

Impacts of g-g-v Constraints Formulations on Online Minimum-Time Vehicle Trajectory Planning^{*}

Mattia Piccinini^{*} Sebastiano Taddei^{*,**} Mattia Piazza^{*}
Francesco Biral^{*}

^{*} *Department of Industrial Engineering, University of Trento, 38123
Trento, Italy (e-mail: name.surname@unitn.it).*

^{**} *Department of Electrical and Information Engineering, Politecnico
di Bari, 70125 Bari, Italy.*

Abstract: The g-g-v diagram is a popular representation of a vehicle's maximum longitudinal and lateral accelerations, as a function of its speed. Recently, g-g-v diagrams have been used as performance constraints for online minimum-time vehicle trajectory planning with economic nonlinear model predictive control (E-NMPC). However, in the related literature, the modeling accuracy of the g-g-v formulations was often compromised in favor of computational efficiency for online E-NMPC. This paper compares various formulations of the g-g-v constraints, evaluating their accuracy, computational efficiency and impact on the lap times, when applied by an artificial race driver (ARD) to control a sports car model on a circuit. Also, we propose a new g-g-v diagram formulation, based on convex polytopic and non-convex polynomial constraints. Our formulation yields improved modeling accuracy while preserving a low computational burden for online E-NMPC. Finally, we analyze the vehicle trajectories, to understand how ARD achieves lower lap times through an improved knowledge of the maximum performance.

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Keywords: Autonomous vehicles, autonomous racing, g-g diagram, minimum-time problem, optimal control, model predictive control, trajectory planning.

1. INTRODUCTION

The g-g and g-g-v diagrams are well-known tools to model the maximum performance of a vehicle (Rice, 1973; Biral and Lot, 2009; Tremlett et al., 2014). Specifically, the g-g diagram is a two-dimensional (2D) plot of the maximum reachable longitudinal and lateral accelerations; the g-g-v is a 3D chart, which extends the g-g by showing the dependency on the vehicle speed (Biniewicz and Pyrz, 2023).

Recently, g-g-v diagrams have been applied as performance constraints for online (*i.e.*, real-time) minimum-time trajectory planning (Pagot et al., 2020; Rowold et al., 2023; Piccinini et al., 2023a,b), based on economic nonlinear model predictive control (E-NMPC) (Faulwasser et al., 2018). Using the g-g-v diagram, which collects various non-linearities like aerodynamic drag and tire force saturation, can help reduce the computational burden of online minimum-time E-NMPC.

The mathematical formulation of the g-g-v constraints for minimum-time E-NMPC can hugely impact the final lap time (Rowold et al., 2023). In the literature, most of the g-g-v formulations were either inaccurate in modeling the real shape of the g-g-v diagram, or their computational

complexity was too high for online trajectory planning. The main contributions of this paper are:

- A comparison of different formulations of the g-g-v constraints, in terms of (a) accuracy in modeling the g-g-v diagram, (b) computational efficiency for online minimum-time E-NMPC, and (c) impact on the lap time and trajectories achieved by an artificial race driver, which uses E-NMPC to drive a high-fidelity sports car model.
- A new formulation of the g-g-v constraints, combining convex polytopic and non-convex polynomial constraints. Our approach is shown to be more accurate than other formulations in the literature, while maintaining a low computational cost for E-NMPC.

To assess the lap times achievable with different g-g-v formulations, we integrate them into the E-NMPC planner of the artificial race driver (ARD) introduced by Piccinini et al. (2023b), for the control of a sports car model.

1.1 Related Work

Some authors recently combined g-g-v constraints with kinematic or kineto-dynamical vehicle models, for online E-NMPC trajectory planning near a vehicle's performance limits (Betz et al., 2022). Various formulations of the g-g-v constraints were used. For example, Rowold et al. (2023) used *diamond shapes* to model the g-g-v constraints. However, their method underestimated the real g-g-v diagram. Montani et al. (2021), Duhr et al. (2022) and

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Novi et al. (2020) combined elliptic and linear g-g-v constraints; however, Novi et al. (2020) overestimated the actual diagram. Elliptic and super-elliptic constraints were used by Pagot et al. (2020) and Piccinini et al. (2023a,b), but they did not analyze the accuracy of the g-g-v diagram approximation.

Veneri and Massaro (2020), Biniewicz and Pyrz (2023) and Lovato and Massaro (2022) employed polar splines to accurately model the g-g-v diagram on 2D and 3D circuits. However, the computational burden of their technique did not allow online E-NMPC.

Critical Summary To our knowledge, no existing paper thoroughly compared the impact of various g-g-v constraint formulations for online minimum-time trajectory planning with E-NMPC. Moreover, many proposed formulations failed to accurately model the actual g-g-v diagram or lacked computational efficiency for online E-NMPC.

2. SIMULATION FRAMEWORK

2.1 Artificial Race Driver (ARD)

The g-g-v formulations of this paper are compared using the artificial race driver (ARD) framework of Piccinini et al. (2023b), shown in Fig. 1. ARD performs high-level trajectory planning with minimum-time E-NMPC, and uses physics-driven neural networks for low-level trajectory tracking and state estimation (Da Lio et al., 2023). The g-g-v constraints of this work are integrated into the E-NMPC trajectory planner (blue block in Fig. 1), while the tracking controllers (gray blocks) are the same as in Piccinini et al. (2023b).

2.2 Trajectory Planning E-NMPC Problem

Let us briefly describe the trajectory planning E-NMPC problem, which incorporates the g-g-v constraints:

$$\min_{\mathbf{u} \in \mathcal{U}} \sum_{j=1}^{n_s} (\mathbf{x}_j(T) - \mathbf{x}_{f_j})^2 + \int_0^T w_T dt \quad (1a)$$

$$\text{s.t.} \quad \begin{cases} \frac{d\mathbf{x}(t)}{dt} = \mathcal{F}(\mathbf{x}(t), \mathbf{u}(t)), & (1b) \\ \mathcal{B}(\mathbf{x}(0)) = 0, & (1c) \\ \mathcal{C}(\mathbf{x}(t), \mathbf{u}(t)) \leq 0 & (1d) \end{cases}$$

The cost function (1a) minimizes the maneuver time T , with w_T being a tunable weight, and sets soft final conditions for the states $\mathbf{x}(T)$. The states \mathbf{x} and controls \mathbf{u} are subject to the dynamics (1b), which is the kinetodynamical vehicle model of Piccinini et al. (2023b). Equation (1c) sets the initial conditions for \mathbf{x} , while the inequalities in (1d) enforce the g-g-v constraints, the track limits, and the control bounds. The reader is referred to Piccinini et al. (2023b) for a more detailed description of the E-NMPC problem.

The problem (1) is reformulated using the curvilinear abscissa ζ of the circuit center-line as the independent variable. The prediction horizon is set to 300 m, with a total number of 350 mesh points. The E-NMPC is solved in real-time with the optimal control solver PINS (Biral et al., 2016). Other recent examples of PINS use for online vehicle trajectory planning can be found in Piccinini et al. (2023b); Pagot et al. (2023).

2.3 Vehicle Model for Simulation (VMS)

To assess the impact of various g-g-v formulations on the lap times, we use ARD for closed-loop planning and control of a vehicle model for simulation (VMS). The VMS faithfully reproduces the dynamics of a real high-end sports car, with a top speed beyond 300 km/h. The VMS is a double-track vehicle model with roll axis, and is validated with real test-track telemetry data, collected with a professional driver. Additional details and validation plots can be found in Piccinini et al. (2024). The VMS is used for two purposes:

- (1) ARD drives the VMS with *online* E-NMPC, using various g-g-v constraint formulations.
- (2) A minimum-lap-time OCP (named *MLT-VMS*) is solved *offline* with the full VMS, to benchmark the lap times achieved by ARD with different g-g-v formulations.

2.4 g-g-v Diagram of the VMS

The g-g-v diagram of the VMS is estimated by combining open-loop (OL) and closed-loop (CL) maneuvers. First, the OL tests proposed by Piccinini et al. (2023a) are carried out with the VMS, to assess the maximum longitudinal/lateral accelerations $\{a_x, a_y\}$, as a function of the longitudinal speed v_x . Fig. 2 shows the resulting points of the g-g-v diagram. Second, the CL iterative scheme of Piccinini et al. (2023b) is applied, using the g-g-v diagram as a constraint for CL trajectory planning with E-NMPC. During this phase, a limitation of the combined steering-braking performance is learned (brown planes in Fig. 2), to exclude the unstable g-g-v regions (Tremlett et al., 2014) and improve the closed-loop lap times.

We remark that our g-g-v diagram estimation method can also be applied to a real car. Other techniques were proposed to obtain the g-g-v diagram of a VMS, *e.g.*, optimal control (Massaro et al., 2024).

3. FORMULATIONS OF THE G-G-V CONSTRAINTS

This section outlines the new and benchmark formulations of the g-g-v constraints for minimum-time E-NMPC.

3.1 Proposed Formulation

We propose the following g-g-v formulation:

$$\begin{cases} \mathbf{P} [a_y \ a_x \ v_x]^T \leq \mathbf{q} & (2a) \\ \Phi_1(v_x, a_y) \leq a_x \leq \Phi_2(v_x, a_y) & (2b) \\ a_x \geq \bar{s} (|a_y| - \bar{a}_y) & (2c) \end{cases}$$

The inequality (2a) is the convex hull of the g-g-v points in Fig. 2, and is a set of n_p polytopic constraints¹ in a_y , a_x and v_x . The resulting polytope is defined by the matrix $\mathbf{P} \in \mathbb{R}^{n_p \times 3}$ and the vector $\mathbf{q} \in \mathbb{R}^{n_p \times 1}$, which are fitted to the g-g-v points.

In (2b), the nonlinear functions $\Phi_1(\cdot)$ and $\Phi_2(\cdot)$ constrain a_x by modeling the non-convex parts of the g-g-v diagram. Indeed, for race cars and motorcycles, the g-g-v regions where a_x is highest and lowest are typically non-convex

¹ The minimum number n_p of polytopic constraints depends on the shape of a given g-g-v diagram. In our case, $n_p = 263$.

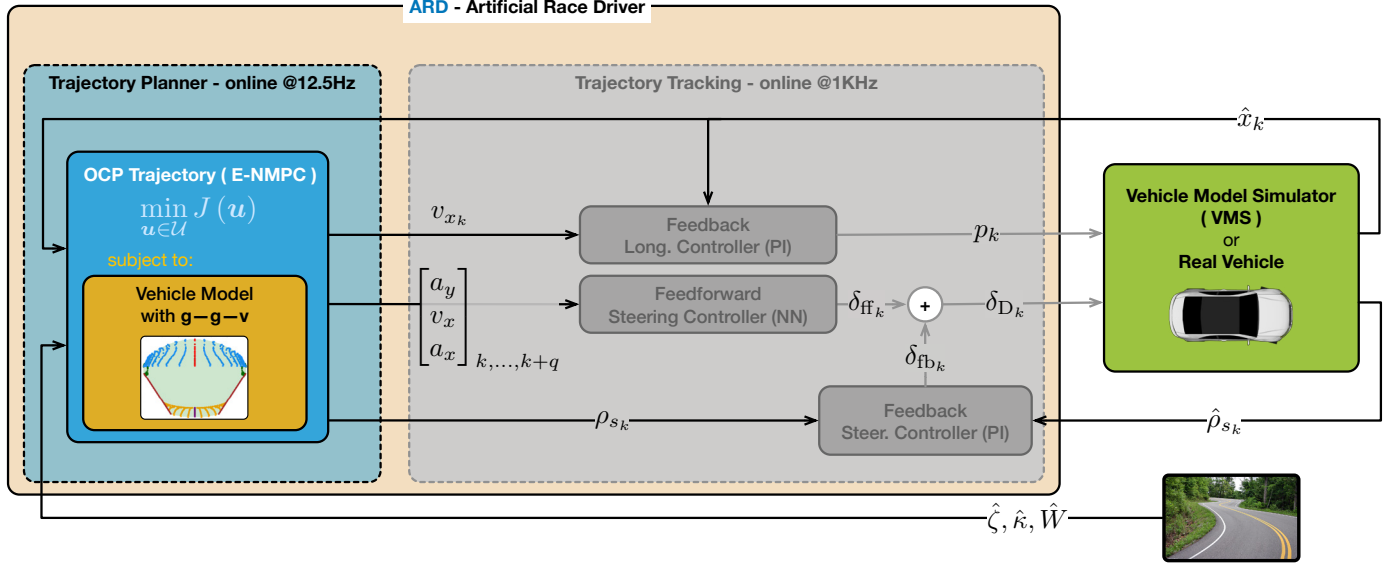


Fig. 1. ARD framework for online minimum-time trajectory planning and control of a vehicle model for simulation (VMS). The g-g-v constraints are integrated into the E-NMPC planner (blue block). The trajectory tracking controllers (gray blocks) are the same of Piccinini et al. (2023b), and they are not in the scope of this paper.

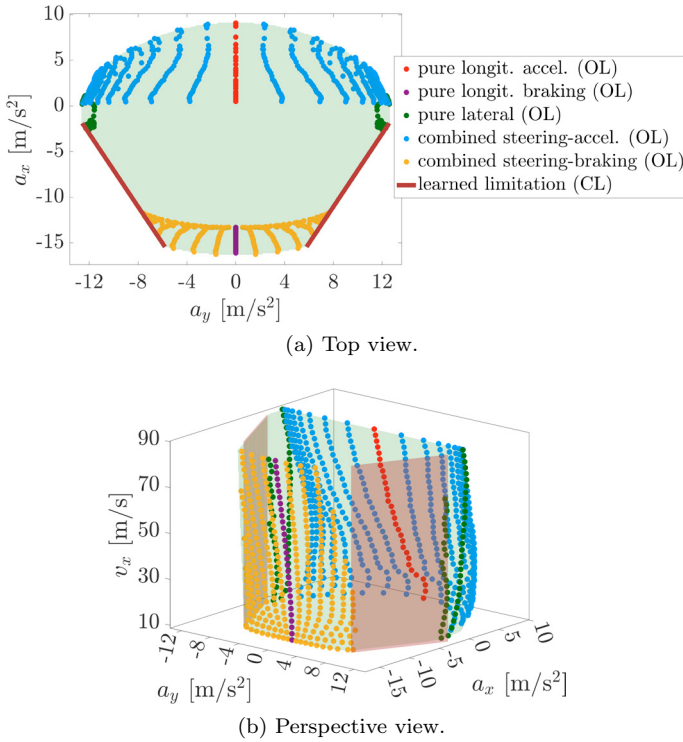


Fig. 2. g-g-v diagram of the VMS, identified with open-loop (OL) and closed-loop (CL) maneuvers. The green shaded volume represents the reachable accelerations.

(Veneri and Massaro, 2020; Biral and Lot, 2009). We employ a numerically-efficient polynomial formulation for $\Phi_1(\cdot)$ and $\Phi_2(\cdot)$:

$$\begin{cases} w_{1i}(v_x) = \sum_{j=0}^{n_v} p_{1ji} v_x^j, & w_{2i}(v_x) = \sum_{j=0}^{n_v} p_{2ji} v_x^j & (3a) \\ \Phi_1(v_x, a_y) = \sum_{i=0}^{n_y} w_{1i}(v_x) a_y^i & & (3b) \\ \Phi_2(v_x, a_y) = \sum_{i=0}^{n_y} w_{2i}(v_x) a_y^i & & (3c) \end{cases}$$

where the coefficients $\{p_{1ji}, p_{2ji}\}$, $i = 0, \dots, n_y$, $j = 0, \dots, n_v$, are fitted to the g-g-v points. The polynomial

degrees n_v and n_y are chosen to obtain a good fit of the g-g-v points, while keeping a low computational cost for E-NMPC. To model the g-g-v of the sports car in Fig. 2, we set $n_v = 6$ and $n_y = 0$; however, we will show that $n_y > 0$ is typically needed to model the g-g-v of a racing motorcycle.

Finally, the constraint (2c) models the stability limitations of the combined steering-braking performance, as learned from the CL maneuvers of Fig. 2. The parameters \bar{s} and \bar{a}_y in (2c) are fitted to the learned limitations.

Fig. 3 plots the g-g-v constraints, using the model (2).

Generalizability of the Proposed Formulation Our formulation (2) is an extension of Piccinini et al. (2024): the constraints (2b) can now model more generic non-convexities in the g-g-v diagrams of race cars and motorcycles. For example, Fig. 4 shows that our approach can accurately capture the racing motorcycle's g-g diagram of Biral and Lot (2009), in which non-convex regions are present due to the wheelie and stoppie effects. Super-ellipses and diamond shapes (described next) would not be suitable for these types of diagrams.

3.2 Benchmark: g-g-v Formulation with Super-Ellipses

The benchmark g-g-v formulation with super-ellipses is taken from Piccinini et al. (2023b):

$$\begin{cases} (a_x - x_o(v_x))^+ \cdot \left[\left(\frac{|a_y|}{Y(v_x)} \right)^n + \left(\frac{|a_x - x_o(v_x)|}{X_M(v_x)} \right)^n \right] - \\ (a_x - x_o(v_x))^- \cdot \left[\left(\frac{|a_y|}{Y(v_x)} \right)^n + \left(\frac{|a_x - x_o(v_x)|}{X_m(v_x)} \right)^n \right] \leq 1 & (2c) \end{cases} \quad (4)$$

The first inequality in (4) constrains a_x and a_y with v_x -dependent super-ellipses, whose tunable quantities are the exponent n and the functions $X_M(v_x)$, $X_m(v_x)$, $Y(v_x)$, $x_o(v_x)$, implemented as polynomials in v_x . $(\cdot)^+$ and $(\cdot)^-$ are smooth functions enabling the use of super-ellipses

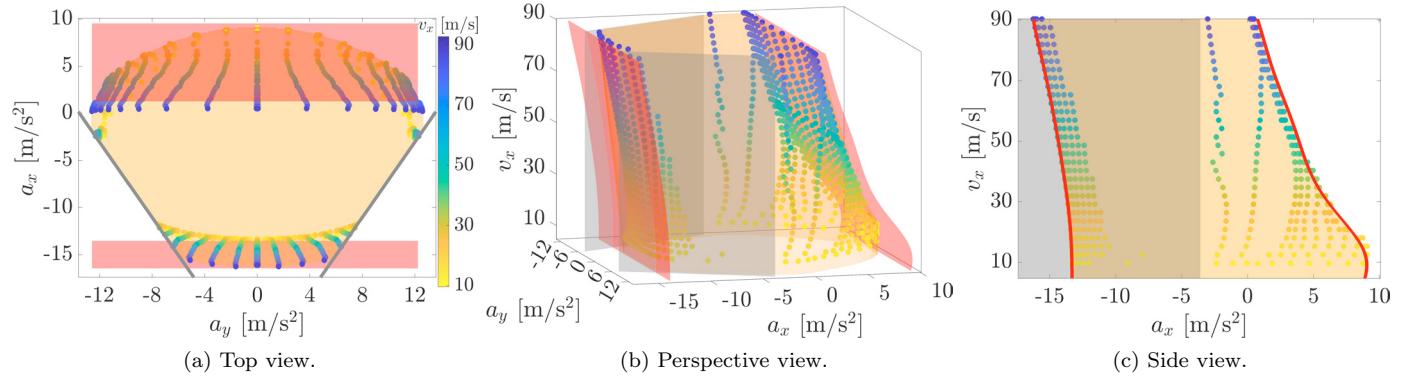


Fig. 3. Modeling the g-g-v diagram of Fig. 2 with our formulation (2): polytopic constraints (2a) (not shown), non-convex constraints (2b) (red curves), stability limitations (2c) (gray planes). The resulting constraint is the orange volume. The colored g-g-v points are the same as in Fig. 2.

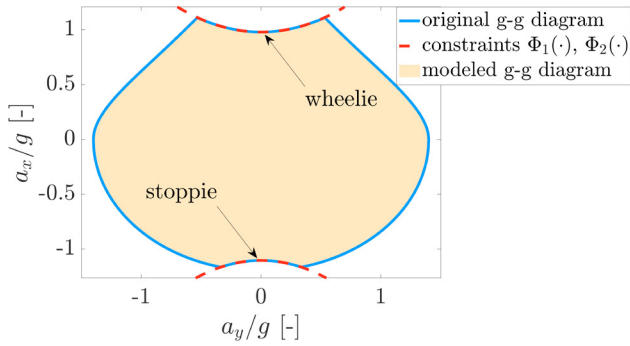


Fig. 4. Applying our formulation (2) to the g-g diagram of the racing motorcycle model in Biral and Lot (2009). The figure shows that our approach can model the non-convex g-g regions due to wheelie and stoppie.

with different sizes, for acceleration and braking. The absolute values in (4) are approximated with a smooth function, defined in Piccinini et al. (2023b). The stability limitation (2c) is added in (4) to constrain the combined steering-braking performance, like in our formulation.

3.3 Benchmark: g-g-v Formulation with Diamond Shapes

The benchmark g-g-v formulation with diamond shapes is taken from Rowold et al. (2023):

$$\begin{cases} a_x \leq a_{x_{\max}}(v_x), & |a_y| \leq a_{y_{\max}}(v_x) \\ |a_x| \leq |a_{x_{\min}}(v_x)| \left[1 - \left(\frac{|a_y|}{a_{y_{\max}}(v_x)} \right)^{n(v_x)} \right]^{\frac{1}{n(v_x)}} \end{cases} \quad (2c) \quad (5)$$

where the tunable quantities are $a_{x_{\max}}(v_x)$, $a_{y_{\max}}(v_x)$, $a_{x_{\min}}(v_x)$ and the exponent $n(v_x)$, implemented as linear splines in v_x . Note that, in comparison with Rowold et al. (2023), (5) neglects the terms related to a three-dimensional road geometry, since in this work we focus on the g-g-v constraints for a flat road.

3.4 Benchmark: g-g-v Formulation with Polar Splines

The polar spline benchmark formulation of the g-g-v constraint is inspired by Veneri and Massaro (2020):

$$\begin{cases} \rho = \sqrt{a_x^2 + a_y^2}, & \alpha = \text{atan2}(a_x, a_y) \end{cases} \quad (6a)$$

$$\begin{cases} \rho \leq \rho_{\max}(\alpha, v_x) \end{cases} \quad (6b)$$

where ρ is named adherence radius, α is the polar orientation angle, and $\text{atan2}(\cdot, \cdot)$ is the two-arguments arctangent. The reachable accelerations are defined by the function $\rho_{\max}(\alpha, v_x)$, which is implemented as a bi-cubic spline.

4. RESULTS

4.1 Approximation of the g-g-v Diagrams

Let us analyze the accuracy of the g-g-v formulations in approximating the g-g-v diagram. The computational efficiency will be discussed in Section 4.2.

Fig. 5 plots the g-g-v diagrams at a low and medium-high vehicle speed. Polar splines provide the best approximation of the g-g-v diagram, since they exactly interpolate the g-g-v points. Super-ellipses are less accurate, especially in the g-g-v regions where $a_x > 0$: they overestimate the g-g-v at low speeds, and underestimate it at high speeds. Indeed, due to their constant² exponent n in (4), super-ellipses cannot adequately model the local curvature of the diagram, which changes considerably with v_x . Diamond shapes provide a better approximation of the braking performance, but they overestimate the regions with $a_x > 0$, especially at low speeds. Finally, our formulation provides good accuracy at both low and high speeds, with only a slight overestimation of the braking performance.

4.2 Use of the g-g-v Constraints for Closed-Loop E-NMPC

In this section, we integrate the g-g-v constraints into the E-NMPC problem (1), to compare the computational efficiency of the g-g-v formulations, and their impact on the lap time achieved by our ARD (artificial race driver). As explained in Section 2.1, ARD performs online minimum-time trajectory planning with E-NMPC, and drives the sports car VMS with neural low-level controllers. We test ARD on the Catalunya circuit, a challenging track with low and high-speed corners.

To assess the quality of the lap times achieved *online* by

² A better accuracy could be achieved using two different v_x -dependent exponent functions n for the acceleration and braking super-ellipses in (4). However, this would excessively increase the computational complexity, and the E-NMPC could not be solved online with low computational times.

g-g-v formulation	Mean CPU time E-NMPC (<i>online</i>)	Std dev CPU time E-NMPC (<i>online</i>)	Lap time $T_{\text{MLT}_{\text{VMS}}}$ (<i>offline</i> MLT-VMS)	Lap time $T_{\text{MLT}_{\text{ard}}}$ (<i>offline</i> MLT-ARD)	Lap time T_{ARD} (<i>online</i> ARD)
polar splines Veneri and Massaro (2020)	192.7 ms	150.7 ms	114.054 s	114.072 s	ARD cannot operate online
super-ellipses Piccinini et al. (2023b)	15.1 ms	7.6 ms		115.202 s	115.428 s
diamond shapes Rowold et al. (2023)	19.1 ms	10.8 ms		113.025 s	114.841 s
proposed	28.5 ms	12.6 ms		114.076 s	114.348 s

Table 1. Comparing the g-g-v formulations in terms of computational efficiency for E-NMPC; lap time $T_{\text{MLT}_{\text{ard}}}$ of the *offline* MLT-ARD OCP; lap time T_{ARD} achieved by ARD, which drives *online* the VMS with E-NMPC on the Catalunya circuit. Finally, the *offline* MLT-VMS OCP provides our best estimate of the minimum-lap-time, and is used to evaluate the ARD’s results.

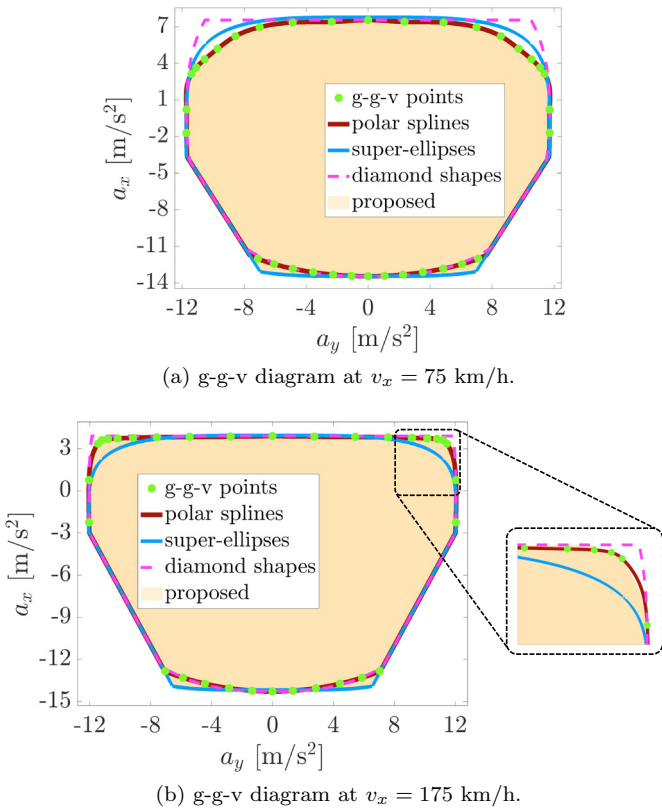


Fig. 5. Comparing the g-g-v formulations at $v_x = 75$ km/h (a), $v_x = 175$ km/h (b). Polar splines offer the best approximation but are computationally expensive for online E-NMPC; super-ellipses and diamond shapes are computationally efficient but less accurate. Our method strikes a balance between accuracy and computational efficiency, resulting in the best lap time T_{ARD} (Table 1).

ARD with each g-g-v formulation, we solve *offline* the following OCPs, on a full lap:

- The MLT-VMS OCP of Section 2.3, which is our best estimate of the minimum-lap-time.
- A new OCP, named *MLT-ARD*, with the same formulation and constraints used for E-NMPC (1). The MLT-ARD provides the *theoretical* lap time that a certain g-g-v formulation can achieve, without considering the trajectory feasibility for the VMS.

Lap Times and Computational Efficiency

Table 1 compares the computational efficiency and lap times obtained with each g-g-v formulation. The computational burden is measured by the CPU times (mean and standard deviation) required to solve the E-NMPC problems on one lap of the Catalunya circuit. The E-NMPC problems are re-computed online every 80 ms, on a MacBook Pro laptop with a 2.6 GHz 6-Core Intel i7 processor.

Polar splines: The polar spline approach provides the best modeling accuracy: the MLT-ARD offline solution sets a lap time very close (18 ms) to the MLT-VMS reference. However, polar splines are computationally inefficient for E-NMPC, with a high mean CPU time (192.7 ms) and standard deviation (150.7 ms). Considering our re-planning sample time of 80 ms, polar splines do not allow ARD to operate online. Indeed, Veneri and Massaro (2020) used their polar spline approach only for *offline* trajectory optimization.

Super-ellipses: The formulation with super-ellipses is the most computationally efficient, with the lowest mean CPU time (15.1 ms) and standard deviation (7.6 ms). However, the lap time T_{ARD} is 1.374 s higher than the MLT-VMS benchmark, due to the worse approximation of the g-g-v diagram, which can be noticed in Fig. 5.

Diamond shapes: As discussed in Section 4.1, diamond shapes overestimate the combined steering-acceleration regions of the g-g-v. This leads to a lower *theoretical* lap time $T_{\text{MLT}_{\text{ard}}}$ (1.029 s below the MLT-VMS). However, due to the g-g-v overestimation, the trajectories planned by E-NMPC are locally infeasible for the VMS, and ARD achieves a higher lap time when driving in closed-loop (0.787 s worse than the MLT-VMS). The advantage of diamond shapes is the computational efficiency: the mean CPU time for online E-NMPC is 19.1 ms.

Proposed formulation: Our approach yields the best lap time T_{ARD} , which is only 0.294 s higher than the MLT-VMS benchmark. The mean CPU time is 28.5 ms, which is slightly higher than the diamond shapes formulation, but still allows ARD to operate online with a re-planning sample time of 80 ms. As discussed in Section 4.1, our formulation provides a good approximation of the g-g-v diagram, both at low and high vehicle speeds.

Fig. 6 plots the accelerations achieved online by ARD with

our g-g-v model, and by the MLT-VMS offline solution, on the Catalunya circuit. The figure shows that ARD and the MLT-VMS solution achieve similar maximum accelerations, which confirms that our formulation allows ARD to drive close to the handling limits.

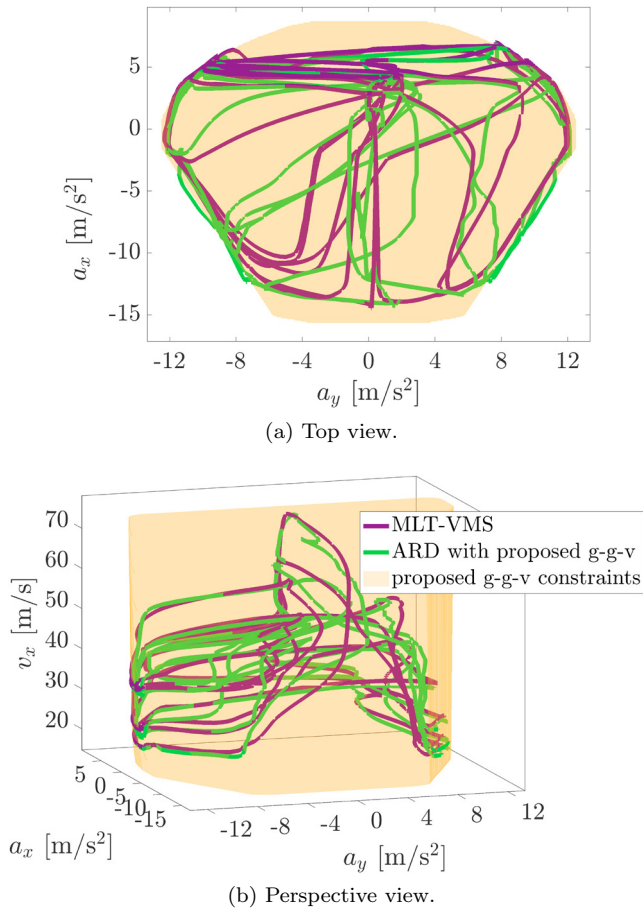


Fig. 6. g-g-v diagram, Catalunya circuit: comparing the accelerations achieved online by ARD with the proposed g-g-v constraints, and by the benchmark offline MLT-VMS, which uses the full VMS in the OCP.

Analysis of the Vehicle Trajectories

Fig. 7 analyzes the vehicle trajectories, velocity profiles and maneuver times executed by ARD at the corner n.5 of the Catalunya circuit, with the three g-g-v formulations enabling online E-NMPC. In the corner entry (sector A-B), our formulation leads to a similar trajectory and speed as the super-ellipses. Conversely, diamond shapes locally overestimate the vehicle performance (Fig. 5), so ARD carries more speed into the corner. However, the trajectory planned with diamond shapes is locally infeasible: ARD needs to re-plan and execute a wider and slower maneuver (point C), which increases the travel time. In the corner exit (sector C-D), super-ellipses slightly underestimate the combined steering-acceleration performance, while our approach results in a higher local speed, similar path and lower maneuver times.

Considering the whole maneuver A-D, our g-g-v formulation enables ARD to drive faster than super-ellipses (by 0.03 s) and diamond shapes (by 0.11 s), which is not negligible on a sector of only 150 m.

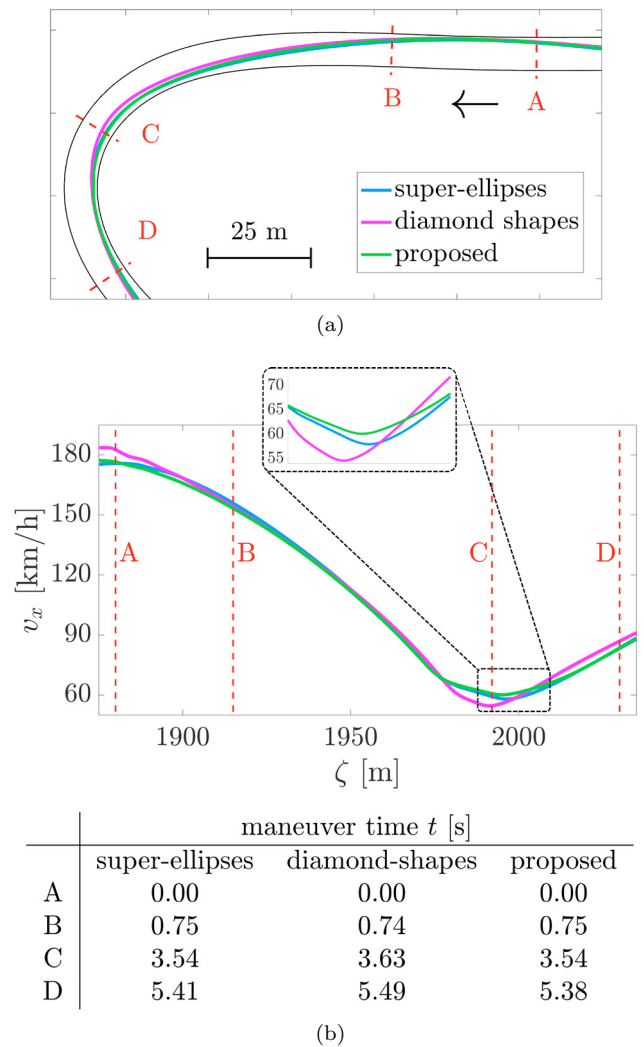


Fig. 7. Vehicle trajectories, velocity profiles and maneuver times executed by ARD at the corner n.5 of the Catalunya circuit, with different g-g-v formulations for E-NMPC.

5. CONCLUSIONS

This paper compared existing and new formulations of the g-g-v performance constraints for online minimum-time trajectory planning with E-NMPC. We evaluated the literature techniques that used polar splines, super-ellipses and diamond shapes. Also, a new g-g-v formulation was introduced, based on polytopic and polynomial constraints. The g-g-v constraints were integrated into the E-NMPC problem of our ARD agent, which was applied to drive a high-fidelity VMS on a test circuit.

The results indicate that our approach provides a better approximation of the g-g-v diagram, and it enables online E-NMPC with low computational times. By analyzing the vehicle trajectories, we discussed how our method allowed ARD to be faster in many corners. As a result, ARD achieved a lap time very close to the MLT-VMS OCP solution, which was our best lap time estimate.

Finally, our modeling technique showed good generalizability when applied to a racing motorcycle's g-g diagram, providing an accurate approximation of its non-convex regions.

Future work will apply the g-g-v formulation to the graph-

based trajectory planner of Gottschalk et al. (2024), for dynamic obstacle avoidance.

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