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January 2011

Technical Report # DISI-11-158

Beam Pattern Optimization in Time-Modulated Linear Arrays

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Time-modulated arrays exploit time as an additional degree of freedom [1] where a set of radio-frequency (RF) switches are used to modulate the static excitations to arbitrarily shape the radiation pattern. Firstly, time-modulation has been considered for the synthesis of radar array to obtain low and ultra-low sidelobe patterns [2]. Unfortunately, the periodic on-off of the RF switches causes the generations of undesired radiations, the so-called sideband radiation (SR) [3], which represents a loss of the radiated power. Such a drawback has limited the use of time-modulated array in real world applications.

However, in the last decades, thanks to the significant development of evolutionary-based algorithms boosted by the growing computational capabilities of modern personal computers, several approaches have been proposed to deal with the synthesis of time-modulated array while limiting the losses in the SR. Among them, strategies based on the differential evolution (DE) method [4], simulated annealing (SA) [5], and genetic algorithms (GAs) [6] have been used for the optimization of the time sequence controlling the RF switches.

Recently, an innovative approach based on a particle swarm optimizer (PSO) [7] has considered the optimization of the switch-on instants of the time pulses to minimize the sideband level (SBL), namely the level of the harmonic radiations with respect to the peak value of the pattern at the central frequency. There, it has been also shown that the instantaneous directivity is maintained almost constant during the modulation period although not involved in the optimization problem when considering these additional variables.

In the present study, the same optimization approach used in [7] is considered but in this case the cost function is defined to guarantee the instantaneous directivity to remain as constant as possible during the modulation period.

Towards this aim, let us consider a time-modulated linear array of N isotropic and equally-spaced elements lying on the z-axis. By considering a set of static excitation, α_n, $n = 0,...,N-1$, the array factor of a time-modulated linear array is expressed as

$$AF(\vartheta,t) = \exp(j\omega t)\sum_{n=0}^{N-1}\alpha_n U_n(t)\exp(j\beta dn\cos\vartheta) \qquad (1)$$

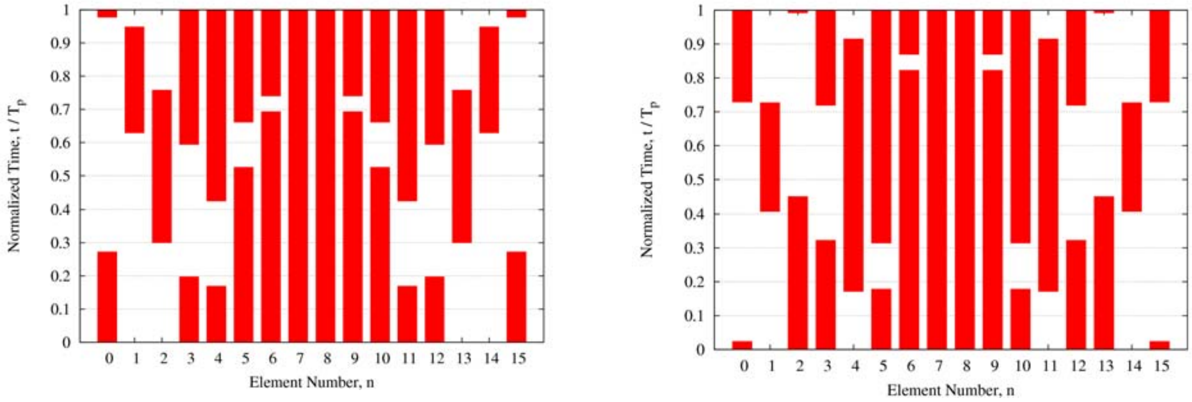
where $\beta = 2\pi/\lambda$ is the background wave-number, d is the inter-element spacing, and ϑ is the angular direction computed from the array axis. Moreover, $U_n(t)$, $n = 0,...,N-1$, are periodic rectangular functions, with period equal to T_p, mathematically modeling the on-off sequence enforced by means of the RF switches. The modulating function are equal to $U_n(t) = 1$ for $t_n^1 \le t \le t_n^2$, t_n^1 and t_n^2 being the switch-on instant and switch-off instant, respectively, and $U_n(t) = 0$ for the rest of the period where $t_n^1 \le t_n^2$ is assumed.

By expanding $U_n(t)$, $n = 0,...,N-1$, through the corresponding Fourier series, (1) can be rewritten as

$$AF(\vartheta,t) = \sum_{k=-\infty}^{\infty}\exp\left[j\left(\omega + k\omega_p\right)t\right]\sum_{n=0}^{N-1}\alpha_n u_{kn}\exp(j\beta dn\cos\vartheta) \qquad (2)$$

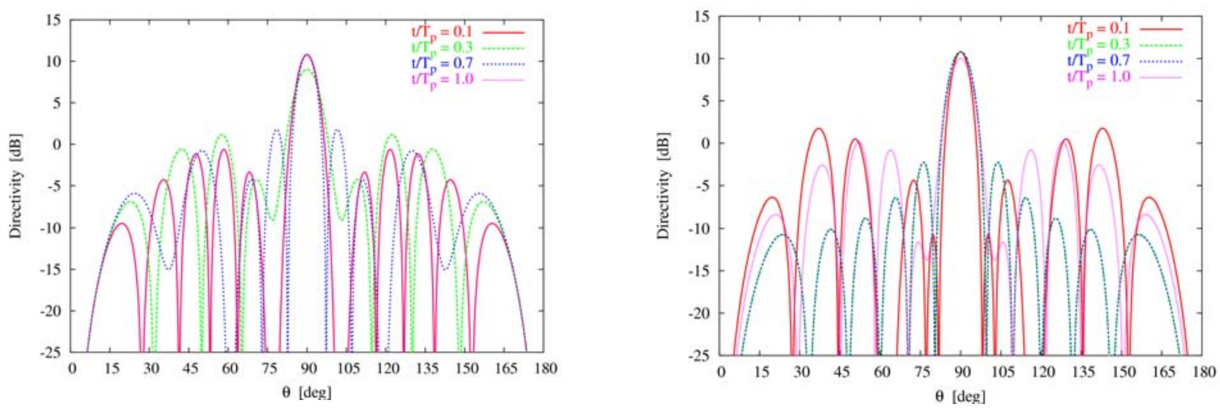
where $\omega_p = 2\pi/T_p$ and the Fourier coefficients are computed as $u_{kn} = \frac{1}{T_p}\int_0^{T_p}U_n(t)\exp\left(-jk\omega_p t\right)dt$. It is worth noting from (2) that the array factor turns out being composed by an infinite summation of harmonic contributions [3]. As a matter of fact, (2) represents both the power radiated by the antenna in the SR as well as the radiation pattern at the central frequency which is given by

$$AF_0(\vartheta,t) = \exp(j\omega t)\sum_{n=0}^{N-1}\alpha_n\tau_n\exp(j\beta dn\cos\vartheta) \qquad (3)$$



(*a*) (*b*)

Figure 1: Pulse sequence obtained when optimizing (*a*) the SBL and (*b*) the instantaneous directivity.



(*a*) (*b*)

Figure 2: Directivity patterns at the central frequency obtained at different time instants when considering the pulse sequence achieved optimizing (*a*) the SBL and (*b*) the instantaneous directivity.

where $\tau_n = \left(t_n^2 - t_n^1\right)/T_p$, $n = 0,...,N-1$, are the normalized pulse durations. As in [7], $\alpha_n = 1$, $n = 0,...,N-1$, the values $\tau_n = \hat{\tau}_n$, $n = 0,...,N-1$, are set to obtain a desired radiation pattern at the central frequency, and only the switch-on instants, t_n^1, $n = 0,...,N-1$, are considered in the optimization process. Unlike [7], the PSO is here used to maintain the peak directivity value in the desired direction as constant as possible during the modulation period. Towards this aim, the following cost function is minimized

$$\Psi\left(t_n^1, \hat{\tau}_n\right) = \sum_{l=1}^{L} \frac{\left[D_l\left(t_n^1, \hat{\tau}_n\right) - D_{av}\right]\Delta t_l\left(t_n^1, \hat{\tau}_n\right)}{D_{av}} \tag{4}$$

where the instantaneous directivity is computed at each time instant as

$$D(t) = \frac{\left|\sum_{n=0}^{N-1} \alpha_n U_n(t)\right|^2}{\sum_{n=0}^{N-1} \left|\alpha_n U_n(t)\right|^2}. \tag{5}$$

Moreover, in (4) the value L identifies the number of directivity variations within the modulation period, $D_l\left(t_n^1, \hat{\tau}_n\right)$ is the instantaneous directivity during the l-th time interval of normalized duration $\Delta t_l\left(t_n^1, \hat{\tau}_n\right)$, and D_{av}

is the average directivity equal to $D_{av} = \frac{1}{L}\sum_{l=1}^{L} D_l\left(t_n^1, \hat{\tau}_n\right)\Delta t_l\left(t_n^1, \hat{\tau}_n\right)$. The optimization of (4) has been carried out using the

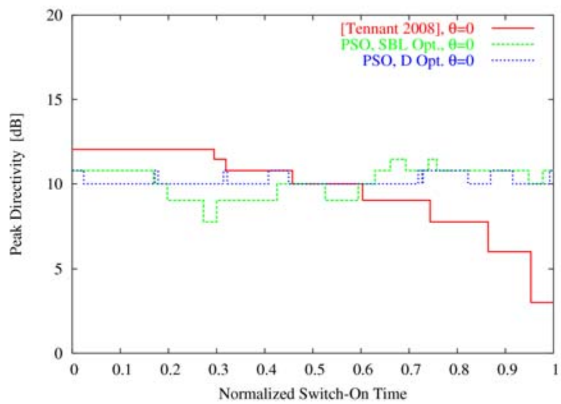


Figure 3: Behavior of the peak directivity in the modulation period.

same PSO strategy and control parameters reported in [7].

As a representative result, the same array considered in [7] with $N = 16$ elements half wavelength spaced is taken into account. The pulse durations, $\hat{\tau}_n$, $n = 0,...,N-1$, are set to generate a Dolph-Chebyshev pattern with sidelobe level at $-30dB$. Fig. 1(*a*) and Fig. 2(*a*) show the pulse sequence and the corresponding directivity pattern at different time instants obtained when optimizing the SBL as in [7]. Fig. 1(*b*) and Fig. 2(*b*) report the result achieved optimizing the instantaneous directivity according to the proposed approach. As shown in Fig. 2 for a set of time instants and in Fig. 3 for the peak directivity value throughout the modulation period, the proposed technique allows to maintain the peak directivity more constant as compared to the solution of [7].

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