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### ANT COLONY OPTIMIZATION FOR TREE-SEARCHING BASED SYNTHESIS OF MONOPULSE ARRAY ANTENNA

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# Ant Colony Optimization for Tree-Searching based Synthesis of Monopulse Array Antenna

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The design of simple feed networks is of great interest in synthesizing monopulse radar array antennas in order to reduce the complexity of the antenna architecture, the costs as well as the occupied physical space (e.g., on aircrafts). Sub-arraying techniques have been proposed to properly address such a task. In this letter, starting from the formulation of the sub-arraying problem in terms of a combinatorial one, the final compromise solution is obtained by looking for the minimum cost path inside a binary tree graph through an Ant Colony Optimizer.

Introduction: The synthesis of the optimal compromises between sum and difference patterns in monopulse radar arrays has been widely studied in the literature. Various methods based on the sub-arraying strategy [1] have been proposed to reduce both the complexity of the feeding network and the costs, thus obtaining cheap and compact antenna devices suitable for mobile applications. The synthesis problem is recast as the optimization of the sub-array configuration and of the sub-array weights to afford a compromise difference pattern that satisfies a set of user-defined constraints for a given optimal sum mode. Several approaches based on simulated annealing [2][3], genetic algorithms [4], and differential evolution [5] have been proposed to deal with the non-linearity of the synthesis problem and to avoid the ill-conditioning of the system-matrix-based approach described in [1]. Recently, an excitation matching approach has been proposed [6] where the array optimization has been reformulated into a combinatorial problem by exploiting the relationships between the optimal and independent sum and difference excitations. Since the set of

admissible solutions can be properly reduced by considering only contiguous partitions, the solution space has been represented by means of a non-complete binary tree. To look for the best compromise solution (i.e., the minimum cost path), an *ad-hoc* local search strategy, called border element method (BEM), has been implemented. Nevertheless, since the combinatorial optimization deals with a non-convex functional, the BEM can get stuck into local minima. In order to overcome this drawback, the ant colony optimizer (ACO) [7] is considered in this work because of its "hill climbing" properties and since, unlike other evolutionary-based optimizers, it fully exploits the graph representation of the solution space. Inspired by the foraging behavior of ant colonies, where the ants look for the shortest path between the food sources and the nest, the ACO was proposed by Dorigo *et al.* [7] as a viable approach to stochastic combinatorial optimization.

Ant colony optimization for monopulse array antenna synthesis: Let us consider a linear uniform array with N = 2M,  $m = \pm 1,...,\pm M$ , radiating elements spaced by d. With reference to the sub-arraying strategy [1], the optimal sum pattern is given by a set of symmetric real excitations  $\underline{A}^{opt} = \{\alpha_m = \alpha_{-m}; m = 1,...,M\}$ , while the coefficients of the compromise difference pattern are defined as  $\underline{B} = \{b_m = -b_{-m}; m = 1,...,M\}$  where  $b_m = \alpha_m \delta_{c_m q} w_q$ . Moreover,  $\underline{W} = \{w_q; q = 1,...,Q\}$  is the set of sub-array weights,  $\delta_{c_m q}$  is the Kronecher delta, and  $\underline{C} = \{c_m \in [1,Q]; m = 1,...,M\}$  defines the sub-array memberships of the array elements. According to the excitation matching approach proposed in [6], let us consider the following matching function

$$\Psi(\underline{C},\underline{W}) = \frac{1}{M} \sum_{q=1}^{Q} \sum_{m=1}^{M} \alpha_m^2 \left| \frac{\beta_m}{\alpha_m} - \delta_{c_m q} w_q(\underline{C}) \right|^2$$
(1)

where  $\underline{B}^{opt} = \{\beta_m = -\beta_{-m}; m = 1, ..., M\}$  is the set of reference/optimal difference excitations,  $g_m = \frac{\beta_m}{\alpha_m}$ , and  $w_q(\underline{C})$ , q = 1,...,Q, is given by the weighted (with weights  $\alpha_m^2$ ) arithmetic mean of the optimal gains  $g_m$  of the elements belonging to the q-th sub-array [6]. Accordingly, the definition of the best compromise difference pattern close as much as possible to the optimal one is obtained by determining the unknown aggregation array C. Towards this end, the solution space is represented in an effective fashion by means of a non-complete binary tree [6] of depth M-1. Each path from the root to a leaf codes an admissible subarray configuration. A sketch of the solution tree when M = 5 and Q = 3 is shown in Fig. 1, U = 6 being the number of admissible solutions. Unlike [6], where the BEM is used to minimize (1), the ACO is adopted here to avoid the local minima problem. Each ant  $\underline{a}^{(i)}$ ,  $i = 1, \dots, I$ , is a collection of M integer values that codes a trial aggregation vector  $\underline{C}^{(i)}$  filled in while the ant is moving throughout the binary tree, from the root towards the leafs, as schematically depicted in Fig. 1. At the first iteration (k = 0, k being the iteration index), the branches of the tree have the same probability of being explored. At each node, the probability of choosing a path is given by  $p_{b,h}^k = \frac{\tau_{b,h}^k}{\tau_{1,h}^k + \tau_{2,h}^k}$ , b = 1,2; h = 1,...,M-1. At the end of each iteration,

the pheromone level  $\tau_{b,h}^{k}$  is updated as follows

$$\tau_{b,h}^{k+1} \leftarrow \tau_{b,h}^{k} + \frac{H}{\Psi_{c}^{k}}$$
(2)

where  $\Psi_k^{(i)} = \Psi(\underline{C}_k^{(i)}, \underline{W}_k^{(i)})$  is the fitness value of the *i*-th ant at the *k*-th iteration and *H* is a positive constant. Afterwards, the evaporation operation is applied

$$\tau_{b,h}^{k+1} \leftarrow (1-\rho)\tau_{b,h}^{k+1} \tag{3}$$

 $\rho \in (0,1]$ . The procedure is iterated until a maximum number of iterations, *K*, or when a fixed threshold on the percentage of ants going through a path is reached.

*Numerical assessment*: Let us consider a 100-elements array (M = 50) with  $d = \frac{\lambda}{2}$ . The optimal sum and difference coefficients have been set to those of a Dolph-Chebyshev sum pattern with SLL = -25 dB [8] and of a Zolotarev difference pattern with SLL = -30 dB, respectively. Q = 6 sub-arrays are considered in the compromise synthesis. As far as the BEM is concerned, a uniform aggregation  $\underline{C}_{0}^{BEM}$  has been used at the initialization (k = 0), while the following setup has been adopted for the ACO: I = 1000, K = 500, H = 0.1, and  $\rho = 0.01$ . Moreover, the initial values of the pheromone on each branch have been fixed to  $\tau_{b,h}^{0} = 0.5$ , b = 1,2; h = 1,...,M-1.

Figure 2 shows the behaviors of the minimization algorithms versus the iteration number *k*. As it can be noticed, the BEM gets stuck after  $K^{BEM} = 5$  iterations reaching a convergence fitness value equal to  $\Psi_{opt}^{BEM} = 1.96 \times 10^{-2}$ . On the other hand, the ACO outperforms the BEM in matching the optimal difference pattern by achieving a better compromise solution with  $\Psi_{opt}^{ACO} = 1.70 \times 10^{-2}$  thanks to its effectiveness in sampling the  $U \cong 2 \times 10^6$  paths of the solution tree. For completeness, the compromise difference beams obtained by the BEM and the ACO as well as the optimal difference pattern are shown in Fig. 3. It is worth notice that, although the ACO optimization is aimed at the best matching with the optimal difference pattern and not to minimize the SLL, the SLL value synthesized with the ACO is below of more than 3 *dB* to that of the BEM.

*Conclusion*: In this paper, the ACO is used for graph searching instead of BEM to properly address the local minima problem and to fully exploit the tree-structured representation of the solution space. Preliminary results have been reported to assess the effectiveness of the proposed ACO-based excitation matching approach in dealing with the compromise synthesis problem.

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#### Figure captions:

- Fig. 1 Non-complete binary solution-tree and ANT solutions.
- Fig. 2 Plots of the matching cost function  $\Psi$  versus the iteration index: ACO (best and mean values) and the BEM.
- Fig. 3 Reference optimum and compromise difference patterns obtained with the ACO and the BEM (M = 50 and Q = 6).

Figure 1



Figure 2



Figure 3

