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Abstract

In this paper, superconducting transmission lines are analyzed by means of a numerical method which takes into account the experimentally-verified numerical dependence to the magnetic field. In particular, in the framework of the classical electrodynamics, the high-temperature superconducting material is described by recurring to an enhanced two-fluid model. Striplines and microstrip lines are considered and the attenuation constants of these guiding structures are computed by assuming quadratic and cubic nonlinearities.

Key words:

Superconducting Materials, Transmission lines, Numerical Methods.

1 Introduction

It is generally recognized that the development of high-temperature superconducting (HTS) transmission lines is strongly connected to the accurate modeling of the electromagnetic behavior of the superconducting material. In recent years, the use of HTS superconducting transmission lines has been widely studied in the framework of the classical guided-wave propagation [1]-[4]. While experimental results have been obtained to understand the physical behavior of superconducting materials, phenomenologically plausible macroscopic models have been devised to be inserted in computational tools [5]. The authors of this paper developed in [6] a numerical method in order to take into account the experimentally-verified nonlinear dependence of the surface resistance to the peak amplitude of the magnetic field [2][3]. The approach was applied in [4] in order to study some guiding structures commonly used in practical applications. However, in [6] and [4], the HTS-modeling was developed in the framework of the two-fluid model, whereas other more sophisticated macroscopic models (taking into account the effect of losses and impurities of superconducting films) have been recently devised. In particular, in [7][3] an "*enhanced*" version of the two-fluid model has been proposed. In the new approach, a model parameters τ_s (called *relaxation time*) has been introduced in the original two-fluid model to deal with the effects of losses due to material-impurities and possible anisotropy of the superconducting film. Consequently, in the present paper, the enhanced two-fluid model is combined with the nonlinear approach proposed in [4], by considering both quadratic and cubic nonlinearities. Several comparisons are reported showing sometimes significant differences with the classic two-fluid model in terms of the computed attenuation constant of the structure, especially at high field levels. In particular, a stripline and a microstripline are analyzed and the results are also compared, in the linear case, with those obtained by another proposed method, i.e. the bipolaron model [8].

2 Mathematical Formulation

The classical model usually adopted for describing superconducting transmission lines is the two-fluid model [5] where two different carriers (i.e., the normal electrons and the super-electrons) do not interact each other. According to the classical two-fluid model, a superconducting material can be modeled as a material characterized by a complex conductivity σ_c^{tc} defined as

$$\sigma_c^{tc} = \sigma_1 + j\frac{1}{\omega\mu_0\lambda^2(T)} \tag{1}$$

where σ_1 is the normal-state conductivity for a temperature $T = T_c$ (T_c being the critical temperature), ω is the angular frequency, μ_0 is the free space magnetic permeability, and $\lambda(T)$ is the effective penetration depth of the superconductor.

Since the results of the classical two-fluid model deviate from experimental results [3], it was necessary to introduce more accurate models for applications in microwave engineering. The main approximation used in the two-fluid model assumes that the two carrier-types are scattering free and lossless. In order to improve the classical two-fluid model, the conductivity of a HTS material is calculated taking into account the superconducting-electrons scattering and loss by defining an "*effective relaxation time constant*" [7]. From this enhanced two-fluid model, the predicted values of the surface resistance better agree with measured data.

In the enhanced two-fluid model, the electric conductivity is modeled as follows

$$\sigma^{tfe} = \left(\sigma_1 + \frac{\frac{1}{\omega\mu_0\lambda^2(T)}}{\omega\tau_s}\right) + j\frac{1}{\omega\mu_0\lambda^2(T)} \tag{2}$$

where τ_s is the quasi-particle relaxation time empirically obtained. It should be pointed out that the real part of the conductivity, σ_1, is now dependent on the driving frequency and τ_s changes if impurities are present in the superconducting film.

In order to introduce the enhanced two-fluid model in the nonlinear model developed in [6] for the analysis of propagating structures, the following nonlinear complex electric conductivity is defined

$$\sigma(H)_{nl}^{tfe}=\left(\left(\sigma_1+\frac{\frac{1}{\omega\mu_0\lambda^2(T)}}{\omega\tau_s}\right)+j\frac{1}{\omega\mu_0\lambda^2(T)}\right)+L(H) \tag{3}$$

where $L(H)$ is a nonlinear operator derived starting from the experimental characterization of superconducting materials. In particular, two kinds of nonlinear operators (second-order and third-order operators), developed in [4], are used. Consequently, superconducting propagating structures, characterized by rectangular cross-sections, can be suitably analyzed by considering the approach developed in [9] - extended to superconducting structures in [4] - and taking into account the effects of the nonlinearity on the attenuation constant by means of (3).

3 Numerical Results

In order to evaluate the effects of the more realistic model, the propagation characteristics of some propagating structures are analyzed.

The first example is concerned with a stripline embedded in vacuum. The structure is characterized by the following parameters configuration: $S = 150\,\mu m$ (width of the signal line), $W = 8000\,\mu m$ (width of the return lines), and $d = 864\,\mu m$ (distance between return lines). The structure is the same considered as in [3]. The attenuation constant (computed according to [10] from the knowledge of the circuit parameters) versus the penetration depth is reported in Figure 1 for the linear case and in correspondence with different values of the film thickness: $t = 0.1\,\mu m$ and $t = 0.8\,\mu m$. Three different models are compared: the classical two-fluids model, the enhanced two-fluid model, and the bipolaron model [8]. The parameters for the enhanced two-fluid model are chosen by considering a low-quality superconducting film. The effective relaxation time, τ_s, is computed as in [3]. As can be seen, when $t = 0.8\,\mu m$, there are not significant discrepancies between the classical two-fluid model and the modified models. On the contrary, notably different behaviors are predicted for $t = 0.1\,\mu m$, even in the linear case. The plots for the two-fluid model are in good agreement with the results in [2]. Whereas, as expected, the attenuation constant predicted by means of the enhanced two-fluid model is significantly higher.

The nonlinear case is dealt with in Figures 2 and 3, in correspondence with two values of the peak-amplitude of the magnetic field: $H = 25\,Oe$ and $H = 500\,Oe$, respectively. In particular, nonlinearities of second- and third-order are taken into account by recurring to the approach described in detail in [4]. It results that the enhanced two-fluid model predicts attenuation-constant values which are about $10\,\%$ higher than the corresponding values of the two-fluid model for $H = 25\,Oe$ (Figure 2). On the other hand, more significant differences are obtained in correspondence with $H = 500\,Oe$. In particular, large deviations occur when $t = 0.1\,\mu m$ and $\lambda > 0.8\,\mu m$ (e.g., when $\lambda = 1\,\mu m$, the difference is about $35\,\%$) (Figure 3).

Finally, Figure 4 shows the plots of the (normalized) attenuation constant of a HTS microstrip line at $T = 77\,K$. The dielectric substrate is made by $LaAlO_3$ ($\varepsilon_r = 25$) with a thickness $d = 500\,\mu m$. The width of the signal line is equal to $s = 150\,\mu m$ and the return line is $w = 5000\,\mu m$ wide. This propagation structure is the same considered by Liu and Itoh in [11]. Also in this case, the enhanced two-fluid model (in correspondence with second-order as well as third-order nonlinearities) estimates a slight increase in the attenuation-constant values with respect to the two-fluid model.

4 Conclusions

In this paper, the attenuation constant of some superconducting transmission lines has been computed starting from an enhanced two-fluid model and taking into account the experimentally-verified nonlinear dependence between the surface resistance and the peak magnetic field. By comparison with the classic two-fluid model, significant differences have been revealed in the attenuation constant of striplines and microstrip lines especially in correspondence with high-level fields. These results confirm the no-negligible effect of the nonlinearity of the material to be suitably taken into account for the development of high-performance superconducting devices.

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FIGURE CAPTIONS

- Figure 1.

  Attenuation constant of a HTS stripline versus the penetration depth λ. Linear case.

- Figure 2.

  Attenuation constant of a HTS stripline versus the penetration depth λ. Nonlinear case $(H = 25\,Oe)$.

- Figure 3.

  Attenuation constant of a HTS stripline versus the penetration depth λ. Nonlinear case $(H = 500\,Oe)$.

- Figure 4.

  Normalized attenuation constant $\frac{\alpha}{\omega\sqrt{\mu_0\epsilon_0}}$ of a HTS microstrip line versus the film thickness t. Nonlinear case.

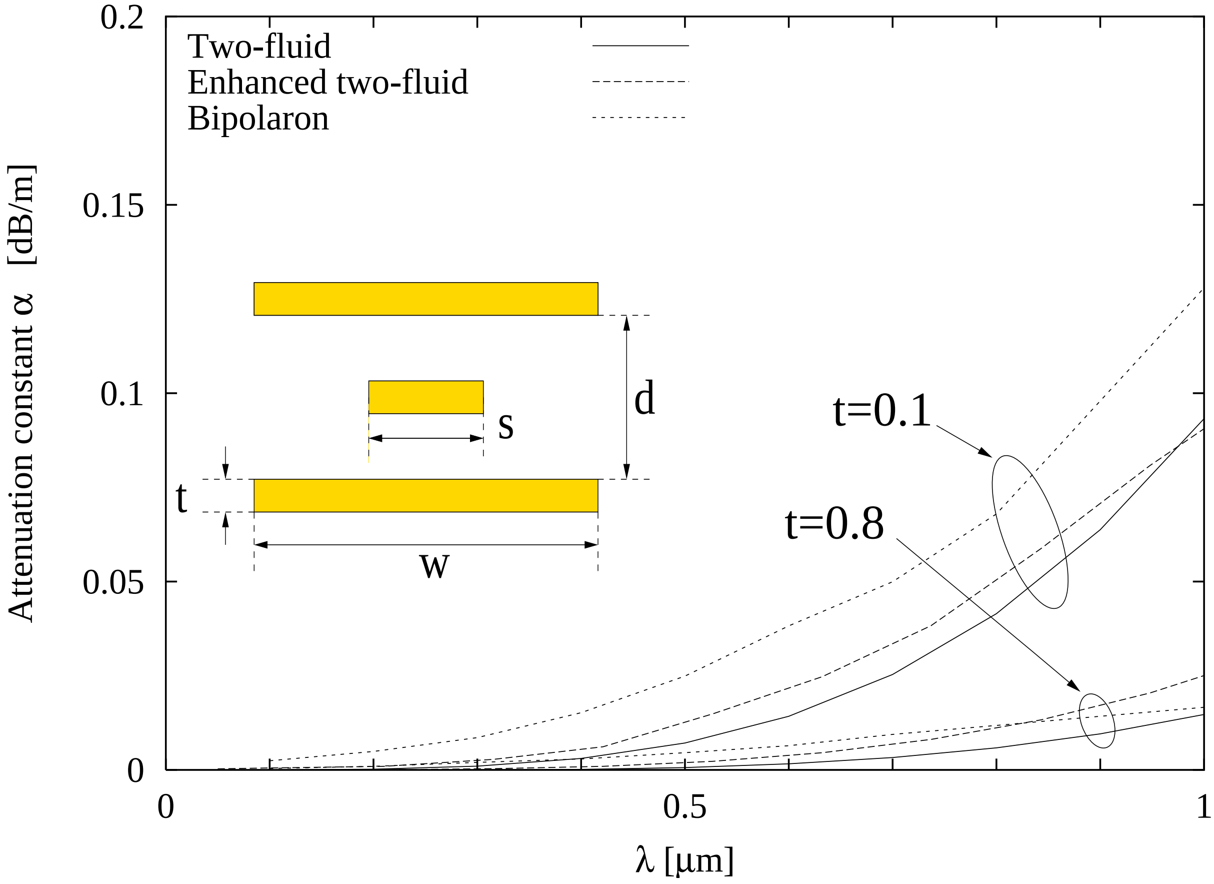
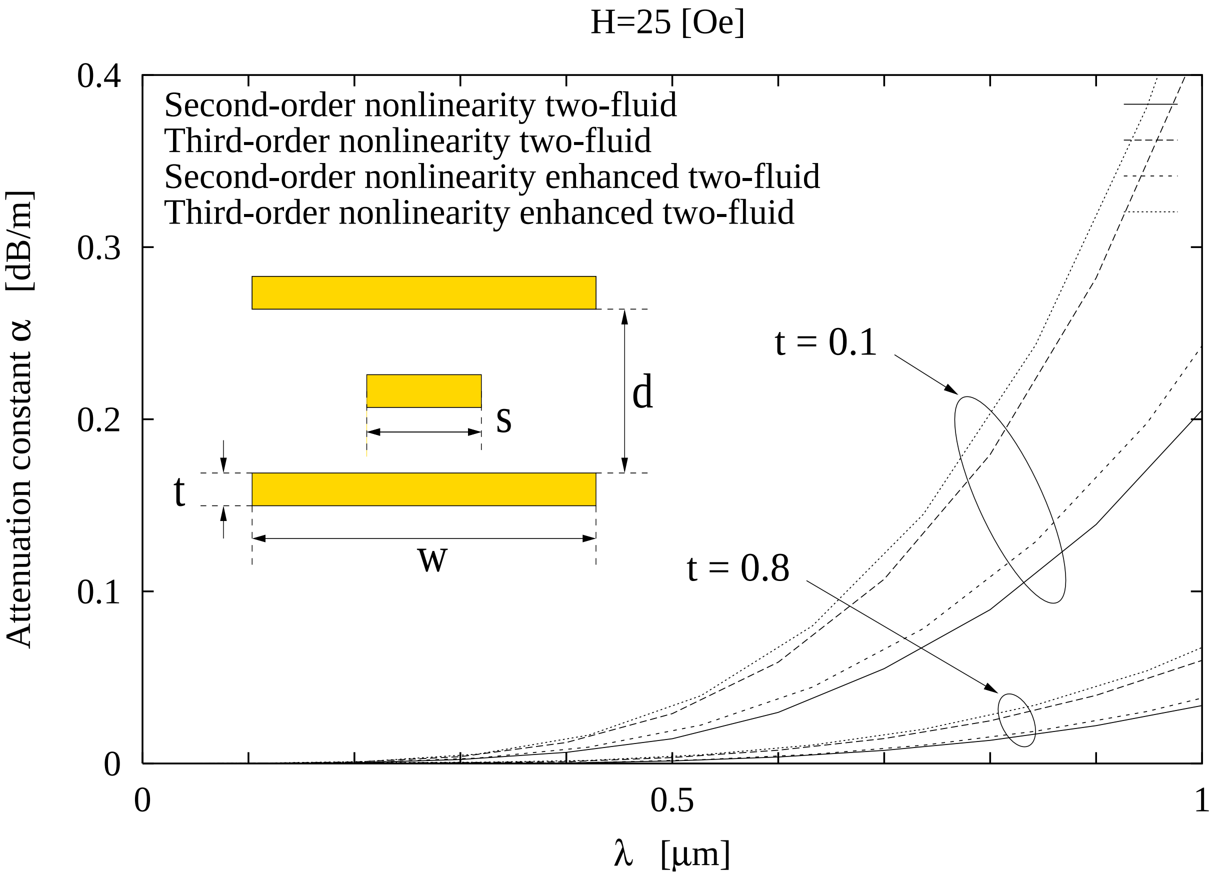


Fig. 1 - S. Caorsi *et al.*, "Assessment of a numerical ..."



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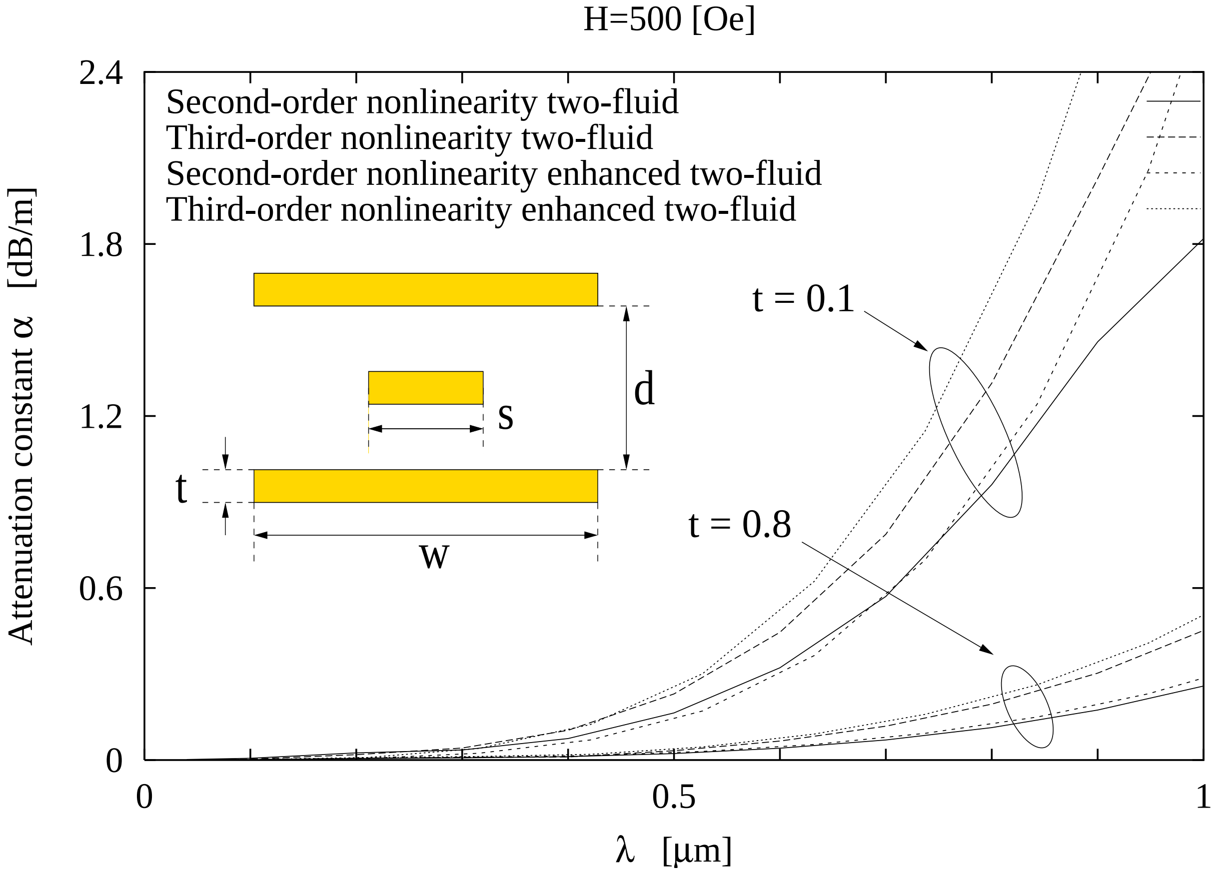

Fig. 3 - S. Caorsi *et al.*, "Assessment of a numerical ..."

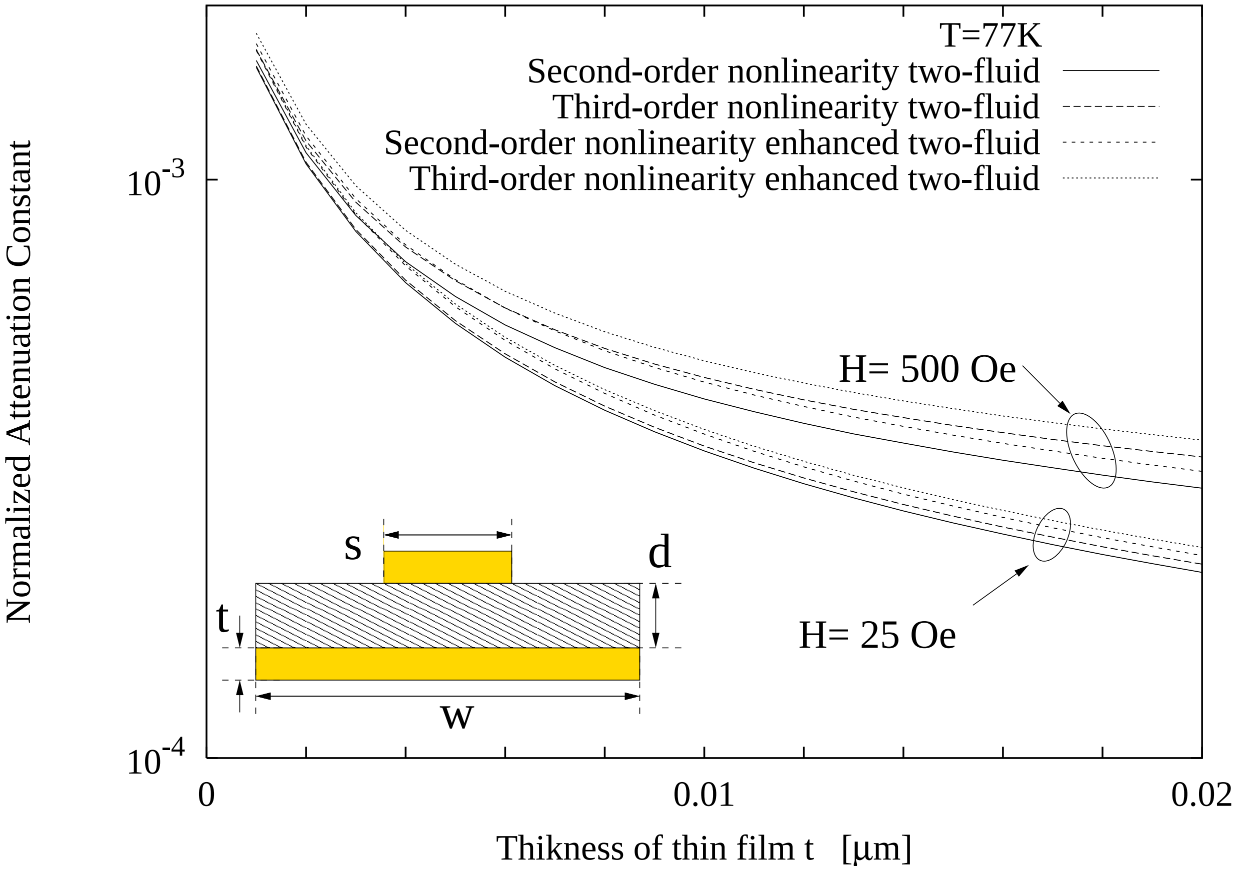

Fig. 4 - S. Caorsi *et al.*, "Assessment of a numerical ..."