UNIVERSITY
OF TRENTO

DIPARTIMENTO DI INGEGNERIA E SCIENZA DELL'INFORMAZIONE

38123 Povo – Trento (Italy), Via Sommarive 14
http://www.disi.unitn.it

ADAPTIVE ANTENNA ARRAY CONTROL IN PRESENCE OF INTERFERING SIGNALS WITH STOCHASTIC ARRIVALS – ASSESSMENT OF A GA-BASED PROCEDURE

C. Sacchi, F. De Natale, M. Donelli, A. Lommi, and A. Massa

January 2011

Technical Report # DISI-11-088

Adaptive Antenna Array Control in the Presence of Interfering Signals with Stochastic Arrivals: Assessment of a GA-based Procedure

Claudio Sacchi, Francesco De Natale, Massimo Donelli,
Andrea Lommi, and Andrea Massa

Department of Information and Communication Technology
University of Trento
Via Sommarive, 14 - 38050 Trento, ITALY

Abstract

In this letter, a real-world working case of interfering signals coming to an antenna array with random arrivals modeled as a Poisson process is considered. A procedure based on a suitable Genetic Algorithm for adaptive array control is assessed by means of numerical simulations. Selected results clearly demonstrate the effectiveness and flexibility of the proposed procedure.

Indexing terms: Antenna arrays, Adaptive Control, Genetic algorithms, Poisson Modeling.

1. INTRODUCTION

The use of adaptive antenna arrays in wireless communications is dated since late ‘60s. Applebaum’s paper [1] provided the mathematical basis for the optimization of signal-to-interference-plus-noise-ratio (*SINR*) in the presence of different spatial configurations of noise sources, considering both background noise and jamming signals.

Adaptive arrays are aimed at separating a desired signal from interfering ones impinging the array itself. Moreover, the continuous tuning of array weights [1] can theoretically face any variation of noise and interference occurring in the communication environment, thus ensuring optimal performance in any situation.

The optimal solution to the antenna array control problem, proposed by Applebaum in [1], requires the full inversion of the signal covariance matrix. Generally, this is not a trivial task [2]. Consequently, many alternative solutions based on dynamic programming in order to avoid the matrix inversion [2] [3] have been proposed. Least-Mean-Square (LMS) and Recursive-Least-Square (RLS) algorithms are well-known examples of mathematical solutions to array optimization (see [3] for a thorough overview). Despite their mathematical elegance, such methods present some drawbacks, hindering their practical implementation [4] [5] [6]. In more detail, LMS and RLS require analog amplitude and phase weights at each element. Although very attractive from a theoretical viewpoint the implementation of amplitude control turns out to be very expensive. For this reason, antenna arrays usually adopt only digital phase shifters for beam steering [4] [7]. The resulting weight quantization actually limits null placement. In general, the determination of the number of bit of the digital phase shifter presents a trade-off between the array performance requirements and the need of an economical design. The analysis presented in [7] points out that a choice of 8 bits for weight quantization could be satisfactory in several practical applications. On the other hand, the convergence of conventional approaches for array optimization strongly depends upon the eigenvalue spread [3] and on the external noise environment. Moreover, these techniques result very slow in severe jamming situations [8] and do not prevent the solution be trapped in local minima [4] [6]. In this framework, the use of Genetic Algorithms (GAs) can be regarded as a valuable solution for the array optimization problem [4] [5] [9]. In [9], it has been shown that GA-based approaches outperform conventional solutions based on LMS strategy (being RLS an efficient variant of LMS providing faster convergence [3]).

In [5], Weile and Michielssen proposed the use of a GA with a population characterized by double chromosomes (diploid structure) for the adaptive control of antenna arrays. The effectiveness of such an approach has been assessed by considering some specific working conditions. Nevertheless, more general test cases should be dealt with in order to assess the robustness and reliability of GA-based approaches. In particular, some restrictive hypotheses usually made in literature about the deterministic nature of the interference should be removed. For instance, in [5] the arrivals of interfering signals to the antenna array are deterministic and synchronous in time. Moreover, the angles of arrival of received signals (both desired and interfering) are deterministically fixed, too. These assumptions are not realistic in actual wireless communication systems, where the angle of arrival is a random variable (in [10] a Student's-t-distribution has been proposed as a possible statistical model), and the arrival of interfering signals is asynchronous and can be modeled as a random process. In order to define a suitable interference model, a more realistic assumption consists in modeling the arrival of interfering signals as a *Poisson process*. A Poisson process [11] [12] provides a statistical description of phenomena such as the counting of occurrences of specific events within a fixed observation time interval. It is therefore a *discrete-state process* characterized by a monotonic distribution function with discontinuity points coinciding with the discrete observation intervals [11]. Poisson processes are used to model many situations of interest in communication systems, including telegraphic signals [11], telephone calls [12], packet arrivals in computer networks [13], etc. Middleton's paper about urban radio noises [14] pointed out that the probability of generating an impulsive transient interference in a space region is subject to a Poisson law. Consequently, the arrival of an interfering signal to the antenna array can be modeled as a Poisson process. Following Middleton's idea, Sacchi et *al.* [15] modeled the ingress-noise affecting coaxial cable lines as a sum of time-limited CW sinusoidal pulses, whose arrival times are Poisson-distributed. This allowed testing the performance of different cable modem transmission techniques in remote video-based surveillance applications, regarding ingress-noise as the main capacity limitation factor in digital transmission over cable channels. In this paper the effectiveness of a suitable GA (called *Learned Real-Time Genetic Algorithm* (LRTGA) [16]) targeted to solve the optimal array control problem is assessed by considering a realistic simulation scenario. In particular:

i) a Poisson statistical model is used to describe the *random arrival* of *time-limited* interfering signals, thus emulating the usual behavior of data transmission in civil applications (burst transmission [12]);

ii) a random uniform distribution is assumed to describe the angles of arrival of jamming signals.

In addition, some deterministic hypotheses are maintained, namely:

iii) each stochastic arrival is assumed synchronous with the generation period of the GA;

iv) the duration of interfering signals is assumed here as a deterministic multiple integer of the GA generation period.

The use of the GA generation period as a time reference for the overall transmission system is not realistic, since burst transmissions of external users are in general fully asynchronous. Nevertheless such an assumption can be considered reasonable in the specific context of performance evaluation of GA-based array control strategy, as clearly stated in [5]. As far as a deterministic duration of interfering signals is concerned, the study of a more sophisticated model taking into account a random duration will be matter for future works.

The paper is structured as follows: Section 2 is devoted to describe the antenna system. Section 3 briefly outlines the GA-based strategy for adaptive array control. Section 4 describes the statistical modeling of the interference and Section 5 presents selected simulation results. Finally, Section 6 draws the conclusions.

2. ANTENNA SYSTEM DESCRIPTION

Let us consider an array antenna of *S* isotropic equally-spaced elements (being *d* the inter-element distance). According to [1], the m-*th* signal received at the i-*th* array element $S_{m,i}(t)$ $m \in \{1,...M\}$ can be expressed as follows:

$$S_{m,i}(t) = \mathrm{Re}\{\gamma_m(t)\Theta_i(\theta_m)\exp(j\omega_c t)\} \quad i = 1,..,S \tag{1}$$

where $\gamma_m(t)$ is the signal envelope, regarded as a slowly time-varying, ergodic random process, and ω_c is the carrier radian frequency (common to each signal, thus considering the case of *co-channel interference*).

Furthermore, $\Theta_i(\theta_m)$ is a term taking into account the signal phase shift:

$$\Theta_i(\theta_m) \hat{=} \exp\left(j \frac{2\pi id}{\lambda} sin(\theta_m) \right) \quad i = 1,...,S \tag{2}$$

where λ is the free-space wavelength, and θ_m is the arrival angle with respect to the broadside direction. For the sake of simplicity, let us assume the first signal of the set (*m* = 1) as the desired signal. Consequently, the other $N = M - 1$ signals are regarded to as interfering ones. Commonly, *N* is assumed to be a deterministic value. In this paper, a more realistic assumption is made by considering the number of interfering signals as a random process $N(t)$. Details on the statistical properties of $N(t)$ will be reported in the next section.

The desired and interfering signals are represented by their complex envelopes $\gamma_m(t)$ $m = 1,..., M$, which modulate the carrier. The signal envelopes never appear explicitly in the optimum array control problem solution (as clearly stated in [1]) so they can be regarded as irrelevant in the proposed analysis. This is not true for what concerns signal phase terms, for they depend on arrival angles. The power of the desired signal is assumed to be lower than the power of interfering sources. Moreover, the background noise is modeled as a gaussian process $n(t)$, added to the signals at the receiver (AWGN).

The problem consists in the optimal choice of array weights in order to maximize the signal-to-interference-plus-noise ratio at the receiver, defined as [1] [5]:

$$SINR = \alpha^2 \frac{\left| \underline{w}^T \, \underline{\Theta}(\theta_1) \right|^2}{\underline{w}^{T^*} C_u \, \underline{w}} \tag{3}$$

where α^2 is the squared mean value of the signal envelope $\gamma_m(t)$, $\underline{w}$ is the complex vector of weights $w_i = a_i \exp\{j\psi_i\}$, $i = 1,...,S$, $\underline{\Theta}(\cdot)$ is the vector of phase-related terms, and C_u is the *undesired signal covariance matrix* related to both Gaussian background noise and undesired (interfering) signals. The knowledge of the matrix C_u is troublesome, nevertheless it can be proven [5] that the cost function $f(\underline{w})$ reported in (4) has a maximum for the same vector of weights $\underline{w}^{opt}$ that maximizes (3):

$$f(\underline{w}) = \frac{\left| \underline{w}^T \, \underline{\Theta}(\theta_1) \right|^2}{\underline{w}^{T^*} C_y \, \underline{w}} \tag{4}$$

The matrix C_y appearing in (4) is the *covariance matrix* related to the observation vector $\underline{y}$ (whose i-*th* component is equal to $y_i(t) = s_{1,i}(t) + I_i(t) + n_i(t)$, being $I_i(t) = \sum_{m=2}^{M} s_{m,i}(t)$ and $n_i(t)$ the background-noise at the i-*th* array element). It can be computed on-the-fly on the basis of the received signals. For this reason, the cost function (4) will be used as *fitness function* [17] in the iterative GA-based optimization procedure aimed at computing $\underline{w}^{opt}$.

3. Adaptive optimization strategy

A suitable Genetic Algorithm is used to solve the optimization problem so far defined. GAs are multi-agent methods inspired on the principles of natural evolution [17]. Standard GA implementations (SGA) [18] represent feasible solutions as a set of individuals (called *population*), each of which is usually encoded with chromosome-like bit strings. At each iteration k, the genetic operators of *crossover* and *mutation* are applied on selected chromosomes with probability P_C, and P_M respectively, in order to generate new solutions belonging to the search space. The population generation terminates when a satisfactory solution has been produced or when a fixed number of iterations, $K_{\max}$, has been completed.

Although GAs have been applied with success to a wide variety of electromagnetic problems (see [18] for a list of applications), their application to wireless-communications is quite recent. A reason of this accounts for the fact that while standard GAs are powerful tools in off-line applications like antenna design, they are not well suited for real-time applications such as adaptive array control. As a matter of facts, the re-adaptation of the numerical procedure (i.e., convergence of population towards one or more solutions fitting the new environment conditions) is usually very slow, thus penalizing the performance of the system.

In order to provide a more efficient scheme for real-time control, SGAs basic strategy has to be enhanced. To this end, a suitably-defined GA has been proposed in [16]. The main features characterizing the LRTGA with respect to a Standard Genetic Algorithm are:

- the chromosome ($\overline{\Psi}$) codes only discrete phase coefficients, $\{\psi_i ; i = 1,...,S\}$, whereas amplitude coefficients, $\{a_i ; i = 1,...,S\}$, are fixed according to the Dolph-Chebyschev criterion [19];
- the application of genetic operators is determined according to the following *population variance measure*:

$$\sigma_k^2 = \sqrt{\frac{\sum_{p=1}^{P}\left\{ f\left(\overline{w}_k^{\,p}\right) - \frac{1}{P}\sum_{p=1}^{P} f\left(\overline{w}_k^{\,p}\right)\right\}^2}{P}} \qquad (5)$$

computed at each iteration of the genetic procedure, being P the dimension of the population. Consequently, probability values ($P_M = P_M(\sigma_k^2)$ and $P_C = P_C(\sigma_k^2)$) and discard rate of the GA ($P_R = P_R(\sigma_k^2)$, where $P_R \times P$ is the number of worst fit chromosomes that are replaced by random ones at each iteration) are heuristically tuned according to the patterns shown in Fig. 1;

- Two new genetic mechanisms are defined in order to improve the “reaction” of the algorithm to environmental changes. At each iteration, on the basis of the following *improvement measure*:

$$\Omega_k = \frac{\sum_{l=k-(L-1)}^{k} f\left(\overline{w}_l^{\,opt}\right)}{L} \qquad (6)$$

being L a fixed number of iterations, the best chromosome, $\overline{\Psi}_k^{\,opt}$, is marked as “inactive” (and temporarily excluded from the iterative process) with a probability $P_I = P_I(\Omega_k)$ proportional to the improvement of the fitness function, as indicated in Fig. 1. On the other hand, when $f\left(\overline{w}_l^{\,opt}\right)$ decreases, the fitness of inactive chromosomes is evaluated with probability $P_E = P_E(\Omega_k)$ (Fig. 1). If the fitness of an inactive chromosome results greater than $f\left(\overline{w}_l^{\,opt}\right)$, such a chromosome is activated in the next generations.

4. STATISTICAL MODELING OF INTERFERENCE ARRIVALS

Let us focus now on the statistical modeling of interference arrivals. The number of arrivals during a time interval $(0, \tau)$ is modeled with a discrete-state *Poisson process* [11], $Q(\tau)$, with cumulative distribution function given by:

$$P\{Q(\tau)=q\}=\frac{e^{-\Lambda\tau}(\Lambda\tau)^q}{q!} \qquad (7)$$

For a fixed τ, $Q(\tau)$ is a Poisson-distributed random variable, with mean value given by [11]:

$$E\{Q(\tau)\}=\Lambda\tau \qquad (8)$$

being Λ the *Poisson rate*, or *Poisson frequency*. In our specific context, the observation time interval is a multiple of the generation period of GA, so that at each generation a different random number of interfering signals may arrive to the array. By indicating with T_G the generation period of the GA, the parameter τ results equal to $\tau = jT_G$, $j = 0,1,2,..., J$ (a finite-length observation time interval is assumed). Consequently, the total interference that affects the i-*th* array element is given by:

$$I_i(t)=\sum_{j=0}^{J}\sum_{m=2}^{M_j\neq 0} s_{m,i}(t-jT_G)\Pi_{T_I}(t-jT_G) \quad i=1,..,S \qquad (9)$$

where $\Pi(t)$ is the rectangular pulse function [12] of unit amplitude and duration T_I, and M_j is the *effective number* of signals arriving to the i-*th* array element at the beginning of the j-*th* generation period. M_j can be written as:

$$M_j \,\hat{=}\, Q(jT_G)-R(jT_G) \qquad (10)$$

being $R(jT_G)$ the number of time-limited interfering signals arrived during past iterations and switched-off at the current one. Due to causality and finite duration assumed for all signals, $M_j \geq 0\ \forall j$. In the proposed approximated model the interference arrival results synchronous in time with respect to GA generation period. In real-word applications, the arrival of interfering signals to a receiving station is asynchronous. Nevertheless, the proposed time-synchronous model is reasonable, because the GA-based array control algorithm can react and adapt itself to a new scenario only in correspondence with a new GA generation. Moreover, the duration of the interfering signals has been fixed equal to T_I, being T_I an integer multiple of T_G. As an example, Figure 2 shows a sample of the random process that represents the arrivals of interfering signal, obtained by setting the following parameters: $\Lambda = 1/T_G$ and $T_I = 5T_G$. The observation interval has been set to 900 GA-generations.

As far as the impinging direction of jamming signals is concerned, arrival angles are randomly generated with a uniform distribution in the range $\left(-90°,90°\right)$. In Figure 3, a simulated sample of arrival angles of interfering signals at each iteration of the genetic algorithm is depicted.

5. SIMULATION RESULTS

In this section, selected simulation results are provided in order to demonstrate the robustness of LRTGA-based array control procedure when jamming signals characterized by stochastic arrivals and stochastic arrival angles occur. For comparison purposes, a state-of-the-art SGA-based control strategy, the optimal Applebaum's weighting strategy [1] (applied to a continuously-weighted array as a reference on the optimal solution), and a modification of Applebaum's method (which considers continuous module and discrete phase coefficients as in the genetic process) are used as touchstone methods. Following the approach in [1], it is possible to obtain very accurate nulls in the radiation diagram, whose depth reflects into high values of SINR. Unfortunately, as already pointed out in Section 1, this control method is very efficient but difficult to be implemented for real-world applications. Fig. 4 shows the running averages of the SINR over 100 past iterations (being $K_{\max} = 900$) considering a signal-to-single interference power ratio *C/I* = $-30\,$dB (the signal-to-background noise ratio has been set to 30dB in overall simulations). For such a simulation, an 8-bit phase shifter (corresponding to *L*=256 phase quantization levels) has been considered. It can be observed that LRTGA significantly improves the capability of adaptation of SGA-based control algorithm, resulting in an effective processing of stochastic arrivals of interfering signals. It is to be pointed out that although the achieved *SINR* decreases for an increasing number of interfering signals, the *SINR* values provided by LRTGA are almost positive even when the number of interfering users is quite high. This is not true for SGA-based procedure, which often provides negative *SINR* values. Moreover, despite the optimality of the method in [1], the SINR attained by LRTGA is almost equivalent to the one achieved by Applebaum's method with discrete phases, even if the latter considers continuous modules. The statistics deriving from some hundreds of LRTGA and SGA executions are reported in Table 1 in correspondence of three different scenarios (*C*/*I* ratio equal to $-10, -20$ and -30 dB). The LRTGA-based method again confirms its effectiveness with respect to the SGA-based method, slightly overcoming the performances (in terms of mean value of SINR) of Applebaum's method with discrete phases.

For completeness, Table 2 reports the results of a numerical assessment aimed at evaluating the dependence of LRTGA performances on the numbers of phase quantization levels. It is worth noting that LRTGA outperforms discrete-phases Applebaum's method when digital phase shifters using few bits for phase quantization (i.e., from 4 to 9 bits) are employed. As expected, discrete-phases Applebaum's method provides better results when digital phase shifters using more than 10 bits are considered. Nevertheless, as clearly stated in [7], phase shifters with an increased number of quantization levels can involve higher hardware costs, strongly limiting the practical implementation.

Finally, let's introduce some notes about convergence rate and computational load of the proposed algorithm. The convergence rate of GA-based array control strategies only depends on the population dimension *P*, [17], being independent of the specific parameters of the optimization problem to be faced. This is not true for LMS algorithm, whose convergence rate directly depends on the eigenvalue spread of the covariance matrix [3]. Better performances can be achieved by RLS, as clearly stated in [3].

On the other hand, the computational complexity of GA-based methods is rather similar to that of conventional approaches. The number of elementary operations, υ_{op}, required by RLS and

LMS algorithms is approximately equal to $MK_{\max}$ [20]. As far as GAs are concerned , υ_{op} , results equal to [17]:

$$\upsilon_{op} = (P_C + P_M)PK_{\max} \tag{11}$$

where P_C and P_M are crossover and mutation probability respectively, and usually $(P_C + P_M)P \le M$.

6. CONCLUSIONS

In this letter, the effectiveness of a suitable GA-based strategy for adaptive antenna array control was assessed, in the presence of stochastic arrivals of time-limited interfering signals. The arrival time of interfering signals was modeled as a discrete-time Poisson process, whereas a deterministic duration for the interfering sources (multiple of the generation period of the genetic algorithm) was considered. Also the angles of arrival of interfering signals were modeled as random variables. In this framework, the robustness of LRTGA has been compared with the optimal solution, by enforcing randomly time-varying working conditions quite similar to real world environment. Future developments of the proposed analysis should assess the efficiency of LRTGA in the presence of interfering signals with random duration (i.e.: a random multiple of the generation period), different for each interfering signal.

REFERENCES

[1] S. P. Applebaum, “Adaptive arrays,” *IEEE Trans. Antennas Propagat.*, vol. A.P-24, No. 5, pp. 585-598, September 1976.

[2] R.L. Haupt, “The Development of Smart Antennas”, *2001 IEEE International Symposium of Antennas and Propagation Society*, Vol. 4, 2001, pp. 48-51.

[3] I. Pokrajac, M. Sunjevaric, B. Zrnic, “The Base Station Antenna Array System Output SINR Determination for Different Spatial Channel Models”, *5th Conf. on.*

Telecommunications in Modern Satellite Cable and Broadcasting Service (TELSIKS 2001), Nis (YU), September 19-21, 2001, Vol. I, pp. 141-144.

[4] R. L. Haupt, "Phase-only adaptive nulling with a genetic algorithm," *IEEE Trans. Antennas Propagat.*, vol. 45, No.6, pp. 1009-1015, June 1997.

[5] D. S. Weile and E. Michielssen, "The control of adaptive antenna arrays with genetic algorithms using dominance and diploidy," *IEEE Trans. Antennas Propagat.*, vol. 49, No. 10, pp. 1424-1433, October 2001.

[6] R.L. Haupt, H. Southall, "Experimental Adaptive Cylindric Array", *Proc of 1999 IEEE Aerospace Conference*, Vol. 3, 1999, pp. 291-296.

[7] H.J. Im, S. Choi, B. Choi, H. Kim, J.Choi, "A survey of essential problems in the design of smart antenna system, *Microwave and Optical Technology Letters*, Vol. 33, No. 1, April 5, 2002.

[8] C.C. Ko, "Power Inversion Array in a Rotating Source Environment", *IEEE Trans. on Aerosp. Elecron. Syst.*, Vol. AES-16, No. 6, November 1980, pp. 755-762.

[9] Y. Chung, and R.L. Haupt, "Amplitude and phase adaptive nulling with a genetic algorithm", *1998 IEEE/URSI International Symposium, Atlanta (GA)*, June 1998.

[10] J. Bach-Andersen, K. Ingemann-Pedersen, "Angle-of-Arrival Statistics for Low Resolution Antennas," *IEEE Trans. Antennas Propagat.*, vol. 50, No. 3, pp. 391-395, March 2002.

[11] A. Papoulis, "*Probability, Random Variables and Stochastic Processes*" McGraw-Hill, New York: 1991.

[12] A. Carlson, "*Communication Systems*", McGraw-Hill, New York: 1987.

[13] A. Tannenbaum, "*Computer Networks*", Prentice-Hall, New Jersey: 1996.

[14] D. Middleton, "Statistical-physical models of urban radio-noise environments-Part I: Foundations," *IEEE Trans. Electromagn. Compat.*, Vol. 14, No. 2 pp. 38-56, May 1972.

[15] C. Sacchi, G. Gera, and C. Regazzoni, “Actual high-speed modem solutions for multimedia transmission in remote cable-based video-surveillance systems,” in: *Multimedia Video-Based Surveillance Systems*, C. Regazzoni, G. Foresti, P. Mahonen (eds.), Kluwer Academic Publishers, Norwell (MA): 2000, pp. 197-206.

[16] S. Caorsi, M. Donelli, A. Lommi, and A. Massa, “A new GA-based strategy for adaptive antenna array control,” *Proc. JINA2002 Conference*, Nice, France, pp. 447-450, 12-14 November 2002.

[17] D. E. Goldberg, “*Genetic Algorithms in Search, Optimization and Machine Learning”*, Addison-Wesley, Reading, MA: 1999.

[18] J.M. Johnson, Y. Rahmat-Samii, “Genetic algorithms in engineering electromagnetics”, *IEEE Antennas Propagat. Mag.*, Vol. 39, No. 4, pp. 7-21, August 1997.

[19] C. L. Dolph, “A current distribution for broadside arrays which optimizes the relationship between beamwidth and sidelobe level,” *Proc. of IRE*, vol. 34, pp. 335-345, 1946.

[20] V.V. Zaharov, F.S. Casco, O.A. Amin, M.L. Villasenor, “Smart Antenna Technique for Wireless Communications”, *1st International Conference “Microwave & Telecommunications Technologies” (CriMiCo 2001)*, Sevastopol (UKR), September 10-14, 2001, pp. 318-320.

FIGURE CAPTIONS

- Figure 1

  Patterns of LRTGA probabilities.

- Figure 2

  Example of stochastic realization of Poisson-distributed interference arrivals ($\Lambda = 1/T_G$, $T_I = 5T_G$).

- Figure 3

  Distribution of the arrival angles of the interfering signals versus GA iteration number.

- Figure 4

  Running average *SINR* in presence of stochastic interference arrivals ($K_{\max} = 900$), computed by considering 100 past iterations. Comparison among results provided by: LRTGA (solid line), SGA (dashed line), optimal method reported in [1] (dash-dotted line), and the same method with phase-coefficients constrained to discrete values (dotted line). A digital phase shifter with *L*=256 phase quantization levels has been considered in overall simulations.

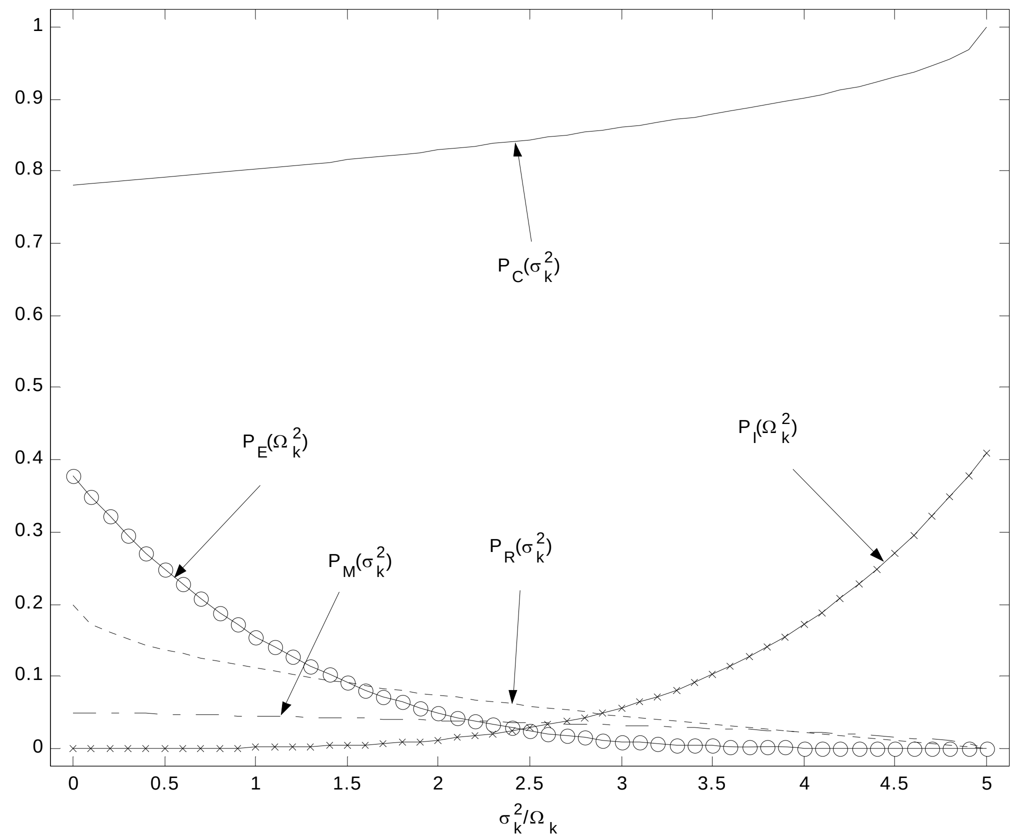

Fig. 1 – C. Sacchi et *al.*, “Adaptive Antenna Array Control ...”.

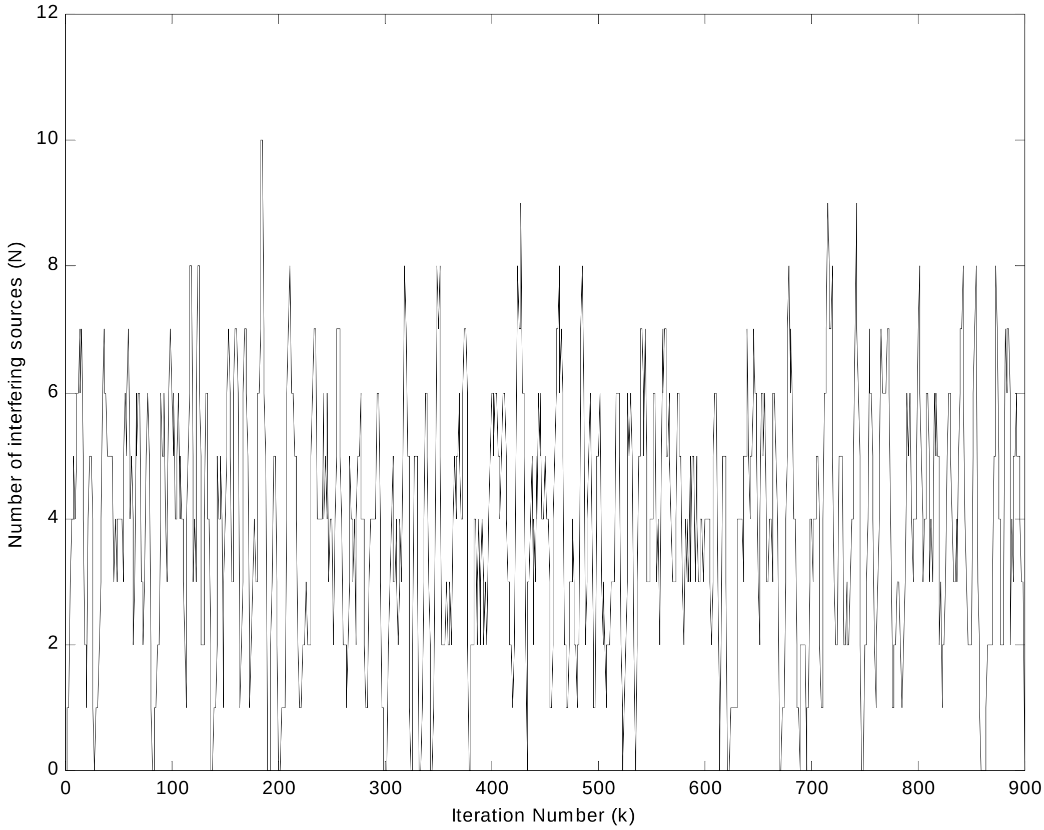

Fig. 2 – C. Sacchi et *al.*, “Adaptive Antenna Array Control ...”.

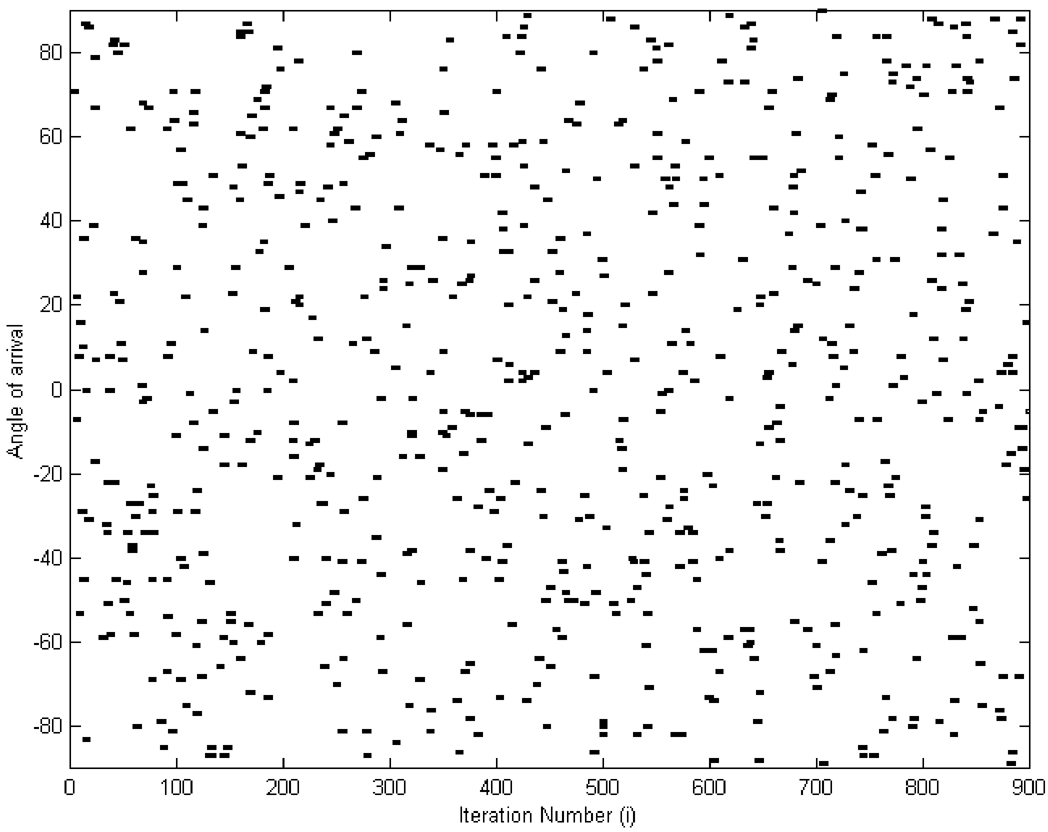

Fig. 3 – C. Sacchi et *al.*, “Adaptive Antenna Array Control ...”

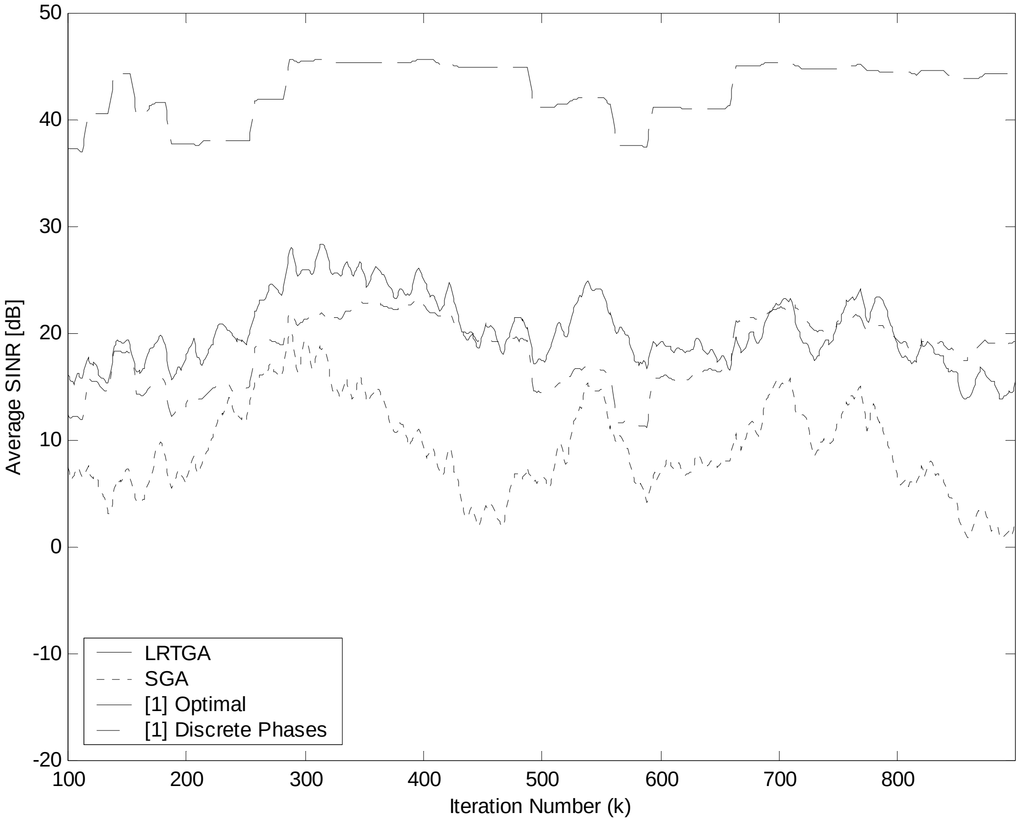

Fig. 4 – C. Sacchi et *al.*, “Adaptive Antenna Array Control ...”

TABLE CAPTIONS

- Table 1.

  SINR statistics deriving from hundreds of executions of LRTGA and SGA, compared with the optimal method in [1], and the same method with phase coefficients constrained to discrete values for different C/I ratios, and L=256 phase quantization levels.

- Table 2

  Average *SINR* deriving from hundreds of executions of LRTGA and SGA, compared with the optimal method in [1], and the same method with phase coefficients constrained to discrete values for C/I = -30dB and L=16, L=64, L=128, L=256, L=512, and L=1024 phase quantization levels.

	C/I = - 10dB		C/I = - 20dB		C/I = - 30dB	
Method	**Mean**	**σ**	**Mean**	**σ**	**Mean**	**σ**
LRTGA	34.86	15.25	29.93	18.42	19.92	21.66
SGA	16.62	16.81	13.65	19.16	10.91	20.16
Applebaum [1]	43.17	10.23	42.84	11.97	42.50	13.73
Applebaum [1] with discrete phases	35.38	15.09	27.16	15.73	17.94	16.47

Table 1 – C. Sacchi et *al*., “Adaptive Antenna Array Control ...”.

	Average SINR (dB)		
Phase quantization levels	**Applebaum [1] with discrete phases**	**LRTGA**	**SGA**
16	1.81	10.52	4.92
32	4.26	13.97	6.73
64	7.57	15.37	6.78
128	12.38	17.39	8.33
256	17.94	19.92	10.91
512	24.28	24.54	12.84
1024	29.26	25.85	14.02

Tab. 2 – C. Sacchi et *al*., “Adaptive Antenna Array Control ...”.