TIME-DOMAIN INVERSION WITH THE IMSA-FBTS APPROACH

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Abstract: In this paper, the problem of localization, shaping, and reconstructing the dielectric permittivity of a dielectric target is addressed. The inversion technique processes the time-domain scattered field data to reconstruct with an increasing degree of accuracy the unknown scatterer by exploiting an iterative multiscaling procedure. Preliminary numerical results are presented to validate the time-domain multi-resolution multi-step approach.

Keywords: Microwave Imaging, Forward-Backward Time-Stepping Method, Iterative Multiscaling Method.

1. **Introduction**

Microwave imaging techniques based on inverse-scattering methodologies are aimed at localizing and reconstructing the dielectric properties of unknown scatterers from the measurement of the scattered field. An inverse scattering problem is usually very difficult because its nonlinearity, the ill-posedness, and the non-uniqueness of the solution [1]. Several inversion methods have been proposed in the frequency-domain by assuming a monochromatic illumination of the scenario under test [2-3]. Although frequency-based techniques have been successfully applied in different applicative domains (e.g., medical imaging and nondestructive testing) with satisfactory results [4-6], they present some drawbacks. More specifically, the use of higher frequencies enables an improvement of the spatial resolution achievable from the inversion, but it leads to a highly nonlinear formulation and a growing complexity in measuring the phase of the scattered field. On the other hand, the exploitation of single-frequency data only allows the collection of a limited amount of information. To properly address these issues, multiple acquisitions at different frequencies are necessary [7] or broadband probing fields should be adopted [8-9]. This latter is usually preferred even if it requires a time-domain field analysis with a non negligible computational burden (i.e., storage resources and computational time). As a matter of fact, the numerical solution of the inverse problem at hand needs the discretization of both the investigation domain (i.e., the area where the unknown scatters are located) and the region surrounding the transmitters and the receivers [10].

To limit the computational burden of time-domain inversions as well as enhance the spatial resolution, this work presents an innovative approach based on the time-domain application of the iterative multiscaling methodology [11].

The outline of the paper is as follows. Section 2 is devoted to the mathematical formulation of the time-domain inverse scattering problem and the IMSA solution is presented. Successively (Sect. 3), a representative test case is discussed to preliminary validate the proposed approach. Finally (Sect. 4), some conclusions are drawn and future developments envisaged.

2. **Mathematical Formulation**

Let us consider a two-dimensional unknown scatterer belonging to an investigation domain D_{inv}. The scatterer is illuminated successively by V short pulses waves generated by a set of sources located at $r_v^t \left(v = 1, 2, \ldots, V\right)$. The scattered field is collected at $M^{(v)}$ different measurement points $r_m^r \left(m = 1, 2, \ldots, M^{(v)}\right)$ placed in an observation domain D_{obs} enclosing the investigation domain. The material properties of the scatterer are modeled by means of a parameter vector function $\underline{\tau}(x, y)$ defined as

$$\underline{\tau}(x, y) = \left[\varepsilon_r(x, y) - 1; \sigma(x, y)\right] \tag{1}$$

where $\varepsilon_r(x, y)$ and $\sigma(x, y)$ are the dielectric permittivity and electric conductivity of the scatterer, respectively. The background medium is assumed to be free space.

As far as the multi-scaling procedure is concerned, it is applied as follows. At the initialization step ($s = 0$), a "coarse" profile of the object function is reconstructed by applying the Forward-Backward Time Step (FBTS) algorithm [9]. Towards this end, the investigation domain $D_{inv}^{(0)}$ is discretized into N cells of size $\Delta x_{inv}^{(0)} = \Delta \Delta_{inv}^{(0)} = L_{inv}^{(0)} / \sqrt{N}$ and the profile of the object function is retrieved by minimizing of the following cost function [9]:

$$Q\left(\underline{\tau}^{(s)}(x, y)\right) = \int_0^T \sum_{v=1}^{V} \sum_{m=1}^{M^{(v)}} K_{mv}(t) \left| E_m\left(\underline{\tau}^{(s)}(x, y); r_v^t, t\right) - \widetilde{E}_m\left(r_v^t, t\right) \right|^2; \quad s = 0 \tag{2}$$

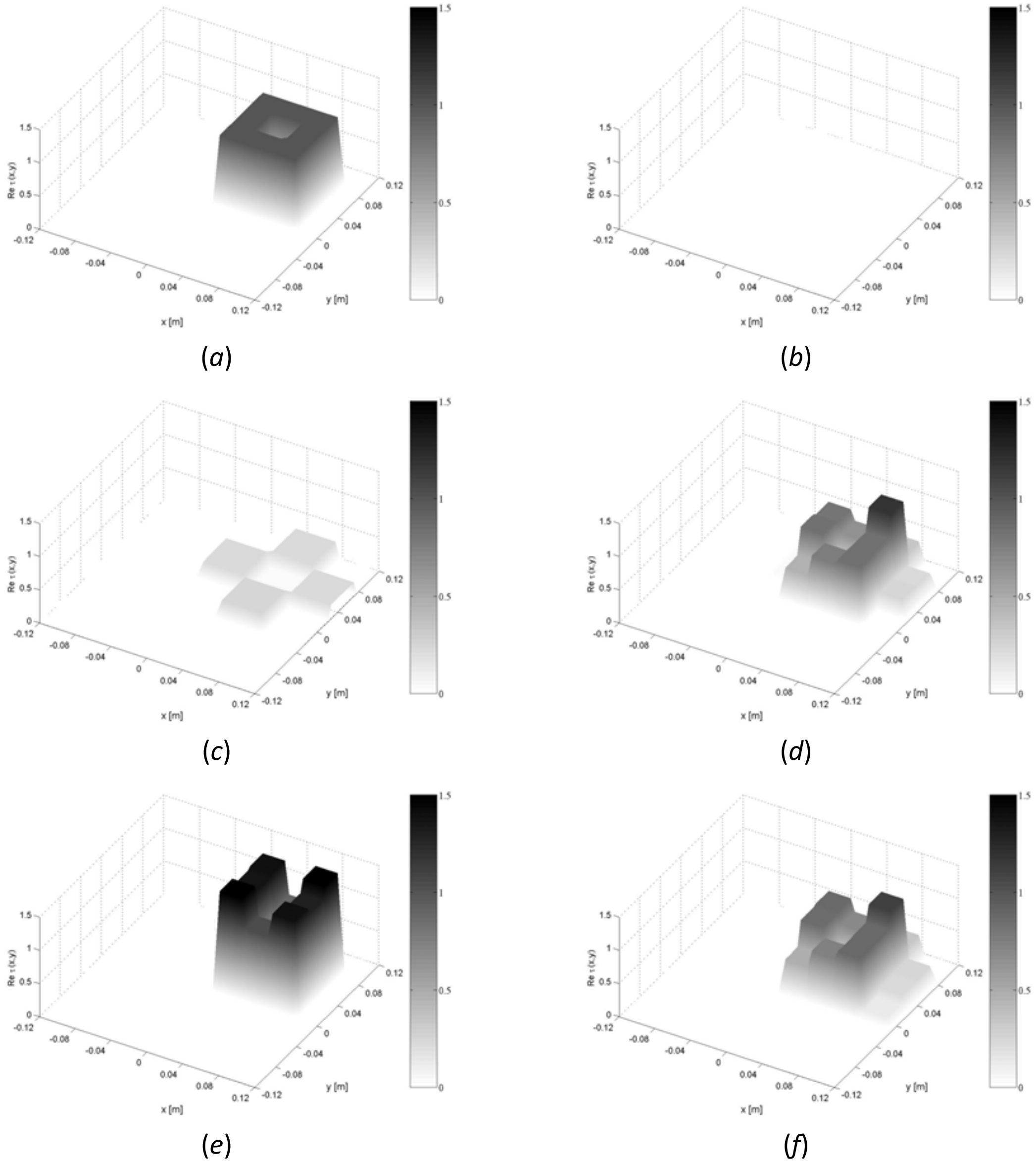
where $\widetilde{E}_m\left(r_v^t, t\right)$ is the measured electric field at the m^{th} receiver positions due to the pulse radiated by the source located at $r_v^t \ \left(v = 1, 2, \ldots, V\right)$ and $E_m\left(\underline{\tau}^{(s)}(x, y); r_v^t, t\right)$ is the field due to the parameter vector function estimated at the *s*-th step, $\underline{\tau}^{(s)}(x, y)$. The weighting factor $K_{mv}(t)$ is a nonnegative function which takes the value of zero at $t = T$, T being the time duration of the measurement.

The estimated dielectric profile is processed to determine a reduced investigation domain $D_{inv}^{(1)}$ (called "Region-of-Interest") as described in [11][12]. Again, the reconstruction is carried out by means of the FBTS now considering a finer spatial grid. The iterative procedure is repeated until stationary conditions hold true [13].

3. **Numerical Assessment**

In order to validate the proposed approach, let us consider the reconstruction of a dielectric hollow square cylinder centered in $(0.05, 0.05)\ [m]$ with a relative permittivity equal to $\varepsilon_r = 2.0$. The outer side of the cylinder has dimension $0.09[m]$, while the inner side is equal to $0.03[m]$ respectively [Fig. 2(*a*)]. Sixteen transmitters and sixteen receivers have been located at equally spaced points on a circle of radius $R = 0.5\ [m]$ around the object. A band-pass Gaussian incident pulse with center frequency $f_0 = 2[GHz]$ and band-width $1.3[GHz]$ has been adopted for the illumination. The time duration *T* of the measurement has been set to $217 \Delta t$ with a time step size of $\Delta t = 93.74\ [p\sec]$. The initial guess solution has been chosen equal to the free-space [Fig. 2(*b*)].

Figures 2(*c*)-2(*e*) show the evolution of the reconstruction throughout the iterative multiscaling approach. At the initial step $(s=0)$, the investigation domain is sized as follows $L_{inv}^{(0)} \times L_{inv}^{(0)} = 0.25[m] \times 0.25[m]$ and it has been discretized into $N=25$ cells. After the FBTS reconstruction, the scatterer has been roughly localized at $(0.044, 0.044)\,[m]$ [Fig. 2(*c*)] and a Region-of-Interest of size $L_{\mathrm{inv}}^{(1)} \times L_{inv}^{(1)} = 0.15[m] \times 0.15[m]$ and center $\left(\mathrm{x}_{\mathrm{c}}^{(1)}, \mathrm{y}_{\mathrm{c}}^{(1)}\right) = (0.04, 0.04)\,[\mathrm{m}]$ has been determined. At the successive step $(s=1)$, the retrieved object profile is shown in Fig. 2(*d*). At the stationary condition of the IMSA, the final result shown in Fig. 2(*e*) has been reached.



(*a*) (*b*)

(*c*) (*d*)

(*e*) (*f*)

Figure 2. (*a*) Hollow dielectric square cylinder (true object), (*b*) Initial guess for both "bare" – IMSA, (*c*) reconstruction after the 1st IMSA step, (*d*) reconstruction after the 2nd IMSA step, (*e*) reconstruction after the 3rd IMSA step (final result) and (*f*) reconstruction of the "bare" approach.

In order to point out the effectiveness and reliability of the proposed approach, the solution has been compared with that obtained with the "bare" FBTS approach [Fig. 2(*f*)]. In such a strategy, a uniform discretization, equal the finest spatial resolution achieved with the IMSA, has been chosen within the whole investigation domain. As expected, although the "bare" method correctly locates the scatterer, the IMSA performs a better reconstruction of the unknown object.

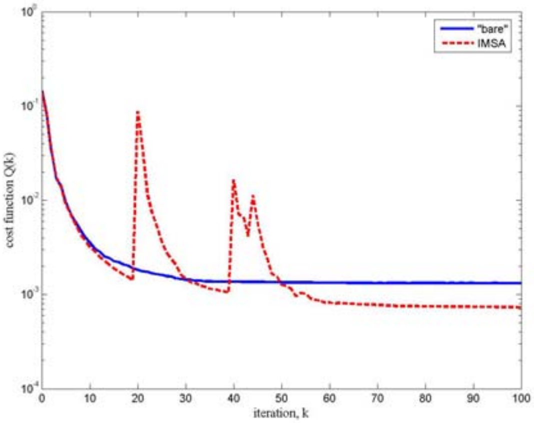


Figure 3. Cost function evolution.

For completeness, the evolution of the cost function during the iterative minimization is reported in Fig. 3 for both the *IMSA* and the "bare" approach.

As regards the computational costs, Table I summarizes the obtained results. The number of iterations to achieve the convergence, the total amount of time, T_{tot}, the average time $\langle t \rangle$ for each iteration, and the number of cells N used to discretize the investigation domain are reported. The computational indexes confirm that the *IMSA* outperforms the "bare" approach with a reduction of the computational time of about $\frac{1}{3}$ (2288 sec vs. 3301 sec – Tab. I) and of the complexity of about ½ (25 cells vs. 64 cells – Tab. I)

	"Bare" FBTS	*IMSA – FBTS*			
Step No.	-	0	1	2	-
Iterations	100	20	20	60	100
T_{tot} [sec]	3301.82	306.94	335.58	1646.29	2288.81
$\langle t \rangle$[sec]	33.02	15.35	16.77	27.44	22.89
N	64	25	25	25	

Table I. Computational indexes.

4. **Conclusions**

In this paper, a procedure based on the integration of the iterative multiscaling approach and the forward-backward time-stepping algorithm has been presented to address the inversion of time-domain data. Selected numerical results confirm, although in a preliminary fashion, the effectiveness and efficiency of the proposed reconstruction strategy. As a matter of fact, the method reduces the computational burden, decreasing both the storage resources and the time needed to perform the inversion, enabling a more detailed retrieval of the scatterer properties. However, further and deeper analyses are mandatory to assess in a more exhaustive way both features and limitations of the time-domain multi-resolution technique.

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