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January 2011

Technical Report # DISI-11-227

Computationally-Effective Optimal Excitation Matching for the Synthesis of Large Monopulse Arrays

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Introduction

Antenna arrays able to generate two different patterns are widely used in tracking radar systems [1]. Optimal (in the Dolph-Chebyshev sense) sum [2] and difference patterns [3] can be generated by using two independent feed networks. Unfortunately, such a situation generally turns out to be impracticable because of its costs, the occupied physical space, the circuit complexity, and electromagnetic interferences. Thus, starting from the optimal sum pattern a sub-optimal solution for the difference pattern is usually synthesized by means of the sub-array technique. The array elements are grouped in sub-arrays properly weighted for matching the constrains of the difference beam.

Finding the best elements grouping and the sub-array weights is a complex and challenging research topic, especially when dealing with large arrays. As far as linear arrays are concerned, *McNamara* proposed in [4] an analytical method for determining the "best compromise" difference pattern. Unfortunately, when the ratio between the elements of the array and sub-arrays increases, such a technique exhibits several limitations mainly due to the ill-conditioning of the problem and the computational costs due to exhaustive evaluations. A non-negligible saving might be achieved by applying optimization algorithms (see for instance [5] and [6]) aimed at minimizing a suitable cost function. Notwithstanding, optimization-based approaches still appear computationally expensive when dealing with large arrays because of wide dimension of solution space to be sampled.

In order to properly deal with these computational issues, this contribution presents an innovative approach based on an optimal excitation matching procedure. By exploiting the relationship between independently-optimal sum and difference patterns, the dimension of the solution space is considerably reduced and efficiently sampled by taking into account the presence of array elements more suitable to change sub-array membership. In the following, the proposed technique is described pointing out, through a representative case, its potentialities and effectiveness in dealing with large arrays.

Problem Statement and Mathematical Formulation

Let us consider a linear array of $N = 2M$ elements. According to the standard sub-array technique, the sum pattern is generated by means of the symmetric set of the optimal excitations $\underline{A} = \{\alpha_m = \alpha_{-m}; m = 1, \dots, M\}$, while the difference pattern is synthesized by grouping the array elements in Q sub-arrays and associating to each of them a weight $w_q;\ q=1,\dots,Q$ (Fig 1).

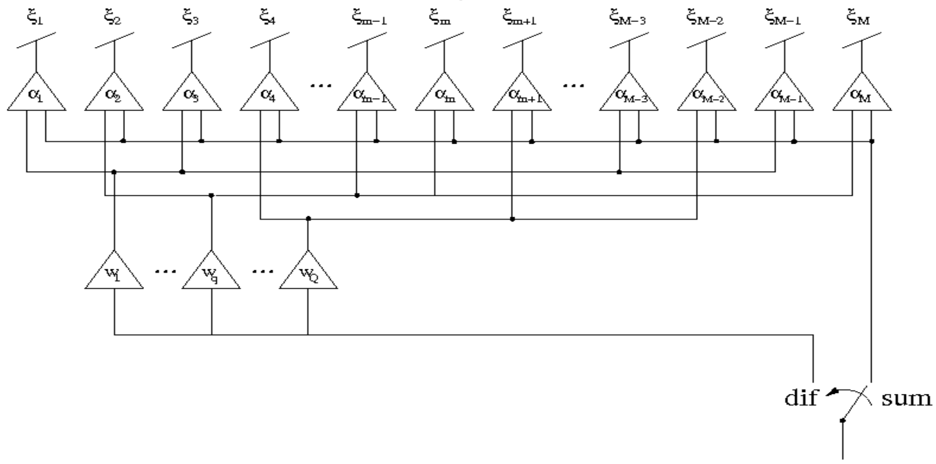

Figure 1 - Antenna feed network using the sub-arraying technique.

Accordingly, the difference pattern is characterized by a set of anti-symmetric excitations $\underline{B}=\{b_m=-b_m;\ b_m=w_{mq}\alpha_m; m=1,\dots,M;\ q=1,\dots,Q\}$. In order to determine the sub-array configuration and the corresponding weights, the set of optimal difference excitations $\underline{B}^{opt}=\{\beta_m=-\beta_{-m}; m=1,\dots,M\}$ [3] is used as a reference "*target*" and the closeness of a trial solution $\underline{B}^t$ to the optimal one is quantified through the following cost function

$$\Psi\left(\underline{C}^t\right)=\sum_{m=1}^{M}\left[v_m-d_m\left(\underline{C}^t\right)\right]^2, \qquad (1)$$

where $\underline{C}^t=\{c_m^t;\ m=1,\dots,M\}$ is the grouping vector that describes the corresponding trial sub-array configuration, $c_m^t\in[1,Q]$ being the sub-array index of the *m*-th element of the array. The reference parameters $\{v_m,\ m=1,\dots,M\}$ can be defined according to two different strategies, namely the *Gain Sorting* (GS),

$$v_m^{GS}=\frac{\alpha_m}{\beta_m}, \quad m=1,\dots,M, \qquad (2)$$

and the *Residual Error Sorting* (RES),

$$v_m^{RES}=\frac{\alpha_m-\beta_m}{\beta_m}, \quad m=1,\dots,M, \qquad (3)$$

respectively. On the other hand, the estimated parameters $\{d_m(\underline{C}),\ m=1,\dots,M\}$, are defined as follows:

$$d_m\left(\underline{C}^t\right)=\frac{\sum_{m=1}^{M}\delta_{c_m q}v_m}{\sum_{m=1}^{M}\delta_{c_m q}}, \quad m=1,\dots,M, \qquad (4)$$

where $\delta_{c_m q} = 1$ if $c_m = q$, $\delta_{c_m q} = 0$ otherwise. Concerning the sub-array weights $\{w_q,\ q = 1,\ldots,Q\}$, they are not directly optimized, but they are analytically-computed as function of $d_m\left(\underline{C}^t\right)$, $m = 1,\ldots,M$, once the configuration $\underline{C}^{opt} = \arg\left\{\min_{\underline{C}^t}\left[\Psi\left(\underline{C}^t\right)\right]\right\}$ is determined.

Towards this purpose, a reduced set of candidate solutions belonging to the so-called "*essential solution space*" $S^{(e)}$ is considered. As a matter of fact, it can be easily shown that once the reference parameters have been ordered in a sorted list, the grouping of the array elements that minimizes the cost function defines a contiguous partition of the ordered list [7]. Accordingly, the dimension of the solution space is reduced from $U = Q^M$ to $U^{(e)} = \begin{pmatrix} M-1 \\ Q-1 \end{pmatrix}$. As far as the sampling of $S^{(e)}$ is concerned [i.e., the search of $\underline{C}^{opt}$ through the minimization of (1)], the convergent succession of trial solutions $\left\{\underline{C}^t \rightarrow \underline{C}^{opt};\ t = 1,2,\ldots,t^{opt}\right\}$ is generated by taking into account that only some elements, indicated as "*border elements*", are candidate to change sub-array of membership without generating non-admissible solutions.

Results

In the following, a representative test case is dealt with in order to assess the effectiveness of the proposed approach when synthesizing large monopulse arrays. Let us consider a linear array of $N = 500$ elements and optimal sum and difference excitations generating a Dolph-Chebyshev pattern with $SLL = -25\,dB$ and a Zolotarev pattern characterized by a $SLL = -30\,dB$, respectively.

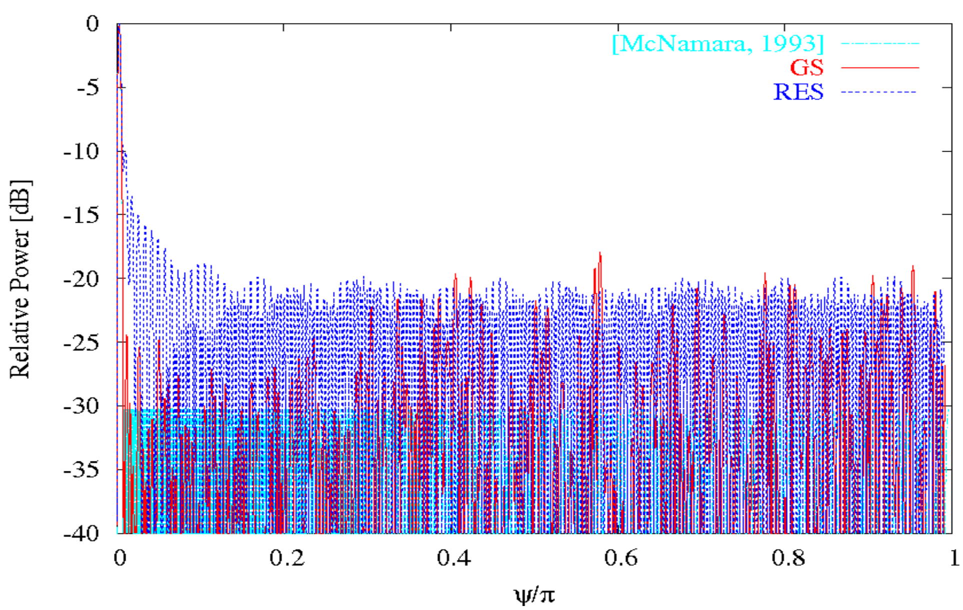

Figure 2 - Synthesized difference patterns (N = 500, Q = 4).

Figure 2 shows the patterns synthesized whit the *GS* and the *RES* when $Q = 4$. As it can be noticed, the *GS* algorithm outperforms the *RES* algorithm, satisfactorily approximating the optimal main lobe characteristics and achieving a maximum SLL of $-17.5\,dB$, two peaks away from the central null position.

As far as the computational costs are concerned, the values of the computational indexes reported in Tab. I clearly point out the non-negligible reduction of the solution space as well as the efficiency of the proposed approach in exploring the set of the admissible solutions.

U	$U^{(e)}$	t^{opt}		Iteration Time [sec]	
		GS	RES	GS	RES
3.27×10^{150}	2.54×10^{6}	47	3	1.007	2.297

Table I - Computational indexes.

Conclusions

An innovative approach for the synthesis of large linear monopulse arrays has been presented. By exploiting some features of the set of admissible solutions, the dimension of the solution space has been considerably-reduced and efficiently explored through a simple and computationally effective approach, thus making the proposed technique very useful and attractive when dealing with large arrays as well as planar and conformal structures.

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