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Introduction

Thinning is a standard technique to reduce the power consumption, hardware complexity, cost, and weight of large arrays [1][2]. However, array thinning does not allow a suitable control of the peak sidelobe level (*PSL*), as for filled arrays, with a reduction of the array performances [1][2].
In the literature, several techniques have been proposed for the design of thinned arrays with low sidelobes [1]-[4]. In a chronological order, arrays were at first thinned by deterministic techniques, but no significant advantages on the PSL reduction have been obtained over random approaches [1]. Successively, stochastic optimization techniques have been widely and successfully applied [3][4]. However, such techniques present high computational costs when applied to large arrays, and they do not allow *a-priori* estimates of the expected performances for a given aperture size and thinning factor [2].
More recently, thinned arrays have been synthesized by exploiting the autocorrelation properties of binary sequences derived from Difference Sets (DSs) [2][5]. Such an approach has several advantages over traditional thinning techniques. It is computationally efficient and its performances are predictable [2]. However, DSs exist only for a limited subset of configurations, and their features cannot be exploited in a general way [2][5].
In order to apply the same strategy to a wider set of array configurations, sequences which are sub-optimal with respect to DSs can be considered. For instance, Almost Difference Sets (ADSs) [6] are good candidates for large array thinning, since they represent a generalization of DSs with several similar properties [6].
This paper is aimed at analyzing the applicability of ADSs to the design of thinned linear arrays, as well as the effectiveness of such an array synthesis technique in terms of beam-pattern features. Selected numerical results are provided to assess the performances of the proposed approach also in comparison with state-of-the-art thinning techniques with predictable PSL behaviour.

Array Thinning by Using Almost Difference Sets

A finite thinned array can be derived from whatever binary sequence of length D by defining the array element location function [2]

$$E(x)=\sum_{i=0}^{D-1} s_i \delta(x-ix_0) \qquad (1)$$

where $s_i \in \{0,1\}$, $i=1,...,D-1$ are the excitation coefficients and x_0 and x are the lattice spacing and the spatial coordinate expressed in wavelengths along the array, respectively.

By following the same mathematical guidelines in [2], it turns out that the power pattern of the ADS-based finite linear array satisfies the following

$$|a(u_n)|^2 = F[\mathrm{X}(\tau)] \qquad (2)$$

at fixed known sampling points $u_n = \frac{n}{Dx_0}$, where *F* is the Fourier transform operator, *a(u)* is the array factor, and $\mathrm{X}(\tau)$ is the periodic autocorrelation function of $E(x)$ [2] given by

$$\mathrm{X}(\tau)=\sum_{i=0}^{N-1} s_i s_{(i+\tau) \bmod D} \,. \qquad (3)$$

The periodic autocorrelation function associated to a *(D,P,L,r)*-ADS is [6]

$$\mathrm{X}(\tau)=\begin{cases} P & \text{if } \tau=0 \\ L & \text{for } r \text{ values of } \tau \\ L+1 & \text{otherwise} \end{cases} \qquad (4)$$

where *P* is the number of active elements in the lattice of array elements. As it can be observed, the three-level autocorrelation function of ADSs is close to the binary autocorrelation [2] of DSs. Therefore, it is expected that the arrays derived from ADSs, although sub-optimal, will provide performances close to that of DSs also in terms of PSL

$$PSL=\frac{\max_{u \notin M} |a(u)|^2}{|a(0)|^2} \qquad (5)$$

where M identifies the mainlobe region. Unlike, finite linear thinned arrays based on DSs, a fixed PSL cannot be *a-priori* determined, but the value of the PSL of an ADS array arrangement belongs to the following confidence range

$$PSL^{\min} \le PSL^{opt} \le PSL^{\max} \qquad (6)$$

where

$$PSL^{\min}=\max\left\{\max_{n\neq 0}\frac{|a(u_n)|^2}{|a(u_0)|^2},(0.8488+1.128\log_{10}D)\times\min_{n\neq 0}\frac{|a(u_n)|^2}{|a(u_0)|^2}\right\} \qquad (7)$$

and

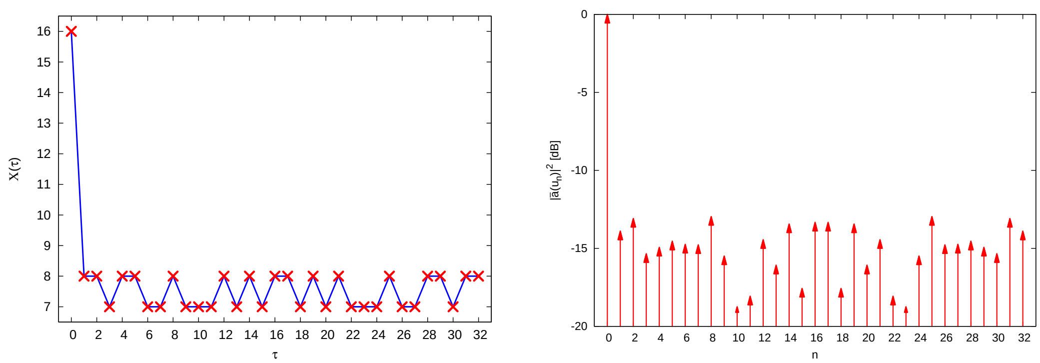
$$PSL^{\max}=(0.8488+1.128\log_{10}D)\times\max_{n\neq 0}\frac{|a(u_n)|^2}{|a(u_0)|^2}\,. \qquad (8)$$

It is worth noting that (7) and (8) can be estimated by the Fourier transform of $\mathrm{X}(\tau)$ through Eq. (2).

In order to evaluate the performances of ADS-based arrays, their actual PSL and the associated bounds (7) and (8) will be compared to that expected from random array theory [1] and from DS-based array theory [2].

Numerical Results

In order to provide a preliminary assessment of the performances of ADS-based arrays, the following ADS is considered [6]
$\overline{A} = \{0, 1, 2, 3, 4, 5, 6, 8, 13, 14, 18, 20, 22, 25, 28, 29\} \quad D = 33, P = 16, L = 7, r = 16$.
Figure 1(*a*) shows the pictorial representation of the periodic autocorrelation function $\mathrm{X}(\tau)$ of the characteristic sequence of $\overline{A}$. As expected, $\mathrm{X}(\tau)$ exhibits a three-level behavior which, unlike DSs (where the same level behaviour is kept both in $\mathrm{X}(\tau)$ and $F[\mathrm{X}(\tau)]$) in not present in its Fourier transform in Fig. 1(*b*).



(*a*) **(*b*)**

Figure 1 – Autocorrelation function (*a*) and its Fourier transform for $\overline{A}$ (*b*).

Despite such a fact, the radiation characteristics of ADS-based finite arrays still present some interesting features to be exploited for improving the array performances with respect to those of the corresponding random arrays [1] or DS-based arrays [2]. To evaluate such comparison, the power pattern and the PSL of the best ADS-based thinned array (among those defined by cyclically shifting the ADS $\overline{A}$, as it is done for DSs [2]) is reported in Fig. 2, along with the PSL of random arrays [1]

$$[PSL^{R}]_{dB} = 10\log_{10}\frac{1}{P} + 10\log_{10}\left\{-\ln\left[1-\beta^{\frac{\lambda}{l(D-1)}}\right]+1+2\left[-\ln\left(1-\beta^{\frac{\lambda}{l(D-1)}}\right)\right]^{-1}\right\} \quad (9)$$

(where β is the probability that no sidelobe exceed the considered PSL^{R} for a random array of size $x_0(D-1)$ with *P* active elements), and DSs arrays [2]

$$[PSL^{DS}]_{dB} = 10\log_{10}\left(\frac{1}{P}\left(1-\frac{P}{D}\right)\right) + 10\log_{10}\left[0.8488 + 1.128\log_{10}(D)\right]. \quad (10)$$

As it can be observed, the PSL obtained from the array based on $\overline{A}$ is significantly below the value predicted by (9) for a corresponding random array, and is comparable to the value predicted by (10) for a corresponding DS-based array.

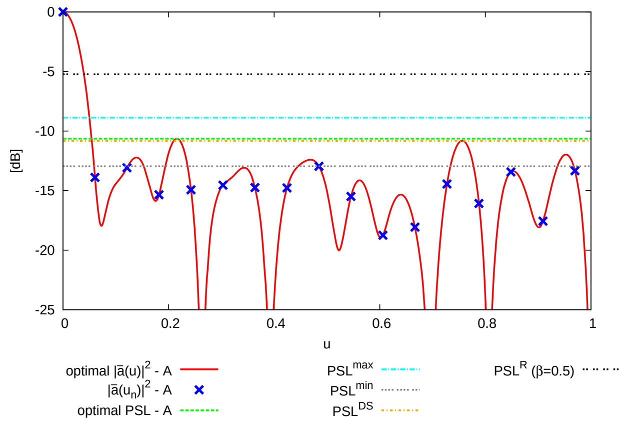


Figure 2 – Power pattern of the optimal finite array derived from $\overline{A}$.

Conclusions

In this contribution, Almost Difference Sets are applied to the design of linear thinned arrays with low sidelobes. It is numerically shown that thinned arrays based on ADSs can exhibit a PSL which is significantly better than that obtained by corresponding random arrays, and is comparable (although achievable in a wider set of array configurations) to that theoretically predicted for corresponding DS-based arrays.

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