Sustainable Futures 7 (2024) 100157

Contents lists available at ScienceDirect

Sustainable Futures

journal homepage: www.sciencedirect.com/journal/sustainable-futures

A practical method for assessing sustainable traffic levels of roundabouts in urban areas using the macroscopic fundamental diagram

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A R T I C L E I N F O

Keywords:
Macroscopic fundamental diagram
Roundabouts
Capacity
Measures of effectiveness

A B S T R A C T

In traffic engineering, queue lengths, waiting times, and delay values are denoted as measures of effectiveness (MOEs). In the case of roundabouts, MOEs are generally calculated starting from the gap-acceptance theory in which the driver behaviour needs to be properly modelled. This research presents an unconventional technique, based on Macroscopic Fundamental Diagrams (MFDs), to describe sustainable traffic levels and (MOEs) of roundabouts, in terms of capacity, speed, density and delay. The deduction of MFDs is based on relatively simplified theoretical hypotheses, but the outcomes could be of interest in several practical applications. MFDs are estimated in the cases of single and double-lane roundabouts, modelled by microscopic simulations starting from a real-world case study in Italy. MFDs are obtained for a wider sample size of origin-destination traffic matrices taking into consideration time intervals of 5 min and 1 h. The results of this research demonstrate that the traffic distribution between arms can influence the shape of a roundabout-MFD. However, the proposed procedure allows us to evaluate the performance of a roundabout as a whole, taking into account numerous MOEs including the total capacity, the critical density and the Level of Service (LOS). Therefore, the use of MFDs could represent a complementary procedure for evaluating the performance of roundabouts.

1. Introduction

The capacity of road networks and intersections has always been a theme of interest for civil engineers, planners, policymakers and government authorities in recent decades. Capacity is a fundamental element for evaluating traffic conditions and estimating the performance of road networks, highway facilities and road intersections [1]. As it is well known, the most important traffic variables that characterize traffic streams are the flow (number of vehicles per time unit), the mean speed and the density (number of vehicles per lane length unit). For a section of road infrastructure in uninterrupted traffic conditions, the relationships between flow q, mean speed v and density k are called Fundamental Diagram (FD). When the macroscopic traffic flow variables are calculated as averages at the road network level, the relations that arise between q, v and k are named Macroscopic Fundamental Diagram (MFD). Therefore, the MFD is similar to the FD but rather than concerning a cross-section, it relates space-averages speed, flow and density for road segments or networks. The macroscopic fundamental diagram (MFD) is a simple method by which traffic-control effects can be evaluated. The idea of MFD was first suggested by Godfrey [2], but its theoretical assumption was developed in 2007 by Geroliminis and Daganzo [3,4] demonstrated the usefulness of the MFD using real traffic data from the city of Yokohama. MFD has been widely used to assess traffic control effects on urban road network performance [5,6]. However, studies on the existence and the properties of MFDs, have been expanded to arterial and motorway networks [7–9]. Some studies investigated the existence of MFD using traffic simulation tools [10]. For instance, in [10] the MFD of Thessaloniki's Road network was simulated using Vissim. For each simulated traffic period, the weighted traffic volume q_w and density k_w were estimated as follows:

$$q_w = \frac{\sum_i q_i \cdot l_i}{\sum_i l_i} \tag{1}$$

$$k_w = \frac{\sum_i k_i \cdot l_i}{\sum_i l_i} \tag{2}$$

where q_i, k_i and l_i are flow, density and length values of the road section i. According to [7,8,10], MFD is an aggregate property of a road network's structure that does not depend on the traffic demand in terms of origin-destination matrices. Therefore, MFDs can be a reliable tool for predicting the effects of different types of traffic management policies on road network performances. To estimate the MFD, the Loop detector data estimation (LDD estimation method) and the floating car data estimation (FCD estimation method) can be applied [11,12].

As for road intersections, queue lengths, waiting times, and delay values are also denoted as measures of effectiveness (MOE). Control delay is the most important Measure of Effectiveness for roundabouts, and it is the basis of the procedures proposed by the Highway Capacity

https://doi.org/10.1016/j.sftr.2024.100157
Received 29 September 2023; Received in revised form 4 January 2024; Accepted 13 January 2024
Available online 15 January 2024

Manual (7th Edition) for estimating the Level of Services (LOS). Generally, control delay is calculated starting from the gap-acceptance theory in which the driver behaviours need to be properly modelled, also in function of traffic flow regulation systems (priority rules). According to the *gap acceptance theory*, the capacity of an entry stream from a secondary lane to a main lane (in the case of a roundabout, entry lane and circulating lane, respectively) is a function of type: $c = c(q_1, T_c, T_f)$, being, q_i the flow of the main lane, T_c the critical gap and T_f the follow-up time (T_c and T_f are the so-called drivers' psychotechnical parameters). However, T_c cannot be measured but only estimated with different techniques (e.g. Drew's method; Raff's method; Ashworth's method, Miller's method, etc.) using a very simplified user behaviour hypothesis:

- T_c is heterogeneous: the critical gap is variable in the driver population;
- T_c is a random variable with unknown probability law;
- T_c is constant for each driver (namely, it does not change over time).

In addition, control delay at roundabouts is estimated with the models of the queue theory. Since in under-saturated traffic conditions (volume-to-capacity ratio $x < 1$) stochastic models do not permit the analysis of numerous non-stationary phenomena that happen in reality (e.g. $x \geq 0.7$–0.8) and in saturated traffic conditions ($x \geq 1$) the deterministic model is not completely suitable to be utilised, in technical applications heuristic solutions are used. For instance, the HCM model for delay estimation is based on the Akçelik and Troutbeck heuristic solutions under the condition that the arrival process follows the Poisson law and service time follows an exponential probability distribution. Finally, closed-form capacity models, including the HCM model, are based on the hypothesis that no congestion phenomena may occur at circulating lanes. All these assumptions about traffic conditions and user behaviour can only rarely occur together and at the same time during the real operating conditions of roundabouts. Therefore, MOEs estimation based on the gap-acceptance theory can often be affected by errors.

To partially solve these problems, MFD can be applied to predict average speed, average delay and the Level of Service (LOS) at roundabouts under unsaturated traffic conditions. In this research, MFDs are estimated for single-lane and double-lane roundabouts, starting from empirical data investigated for an existing roundabout in Trento (Italy). To estimate the characteristics and the shape of MFDs several microscopic traffic simulations were performed using Aimsun as the simulation tool. The microscopic traffic model was first developed and calibrated for the existing roundabout in Trento and then applied to the case studies under consideration (single-lane and double-lane roundabouts). This research demonstrates that the MFD can provide an unconventional method for estimating the measures of effectiveness (MOEs) for a single road intersection [13].

The rest of the article is organised as follows. In Section 2 the concept of the macroscopic fundamental diagram is introduced. Section 3 presents the simulation and the calibration of the microscopic traffic model in Aimsun for the case study of an existing roundabout in Trento (Italy). Section 4 describes discussions and results in terms of MFDs, capacity and levels of service for the single-lane roundabout and double-lane roundabout respectively. Section 5 gives the conclusions of this study.

2. Theory: the macroscopic fundamental diagram for a roundabout

As it is well known, the fundamental flow relationship links the macroscopic traffic variables, i.e. flow, density and space mean speed, to each other with a state equation. The fundamental flow relationship can be demonstrated in several ways. A simple and easily understandable inference of the physical meaning of this relationship is given below. Consider the generic time (t) – space(x) domain D_t in Fig. 1 and a road section (ΔX) long, away from the circulating lane. All the n vehicles of the traffic stream entering the segment Δx long and exit from it in the time interval ΔT. This is a realistic assumption because Δx is very small. Therefore, the flow is:

$$q(x) = n(x)/\Delta T \tag{3}$$

for the n vehicles in ΔT the space mean speed is:

$$\overline{v}_s(x) = \frac{n \cdot \Delta x}{\sum_{i=1}^{n} \Delta t_i} \tag{4}$$

in which Δt_i is the travel time of the i-th vehicle.

The mean vehicle density k(x) on segment Δx long can be determined and calculated for each instant t of ΔT. To calculate this density, the number of times which every vehicle i - i =1,2,…n - is present in the domain D_t, is measured by the interval Δt_i taken by the vehicle "i" during ΔT travelling the segment Δx long. Thus, in the observation interval ΔT the measurement of the vehicle presence in the segment Δx long is given, on average, by:

$$\Theta = \frac{\sum_{i=1}^{n(x)} \Delta t_i}{\Delta T} \tag{5}$$

Θ is called *occupation*.

For the density on the segment Δx long (straddling the road cross section with abscissa x) thus we have:

$$k(x) = \frac{\Theta}{\Delta x} = \frac{\sum_{i=1}^{n(x)} \Delta t_i}{\Delta T \cdot \Delta x} \tag{6}$$

If we now divide the volume q(x) constant through Δx in the interval ΔT, $q(x) = n(x)/\Delta T$, by the mean-travel speed $\overline{v}_s(x)$, it results:

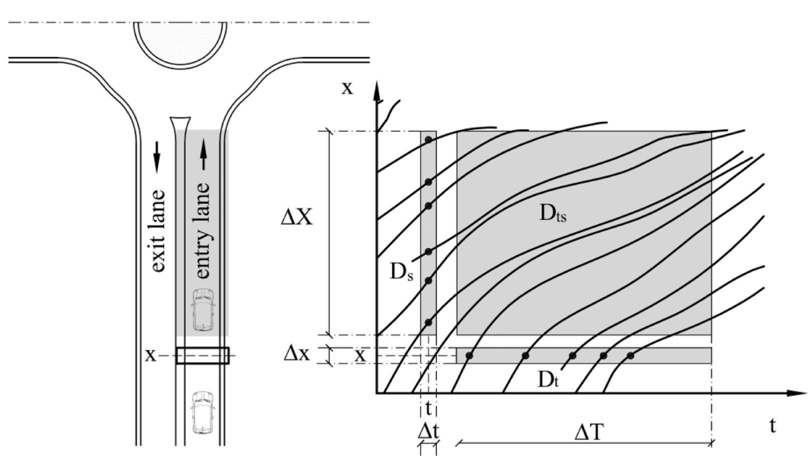

Fig. 1. Vehicle trajectories in an elementary time/space domain related to the entry lane.

$$\frac{q(x)}{\overline{v}_s(x)} = \frac{n(x)}{\Delta T} \cdot \frac{\sum_{i=1}^{n(x)} \Delta t_i}{n(x) \cdot \Delta x} = \frac{\sum_{i=1}^{n(x)} \Delta t_i}{\Delta T \cdot \Delta x} \tag{7}$$

coinciding with expression of the vehicle density k(x) (cf. Eq. (6)), therefore it results:

$$q(x) = k(x) \cdot \overline{v}_s(x) \tag{8}$$

where $\overline{v}_s(x)$is:

$$\overline{v}_s(x) = \frac{1}{\frac{1}{n}\sum_{i=1}^{n} \frac{1}{v_i(x)}} \tag{9}$$

in which $v_i(x)$, i=1,2, …n, is the instantaneous speed of the vehicles in ΔT, in any location x of the segment ΔX long and $\overline{v}_s(x)$ is the harmonic mean of the instantaneous vehicle speeds v_i measured at the location x.

Eq. (8) is the fundamental flow relationship referred to the road sections x, or to the elementary segment Δx straddling x observed during the elementary time interval Δt. For the sake of simplicity, we can adopt the following notation:

$$q = k \cdot v \tag{10}$$

where v denotes the space mean speed.

Haight [14] denoted the flow-density diagram as the “Fundamental Diagram of Road Traffic”. Hence, the traditional fundamental diagram (FD) represents graphically the Eq. (10) in the form v=v(q) or q=q(k). The FD has a concave functional shape, often approximated by a shape-bell curve; it displays a peak flow (capacity) at a particular vehicle density k_c (called critical density) and a zero flow at zero density and jam density k_{jam}. Therefore, traffic flows should be regulated to minimize congestion (i.e. to avoid $k > k_c$) and jam phenomena (i.e. to avoid $k = k_{jam}$) and these conditions require a clear understanding of the relationships among traffic flow, density, and speed under a plethora of traffic conditions.

Theoretically, FD should only be used in the case of uninterrupted and stationary traffic conditions, but for technical purposes, it is also applied in other traffic circumstances. The macroscopic fundamental diagram (MFD) spreads the meaning of FD over a road network within a given cordon, that includes multiple road segments and intersections. However, the MFD is an empirical diagram that is generally different from the theoretical FD due to the presence of interrupted and non-stationary traffic conditions (e.g. generated by at-grade road intersections, unsignalized intersections, bottlenecks, etc.). Many studies have identified MFDs in different types of road infrastructures and road networks, including urban and rural roads [15]. In all these cases, it can be found an empirical formulation similar to that of Eq. (10):

$$Q = K \cdot V \tag{11}$$

If a speed-density relationship is adopted such that speed is a function of density only, then one may write:

$$Q(K) = K \cdot V(K) \tag{12}$$

where Q is the sum of all vehicles that enter the cordon during the observed time interval (veh/h), K is the traffic density averaged over the area of the cordon (veh/km) and V is the mean speed of vehicles within the cordon (km/h).

It should be noted that Eq. (11), unlike Eq. (10), is only an empirical formulation and cannot be mathematically demonstrated because road networks and road intersections (including roundabouts) do not operate under uninterrupted and steady-state traffic conditions. The MDF can also be estimated for a roundabout since it can be considered as a small road network. In general, roundabouts are analysed in terms of capacities, delays and levels of service of each entry, but understanding the relationships between the macroscopic traffic variables by analysing the MFD for the entire intersection could be an additional tool for traffic modelling and management. This is the main reason why the present research investigates the deduction of MFDs for roundabout intersections. In this regard, it has been demonstrated [16] that the macroscopic traffic laws, and therefore the MFD can be deduced directly from microscopic traffic models (i.e. non-linear car-following model) as those implemented in microscopic traffic software. This is why the macroscopic description of traffic phenomena can be derived from a microscopic analysis of traffic streams.

3. Model simulation and calibration

In this paper, a traffic dataset is analyzed based on microscopic simulations to estimate the MFD for one-lane and two-lane traffic roundabouts in an urban context. The research starts from a case study in Trento city that allowed the calibration of the traffic model in the Aimsun environment. Trento is an Italian city with about 118,000 inhabitants and is considered to be one of the most sustainable cities in Italy in terms of transport, energy and environment. Nevertheless, some congested intersections can be easily found. Is this the case of the four arms roundabout between the streets “via del Brennero”, “via dei Caduti di Nassiriya” and “via Antonio Pranzelores”. It is a roundabout with a diameter of 50 m and four two-lane entries and exits (Fig. 2), except “via Antonio Pranzelores which has one lane in each direction. The roundabout is located in an urban area with a speed limit of 50 km/h. The layout is almost symmetric with angles of nearly 90° between the arms.

Traffic simulations were performed by using AIMSUN Next which is a tool widely applied in technical activities and experimental research since it is a versatile tool that allows estimating several traffic indices including capacity and level of services of roads and intersections.

Aimsun is based on the microscopic traffic model developed by Gipps [17,18].

The Gipps model comprises a system of two relationships with one governing the free-flow regime and the other governing the car-following regime. The free-flow relationship is an outcome of fitting empirical data, and its function is to accelerate a vehicle from its initial speed asymptotically toward its desired speed without oscillation. The car-following states that the maximum speed to which a vehicle (n) can accelerate during a time interval (t, t+T) is given by the relationship [19]:

$$V_a(n, t+T) = V(n, t) + 2.5a(n)T\left(1 - \frac{V(n, t)}{V*(n)}\right)\sqrt{0.025 + \frac{V(n, t)}{V*(n)}} \tag{13}$$

in which:

- V(n, t) is the speed of the vehicle n at time t;
- V*(n) is the desired speed of the vehicle (n) for the current position;
- a(n) is the maximum acceleration for the vehicle n;
- T is the reaction time.

Instead, the maximum speed that the same vehicle (n) can reach during the same time interval (t, t+T), according to its characteristics and the limitations imposed by the presence of the lead vehicle (n−1), is [18]:

$$V_b(n, t+T) = d(n)T + \sqrt{d(n)^2T^2 - d(n)\left[2\{x(n-1, t) - s(n-1) - x(n, t)\} - V(n, t)T - \frac{V(n-1, t)^2}{d'(n-1)}\right]} \tag{14}$$

(a) (b)

Fig. 2. (a) Example roundabout in Trento, Italy. (b) the roundabout network model built in Aimsun.

in which:

- d(n) (< 0) is the maximum deceleration desired by vehicle n;
- x(n, t) is the position of the vehicle n at time t;
- x(n−1, t) is the position of the preceding vehicle (n−1) at time t;
- s(n−1) is the effective length of the vehicle (n−1);
- d'(n−1) is an estimation of the vehicle (n−1) desired deceleration.

The speed of the vehicle (n) during the time interval (t, t+T) is the minimum of the values obtained with expressions (13) and (14):

$$V(n, t+T) = \min\{V_a(n, t+T), V_b(n, t+T)\} \quad (15)$$

The position of the vehicle n inside the current lane is updated by taking this speed into the movement equation:

$$x(n, t+T) = x(n, t) + V(n, t+T)T \quad (16)$$

In short, the Gipps car-following model is derived from a safety condition related to the distance between the leader vehicle and the follower one.

According to [20] the Gipps model is “overly safe” because excessive car-following distances result. Besides the car-following model (Eqs. (11)–(14)), Aimsun includes the following ones [19]: lane-changing model; look-ahead model; and microscopic gap-acceptance model.

According to [19], microsimulation analyses require five steps:

- project scope;
- package selection;
- data assembly and input;
- verification and calibration;
- alternatives analysis and conclusions.

Other authors claim that there should be seven steps to achieve good results in traffic simulations [21,22].

The development of a traffic model in Aimsun requires four main key phases: (1) creation of the infrastructure and traffic demand models; (2) parameter selection criteria; (3) model calibration and validation (so that the model better matches the field measurements and traffic performance can be estimated more accurately); (4) definition of scenario and modeling.

The calibration process allows us to achieve the best fit between the simulated traffic flow variables. Then, the calibration process is essential for obtaining simulated values of the traffic flow variables that reflect the reality of the values observed in the real world.

This process consists of modifying the values of numerous traffic parameters to match the results of the simulation to the observed data. The most commonly used parameters are delays, queues and travel times. In this research, the calibration process was performed considering simulated and observed queues.

Fig. 2a shows the current roundabout layout that was modelled in Aimsun and used to simulate the so-called Scenario 0 (Fig. 2b).

To calibrate the model, several traffic measurements were performed through aerial videos using a drone (model “DJI Mavic mini 2″) with a survey duration of about 6 h. Due to the limited flight altitude allowed in urban areas, a zone with a diameter of 100 m plus the roundabout diameter (cf. Fig. 3) was recorded, resulting in a total length of the entry arms of about 400 m (4 × 100 m).

Entering, leaving and traffic flow distributions in the road network behind the observed cordon were measured by using computer vision and deep learning procedures [23,24] at 15 min time intervals. A total of 24-time intervals were collected and the corresponding Origin-Destination (O/D) matrices were estimated and the queues were measured.

A sensitivity analysis was carried out to explore the impacts of different combinations of model parameters on the modelled outputs ensuring they matched or were comparable with real-world observations. In this regard, among the most commonly used measures of goodness of fit are the root mean square error (RMSE), the root mean squared normalized error (RMSNE), the mean error (ME), the mean normalized error (MNE) and the GEH (Geoffrey E. Havers) index [25] applied in this research. The GEH index is obtained as:

$$GEH_i = \sqrt{\frac{2(x_i - y_i)^2}{x_i + y_i}} \quad (17)$$

where x_i and y_i are the *i*th simulated and observed values of the considered traffic variable (e.g. flow, density, queue, delay, etc.).

It then estimates an aggregated index using the following algorithm [19]:

For i = m (number of counting stations)
If $GEH_i \leq 5$, then set $GEH_i = 1$
Otherwise set $GEH_i = 0$
End if;
End for;
Let:

$$GEH = \frac{1}{m}\sum_{1=1}^{m} GEH_i \quad (18)$$

If GEH ≥ 85 % then accept the model, otherwise reject the model
Endif.

The key aspect of any calibration process is the comparison between simulation outputs and observed measurements of various traffic variables [26–28]. According to [19,29–32] some model parameters should be adjusted in Amisun through an iterative process to obtain simulation

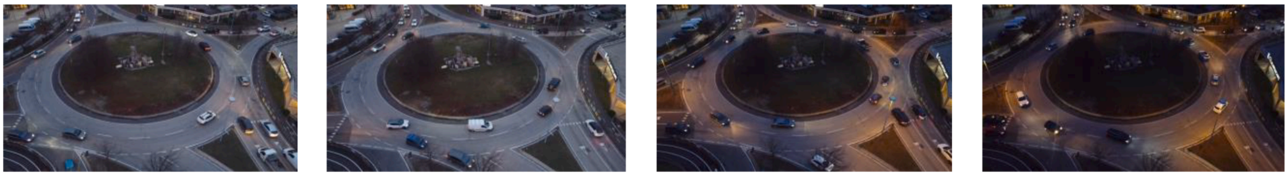

Fig. 3. Screenshot examples of traffic measurements taken from aerial videos.

output close to the observed traffic variables.

The primary kinematic variable taken into account in this study for calibrating the model was the vehicle acceleration. In this regard, the values of real accelerations and speed limit acceptance were measured in the urban road network, near the intersection of interest using a test vehicle equipped with accelerometers and recording systems.

The resulting values deduced during the real-world measures are:

- maximum acceleration: 3.10–4.10 m/s^2 (default values in Aimsun: 2.60–3.40 m/s^2);
- normal deceleration: 3.70–4.90 m/s^2 (default values in Aimsun: 3.50–4.50 m/s^2).

The main outcomes of the fine-tuning parameter process are: speed acceptance = 1.00; time gap = 1.00; max acceleration = 4.10 m/s^2; safety margin factor = 1.00; sensitivity factor = 1.00.

Using the previous values, the current roundabout layout (Fig. 2a) was simulated in the Aimsun environment (the so-called Scenario 0) for each of the 24-time intervals under consideration.

The outcomes confirmed the ability of the traffic model to reproduce the observed queue length throughout 15 min sampling intervals. As shown in Table 1, the GEH index, based on the deviation between simulated queue (x_i) and observed queue (y_i) values allows us to accept the model since it satisfies the limits described above (cf. Eq. (18)). As a matter of fact, it results: GEH = (23.00+23.00+21.00+20.00)/(4*24) *100 = 91 % (more than the minimum acceptable value, i.e. 85 %).

3.1. MFD estimation for single-lane and double-lane roundabouts

To estimate the main properties of a MFD, microscopic traffic simulation studies were performed using the Aimsun tool by analysing two layouts: a) a single-lane roundabout (Fig. 4a) with four single-lane entries and exits, b) a double-lane roundabout with four double-lane entries and exits (Fig. 4b).

The choice to analyse single-lane and double-lane roundabouts results from the fact that these are the most frequently used conventional roundabouts in the world [33–35]. Both roundabout layouts (outer diameter of 50 m) are based on the shape of the existing roundabout of Fig. 2a that was studied to calibrate the traffic model, as explained in the previous Section. The speed limit is 50 km/h for all approaching streets as in the existing roundabout. Two very different traffic scenarios were defined for investigating the MFD, as follows:

- *Scenario 1* ("balanced – b"): equal volumes at the four entries and a traffic distribution matrix (i.e. origin/destination percentage matrix) $P^1_{O/D}$ in which traffic streams were set to 33.33 % for right turns, 33.33 % for left turns and 33.33 % for through movements;
- *Scenario 2* ("unbalanced – ub"): equal volumes at the four entries and a traffic distribution matrix $P^2_{O/D}$ in which traffic streams were set to 60 % for right turns, 20 % for left turns and 20 % for through movements.

For each entry traffic demand (expressed by a vector) $Q_e(m) = [Q_{ei}(m)]$ - with i = 1, 2, 3, 4 – we obtain different origin/destination matrices:

Table 1
GEH_i index, calculated with the measured (y_i) and estimated (x_i) queue values (24 intervals of time).

Simulation n.	Arm 1				Arm 2				Arm 3				Arm 4			
	y_i	x_i	GEX_i	Set value of GEX_i	y_i	x_i	GEX_i	Set value of GEX_i	y_i	x_i	GEX_i	Set value of GEX_i	y_i	x_i	GEX_i	Set value of GEX_i
1	5	5	0.00	1.00	6	2	2.00	1.00	4	3	0.53	1.00	10	5	1.83	1.00
2	5	4	0.47	1.00	6	4	0.89	1.00	4	5	0.47	1.00	6	7	0.39	1.00
3	4	4	0.00	1.00	4	4	0.00	1.00	7	8	0.37	1.00	9	6	1.10	1.00
4	4	2	1.15	1.00	5	2	1.60	1.00	3	3	0.00	1.00	9	5	1.51	1.00
5	3	5	1.00	1.00	4	5	0.47	1.00	5	7	0.82	1.00	12	8	1.26	1.00
6	6	7	0.39	1.00	7	3	1.79	1.00	5	4	0.47	1.00	11	6	1.71	1.00
7	5	5	0.00	1.00	6	5	0.43	1.00	4	7	1.28	1.00	22	3	5.37	1.00
8	5	3	1.00	1.00	7	3	1.79	1.00	8	4	1.63	1.00	11	6	1.71	1.00
9	7	5	0.82	1.00	8	5	1.18	1.00	6	9	1.10	1.00	17	7	2.89	1.00
10	7	8	0.37	1.00	12	8	1.26	1.00	10	9	0.32	1.00	16	11	1.36	1.00
11	9	8	0.34	1.00	10	8	0.67	1.00	8	9	0.34	1.00	15	13	0.53	1.00
12	9	7	0.71	1.00	13	4	3.09	1.00	9	15	1.73	1.00	17	7	2.89	1.00
13	9	8	0.34	1.00	12	8	1.26	1.00	8	9	0.34	1.00	17	16	0.25	1.00
14	10	7	1.03	1.00	11	7	1.33	1.00	11	13	0.58	1.00	15	11	1.11	1.00
15	12	10	0.60	1.00	13	10	0.88	1.00	11	18	1.84	1.00	19	18	0.23	1.00
16	15	14	0.26	1.00	16	14	0.52	1.00	13	15	0.53	1.00	20	3	5.01	0.00
17	15	13	0.53	1.00	12	13	0.28	1.00	18	19	0.23	1.00	16	14	0.52	1.00
18	14	15	0.26	1.00	15	15	0.00	1.00	13	16	0.79	1.00	22	21	0.22	1.00
19	15	16	0.25	1.00	19	16	0.72	1.00	19	2	5.25	0.00	29	7	5.19	0.00
20	14	14	0.00	1.00	22	14	1.89	1.00	13	15	0.53	1.00	29	26	0.57	1.00
21	18	16	0.49	1.00	19	16	0.72	1.00	22	31	1.75	1.00	23	18	1.10	1.00
22	22	23	0.21	1.00	24	23	0.21	1.00	29	5	5.82	0.00	35	26	1.63	1.00
23	22	35	2.44	1.00	24	35	2.03	1.00	36	14	4.40	1.00	28	4	6.00	0.00
24	24	3	5.72	0.00	35	7	6.11	0.00	35	6	6.41	0.00	41	7	6.94	0.00
TOTAL				**23.00**				**23.00**				**21.00**				**20.00**

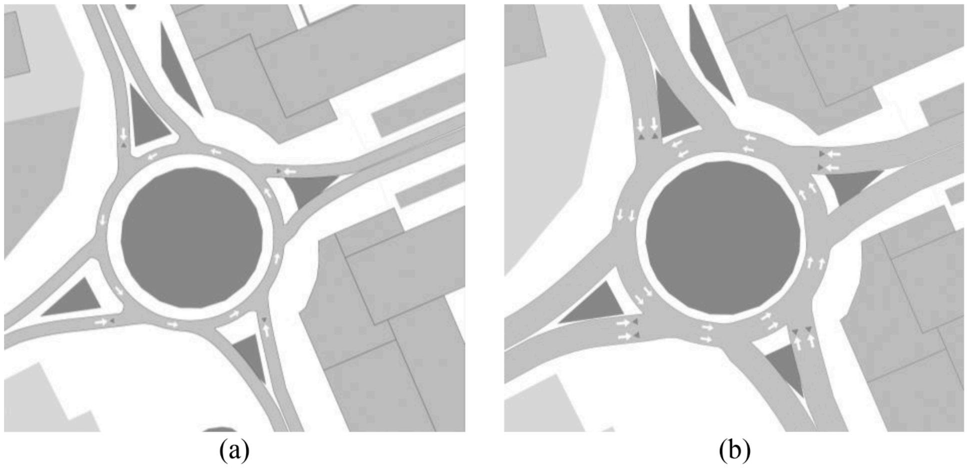

Fig. 4. Single-lane roundabout (a) and double-lane roundabout (b) network model built in Aimsun.

- Scenario 1: $M^{O/D1}(m) = P^{O/D1} \bullet Q^{e}(m) = [Q_{ij}(m)]$, i, j = 1, 2, 3, 4;
- Scenario 2:$M^{O/D2}(m) = P^{2}_{O/D} \bullet Q_{e}(m) = [Q_{ij}(m)]$, i, j = 1, 2, 3, 4;

where $Q_{ij}(m)$ is the flow that enters from the generic arm "i" and exits from the generic arm "j" calculated considering the entry traffic demand $Q_e(m)$.

In total m = 30 origin-destination matrices were considered for Scenario 1 and m = 30 for Scenario 2, by a variation of $Q_e(m)$ in the range from 1 pc/h to 1.215 pc/h (with progressive increments of 25 pc/h).

The macroscopic flow parameters – flow, speed (harmonic mean) and density – were calculated with reference to time intervals of 15 min and 1 h. This approach allowed the generation of large samples of traffic data under prefixed input flows in terms of origin-destination matrices.

Figs. 5a and 5b illustrate that the total inflow and total outflow volumes are similar above all in the low-medium total flow range (approximately 2000 pc/h and approximately 2500 pc/h for the single-lane roundabout and double-lane roundabout respectively) because vehicles travel throughout the roundabout without noteworthy delay. Instead, in the higher flow range, the deviations between outflow and inflow volumes increase more and more as the flow approaches the capacity value, even if these differences are not remarkable. According to [13], MFDs and capacities should be rigorously estimated by referring to outflow volumes due to the circumstance that for large density numerous vehicles are temporarily stored within queues and consequently it results that outflow < inflow as clearly shown in Figs. 6 and 7.

Nevertheless, for comparison purposes with respect to conventional procedures for roundabout capacity estimation (i.e entry capacity, simple capacity and total capacity, [33,34]), in this research MFDs and capacities will be related only to the total inflow Q_T.

In fact, inflow volume can be more appropriate for roundabout performance analysis evaluation (e.g., calculation of capacity, delay, queue) because generally traffic demand, which determines arriving vehicles and therefore the total inflow volume, is considered as the main input data.

All this considered, a series of simulation runs were performed to evaluate several traffic variables including the inflowing volume (Q_T) and outflowing traffic volumes, the vehicle density (K), the harmonic mean speed (V), the vehicles inside and outside the cordon, the delay time and the total travel time. These values allowed us to identify and collect the pairs (V; K), (Q_T; K), (V; Q_T) for time intervals $\Delta T = 5$ min and $\Delta T = 1$ h and consequently the relations of interest (i.e. speed-density, inflow-density and speed-inflow relations).

4. Results and discussions

The results are depicted in Figs. 8–11 for the roundabouts and traffic scenarios under consideration.

As a main result of simulations, the Macroscopic Fundamental Diagrams are exhibited in the form of the (V; Q_T) -diagram in Figs. 8b, 9b, 10b, 11b and also in the form of the (Q_T; K)- diagram in Figs. 8c, 9c, 10c, 11c. The shape of the experimental points complies with the upper

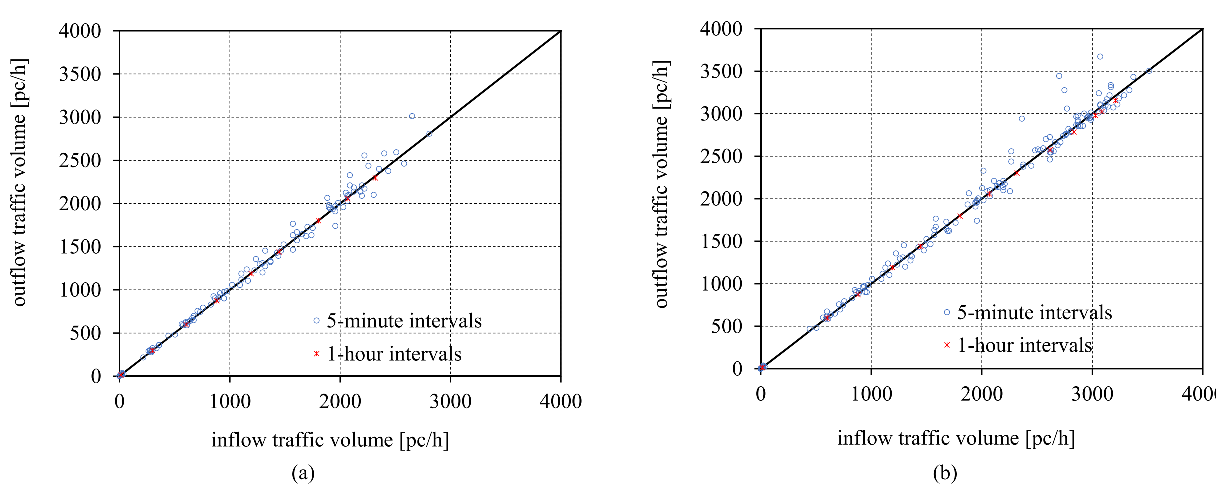

Fig. 5. Outwflow versus inflow traffic volumes (a) single-lane roundabout - *Scenario 1* ("balanced – b"). (b) double-lane roundabout - *Scenario 1* ("balanced – b").

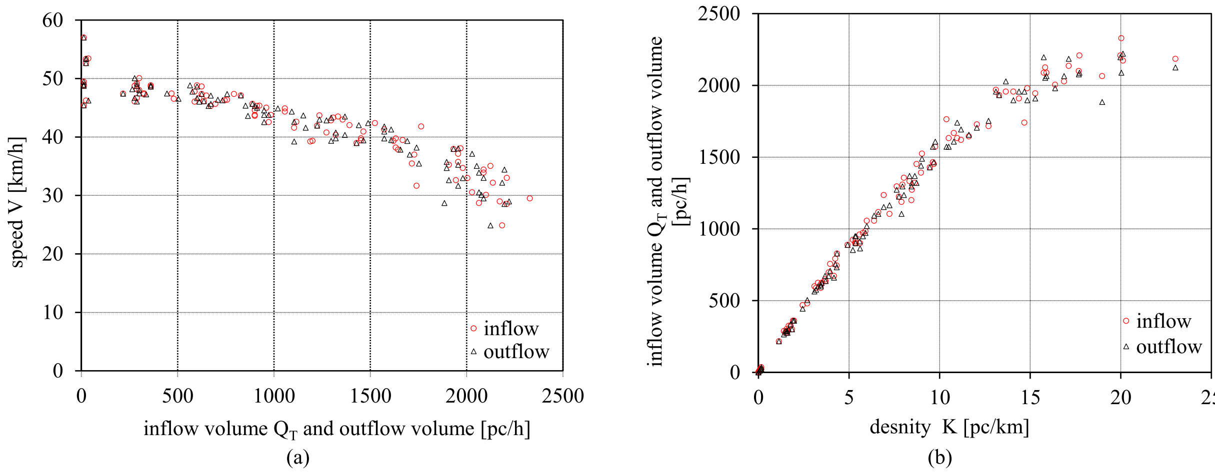

Fig. 6. Speed-flow and flow-density scatter points - single-lane roundabout, Scenario 1 (“balanced – b”).

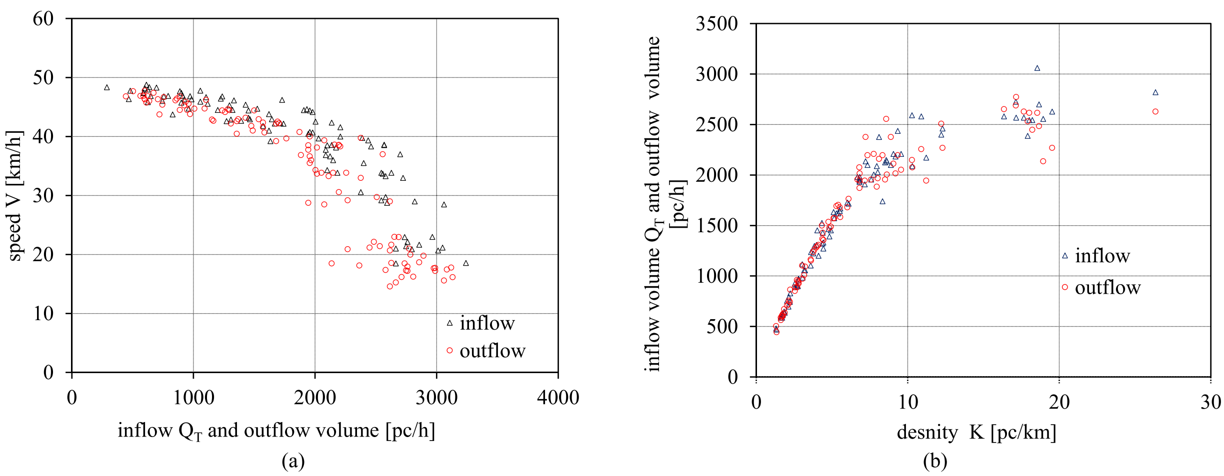

Fig. 7. Speed-flow and flow-density scatter points - double-lane roundabout, Scenario 1 (“balanced – b”).

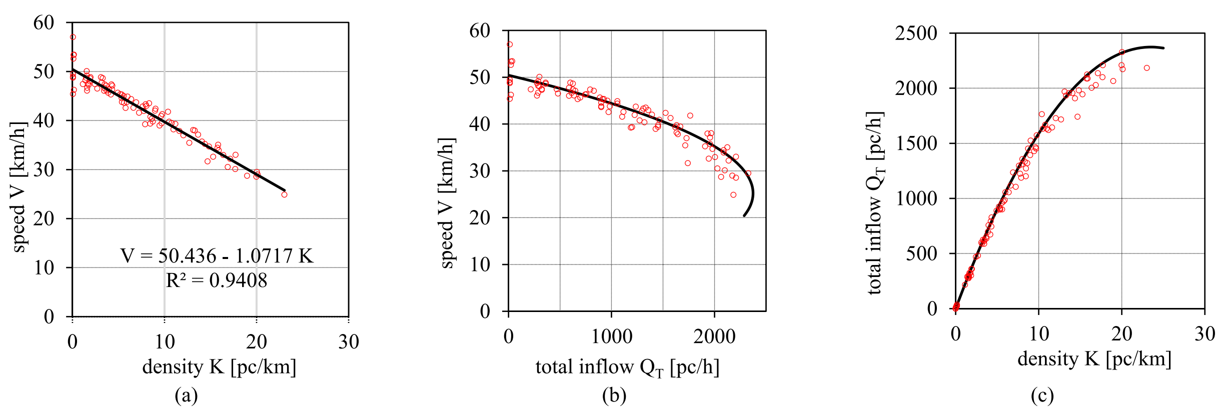

Fig. 8. Speed-density, speed-flow and flow-density scatter points and calibrated Greenshields model for the single-lane roundabout - *Scenario 1* (“balanced – b”).

portion (stable traffic conditions) of conventional fundamental diagrams (FDs). Nevertheless, some differences appear: in the MFDs obtained in this research the harmonic mean speed (V) and the inflowing volume (Q_T) do not drop to zero under high-density values. This is because V is calculated taking into consideration the speed of vehicles within the cordon (i.e. entering and exiting the roundabouts) and the vehicle speed on exit lanes tends to a value close to the free-flow speed. The scatter points (V; Q_T) and (Q_T; K) show a limitation of the total inflow Q_T that indicates the total roundabout capacity (C). To estimate the total capacity and the critical density (K_c) values we opted for the bell-shaped curve model proposed by Greenshields’ which proved to be the best in interpreting the available data. As a matter of fact, the simulated pairs speed-density (V; K) are linearly related as shown in Figs. 8a, 9a, 10a, 11a.

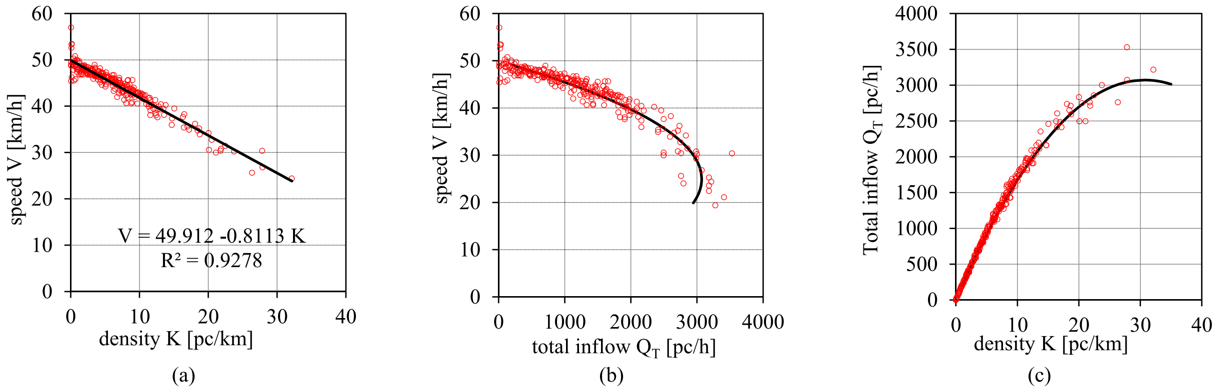

Fig. 9. Speed-density, speed-flow and flow-density scatter points and calibrated Greenshields model for the single-lane roundabout - *Scenario 2* ("unbalanced – ub").

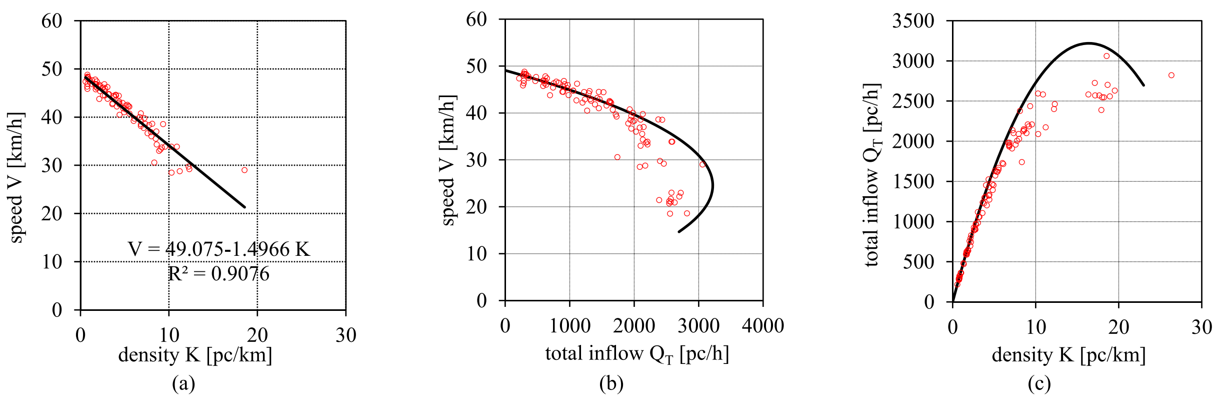

Fig. 10. Speed-density, speed-flow and flow-density scatter points and calibrated Greenshields model for the double-lane roundabout - *Scenario 1* ("balanced – b").

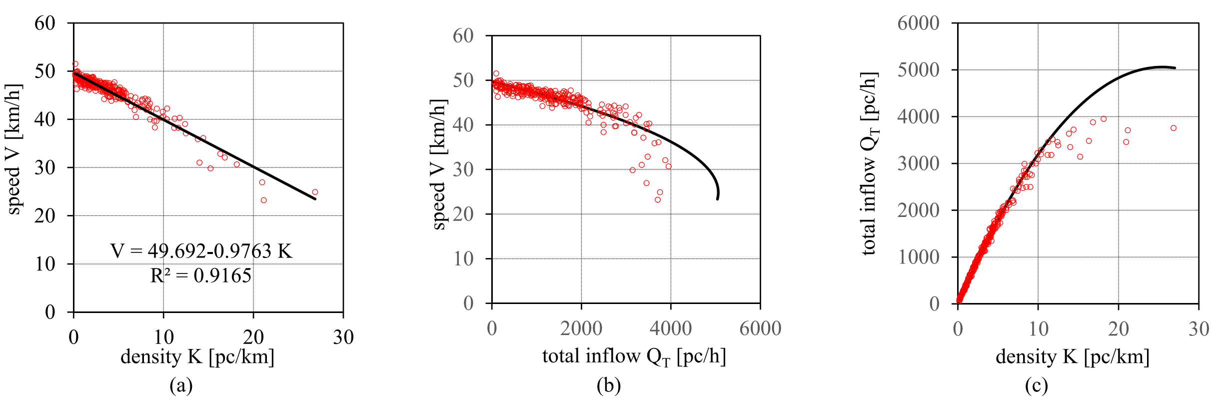

Fig. 11. Speed-density, speed-flow and flow-density scatter points and calibrated Greenshields model for the double-lane roundabout - *Scenario 2* ("unbalanced – ub").

By analysing Figs. 8a, 9a, 10a, 11a we can write a typical Greenshields' relationship between the mean speed V and the density K, as follows:

$$V = V_f - |\beta| \cdot K \tag{19}$$

Considering a roundabout with n arms each with l lanes, and taking into consideration the fundamental flow equation (Eq. (11)), the total inflow Q_T can be calculated with the following relationship $Q_T = Q_T(K)$:

$$Q_T = n \cdot l \cdot V \cdot K \tag{20}$$

$$Q_T = n \cdot l \cdot (V_f - |\beta| \cdot K) \cdot K = n \cdot l \cdot V_f \cdot K - n \cdot l \cdot |\beta| \cdot K^2 \tag{21}$$

The critical density K_c is obtained by deriving the previous relationship and by imposing the condition $Q_T^{'} = 0$

$$Q_T^{'} = n \cdot l \cdot V_f - 2 \cdot n \cdot l \cdot |\beta| \cdot K \tag{22}$$

$$Q_T^{'} = n \cdot l \cdot V_f - 2 \cdot n \cdot l \cdot |\beta| \cdot K = 0 \tag{23}$$

$$K = K_c = \frac{V_f}{2|\beta|} \tag{24}$$

Therefore, the total capacity of the roundabout can be estimated by Eqs. (21) and (24). Under the condition $K = K_c$, it results $C = \max(Q_T)$:

$$C = n \cdot l \cdot (V_f - |\beta| \cdot K_c) \cdot K_c \quad (25)$$

$$C = \frac{n \cdot l \cdot V_f^2}{4|\beta|} \quad (26)$$

that allows us to estimate the total roundabout capacity C. With Eq. (26) the following relationships can be derived:

- 4 arms single-lane roundabout (n = 4, l = 1):

$$C = \frac{V_f^2}{|\beta|} \quad (27)$$

- 4 arms double-lane roundabout (n = 4, l = 2):

$$C = \frac{2 \cdot V_f^2}{|\beta|} \quad (28)$$

Figs. 8–11 represent the calibrated Greenshield model with continuous lines, instead, Table 2 summarizes the estimated speed-density relationships along with the critical density K_c and total capacity C of each configuration. As expected, it can be observed that the layout and traffic scenario notably affect the roundabout total capacity, which ranges from 2374 pc/h to 5058 pc/h for the cases under consideration. Outcomes in terms of Average delay time (DT), as a function of the total inflow volume Q_T, are shown in Figs. 12 and 13. DT is the difference between the expected travel time (the time it would take to traverse the system under ideal conditions) and the actual travel time. It is calculated as the average of all vehicles and then converted into time per kilometer. It does not include the time spent in a virtual queue.

The Average delay time may be considered as a measure of effectiveness (MOE) for a road intersection as a whole. As expected, the double-lane roundabout gives the best performance in terms of DT compared to the single-lane roundabout (Figs. 12 and 13) in all ranges of total flow and scenarios.

As shown, the dispersion degree of Delays is relatively high. Regression analysis of the delays data in Figs (12) and (13) is carried out by the least square method and R^2 – which ranges from 0.75 to 0.87 - is acquired for the exponential fitting curve. However, Delay Time can be estimated with an exponential function in the form:

$$DT = \psi \cdot e^{\phi \cdot Q_T} \quad (29)$$

Therefore, from Eq. (29), the total inflow that produces a prefixed value of delay time can be approximately calculated with the expression:

$$Q_T = \frac{\ln \frac{DT}{\psi}}{\phi} \quad (30)$$

According to the HCM 7 h Edition [36], the Level of Service (LOS) for motorized vehicles in roundabouts is calculated as a function of the control delay. However, when considering an urban road network that includes roundabout intersections through-vehicle travel speed is used to characterize the LOS. This speed reflects the factors that influence running time along each link and the delay incurred by through vehicles at each boundary intersection. We propose a different method for assessing LOS based on the Macroscopic Fundamental Diagram. In particular, the MFD curve can be divided into different portions, analogously to those established for road infrastructures under uninterrupted traffic conditions (freeways). According to the HCM 7 h Edition the saturation degrees (volume-to-capacity ratios) that define each LOS limit can be calculated as follows:

- $x_A = Q_{TA}$ / C = density LOS A / density LOS F = 0.25;
- $x_B = Q_{TB}$ / C = density LOS B / density LOS F = 0.39;
- $x_C = Q_{TC}$ / C = density LOS C / density LOS F = 0.57;
- $x_D = Q_{TD}$ / C = density LOS D / density LOS F = 0.78;
- $x_E = Q_{TE}$ / C = density LOS E / density LOS F = 1.0.

Therefore, based on the roundabout MFD we can describe the LOS as follows (Table 3):

- LOS A primarily describes free-flow operation. Vehicles are completely unimpeded in their ability to maneuver within the traffic stream. Control delay at the roundabout is minimal. The volume-to-capacity ratio is no greater than 0.25;
- LOS B describes reasonably unimpeded operation. The ability to maneuver within the traffic stream is only slightly restricted, and control delay at the intersection is not significant. The volume-to-capacity ratio is no greater than 0.39;
- LOS C describes stable operation. Longer queues at the roundabout may contribute to lower travel speeds. The volume-to-capacity ratio is no greater than 0.57;
- LOS D indicates a less stable condition in which small increases in total inflow may cause substantial increases in delay and decreases in travel speed. The volume-to-capacity ratio is no greater than 0.78;
- LOS E is characterized by unstable operation and significant delay. The volume-to-capacity ratio is no greater than 1.0;
- LOS F is characterized by flow at extremely low speed. Congestion, high delay and extensive queuing are associated with this LOS. The volume-to-capacity ratio is greater than 1.0.

Fig. 14 shows the Macroscopic Fundamental Diagrams and Levels of Service for the cases under consideration; instead, Tables 4–7 summarize the values of total inflow volume, mean speed and density intervals associated with each LOS.

It is worth underlining that an accurate measurment of traffic variables is essential for the correct deduction of MFDs. In this regard, at present, traditional and modern traffic variable measure techniques, including the Loop detector data estimation (LDD estimation method) and the floating car data estimation (FCD estimation method), guarantee the estimation of traffic variables in a very precise way.

Table 2
Speed-density relationships and values of free-flow speed V_f, critical density K_c and total capacity C.

Geometric and Traffic parameters	Single-lane roundabout		Double-lane roundabout	
	Scenario 1	Scenario 2	Scenario 1	Scenario 2
number of entry lanes l	1	1	2	2
number of exit lanes l	1	1	1	1
Number of arms n	4	4	4	4
Speed-density relationship	V=50.436–1.0717K	V=49.912–0.8113K	V=49.075–1.4966K	V=49.692–0.9763K
R^2	0.9408	0.9278	0.9076	0.9165
Free-flow speed V_f [km/h]	50.44	49.91	49.08	49.69
β [km^2/(pc•h)]	-1.0717	-0.8113	-1.4966	-0.9763
Critical density K_c [pc/km]	23.57	30.76	16.40	25.45
Total capacity C [pc/h]	2374	3071	3233	5058

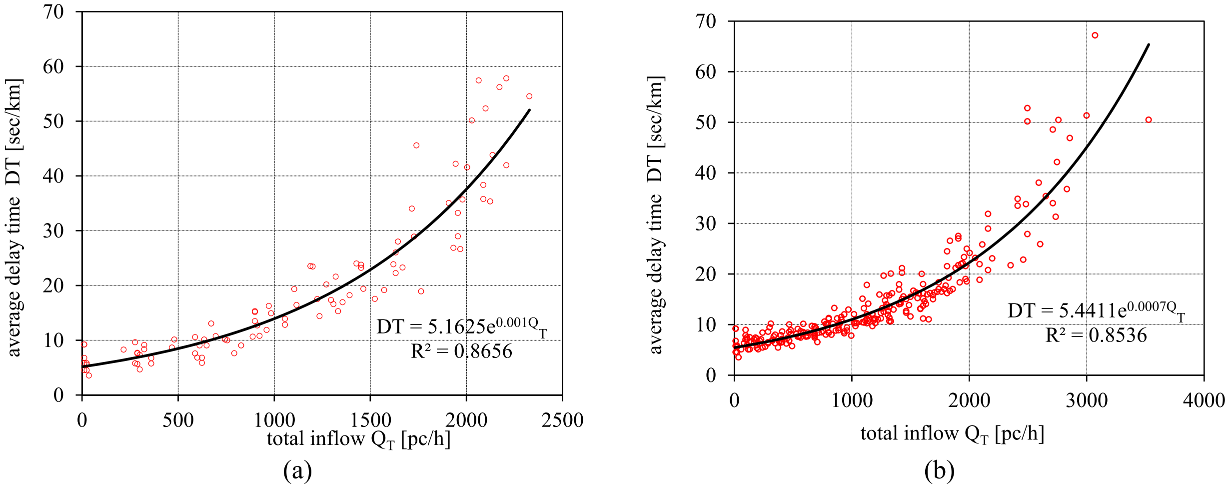

Fig. 12. Delay Time as a function of the total inflow volume for the single-lane roundabout. (a) Scenario 1; (b) Scenario 2.

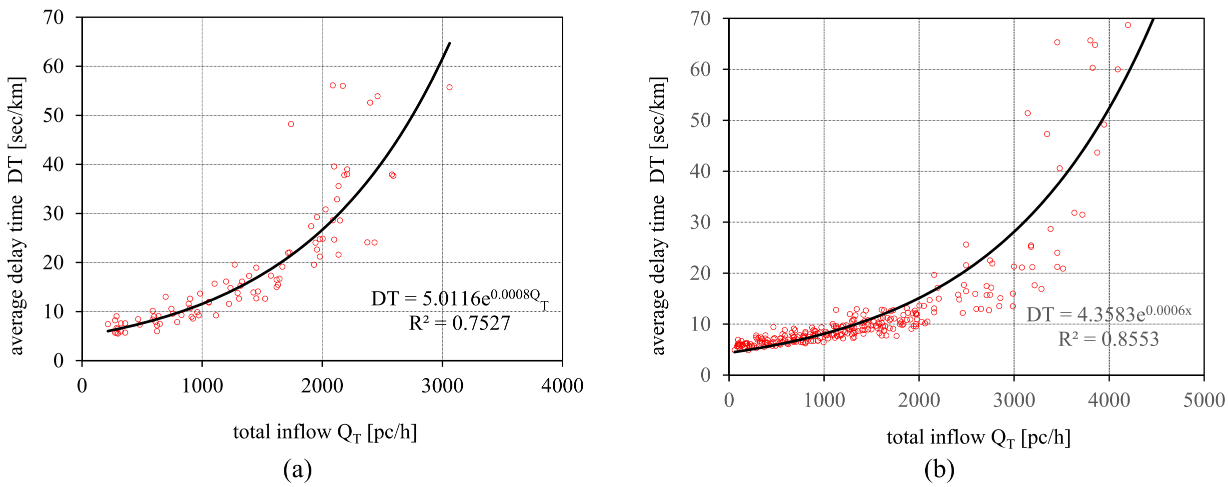

Fig. 13. Delay Time as a function of the total inflow volume for the double-lane roundabout. (a) Scenario 1; (b) Scenario 2.

Table 3
LOS Criteria based on the MFD.

Level of Service LOS	saturation degree x (volume-to-capacity ratio)
A	≤ 0.25
B	≤ 0.39
C	≤ 0.57
D	≤ 0.78
E	≤ 1.00
F	> 1.00 (Demand exceeds capacity)

5. Conclusions

The macroscopic fundamental diagram (MFD), which relating flow, density, and speed, has been analysed in some empirical studies that have shown how the MFD can be an efficient tool for traffic management and control. In this article, we analyze an extensive traffic data set based on microsimulations to estimate the MFD for single-lane and double-lane roundabouts in an urban context. The research starts from a case study in Italy that allowed the calibration of the traffic model in the Aimsun environment. Two very different traffic scenarios were studied: (a) *Scenario 1* in which traffic streams were set to 33.33 % for right turns, 33.33 % for left turns and 33.33 % for through movements; (b) *Scenario 2* in which traffic streams were set to 60 % for right turns, 20 % for left-turns and 20 % for through movements. In addition, 30 different origin-destination traffic matrices were considered for each scenario. Among others, the following traffic variables were estimated for time intervals $\Delta T = 5$ min and $\Delta T = 1$ h: inflowing volume (Q_T) outflowing traffic volumes, vehicle density (K), harmonic mean speed (V), vehicles inside and outside the cordon, delay time and total travel time. In particular, the scatter points (V; K), (Q_T; K), (V; Q_T) allow the deduction of MFDs for all the case studies under consideration.

The main outcomes of this research are briefly explained below:

- the total inflow and total outflow values are very close to each other in the low-medium flow range for the reason that vehicles travel throughout the road intersection without significant delay;
- for performance analysis purposes, the inflow volume is more suitable with respect outflow volume;
- in MFDs the harmonic mean speed (V) and the inflowing volume (Q_T) do not drop until zero for high-density values;
- the shape of a MFD is dependent on the traffic demand and is not only an inherent property of the roundabout itself;
- MFDs are a useful tool for estimating the critical density K_c, total capacity C and the Level of Service (LOS).

In conclusion, the MFD provides an efficient technique to define the performance of a roundabout as a whole, starting from simple traffic

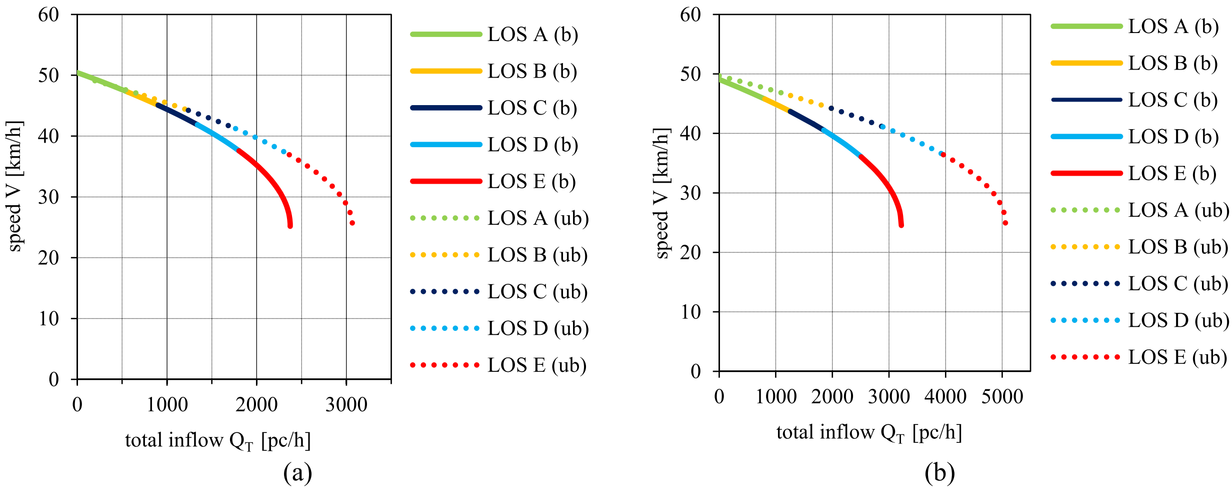

Fig. 14. MFDs and LOS limits. (a) single-lane roundabout; (b) double-lane roundabout.

Table 4
Macroscopic traffic variables as function of LOS - Single-lane roundabout - Scenario 1 (“balanced – b”).

LOS	Degree of saturation x		Total inflow Q_T [pc/h]		Mean Speed V [km/h]		Density K [pc/km]	
	min	max	min	max	max	min	min	max
A	0	0.25	0	567	50	47	0	3
B	> 0.25	0.39	567	902	47	45	3	5
C	> 0.39	0.57	902	1340	45	42	5	8
D	> 0.57	0.78	1340	1804	42	38	8	12
E	> 0.78	1.00	1804	2374	38	25	12	24
F	> 1		>2374		< 25		>24	

Table 5
Macroscopic traffic variables as function of LOS - single-lane roundabout - *Scenario 2* (“unbalanced – ub”).

LOS	Degree of saturation x		Total inflow Q_T [pc/h]		Mean Speed V [km/h]		Density K [pc/km]	
	min	max	min	max	max	min	min	max
A	0	0.25	0	747	50	47	0	4
B	> 0.25	0.39	747	1239	47	44	4	7
C	> 0.39	0.57	1239	1765	44	41	7	11
D	> 0.57	0.78	1765	2364	41	37	11	16
E	> 0.78	1.00	2364	3071	37	25	16	31
F	> 1		>	3071	< 25		>31	

Table 6
Macroscopic traffic variables as function of LOS - double-lane roundabout - *Scenario 1* (“balanced – b”).

LOS	Degree of saturation x		Total inflow Q_T [pc/h]		Mean Speed V [km/h]		Density K [pc/km]	
	min	max	min	max	max	min	min	max
A	0	0.25	0	806	49	46	0	2
B	> 0.25	0.39	806	1258	46	44	2	157
C	> 0.39	0.57	1258	1849	44	41	157	6
D	> 0.57	0.78	1849	2509	41	36	6	9
E	> 0.78	1.00	2509	3218	36	25	9	16
F	> 1		>	3218	< 25		>16	

Table 7
Macroscopic traffic variables as function of LOS - double-lane roundabout - *Scenario 2* (“unbalanced – ub”).

LOS	Degree of saturation x		Total inflow Q_T [pc/h]		Mean Speed V [km/h]		Density K [pc/km]	
	min	max	min	max	max	min	min	max
A	0	0.25	0	1261	50	46	0	3
B	> 0.25	0.39	1261	1981	46	44	3	6
C	> 0.39	0.57	1981	2893	44	41	6	9
D	> 0.57	0.78	2893	3962	41	36	9	14
E	> 0.78	1.00	3962	5058	36	25	14	25
F	> 1		>	5058	< 25		>25	

data acquisition techniques that nowadays are simplified by the use of automated counting techniques (e.g. floating cars). MFDs may play a crucial role because they represent an alternative and complementary method compared to traditional ones for roundabout performance evaluation. Moreover, in traditional methods (e.g. HCM model), the measures of effectiveness are estimated separately for each arm and this often leads to over-optimistic performance assessments, especially at higher saturation degrees. The MFD overcomes these disadvantages because it considers, implicitly and simultaneously, numerous geometric and traffic elements. Therefore, it is reasonable to expect that soon the MFD will become an instrument for traffic monitoring and control. However, further research needs to be conducted to better investigate the impact of roundabout diameter, road network characteristics, traffic components - which can vary depending on the context of reference (urban or rural) - and length within the cordon on the shape of the MFD.

CRediT authorship contribution statement

Marco Guerrieri: Conceptualization, Data curation, Formal analysis, Investigation, Methodology, Resources, Writing – original draft, Writing – review & editing.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Data availability

Data will be made available on request.

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