanimity. If instead the condition is met, the allocation of x to all the agents is indeed an equilibrium. Notice also that the condition is both sufficient and necessary for the existence of an unanimity equilibrium. The condition, however, does not imply that the equilibrium is either unique or that it is reached by a process of choice revision. For instance, if one consider the following situation $X = \{x_i\} \cup \{u\}$ where $i \in N$ and $A_i^* = \{x_i, u\}$, then both the allocation where u is given to all the agents and the allocation where x_i is assigned to agent i are equilibria.

Similar considerations can be made concerning the conditions for the existence of an alternative that is chosen by the majority of the agents. A necessary and sufficient condition for the existence of a majority equilibrium is the presence of an alternative x such that $\#(\{i \in N | x \in A_i^*\}) > N/2$. However, also in this case the equilibrium may be neither unique nor reached. A more interesting condition is the following one: if $x \in A_i^*$ for some $i \in N$, then $\#(\{i \in N | x \in A_i^*\}) > N/2$. This condition guarantees that an equilibrium where the majority of the agents chooses the same alternative is reached whenever there is an alternative that is relatively more popular than the others. That is, a process of choice revision converges to a majority equilibrium from all the situations f where the set $\{x \in X | f(x) \geq f(y)$ for all $y \in X\}$ is a singleton.

A different interpretation of the model is related to the issue of selecting a qualified elite from a group of individuals. Recent models of Elitist qualification (Ballester and García-Lapresta, 2008b,a) analyse various rules of sequential identification of an elite of individuals. In these models, the current elite has to designate the next one until the process reaches a stable selection. At the beginning of the process, each member i provides an assessment p_{ij} for each individual j in the group. Given the assessment profile $P = \{p_{ij}\}$ and an initial set of qualified people S the authors consider an aggregation function $v(S, P, k)$ that provides a synthetic evaluation of each agent k based on the opinions of the current elite S of members of the group. A new elite S' is then selected on the basis of the synthetic evaluation $v(S, P, k)$ and of a threshold α_S: an agent k is part of the new elite S' if $v(S, P, k) \geq \alpha_S$. These papers explore various hypothesis regarding the aggregation functions and provide conditions under which a stable elite can be reached.

Notice that also our model can be reinterpreted as a problem of elitist qualification. Indeed, one can equate the universal collection of alternatives X with the set of agents N and consider the choice function c_i as a very simple assessment function where each individual i can only indicate one member of the group as a candidate for the selection. According to this interpretation, the sum of the votes received by each agent can be seen as a natural aggregation function for the various opinions, and the new elite may be selected by setting a family of thresholds α_S. Thus by simply adding a family of thresholds α_S to the process of choice revision presented in theorem 3 one has a model of elitist selection that is always convergent.

Despite our model and the models of elitist qualification have important similarities, especially for what concerns the recursive process employed in theorem 3, they possess some distinctive features. While in the models proposed by Ballester and García-Lapresta (2008b,a) the assessments of the agents are encoded in the given assessment profile and remain constant during the recursive process of elite selection, in our model the assessment of each agent depends on the choices of all the members of the group, and hence it is changing during the process. Another difference is that, while in our model the assessments of all the individuals at each stage of the process are relevant to the selection of the new elite, in the models of elitist qualification only the assessments of the current elite are relevant. Concerning these points, allowing for changes in the assessment and in the set of agents whose assessment is relevant may be worth to be explored in future work.

The model presented in this chapter assumes that the set of alternatives available to each agent is fixed. However, the effects of changes in the set of available alternatives are an interesting object of future research. Indeed, since both Monotonicity and Maximality are silent about the behaviour of the choice function when the set of available alternatives varies, one can investigate the effects of including different expansion or contraction properties of the choice function while keeping both Monotonicity and Maximality. For example, one can include in the present model the Conditional IIA property used in Chapter 2 and analyse what happens when the set of available alternatives expands because a new item becomes available at a certain point of the choice–revision process.

A final comment refers to the choice of allowing agents to imitate people with sets of available alternatives that differ from their own. In the model the agents do not have any information about the composition of the others' sets, therefore, it is not relevant to the agents whether they have the same sets of the others or not. Assuming this lack of information is realistic, indeed, in many real economic situations where imitation is at work, people do not have that piece of information. For instance, when buying an Mp3 player, one can buy the model bought by the majority of his reference group without knowing their budget constraint.

Chapter 4

"Do What The Majority Do" heuristic over lotteries: An experimental test

4.1 Introduction

There are many social and economic situations in which we are influenced by what others around us do. Like many other animals, humans tend to use imitation as both a learning tool and a decision tool (Laland and Williams, 1998; Laland, 2001). The most common examples of the presence of imitative behaviour can be found when looking at everyday life. We often decide what to eat, buy, or do on the basis of how much the options are popular/widespread among the people we know. For instance, there is evidence that voters are influenced by exit polls, and they tend to vote for the candidate that is indicated as more probable winner (McAllister and Studlar, 1991). There is also evidence suggesting that the buy and sell recommendations of security analysts are positively influenced by the recommendations of the preceding two analysts (Welch, 2000).

As Bikhchandani et al. (1992) pointed out, there are many explanations of the insurgence of imitative behaviour. People may imitate because the behaviour of their peers can be informative about the consequences of the actions, or because of the presence of payoff externalities, or because of sanctions on deviants. Moreover, there may be other sources of imitative behaviour: imitation is also a powerful learning tool (Laland, 2004) and an indecisiveness breaking rule (Gigerenzer, 2004).

There is a long tradition in both the theoretical and the experimental literature studying imitation as a rational response to lack of precise information. The models studying informational cascades identify the conditions under which agents rationally disregard private information about the consequences of an action and choose to undertake the behaviour exhibited by agents that have chosen before them.[1] Besides this rational herding approach, there are other models that maintain the idea of non–rational herding. For instance, in the tradition of models of heuristic behaviour, Gigerenzer (2004) proposes the idea of the "Do What The Majority Do" heuristic. According to this rule of behaviour, when people are unable to decide what to do or unable to calculate the consequences of the available actions, they look at what other people do and engage in the behaviour displayed by the majority. Another rule studied by the same tradition of Fast and Frugal heuristics is the "Do What the Successful Do" that prescribes to imitate the behaviour of successful peers instead of the one exhibited by the majority (Laland, 2001). The Fast and Frugal approach to heuristic behaviour underlines the importance to study simple behavioural rules that provide a more realistic description of how boundedly rational decision makers cope with a complex environment.

This chapter focuses on the role of imitative behaviour as an indecisiveness breaking rule. Specifically, we experimentally test the choice–theoretic characterisation of imitative behaviour proposed in Chapter 4 of this thesis which provides a formal version of the "Do What The Majority Do" heuristic (hereafter DWMD). According to the heuristic, imitation is used to break indecision: whenever there is a subset of alternatives the decision maker is unable to discriminate among, he

[1] For a theoretical analysis of herding in finance see: Welch (1992); Bikhchandani et al. (1992); Banerjee (1992); Devenow and Welch (1996); Avery and Zemsky (1998); Goeree et al. (2007). For empirical analysis see:Welch (2000); Drehmann et al. (2005); Alevy et al. (2007); Goeree et al. (2007); Weizsäcker (2008)

will break the indecision by choosing one alternative among those chosen by the highest number of people. More precisely, the heuristic can be thought as a second stage of a two stage procedure in which, at the first stage, the agent identifies a subset of alternatives that are potentially good, i.e., are "deemed choosable", and at the second stage, uses the number of people choosing the alternatives to break indecisiveness.

The model proposed in the previous chapter considers a finite collection of alternatives (or actions) X and a finite set of agents N. According to the model, each agent i faces a subset of alternatives $A_i \subset X$ and chooses one alternative $c(A_i, f_i)$ from the set A_i depending on both the set A_i and the number $f_i(x)$ of people choosing each of the alternatives x in X. In this model, choices are a function not only of the set of available alternatives like in the traditional choice theoretic models, but also of the number of people $f_i(x)$ choosing each alternative x in the collection X.

In order to capture the behavioural implications of the DWMD heuristic, in Chapter 4 we constrained the behaviour of the choice function by imposing two axioms: the Maximality and the Monotonicity axiom.

Def (Maximality)**.** if $x = c_i(A_i, f_i)$ for some (A_i, f_i), then $x = c_i(A_i, f_i')$ for all f_i' such that $f_i'(x) > f_i(z)\ \forall z \in X - \{x\}$.

Def (Monotonicity)**.** If $x = c_i(B_i, f_i) \Rightarrow \forall f_i'$ such that $f_i'(x) \geq f_i(x)$ and $f_i'(z) = f_i(z)\ \forall z \in X - \{x\}$, $x = c_i(B_i, f_i')$ and;

If $x \neq c_i(B_i, f_i) = y \Rightarrow \forall f_i'$ such that $f_i'(x) \leq f_i(x)$ and $f_i'(z) = f_i(z)\ \forall z \in X - \{x\}$, $y = c_i(B_i, f_i')$.

Both the first and the second axiom capture the idea that, if an alternative is chosen, it means that it is the deemed choosable alternative with the highest index and hence, according to the two stage interpretation of the DWMD heuristic, the alternative must be chosen whenever it has the globally maximal index (Maximality) or when its index remains maximal among the deemed choosable alternatives (Monotonicity). More explicitly, the first axiom prescribes that if an alternative is chosen, it belongs to the deemed choosable alternatives and hence, it has to be chosen when the number of people choosing it is strictly higher than the number of people choosing all the other alternatives in X. The second axiom captures the idea that if an alternative is chosen, it has to be necessarily a maximal alternative among the deemed choosable ones, and hence it has to be chosen when its index increases or the index of another alternative decreases. This because the alternative still possesses the highest index among the deemed choosable alternatives.

This chapter is aimed to test the empirical validity of the two axioms proposed defined above. Since this model, like all the choice theoretic models, provides clear implications about the properties that choices should possess, it allows for a straightforward check of the congruence of choice with theory. The idea to test directly the properties of choices is not new. For instance, Andreoni and Miller (2002) adopted this approach to check the robustness of the GARP in other–regarding behaviour, and Harbaugh et al. (2001) tested the compatibility of children's' choices with the GARP. In our case we will experimentally check whether the choices of a group of people are influenced by what the members of the group do and whether this influence satisfy Maximality and Monotonicity.

A first issue of our test is the choice of the type of alternatives to use in the experimental task. According to the definition of the DWMD heuristic, indecisiveness is a necessary premise for the application of the heuristic rule (Gigerenzer, 2004). Indeed, if the agent were not indecise, he would not use the rule. Hence, in order to make the agent indecisive, we decided to use lotteries because they permit to select sets of alternatives difficult to compare. In particular, we have chosen lotteries that do not first order stochastically dominate each other and that have similar expected value and variance. The selection of such lotteries is motivated by the evidence supporting the idea that people are relatively good at avoiding first order stochastically dominated lotteries. Notice that, in the light of the choice model tested here, one can interpret the selection of alternatives that do not stochastically dominate each other like the assumption that the agents' preferences contain at least a partial ordering of the lotteries according to First Order Stochastic Dominance. That is, given two lotteries l_1 and l_2 such that l_1

stochastically dominates l_2, it is equivalent to assume that the agent prefers l_1 over l_2 and hence he excludes l_2 from the set of the deemed choosable alternatives whenever l_1 is present.

Moreover, if read through the lenses of standard Expected Utility Theory, the assumption that people discard stochastically dominated lotteries implies that the agents prefers more money than less, indeed the degenerate lotteries are all linearly ordered by stochastic dominance. However, assuming that the alternatives are only partially ordered by F.O. Dominance, one cannot discriminate between lotteries that are not dominating each other and hence attitudes toward risk become endogenously determined. This because the choice depends by both the set of available options and the choices of the other people in the group.

Another issue is about how the groups of people are formed. Indeed, the theory we want to test is silent about what group of persons influences choices. Thus, it may reasonably be true that only the choices of friends or people that one knows influence one's own choice, while choices of peers in an experiment do not influence one's decision in any way. Considering this, we decided to keep the sentiment of membership weak by randomly grouping participants to the experiment. This choice is motivated by the unpredictability of the effect of knowing the members of the group. Indeed, knowledge about the identity of the peers may trigger either imitation of the majority or imitation of some specific member, or contrarian behaviour, i.e., a snob effect. All these effects are ruled out by the model that assumes only knowledge about the overall diffusion of each alternative in the group. In a sense, the model assumes the mere knowledge of the overall demand for that good and not the knowledge about the identity of the people demanding the good.

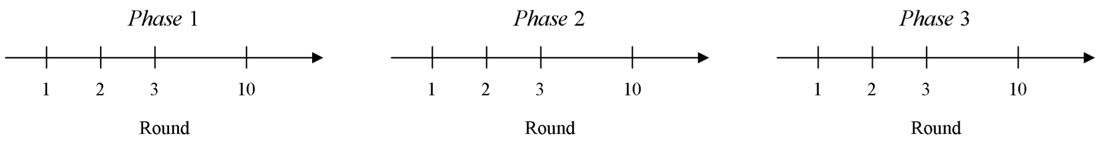
We investigate the presence of imitation in a simple choice task where no payoff externalities and informational structure are present because this simple environment is one of the least studied. As reported above, there is a considerable number of experiments addressing imitation as formation and stability of informational cascades, but, at the best of our knowledge, there are few studies tackling the phenomenon of herding in choice environments like the ones prescribed by the "Do What the Majority Do Heuristic". The most similar studies are the ones about bandwagon effects in elections that study whether exit–polls communications alter the choices of voters (McAllister and Studlar, 1991; Mehrabian, 1998).

4.2 Experimental design

The experiment is aimed to test whether the axioms proposed in Chapter 4 hold or not. Because of the nature of the model, the experimental task must involve a group of subjects each of which has to choose more than once from a given set of alternatives. While the use of groups is clearly motivated by the dependence of choices by what the other people do, the necessity to have multiple choices from the same set of alternatives depends on the nature of the axioms. Indeed, the axioms constrain the choice only given the premise that an alternative has been already chosen. Hence, in order to test an axiom one needs to observe at least two choices. Therefore, we decided to employ groups of 7 participants that face an experimental task with three choice phases.

In the first phase each member of the group has to choose one lottery from a set of 3 lotteries. In this phase subjects choose without having any information about the choices of the other members of the group. In the second phase the agents face the same set of lotteries they faced during the first phase, but they can be informed about the number of members of the group that have chosen each of the alternatives in phase one. In particular, subjects receive an index associated to each alternative. The indexes represent the number of group members that have chosen each alternative in the first phase, but the indexes associated to the alternatives may be either the true number of subjects choosing each alternative or some fictitious index previously determined by the experimenter. The participants are obviously informed of this possibility, but not of the procedure adopted by the experimenter for the determination of the fictitious indexes. In the third phase the subjects face for the third and last time the three lotteries they faced during the previous phases, and they may be informed about the number of group

Figure 4.1: Experiment timeline



The experiment is composed of 3 phases, each of which is composed of 10 rounds. In the same round of the three phases subjects face the same set of alternatives. Thus, for instance, subjects choose from the same set of lotteries in round 1 of Phase 1, round 1 of Phase 2, and round 1 of Phase 3. The differences between the phases concern the information provided to the subjects (i.e., the index function attached). In Phase 1 subjects choose without information about the choices of the group; in each round of Phase 2 they are informed about the choices of the group during the same round of Phase 1 (or they receive fictitious indexes), and in each round of Phase 3 they are informed about the choices of the group during the same round of Phase 2 (or they receive fictitious indexes).

members choosing each alternative in the previous phase. That is, subjects receive again a set of indexes that may be either the true number of group members choosing each alternative or some fictitious index previously determined by the experimenter.

To summarise, each subject faces the same sequence of 10 different situations, i.e., 10 different sets of lotteries L (defined as A_i in Chapter 3) for three times (phases). In each phase the 10 situations are associated with different index functions: in the first phase the subject chooses without any information about the choices of the other group members, while in the second and third phases he chooses knowing the number of group members that have chosen each alternative in the previous phase. Figure 4.1 sketches the timeline of the experimental task.

The lotteries used in the experiment were randomly selected from a set composed of 59 lotteries with the following properties:

- each lottery has 3 distinct outcomes belonging to the integers between 1 and 10 (1 and 10 included);

- the probability of each outcome belongs to the set $\{\frac{1}{10}, \frac{2}{10}, \frac{3}{10}, \frac{4}{10}, \frac{5}{10}, \frac{6}{10}, \frac{7}{10}, \frac{8}{10}\}$;
- the expected value is between 4.9 and 5.1;
- the variance is between 4 and 8.

Each set L of 3 lotteries has been drawn without replacement from the set defined above. After the drawing of the 3 lotteries, we checked if the lotteries first order stochastically dominate each other. If so, the 3 lotteries were dropped and another draw was done, else the 3 lotteries was kept. The selected lotteries are reported in Table 4.1.

The reason motivating the selection of the lotteries from this very specific set is to make the participants indecise about the choice of one of the 3 lotteries. According to the DWMD heuristic, imitation is used to break indecision, and, hence, a situation where the decision of what to choose is complicate is a crucial prerequisite. We assume that the use of lotteries with similar expected values, similar variances, and not stochastically dominating each other is a sufficient condition to produce a complex choice task.

To test Maximality or Monotonicity, it is necessary to observe for each participant, a minimum number of choices under the conditions prescribed by the axioms. However, if only true information about the number of people choosing each alternative were provided, it would be necessary to observe an exaggerated number of decisions to obtain an adequate sample of observations for each axiom. Indeed, given the size of the index space, nothing guarantees that the conditions prescribed by the axioms are realized within few trials. To overcome this problem, we employ a modified version of the Conditional Information Lottery (CIL hereafter; Bardsley, 2000; Bardsley and Moffatt, 2007). This technique makes use of dummy tasks that are aimed to test, without using deception, subjects' behaviour in situations that are unlikely to spontaneously occur during the experiment. According to the CIL a given task is repeated n times but in only one out of the n repetitions the true information about the task (e.g., about other participants' behaviour) is provided. Each participant is rewarded according to the choices made during the real task, but he does not know which task is the real

Table 4.1: Lotteries.

Round	Lottery	Outcome (€)	Probability	Round	Lottery	Outcome (€)	Probability
		1	0.1			1	0.2
	l_1	2	0.3		l_1	5	0.5
		7	0.6			8	0.3
		3	0.5			3	0.5
1	l_2	6	0.2	6	l_2	5	0.3
		8	0.3			10	0.2
		3	0.4			4	0.8
	l_3	5	0.4		l_3	8	0.1
		9	0.2			10	0.1
		2	0.3			2	0.3
	l_1	6	0.5		l_1	6	0.5
		7	0.2			7	0.2
		3	0.5			3	0.5
2	l_2	6	0.3	7	l_2	6	0.2
		8	0.2			8	0.3
		2	0.1			3	0.4
	l_3	4	0.7		l_3	5	0.4
		10	0.2			9	0.2
		1	0.3			1	0.3
	l_1	4	0.1		l_1	6	0.3
		7	0.6			7	0.4
		2	0.2			3	0.6
3	l_2	4	0.5	8	l_2	5	0.1
		9	0.3			9	0.3
		1	0.2			3	0.6
	l_3	5	0.6		l_3	7	0.3
		9	0.2			10	0.1
		2	0.3			2	0.1
	l_1	5	0.2		l_1	3	0.4
		7	0.5			7	0.5
		2	0.1			3	0.6
4	l_2	4	0.7	9	l_2	7	0.2
		10	0.2			9	0.2
		2	0.2			3	0.6
	l_3	5	0.7		l_3	8	0.3
		10	0.1			9	0.1
		2	0.1			3	0.6
	l_1	3	0.5		l_1	5	0.1
		8	0.4			9	0.3
		2	0.4			3	0.3
5	l_2	5	0.2	10	l_2	4	0.5
		8	0.4			10	0.2
		3	0.6			3	0.5
	l_3	5	0.1		l_3	5	0.3
		9	0.3			10	0.2

one. This is equivalent to implement a lottery over the information provided to the agents. Notice that this technique is a combination of the Strategy Method and the Random Lottery designs. The former is used in strategic interaction experiments in which agents are asked to provide a strategy for each possible action chosen by their partners. Payoffs are determined by the intersection of the strategies provided by the participants. The latter is used in individual decision making experiments in which subjects are asked to complete a sequence of tasks and only one randomly selected task is payoff relevant. The CIL mixes the two designs since it requires subjects to provide a strategy for a subset of possible situations that are of interest to the experimenter and randomly selects the task in which true information is provided. The payoff relevant task is the one in which true information is provided, but it remains unknown to the subjects in order to incentivize revelation of truthful behaviour in all the tasks.

Bardsley (2000) discusses the theoretical validity of the CIL design and shows that in Public Good Games it produces results that are in line with typical results obtained in these games. The author points out how the CIL design allows the observation of behaviour in unlikely histories of the game keeping both the number of subjects and the number of repetitions low.

In order to improve its effectiveness in our particular task, we modify the CIL design in two aspects. First, instead of providing true information in only one task, we provide it in half of the tasks. A low probability of having true information, in addition to the fact that this piece of information is payoff irrelevant, could lead participants to consider it meaningless. Increasing the probability of facing true information should enhance the credibility of the information provided. Notice however that, in case this modification were not effective, it would not have any effects, neither in favour of nor against our model. Indeed, if they are not convinced that the indexes are informative, subjects are expected to disregard the indexes and focus only on the lotteries.

Second, the payoff relevant decisions are randomly drawn from the set of all the tasks instead of being drawn from the set of the tasks where true information is provided. In the CIL design only the task with true information is paid; indeed, if tasks with fictitious information were paid, the experimenter could directly determine the final payoff of the subjects, hence distorting the incentive scheme. Notice that, the same distortions are not present in our experiment since indexes do not determine directly the subjects' payoffs.

As already mentioned, participants are asked to choose more than once from each set of alternatives. Indeed if one wants to test Maximality or Monotonicity one has to observe the choices of the participants in the conditions prescribed by the axioms. The following subsections discuss in detail how the test of the axioms is implemented.

4.2.1 Test of the Maximality axiom

According to the definition of Maximality, if something is chosen, then it has to be chosen whenever it is chosen by the highest number of people. So in order to test Maximality, one has to know that an alternative, say l, is chosen, and then, one has to propose to the participants a situation in which l possesses the strictly maximal index, i.e., is chosen by the highest number of people.

Thus, the test of the axiom requires at least two choices from the same set of lotteries but with different indexes. The choices of phase 1 and phase 2 are aimed at this: in phase 1, participants choose without any information about other participants' choices; in phase 2, they choose either knowing the actual number of members of the group choosing each alternative or knowing a fictitious set of numbers. In this case the fictitious numbers attached to the alternatives are determined in such a way to guarantee the test of Maximality for each player.[2]

Specifically, in rounds 1, 3, and 10 of phase 2 each player receives a different set of indexes. These indexes are determined so that the index of the alternative that has been chosen by the subject from the same set during the previous phase is strictly higher than the index of the other alternatives belonging to the set. For

[2]Notice that the choice during phase 1, i.e., without indexes, can be interpreted either as a choice with index 0 attached to each alternative or as a choice made according to some belief regarding the indexes. However, this is unimportant for the test of the Maximality axiom which only requires the lottery being chosen for some indexes.

instance, if a subject chooses l_1 form $\{l_1, l_2, l_3\}$ in the first phase in the second phase the index of l_1 will be strictly greater than the indexes of l_2 and l_3 despite phase one's choices of the other members of the group. Table 4.2 reports the indexes proposed to the subjects in rounds 1, 3, and 10. If Maximality holds, during round 1, 3, and 10, subjects should confirm the choice made in the previous phase when facing the same set of alternatives.

Notice that, since the independence of choices from the indexes is compatible with the axioms, if the participant chooses the same alternative in all the 3 phases it is impossible to discriminate between the model tested here and the more parsimonious model where choices depend only upon the set of alternatives and not upon the choices of the other participants. Indeed, according to the two stage interpretation of procedure, if the participant always chooses the same alternative, he is supposed to have clear preferences regarding the alternatives in the set, and he does not need to break indecision applying the second stage of the procedure, i.e., there is no need to apply the heuristic.

In order to tackle this problem we introduced some situations for which the behavioural implications of the two models can be different. The indexes presented to the subjects during rounds 2 and 5 of phase 2 were aimed to check the presence of choices compatible with Maximality and not with the more standard model of choice. In those rounds, like in rounds 1, 3, and 10, each player receives a different set of indexes, but in this case the alternative chosen by the subject during the previous phase does not receive the maximal index. On the contrary, the highest index is assigned to an alternative that has not been chosen during the first phase. For example, suppose the subject chooses the lottery l_1 from $L = \{l_1, l_2, l_3\}$ in phase 1, then the index attached to l_1 in the second phase will be lower than the index of l_2 or l_3 despite phase one's choice of the other members of the group. Table 4.2 presents the indexes proposed to the subjects in rounds 2 and 5 of phase 2. In those two rounds the choice of any alternative is compatible with Maximality, while only a confirmation of the previous choice is compatible with independence of choices from the indexes.

Notice however, that the alternative chosen in phase 2 implies some constraints

Table 4.2: Fictitious indexes.

Round 1

		Phase 1 l_1	Phase 1 l_2	Phase 1 l_3
Phase 2	l_1	[4,2,1] [6,0,1]	[2,4,1] [4,2,1]	[1,2,4] [3,0,4]
Phase 2	l_2	[4,2,1] [3,4,0]	[2,4,1] [1,6,0]	[1,2,4] [1,4,2]
Phase 2	l_3	[4,2,1] [2,2,3]	[2,4,1] [0,4,3]	[1,2,4] [0,1,6]

Round 2

		Phase 1 l_1	Phase 1 l_2	Phase 1 l_3
Phase 2	l_1	[1,5,1] [2,4,1]	[1,1,5] [2,0,4]	[5,1,1] [6,0,1]
Phase 2	l_2	[1,5,1] [1,6,0]	[1,1,5] [1,2,4]	[5,1,1] [5,2,0]
Phase 2	l_3	[1,5,1] [0,5,2]	[1,1,5] [0,1,6]	[5,1,1] [4,1,2]

Round 3

		Phase 1 l_1	Phase 1 l_2	Phase 1 l_3
Phase 2	l_1	[3,2,2] [4,1,2]	[2,3,2] [3,2,2]	[2,2,3] [3,1,3]
Phase 2	l_2	[3,2,2] [3,3,1]	[2,3,2] [2,4,1]	[2,2,3] [2,3,2]
Phase 2	l_3	[3,2,2] [2,2,3]	[2,3,2] [1,3,3]	[2,2,3] [1,2,4]

Round 5

		Phase 1 l_1	Phase 1 l_2	Phase 1 l_3
Phase 2	l_1	[2,1,3] [4,0,2]	[1,2,4] [3,0,4]	[1,4,2] [3,2,2]
Phase 2	l_2	[2,1,3] [2,3,1]	[1,2,4] [1,4,2]	[1,4,2] [1,6,0]
Phase 2	l_3	[2,1,3] [0,1,5]	[1,2,4] [0,1,6]	[1,4,2] [0,3,4]

Round 10

		Phase 1 l_1	Phase 1 l_2	Phase 1 l_3
Phase 2	l_1	[5,1,1] [6,0,1]	[1,5,1] [2,4,1]	[1,1,5] [2,0,5]
Phase 2	l_2	[5,1,1] [5,2,0]	[1,5,1] [1,6,0]	[1,1,5] [1,2,4]
Phase 2	l_3	[5,1,1] [4,1,2]	[1,5,1] [0,5,2]	[1,1,5] [0,1,6]

The table reports the indexes used during rounds 1,2,3,5, and 10. There is one distinct table for each round where fictitious indexes are used. Each those tables contains the indexes provided to the participants during phase 2 and 3 according to the choice they made during phase 1 (in columns) and phase 2 (in rows). A cell of the table contains two vectors representing the indexes provided to the subjects during phase 2 and phase 3 respectively. Each of the two vectors contains the indexes of lottery 1, lottery 2, and lottery 3 in order. Thus, for example, if during phase 1 of round 1 the subject chooses l_2 then during phase 2 he will face the index 2 attached to l_1, the index 4 attached to l_2, and the index 1 attached to l_3 (the first vector of the second column of one arbitrary row of Round 1 table). If the same subject then chooses l_1 during the second phase of round 1, he will face the index 4 attached to l_1, 2 attached to l_2, and 1 attached to l_3 (the second vector of the second column of the first row of Round 1 table).

on the choice of phase 3. Suppose for instance, that l_1 was chosen in phase 1 and that l_2 is the strictly maximal alternative in the same situation of phase 2, then if the agent chooses l_1, Maximality implies that he will not choose the alternative l_2 in phase 3. Hence, despite Maximality is compatible with the choice of any alternative during phase 2, it implies some restrictions on the choice behaviour of phase 3.

Concerning the remaining rounds of phase 2—i.e., round 4, 5, 6, 7, 8, and 9—, true information about the number of members of the group choosing each alternative is provided. That is, participants are informed about the number of subjects in the group that have chosen each alternative during the first phase. Notice that, also from the comparison of choices made during phase 1 and phase 2 of these rounds, one can observe situations in which Maximality is tested. Indeed, if in the first phase, 4 or more members of the group choose the same alternative, say l_1, then in phase 2 they face the same conditions imposed during rounds 1, 3, and 10—i.e., l_1 has the strictly maximal index. Moreover, who in phase 1 choose something different from l_1, is exposed to the same conditions of rounds 2 and 5.

After these considerations one may consider the use of the fictitious indexes an unnecessary complication of the experimental test. However, the decision to use fictitious indexes in some rounds is motivated by two reasons, the first is to have a minimal number of situations for each participant in which Maximality is tested, the second one is to have an initial feasible situation for the test of the Monotonicity property, but this second issue will be discussed in the next subsection.

4.2.2 Test of the Monotonicity axiom

This subsection focuses on the implementation of the test of the Monotonicity axiom. Like Maximality, Monotonicity imposes the choice of a specific alternatives only if that alternative has been already chosen under some precise circumstances. More specifically, if l_1 has been chosen from $\{l_1, l_2, l_3\}$ given a set of indexes, then Monotonicity requires l_1 being chosen in all those situations in which the number of individuals choosing l_1 is increased or the number of individuals choosing l_2 (or l_3) is decreased.

Testing this axiom requires the observation of two choices under precise conditions regarding the indexes, but these conditions are unlikely to occur in the lab. Therefore, the simplest way to obtain them is by using fictitious indexes. Specifically, Monotonicity is tested confronting the choice made by the subject in the second phase with the choice made in the third phase. Indeed, given the choice made in the second phase, one can choose the indexes for the third phase in the convenient way. In phase 3 the fictitious indexes are used in 5 out of 10 rounds and those rounds, i.e., round 1, 2, 3, 5, and 10. In these rounds, each participant receives a different set of indexes that is based upon the choice made in the same situation during phase 2. That is, if a participant chooses l_1 from $\{l_1, l_2, l_3\}$ during phase 2, then phase 3 index of l_1 is incremented (by 1 or by 2) and the indexes of l_2 and l_3 are reduced in order to keep the sum of the indexes equal to the number of people in the group (i.e., 7). The fictitious indexes used in phase 3 are reported in table 4.2.

The choice to use fictitious indexes in the same rounds where they were used in phase 2 is necessary in order to be sure that the required movements of the indexes are feasible. The indexes of rounds 1, 2, 3, 5, and 10 of phase 2 assure that, for each alternative that can be chosen during the second phase, there is always the possibility to increase its index and to reduce the index of the other alternatives in order to keep their sum equal to 7. This would not always be the case if the same rounds of phase 2 were not chosen. For example, if during the second phase all the people choose the same alternative, then it is impossible to test Monotonicity without altering the size of the group.

Regarding behavioural predictions, if Monotonicity holds true, participants should confirm the choice they made during the same round of phase 2. However, the same remark made for Maximality applies. In the rounds where Monotonicity is tested, whenever the participant chooses the same alternative during all the three phases, he conforms to the axiom, but the same choice may be explained also by the simpler model where choices are independent of the indexes. This feature is an unavoidable since, according to the model proposed here, the independence of choice from the indexes is a special case of the DWMD heuristic. However one can discriminate between the two explanations by looking at the choice made during the first phase. Indeed, Monotonicity implies that agents choose the same alternative during phase 2 and 3, but they can choose something different during phase 1.

4.2.3 Further issues

The model proposed distinguishes two cases for how the indexes are computed. In the first case the indexes of each agent are computed considering only the choices of the other members of the group, while in the second case indexes are determined considering also one's own choice. However, the theory is silent about how to inform the subjects regarding the number of people choosing each alternative. Specifically, one can either display the choices of each agent, or one can display the number of people choosing each alternative. In the former case, the subject is supposed to "compute" the number of people by himself, while in the latter case he has the numbers already computed. We already discussed the drawbacks related to the information about the identity of the other members in the previous section, where we have explained the decision to use anonymous group membership. However there is another issue related to how information should be presented: one should decide whether to include in the count also the choice of the agent who is presented the information. This is a crucial point for the model. Indeed, the model provides two different prediction concerning the existence of an equilibrium depending upon whether the agents include or not their own choice in the count. As shown in the previous chapter, including one's own choice in the count has the same effect of a Status Quo Maintenance strategy: in case of ties in the indexes the one's own choice gives a bonus to the previously chosen alternative. As reported in theorem 1 of the previous chapter, counting one's own choice is a necessary condition with lemma 1 to guarantee the existence of an equilibrium in the choices.

Concerning our experimental test of the axioms, we decided to inform people about the overall number of agents choosing each alternative in the set. This because one cannot really control for how the indexes are interpreted by the subjects. Indeed, a person informed of the overall number of people choosing each alternative can subtract his own choice from the indexes and a person informed of the number of other people choosing each alternative can always add its own choice in the indexes. Moreover, counting one's own choice has some effect only in case of ties of the indexes, thus choice behaviour in the two cases may be different only in a small number of situations.

4.2.4 Implementation

The experiment was run at the Computable and Experimental Economics Laboratory (CEEL) of the University of Trento. It was a computerised task and the software was written using Delphi. Overall, 28 subjects participated to the experiment. All the participants were students of the University of Trento that were recruited by email.

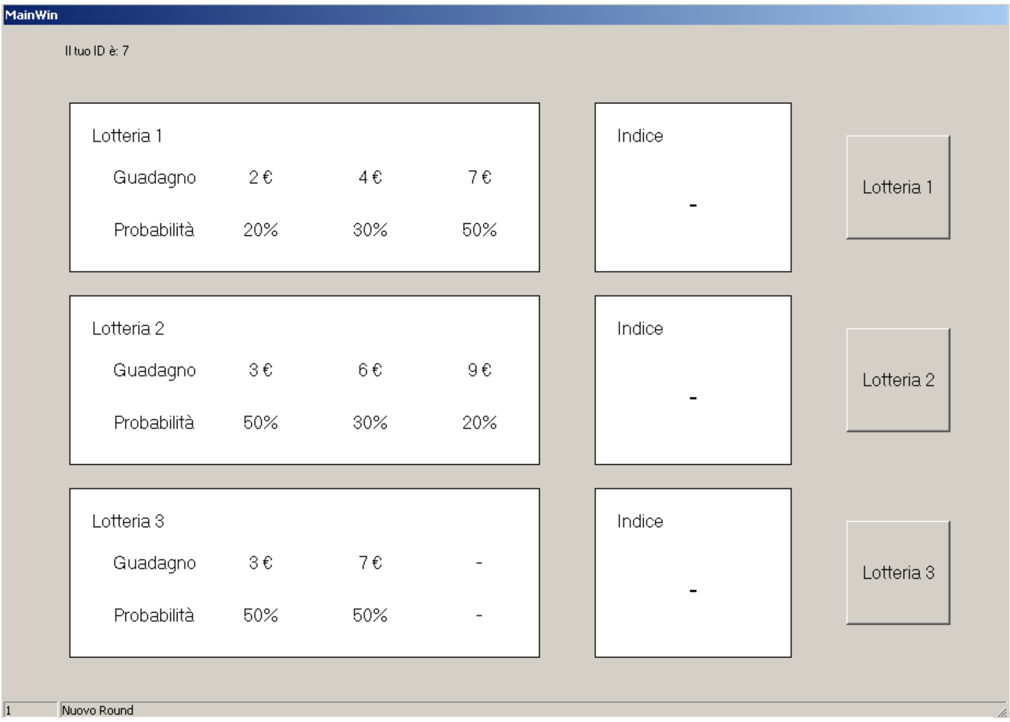
Two sections of 14 subjects each were run. Before the beginning of the experiments participants were randomly assigned to a cubicle and randomly matched in order to form two groups of 7 people.[3] Subjects were not informed about the identity of the other participants in the group.

When all the participant reached their cubicles, a copy of the instructions (see Appendix) was presented to them on the screen of the computer and they were given five minutes to read the instructions privately. After the private reading, instructions were read aloud and possible questions were answered, then phase one of the experiment started.

During phase one, at the beginning of each round the three lotteries they had to choose from were presented to the participants on the computer's screen (see Figure 4.2). Participants then chose one lottery among the three available

[3]Each participant was randomly assigned to a cubicle drawing from an urn a numbered token representing the cubicle's number. This draw both assigned the subject to a cubicle and to a group because even and odd cubicles were grouped together respectively.

Figure 4.2: An example of experiment's screenshot



by clicking on the corresponding button on the right of the screen. After all the participants made their choice, the experimental software automatically advanced to the next round. At the end of the ten rounds of the first phase subjects were informed that the second phase was starting.

In the second phase the subjects faced the sets of lotteries in the same order of phase one. The only difference was that in the second phase subjects were informed about the indexes. In the squares named "Indice", on the right of the description of each lottery (see Figure 4.2), either the number of group members choosing that lottery during the previous phase or the fictitious index attached to that lottery was presented. Like in phase one, after all the participants made their choice, the software automatically advanced to the next round. When all the 10 rounds of phase two were completed, subjects were informed that the third and last phase was starting. The third phase was identical to the second phase but the index attached to each alternative was either the number of members of the group choosing that lottery during the second phase or the phase's three fictitious index attached to that lottery.

In order to determine the payoff of each participant, at the end of the experiment two rounds—one belonging to phase 1 and one belonging either to phase 2 or to phase 3—were selected, and the lottery chosen by the agent during that round was played by the computer. On average, one sessions lasted for about 40 minutes and subjects received a payoff of about 10 euros.

4.3 Results

Overall we recorded 840 choices made by 28 subjects (28 subjects $\times$ 10 sets of lotteries $\times$ 3 choices). As pointed out in the experimental design, each subject chooses three times for each situation, i.e., for each set of lotteries presented in Table 4.1. The comparison of the three choices allows to check whether individual behaviour satisfies Monotonicity and Maximality.

Before proceeding with the analysis, we need to set some terminology. If the three choices from, e.g., the set of lotteries used in round 4 do not contradict Monotonicity, we say that situation 4 satisfies Monotonicity. The same applies to Maximality. Moreover, if in a given situation both Maximality and Monotonicity are satisfied, then we say that the DWMD heuristic is satisfied. Finally, if in a given situation the participant always chooses the same lottery, we say that the situation satisfies independence of the indexes.

In the experimental design we discussed the need to introduce fictitious indexes in order to be sure each agent faces a situation where the prerequisites of both Maximality and Monotonicity are met. Indeed, in rounds different from 1, 3, and 10, the prerequisites for the test of Monotonicity and Maximality are not always met, when this is the case the property cannot fail, and hence the property is vacuously true. Tables 4.3 and 4.4 report the frequency of axiom satisfaction by player and by situation, respectively. These tables consider the number of times a property is satisfied, restricting the count only to those cases where both Maximality and Monotonicity are testable.

Table 4.3: Axiom satisfaction by player.

ID	Maximality	Monotonicity	DWMD	Indep. from Ind.
0	5 / 7 (71.4%)	5 / 7 (71.4%)	5 / 7 (71.4%)	4 / 7 (57.1%)
1	1 / 7 (14.3%)	2 / 7 (28.6%)	1 / 7 (14.3%)	0 / 7 (0.0%)
2	2 / 4 (50.0%)	2 / 4 (50.0%)	2 / 4 (50.0%)	2 / 4 (50.0%)
3	3 / 5 (60.0%)	5 / 5 (100.0%)	3 / 5 (60.0%)	2 / 5 (40.0%)
4	2 / 5 (40.0%)	4 / 5 (80.0%)	2 / 5 (40.0%)	2 / 5 (40.0%)
5	5 / 7 (71.4%)	6 / 7 (85.7%)	5 / 7 (71.4%)	3 / 7 (42.9%)
6	4 / 5 (80.0%)	4 / 5 (80.0%)	4 / 5 (80.0%)	3 / 5 (60.0%)
7	5 / 8 (62.5%)	7 / 8 (87.5%)	5 / 8 (62.5%)	4 / 8 (50.0%)
8	4 / 8 (50.0%)	5 / 8 (62.5%)	4 / 8 (50.0%)	4 / 8 (50.0%)
9	4 / 7 (57.1%)	5 / 7 (71.4%)	4 / 7 (57.1%)	3 / 7 (42.9%)
10	2 / 7 (28.6%)	5 / 7 (71.4%)	2 / 7 (28.6%)	2 / 7 (28.6%)
11	5 / 8 (62.5%)	5 / 8 (62.5%)	4 / 8 (50.0%)	3 / 8 (37.5%)
12	3 / 5 (60.0%)	3 / 5 (60.0%)	3 / 5 (60.0%)	2 / 5 (40.0%)
13	5 / 6 (83.3%)	6 / 6 (100.0%)	5 / 6 (83.3%)	4 / 6 (66.7%)
14	4 / 5 (80.0%)	5 / 5 (100.0%)	4 / 5 (80.0%)	2 / 5 (40.0%)
15	3 / 4 (75.0%)	3 / 4 (75.0%)	3 / 4 (75.0%)	3 / 4 (75.0%)
16	1 / 5 (20.0%)	2 / 5 (40.0%)	1 / 5 (20.0%)	1 / 5 (20.0%)
17	5 / 6 (83.3%)	5 / 6 (83.3%)	5 / 6 (83.3%)	3 / 6 (50.0%)
18	2 / 6 (33.3%)	3 / 6 (50.0%)	2 / 6 (33.3%)	2 / 6 (33.3%)
19	3 / 6 (50.0%)	4 / 6 (66.7%)	3 / 6 (50.0%)	2 / 6 (33.3%)
20	4 / 7 (57.1%)	4 / 7 (57.1%)	4 / 7 (57.1%)	4 / 7 (57.1%)
21	7 / 7 (100.0%)	7 / 7 (100.0%)	7 / 7 (100.0%)	7 / 7 (100.0%)
22	5 / 9 (55.6%)	7 / 9 (77.8%)	5 / 9 (55.6%)	5 / 9 (55.6%)
23	4 / 4 (100.0%)	4 / 4 (100.0%)	4 / 4 (100.0%)	4 / 4 (100.0%)
24	3 / 7 (42.9%)	4 / 7 (57.1%)	3 / 7 (42.9%)	1 / 7 (14.3%)
25	3 / 8 (37.5%)	4 / 8 (50.0%)	3 / 8 (37.5%)	2 / 8 (25.0%)
26	3 / 7 (42.9%)	6 / 7 (85.7%)	3 / 7 (42.9%)	1 / 7 (14.3%)
27	4 / 7 (57.1%)	5 / 7 (71.4%)	4 / 7 (57.1%)	3 / 7 (42.9%)
Tot	101 / 177 (57.1%)	127 / 177 (71.8%)	100 / 177 (56.5%)	78 / 177 (44.1%)

The aggregate result emerging from these tables is that, out of the 280 situations observed (28 subjects $\times$ 10 situations), there are 177 situations where both

Monotonicity and Maximality can be tested. Of these 177 situations, 57% satisfies Maximality, 72% satisfies Monotonicity, and 56% satisfies both Maximality and Monotonicity. Thus, more than half of the times in which DWMD heuristic is under examination choices satisfy it. However, it seems that Monotonicity is more robust than Maximality. Looking at the overall number of situations that are compatible with independence of the indexes, 78 out of 177 occurrences meet this requirement (44%). As pointed out in the previous section, the set of situations satisfying independence of the indexes is necessarily a subset of the set of situations satisfying DWMD heuristic, so it is very hard to discriminate between the two models. At the aggregate level, the percentage of situations explainable with DWMD heuristic and not with independence of the indexes is about 12%, thus DWMD explains about 27% more situations than independence of the indexes.

Looking at the individual level (table 4.3), there is an high variability between subjects regarding the number of situations explainable by the DWMD heuristic. Indeed, the satisfaction of both Monotonicity and Maximality ranges from 14.3% to 100% of the cases where the subject faces a test of both the properties. Also in this case, like in the aggregate case, the satisfaction of Monotonicity seems more robust than the satisfaction of Maximality. In more detail, 5 people out of 28 always satisfy Monotonicity, while 2 out of 28 do the same for Maximality.

Looking at the situation level (table 4.4), one can comment about behaviour where fictitious indexes were used. Recall that situations 1, 3 and 10—i.e., rounds 1, 3, and 10 of phase 2 and 3—are rounds where each subject faces a test of both Monotonicity and Maximality. With situations 2 and 5 an attempt to discriminate between DWMD heuristic and independence of the indexes was made. The rate of satisfaction of Maximality in situation 1, 3, and 10 is of 46%, 43%, and 64% respectively. For the same situations, the rate of satisfaction of Monotonicity is considerably higher (82%, 78%, and 75%, respectively). Concerning the rate of satisfaction of independence of the indexes, by construction of the fictitious indexes, this must be equal to the rate of satisfaction of both Maximality and Monotonicity (i.e., equal to the rate of satisfaction of DWMD).

Table 4.4: Axiom satisfaction by situation.

Sit	Maximality	Monotonicity	DWMD	Indep. from Ind.
1	13 / 28 (46.4%)	23 / 28 (82.1%)	13 / 28 (46.4%)	13 / 28 (46.4%)
2	5 / 12 (41.7%)	5 / 12 (41.7%)	5 / 12 (41.7%)	0 / 12 (0.0%)
3	12 / 28 (42.9%)	22 / 28 (78.6%)	12 / 28 (42.9%)	12 / 28 (42.9%)
4	3 / 9 (33.3%)	6 / 9 (66.7%)	3 / 9 (33.3%)	3 / 9 (33.3%)
5	20 / 27 (74.1%)	20 / 27 (74.1%)	20 / 27 (74.1%)	11 / 27 (40.7%)
6	8 / 11 (72.7%)	8 / 11 (72.7%)	8 / 11 (72.7%)	7 / 11 (63.6%)
7	10 / 13 (76.9%)	10 / 13 (76.9%)	10 / 13 (76.9%)	7 / 13 (53.8%)
8	4 / 11 (36.4%)	4 / 11 (36.4%)	3 / 11 (27.3%)	2 / 11 (18.2%)
9	8 / 10 (80.0%)	8 / 10 (80.0%)	8 / 10 (80.0%)	5 / 10 (50.0%)
10	18 / 28 (64.3%)	21 / 28 (75.0%)	18 / 28 (64.3%)	18 / 28 (64.3%)
Tot	101 / 177 (57.1%)	127 / 177 (71.8%)	100 / 177 (56.5%)	78 / 177 (44.1%)

Looking at situations 5 and 10 allows us to collect some evidence about the gap between independence of the indexes and DWMD. Indeed, in these situations the fictitious indexes in phase 2 presented a maximal alternative that was different from the alternative chosen in phase 1. This means that the gap between DWMD and independence of the indexes captures the fraction of people satisfying Maximality and Monotonicity that switched from the alternative chosen in phase 1 to another alternative in phase 2.[4] Both in situation 2 and 5, the percentage of situations explained by DWMD exceeds the percentage of situations explained by independence of the indexes by more than 30 percentage points.

Moving toward a test of the axioms, a preliminary check that has to be performed is whether the position of the alternatives on the screen affects choices. In each round the lotteries are presented as reported in Figure 4.2. So the lotteries are listed vertically with the choice buttons on the right side of the form: Lottery 1 (l_1) is the top one, lottery 2 (l_2) is the middle one, and lottery 3 (l_3) is the bottom one. Since lotteries l_1, l_2, and l_3 have been randomly chosen in each

[4]This may measures the fraction of people that switched from the alternative chosen in phase 1 to the maximal alternative of phase 2, but also something more subtle than this. Indeed, subjects can also choose the third alternative, i.e., the one that is both not chosen in phase 1 and not maximal in phase 2. But in this case, choice of phase 2 and situation of phase 3, must be also a test of Maximality and the subject must have respected it.

Figure 4.3: Frequency of lottery's choice (Phase 1)

situation, one expects that being in one position does not affect choices. In order to test this hypothesis only the choices of phase 1 are used because other phases' choices may be affected by the indexes attached to the alternatives. Figure 4.3 reports the choice frequency of l_1, l_2, and l_3 during the first 10 rounds. At a first glance, upper lotteries seem to be more attractive than lower ones. However, a goodness–of–fit test against the uniform distribution leads to a failure of the rejection of the null hypothesis at the conventional 5% significance level ($\chi^2_2 = 5.5143$; P-Value $= 0.06347$). Notice however that, under the null hypothesis, the probability to observe data farther from uniform is quite small and not distant from 0.05.

Table 4.5 helps understand the nature of the bias toward lottery one. The table reports the choice frequency of the lotteries by time needed to choose. Thus, the first row of the table reports the distribution of the choices made in less than 20 second, the second row reports the distribution of the choices made between 21 and 40 seconds, and so on. As one can see, for fast choosers lottery l_1 is the most preferred, while with the increase in the time spent to choose the preferences are more distributed. This may suggest that, when people do not put a lot of effort in their choice, they tend to pick the first lottery by default. However a more accurate analysis should be performed to corroborate this hypothesis.

Table 4.5: Frequency of choices by time (phase 1).

Time (secs)	$\#l_1$	$\#l_2$	$\#l_3$
0–20	29	15	10
21–40	60	48	49
41–60	18	20	15
> 60	3	9	4

Concerning the test of DWMD versus independence of the indexes we run a Multinomial Mixed Model (Cameron and Trivedi, 2005) where we estimated the probability to choose the lotteries in the various position during phase 2 and 3 as a function of the choice in phase 1 and of other regressors such as the indexes of the alternatives. The model is the following one:

$$P(y = l_i) = \frac{e^{\beta_1 Prev_i + \beta_2 Max_i + \gamma_{1i} I_1 + \gamma_{2i} I_2 + \gamma_{3i} I_3 + \gamma_{4i} Time}}{\sum_{k=1}^{3} e^{\beta_1 Prev_k + \beta_2 Max_k + \gamma_{1k} I_1 + \gamma_{2k} I_2 + \gamma_{3k} I_3 + \gamma_{4k} Time}} \tag{4.1}$$

Where $Prev_i$ and Max_i are alternative specific dummies equal to one if, respectively, alternative l_i is the alternative chosen by the subject during phase 1 of the same situation and alternative l_i is the alternative that is maximal in the current situation. I_1, I_2, I_3, and $Time$ are observation specific variables, i.e., they remain constant for all the l_i given the subject and the alternative. These regressors represents the indexes attached to the alternatives (I_1, I_2, I_3) and the time used to choose ($Time$), respectively.

This model estimates the probability that a lottery in a given position is chosen as a function of the initial choice and of the indexes. Since the lotteries in the various positions are randomly determined in each round, the position does not capture specific characteristic of the lottery. Thus, if participants choose randomly one should expect a probability of 1/3 to choose a lottery in a given position. If instead, one expects participants choose consistently with independence of the indexes, the estimate of $Prev_i$ should have a positive impact on the choice probability and the estimates of Max_i, I_1, I_2, I_3, and $Time$ should be not significantly different from zero. If one finally expects that participants choices satisfy Maximality and Monotonicity the variables of the indexes and Max_i should have a significant effect on choice probabilities. More specificly, if Maximality holds true one expects that, on average, the impact of being the strictly maximal ($Max_i = 1$) alternative is significant and with positive sign. Concerning Monotonicity, one expect that, on average, the probability to choose a given lottery, l_i, is positively influenced by its index, i.e. I_i, and negatively influenced by the indexes of the other lotteries, i.e., I_j where $j \neq i$.

Data used in the regression do not include choices of phase 1, that are used to determine the initial choice, i.e., the value of $Prev_i$, thus 560 choices made by 28 players are considered. At this stage of the analysis, the panel structure of the dataset is ignored. Further work is needed to include individual random effects to control for idiosyncratic features of the subjects.

Model's estimates are presented in table 4.6. The impact of $Prev_i$ on probability is positive and significant, thus alternative l_i has higher probability of being chosen when it is chosen during phase 1 rather than when it is not chosen. Concerning the effect of Max_i, this not significant and very close to zero. Thus, one fail to reject the hypothesis that the alternative with the highest index has an higher probability of being chosen when controlling for the choice made when no index was presented. However, the choice of the alternative possessing the maximal index often coincides with the choice of the alternative that was chosen during phase 1—in situations 1, 3, and 10 this is always the case—thus Max_i and $Prev_i$ are highly correlated and hence it is difficult to estimate correctly their effect on the probability of choice. Turning to the parameters of the variables I_1, I_2, andI_3, we have different estimates for each lottery. The regression estimates confirm that there is some significant impact of the indexes on the probability of choice. Also the Likelihood Ratio Test testing the restricted model where only $Prev_i$ and the control variable $Time$—i.e., the joint hypothesis that all the coefficients of I_1, I_2, I_3, and of Max_i are equal to zero—leads to a rejection of the null hypothesis that the indexes have no impact on choices ($\chi^2_7 = 82.99$; P-value $= < 0.01$).

Table 4.6: Lottery choices: Logit Estimates (with ChTime).

Regressors		Coefficient	Std. Error	t-value	P-value	
Previous		0.9170524	0.0985239	9.3079	< 2.2e-16	***
Maximal		-0.0964497	0.1864099	-0.5174	0.6048724	
I1	Alt2	-0.3188343	0.0845702	-3.7701	0.0001632	***
	Alt3	-0.4059347	0.0825532	-4.9173	8.777e-07	***
I2	Alt2	0.3113180	0.0830464	3.7487	0.0001777	***
	Alt3	0.1244005	0.0696031	1.7873	0.0738918	.
I3	Alt2	-0.1965366	0.0817798	-2.4032	0.0162505	*
	Alt3	0.0213501	0.0895402	0.2384	0.8115392	
ChTime	Alt2	0.0237209	0.0104797	2.2635	0.0236039	*
	Alt3	0.0389838	0.0096028	4.0596	4.915e-05	***
Log-Lik:		-487.75				

Concerning the sign of the indexes' effects, from table 4.6 one can only infer the effect of the indexes on the probability of choice of l_2 and l_3 with respect to the base alternative l_1. It can be clearly notices, that an increase in the index of alternative l_1 (an increase in I_1) produces a reduction in the probability of choice of l_2 and l_3 with respect to the probability of choice of l_1. Unfortunately, other effects are harder to interpret. A more natural interpretation of the effects of a change in the indexes on probabilities can be derived from table 4.7 which reports the marginal effects.[5] Notice that, with the exception of the effect of I_3 on $P(l_1)$ that is positive, all other marginal effects have the expected sign, i.e., $P(l_i)$ increases when its index increases and decreases when the index of another

[5]The marginal effect of the index I_j on $P(y = l_i)$, i.e.,

$$\frac{\partial P(y = l_i)}{\partial I_j} = P(y = l_i)\left(\gamma_{ij} - \sum_{k=1}^{3} \gamma_{jk} P(y = l_k)\right)$$

is calculated using the predicted probabilities and the estimated parameters as the average marginal response over the 560 observations, i.e.

$$\sum_{q=1}^{560} \hat{p_{iq}} \left(\hat{\gamma_{ij}} - \sum_{k=1}^{3} \hat{\gamma_{jk}} \hat{p_{kq}}\right)$$

alternative increases.

Table 4.7: Marginal Effects for Logit Model.

	Change in I_1	Change in I_1	Change in I_3
Change in $P(l_1)$	0.05942928	-0.033167340	0.01180801
Change in $P(l_2)$	-0.01408819	0.039464988	-0.03435658
Change in $P(l_3)$	-0.04534108	-0.006297648	0.02254857

Interestingly, the control variable $Time$ has a significant effect on the probability of choice. In particular, both the coefficients of $Time$ suggest that an increase in the time spent produces an increase in the probability of choosing lotteries l_2 and l_3 compared to the probability of choosing l_1. This corroborates the evidence drawn from Table 4.5.

Overall, the results of the regression confirm the considerations drawn when looking at tables 4.3 and 4.4. To summarise, there is evidence that indexes have some impact on choices, but there is mixed evidence about the robustness of the DWMD heuristic. In particular, while Monotonicity seems to be satisfied, at least at population level, Maximality seems not to hold.

4.4 Summary and conclusions

We have explored the empirical validity of the choice–theoretic model proposed in the previous chapter using lotteries and randomly generated groups of people. The results obtained from this first experimental test show mixed evidence about axioms' validity: on one hand, there is evidence that people react to changes in the indexes; on the other hand, we obtain only weak support to the Monotonicity axiom, while Maximality must be rejected.

As previously discussed, due to the impossibility of easily discriminate between the model proposed and the simpler model where choices are independent of the indexes, the results obtained are not conclusive both regarding the validity of Monotonicity and Maximality. Indeed, independence of the indexes is a special case of the proposed model. A crucial prerequisite to be able to discriminate between the two models is the presence of some indecision, but it would be possible that participants were not indecise and hence not influenced by the indexes despite the selection of the lotteries used in the experiment.

Another crucial issue is the selection of people influencing the choice. The theoretical model assumes the group of people to be given. However, group composition, identity of the members, and size of the group, may influence the use of the heuristic by the participants. The experiment induces the weakest form of group membership by randomly grouping people and keeping anonymity about peers' identity. The motivation is that when people know each other, the direction of the effect is unpredictable. Indeed, there are possible reputational effects at work: if one knows the members of the group one can attach different values to the choices of different people, and react differently according to the identity of the peers. Thus the purpose of anonymous group membership is to guarantee the condition prescribed by the model, but, on the other hand, it may be too weak to induce significative effects on behaviour.

Concerning the theoretical result proposed in the previous chapter, if both Maximality and Monotonicity hold, and each agent considers also his own choice, then there exist a choice equilibrium where all the agents keep their alternative given the alternatives chosen by the other group members. Obviously, if only Monotonicity is confirmed, then this theoretic result fails to be empirically supported.

In conclusion, the chapter represents a first attempt to study imitative behaviour in simple situations where no payoff externalities, reputation effects, and informational learning are at work. The results weakly confirm that, the more an option is popular, the higher the likelihood of it being chosen. However, an important remark about the relationship with the standard results about framing effect should be made (for a review see: Levin et al., 1998; Kuhberger, 1998). Indeed, there is evidence that people's choices react also to very uninformative stimuli. For instance, Ariely et al. (2003) show that the subject's Willingness To Pay for a given product is influenced by the exposition to the social security number. Before the elicitation of the WTP for a given product, the authors ask subjects if they want to buy that product for the same price as the last two digits of their social security number and they find that the subsequent elicitation of the WTP is positively correlated to these numbers. Thus a crucial point of the experiment is whether participants attached meaning to the indexes or whether they were simply seen as numbers attached to the lotteries. The necessity to have fictitious indexes may pose some doubts about the participants' interpretation of the indexes, thus a deeper analysis to disentangle these effects is needed. Future work may try to use mouselab in order to explore the process of information gathering used by the participants. This may help understand which index influence the choice. In addition to this, repeating the experiment using only fictitious indexes, may help understand whether choices react to indexes even when they are devoid of any meaning.

Appendix: Instructions of the Experiment

This appendix contains the instructions (originally in Italian) that has been used in the experiment.

Thank you for taking part in this experiment. From now on, and until the end of the experiment, any communication with other participants is not allowed. If any type of communication is detected by the experimenter you will loose your earnings and will be sent out of the room. If you have some questions, please raise your hand and one of the experimenters will come to your desk to answer.

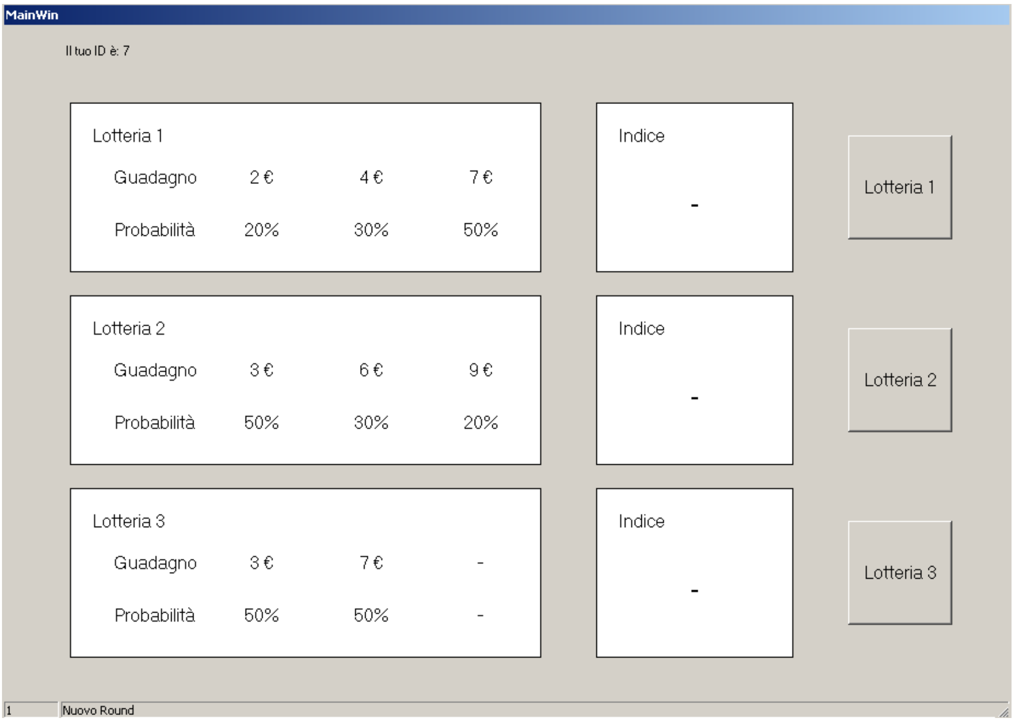
Structure of the Experiment

The experiment consists of three phases, each phase consists of 10 rounds. In each round you have to choose one lottery among three available lotteries. Each lottery has 3 possible outcomes, each of which happens with a given probability. The lotteries will be displayed to you on the computer's screen. An example of the experiment's screenshot is reported in Figure 4.4. The screen is divided into 3 columns. The column on the left displays the description of the three lotteries, the central column displays an index, whose meaning will be explained below, attached to each lottery, and the right column displays the three choice buttons.

Consider for instance Lottery 1 reported in Figure 4.4. This lottery gives you the chance to gain 2 euros in the 20% of the cases, 4 euros in the 30% of the cases and 7 euros in the remaining 50% of the cases. You can imagine Lottery 1 like an urn containing 100 balls, 20 balls reporting the script "2 euros", 30 balls reporting the script "4 euros" and the remaining 50 balls reporting the script "7 euros". The outcome of the lottery is determined by drawing one ball from the urn. Thus, in the situation reported in Figure 4.4 it is as if you choose the urn from which you want the ball being drawn. You can choose your favourite lottery by clicking the corresponding button.

At the beginning of the experiment you will be randomly matched with 6 other participants so that you will form a group of 7 participants. The groups of

Figure 4.4: An example of experiment's screenshot



participants will be used during the second and third phase of the experiment.

Phase One

The first phase consists of 10 rounds. In each round you have to choose one lottery among the three available by clicking on the corresponding button on the right part of the screen. When all the members of your group have chosen a lottery, the software will automatically pass to the next round. When all the 10 rounds of the first phase will be played, the second phase of the experiment will automatically start.

Phase Two

The second phase is similar to the first phase. You will face the 10 situation you faced during the first phase, but in this phase you may also be informed of the number of group members choosing each lottery during the previous phase. In the square labelled "Indice" (See Figure 4.4) on the right of the lottery's description you will read a number. These numbers will be determined in the following way:

- In 5 out of 10 rounds composing the second phase, the index associated to each lottery is the number of members of your group that chose that lottery in the same situation of **phase 1** (Obviously the sum of the indexes is 7 since the group is composed by 7 persons).
- In the remaining 5 round the proposed indexes are three numbers whose sum is 7.

During the game you will not know whether the indexes are simply three numbers whose sum is 7 or are the actual number of members of your group that chose each alternative during the previous phase. At the end of the 10 rounds of the second phase the third phase will automatically start.

Phase Three

The third phase is similar to the second phase. You will face for the third time the 10 situation you faced during the first phase. Like in the second phase, you may also be informed of the number of group members choosing each lottery during the previous phase. In the square labelled "Indice" (See Figure 4.4) on the right of the lottery's description you will read a number. In this case the numbers associated to the lotteries will be determined in the following way:

- In 5 out of 10 rounds composing the third phase, the index associated to each lottery is the number of members of your group that chose that lottery in the same situation of **phase 2** (Obviously the sum of the indexes is 7 since the group is composed by 7 persons).

- In the remaining 5 round the proposed indexes are three numbers whose sum is 7.

During the game you will not know whether the indexes are simply three numbers whose sum is 7 or are the actual number of members of your group that chose each alternative during the previous phase. At the end of the 10 rounds of the third phase the experiment will end and you will be informed of your earnings.

Earnings

At the end of the experiment, the computer will randomly draw one round of the 10 belonging to the fist phase and one round of the 20 belonging to the second and third phase for each participant. The lotteries you chose in these rounds will be "played" by the computer. At the end of the 10 rounds of the third phase, after the drawing of the outcomes of the lotteries, you will be informed of your earnings and the experiment will be concluded.

Chapter 5

Conclusions

5.1 Summary of the results

This thesis has taken seriously the issue posed by empiricists and theorists, from both the fields of economics and psychology, regarding the presence of alleged irrelevant aspects that are crucial for choices. Existing models suggesting the necessity to consider additional components of the choice function describe these additional components as elements of an abstract set (Salant and Rubinstein, 2008; Bernheim and Rangel, 2008, 2009). This thesis provides a more specific structure for these elements: each alternative is coupled with an index that may capture either psychological attitudes towards the alternative—e.g., the salience, popularity, or perceived availability of the alternative—or environmental information that does not enter the description of the alternative—e.g., the number of peers choosing that alternative or the amount of time the alternative has been advertised.

Chapter 2 considers an index–independent choice correspondence that results from the aggregation of single–valued index–dependent choice functions. Two axioms concerning how indexes affect choices are provided: Conditional IIA and Monotonicity. These two axioms capture the intuition about how the proposed interpretations of the indexes should affect choices. The main result presented in

Chapter 3 is that, when the choice conditioned to the indexes satisfies Monotonicity and Conditional IIA, the unconditional choices, i.e., the generated choice correspondence, can be rationalised by a quasi–transitive preference relation. This result, in analogy with the classical rationalisation, permits to interpret choices "as if" the agent maximises a preference relation and then uses the indexes to break ties (or indecision). The chapter provides also a discussion of the related literature and, in particular, it shows the equivalence of the presented model and the model proposed by Salant and Rubinstein (2008). In addition, the issue of welfare analysis with behavioural models is discussed showing that, also without well–behaved preferences, it is still possible to draw some basic welfare conclusions.

Chapter 3 provides a narrower interpretation of the indexes—specifically, each index represents the number of people in a group that choose each alternative—and discusses the properties an index–dependent choice function should satisfy in order to capture the behavioural implications of the "Do What The Majority Do" heuristic (Gigerenzer, 2004). This rule of thumb prescribes that, when unable to choose, one should follow the behaviour exhibited by the majority. The behavioural implications of the heuristic are summarised by means of two axioms: Monotonicity, which is a simplified version of the Monotonicity property presented in chapter 3, and Maximality. The chapter discusses the implications of the heuristic when it is applied by all the members of a group of individuals. In particular, the chapter defines the concept of equilibrium of choices—i.e., an allocation of alternatives such that, for each member of the group, the alternative allocated to him is also his choice given the alternatives allocated to the members of the group—and identifies the conditions for the existence of an equilibrium. In addition, the chapter shows that under these conditions, if all the members of the group review their choices according to the axioms, then an equilibrium is reached independently of the initial allocation of the alternatives.

Chapter 4 is an experimental test of the use of the "Do What the Majority Do" heuristic in the domain of lotteries. It provides a test of the empirical content of the two axioms proposed in chapter 3 with randomly grouped people choosing among lotteries. The results seem to support the idea that people are influenced by what the others do. However, the empirical validity of the axioms is unclear: while Monotonicity is compatible with the observed behaviour, Maximality receives very little support.

5.2 Concluding remarks and future work

To conclude I want to discuss some limitations of the present work, and to suggest some potential developments. The first comment is about the issues discussed in Chapter 1. The present work provides an explanation only for one of the issues presented in that chapter, namely the instability of choices. Other phenomena, such as the assumption that the decision maker is able to take into consideration all the alternatives present in the choice set, are not considered in the model presented in the thesis. While modelling multiple issues permits a more realistic picture of the decision maker and, hence, increases the descriptive accuracy, it also complicates the analysis. However, despite the increased complexity, there are some promising attempts to consider multiple issues in the same model. For instance, Tyson (2008) proposes a model where both salience and cognitive constraints are at work. I think that future developments of the present model in this direction can be achieved. For instance, one can consider assumptions alternative to the Conditional IIA which allow for more flexibility of choices when the set of alternatives changes.

Another aspect that has not been treated directly in this dissertation is the role of errors in choice. Almost all the choice–theoretical models consider deterministic choices an also rule out the possibility that the decision maker commits some mistake when choosing. In the same way behavioural models usually do not consider the possibility that the agent makes errors during the choice process. This topic however, becomes crucial when relaxing rationality assumptions: if errors are considered random, then not modelling them can be a minor issue[1];

[1] Models of choices with random errors has been largely explored in econometrics using random utility models (See, e.g., Cameron and Trivedi, 2005, Ch. 14-15)

if instead there is a systematic discrepancy between what people want and what people choose, then errors become an important behavioural pattern that should be incorporated into the model.

The analysis proposed here has been more positive than normative, but this is a common feature of many recent models relaxing the classical assumptions. This does not mean that normative implications are unimportant; on the contrary, they are a fundamental component of economic theory. However, relaxing the classical assumptions makes it more difficult to observe what people like and to draw welfare conclusions (Bernheim, 2009). For instance, when considering agents that use procedures and heuristics, one does not assume anything about the preferences of the chooser. Indeed, the chooser may stick to the procedure even if it selects a suboptimal or unwanted alternative. Recently, new analytical tools concerning welfare analysis in behavioural models have been proposed (Green and Hojman, 2007; Bernheim and Rangel, 2008, 2009; Bernheim, 2009) and also in this work basic welfare considerations have been discussed (in chapter 2). However, future work is needed to improve the normative content of the model, but unavoidable difficulties are present: for instance, an outside observer cannot infer the well being delivered by one alternative picked from a given set under two different index functions without imposing additional assumptions.

A final comment concerns the observability of the indexes. This issue has been discussed in the conclusion of Chapter 2, where I pointed out that in many cases it is impossible to directly observe the indexes because of the psychological nature of the measure. However, some of these psychological attitudes can be operationalised by offering a more operational definition. For instance, brand salience can be operationalised in various ways. In marketing research experiments, subjects are asked to list the brands that come up to their mind in a given product category, and then the salience of each brand is measured by the position of the brand in the list (Alba and Chattopadhyay, 1986). A more complex measure of brand salience can be obtained using eye–tracking techniques and equating salience with the probability to fixate a target product in pictures of supermarket shelves (van der Lans et al., 2008). The use of these operational measures suggests possible developments for future work and provides a starting point for testing the empirical content of the axioms presented in this thesis.

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