Doctoral School of Social Sciences

Doctoral programme in Economics and Management

Essays on the Prelude to the Critique of Economic Theory

a dissertation submitted in partial fulfillment of the requirements for the Doctoral degree (Ph.D.) doctoral programme in Economics and Management

Giacomo Bracci

Academic Year 2021/2022

Supervisor:

Prof. Stefano Zambelli
University of Trento

Doctoral Committee:

Prof. Anna Maria Variato
University of Bergamo

Prof. Ragupathy Venkatachalam
Goldsmiths, University of London

Prof. Enrico Zaninotto
University of Trento

Contents

1 On the importance and the unimportance of basic and non-basic goods **13**
- 1.1 Introduction . . . 13
- 1.2 Related literature . . . 15
- 1.3 The model . . . 19
- 1.4 A numerical example . . . 23
- 1.5 Conclusions . . . 28

2 Piero Sraffa and Monetary Theory: a Reappraisal **30**
- 2.1 Introduction . . . 30
- 2.2 The building blocks of Sraffa's monetary theory in the 1920s . . . 33
- 2.3 Friedrich Hayek's *Prices and Production* . . . 36
- 2.4 Sraffa's 1932 critique of Hayek's *Prices and Production* . . . 41
- 2.5 Sraffa's innovative insights on money . . . 47
- 2.6 Potential and limitations of the 'commodity rates of interest' . . . 53
- 2.7 The integration of money in Sraffian schemes . . . 59
- 2.8 Concluding remarks . . . 66

3 Walrasian General Equilibrium and the Sraffian critique **70**
- 3.1 Introduction . . . 70
- 3.2 Hahn's 'island' experiment . . . 71
- 3.3 The neoclassical economist in the island . . . 74
- 3.4 The Sraffian economist in the island . . . 78
- 3.5 Special and general theories . . . 84
  - 3.5.1 Flexible production coefficients . . . 85
  - 3.5.2 Non-uniform profit rates . . . 87
  - 3.5.3 Constant relative prices . . . 89
  - 3.5.4 Aggregation problems, constructivity and computational difficulties . . . 91
- 3.6 Conclusions . . . 96

4 Output gap, participation and minimum income: a proposal for Italy **97**
- 4.1 Background and motivation . . . 97

4.2 The output gap and the fiscal framework of the Euro area 101
4.3 A bird's-eye view of Italy 103
4.3.1 GDP stagnation and the labor market 103
4.3.2 Actual vs. structural unemployment 108
4.4 The proposal: simulation results and policy implications 110
4.5 Criticism and the case of Germany 116
4.5.1 Main points of criticism 116
4.5.2 Labor supply shocks and the case of Germany 118
4.6 Conclusions 122

List of Figures

1.1 Domain of the self-replacing prices (left) and the labor share of output (right) . . . 24
1.2 Domain of the self-replacing profit rates (left) and the labor share of output (right) . . . 25
1.3 Domain of the aggregate value of capital, prices (iron, coal, wheat, beans and horses) and the wage rate . . . 26
1.4 Domain of the sectoral value of capital, prices (iron, coal, wheat, beans and horses) and the wage rate . . . 28

4.1 Broader estimates of labor underutilization in the Euro area. Source: elaborations on OECD and Eurostat. All components are expressed as a percentage of the active population. The latest observations are for the yearly average of 2017. . . . 98
4.2 Broader estimates of labor underutilization in Italy. Source: elaborations on OECD and Eurostat. All components are expressed as a percentage of the active population. The latest observations are for the yearly average of 2017. . . . 99
4.3 Italy's real GDP dynamics. GDP, billions of Euro at constant 2010 prices, bold line, left scale. Per capita GDP, Euro at constant 2010 prices, dashed line, right scale. Source: elaborations on AMECO. . . . 104
4.4 Gaps in participation. The figure reports the dynamic pattern of the difference between Italy's participation rate and the average participation rate of two aggregates. Source: elaborations on OECD Stats, Labour Force Statistics, labour force participation rate (15-64). . . . 105
4.5 Gaps in output gap. The figure reports the dynamic pattern of the difference between Italy's output gap and the output gap of different countries or aggregates. Source: elaborations on AMECO. 107
4.6 Italy's actual unemployment rate and NAWRU. Source: AMECO (data extracted on March 2018). . . . 109
4.7 Germany's cyclically adjusted budget balance and output gap. Note: both measures are expressed in % of potential GDP. Source: AMECO (data extracted on March 2018). . . . 120

List of Tables

1.1 Uniform shares across industries with $d_w = 0.7$. The numeraire is the net product. (*) Multiplied by 10^{-3} 27
1.2 Uniform profit rates across industries with $d_w = 0.7$. The numeraire is the net product. (*) Multiplied by 10^{-3} 27

4.1 Comparative statics model for 2016 (different policy scenarios). . 112
4.2 Simulation model for 2017 (single policy scenario). 113
4.3 Actual and policy macroeconomic scenarios for 2017. Source: AMECO (data extracted on March 2018). 115

Abstract

This PhD thesis consists of four chapters on different topics in economic theory and empirical applications. The research questions in the first three chapters are principally based on the works of Piero Sraffa. The first chapter discusses the distinction between basic and non-basic commodities in Piero Sraffa's *Production of Commodities by Means of Commodities* (Sraffa, 1960) and finds that once the distribution of the whole surplus (which includes basic and non-basic goods) is treated as in Zambelli (2018b), the computation of self-replacing prices and the profit rates can be made by considering the whole production system (i.e., in terms of the distribution of a surplus which is made of both basics and non-basics) and not only the basic industries. Therefore, the methodology in Zambelli (2018b) enables the computation of all the self-replacing prices and profit rates as a function of the wage rate measured in terms of the whole surplus and not only as a function of the surplus of the basic system.

The second chapter aims at reconstructing Piero Sraffa's monetary theory through an analysis of Sraffa's published and unpublished works. A coherent approach to monetary theory emerges, according to which money is inherently non-neutral with respect to economic outcomes, due to the institutional forces establishing the monetary standard of deferred payments. Moreover, it is shown that Sraffa was able to anticipate monetary problems that would have been discussed only several decades later in the economic literature.

The third chapter investigates the conditions under which Sraffian schemes can be reduced to a special case of the Walrasian general equilibrium model, as argued by Hahn (1982). By devising a thought experiment, a neoclassical model and a Sraffian model are compared and analyzed from the perspective of Computable Economics. It is found that arguments based on the flexibility of production coefficients, the uniformity of profit rates across sectors, intertemporal specification of Sraffian schemes, and topological existence theorems of Walrasian general equilibrium are not sufficient in order to prove that Sraffian schemes arise out a specialization of neoclassical theory as claimed by Hahn (1982). Moreover, it is shown that when Sraffian schemes are set in a fully-fledged neoclassical model, including a neoclassical choice of technique and a set of consumer preferences mapped by utility functions, income distribution is still indeterminate.

The fourth chapter builds on the debate on the soundness of neoclassical methods for the estimation of potential output adopted by the European Commission in pre-Covid years. The introduction of a deficit-financed conditional minimum income (CMI) to discouraged people which are outside the labor force is proposed and discussed within the commonly agreed European methodology for the estimation of potential output. By stimulating participation, this measure would bring about an upward revision of Italy's potential output, and this in turn will contribute to generate a greater fiscal stance. It is shown that policies targeting discouraged individuals are susceptible of having a positive

impact on the growth of the productive capacity in the economy. The chapter also discusses the theoretical problems and empirical pitfalls of the commonly agreed methodology.

Keywords: Piero Sraffa, monetary theory, Sraffian economics, Computable Economics, output gap, conditional basic income

Acknowledgements

This PhD thesis is the result of several years of work during which my research project has undergone numerous transformations over time.

I am deeply grateful for the teaching, guidance and inspiration provided by Professor Stefano Zambelli. Indeed, Stefano Zambelli allowed me to develop a strong interest both in deep theoretical debates on value and distribution and in the fascinating world of the mathematical foundations of economic theory. His passionate way of teaching and researching shaped and transformed my way of thinking about economic problems.

The insights developed by Professor Zambelli throughout his *oeuvre* are certainly the greatest source of inspiration for the works included in this thesis and for all the ideas that I have developed beyond this dissertation. All the errors in the following chapters fully remain my own.

Furthermore, I am deeply indebted to Professor Zambelli for his perpetual support, friendship and encouragement, without which this thesis would not have been completed, and to Professor Ragupathy Venkatachalam for his illuminating insights, which particularly influenced the third chapter of this thesis. I am also indebted to Professor Luigi Mittone and the Economics and Management Department at the University of Trento for their help and support.

I am also especially grateful to Alessandro Goracci, Piero Cipollone, Paola Ansuini, Carlo Chiattelli, Emiliano Micucci and Alessandro Nardi for allowing me to complete this research project alongside my work experiences. For the time and energy that I have been able to spend in completing this project, I have to thank you.

Pasquale Tridico and Walter Paternesi Meloni deserve special thanks for the effort and passion they put in devising our joint paper, which is included in this thesis as the fourth chapter. The idea behind this paper enabled me to be part of an exciting journey at the crossroads between economics and policy applications.

Moreover, I thank Warren Mosler, Randall Wray, Emiliano Brancaccio, Massimo Ricottilli, Francesco Ruggeri, Marco Veronese Passarella, Andrea Terzi, Daniele Santolamazza, Daniele Della Bona, Simone Gasperin, and Guendalina Anzolin for the numerous conversations which decisively contributed to shaping my knowledge of economics.

Finally, I thank my parents and Carlotta for their unvaluable patience, support and understanding.

Overview

This PhD thesis consists of four chapters on different topics in economic theory and empirical applications. The first three chapters share the works and insights of Piero Sraffa as a common inspiration for the different investigations carried out therein.

Piero Sraffa's *oeuvre* is mainly concerned with the theory of value and distribution, but it also encompasses different - albeit interrelated - topics such as monetary theory, banking and finance, and the theory of production. On top of the author's published works, the publication of his manuscripts, correspondence and unpublished preparatory material at the Wren Library of Trinity College in Cambridge, UK (henceforth, the "*Sraffa Papers*") enabled a more complete understanding of his thinking.

Sraffa's seminal book *Production of Commodities by Means of Commodities* (Sraffa, 1960) set off a lively debate which has shook the foundations of economic theory and has not been settled ever since. The subtitle, "*Prelude to a Critique of Economic Theory*", hints at the critical nature of the book with respect to the prevailing economic theory at the time the author was writing - i.e., the neoclassical theory of value and distribution. In Sraffa's words:

> "*It is, however, a peculiar feature of the set of propositions now published that, although they do not enter into any discussion of the marginal theory of value and distribution, they have nevertheless been designed to serve as the basis for a critique of that theory. If the foundation holds, the critique may be attempted later, either by the writer or by someone younger and better equipped for the task*". (Sraffa, 1960, p. vii)

Hence, the first three chapters of this thesis aim at making small steps in the direction hoped by Piero Sraffa. These chapters - and the research questions which they address - stem from the approach and the interpretation of Sraffian economics undertaken by Prof. Stefano Zambelli, Prof. Ragupathy Venkatachalam, Prof. Vela Velupillai, and other distinguished scholars within the *Algorithmic Social Sciences Unit* (ASSRU).

This approach is based on two fundamental pillars. First, Piero Sraffa's study of the production, measurement and distribution of income shows that economic theories do not possess sufficient elements in order to explain income

distribution as a consequence of the mere action of market forces. To do so, Sraffa (1960) constructs a set of thought experiments aimed at demonstrating that the prices which would allow an observed economic system to reproduce itself (i.e., the *self-replacing* prices) are associated to a certain way of distributing the net product (or *surplus*) of such a system.

Sraffa's point, however, is that forces outside the economic system have a crucial role in determining income distribution and economic values (prices, wages and profits). Hence, Sraffa (1960) regards distribution as fundamentally *indeterminate* and offers no theory of its determination (Bharadwaj, 1963, p. 1450): production relations determine the net product but do not contain sufficient information to derive the prevailing distributive shares.

Secondly, Sraffa's method is consistent with the principles of *Computable Economics*, which embraces classical recursion theory and constructive mathematics in economic theory and applications (Velupillai, 2021).

According to Computable Economics, the pervasive use of classical mathematics - especially, but not uniquely, in neoclassical models - has led the economic profession to rely on non-constructive existence proofs of objects for which there is no algorithm capable of generating them in a finite number of steps.

For instance, the existence proofs of Walrasian general equilibrium presented in classics such as Debreu (1959) and Arrow et al. (1971) are neither constructive nor computable (Velupillai, 2006); moreover, the first and the second welfare theorems are not computationally feasible (Velupillai, 2007), and the attempts to provide Walrasian general equilibrium theory with a computational content have not yet succeeded in the task.

In the view of Computable Economics, the lack of computational content in most economic models is due to several problems in the mathematics underpinning them. First, the closure of neoclassical models is often provided by assuming a structure of agents' preferences mapped by utility functions for which, however, there is no effective universal procedure to generate preference orderings - i.e., preference orderings are *undecidable* (Velupillai, 2000, p. 38).

Furthermore, values and quantities are often defined in the domain of real numbers. The latter includes the subset of irrational numbers which is uncountably infinite, and thus composed of numbers for which there is one-to-one to natural numbers. Hence, there is no effective procedure to pick a a computable number from a non-computable set. As a consequence, redefining economic theorems in the context of natural or rational numbers implies that their general validity is no longer ensured[1].

Finally, several crucial results in Walrasian general equilibrium theory - but also in game theory [2] and in some strands of 'heterodox' economics - lack al-

[1] See, for instance, Velupillai (2014) on how, despite the possibility to create a constructive alternative to the non-constructive *Hahn-Banach theorem* used in the proof of the second fundamental welfare theorem (Arrow et al., 1971, p. 382), the impossibility to constructivize the *Uzawa equivalence theorem* (Uzawa, 1962) used in the derivation of excess demand functions makes it impossible to apply the constructive version of the theorem to compute a Walrasian equilibrium.

[2] See Velupillai (2016) on the constructive indeterminacy of Nash equilibria of finite games.

gorithmic content, in that they are not produced by a finite set of instructions culminating in a halting of the procedure as soon as the solution is found.

On the other hand, Computable Economics recognizes that Piero Sraffa's works entail propositions which are based on definitions and assumptions on the economic system, and which are proved to exist via rules of inference, non-ambiguous and finite sets of instructions and computation rules (Velupillai, 2021). For instance, the the Standard system in pages 26 and 27 of Sraffa (1960) is obtained via a complete algorithm which is the recursive application of a simple two-step procedure, backed by assumptions, and endowed with a clear stopping rule (Velupillai, 1989); indeed, this procedure can be fully programmed in any simulations software.

Moreover, throughout his *oeuvre*, Sraffa does not rely on non-constructive methods such as *tertium non datur* (i.e, the law of excluded middle), the axiom of choice (or Zorn's lemma), the *Bolzano-Weierstrass theorem*, any notion of the *Brouwer fixed point theorems*, or any notion of the *Perron-Frobenius theorem* on non-negative square matrices. The absence of such mathematical tools in Sraffa (1960) led some reviewers to conclude that the book suffered from a lack of formalization and that it could be restated in more rigorous mathematical terms (Quandt, 1961; Burmeister, 1968). However, from the point of view of Computable Economics, Sraffa's approach lacks nothing in mathematical terms, as it does not need any notion of axiom or theorem in order to derive its results.

The absence of non-constructive topological arguments in Sraffa has also been interpreted as a lack of general validity of his propositions[3].

In this regard, it could be argued that the benefit of using theoretical results endowed with higher general validity may outweigh the cost of adopting non constructive and non computable mathematical instruments if applied methodologies and approximation procedures are available.

Sraffa's approach, however, does not allow such imprecision when it comes to theory; nor does Computable Economics allow statistical approximation to fill existing gaps in theoretical explanations of reality. Sraffa's intervention at the 1958 conference of the International Economic Association at Corfu provides an illuminating account of the purpose of economic theory and the difference between theory and measurement:

> "*[O]ne should emphasize the distinction between two types of measurement. First, there was the one in which the statisticians were mainly interested. Second there was measurement in theory. The statisticians' measures were only approximate and provided a suitable field for work in solving index number problems.* ***The theoretical measures required absolute precision. Any imperfections in these theoretical measures were not merely upsetting, but knocked down the whole theoretical basis.*** *[· · ·] The work of J. B. Clark, Böhm-Bawerk and others was intended to produce pure definitions of capital, as required by their theories, not as a guide to*

[3] An example is the famous criticism in Hahn (1982), to which the third chapter of this thesis is dedicated

> *actual measurement. If we found contradictions, then these pointed to defects in the theory, and an inability to define measures of capital accurately.* ***It was on this - the chief failing of capital theory - that we should concentrate, rather than on problems of measurement.***" (Lutz and Hague, 1961, pp. 305–306; emphasis added)

The approach to Sraffian economics underpinning the first three chapters of this thesis builds on a body of previous works, which is based upon the two aforementioned pillars.

Zambelli (2018b) discusses the critical purpose of Sraffa (1960) with regards to several theoretical concepts in economic theory, such as the incorporated theory of value in classical economics, the period of production measurement of capital in the Austrian School, the assumption of a monotonic relation between the interest rate and the value of 'capital', and - in general - the 'marginal theory of value and distribution'. Therefore, it is argued that the assumption of uniform rate of profits may be sufficient for the purposes of Sraffa's critique of economic theories, but not necessary for the critique to hold in general[4].

Indeed, unless further structure is imposed - such as the hypothesis of a convergence to a long-run equilibrium consistent with a uniform rate of profit across sectors - the economy postulated in Sraffa (1960) is able to reproduce itself with self-replacing prices associated to both uniform and non-uniform profit rates (Zambelli, 2018b, p. 802).

The result is proven by devising a constructive algorithmic procedure through which the domain of self-replacing prices, wages and profit rates is obtained by spanning all possible combinations of the distributive shares, given an arbitrary degree of precision (Zambelli, 2018b, pp. 802-805). The procedure is also simulated using the numerical example of an economy producing a surplus in p. 19 of Sraffa (1960).

An important corollary of the proof is that Sraffa's critique retains its validity when non-uniform rates of profit are considered. Moreover, the proof also lays the foundations of a *pars construens* in which the Sraffian method can be used for policy prescriptions and empirical applications, where non-uniform rates of profit are observed.

Against this background, Chapter 1 extends the methodology devised in Zambelli (2018b) to the introduction of non-basic commodities in the system.

The contribution of this Chapter is that - once the distribution of the whole surplus (which includes basic and non-basic goods) is treated as in Zambelli (2018b) (eq. 4,3 and 4.4, p. 804) - the computation of self-replacing prices and the profit rates is made by considering the whole production system (i.e., in terms of the distribution of a surplus which is made of both basics and non-basics).

[4] See Zambelli (2018a) for a reconstruction of the 1960s Cambridge Capital Controversies on the 'neoclassical postulate' according to which the rate of interest is a monotonically decreasing function of the capital intensity of production, and an investigation of the empirical validity of the postulate.

This is relevant because both workers and the producers consume both basic commodities and non-basic ones. Once the wage rate becomes central for the analysis of the economic system, it is important to be able to compute all the self-replacing prices and profit rates as a function of the wage rate measured in terms of the whole surplus and not only as a function of the surplus of the basic system.

Furthermore, the result is relevant for the theoretical and empirical analysis based on Sraffian schemes, because both the critique to economic theory and the reconstruction based on Sraffian premises do not depend on the assumption of a uniform rate of profits and on the characteristics of the production system.

Chapter 2 discusses the relationship between Sraffian schemes and the realm of money and credit. The economic systems analyzed in Sraffa (1960) are in a self-replacing state, where all industries recoup the sufficient purchasing power to reproduce their production processes. In such economies, there is no accumulation of debt or credit positions between agents at the end of the market day in each period, as the self-replacing prices are also consistent with market clearing.

However, Venkatachalam and Zambelli (2022) extend the methodology in Zambelli (2018b) to the case of non-self-replacing states where output may be sold against IOUs (*I Owe You*-s) to be settled in the future. In this framework, it is shown that financial factors such as the level of interest rates on debt positions accumulated at the end of the period and the length of the repayment periods shape the distribution of the physical net surplus across industries.

These results are consistent with the argument put forward by Sraffa in his debate with Hayek on the existence of a 'natural' interest rate (Sraffa, 1932a,b). Moreover, they are also consistent with the famous remark by Piero Sraffa on the possible relationship between rates of interest and rates of profit:

> "*The rate of profits* [...] *is accordingly susceptible of being determined from outside the system of production,* ***in particular by the level of the money rates of interest***". (Sraffa, 1960, p. 35; emphasis added)

Although several authors have interpreted this remark as pointing to a possible closure of Sraffian schemes via a unique relationship between the forces determining monetary interest rates and distribution, Piero Sraffa himself dismisses any interpretation of the above passage as an attempt to construct a theory of income distribution which is mechanically based on monetary interest rates (Venkatachalam and Zambelli, 2021, pp. 464–469).

Building on the contribution by Venkatachalam and Zambelli (2021), Chapter 2 discusses Piero Sraffa's contribution to monetary theory through a textual analysis of his published and published works, and a critical reappraisal of the 'trilateral' debate on money and capital among Piero Sraffa, John Maynard Keynes and Friedrich Hayek in the 1930s.

This Chapter builds on the interpretation of Piero Sraffa's *oeuvre* as a coherent whole, put forward by Venkatachalam and Zambelli (2021). Despite Sraffa's

interest in banking and finance in the beginning of his career in the 1920s, Sraffa (1960) does not encompass money and credit.

Hence, it has been argued that Sraffa's research interests shifted towards the analysis of value and distribution in a real economy, thereby leading to the conclusion that monetary problems have not played a major role in the economic thinking of the author.

Hence, the paper aims at reconstructing Sraffa's monetary theory through an analysis of Sraffa's published and unpublished works. A coherent approach to monetary theory emerges, according to which money is inherently non-neutral with respect to economic outcomes, due to the institutional forces establishing the monetary standard of deferred payments. The Chapter also shows that Sraffa was able to anticipate monetary problems that would have been formally discussed only several decades later in the economic literature.

The methodology in Zambelli (2018b) is also a suitable environment for discussing and testing several arguments about the consistency and the scope of Sraffian schemes. Economists in the neoclassical tradition have responded to the various points of criticism from Sraffian economists in different ways.

The famous 1960s Cambridge Capital Controversies focused on the results in Sraffa (1960) concerning the aggregated versions of neoclassical theory, such as the critique of the 'neoclassical postulate' according to which commodity prices are monotonic functions of wage rates or profit rates (Pasinetti, 1966). According to Sraffa, the postulate is theoretically meaningless, thereby depriving the notion of capital-intensive or labour-intensive notion of any meaningful theoretical content:

> "[...] *as the wages fall the price of the product of a low proportion* [...] *may rise or it may fall, or it may even alternate in rising and falling, relative to its means of production; while the price of the product of a high-proportion* [...] *industry may fall, or it may rise or it may alternate. What neither of such products can do* [...] *is to remain stable in price relative to its means of production throughout any range, long or short, of the wage-variation.* (Sraffa, 1960, pp. 14–15)

While some neoclassical authors acknowledged the existence of this problem for aggregate models (Samuelson, 1966; Ferguson, 1969), other authors have regarded it as empirically irrelevant (Sato, 1974).

On top of that, other prominent economists have argued that Sraffa succeeded at highlighting problems in aggregate neoclassical models but do not affect the Arrow-Debreu version of Walrasian general equilibrium (Bliss, 1975; Burmeister, 1968; Hahn, 1982).

According to this strand of literature, a fully-fledged Walrasian general equilibrium does not require any aggregation of 'capital' and it is thus not subject to the Sraffian critique. Moreover, according to these authors, the Sraffian system can be derived as a special case of a general equilibrium system where a linear production technique with constant returns to scale is assumed and the demand side of the economy is not specified (Hahn, 1982).

Against this background, Chapter 3 discusses the proposition put forward by Hahn (1982) on Sraffian schemes as a special case of Walrasian general equilibrium in which consumers' preferences are not specified and set of methods of production available to firms is composed of one element only. Hahn's argument has sparked a lively debate between neoclassical and Sraffian economists, which is reviewed in the Chapter.

This Chapter constructs a 'thought experiment' inspired by Hahn (1982), in order to show that Sraffian schemes cannot be reduced to a special case of Walrasian general equilibrium. On the other hand, it is found that the market-clearing condition required for a price vector to be Walrasian is embedded in Sraffian schemes. As a consequence, Sraffian indeterminacy cannot be obviated. unless additional structure is imposed. However, the additional structure of agents' preferences introduced by Walrasian general equilibrium theory does not encompass a methodology for effectively constructing the second existence condition of a Walrasian general equilibrium - i.e., the maximization of each agent's objective function.

This Chapter compares two models used by a hypothetical neoclassical economist and a hypothetical Sraffian economist building on two key insights derived from extensions of the methodology in Zambelli (2018b). First, Zambelli (2022a) adds differentiable production functions to the specification of a Sraffian system and shows that Sraffian schemes are not based on the assumption of linear activity production function and constant returns to scale. Applying the algorithm in Zambelli (2018b), the paper shows that Sraffa's critique holds independently of any assumption about returns to scale and the flexibility of production coefficients, both at the industry level or at the firm level.

Secondly, Zambelli (2022b) introduces utility functions associated to a number of consumers in the model specified in Zambelli (2018b) and devises a constructive methodology to compute prices associated with a Walrasian general equilibrium for each distributional pattern and for each combination of the distribution of firms across sectors. The investigation shows that market clearing equilibria in the market for consumption goods do not occur for most of the economies considered; moreover, multiple equilibria are associated to several instances in which a Walrasian price vector is found.

Hence Sraffa's critique of economic theory that economic forces alone are not sufficient in determining self-replacing prices and hence in determining distribution among agents is confirmed and reinforced by this analysis.

Chapter 4 is based on a different methodology and examines methodological issues about the estimation of potential output according to the production function methodology developed within the budgetary framework of the Euro area. The Chapter derives from a paper which I co-authored with Walter Paternesi Meloni and Pasquale Tridico and which was published in the *Journal of Post Keynesian Economics* in 2021.

The conceptualization of potential output traces back to the seminal contribution by Okun (1962). Potential output is often referred to as an estimate of the maximum amount of goods and services an economy can produce when all the available resources, including labor and capital, are utilized at their maxi-

mum sustainable (i.e., non-inflationary) level[5].

Okun's original definition of potential output was explicitly proposed as a short-run concept, taking the state of technology and the level of installed capacity as given (Okun, 1962, p. 98-99). However, the concept has often been used in the context of the definition of an appropriate medium-term fiscal stance in order to smooth business cycle fluctuations.

In fact, the idea of a cyclically-adjusted budget - which is supposed to balance over the business cycle - predates Okun's definition of potential output, as it was first conceived in the 1930s by the economists of the Stockholm School and thus implemented by the Swedish government. In the 1960s, Keynesian economists associated with the Kennedy and Johnson administrations reformulated the notion, building on the conceptualization of potential output - and its empirical estimates - developed by Okun (1962)[6]. as a guidance for the implementation of macroeconomic stabilization policies.

After the 1970s, the influence of Monetarist and New Classical schools promoted significant methodological changes in the conceptualization and estimation of potential output, which came to be understood as a long-run attractor of actual output through changes in prices and expectations, as a consequence of the postulated ineffectiveness of fiscal and monetary policies in boosting output (besides marginal short-run effects of non-anticipated policies)[7].

As a result, contemporary mainstream approaches to the estimation of potential output generally involve statistical methods which analyze past trends in economic magnitudes - such as output, employment and various measures of efficiency - and project these trends into the future. The most common methods used by research and policy institutions also involve the definition of a functional form for an aggregate production function which describes how the trend component of the time series of labor and 'capital' inputs relate to the level of potential output.

Moreover, building on various versions of the Phillips Curve, potential output is related to the 'non-inflationary' level of unemployment. Therefore, the output gap - i.e., the difference between potential output and actual output - aims at capturing the amount of slack in the economy which can be mobilized by fiscal or monetary policies. In this context, the output gap is a key indicator for defining the appropriate fiscal stance which should be set by policy makers in order to prevent inflation.

Although the background and the research questions in Chapter 4 are rather

[5] "*Potential GNP is a supply concept, a measure of productive capacity. But it is not a measure of how much output could be generated by unlimited amounts of aggregate demand. The relation would probably be most productive in the short-run with inflationary pressure pushing the economy. The social target of maximum production and employment is constrained by a social desire for price stability and free markets. The full employment goal must be understood as striving for maximum production without inflationary pressure*" (Okun, 1962, p. 98)

[6] See Costantini (2018) for a thorough reconstruction of the development of the cyclically-adjusted budget notion

[7] See Costantini (2015) for a reconstruction of the transition to contemporary mainstream definitions and estimation methodologies of potential output.

different from Chapter 3, the methodology can also be discussed in light of the framework underpinning the investigations carried out in the first three chapters.

A different methodology for computing estimates of potential output could be developed building on key elements of the Sraffian framework. While statistical methods require the imposition of strong priors on the smoothness of the trend or cycle components of economic variables, the studies underpinning the framework adopted in the first three Chapters question many of the key assumptions and concepts behind the theoretical-based methods for estimating potential output (such as the production function approach).

The production function approach is based upon the assumption of an aggregate Cobb-Douglas production function for the whole economy, where the demands for labour and capital are negatively related to the factor prices (i.e., the wage rate and the interest rate). While several studies have disputed the statistical validity of estimations based on aggregated functions (Simon and Levy, 1963; Shaikh, 1974; Fisher, 1969; Simon, 1979; Felipe and Fisher, 2003), Zambelli (2018a) has provided a rigorous computation of a surrogate '*as-if*' aggregated production function starting from a multi-commodity space following the methodology outlined in Samuelson (1962), and deriving the *wage-profit frontier* for such an economy.

It is found that the aggregate production function does not exhibit the fundamental neoclassical properties associated to CES and Cobb-Douglas production functions, such as the inverse relationship between the rate of interest and the capital per unit of employed labor - i.e., the 'neoclassical postulate of production theory' (Zambelli, 2018a, p. 405). Hence, notions such as the aggregate marginal productivity of factors are found to be meaningless for the case of heterogeneous production, thereby casting serious doubt on the validity of production function approaches for efficiency measurement.

Likewise, Zambelli et al. (2017) use the information included in multi-regional input-output tables to derive an efficient set of methods of production based on the computation of production prices. Each combination of methods of production (i.e., the triple composed of material inputs, labor inputs and the output of goods and services) is evaluated with respect to another set of methods of production by studying the prices that would allow the system to reproduce itself in the future (i.e., production prices).

The evaluation of the relative efficiency of different combination of methods is carried out via the construction of *wage-profit curves* (Zambelli et al., 2017, p. 40) associated to each set of methods. A method is said to exhibit a technological improvement with respect to another when its production prices are lower than the other or when, for given profit rates r, the associated wage rate is higher than the other.

The *wage-profit frontier* is then constructed as the outer envelope of all wage-profit curves through an original algorithm. The domain of this frontier (yearly or intertemporal) is composed of a set of intervals which represent the most efficient methods of production for given values of the profit rate (Zambelli et al., 2017, p. 41). Subsequently, this set of methods is used to construct indexes for evaluating systemic and sectoral technical efficiency and technological progress.

The method in Zambelli et al. (2017), originally implemented to measure technological progress through a robust international comparison, could also be extended to identify the most efficient production technique out of the information embedded in existing input-output tables for a given set of countries.

In this framework, country-specific wage-profit curves and the wage-profit frontier for the target set of countries could be constructed in order to evaluate the gap between 'actual' methods of production and the most efficient methods of production belonging to the wage-profit frontier, which - if adopted by a given country - could yield the 'potential' output.

Applying the method in Zambelli et al. (2017) would involve a different goal with respect to the mainstream conceptualization of potential output. While the most common mainstream approaches - such as the framework discussed in Chapted 4 - aim at identifying an inflation barrier for the purpose of fine-tuning fiscal and monetary policies, such an alternative approach based on Zambelli et al. (2017) would be used to construct robust measures of efficiency for the purpose of implementing policies aimed at achieving structural change.

Such a methodology would not entail the assumption of an aggregate neoclassical production function (or convexification techniques) and the associated notion of Total Factor Productivity. Furthermore, it would enable an evaluation of efficiency gaps in each sector, thereby allowing for more operational and targeted policy implications with respect to an aggregated framework. Moreover, as the procedure is grounded in discrete mathematics, a computable definition of potential output can be obtained.

Hence, the Sraffian framework underpinning the first three Chapters enables a rigorous critique of the neoclassical production function approach to the measurement of potential output which is discussed in Chapter 4. The construction of an alternative Sraffian framework for the analysis of potential output is left for future work.

Chapter 4 starts by briefly reviewing the controversies related to the definition of potential output. A debate around the estimation methodology of the potential output developed in the 2010-2020 decade, culminating in a number of empirical works casting doubt on the plausibility of the official estimations of output gaps (i.e., the difference between potential and actual output), particularly due to pro-cyclical biases in the estimation methodology.

This debate had an impact on the reassessment process of Euro area fiscal rules which is still currently underway. For instance, an official accompanying document of a Communication from the European Commission highlights that the framework "*relies heavily on variables that are not directly observable and are frequently revised, such as the output gap and the structural balance, which hampers the provision of stable policy guidance*". (European Commission, 2020).

In the Euro area, the estimation of potential output is a crucial step in the fiscal surveillance process of member States. In order to supervise fiscal policies in European countries and provide recommendations to countries related to medium-term budgetary objectives, the European Commission uses a Cobb-Douglas production function methodology to provide estimates for potential output of supervised countries (Havik et al., 2014). In this framework, potential

output is defined as the the level of output which is consistent with the maximum 'non-inflationary level' of employment (European Commission, 2019).

This unobservable variable is aimed at capturing the 'structural' unemployment level marking a limit under which, if the economy is subject a to a positive demand shock, an inflationary process would begin - i.e., the *Non-Accelerating Wage Rate of Unemployment* (NAWRU).

Against this background, Chapter 4 discusses the impact of the introduction of a deficit-financed conditional minimum income (CMI) in Italy to discouraged people which are outside the labor force on potential output estimates. The intuition is that if such a measure is able to stimulate the participation rate, an upward revision of Italy's potential output would result, and this - in turn - would contribute to generate a greater fiscal stance. The impact of this measure is assessed by using both comparative statics and empirical estimations carried out via the simulation procedure used by the *Output Gaps Working Group* of the European Commission.

Assuming one million newcomers in the labor force, the findings indicate that the measure would have produced a greater fiscal space of approximately 19 billion euros in 2016 and 12 billion euros in 2017. The impact of the introduction of the deficit-financed CMI on real GDP and public finance indicators is also assessed.

The aim of the exercise is to show that policies targeting discouraged individuals are susceptible of having a positive impact on the growth of the productive capacity in the economy, even according to the official methodology implemented by the European Commission. The exercise stimulated a lively debate in the academic profession and in economic journalism in Italy.

Moreover, the Italian Ministry of Finance (MEF) applied the intuition in the paper to the case of the '*Reddito di Cittadinanza*', a program of conditional income support started in the first half of 2019 in Italy, which supports the household incomes which are below the national poverty line. The labor supply shock stemming from RDC has been estimated by the Italian Ministry of Finance (MEF) to widen the output gap in the first five years, due to an increase of the labor gap in absolute value (as a result of the upward revision of the participation rate) which is partly offset by a reduction of the Total Factor Productivity gap (MEF (Italian Ministry of Finance), 2019, p. 33).

Chapter 1

On the importance and the unimportance of basic and non-basic goods

1.1 Introduction

In his seminal book *Production of Commodities by means of Commodities* (hereafter *PCMC*), Piero Sraffa introduces the concept of basic and non-basic commodities after allowing a simple single-product, multisectorial economic system to produce a positive surplus in a *self-replacing condition* (Sraffa, 1960, p. 7).

In the case of subsistence analyzed in Chapter I of Sraffa's book all goods are basic - i.e., they are directly or indirectly entering the production of all other commodities. However, with the emergence of a surplus there is room for non-basic commodities that "*are not used, whether as instruments of production or as articles of subsistence, in the production of others*" (Sraffa, 1960, p. 7).

Since non-basic commodities do not directly or indirectly enter the production processes of all other commodities, Sraffa points out that any innovation in the production process of these commodities would not have any impact on the prices of basic commodities.

This occurs because the price of a non-basic product depends on the prices of the intermediate inputs used up in their production process, but the prices of the intermediate inputs do not depend on the price of non-basics[1].

The price vector that allows the system to reproduce itself at a given rate of profit, assumed to be uniform, is fully determined by the conditions of production of basic goods; moreover, the Maximum rate of profit that the system can afford - given the available techniques of production - also depends uniquely on

[1] Hence, Sraffa points out that the classical term 'cost of production' is appropriate for non-basic goods, whereas the term 'price' or 'exchange-value' is preferred for basic goods (Sraffa, 1960, p. 9).

basic commodities.

Hence, non-basics may seem *unimportant* for the determination of overall prices and distribution, as they apparently only include 'luxury' or non necessary goods.

Sraffa focuses his attention to a system composed only of basic commodities. While this is justified by the scope of his work, which was that of demonstrating the indeterminacy of the system and the problems related with measurement of capital - see Ch. 6 of Sraffa (1960), Zambelli (2018b), Zambelli (2022a), Zambelli (2022b), Venkatachalam and Zambelli (2021) and Venkatachalam and Zambelli (2022) - it is also the case that the surplus of an economic system is composed, almost always, of commodities which are basic and non-basic commodities.

To restrict the search of the prices that would allow self-replacing of the basic system and not of the whole system - which is composed of both basics and non-basics - although justified in the context of Sraffa (1960) - means imposing a limit to the notion of self-replacing.

As we can see below, this limit is removed when the prices are self-replacing prices for the whole economic system. Hence are also the prices associated to the distribution among producers and workers of the whole surplus.

When treating the whole wage rate as variable, non-basics will also include the necessaries of consumption, as long as they do not feed back onto the production of intermediate inputs. The problem is explicitly recognized by Sraffa:

> "... [we] *shall follow the usual practice of treating the whole of the wage as variable. The drawback of this course is that it involves relegating the necessaries of consumption to the limbo of non-basic products. This is due to their no longer appearing among the means of production on the left-hand side the equations: so that an improvement in the methods of production of necessaries of life will no longer directly affect the rate of profits and the prices of other products. Necessaries however are essentially basic and if they are prevented from exerting their influence on prices they must do so in devious ways (e.g. by setting a limit below which the wage cannot fall ...)*" (Sraffa, 1960, p. 10)

Including non-basics in the analysis of the economic system is both theoretically and empirically necessary. Non-basics directly arise out of the emergence of a surplus and they are part of workers' and producers' consumption basket.

The purpose of this paper is to reconsider some crucial assumptions in the treatment of Sraffian schemes. Following the methodology in Zambelli (2018b), we are able to remove non-basic products from the 'limbo' in which Sraffa was placing them and are consequently in the position of dealing with the consumption necessities of the workers (and producers) which are to be expressed in terms of all the goods composing the system.

It will be shown that the treatment of Sraffian schemes under this methodology allows to overcome some of the difficulties highlighted in the debate, thereby paving the way for an appropriate consideration of non-basic goods in empirical

analysis. In Section 1.2, the most relevant literature related to the economic meaning and the conditions of reproduction of non-basic goods is reviewed. Section 1.3 presents the model including non-basic goods and Section 1.4 provides a numerical example. Section 1.5 concludes.

1.2 Related literature

In Sraffa (1960), two examples of analytical difficulties in the treatment of non-basic goods are discussed.

With regards to single-product industries, in Appendix B of *PCMC* Sraffa discusses a case of a self-reproducing non-basic commodity which enters to a large extent into the production of itself. In this case, the Maximum rate of profits of the system set by the production conditions of basic commodities may be incompatible with the production conditions of the non-basic commodity.

For instance, if a non-basic commodity is subject to a wastage such that for every 100 units sown no more than 110 are reaped, the industry of that given commodity would not admit a profit rate higher than 10%, since other commodities are also required for production (Sraffa, 1960, p. 90).

If the wage rate is reduced, the relative price of beans would have to increase without bound, as a greater and greater share of the net product of beans would be required in order to keep pace with the resulting increase in the (uniform) profit rate.

If beans are taken as the numeraire of the system when their relative price goes to infinity, the prices of all other commodities would be zero. This would not occur in the case of basic commodities, because there is at least another commodity in the means of production of which the basic commodity enters. Furthermore, at a rate of profit above 10%, the only possible solution for such a system would require a negative price for beans (Sraffa, 1960, p. 91).

The economic significance of a system in which prices of basic commodities are all equal to zero is uncertain. While discussing the construction of the Standard product for multiple-product equations, Sraffa also explains that in a Standard system containing non-basic commodities there would be values of the Maximum rate of profit such that all basics would have a zero prices. According to Sraffa, "*such values of R [the Maximum rate of profit] are meaningless from the standpoint of an economic system and must be rejected*" (Sraffa, 1960, p. 92).

The rationale for Sraffa's rejection seems to be the following. If basic commodities have a zero price, it means that they can be conceived as 'free goods' that could be acquired by non-basic industries without any cost. Moreover, basic commodities with zero prices would no longer enter directly or indirectly all the commodities in the system, as they would disappear from the accounting of the means of production for each industry, thereby losing their fundamental characteristic.

Three straightforward implications arise. First, choosing a basic commodity, a linear combination of basic commodities or the basic net product as a

numeraire of the system is the only way to analytically rule out the case in which all basic commodities have a zero price.

Furthermore, given that the prevailing profit rate and its Maximum value are fully determined by the production conditions of basic commodities, excluding non-basics from the analysis would not alter the determination of meaningful self-replacing price vectors. Finally, considering multiple-product systems, the construction of the Standard system requires getting over the unpleasant possibility of zero prices for basic commodities, which would be equal to eliminating basic commodities from the construction of the Standard product.

These complications arise because, as Sraffa points out, "*we are all the time concerned merely with the implications of the assumption of a uniform price for all units of a commodity and a uniform rate of profits on all the means of production*" (Sraffa, 1960, p. 90).

Several authors have discussed the role of non-basic goods in *PCMC*. In a famous critique of *PCMC*, Newman (1962) argues that the presence of non-basic goods will often not imply a positive price vector; hence, either one of Sraffa's assumptions - a uniform profit rate and the self-replacing condition - should be abandoned, or it should be assumed that non-basic goods do not exist.

Moreover, Newman (1962) suggests to assume that all commodities are basic because "*whether a good is non-basic is partially a matter of the degree of aggregation in the system*".

The treatment of non-basic goods is discussed in letters exchanged between Sraffa and Newman which are published in the Appendix of Bharadwaj (1970). One of the main results highlighted by Newman (1962) (p. 66-67), in a two-commodity example, is that a non-basic good with a lower rate of reproduction than the (uniform) profit rate set by the production conditions of the basic system will not have a positive price. Sraffa answers by reminding that this case is exactly the one discussed in Appendix B of *PCMC*.

Moreover, although Sraffa finds it "*obvious how rare (if any) such cases must be in the real world*" (Bharadwaj, 1970, p. 426), he is careful not to dismiss theoretical possibilities due to empirical considerations. As a consequence, Sraffa responds to the second point of criticism by Newman (that the distinction between basics and non-basics is partly a matter of the degree of aggregation in the system) stressing that "*aggregation is the act of the observer, whilst the distinction is based on a difference in objective properties*".

In a second letter, Newman generalizes the discussion to a multiple-commodity system. Recalling Gantmacher's treatment of decomposable matrices, Newman finds that a positive price vector exists if and only if the rate of profit for the basic system is lower than the rate of profit for the *internal* non-basic system (Bharadwaj, 1970, p. 427). Newman presents an economic system with basic and non-basic goods as a decomposable matrix in which different indecomposable sub-matrices can be identified. In this form, two dominant latent roots can be computed and associated to different *internal* rates of profit for basic and non-basic goods.

Sraffa's second reply is a based on a methodological point. While the mathematical representation of a basic system has a meaningful economic interpreta-

tion - as basic goods are interconnected by definition - Sraffa argues that "*there is no such thing as a non-basic system*" (Bharadwaj, 1970, p. 428) whose internal rate of profit can be compared with that of the basic system.

According to Sraffa, the peculiarity of non-basics is to be incapable of forming an independent system, as they are not all interconnected directly or indirectly among each other. Hence, the only meaningful formal treatment is to compare the rate of reproduction of each non-basic good with the rate of profit of the basic system. This is, in fact, the methodology used by Sraffa in Appendix B of *PCMC*, which makes it *a priori* very unlikely that any individual rate of profit is smaller than the prevailing rate of profit of the basic system, the exception being the case of self-reproducing non basics with a very low rate of reproduction such as in the 'beans' example created by Sraffa.

In her paper, Bharadwaj (1970) also argues that the classification of commodities into basics and non-basics in a given economic system uses more of the available information than the formal treatment of a decomposable or indecomposable system. The presence of non-basics in the system implies in itself that that the system is decomposable. Moreover, the distinction between basic and non-basic goods provides additional information that is relevant to the discussion of switching possibilities between different production techniques (Bharadwaj, 1970, p. 420).

In particular, the paper proves that the maximum number of switches among alternative methods of production coincides with the number of different goods, without double counting, that enter directly or indirectly into at least one of the alternative methods of production.

Hence, if the alternative methods produce a good that is basic with respect to both techniques, then non-basics in both techniques play no role. On the other hand, if the alternative methods produce a non-basic commodity in at least one of the techniques, then a role is played also by those non-basics that enter directly or indirectly into the production of at least one of the alternative methods of production.

Zaghini (1967) and Varri (1979) discuss the classification between basics and non-basics by focusing on the economic meaning of necessary and sufficient conditions for non-basics to have positive prices. In particular, Zaghini (1967) recalls that non-basic goods cannot be neglected as they represent a large part of consumer goods when workers earn a wage above subsistence levels and hence are entitled to consume a part of the net product. In response to Newman's claim that no economic rationale exists for the consideration of non-basic commodities, Zaghini highlights the centrality of non-basic commodities for workers' consumption:

> "*At this point my opinion diverges radically from that of Newman's, as it seems to me that his assumption is far more restrictive than the one he seeks to avoid. In fact it certainly implies the exclusion of a relevant number of commodities which cannot be ignored. When no Ricardian hypothesis is made of a natural wage-rate represented by a fixed basket of goods, in the category of non-basics there enters*

> *besides the luxury goods, strictly speaking, also the great majority of consumer goods. Taking this into account I cannot absolutely agree with Newman when he states that non-basic commodities are of greater mathematical interest than economic*". (Zaghini, 1967, p. 257)

In a subsistence economy - i.e., when the net product is zero - the possibility of obtaining a positive price vector for all industries is limited to the case of a basic system, as the only solution consistent with self-replacing in the presence of non-basics would imply zero prices for all basic industries (Varri, 1979, p. 61).

When production with a surplus is considered, Varri (1979) formally extends Sraffa's insight in Appendix B, showing that the wage rate and the price vector are positive if, and only if, the maximum rate of profits of the non-basic systems are greater than the maximum rate of profits in the basic industries of the economic system. As in *PCMC*, Varri (1979) acknowledges that this finding is a necessary condition for the existence of "*a range of uniform rates of profits*" (p. 71).

The thorough analytical treatment of Sraffian schemes in Kurz and Salvadori (1995) also deals with the conditions of reproduction of systems with non-basic goods. A set of common assumptions in the Sraffian tradition is considered: the rate of profit is uniform across industries; the economy is *viable* - i.e., industries are capable of reproducing themselves; there is at least one basic commodity in the system; labour is required directly or indirectly in the production of all commodities.

Building on this set of assumptions, Kurz and Salvadori (1995) find that a set of properties must hold: only basic goods are indispensable for the system; the viability of the economy and the maximum rate of growth depend on the conditions of production of the basic commodities only; the prices of basics are determined independently of the prices of non-basics and the relationship between the wage rate and rate of profit is determined by the conditions of production of basics (if the numeraire consists of basic commodities only).

In a recent paper, Kurz and Salvadori (2021) reconstruct the intellectual journey that led Sraffa to the elaboration of a distinction between basic and non-basic commodities. The authors analyze Sraffa's notes written before the publication of *PCMC*, showing that Sraffa realizes the importance of non-basics in a modern economy, characterized by a sophisticated division of labour, a multiplicity of ways to distribute the surplus between capital and labour and the availability of 'luxury' goods that serve as an incentive to prevent capitalists from withdrawing their investments.

Although Sraffa acknowledges that a part of the wage is a necessity of the economic system, "*on the same footing as the fuel for the engines or the feed for the cattle*" (Sraffa, 1960, p. 9), the decision to treat the wage rate as paid *post-factum* in *PCMC* - allowing workers to participate in the sharing of the surplus - reflects Sraffa's awareness of these features of modern capitalism.

The authors also show that Sraffa is already aware of the difficulties with

self-reproducing non-basics in two notes written five years before the publication of *PCMC* (Kurz and Salvadori, 2021, p. 16). In these notes, Sraffa illustrates the same problem later described in Appendix B by recurring to examples such as an upper limit on the rate of reproduction of oaks and the limits to the fertility of 'white elephants', which constrain their rate of reproduction.

1.3 The model

The debate reviewed in the previous Section revolves around properties and analytical difficulties of an economic system with non-basic commodities, assuming that the system is in a *self-replacing* condition and that the rate of profit is *uniform* across industries (both basic and non-basic).

Here we extend the case of self-replacing of the system of basic goods to the case where self-replacing is associated to the whole system, i.e. where the equations related with basic and non-basic goods are made explicit. In doing so we remove the non-basic goods from the 'limbo' (see the quotation above, p. 12, from Sraffa (1960), sec. 8, p. 10).

As a matter of fact, the economic system described in *PCMC* has one degree of freedom. There is no automatic mechanism which links the distribution of output between wages and profits to the contribution made by workers and owners of the means of production in the production process. Hence, in this system, the value of output and the means of production cannot be measured independently of distribution, and distribution does not depend on market dynamics but rather on political and institutional factors; it is, in a sense, set from *outside* the economic system.

Hence, in order to solve the equations in the Sraffian system, one or more distributive variables (prices, wages and profits) must be set as parameters; moreover, a numeraire must be set in order to define values in terms of a unit of account. Therefore, if the rate of profit is assumed to be uniform and one or more of the goods in the system are chosen as a numeraire, it is possible to compute the price vector and the wage rate for each level of the rate of profit.

Sraffa has picked the uniform rate of profits as an essential distributive variable (which is determined outside the economic system of production and for this reason is considered to be an independent variable). Here, following Zambelli (2018b), we pick as an independent element of the analysis the whole vector of distribution of the shares of the surplus (the vector $\mathbf{d}$).

As we can see from Zambelli (2018b) and Venkatachalam and Zambelli (2022) and as it will also be clear from the analysis developed below, here the wage shares of the whole surplus are explicitly defined and are independent of the system of production: the profit rates - and the prices - the dependent variables.

A substantial improvement with respect to *PCMC* is that now the 'necessities' of the workers may be set directly by defining 'subsistence levels' directly as necessities of the workers in terms of workers' physical share of the surplus. Obviously, in this way the necessities of the workers are defined explicitly and

not in a 'devious' way, by setting a limit to the value of the wage rate (whatever is the numeraire used).

Different choices for the numeraire will yield different sets of price vectors, wage rates and rates of profit; hence, changes in these values are changes in *relative* values, and it is impossible to distinguish parts of these changes which can be related to the features of the numeraire from parts related to the features of other goods.

Following Zambelli (2018b), an 'agnostic' position is taken on the theory of distribution. In line with the review of Sraffa (1960) provided by Bharadwaj (1963), according to whom "*no theory of distribution is offered in the book*", the model in this Section does not assume a uniform profit rate across sectors. Hence, it broadly encompasses all possible theories of distribution, as it is merely concerned with finding those price-wage vectors such that all expenditures are matched by revenues in all industries as a self-replacing condition.

Our investigation considers the simple case of n single-product industries and, as in Sraffa (1960), aims at computing the set of prices which restores the original distribution of the products brought to the market by industries in each yearly production cycle, under the assumption that the system is 'viable' (Chiodi, 2010).

In the sequel, we consider an economic system where both basic and non-basic goods are produced. The self-replacing condition is that all revenues are matched by expenditures in all industries. Each industry sells a given quantity of a commodity at a price which makes the sectoral revenue equal to the sum of sectoral expenditures, composed by the value of the purchased means of production, the quantity of labor inputs multiplied by the real wage rate and the consumption of the product by capitalists.

The value of the product consumed by capitalists for each industry must be equal to the sectoral profit under the assumption of given input-output relations: hence, all profits are consumed and there is no saving by capitalists and workers in each period[2]. Denoting by $\mathbf{A}$ the $n \times n$ matrix of inputs, by $\mathbf{B}$ the $n \times n$ matrix of outputs and by $\mathbf{L}$ the $1 \times n$ vector of labor inputs, the vector of self-replacing prices is such that:

$$(\mathbf{I} + \mathbf{R})\mathbf{A}\mathbf{p} + \mathbf{L}w = \mathbf{B}\mathbf{p} \tag{1.1}$$

where w is the real wage rate distributed to workers, $\mathbf{R}$ is an $n \times n$ diagonal matrix whose diagonal contains the vector of self-replacing profit rates and $\mathbf{I}$ is an $n \times n$ identity matrix. While Zambelli (2018b) considers an indecomposable matrix $\mathbf{A}$, here - given the assumption of coexistence of basic and non-basic goods - we consider basic and non-basic commodities, thereby allowing the matrix $\mathbf{A}$ to be decomposable:

$$\mathbf{A} = \begin{bmatrix} \bar{\mathbf{A}} & 0 \\ \bar{\bar{\mathbf{A}}} & \bar{\bar{\mathbf{A}}}^* \end{bmatrix} \tag{1.2}$$

[2]The self-replacing condition is relaxed in a companion paper by Venkatachalam and Zambelli (2021), where exchanges are allowed to be settled either by barter and the issuance of IOUs.

where $\bar{\mathbf{A}}$ is the indecomposable submatrix of inputs in the basic industries, $\bar{\bar{\mathbf{A}}}$ is the submatrix of inputs of basic commodities entering into the production process of non-basic commodities, and $\bar{\bar{\mathbf{A}}}^*$ is the submatrix of non-basic inputs entering into the production process of non-basic commodities.

Given the presence of non-basic commodities, the output matrix is also decomposable:

$$\mathbf{B} = \begin{bmatrix} \bar{\mathbf{B}} & 0 \\ \bar{\bar{\mathbf{B}}} & \bar{\bar{\mathbf{B}}}^* \end{bmatrix} \tag{1.3}$$

where $\bar{\mathbf{B}}$ is the indecomposable submatrix of outputs of the basic processes, $\bar{\bar{\mathbf{B}}}$ is the submatrix of output of basic commodities produced by the non-basic industries (its elements must all be equal to zero in the case of single-product industries), and $\bar{\bar{\mathbf{B}}}^*$ is the submatrix of non-basic outputs produced by non-basic industries.

Following Sraffa (1960), Section 12, p. 11, we define the net product of the economy $\mathbf{S} = (\mathbf{B}-\mathbf{A})^T\mathbf{e}$ - or 'surplus' - as the numeraire of the economy: hence, it must be $\mathbf{S}^T\mathbf{p} = 1$. This normalization allows to define a distributional vector $\mathbf{d}$ which measures the shares of the net product distributed to each industry (basic and non-basic) and to workers. The vector is obtained by:

$$\mathbf{d}\mathbf{S}^T\mathbf{p} = \left[\begin{array}{c} (\mathbf{B}-\mathbf{A})\mathbf{p} - \mathbf{L}w \\ \hdashline \mathbf{e}^T\mathbf{L}w \end{array}\right] = \left[\begin{array}{c} \mathbf{R}\mathbf{A}\mathbf{p} \\ \hdashline \mathbf{e}^T\mathbf{L}w \end{array}\right] = \left[\begin{array}{c} \mathbf{d}_{n\times 1} \\ \hdashline d_w \end{array}\right] \tag{1.4}$$

where $\mathbf{d}_{n\times 1}$ is the subvector of distributional shares to all n industries (including basic and non-basic industries) and d_w is the $n+1$-th element of $\mathbf{d}$ expressing the share of the surplus going to workers. The elements d_i in this vector must be such that $d_i \geq 0 \;\forall\; i = 1, \ldots, n+1$ and $\sum_0^{n+1} d_i = 1$.

Recalling that $\mathbf{S}\mathbf{p} = 1$, (1.4) can be reformulated as:

$$\mathbf{d} = \left[\begin{array}{c} (\mathbf{B}-\mathbf{A}) - \mathbf{L} \\ \hdashline \mathbf{e}^T\mathbf{L} \end{array}\right] \left[\begin{array}{c} \mathbf{p} \\ \hdashline w \end{array}\right] \tag{1.5}$$

A constructive solution of the problem can be found by expressing the vector of self-replacing prices and wages as a function of the distributional vector $\mathbf{d}$. By means of a simple algorithm it is possible to span all possible distributional shares of the surplus, thereby computing prices and wages that allow the original distribution of the surplus to be reproduced, in line with the thought experiment explained by Sraffa (1960).

Denoting by $\mathbf{M}$ the matrix in (1.5) which pre-multiplies the price-wage vector in (1.5), we have:

$$\left[\begin{array}{c} \mathbf{p} \\ \hdashline w \end{array}\right] = \mathbf{M}^{-1}\mathbf{d} \tag{1.6}$$

Expanding the inversion of $\mathbf{M}$, as in Equation 4.4 of Zambelli (2018b) we have:

$$\mathbf{M}^{-1} = \left[\begin{array}{c:c} (\mathbf{B}-\mathbf{A}) & -\mathbf{L} \\ \hdashline \mathbf{0}_{1\times n} & \mathbf{e}^T\mathbf{L} \end{array}\right]^{-1} = \left[\begin{array}{c:c} (\mathbf{B}-\mathbf{A})^{-1} & (\mathbf{B}-\mathbf{A})^{-1}\frac{\mathbf{L}}{\mathbf{e}^T\mathbf{L}} \\ \hdashline \mathbf{0}_{1\times n} & \frac{\mathbf{L}}{\mathbf{e}^T\mathbf{L}} \end{array}\right] \tag{1.7}$$

In the case of a fully basic system as in Zambelli (2018b), $\mathbf{M}$ is always invertible because the matrix $(\mathbf{B}-\mathbf{A})^{-1}$ is. Therefore, as all the four elements of $\mathbf{M}$ are all computable, there is always a solution in the case of a fully basic system, so that the price-wage vector is computable.

However, in the presence of non-basic commodities, it should be shown that substituting the submatrices of $\mathbf{A}$ to the complete matrix still yields a solution for (1.6), provided that the economic system is viable. Clearly, also in the case of a system of equations for both basics and non-basics, equation (1.5) has a meaning only if matrix $\mathbf{M}$ is invertible.

In the presence of non-basic commodities, the price vector $\mathbf{p}$ can be decomposed in the subvectors $\bar{\mathbf{p}}$ (prices of basic commodities) and $\bar{\bar{\mathbf{p}}}$ (prices of non-basic commodities); analogously, the vector of labor inputs $\mathbf{L}$ can be decomposed in $\bar{\mathbf{L}}$ (labor inputs in basic industries) and $\bar{\bar{\mathbf{L}}}$ (labor inputs in non-basic industries). The same decomposition can be carried out for the distributional vector $\mathbf{d}$.

Hence, in the presence of non-basic commodities, the vector $\mathbf{d}$ becomes:

$$\mathbf{d}=\left[\begin{array}{c:c:c} (\bar{\mathbf{B}}-\bar{\mathbf{A}}) & \mathbf{0} & -\mathbf{L} \\ \hdashline (\bar{\bar{\mathbf{B}}}-\bar{\bar{\mathbf{A}}}) & (\bar{\bar{\mathbf{B}}}^{*}-\bar{\bar{\mathbf{A}}}^{*}) & -\bar{\bar{\mathbf{L}}} \\ \hdashline \mathbf{0} & \mathbf{0} & \mathbf{e}^{T}\bar{\mathbf{L}} \end{array}\right]\left[\begin{array}{c} \bar{\mathbf{p}} \\ \hdashline \bar{\bar{\mathbf{p}}} \\ \hdashline w \end{array}\right] \tag{1.8}$$

where, as in equation (1.7), the matrix pre-multiplying the price-wage vector can be denoted as $\mathbf{M}$. From (1.8) we can define the vector of prices and wages, provided that the matrix on the right-hand side is invertible. Once the inverse matrix $\mathbf{M}^{-1}$ is expanded we have:

$$\left[\begin{array}{c:c:c} (\bar{\mathbf{B}}-\bar{\mathbf{A}})^{-1} & \mathbf{0} & (\bar{\mathbf{B}}-\bar{\mathbf{A}})^{-1}\frac{\bar{\mathbf{L}}}{\mathbf{e}^{T}\bar{\mathbf{L}}} \\ \hdashline -(\bar{\bar{\mathbf{B}}}^{*}-\bar{\bar{\mathbf{A}}}^{*})^{-1}(\bar{\bar{\mathbf{B}}}-\bar{\bar{\mathbf{A}}})(\bar{\mathbf{B}}-\bar{\mathbf{A}})^{-1} & (\bar{\bar{\mathbf{B}}}^{*}-\bar{\bar{\mathbf{A}}}^{*})^{-1} & -(\bar{\bar{\mathbf{B}}}^{*}-\bar{\bar{\mathbf{A}}}^{*})^{-1}[-\bar{\bar{\mathbf{L}}}+(\bar{\bar{\mathbf{B}}}-\bar{\bar{\mathbf{A}}})(\bar{\mathbf{B}}-\bar{\mathbf{A}})^{-1}\frac{\bar{\mathbf{L}}}{\mathbf{e}^{T}\bar{\mathbf{L}}}] \\ \hdashline \mathbf{0} & \mathbf{0} & \frac{1}{\mathbf{e}^{T}\bar{\mathbf{L}}} \end{array}\right] \tag{1.9}$$

The inverse matrix in (1.9) exists if and only if the inverse submatrices within its elements exist as well. We know that $(\bar{\mathbf{B}}-\bar{\mathbf{A}})$ - related to the basic system - is indecomposable: it is therefore always invertible.

The matrix $(\bar{\bar{\mathbf{B}}}^{*}-\bar{\bar{\mathbf{A}}}^{*})$ is related to the non-basic system and has to be analyzed. In a single-product setting, this matrix has non-zero elements in its diagonal and zero elements in all the other positions. It is straightforward to note that the matrix is singular if and only if there is at least one element equal to zero on its diagonal.

This case occurs when $\bar{\bar{b}}^{*}_{jj}=\bar{\bar{a}}^{*}_{jj}$ in at least one of the j non-basic industries. In economic terms, this is a more extreme example than the one discussed by Sraffa in Appendix B of *PCMC*: the non-basic commodity j enters in a so large extent into the production of itself that the output of commodity j is exactly equal to the input of the same commodity which is required to produce it. In such a condition, the non-basic industry j would not have any real purchasing power, regardless of the values of prices, wages and profit rates.

Hence, this condition would not be compatible with the assumption of a *viable* system that was made initially. Therefore, if viability is assumed for all

industries, the matrix $\mathbf{M}$ is always invertible, and prices and wages can be fully determined also in the presence of non-basic commodities.

For each possible configuration of $\mathbf{d}$, the system is characterized by $n+1$ equations and $n+1$ unknowns, thereby being fully determined. Once each price-wage vector associated with each distributional vector $\mathbf{d}$ is computed, the vector of profit rates for all industries can be computed as follows:

$$\mathbf{r_d} = \mathbf{d}_{n\times 1} \oslash \mathbf{A}\mathbf{p_d} \tag{1.10}$$

where $\mathbf{r_d}$ and $\mathbf{p_d}$ are respectively the vector of profit rates and the vector of prices associated with a given distribution vector $\mathbf{d}$[3]. The vector of profit rates $\mathbf{r_d}$ can be fully determined given the price vector.

1.4 A numerical example

In this section, we augment an example used in (Sraffa, 1960, p. 19), adding two non-basic commodities (called 'beans' and 'horses') to the original three basic commodities (iron, coal and wheat) and we introduce joint production by considering that the second industry produces both coal and wheat:

$$\mathbf{A} = \begin{bmatrix} 90 & 120 & 60 & 0 & 0 \\ 50 & 125 & 150 & 0 & 0 \\ 40 & 40 & 200 & 0 & 0 \\ 0 & 50 & 15 & 30 & 0 \\ 0 & 75 & 20 & 0 & 0 \end{bmatrix}; \mathbf{L} = \begin{bmatrix} 3/16 \\ 5/16 \\ 8/16 \\ 1/16 \\ 1/16 \end{bmatrix}; \mathbf{B} = \begin{bmatrix} 180 & 0 & 0 & 0 & 0 \\ 0 & 450 & 15 & 0 & 0 \\ 0 & 0 & 480 & 0 & 0 \\ 0 & 0 & 0 & 40 & 0 \\ 0 & 0 & 0 & 0 & 5 \end{bmatrix} \tag{1.11}$$

The rows of $\mathbf{A}$ respectively represent the means of production required for the iron industry, the coal industry, the wheat industry, the beans industry and the horses industry. The columns of $\mathbf{A}$ indicate the inputs of iron, coal, wheat, beans, and horses used up in the production processes of the various industries. At the end of the year, industries have produced the outputs composing the diagonal of matrix $\mathbf{B}$, that should be exchanged in order to restore the original distribution of the product. Therefore, at the end of the market day of exchange, the net product to be distributed is $\mathbf{S} = [0, 60, 50, 10, 5]^T$.

By using a constructive algorithm described, it is possible to compute all feasible configurations of the distribution vector $\mathbf{d}$. For all feasible configurations of $\mathbf{d}$, there is a unique price-wage vector and a unique vector of profit rates. Figure 1 shows the pairwise domain of self-replacing prices for iron, wheat, beans and horses, as well as the domain of the labor share of output.

The highlighted yellow region is the set of prices that ensure self-replacing, the red subset is associated with a share of surplus going to workers equal to 70%.[4] and the black line is the subset of all prices consistent with a uniform

[3] The $n \times n$ matrix of profit rates entering equations (1) and (4) would then be obtained by $\mathbf{R} = diag(\mathbf{r_d})$

[4] This distributional pattern is close to the estimated elasticity of output to labor in the Cobb-Douglas production function adopted by several production function-based studies. As

rate of profit. Subsequently, Figure 2 shows the pairwise domain of rates of profits for the same commodities shown in Figure 1.

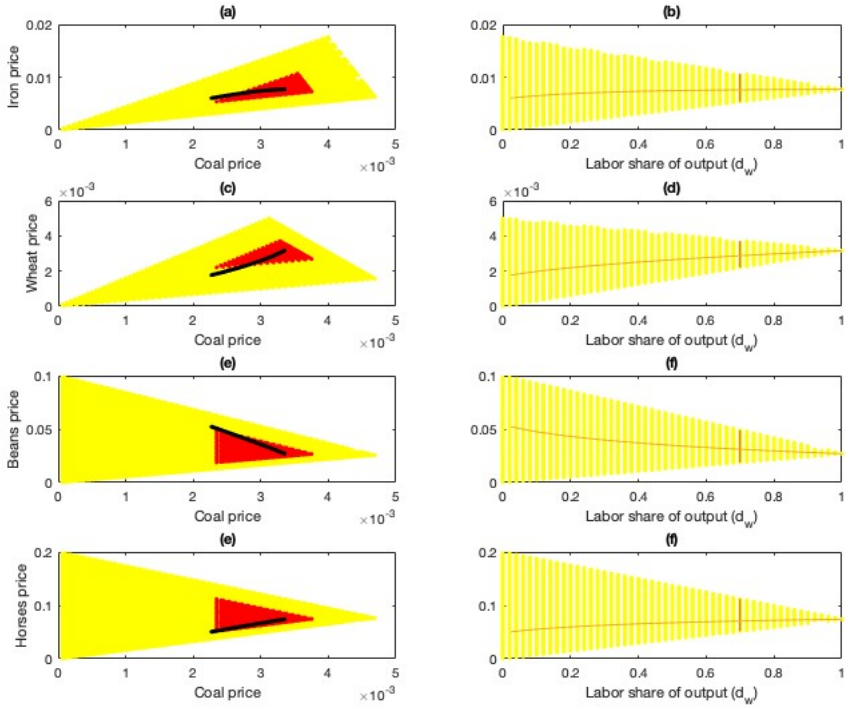

Figure 1.1: Domain of the self-replacing prices (left) and the labor share of output (right)

All the most important results in Zambelli (2018b) are robust to the introduction of non-basic commodities. The movement of relative prices cannot deliver any notion of 'quantity of capital' independent of distribution and prices: Figure 3 plots the values of aggregate capital as a function of prices of iron, coal, beans, horses and the wage rate and Figure 4 plots the capital/output ratio in relation with prices, showing no regularity between the value of capital and the value of output both at the sectoral and the aggregate level. Therefore, the capital intensity of any technique cannot be independent from distribution even in a *free competition* context, as highlighted by the Cambridge Capital Controversies. As in Figures 1 and 2, the yellow regions are those associated with all self-replacing price vectors, the red regions are related to all self-replacing price

McCombie (2001) has shown, parameters of Cobb-Douglas production functions believed to be elasticities of output to factors of production actually measure observed wage and profit shares in the economy.

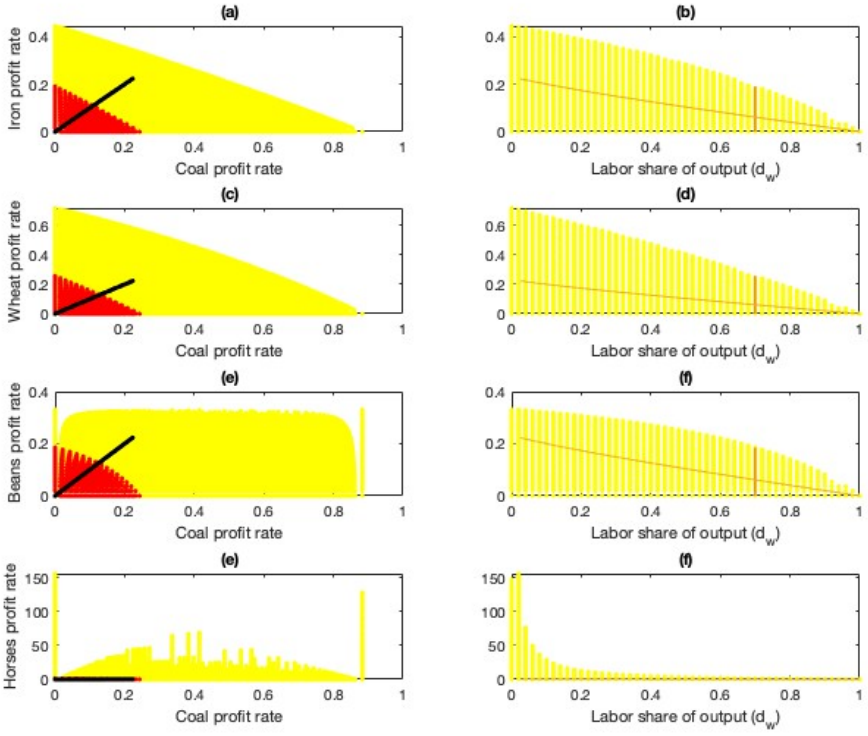

Figure 1.2: Domain of the self-replacing profit rates (left) and the labor share of output (right)

vectors consistent with $d_w = 0.7$ and the black regions are those associated with a uniform rate of profit.

By focusing on all feasible distributive patterns consistent with self-replacing - i.e., beyond the assumption of uniform profit rates - several complications related to the introduction of non-basic commodities are no longer problematic.

The constructive procedure summarized in equation (5) simultaneously associates every feasible physical distribution of the net product to the prices that allow such allocation of the surplus: therefore, each industry has a Maximum rate of profit corresponding to the maximal feasible share of the surplus that each sector can obtain. Thus, the domain of self-replacing profit rates must be such that sectoral profit rates can vary between zero and the Maximum sectoral profit rate.

Moreover, analytically ensuring that the price vector is positive given a uniform profit rate is not strictly necessary in the context of our constructive procedure. Spanning all possible distributive patterns, the cases in which basic

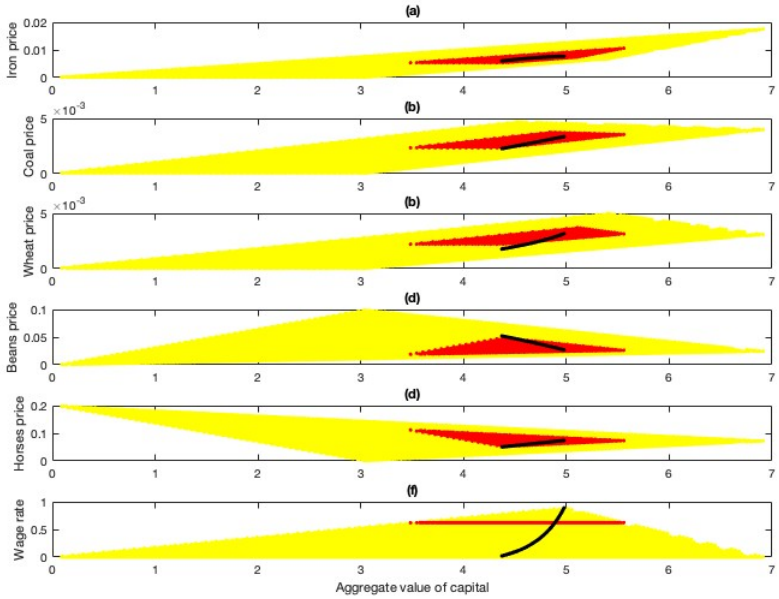

Figure 1.3: Domain of the aggregate value of capital, prices (iron, coal, wheat, beans and horses) and the wage rate

goods have a zero price can be excluded from the domain of the feasible price vectors.

Furthermore, we show the robustness of a conundrum highlighted by Zambelli (2018b) between uniform profit rates across sectors with heterogenous distributional shares, and egalitarian distributional shares corresponding to non-uniform profit rates.

When the more general case is considered, self-replacing price vectors can be associated to uniform as well as non-uniform profit rates, and with equal as well as different shares of the surplus. As it is shown in Zambelli (2018b) is thus unclear whether a condition of uniform profit rates but different shares of the surplus (i.e., given different profit levels) should be considered as a rest position of the economy or not, according to the notion of *free competition*.

By relaxing the assumption of a uniform rate of profit, it is possible to span all feasible distributions of the net product even when non-basic commodities enter the numeraire of the system - i.e., the whole net product. As in the case of single-product industries, the occurrence of 'meaningless' cases in which basic industries have zero prices occurs when the whole net product is distributed to non-basic industries. Therefore, even in the case of multiple-product industries, the exclusion of non-basic commodities from the construction of the Standard commodity is not strictly necessary to cope with the possible analytical difficul-

Industries (means of production)	Prices	Profit rates	Shares of Surplus	Value of Capital	Value of Product
Iron	7.22*	5.07%	**6.00**%	1.18	1.36
Coal	3.05*	5.17%	**6.00**%	1.16	1.42
Wheat	2.79*	6.19%	**6.00**%	0.97	1.34
Beans	29.34*	5.58%	**6.00**%	1.07	1.17
Horses	76.74*	21.06%	**6.00**%	0.29	0.38
Aggregate	···	···	30%	4.67	5.67

Table 1.1: Uniform shares across industries with $d_w = 0.7$. The numeraire is the net product. (*) Multiplied by 10^{-3}

Industries (means of production)	Prices	Profit rates	Shares of Surplus	Value of Capital	Value of Product
Iron	7.62*	**5.33**%	6.64%	1.25	1.44
Coal	3.21*	**5.33**%	6.50%	1.22	1.49
Wheat	2.91*	**5.33**%	5.42%	1.02	1.40
Beans	30.51*	**5.33**%	5.97%	1.12	1.22
Horses	71.27*	**5.33**%	1.60%	0.30	0.36
Aggregate			30%	4.90	5.90

Table 1.2: Uniform profit rates across industries with $d_w = 0.7$. The numeraire is the net product. (*) Multiplied by 10^{-3}

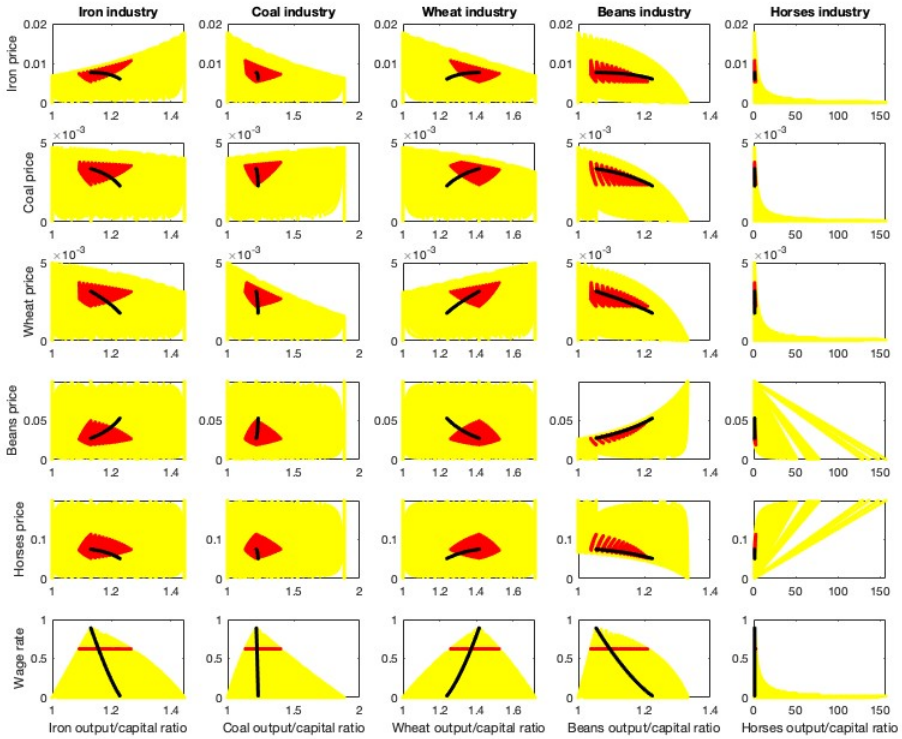

Figure 1.4: Domain of the sectoral value of capital, prices (iron, coal, wheat, beans and horses) and the wage rate

ties implied by the case of uniform rates of profit.

1.5 Conclusions

The paper has discussed the distinction between basic and non-basic commodities in an economic system since the publication of *PCMC* and has extended the model developed in Zambelli (2018b) to the case of an economic system in which both basic and non-basic goods are present.

The literature on basic and non-basic goods in Sraffian schemes focuses on studying the properties of the system under the assumption of a uniform rate of profit across industries.

The domain of prices computed by the algorithm developed in the present paper is compatible with different theories of distribution, on which no assumption is made, in line with the Sraffian approach as described by Bharadwaj

(1963).

If profit rates are not assumed to be uniform, non-basic industries have greater economic viability; moreover, if an invariable measure of value is no longer required to study the ability of the system to reproduce itself, computing the Standard commodity in the presence of simple or joint production is no longer necessary.

In our framework the presence of non-basic commodities is *important* for an explicit inclusion of the basket of commodities consumed by workers and producers, and the resulting distribution of the net product, which is explictly formulated in terms of the necessities of the consumers, which are both basic and non-basic commodities (see Sraffa (1960), sec. 8, p. 10). On the other hand, the presence of non-basic commodities does not change the understanding of the economic viability of the system; analytical complications related to the existence conditions of positive prices consistent with a uniform rate of profit are no longer of concern.

Hence, a full treatment of non-basic commodities in theoretical and empirical analysis is possible in the context of our framework. As Bharadwaj (1970) shows, the distinction between basic and non-basic commodities is crucial for the analysis of switching between different production techniques. Moreover, the use of the whole net national product as a numeraire (including non-basic commodities) for empirical analysis is particularly useful for international and intertemporal comparisons of technical progress.

The framework developed in Zambelli (2018b), and extended to the case of non-basics in this paper, retains all the important properties of Sraffian schemes highlighted in the Cambridge Capital Controversies. Moreover, the model described in the previous Section shows the existence of several possible 'rest positions' of the economy - e.g., uniform profit rates or uniform distributive shares - which are based on different distributive patterns. These properties are robust to the introduction of non-basic commodities.

Further extensions of the framework in Zambelli (2018b) include the possibility for exchanges to take place not only in a self-replacing state, but also through deferred means of payments such as debt and credit (Venkatachalam and Zambelli, 2021, 2022). Moreover, the system can be further generalized by introducing the possibility for industries to save part of their share of surplus or invest it, thereby enabling a complete study of growth and monetary theory in Sraffian schemes.

Chapter 2

Piero Sraffa and Monetary Theory: a Reappraisal

2.1 Introduction

Piero Sraffa is mostly known for *Production of Commodities by Means of Commodities* (henceforth *PCMC*), which is universally regarded as the author's *magnum opus*. In *PCMC*, Sraffa (1960) provides important insights related to the production, measurement, and distribution of the net product in the economy.

Before the publication of *PCMC*, the problem of distribution of the surplus (i.e., the net product of an economy) had been analyzed by three main strands of literature: the classical tradition (Adam Smith and David Ricardo), the neoclassical tradition (marginalism and Austrian theory) and the Marxian approach.

The classical and Marxian approaches do not rely on mechanical explanations for the investigation of how the surplus is distributed among different social groups. According to the classical and Marxian traditions, the structure of market phenomena is particularly influenced by the prevailing power relations. In both frameworks, competition is the realization of power relations *through* market forces. As power is unevenly distributed among these competing social entities, the access of the different social groups to the control of the means of production - given the prevailing technological, environmental, and demographic constraints - is also uneven. In this environment, the more powerful groups can determine the share of the product which is allocated to the less powerful ones.

On the contrary, the neoclassical approach offers an endogenous explanation of distributional outcomes. Given the endowments of material inputs and economic agents' preferences, the choice of techniques implemented in production and the distribution of the surplus are determined by a utility maximization process of rational agents subject to budget constraints (which are assumed to

be mutually consistent). The collective exchanges of every single consumer or producer, which take place inside free-entry competitive commodity markets, determine market-clearing prices that are such that every single agent is able (by definition) to implement her maximization plans.

Under a set of assumptions, this process generates the equilibrium relations between the marginal productivity of the factors of production (capital and labor) and the remuneration of each factor (profits and wages).

Sraffa shows that in an economy which can produce a surplus, and where multiple goods and services are produced and exchanged, it is impossible to define any notion of 'capital' or 'cost of production' independently of the determination of the prices of the net product, which allow the system to reproduce itself, i.e. to restore the original distribution of products among industries[1].

If the economy produces only for subsistence, there is a unique set of exchange-values (prices) that restores the original distribution. However, when a surplus is generated, prices depend on the distribution of the surplus between wages and profits.

Sraffa (1960) does not assume that distribution is endogenously generated by production relations: distributive shares of the product - i.e., wages and profits - are not obtained from a production function through the criterion of marginal productivity. Quantities of goods produced are taken as given and are held constant throughout *PCMC*.

The Sraffian system is thus an *indeterminate* one: prices and the techniques of production only determine the net product (or surplus) of an economy. Hence, one of the distributive variables (prices, wage rates, or profit rates) has to be exogenously defined to be able to compute aggregate economic magnitudes. In the words of Bharadwaj (1963), "*no theory of distribution is offered in the book*" (p. 1450) (see also Venkatachalam and Zambelli (2021), p. 456).

In this context, the notions of 'real cost' and 'quantity of capital', which are widely used in Marshall's marginal productivity theory, can be measured only after an exogenous force has determined the distribution of the surplus in the economy[2].

Moreover, in the context of the Sraffian system, the notion of 'capital intensity' of a given technique of production - i.e., the ratio between the value of capital and labour inputs - cannot be defined before the determination of distribution and prices.

In a multi-product economy, there is no unique 'capital good'. The notion of 'capital' is the result of the aggregation of different means of production; thus,

[1] See the Chapter 1 of this thesis, Zambelli (2004), Zambelli (2018b), Venkatachalam and Zambelli (2021), and Venkatachalam and Zambelli (2022).

[2] This is explicitly stated in *PCMC*: "*It may be added that not only in this case but in general the use of the term 'cost of production' has been avoided in this work, as well as the term 'capital' in its quantitative connotation, at the cost of some tiresome circumlocution. This is because these terms have come to be inseparably linked with the supposition that they stand for quantities that can be measured independently of, and prior to, the determination of the prices of the product.* ***(Witness the 'real cost' of Marshall and the 'quantity of capital' which is implied in the marginal productivity theory.)***" (Sraffa, 1960, p. 10; emphasis added)

the value of capital can be computed only after the prices of all commodities are known, and no monotonous relation can be assumed *a priori* among the value of capital, prices, wages and the profit rate.

Figures 1.3 and 1.4 of Chapter 1 show precisely this point. That is: there is no monotonic relationship between the value of capital and prices. This is so in the aggregate (Figure 1.3) and when considering the individual industries/markets (Figure 1.4).

This insight is at the core of the 1960s Cambridge Capital Controversies, as it is susceptible to invalidating the fundamental *neoclassical postulate* according to which the capital intensity of production is inversely related to the rate of interest (Ferguson, 1969). As a matter of fact, in a multi-product environment such as the Sraffian system, an increase in capital intensity may be related to a decrease as well as an increase in the rate of profit.

This result undermines the theoretical foundation of the aggregate production function as defined by Samuelson (1962). While some authors in this tradition have acknowledged the existence of this problem from the logical point of view (Levhari and Samuelson, 1966; Samuelson, 1966), others have regarded it to be an empirically irrelevant problem (Sato, 1974)[3].

Other strands of literature in the Neo-Walrasian approach do not rely on any notion of capital as a single magnitude (Arrow and Debreu, 1954; Debreu, 1959; Magill and Quinzii, 2002). As a matter of fact, studies in the neoclassical tradition have shown that the 'paradoxical' behaviour of the relationship between the value of capital and the rate of profit can be observed in a multi-product economy unless stronger restrictions are introduced (Mas-Colell, 1989). Moreover, authors in the Walrasian general equilibrium tradition have acknowledged that the results of well-known papers in growth theory such as Solow (1956) are crucially affected by aggregation problems highlighted by the Sraffian approach (Hahn, 1966, 1982).

The implications of the results in Sraffa (1960) for economic theory, value and distribution have contributed to generating well-deserved attention for *PCMC* in the economic profession. However, despite the absence of money, credit and finance in *PCMC*, Sraffa has also provided important contributions to the understanding of the role of money and credit in an economy in both his published and unpublished works (Venkatachalam and Zambelli, 2022); the latter are included in the archive of the *Sraffa Papers* (henceforth SP)[4].

Specifically, Sraffa's *oeuvre* shows that money is an essential institution of the economic system, which is not merely meant to facilitate mutually beneficial trade and that cannot be neutral with respect to the equilibrium economic outcomes.

Hence, the purpose of this Chapter is to reconstruct Sraffa's monetary theory,

[3] See Zambelli (2018a) for an empirical analysis of the neoclassical aggregate production function. The conclusion is that the logical flaws exposed by the 1960s Cambridge Capital Controversies also happen to be the rule in empirical observation, as an appropriately computed aggregate production function does not exhibit the expected neoclassical properties.

[4] Quotes from the *Sraffa Papers* will be henceforth denoted by (SP, *archive code*) and have been retrieved from the following online archive: Papers of Piero Sraffa.

analyzing his published and unpublished writings throughout the 20th century. Section 2.2 analyzes the fundamental elements of monetary theory in Sraffa's lecture notes written in the 1920s (as retrieved in the SP) and compares them to the monetary framework of William Stanley Jevons.

Sections 2.3 and 2.4 review the debate between Friedrich Hayek and Piero Sraffa in the 1930s on the concept of '*neutral money*'. Section 2.5 focuses on the innovative insights developed by Sraffa during the preparation of his critique of Hayek's framework. Section 2.6 discusses the influence of the notion of '*commodity rate of interest*' on the economic debate on monetary theory. Section 2.7 reviews the main attempts to integrate money and finance in Sraffian schemes, discussing their merits and limitations. Section 2.8 provides some concluding remarks.

2.2 The building blocks of Sraffa's monetary theory in the 1920s

The first part of Piero Sraffa's *oeuvre* is dedicated to issues related to the study of money and banking and it is generally applied in character. The author's first contribution, derived from his graduate thesis, was titled *L'inflazione monetaria in Italia durante e dopo la guerra* (Sraffa, 1920) and studied the causes and effects of inflation in Italy.

Sraffa's second important block of papers is also dedicated to Italian monetary problems (Sraffa, 1922a,b). These articles showcased a significant familiarity with banking operations, the regulatory framework, and practical workings of the banking sector, which Sraffa had also got acquainted with during his apprenticeship at a local bank after his graduation (Venkatachalam and Zambelli, 2021).

Still, Sraffa's most famous contribution in the field of monetary economics is a review of Friedrich Hayek's *Prices and Production* (Hayek, 1931) published in 1932 in the *Economic Journal* (Sraffa, 1932a).

In the paper, Sraffa provides a detailed critique of the concept of 'neutral money' put forward by Hayek in his lectures and displayed a nuanced understanding of interest rate policy and central banking. Sraffa's critique induced Hayek to reply (Hayek, 1932), and a further response by Sraffa followed shortly after (Sraffa, 1932b). The Sraffa-Hayek debate received much attention in the literature and will be thoroughly analyzed in Section 2.3.

Alongside published writings, the archive of unpublished notes and articles in the *Sraffa Papers* shows that a coherent theory of money can be inferred from Sraffa's *oeuvre*. According to Sraffa, money is conceived as both *essential* to the workings of the economic system and *non-neutral* concerning economic outcomes, such as output, employment and the price system.

In his 1927 lecture notes, Sraffa introduces the motivation behind studying economic theories, put forward a compelling case for methodological pluralism in economic analysis and lays the foundations of his theory of money. In the

initial paragraphs of the notes, Sraffa explains that economic theory aims at reducing all economic phenomena that continuously manifest in reality to a set of generalized and coordinated relations, which allow the theorist to identify common patterns and regularities[5].

However, Sraffa admits that each generalization implies a loss of precision, so that each theory has to choose the facts that are to be eliminated and those that are to be considered; hence, each theory includes both elements of truth and mistakes. According to Sraffa, 'exact sciences' can neglect few important facts while generalizing; on the contrary, economic theory regularly neglects important facts while generalizing, due to "*the complexity of facts and the backward state of* [this] *science*" (SP, D2/1/2r).

Having put forward a careful reminder of the limitations of economic theory as a whole, Sraffa claims that monetary theory is the most advanced field within economics:

> "*Within the different parts of economics,* ***the monetary one is among the least inaccurate: the theory of money is perhaps the one that overlooks the fewest essential facts****. This is probably partly due to the relative simplicity of the subject, partly to the fact that, given its practical importance, it has been studied more anciently and has undergone greater elaboration. Consequently, monetary theory (together with finance) is perhaps the only economic theory that practitioners must take into account when acting.*" (SP, D2/1/2v, emphasis added) [6].

This passage provides a clue as to why Sraffa would dedicate most of his subsequent works to the analysis of 'pure theory' - i.e., the theory of value and

[5] In the Italian original: "*La realtà è estremamente complicata: le ragioni per cui la società produce, consuma, attribuisce certi valori a determinate cose, ecc. sono infinite: Se volessimo esaminarle tutte, tenendo conto di tutte le condizioni che determinano i fatti economici, dovremmo scendere ad analizzare le condizioni particolari di ogni singolo individuo, i suoi gusti, i suoi bisogni, ecc. (es.: Come sono determinati i prezzi dei biglietti del tram? Per quale motivo ho preso il tram stamattina, ecc. ecc.: che cosa c'è di comune fra tutti gli utenti del tram? Questo è l'essenziale). Non solo, ma il prezzo di ciascuna merce subisce l'influenza dei prezzi di tutte le altre merci (se il pane è più caro si devono pagare di più i tramvieri, e quindi si deve aumentare il biglietto) e quindi la nostra ricerca si estenderebbe sempre più, talché, per ogni minima questione dovremmo indagare tutto l'universo. Per conoscere le cause del prezzo di una merce ci vorrebbe tanta conoscenza da riempire 10.000 volumi: ciò è evidentemente impossibile. Se anche fosse possibile, non ci sarebbe mente umana capace di contenere e di coordinare tutte quelle notizie, e quindi si ricadrebbe nella confusione, nell'ignoranza da cui si è partiti. Compito della teoria è di ridurre quei 10.000 volumi a 10 pagine: non per economia di carta, ma per poterle abbracciare con un solo sguardo, vederne tutti insieme i rapporti, poterle coordinare. In ciò consiste la teoria: riassumere, semplificare, cioè generalizzare.*" (SP, D2/1/1-2))

[6] In the Italian original: "*Fra le diverse parti dell'economia,* ***quella monetaria è fra le meno inesatte: la teoria della moneta è quella forse che trascura il minor numero di fatti essenziali****. Ciò è dovuto probabilmente in parte alla relativa semplicità della materia, in parte al fatto che, per la sua importanza pratica è stata più anticamente studiata ed ha subito una maggior elaborazione. In conseguenza di ciò, la teoria monetaria (insieme con la finanziaria) è forse la sola teoria economica di cui i pratici, nell'agire, devono tener conto.*"

distribution. When the lecture notes were written, Sraffa felt that monetary theory was the most advanced field in economic analysis, whereas other important problems (e.g., the theory of value and distribution) remained to be settled.

But the 1927 lecture notes do not only deal with methodological issues; in a very dense fashion, they also deal with the essential role of money for production and exchange.

Following Adam Smith, Sraffa stresses that the division of labour creates room for a widening of trade opportunities. To properly function, the market requires a system of accounting for debts and credits that arise out of mutually beneficial exchanges among economic agents[7].

As the specialization induced by the division of labour causes each economic agent to produce different goods and services, agents in a barter economy may struggle to engage in all possible mutually beneficial exchanges.

Some of these problems had been already identified by prominent monetary theorists such as Jevons (1876), who observed:

> "*The first difficulty in barter is to find two persons whose disposable possessions mutually suit each other's wants. There may be many people wanting, and many possessing those things wanted;* ***but to allow an act of barter, there must be a double coincidence, which will rarely happen.***" (Jevons, 1876, pp. 3-4, emphasis added)

The 'first difficulty' identified by Jevons is related to the usefulness of money as a *medium of exchange*. The second difficulty of barter, according to Jevons, is related to the complexity of computing relative prices without a *common measure of value*: hence, the second difficulty makes room for the existence of money as a *unit of account* for economic exchanges.

Therefore, according to Jevons, it is rational for economic agents to elect one of the commodities available for trade as the *commodity money*. This monetary instrument can act as a unit of account of debits and credits and as a means of payment, in order to save transaction costs associated with the absence of a double coincidence of wants and the complexity of computing relative prices.

Hence, according to Jevons, the use of a commodity as money in a given community is the result of a strategy aimed at minimising transaction costs incurred in a barter environment. In this framework, money is *essential* for the proper functioning of the market; however, money can be supposed *neutral* for the determination of economic outcomes such as output, employment and the price system.

The use of the expression 'neutrality of money' was introduced into the English-speaking academic debated by Friedrich Hayek in his lectures at the London School of Economics, which were published as *Prices and Production*

[7] "*Baratto: difficoltà di trovare l'esatto compenso negli scambi. Accreditamenti verso la società: scambi senza moneta. Merce universale. Debiti e crediti, conseguenti agli scambi. Divisione del lavoro: ognuno produce una merce: necessità di scambi*" (SP, D2/1/3r)

(Hayek, 1931). In the second edition of *Prices and Production*, Hayek also acknowledged the influence of the Dutch economists Johan Koopmans, who had introduced the term in a discussion with Verrjin Stuart, and the German economists Bortkieweicz and Behrens (see Patinkin and Steiger (1989) for a detailed reconstruction of the origin of the 'neutrality' term).

However, the notion of neutrality traces back to the proposition stated by David Hume in his 1752 essays, where Hume argues that changes in the total quantity of money only affect the absolute price level of an economy, and not real output or relative prices - e.g., see the chapters *Of Money*, *Of Interest* and *Of the Balance of Trade* in Hume (1752).

In his notes, Sraffa points out that money is the result of *social contrivance*. Various commodities have been elected as the commodity money in history, depending on the availability of the commodity itself, and subsequently paper money - i.e., non-convertible money or *fiat money* - appeared.

However, both commodity money and fiat money have been regulated by law: political institutions have been able to arbitrarily define both money as a unit of account and, as a consequence, to steer agents' behaviour towards the use of the same money as a means of exchange[8]

Thus, in Sraffa's view, the essential role of money in economic exchanges is the result of an institutional arrangement and not the product of a market-based endogenous process.

2.3 Friedrich Hayek's *Prices and Production*

Sraffa's most famous work in the field of monetary theory is his review of Friedrich Hayek's *Prices and Production* published in the *Economic Journal* in 1932 (Hayek, 1931; Sraffa, 1932a). Sraffa's article sparked a response by Hayek (1932) and a further rejoinder (Sraffa, 1932b).

In his review, Sraffa aims at deconstructing the theoretical framework put forward by Hayek (1931), highlighting several inconsistencies related to Hayek's concept of *neutral money*. Sraffa's 1932 article and the unpublished preparatory notes available in the Sraffa Papers archive show that the preparation of the review allowed Sraffa to develop original insights, sometimes anticipating problems that would have been discussed only decades later in the economic profession.

Hayek's contribution provides a theory of the business cycle which relies upon the Austrian theory of capital developed by Böhm-Bawerk (1923) and the contribution by Von Mises (1912), who had in turn reworked the monetary theory of Wicksell (1898). To understand Sraffa's 1932 critique, it is useful to review the theoretical roots behind Hayek's framework and to summarize the main argument in Hayek (1931).

[8] In the Italian original: "*Il denaro sorge per costume: varie merci - denaro: oro, ragioni della preferenza, ragioni di abbondanza di argento non in Oriente.* [...] *Regolato dalla legge: poi, carta moneta.* [...] *Il denaro è un'unità di misura: definizione arbitraria dell'unità [stock esistente di oro].* [...] *Stabilità dell'unità di misura.*" (SP, D2/1/3r)

In the competitive general equilibrium described in Wicksell (1901), money is inessential: agents' behaviour and equilibrium allocations only depend on relative prices and trade occurs without a monetary medium of exchange. The notion of equilibrium employed in this framework is defined as a *stationary equilibrium*: as long as consumers' preferences, initial endowments and productive technologies are unchanged, equilibrium prices and output remain constant in each period.

Following Böhm-Bawerk (1923), productive processes - through which factors of production are transformed into capital (or intermediate) goods, and then into final goods and services - can be distinguished in terms of the length of the period of production.

In general, productive processes which require the production of more capital goods to be finalised need longer periods of production on average; processes with a longer (shorter) average period of production are defined as more (less) *roundabout*. Hence, capital goods can be reduced to dated quantities of initial endowments[9]

Therefore, in a stationary equilibrium, the temporal distribution of dated quantities of inputs employed in production has to be held constant.

The monetary component of the Wicksellian framework is a model of a *pure credit* economy. The banking system creates money by extending loans to entrepreneurs at an exogenously defined 'monetary' interest rate; banks are not able to set the interest rate and have no reserve constraints, thereby allowing the banking system to create unlimited quantities of credit.

In Wicksell (1898) and Wicksell (1906), the economic system is inherently unstable due due to the presence of money and the influence of monetary aggregates. While Say's Law determines a fundamental dichotomy between real and nominal magnitudes - as aggregate demand cannot exceed aggregate supply under all circumstances - Wicksell argues that the money supply must be allowed to bear an influence on the demand for goods and services in order to cause an increase in the general price level, according to the Quantity Theory of Money: "[...] *Any theory of money worthy of the name must be able to show how and why the monetary or pecuniary demand for goods exceeds or falls short of the supply of goods in given conditions*" (Wicksell, 1906, pp. 159–160).

Therefore, the availability of credit in excess of existing savings in the economy is the means through which aggregate demand can outstrip aggregate supply, thereby producing a "cumulative process". It must be noted that, according to Wicksell, *endogenous* increases in the demand of credit by entrepreneurs, due to the relative conditions of the real and financial sectors, are the main drivers of instability[10].

Hence, in Wicksell's monetary model, other conditions must also be fulfilled in equilibrium: first, the gross supply of 'capital' (defined as *savings*) has to be equal to the gross demand of 'capital' (defined as *investments*); secondly, net

[9] See *PCMC* (Sraffa, 1960, pp. 34-38) for a critique of the notion of roundaboutness of capital advocated by Austrian capital theory.

[10] Wicksell considers increases in the money supply as a less common possibility (Wicksell, 1898, p. 167)

savings and net investments must be equal.

The integration of the real and the monetary component in Wicksell produces a theory of business cycle fluctuations. According to Wicksell, the stationary equilibrium requires the 'monetary' interest rate to be equal to the 'natural' interest rate - i.e., the interest rate which clears the market for savings and investments.

When the monetary interest rate is lower than the natural interest rate, entrepreneurs increase their borrowing to expand production; however, the supply of capital cannot catch up with the increase in the demand for capital, thereby sparking an inflationary process. A deflationary process, instead, occurs when the monetary interest rate exceeds the natural interest rate.

Therefore, according to Wicksell, price stability is not ensured through inherent mechanisms in the system, and it can only be maintained if the monetary authorities ensure that the monetary interest rate is equal to the natural interest rate. Hence, since the natural interest rate equalises the demand and the supply of capital and keeps prices stable, it can be conceived as the interest rate that would prevail in a market of loans in kind. The definition of the natural interest rate requires a unique measure of the quantity of 'capital', which is not provided in the model.

Böhm-Bawerk's average period of production, initially adopted by Wicksell (1901), can be employed to order processes in terms of their roundaboutness: more (less) roundabout processes are associated with a higher (lower) natural interest rate. However, this notion is abandoned in Wicksell's subsequent works.

Building on the Wicksellian framework, Von Mises (1912) studies the effect of the divergence of the monetary interest rate from the natural interest rate on the dynamics of relative prices of consumption goods and intermediate (capital) goods.

The contribution of Hayek (1931) is a further extension of Von Mises (1912). Hayek's model envisages a Wicksellian pure credit economy and extends it to the analysis of the choice of technique by entrepreneurs. Building on the theory of capital in Böhm-Bawerk (1923), Hayek argues that the excess of the natural rate of interest over the monetary rate of interest induces entrepreneurs to increase their borrowing. However, this additional borrowing is not meant to increase production in general - as in Wicksell - but aims at adopting more capital-intensive techniques, i.e. more roundabout production processes. In particular, Hayek rejects the notion of an 'average price level' and assumes that the economy produces a set of consumption goods and a set of investment goods.

Hence, in Hayek's framework, a banking policy which envisages a low monetary interest rate determines a change in the structure of production, inducing the adoption of techniques associated with a longer period of production.

Such a change in the productive structure of the economy initially drives up the prices of investment goods and reduces the output of consumption goods, due to the increase in the average period of production: the 'elastic' banking policy induces a change in relative prices of consumption and investment goods. However, if consumers' preferences are unchanged, the demand for consumer goods does not adapt to the decrease in the supply, thereby driving up the price

of consumption goods as well.

Hence, the change in the productive structure of the economy cannot be permanent. When the price of consumption goods rises, reflecting the underlying pattern of consumers' preferences, entrepreneurs have to abandon the more roundabout productive processes they have adopted: as a consequence, the economy undergoes a contraction until the productive structure returns in line with consumers' preferences.

Finally, although the structural change induced by the 'elastic' banking policy turns out to be transitory, the final equilibrium - reached by the economy after the perturbation of the initial equilibrium - is characterized by higher prices for consumption and investment goods with respect to the initial position.

According to Hayek, such a perturbation of the stationary equilibrium could not occur in a non-monetary economy. Absent money, there would be no artificial price signal sent by the banking system and the prevailing interest rate in loans in kind would be the natural interest rate - i.e. the interest rate which equalises the demand of capital (investments) and the supply of capital (savings).

Hence, Hayek argues that monetary authorities should follow a 'neutral money' approach in order to prevent such fluctuations in the business cycle. In the words of Hayek, if we "*eliminate all monetary influences on production* [...] *we may treat money as non-existent*" (Hayek, 1931, p. 109). Two different - albeit interrelated - definitions of 'neutral money' can be found:

a) the banking system should keep the quantity of money constant;

b) the banking system should set the monetary interest rate at the level of the natural interest rate. The two definitions are interrelated in that when banks lower the monetary interest rate below the 'natural' level, the quantity of money in the economy increases.

In his review of *Prices and Production*, Sraffa takes issue with both definitions. Hayek's definition *a*) is related to traditional statements of the Quantity Theory of Money, according to which changes in the money supply ultimately lead to increases in prices. In his 1920 dissertation ('*L'inflazione monetaria in Italia durante e dopo la guerra*'), Sraffa had relied on the Quantity Theory for explaining monetary inflation in Italy during and after the First World War (Harcourt and Sardoni, 1993, pp. 12–15)[11], supporting the argument of a mainly fiscal origin of inflation.

Interestingly, as Marcello de Cecco pointed out (De Cecco, 1993, pp. 2–3), the orthodox argument developed by Sraffa did not lead him to support a return to the pre-war lira-gold parity, nor any deflationary therapy for the Italian economy, predating Keynes's argument in his 1923 *Tract on Monetary*

[11] "*Both monetary and bank inflation have created new purchasing power without a corresponding increase in the quantity of goods. On the contrary, while the quantity of goods probably declined, the result could not fail to be such a large price increase as to re-establish the balance between the total purchasing power and the quantity of available goods*" (Harcourt and Sardoni, 1993, p. 15)

Reform. Sraffa combined insights from Irving Fisher and Gustav Cassel to stand for the maintenance of a non-convertible currency and explains the asymmetrical nature of deflation (Harcourt and Sardoni, 1993, pp. 21–23): while deflation surely benefits debt holders, workers benefit from a greater real wage only if unemployment does not increase due to the collapse of economic activity.

In his 1932 review, instead, Sraffa makes a subtle point with regards to Hayek's use of the term 'quantity of money': "*I follow Dr. Hayek's practice of using "the quantity of money" as short for "the quantity of money multiplied by its velocity of circulation"; although it is a dangerous omission which leads him to overlook that the velocity is bound to change as the direct result of a change in prices*" (Sraffa, 1932a, p. 44, footnote 1). Sraffa's observation includes, in a nutshell, the major challenge to the 'monetarist' interpretation of the Quantity Theory of Money. The direct increase (or decrease) in the price level caused by an increase (or decrease) of the money supply, in fact, can only be observed if the velocity of circulation of money is kept constant.

Hence, in his 1932 review, Sraffa provides a more nuanced interpretation of the Quantity Theory of Money, which has been regarded as a step forward in his intellectual formation, similar to the path of development of a more original monetary theory followed by Keynes.

A typical case in which velocity is bound to be stable is when the economy has already reached a full employment equilibrium. However, changes in prices and savings, which - according to Hayek - can be conceived as an increase in the flow of money directed to the purchase of investment goods, are susceptible of perturbing such equilibrium condition, thereby complicating the task of finding a banking policy which maintains constant under all circumstances the quantity of money multiplied by its velocity of circulation.

Hayek (1931) distinguishes two cases in which the economy enters a disequilibrium adjustment path as a result of the accumulation of capital. In the case of 'voluntary savings', consumers accept to postpone consumption to enjoy a higher output of consumption goods in the future.

Hence, entrepreneurs lengthen the average period of production and the new equilibrium is characterized by a higher ratio of capital over labour, a higher output of consumption goods and - according to Hayek - lower prices for all goods. The effect "*is one which fulfils the object of saving and investing, and is* ***identical with the effect which would have been produced if the savings were made in kind instead of in money***" (Hayek, 1931, p. 49, emphasis added).

Thus, according to Hayek, the act of voluntary savings produces the same effect - i.e., the accumulation of capital leading to a new equilibrium with higher output and lower prices in the future - both in a monetary and in a non-monetary economy.

Hayek then describes a different accumulation process, defined as 'forced saving', which is regarded as typical of a monetary economy. Even in the absence of savings, a credit expansion by the banking system induces entrepreneurs to lengthen the production process and to choose more capitalistic techniques. However, as consumer preferences do not change, an inflationary process be-

gins, and consumers have to involuntarily reduce their consumption due to the increase in prices.

The reduction in consumption brings about a process of accumulation of capital, but such a process is only temporary: entrepreneurs have to return to less capitalistic methods of production to accommodate the unchanged demand for consumption goods until a new equilibrium is reached where all prices are higher. In the words of Hayek, "*such a transition to less capitalistic methods of production necessarily takes the form of an economic crisis*" (Hayek, 1931, p. 53).

The case of 'forced saving' is typical of a monetary economy because of the arbitrary acts of the banking system in the form of an extension of credit to producers. It is evident to note that money is neutral by definition in Hayek's model: changes in the quantity of money only induce a temporary shift in the productive structure of the economy and a permanent increase in prices.

2.4 Sraffa's 1932 critique of Hayek's *Prices and Production*

The analysis of the relation of money to the interest rate in Hayek provides Sraffa with the opportunity to shape his monetary theory, as it can be inferred both from published articles (Sraffa, 1932a,b) and the non-published notes collected in the SP.

Sraffa begins his criticism of Hayek's model by describing the fundamental features of money that had already been included in his 1927 lecture notes:

> "[...] *money is not only the medium of exchange, but also a store of value, and the standard in terms of which debts, and other legal obligations, habits, opinions, conventions, in short all kinds of relations between men, are more or less rigidly fixed. As a result, when the price of one or more commodities changes, these relations change in terms of such commodities; while if they had been fixed in commodities, in some specified way, they would have changed differently, or not at all*". (Sraffa, 1932a, p. 44)

Therefore, money is essential for the smooth functioning of the economy not only as an *intratemporal medium of exchange* but most importantly as an *intertemporal standard of deferred payments*. In the preparatory notes to the 1932 article in the SP, Sraffa argues that the store of value function depends primarily on the choice of money as a standard of deferred payments (SP, D3/9/38.r): the existence of 'monetary constants' (SP, D3/9/49) in terms of which debts, contracts and legal obligations are set is the main driver of value for an otherwise worthless monetary instrument.

Due to this feature of the monetary instrument, Sraffa argues that money is not only essential but also *inherently non-neutral* (SP, D3/9/38.r). Different monetary policies may result in different values of money in terms of other

commodities, which in turn can generate different economic outcomes in terms of output and employment.

If neutrality is defined as the inability of money to influence relative prices of commodities, Sraffa points out that "*money itself is one of the commodities under consideration*" (Sraffa, 1932a, p. 44), so that changes in the value of money in terms of commodities must also be taken into account.

When money is not only considered as an intratemporal medium of exchange which facilitates trade between economic agents, but also as an intertemporal store of value, the existence of money as the record of debit and credit relations can change "*all kinds of relations between men*", whereas "*if they had been fixed in commodities, in some specified way, they would have changed differently, or not at all*" (Sraffa, 1932a, p. 44). Hence, if the stationary equilibrium condition is disturbed, a monetary policy can exactly restore the initial equilibrium position only by a fluke.

Hayek assumes away any real effect of changes in a general fall (or rise) of prices due to the rejection of any notion of a general price level, understood as an index number of an arbitrarily selected bundle of commodities, thus departing from Wicksell's use of such a definition. However, according to Sraffa, studying the value of money in terms of other commodities is crucial to successfully performing Hayek's task of checking whether it is possible to find a 'neutral' monetary policy whose effects are identical to a given non-monetary system.

Definition *b*) of 'neutral money' is where the descriptive and prescriptive parts of Hayek's contribution intertwine. Having described the effects of monetary disturbances on a stationary equilibrium, Hayek puts forward a policy implication which follows directly from his analysis of the monetary system influenced by Wicksell (1898) and Von Mises (1912): to prevent inflationary outcomes generated by unchecked monetary disturbances, the monetary interest rate must be kept equal to the natural interest rate.

A passage of Hayek (1931), also quoted by Sraffa in his review, shows this very clearly:

> "*In a money economy, the actual or money rate of interest may differ from the* ***equilibrium or natural rate****, because the demand for and the supply of capital do not meet in their* ***natural*** *form but in the form of money, the quantity of which available for capital purposes may be* ***arbitrarily*** *changed by the banks.*" (Hayek, 1931, pp. 20-21, emphasis added)

Thus, according to Hayek, in a non-monetary economy, the prevailing interest rate is always bound to be equal to the natural interest rate, which is also the equilibrium interest rate that clears the market for 'capital' - i.e., where agents who are willing to invest meet agents who are willing to save.

When money comes into play, both sides of the market for 'capital' are intermediated by the banking system. When banks create money and lend it to entrepreneurs, the monetary interest rate turns out to be different from the natural rate, thereby disturbing the stationary equilibrium.

Sraffa shows that there are two problems with this statement. First off, in any economy where multiple goods are produced, each interest rate of a commodity calculated in terms of itself can be defined as a 'natural' interest rate.

Hence, the only way to implement the monetary policy suggested by Hayek (setting the monetary interest rate equal to the natural interest rate) is to arbitrarily define a composite commodity (i.e., a bundle of commodities) and compute an index number of prices.

This is precisely Wicksell's method, but it is not available to Hayek due to his rejection of any concept of general price level[12].

Secondly, the absence of money does not ensure that the stationary equilibrium remains undisturbed. To explain this point, Sraffa provides a simple example:

> "*The "arbitrary" action of the banks is by no means a necessary condition for the divergence* [of interest rates]; *if loans were made in wheat and farmers (or for that matter the weather) "arbitrarily changed" the quantity of wheat produced, the* ***actual rate of interest on loans in terms of wheat would diverge from the rate on other commodities and there would be no single equilibrium rate***." (Sraffa, 1932a, p. 49, emphasis added).

Thus, according to Sraffa, the multiplicity of 'natural' interest rates is not a feature of a monetary economy in itself, but rather a typical feature of states of the world outside of equilibrium.

To compare the effects of disequilibrium in a monetary and non-monetary economy, Sraffa puts forward a famous example of a loan market which can be indirectly defined in terms of commodities:

> "*Loans are currently made in the present world in terms of every commodity for which there is a forward market. When a cotton spinner borrows a sum of money for three months and uses the proceeds to purchase spot, a quantity of raw cotton which he simultaneously sells three months forward, he is actually "borrowing cotton" for that period. The rate of interest which he pays, per hundred bales of cotton, is the number of bales that can be purchased with the following sum of money: the interest on the money required to buy spot 100 bales, plus the excess (or minus the deficiency) of the spot over the forward prices of the 100 bales.*" (Sraffa, 1932a, p. 50).

Therefore, according to Sraffa, the existence of a forward market for commodities allows identifying a commodity rate of interest (i.e., a rate of interest

[12] "*This, however, though it meets, I think, Dr. Hayek's criticism, is not in itself a criticism of Wicksell. For there is a "natural" rate of interest which, if adopted as bank-rate, will stabilise a price-level (i.e. the price of a composite commodity): it is an average of the "natural" rates, weighted in the same way as they are in the price-level itself.*" (Sraffa, 1932a, p. 51)

for the commodity defined in terms of itself) even if the cotton spinner does not actually perform the genuine operation of borrowing cotton.

By computing commodities rates of interest for all commodities traded in the economy, Sraffa aims at fulfilling the task that Hayek has put forward in Hayek (1931): this amount to checking whether it is possible to implement a monetary policy that can exactly reproduce the conditions of a non-monetary economy.

Sraffa's example can be formalized as follows. An economic agent wishes to purchase spot a quantity Q of a good j in an n-commodity economy (where $j = 1, \ldots, n$) and wants to simultaneously sell the same quantity Q of good j three months forward. As the agent does not own any initial purchasing power, she receives a 3-month monetary loan from the banking system at the interest rate i_m.

In Sraffa's thought experiment (as well as in Hayek's model), the agent has no liquidity constraints - i.e., she can borrow infinite quantities of money from banks - and there is no risk of default on loans[13].

Let p_j^S be the spot price of the j-th commodity, p_j^F the forward price of j, and M the sum of money corresponding to the interest paid by the agent in terms of cotton. Thus, the definition of M is such that:

$$M \equiv i_m Q p_j^S + Q(p_j^S - p_j^F) \tag{2.1}$$

Having defined the level of interest that the agent has to pay in terms of good j for 'borrowing' good j, it is possible to compute a commodity rate of interest via a suitable choice of the denominator for the commodity rate. As noted by Donzelli (2004) and Naldi (2015), Sraffa does not mention which price should be used as a the denominator to compute the commodity rate of interest: depending on which price is chosen - the spot price, the forward price, or the expected spot price in three months - the sum of money M can purchase different quantities of good j[14]. If the forward price is chosen as denominator of the commodity rate of interest i_j, then the definition of i_j is:

[13] Venkatachalam and Zambelli (2022) discuss the introduction of credit and debt within Sraffian schemes and show that agents (consumers or industries) cannot find themselves with outstanding debt or credit positions at the end of each period in the context of self-replacing (see also Chapter 1 of the thesis). This is so because credit and debt contracts at the end of the period only emerge when agents exchange a real commodity against an *I Owe You* (IOU) to be settled in the future. If agents can perform exchanges through a clearinghouse which requires all positions to be settled at the end of the period, and supply and demand for each commodity meet due to market-clearing prices, IOUs would cancel out at the end of each period and intertemporal credit and debt position would not emerge. In this context, the cotton spinner in Sraffa (1932a) can be seen as 'borrowing cotton', as the seller of raw cotton in the present will purchase the same cotton three months forward at the forward price. The presence of a market for monetary loans in Sraffa (1932a) is needed for Sraffa to compare the outcomes of monetary and non-monetary economies (in order to critize Hayek's framework): However, as it is discussed below, a market for loans 'in cotton' would produce the same outcome. In a self-replacing condition - the environment of analysis in *PCMC* - forward prices would always match current and expected spot prices, so that both 'natural' and monetary interest rates would be equal to zero

[14] See Naldi (2015) for a discussion of the implications of different choices of the denominator for the 'commodity rate of interest' and a review of the correspondence between Sraffa and

$$i_j \equiv \frac{M}{Qp_j^F} = \frac{i_m p_j^S + p_j^S - p_j^F}{p_j^F} = (1 + i_m)\frac{p_j^S}{p_j^F} - 1 \tag{2.2}$$

Hence, equation (2.2) shows that the commodity rate of interest of j depends on three observable magnitudes: the spot price p_j^S, the three months forward price p_j^F, and the monetary interest rate i_m.

Having defined the building blocks of his thought experiment, Sraffa then draws his conclusion on whether the divergence of the natural interest rate from the monetary interest rate is a typical phenomenon of a monetary economy or not. As the following passage shows, Sraffa argues that such a divergence happens both in a monetary and a non-monetary economy, as it stems from the perturbation of equilibrium:

> "*In equilibrium the spot and forward price coincide, for cotton as for any other commodity; and all the "natural" or commodity rates are equal to one another, and to the money rate.*" (Sraffa, 1932a, p. 50)

In a condition of equilibrium, prices and quantities of commodities in the economy are constant through time: hence, spot and forward prices of any commodity cannot diverge. This can be seen clearly by taking the right side of (2.2) and putting $p_j^F = p_j^S = p_j \ \forall \ j = 1, \ldots, n$. In this case, we have the following condition, which matches exactly Sraffa's wording:

$$1 + i_j = 1 + i_m \quad \forall \ j = 1, \ldots, n \tag{2.3}$$

Hence, in equilibrium, the monetary interest rate coincides with the natural interest rate, which is also equal for all commodities. As Sraffa explains in the following passage, the equality of commodity interest rates in equilibrium does not depend on the choice of the standard of deferred payment (money or any other commodity); it rather depends on whether the economy is in a stationary equilibrium or not:

> "*But if, for any reason, the supply and the demand for a commodity are not in equilibrium (i.e. its market price exceeds or falls short of its cost of production), its spot and forward prices diverge, and the "natural" rate of interest on that commodity diverges from the "natural" rates on other commodities. Suppose there is a change in the distribution of demand between various commodities; immediately some will rise in price, and others will fall; the market will expect that, after a certain time, the supply of the former will increase, and the supply of the latter fall, and accordingly the forward price, for the date on which equilibrium is expected to be restored, will be below the spot price in the case of the former and above it in the case of the latter; in other words, the rate of interest on the former will be*

John Maynard Keynes on this point. See Donzelli (2004) for a discussion of the implied theory of expectations formation in the choice of the denominator for the commodity rate of interest.

> *higher than on the latter. It is only one step to pass from this to the case of a non-money economy, and to see that when equilibrium is disturbed, and during the time of the transition,* ***the "natural" rates of interest on loans in terms of the commodities the output of which is increasing must be higher, to various extents, than the "natural" rates on the commodities the output of which is falling****; and that* ***there may be as many "natural" rates as there are commodities****.*" (Sraffa, 1932a, p. 50, emphasis added).

The above passage includes the crucial argument in Sraffa's criticism of Hayek (1931): the source of the multiplicity of natural interest rates is a state of disequilibrium, be it in a monetary economy or not.

Here Sraffa describes a perturbation of equilibrium which is caused by a shock in the distribution of demand between commodities, whereas in a previous passage he mentioned the possibility of a supply shock (e.g., an extreme weather event with an impact on the output of wheat).

In both cases, the shock causes the spot prices of commodities to diverge from their equilibrium values, which are equal to the forward prices. For the sake of the critique, Sraffa moves in Hayek's equilibrium framework and assumes that the market correctly expects that the spot prices will converge to the equilibrium level, and the forward price perfectly anticipates the equilibrium price[15].

The formalization of the problem in (3.2) matches Sraffa's wording. For any pair of goods k and s in $j = 1, \dots, n$, an increase in the demand for s relative to the demand for k implies that the spot price of s will be greater than its forward (equilibrium) price; at the same time, the spot price of k will be lower than its forward (equilibrium) price.

Defining the commodity rates of interest for k and s via (2.2), given a monetary interest rate i_m, the following condition must hold:

$$\frac{p_k^S}{p_k^F} > \frac{p_s^S}{p_s^F} \implies i_k > i_s \tag{2.4}$$

It is straightforward to see that while the stationary equilibrium is perturbed, natural rates of interest of various commodities will diverge from each other and the monetary interest rate.

The multiplicity of natural rates is not necessarily caused by changes in the quantity of money or the monetary interest rate; it is a direct consequence of a state of disequilibrium (demand shocks, supply shocks, or an increase in saving), which can also occur in a non-monetary economy.

Hence, the monetary policy suggested by Hayek (i.e., setting a monetary interest rate equal to the natural interest rate) cannot be implemented in a multi-product economy, be it monetary or not.

Sraffa also suggests a possible way out of the deadlock, that is considering the spot and forward prices of a composite commodity: "*there is a "natural*

[15] See Donzelli (2004) (pp. 9-12) for an analysis of Sraffa's (implied) assumptions on expectations in Sraffa (1932a)

rate of interest which, if adopted as bank-rate, will stabilise a price-level (i.e. the price of a composite commodity): it is an average of the "natural" rates of the commodities entering into the price-level, weighted in the same way as they are in the price-level itself" (Sraffa, 1932a, p. 51).

However, as there is no unique possible price level, for any composite commodity arbitrarily selected "*there is a corresponding rate that will equalise the purchasing power, in terms of that composite commodity, of the money saved and of the additional money borrowed for investment*" (Sraffa, 1932a, p. 51).

The route suggested by Sraffa corresponds to the common practice of constructing an index number of prices of commodities produced in the economy; after computing the price level, a real interest rate can be derived. However, this route is forbidden for Hayek due to his rejection of statistical averages for prices, while it is not forbidden for Wicksell [16].

2.5 Sraffa's innovative insights on money

Sraffa's review of *Prices and Production* has been generally regarded as a successful critique of the theory of the business cycle put forward by Hayek. Moreover, the review has received much attention in the economic literature due to the presence of several hints for a research agenda in monetary economics.

The 1932 article - which was followed by a reply (Hayek, 1932) and a brief rejoinder by Sraffa himself (Sraffa, 1932b) - served a critical purpose, as it can be inferred from Sraffa's preparatory notes in the SP.

However, in the process of deconstructing *Prices and Production*, Sraffa has provided several innovative intuitions which have enriched his understanding of monetary theory and, in some cases, have anticipated issues that were at the centre of the economic debate in subsequent decades.

The most important issue at stake in the Sraffa-Hayek debate is the neutrality of money. As Deleplace (2014) noted (pp. 144-145), Sraffa starts his analysis of Hayek's model by identifying a fundamental confusion between the conditions for *essentiality* and *neutrality* of money

Comparing the conditions of a non-monetary economy and a monetary economy would "*bring out which are the essential characteristics common to every kind of money, as well as their differences, thus supplying the elements for an estimate of the merits of alternative policies*" (Sraffa, 1932a, p. 43).

Instead, Sraffa writes that "*the reader soon realises that Dr. Hayek completely forgets to deal with the task which he has set himself, and that he is only concerned with [a] wholly different problem*" (Sraffa, 1932a, p. 43).

According to Sraffa, Hayek is mainly concerned with putting forward a particular monetary policy aimed at keeping constant the quantity of money multiplied by its velocity of circulation, which would succeed in giving effect to the voluntary decisions of individuals (as opposed to the case in which inflation induces 'forced' savings).

[16] "*This, however, though it meets, I think, Dr. Hayek's criticism, is not in itself a criticism of Wicksell.*" (Sraffa, 1932a, p. 51)

Hence, Hayek is confusing two distinct problems under the same label of 'neutrality of money': the fundamental characteristics which make a monetary economy differ from a non-monetary economy (i.e., what makes money *essential*) and the effects of different monetary policies on the macroeconomic magnitudes (i.e., whether money can be made *neutral* with respect to magnitudes such as relative prices).

Sraffa's approach addresses both problems. The essentiality of money is related to the 'monetary constants' - i.e., institutional arrangements that cause debts, legal obligations, contracts and other relations to be fixed in terms of money[17].

The neutrality of money is the situation in which prices and output are not affected by monetary policies. The previous section reviews Sraffa's proof of the impossibility to implement a 'neutral money' policy advocated by Hayek outside a one-good world.

The same argument put forward by Sraffa allows to single out another important feature of a monetary economy, which makes money intrinsically non-neutral: when money is the standard of deferred payments, there is a *single nominal (risk-free) term structure of interest rates* in the loan market.

In a non-monetary economy, instead, there would be a term structure of interest rates for each commodity, and the relations between each 'commodity interest rate' would be determined by the expected appreciation of a commodity with respect to another. Sraffa explains this point in the following passage of the SP:

> "*Does not disturb. This means "makes not different". From what? Here is where we want the "absolute" invariable standard of comparison. And here it fails (just as "unchanged price level" and "market equal to normal rate of interest" fail: not because of statistical difficulties, but impossible to define the standard). "Not different from what they would be if money didn't exist", presumably. But is this possible? Does not the mere existence of money "affect" relative prices?* ***If no money (nor other single stand.[ard] of def.[erred] pay.[ment] there would be no single rate of interest.*** *Every article would be lent at a different rate, pos.[itive] or neg.[ative]. Perishable art.[icles] might be lent at enormous neg.[ative] rates. Cp. commodity markets: only, this would be extended to every article*". (SP, D3/9/42.r, emphasis added)

Moreover, according to Sraffa, the fundamental feature of monetary analysis is the study of states of disequilibrium. Due to the ability of money to set a single standard of deferred payments in the economy, the transition from

[17] "*It is impossible to avoid prefacing a discussion of neutral money, without analysing however briefly what is the essential feature which distinguishes any monetary from [what] may be called a non monetary economy. This I should describe as the existence of 'monetary constants'. They may be debts or any other legal obligations, habits or fixed decisions of individuals of the kind predilected by H.[ayek]*" (SP, D3/9/49)

different equilibrium positions of the economy and business cycle fluctuations can be studied:

> "*The non monetary theory studies a state of equilibrium, and the conditions which determine it: it goes as far as comparing two, or more states of equilibrium, and measuring the differences in their conditions – but goes no further.* ***Here begins the field of monetary theory: or rather, jumping over the study of the path followed in the transition from one position to another, it sets to study states of disequilibrium****. I suppose that every monetary economist to-day regards trade fluctuations as his exclusive subject.*" (SP, D3/9/181)

In his analysis, Sraffa is also able to anticipate monetary problems that would have been discussed only decades later in the literature. To define the concept of commodity rate of interest, he explicitly considers *forward prices* in the framework of a monetary economy (Deleplace, 2014, pp. 154-155).

The modern Arrow-Debreu version of general equilibrium assumes the existence of a complete system of contingent forward markets (Arrow and Debreu, 1954; Debreu, 1959). In the Arrow-Debreu model, the market opens on only one date in which all transactions are contracted, regardless of the dates on which the commodities will be delivered. This economy can be conceived as a "futures economy" in the sense of Hicks (1939) (Chapter X, Section 5, pp. 139-140): all conceivable contingent future contracts can be agreed upon by agents on the date in which trading is possible, so that mutual consistency between the expectations and the plans of economic agents is always ensured.

This feature has been recognized by some theorists in the neoclassical tradition. In such an economy, there is no need for money, as it was noted by Hahn (1981):

> "*The most serious challenge that the existence of money poses to the theorist is this: the best developed model of the economy cannot find room for it. The best developed model is, of course, the Arrow-Debreu version of a Walrasian general equilibrium.* ***A world in which all conceivable contingent future contracts are possible neither needs nor wants intrinsically worthless money****.*" (Hahn, 1981, p. 1; emphasis added).

The SP reveal that Sraffa is already aware of this problem when writing his 1932 article, as the following passage shows:

> "***If money did not exist, all effects would be identical as if there were perfect forward markets for all commodities****. In this case, money would not be stand.[ard] of d.[eferred] p.[ayments], because everybody would hedge.*" (SP, D3/9/44, emphasis added)

Hence, Sraffa notes that the existence of money as a standard of deferred payments is associated with the absence of complete markets for all commodities

in all contingent states of the world. A monetary standard of deferred payments comes into play only if it is impossible, or too costly, for agents to hedge against all possible variations in prices of traded goods.

The SP also reveal that the example of the cotton spinner used in Sraffa (1932a) is a case of hedging. In his notes, Sraffa discusses his example and wonders whether a monetary loan market, plus a spot and forward market in commodities, is an adequate representation of a 'loan market' in commodities (SP, D3/9/34.r).

Although Sraffa seems initially sceptical of his own example, he adds the following passage[18]:

> "*No. It's adequate. But it means that those conditions enable anyone to borrow in either of the two ways, money or commodity.* ***If a spinner wants to borrow cotton, it may pay him, or not, to hedge*** *- all depends on his expectations of future price: but on this expectation depends also whether it pays him to borrow at all, in order to produce yarn.*" (SP, D3/9/34.r; emphasis added).

This passage shows that the three conditions assumed by Sraffa in the cotton spinner example are also useful to single out the importance of money outside of a complete market environment *à la* Arrow-Debreu.

A monetary standard of deferred payments allows agents to perform borrowing operations to carry out their production or consumption plans over time when it is impossible to hedge against all future states of the world. This role played by the monetary standard is incompatible with a notion of neutrality of money, as different monetary polices will have an impact on agents' ability to borrow.

On top of that, another typical feature of complete markets is simultaneous trading among all agents; only if markets open at each date in time - as in the *sequence economies* defined by Radner (1968) - then agents may hold money balances that have a positive value in equilibrium at each date.

Sraffa is also aware of this problem, as the following passage from the SP shows:

> "*The use of money as a 'medium of exchange' cannot go without its being 'a standard of deferred payments' or a 'store of value',*

[18] It is interesting to compare Sraffa's notes to the following passage in Marshall's *Industry and Trade* quoted in Naldi (2015): "*A British miller [...] [h]aving ordered the purchase of a certain quantity of what he needs [...] "hedges", by selling at once in a central market an equal quantity of standard wheat for delivery at about the time at which he expects that the wheat, which he has just bought, will be in his elevator ready to be made quickly into flour. If wheat falls in the interval, his flour has to compete with that made from cheaper wheat; but, what he loses through that fall, is returned to him almost exactly by his gain on the "future" which he has sold. Conversely, if wheat rises in the interval, he has to pay on the sale of his "future" about as much as he gains from the corresponding upward movement of his flour. By buying a future he does not speculate; he throws on the shoulders of the general market the risks and the chances of the gain that would otherwise have come to him through general movements external to his own business.*" (Naldi, 2015, p. 19).

> *two attributes which are included in the above:* ***this is obvious if money transactions succeed each other in time; and if they are simultaneous, they must be cleared against each other and no money is required****.*" (SP, D3/9/104, emphasis added)

The problem of sequential versus simultaneous trading is also related to the issue of expectations. When markets open at each date, agents must form expectations about market conditions at future dates. In a sequential trading environment, expectations come into play not only in so far as they reflect beliefs regarding states of nature (as in Arrow-Debreu), but also as they must encompass market expectations about prices in each state of nature. In the words of (Hahn, 1981, p. 3), "*no monetary theory without sequences, and no sequences without expectations*".

To study the behaviour of such a sequential trading economy over time, a theory of expectations formation is required. In this context, proving that money should have a positive value in equilibrium can be tricky, as highlighted by the '*Hahn Problem*' (Hahn, 1965).

According to Hahn, if we consider an economy as being of finite duration, rational economic agents will not save intrinsically worthless money in the last period. Moreover, under rational expectations, agents anticipate that the equilibrium price of money in the last period will be zero and thus refuse to be paid in money at the time date before the last period.

The argument can be infinitely repeated backwards so that, in a general equilibrium framework with rational expectations, monetary markets should never activate. This conclusion poses serious problems for the analysis of monetary problems in a general equilibrium setting (Hahn, 1981, p. 5).

Although Sraffa cannot have any notion of the theory of rational expectations, which has been first put forward by Muth (1961), the SP show that he is aware of the difficulties in explaining the phenomenon of the positive value of money in a general equilibrium setting several decades before a formal analysis of this problem. Furthermore, the SP show that Sraffa is also able to anticipate some of the solutions developed by general equilibrium theorists. Clearly, Sraffa was fully aware that disequilibrium and monetary exchanges are the two sides of the same coin where one cannot exist without the other.

In a 1928 notebook of preparatory notes for *PCMC*, where Sraffa tests some mathematical examples of relations between industries in a no-surplus economy, he dedicates a page to "*Hoarding of money in no-surplus*". As noted by Deleplace (2014) (p. 156), the following passage bears a striking similarity to the 'Hahn Problem':

> "*In the case of no-surplus, instead of barter, we might assume monetary exchange. The money would be used during market day, but what would happen to it during the rest of the year? It would be necessary to* ***assume that the farmers want to hoard****: this would be used as an insurance fund against crop failures for the individuals, but of course not for society.* ***If we did not assume hoarding,***

> ***the money would dwindle in value during market day, since every one would be trying to get rid of it before the end of the day***. *The same result might be reached through a* ***clearing home****, for book credits between buyers and sellers: of course everybody ought not to leave the markets with any credits or debts.*" (SP, D3/12/10/27, emphasis added)[19]

Here Sraffa thinks about what would happen in a self-replacing economy in the sense of *PCMC* - where money is absent - if exchanges were to be made in monetary terms. Sraffa's thought experiment seems to entail one period only (the market day), but it is similar to an intertemporal finite horizon environment.

Just as Hahn (1965) concludes through backward induction that rational agents would not use money at any date, given the zero price for money in the last period, Sraffa wonders about what would happen to money during the rest of the year, after it is used during the market day.

Sraffa's wording suggests that, in such an environment, it would be tricky to assign a positive value to money at the end of the market day, because agents will try to get rid of it as the end of the finite horizon approaches ("*the money would dwindle in value during market day, since every one would be trying to get rid of it before the end of the day*").

Hence, to allow monetary exchanges to happen also during the market day, we must assume the willingness of agents to hoard money at the end of the period. In such a dense passage, Sraffa provides several possible reasons for this assumption. First, agents may be willing to save money ("*hoard*") as an insurance fund against changes in individual incomes ("*crop failures*"): this is the precautionary savings driver for money demand as described by Keynes (1936), which is determined by uncertainty about the future.

Moreover, as noted by Deleplace (2014), Sraffa's passage hints at two of the most common options offered by modern general equilibrium theorists for the realization of monetary exchange. Assuming that agents attach a positive value to money balances is the route followed by the *money in the utility function* approach pioneered by Sidrauski (1967). In this case, savings in equilibrium are justified by the presence of a positive shadow price for money balances in the utility function of traders.

On the other hand, the 'clearing home' for traders which requires each trader's account to be balanced at each date is equivalent to the introduction of a liquidity constraint of the 'cash-in-advance' type, as defined by Clower (1967).

[19] Venkatachalam and Zambelli (2022) (p. 454) introduce the notion of barter exchange equivalents and credit exchange equivalents in a context of Sraffa self-replacing economy. The monetary or credit exchanges are necessary because the prices are not relative to an equilibrium condition where supply and demand of commodities take place at market-clearing prices. Hoarding is a necessity for self-replacing: producers sell commodities to other producers and workers - that do not have sufficient purchasing power - in exchange of the promise (IOUs) that the borrowers will pay back at a future time. This is the condition of disequilibrium as defined by Hayek (1931). In this case money is a store of value which is a necessity for the exchanges to take place.

In this class of models, all agents must have enough funds to pay for their consumption at each point in time, so that no debits or credits can be left unsettled at the end of each date. No savings (and no unsettled debits) will arise at the end of the period, but all agents demand money during the market day and have to use it in exchanges due to the liquidity constraint.

These insights, however, do not imply that Sraffa regards the treatment of money as problematic in his framework. As Sraffa (1932a) and the materials in the SP show, Sraffa does not rely on the 'subjective method' followed by Hayek and the Walrasian general equilibrium tradition.

In Sraffa's view, the existence of money and its non-neutrality are not determined by any endogenous market process, but depend on the institutional arrangements that generate the 'monetary constants'. Several points of critique of the 'subjective method' are present in the SP, such as the following:

> "*Dr H. moves from the 'decisions of individuals' (p. 4) as to how much to save. This is natural enough for one who boasts of taking a 'subjective' point of view, and claims to extend the application of this method to monetary economics. Although the present reviewer strongly disagrees with this view, the reader will be spared the interminable methodological disquisition that would be appropriate to the occasion*". (SP, D3/9/22)

2.6 Potential and limitations of the 'commodity rates of interest'

The 'commodity rate of interest' developed by Sraffa (1932a) has been the focus of many discussions in the economic literature.

In the *General Theory*, Keynes (1936) adopts Sraffa's 'commodity rate of interest' in Chapter 17, named *The Essential Properties of Interest and Money*. This chapter aims at generalizing the theory of investment to all real and financial assets traded in the economy.

In Chapter 17, Keynes's aim appears to show that involuntary unemployment is a structural feature of a monetary economy. The rate of interest on money, according to Keynes, plays "*a peculiar part in setting a limit to the level of employment, since it sets a standard to which the marginal efficiency of a capital-asset must attain if it is to be newly produced*" (Keynes, 1936, p. 222).

The basic idea of the Chapter is that entrepreneurs can choose to invest in a set of real and financial assets ('capital-assets' in Keynes's terminology), which includes money. In their search for profit, investors seek the capital assets with the highest expected return ('marginal efficiency' in Keynes's terminology).

The expected return on holding money is the monetary interest rate. To compare the latter with the expected return on other capital assets, Keynes introduces the concept of 'commodity rate of interest', with an explicit reference to Sraffa (1932a) in a footnote (p. 223). Keynes's definition of the 'commodity

rate of interest' is the following[20]:

> "*Let us suppose that the spot price of wheat is £100 per 100 quarters, that the price of the 'future' contract for wheat for delivery a year hence is £107 per 100 quarters, and that the money-rate of interest is 5 per cent; what is the wheat-rate of interest? £100 spot will buy £105 for forward delivery, and £105 for forward delivery will buy £100 (= 98) quarters for forward delivery. Alternatively £100 spot will buy 100 quarters of wheat for spot delivery. Thus 100 quarters of wheat for spot delivery will buy 98 quarters for forward delivery. It follows that the wheat-rate of interest is* minus *2 per cent per annum*". (Keynes, 1936, p. 223)

Keynes's wording in the above passage matches the definition of a commodity rate of interest in Sraffa (1932a). Having defined the commodity rates of interest ('own-rates of interest' in Keynes's terminology) for each commodity, Keynes argues that, in general, they will be different for different commodities.

Furthermore, Keynes states that "*it is the* greatest *of the own-rates of interest which rules the roost (because it is the greatest of these rates that the marginal efficiency of a capital-asset must attain if it is to be newly produced)*" (Keynes, 1936, p. 223).

According to Keynes, the greatest of the own rates of interest is the monetary interest rate. To prove this argument, however, Keynes departs from Sraffa's definition of the commodity rate of interest and introduces a new definition (Keynes, 1936, pp. 225–226), where the commodity rate of interest (r_i) for each asset i is equal to the yield or output of the asset - as the ability to assist some process of production or to supply services - (q_i), minus the carrying cost (c_i), plus the liquidity premium (l_i):

$$r_i \equiv q_i - c_i + l_i \tag{2.5}$$

According to Keynes, the liquidity premium l_i is the amount of asset i (defined in terms of itself) which agents are "*willing to pay for the potential convenience or security given by this power of disposal*" (Keynes, 1936, p. 226). Hence, the liquidity premium is an intrinsic feature of some assets due to their convenience as a store of value.

At this point, Keynes discusses the example of an economy where three assets are available to investors: wheat, houses and money. Based on definition (2.5) and assuming that houses do not have any carrying cost nor any liquidity premium, the commodity rate of interest of houses r_h can be assumed to be equal to q_h. Assuming that the yield and the liquidity premium of wheat are negligible, the commodity rate of interest of wheat is equal to $-c_w$; furthermore,

[20] Although Keynes pays tribute to Sraffa (1932a) in his example of the commodity rate of interest, he chooses wheat as the reference good; a reference to wheat is also present in (Sraffa, 1932a, p. 49). As noted by Donzelli (2006) (p. 21, footnote 19), this may be a clue of the influence of Fisher (1896) on both authors.

money can be assumed to have no yield nor any carrying cost and a positive liquidity premium l_m.

The three commodity rates of interest are measured in terms of each different commodity. Hence, a measure of change in the relative values of commodities is required in order to determine the equilibrium relationships between the commodity rates of interest 'during the year'.

Taking money as the standard of measurement, Keynes obtains monetary rates of interest for houses and wheat by adding an expected percentage appreciation a_j (with $j = \{h, w, m\}$) of houses and wheat with respect to money (the expected appreciation of money in terms of itself a_m will be equal to zero) to the definition in (2.5).

In equilibrium, investors will have to be indifferent between the own rates of houses, wheat and money - i.e., the equilibrium own rate of interest in terms of money will have to be equal. Hence, the following relationship holds in equilibrium:

$$i_j = a_h + q_h = a_w - c_w = l_m \quad \text{with } j = \{h, w, m\} \tag{2.6}$$

Based on (2.6), Keynes concludes that it is the monetary interest rate which 'rules the roost'.

In a Marshallian fashion, Keynes assumes that the commodity rate of interest for each commodity is a decreasing function of its output and that the commodity rate of interest on money is fixed (or declines more slowly than any other).

Therefore, the adjustment mechanism conjectured by Keynes is such that increases in the output of houses and wheat determine a reduction in the commodity rates of interest of houses and wheat until an equilibrium position is reached, in which all commodity rates of interest are equal to the monetary interest rate.

When equilibrium is reached, entrepreneurs stop investing in other commodities and switch to saving money, because the monetary interest rate becomes insensitive - due to the '*liquidity-motive*' - to further changes in the proportion between the quantity of money and other forms of wealth measured in money terms.

Furthermore, money is assumed to have zero or negligible elasticity of production from the point of view of entrepreneurs[21] (i.e., labour cannot be turned to producing more money) and zero or negligible elasticity of substitution with other factors.

Thus, if there are some factors of production that are not fully employed when the monetary interest rate equalizes other commodity rates of interest defined in terms of money, there is no market mechanism which can clear all markets for the factors of production.

[21] Keynes is aware that the total stock of money available to entrepreneurs can be increased by the monetary authority or via a reduction in the wage rate, but decides to assume that the money supply is fixed as a first approximation (Keynes, 1936, p. 230).

Hence, according to Keynes, the 'reluctance' of the monetary interest rate to fall in response to increases in the demand for money is the 'essential' feature of money recalled by the title of the Chapter. This property - which pertains *exclusively* to money - leads to the existence of an equilibrium position without full employment in a monetary economy.

The liquidity-motive is thus a 'psychological' friction that impedes the smooth functioning of the market mechanisms in the case of money; on the contrary, full employment equilibria would be the standard case in a non-monetary economy. The following passage, in which Keynes mentions the 'natural forces' of the market, is consistent with this interpretation:

> "*Thus with other commodities left to themselves,* ***'natural forces', i.e. the ordinary forces of the market, would tend to bring their rate of interest down until the emergence of full employment had brought about for commodities generally the inelasticity of supply which we have postulated as a normal characteristic of money****. Thus in the absence of money and in the absence — we must, of course, also suppose — of any other commodity with the assumed characteristics of money, the rates of interest would only reach equilibrium when there is full employment. Unemployment develops, that is to say, because people want the moon; — men cannot be employed when the object of desire (i.e. money) is something which cannot be produced and the demand for which cannot be readily choked off. There is no remedy but to persuade the public that green cheese is practically the same thing and to have a green cheese factory (i.e. a central bank) under public control.*" (Keynes, 1936, p. 237; emphasis added)

Keynes's analysis in the Chapter, as summarised by the above passage, implies the need for an expansionary monetary policy in order to overcome the friction caused by the liquidity premium.

As we have seen, Sraffa's influence on Chapter 17 of the General Theory is explicitly acknowledged by Keynes. However, Sraffa's comments on the Chapter show several points of disagreement between the two authors. These divergences have been widely discussed in the literature (Kurz, 2000, 2014; Grieve, 2015; Naldi, 2015; Donzelli, 2006).

In his copy of the General Theory, included in the SP, Sraffa discusses Keynes's use of the commodity rate of interest. According to Sraffa, Keynes's definition of the commodity rate of interest actually corresponds to the *marginal efficiency* of an asset[22]:

> "***What K.[eynes] ought to have spoken of throughout (e.g. [page] 229 [of General Theory] top) is marg.[inal] efficiencies of various articles, and not their rates of interest.***

[22]Keynes himself defines the 'own-rate of interest' as the "*total return expected from the ownership of an asset over a period*" (Keynes, 1936, p. 226). The confusion between own-rates of interest and marginal efficiencies is also recognized by Lerner (1952) (p. 181).

Then, if there is one article the marg. eff. of which never falls below say 5% (this being the valuation of the pleasure people derived from hoarding any quantity of it) the production of all other durable assets will stop when their stocks are such that marg. eff. has come down to that level - for otherwise they could not be sold at cost - and all resources saved will be used for producing the hoardable asset. If this asset cannot be produced (paper money), its demand will increase and can only be met by a continuous rise in its value, i.e. fall in general prices. If this hoarding is expected to go on steadily, and all prices are expected to fall in terms of money, the result will be that all own rates of interest of commodities will be higher than the money rate (this is Fisher's case: and the expected appreciation or depr.[eciation] is the only possible cause of divergence in rates of interest). Thus in the K. case, the result on rates of int.[erest] is opposite to K's conclusion."[23] (SP, I 100/11)

Therefore, Keynes borrows the concept of the own rate of interest by Sraffa (1932a) and Fisher (1896) but turns it into something else - i.e., the definition of marginal efficiency of any asset. This is also the reason why Keynes's own rate of interest only applies to durable commodities, whereas Sraffa's definition can be applied to any commodity for which a forward market exists, as noted by Deleplace (2014).

If the consistency between definitions is restored, Keynes's argument could be restated in terms of the same equations implied in Sraffa (1932a) and summarised in (3.2). The 'expected appreciation' of any asset j can be related to the ratio between the spot price and the forward price of j, as follows:

$$a_j = \frac{p_j^S}{p_j^F} - 1 \tag{2.7}$$

Having re-defined a_j in terms of spot and forward prices, substituting (3.7) in (3.2) and manipulating, we have:

$$(1 + i_j) \equiv (1 + i_m)(1 + a_j) \tag{2.8}$$

The above definition is consistent with the notation in Fisher (1896), which is explicitly mentioned by Sraffa in his comment. A shift in the distribution of demand away from any commodity j, and towards money, will cause spot prices of all commodities j to be greater than their forward prices. This means that $a_j > 0$ for all commodities j, so that the commodity rates of all commodities j will be greater than the monetary interest rate i_m.

Hence, the case discussed by Keynes in Chapter 17 should lead to a general deflation of prices - i.e., an increase in the value of money in terms of other commodities - and, as a consequence, to a *fall* of the monetary interest rate

[23] Although here Sraffa seems to be following Keynes's line of reasoning, the quote does not express his way of thinking about prices. Sraffa (1960) (see, in particular, Ch. 6) shows that relations between the value of capital and prices cannot be monotonous.

below the commodities rate of interest of any commodity. This case is also mentioned by Sraffa in the SP, in his preparatory notes to Sraffa (1932a):

> "*Now in our [Hayek's] case it is expected that, in the final equilibrium, the prices of all commodities will be lower in terms of money, though in various degrees, than they can be at any moment during the period of transition; consequently, during that period, all forward prices of commodities will be lower than their spot prices, in other words the rate on money loans, in the market, must be lower than the natural rate on any other commodity*". (SP, D3/9/165)

The interpretation of own rates of interest as marginal efficiencies of assets is confirmed by the fact that Keynes redefines 'own-rates of interest' as 'marginal efficiencies' in his subsequent works on the topic. For instance, in his 1937 paper called *The Theory of the Rate of Interest*, Keynes writes: "*Thus we can conveniently say that interest on money measures the marginal efficiency of money measured in terms of itself as a unit*" (Keynes, 1937).

On top of the inconsistency in the definitions, which Keynes seems to have overcome after the publication of the *General Theory*, Sraffa's comments reveal further divergences in the monetary approach of the two authors. In his comments on the *General Theory* included in the SP, Sraffa takes issue with the liquidity preference theory, which is another crucial point of Keynes's monetary theory.

On the demand side of the money market, according to Keynes, agents' preferences toward money generate a '*liquidity preference curve*', where a higher (lower) demand for money is related to a lower (higher) level of the monetary rate of interest.

On the supply side, the monetary authority (i.e., the central bank) exogenously controls the total money supply in the economy. As a result, the monetary authority can steer the monetary interest rates through variations in the quantity of money in the economy, given a liquidity preference curve[24].

Sraffa takes issue with both sides of the money market as depicted by Keynes (1936). As for the demand side, Sraffa criticizes the notion of a well-behaved liquidity preference curve as a decreasing function of the interest rate:

> "*I am convinced that is not the case in general - L* [liquidity] *an advantage to some people – positive disadvantage to others; therefore impossible to say that in general there is a definite relationship between the quantity of money and r* [the rate of interest]." (SP, I 100)

Hence, agents may have a different preference for liquidity which cannot be aggregated in a monotonous demand curve. Sraffa considers the example of a bank whose profitability depends on loans performed: to prop up its earnings, the bank has to become less liquid, even when the interest rate is low.

[24] This is also the standard representation of the money market in an IS-LM framework.

Keynes's definition of the liquidity preference curve resembles the neoclassical partial equilibrium approach, which is criticized by Sraffa (1960). According to Donzelli (2006) (pp. 61-63), the partial equilibrium approach is also at the core of Keynes's argument in the determination of the equilibrium positions of 'own rates of interest'/marginal efficiencies, because Keynes does not have any theory of the general price system to explain the behaviour of expected appreciation of commodities with respect to money.

Sraffa also takes issue with the supply side of Keynes's money market and anticipates the notion of 'endogenous money' as defined in the Post-Keynesian tradition, as opposed to the exogenous money supply assumed in Keynes (1936):

> "*if the LP curve does not exist - still the fact remains - abundant cash goes along with low r.* [...] [A] *low rate is cause of abundant money - not vv* [vice versa]" (SP, I 100)

Therefore, according to Sraffa, the monetary authority does not control the quantity of money in the economy. The monetary authority can set (or influence) the monetary interest rate, and the quantity of money accommodates to the demand for borrowed money by entrepreneurs and consumers.

This description of the money market closely resembles the 'horizontalist' view of the money supply (Moore, 1988), which is regarded to be a faithful representation of modern monetary policy operations by modern central bank researchers (Bindseil, 2014; McLeay et al., 2014).

2.7 The integration of money in Sraffian schemes

After the debate with Hayek in the 1930s, Sraffa does not pursue the goal of transforming his purely critical insights on monetary theory into a *pars construens* of his theory of value.

In the post-war decades, Sraffa is mostly dedicated to completing the collection of David Ricardo's works and to elaborating a critique of the prevailing theories of value and distribution, which is the rationale behind the publication of *PCMC*.

As a consequence, Sraffa does not further explore the potential implications of the commodity rate of interest for integrating money into the theory of value and distribution.

One of the most important insights behind Sraffa's critique of Hayek is the concept of 'commodity rate of interest'. This concept has influenced Keynes's *General Theory*, which in turn has sparked a lively discussion among scholars on the significance and the potential to provide a different theory of investment with respect to the marginalist approach (Lerner, 1952; Kaldor, 1960; Barens and Caspari, 1997; Donzelli, 2004, 2006).

According to Donzelli (2004), Sraffa might have not continued his work on the 'commodity rate of interest' because the latter does not enjoy a sufficient degree of conceptual autonomy within his theoretical apparatus. While discussing the transition from a stationary equilibrium to another, Sraffa moves within

Hayek's theoretical framework to relate changes in the output of commodities with changes in their spot prices.

However, as shown by Sraffa's previous criticism of Marshall (Sraffa, 1925), changes in prices in response to variations in output can only be computed by making assumptions on the nature of returns to scale: in particular, the 'well-behaved' response of prices to changes in output is only feasible if constant returns to scale are assumed.

In the preface of *PCMC*, Sraffa writes that such an assumption can only be considered as a "*temporary working hypothesis*" (Sraffa, 1960, p. v) and explicitly states that no such assumption is made throughout the book, as no changes in the scale of production or in the proportion of means of production used by each industry are considered.

This suggests that Sraffa had not engaged in disequilibrium analysis at the time of publishing of *PCMC* and, thus, might have decided not to deepen the analysis of commodity rates of interest and economic systems out of equilibrium.

Moreover, Donzelli (2004) also argues that the 'commodity rate of interest' cannot be of interest to Sraffa in the context of a non-monetary economy such as the one in *PCMC*. The cotton spinner example in Sraffa (1932a), while serving the purpose of showing that no monetary policy can reproduce the results of a non-monetary economy, cannot be extended as it is to the case of a non-monetary economy.

After discussing his example involving a monetary loan, a spot market and a forward market for commodities, Sraffa writes that it "*is only one step to pass from this to the case of a non-money economy*" (Sraffa, 1932a) and to see that when equilibrium is disturbed, 'natural' interest rates will diverge with changes in the output of the goods in terms of which loans are defined.

As Sraffa does not formally discuss this case, Donzelli (2004) reconstructs a possible formalization of the problem. The 'commodity rate of interest' in a monetary economy can only be defined *indirectly* via three prices: the interest rate on the money market, the spot price of cotton and the forward price of cotton. Absent a money market for loans, a different contract has to be imagined to define the commodity rate of interest in a non-monetary environment.

According to Donzelli, this case requires a *commodity future* contract, where the future delivery of the commodity and the payment of the future price are not simultaneous; on the contrary, the 'futures' price is paid at the present date and the delivery of the commodity is at the future date. The combination of the two operations is equivalent to a loan of cotton 'in kind' [25] (Donzelli, 2004, p. 20).

By introducing loans 'in kind', the definition of the commodity rate of interest in a non-monetary economy can be constructed with a direct procedure involving only two prices - the spot price of the commodity and the 'commodity futures' price. In such an environment - as Sraffa noted in the SP - there

[25] Interestingly, before introducing the cotton spinner example Sraffa alludes to this possibility without exploring it: "*In order to realise this we need not stretch our imagination and think of an organised loan market amongst savages bartering deer for beavers*" (Sraffa, 1932a).

would be no single interest rate, but many interest rates on commodities defined in terms of themselves that could be brought into equalization only by the movement of expected relative prices.

The absence of a notion of the commodity rate of interest adapted to the case of a non-monetary economy might explain why Sraffa does not mention such an interest rate even in *PCMC* (Donzelli, 2004, pp. 23-24). On top of that, the definition of commodity rate of interest in Sraffa (1932a) is based on a stationary equilibrium environment: it is implicitly assumed that forward prices perfectly anticipate the expected spot prices, which in turn coincide with the stationary equilibrium prices. Different theories of expectations and different notions of equilibrium would require a more complex analysis of adjustment mechanisms, an issue which is not treated by Sraffa (Lachmann, 1986; Donzelli, 2004).

In any case, there is no direct evidence of any clear cause of Sraffa's departure from monetary issues. One possible hint for an explanation is that Sraffa viewed himself as a 'non-monetary' economist specialized in the field of the theory of value and distribution ('pure theory'). While drafting his review of Hayek (1931), Sraffa wrote:

> "*A long review of a short book requires some apology: the more so that the book is a monetary one, and* ***the reviewer is a non-monetary economist***." (SP, D3/9/181, emphasis added)

The above passage, together with Sraffa's opinion on monetary studies being at a more advanced stage than pure theory, suggests that the author felt that more attention should be devoted to developing a sound theory of value and distribution. However, Venkatachalam and Zambelli (2021) (p. 465) argue that Sraffa's *oeuvre* should be interpreted as a *coherent whole*, aimed at pursuing the maximum possible precision in theory, regardless of the different topics covered in different decades.

A trace of such coherence can be found in *PCMC* itself. While discussing the properties of the rate of profits, Sraffa writes in a well-known passage:

> "*The rate of profits* [...] *is accordingly susceptible of being determined from outside the system of production,* ***in particular by the level of the money rates of interest***." (Sraffa, 1960, p. 33)

Hence, even though money is inessential in the models of *PCMC*, Sraffa retains the notion that the monetary interest rate - and thus monetary policy - cannot be neutral with respect to economic outcomes.

As shown in Venkatachalam and Zambelli (2021), Sraffa clarifies in correspondence with Pierangelo Garegnani that the above passage should not be interpreted as advocating a mechanical theory of the determination of the rate of profit independent of distribution, but rather as suggesting that the equilibrium position of the economy can be influenced by monetary policies.

Sraffa's suggestion, alongside the development of the Keynesian and Post-Keynesian tradition after the publication of the General Theory, has led many

scholars to investigate possible ways of integrating money into the Sraffian schemes of *PCMC*.

One strand of literature has focused on the introduction of financial instruments within Sraffian schemes in order to explain the role of money in the determination of distribution. In Pivetti (1991), entrepreneurs can invest their endowments in one of the available productive processes or in 'gilt-edged securities' (Pivetti, 1991, p. 23) which yield a risk-free rate of return that is arbitrarily set by the Central Bank.

In this framework, ignoring idiosyncratic risk, arbitrage conditions will make sure that the 'normal' rate of profit across industries will be equal to the risk-free rate of return in equilibrium. As the structure of interest rates is not explicitly defined, the rate of return on financial or productive assets will also be equal to the interest rate charged by the banking system to firms.

Hence, Pivetti (1991) is able to devise a 'monetary theory of distribution', where the interest rate set by the monetary authority also determines the profit rate and, as a consequence, the wage rate. Hence, the monetary interest rate is conceived as an *opportunity cost* which is able to influence the level of investment in productive sectors. In this approach, the financial sector does not earn any positive profit in equilibrium, thereby not appearing in the equations of Sraffian schemes as an independent sector.

Another strand of literature models the interest rate as an *effective cost of production*, by explicitly introducing the banking system as a sector with a price equation depending on sectoral revenues and expenditures (Panico, 1985, 1988).

In this framework, the banking system's revenues are equal to the loans extended to firms multiplied by the interest rate charged on loans; the banking system's expenditures are labour costs, the set of 'capital goods' required for delivering financial services and the stock of deposits multiplied by the interest rate on bank deposits. Consequently, firms' price equations also take into account the interest rate paid on loans in the expenditure side.

Moreover, it is assumed that the monetary authority issues short-term bonds which yield a risk-free rate of return, and each asset has an 'illiquidity premium' which can be influenced by monetary policy (Panico, 1985, p. 55). It is assumed that bank deposits have a very high degree of liquidity so that the illiquidity premium on deposits can be assumed to be equal to zero.

The model is closed by imposing three arbitrage conditions. First, net returns on financial assets (bank deposits and short-term bonds minus the illiquidity premium) should be equalised; secondly, returns on productive assets (the profit rate minus the illiquidity premium) should be equalised to returns on financial assets (bank deposits); finally, the interest rate on loans must be equal to the yield on short-term bonds.

Through the manipulation of the illiquidity premia, the monetary authority can influence the structure of interest rates and have an impact on the equilibrium distribution. An increase of the illiquidity premia also drives the yield on short-term bonds upwards; as the yield on short-term bonds equals the interest rate on loans to firms, production costs rise. Moreover, due to the second arbitrage condition, the profit rate rises as well, thereby shifting distribution in

favor of entrepreneurs.

In these two classes of models, the structure of the financial system is not formalised and the determination of leverage ratios of firms - measured by the stock of loans entering firms' price equations - is not endogenously determined.

An attempt to formalize the cost structure of the banking system is provided by Ciccarone (1998), through the introduction of the balance-sheet of the banking system and the definition of the profit rate of the financial industry via a price equation such as in Panico (1988).

Having determined the profit rate of the financial industry, and due to the competition among entrepreneurs, the latter behaves as a point of reference which all other profit rates of industrial sectors adapt to. As noted by Dvoskin and Feldman (2021), this is equivalent to considering the banking industry as the only 'basic' sector in the sense of Sraffa (1932a), because the equilibrium profit rate depends exclusively on the technical conditions of the financial sector.

Franke (1988), instead, attempts to provide an endogenous determination of the leverage ratios of firms. Franke implements the notion of borrower's and lender's risk developed by Kalecki (1937), so that the leverage ratio is a function of the loan interest rate charged by banks; however, as the interest rate is not an opportunity cost between productive and financial investments, the distribution of the surplus between capital owners and workers cannot be explained by financial factors.

In order to provide a closure for the model, the real wage rate has to be fixed; the leverage ratio and the profit rate are then determined, given an exogenously fixed interest rate by the central bank. Hence, in this model, financial factors are able to regulate the share of the surplus between industrial and financial sectors, net of wages, but they cannot have an influence on relative prices.

Building on the notions of the interest rate as an opportunity cost and as an effective cost of production for the industrial sectors, Dvoskin and Feldman (2021) provide a model of the banking sector as the source of finance for the promotion of innovation. Here, the role of the banking industry is twofold: on the one hand, banks create the conditions required for the profitability of new productive techniques, via changes in income distribution; on the other hand, the financial means provided by banks enable entrepreneurs to implement the innovative techniques even when they are not endowed with 'capital'.

These classes of models aim at capturing different functions performed by the banking sector in an economic system and relate these features to a 'real' economy described in the terms of Sraffa (1960). However, these models do not manage to effectively generalize Sraffian schemes by introducing money in such a way as to address the crucial theoretical issues raised by Sraffa in the 1932 controversy with Hayek - i.e., the function of money as the standard of deferred payments and its pivotal role in disequilibrium analysis of prices and distribution.

A more recent strand of literature, however, is more directly related with the fundamental theoretical issues associated with the integration of money in Sraffa's theory of value and distribution (Venkatachalam and Zambelli, 2021, 2022).

According to Venkatachalam and Zambelli (2021), the relevant notion of equilibrium for Sraffian schemes is the self-replacing condition as defined in Sraffa (1960). Self-replacing prices are those prices that allow to restore the original distribution of the means of production and the surplus. In this condition, markets for commodities and labour are cleared and revenues of agents exactly match their expenditures[26].

The self-replacing condition is such that the prevailing distribution - which is the result of extra-economic forces related to the interaction of different social classes - is associated with market-clearing prices; hence, all exchanges can happen either through barter or means of payment issued by a clearing house, which requires agents to balance their books at the end of the market day.

However, the rationale for money as a standard of deferred payments arises if the prevailing distribution of the surplus between workers and entrepreneurs produces a price vector which does not allow self-replacing. In these cases, the system would not be able to reproduce itself unless agents are allowed to perform exchanges through deferred means of payments - i.e., *I Owe You*-s (IOUs) which can be settled at later periods in time (Venkatachalam and Zambelli, 2021, p. 475).

This framework allows to consider a rich set of cases in which the production process, at current prices, would not distribute the necessary purchasing power to replicate the production undertaken in the previous period. By introducing money and credit, instead, some industries in 'deficit' condition can purchase the necessary means of production by agreeing to a deferred payment later in time; at the same time, industries in a 'surplus' are able to sell their product by agreeing to receive a deferred payment. Given n industries, a matrix of material inputs $\mathbf{A}$, a vector of labour inputs $\mathbf{L}$, and a matrix of outputs $\mathbf{B}$, such a condition can be written in a compact matrix notation as follows (Venkatachalam and Zambelli, 2021, p. 485):

$$\mathbf{A}\overline{\mathbf{p}} + \mathbf{L}\overline{w} \leq \mathbf{B}\overline{\mathbf{p}} \tag{2.9}$$

where $\overline{\mathbf{p}}$ is the vector of virtual (bookkeeping) prices and $\overline{w}$ is a virtual (bookkeeping) wage rate. The left hand side of (3.9) shows the virtual expenditures of industries, while the right hand side shows the virtual revenues. By definition, the difference between revenues and expenditures in each industry is equal to the sectoral profit level, and the ratio between this difference and the value of the means of production is equal to the sectoral profit rate. Denoting as $\overline{\mathbf{R}}$ the diagonal matrix whose diagonal elements are the profit rates in each industry, the following condition must hold at the end of the market day:

$$(\mathbf{I} + \overline{\mathbf{R}})\mathbf{A}\overline{\mathbf{p}} + \mathbf{L}\overline{w} = \mathbf{B}\overline{\mathbf{p}} \tag{2.10}$$

where $\mathbf{I}$ is an identity matrix. The equation in (3.10) shows an equilibrium condition in the sense of Sraffa (1960); however, profit rates are not assumed to

[26] The nature of Sraffian schemes as budget constraints is also illustrated in works belonging to the Walrasian general equilibrium tradition (Hahn, 1982; Mandler, 1999b), as discussed in depth in Chapter 3 of this thesis

be uniform and cannot be all positive due to the presence of credit and debt.

Venkatachalam and Zambelli (2022) develop the above model by extending the constructive algorithm in Zambelli (2018b) to case of debt and credit. The algorithm spans all possible distributional patterns and stores them in a set of distribution vectors $\mathbf{d}$, whose elements are shares of the surplus going to industries and workers (the surplus is assumed to be the numeraire of the system).

Once distribution vectors have been computed, a set of prices and wage rates that are uniquely associated with each distributional pattern can be computed. When these prices and wage rates are such that not all industries have enough purchasing power to reproduce themselves, debt and credit come into play and allow exchanges to take place not only in the form of barter, but also through the issuance of IOUs.

Hence, the matrices of material inputs, labour inputs and outputs can be divided in two 'barter' and 'credit' components; furthermore, a virtual central bank is introduced as a bookkeeping clearing-house where flows of debt and credit and the changes in stocks are recorded in each time period.

Given this framework, Venkatachalam and Zambelli (2022) can perform thought experiments such as the perturbation of a self-replacing position with uniform and positive rates of profit as in (Sraffa, 1960, p. 19).

The numerical examples provided in (Venkatachalam and Zambelli, 2022, pp. 25-28) show interesting similarities with the disequilibrium transition sketched in Sraffa (1932a).

As a matter of fact, a change in prices of commodities is associated with "*a change in the distribution of demand between various commodities*" (Sraffa, 1932a, p. 50) and a a state in which "*the supply and the demand for a commodity are not equilibrium (i.e. the market price exceeds or falls short of its cost of production)*" (Sraffa, 1932a, p. 50). This is exactly what happens in (2.9) when virtual prices are not the self-replacing prices.

To restore the self-replacing condition and extinguish debt and credit in following periods, Venkatachalam and Zambelli (2022) show that industries have to undertake either a change in their consumption patterns over time or new changes in prices after the first change in the distribution of demand. The introduction of a monetary interest rate on IOUs, administered by the virtual central bank, is susceptible of changing the trajectory of the economy over time, thereby making money inherently non-neutral.

The numerical example provided by the authors shows that the final self-replacing state needs not be associated with uniform rates of profit (Venkatachalam and Zambelli, 2022, p. 28)[27].

Moreover, Venkatachalam and Zambelli (2022) (equation 3.22, p. 458) also hint at the possibility of extending the framework to the case of permanently

[27]This conclusion differs from Sraffa (1932a), where disturbances of the equilibrium are conceived as temporary deviations from an initial position that is bound to be restored (equilibrium). For a general overview of self-replacing prices that are not bound to be related to uniform profit rates, see Zambelli (2018b)

non self-replacing states where the productive structure of the economy changes over time, allowing for the analysis of net investment and saving .

Hence, the emergence of debt and credit is shown to be instrumental in creating flexibility and structural viability in the system, enabling the study of the dynamics of the economy "*during the time of the transition*" (Sraffa, 1932a, p. 50).

As a result, this framework brings together a close understanding of the Sraffian approach to money and credit with the possibility of extending Sraffian schemes in *PCMC* to the study of structural change, disequilibrium analysis and the study of the 'traverse' in the sense of Hicks (1973) (Chapter I, Section 6).

2.8 Concluding remarks

The paper has attempted to provide a reconstruction of Piero Sraffa's monetary theory through an analysis of the author's published and unpublished works.

In the personal notes collected in the SP, Sraffa described himself as a 'non monetary' economist and suggested that the field of monetary economics was more advanced than the field of 'pure theory' (i.e., the theory of value and distribution). Hence, given the absence of money in Sraffa's *magnum opus* (Sraffa, 1960), it could be argued that monetary problems have not played a major role in the author's economic thinking.

However, a contrarian view is supported by textual evidence, both in published works (Sraffa, 1920, 1922a,b, 1932a,b, 1960) and the unpublished notes in the SP.

A thorough analysis of these works shows that Sraffa had a deep understanding of money, credit, the banking industry and financial markets and that it is possible to reconstruct his approach to monetary theory as a coherent whole (Venkatachalam and Zambelli, 2021, 2022). Moreover, as early as the 1930s, Sraffa was able to anticipate monetary problems that would have been discussed only several decades later in the economic literature.

According to Sraffa, the essentiality of money for exchange and production in a capitalist economy is the product of institutional contrivance aimed at regulating "*all kinds of relations between men*" (Sraffa, 1932a, p. 43). Hence, as it is widely recognized, money is not only a unit of account and a means of payment, but it is most importantly a store of value, which allows to transfer purchasing power over time.

This crucial property of money is conceived by Sraffa as a consequence of 'monetary constants': due to institutional forces, money is the standard in terms of which debts, contracts and legal obligations are fixed. Therefore, the presence of money allows the economic system to operate with a single term structure of (risk-free) interest rates which acts as a benchmark for the investment and consumption plans of economic agents; moreover, money enables agents to face uncertainty when complete markets are unavailable.

Sraffa's critique to the framework put forward by Hayek (1931) has important implications not only for the history of the monetary thought in the 20th century (through the influence on Keynes's *General Theory*), but also for modern debates on the nature of money and interest rates. As a matter of fact, Sraffa (1932a) shows that the notion of 'neutrality' of money is not conceivable in the context of an economy in which relative prices, output and distribution are changing.

Furthermore, no monetary policy can restore the equilibrium conditions that would prevail in a non-monetary economy by setting the monetary interest rate equal to the 'natural' interest rate - a concept pioneered by (Wicksell, 1898). As a matter of fact, Sraffa shows that a state of disequilibrium - where spot and forward prices of commodities diverge and supply and demand are not equalized - determines different 'natural' rates of interest for all goods in the economy.

Although a price-index related to a 'composite commodity' can be arbitrarily constructed, the 'natural' rate of interest in a monetary economy is still an unobservable and derived concept, which requires three prices to be computed: spot prices, forward prices and monetary interest rates. Hence, unless markets for loans 'in kind' are available in the economy, it is not possible to treat real interest rates as independent magnitudes that the monetary authority can target via the use of monetary interest rates.

The publication of Sraffa's review of *Prices and Production* is widely regarded to have dealt a serious blow to Hayek's theory of money and the trade cycle. In spite of this, the concepts of 'monetary neutrality' and of 'natural' interest rate as a guidance for monetary policy have had a long legacy in economic theory and central banking practice.

The notion of neutrality of money has been at the core of many famous macroeconomic models in the neoclassical and Real Business Cycle traditions (Friedman, 1956; Lucas, 1972; Barro, 1978), whereas the New Keynesian tradition has rejected such a notion in the short term but preserved it in the long term (Ball and Romer, 1990). The two viewpoints share a similar normative theoretical framework in which nominal magnitudes do not play any role in determining real economic outcomes in the long run; they differ in the formalization and the interpretation of 'frictions' to the equilibrium position.

Similarly, the notion of a 'natural' interest rate determined by 'real' factors is crucial for *New Consensus Macroeconomics* models such as Micheal Woodford's *Interest and Prices: Foundations of a Theory of Monetary Policy* (Woodford, 2003), whose title pays an explicit tribute to Wicksell (1898).

Moreover, the theory of 'secular stagnation' is based on the idea that the weak performance of economic growth in Western countries in the latest decades is due to a divergence between the monetary and the 'natural' interest rate (Gordon, 2012; Eggertsson et al., 2019; Rachel and Summers, 2019).

The use of this concept is also widespread in central banking theory and practice, where the search for a credible estimate of the natural rate is at the core of strategic discussion in the monetary policy steering committees of the major modern central banks (Holston et al., 2017; Lane, 2019).

A careful reappraisal of Sraffa's critique to Hayek's framework (Sraffa, 1932a)

would suggest a reconsideration of this approach as a guidance for monetary policy.

Sraffa's notes and unpublished works also show that the author's insights on the nature of money predate some innovative methodologies in monetary theory that have been put forward in the post-WWII period.

In his preparatory notes for Sraffa (1932a), the author anticipates that money is inessential in an economy where complete markets are available at each future date; this result is at the core of the Arrow-Debreu general equilibrium framework (Arrow and Debreu, 1954; Debreu, 1959).

Hence, the impossibility of agents to hedge against all possible states of the world is a necessary condition for agents to use a monetary standard of deferred payments: such a result is crucial for general equilibrium models with incomplete markets.

On top of that, Sraffa anticipates that sequential trading is the only possible modeling strategy that can encompass a monetary economy - a point which was later independently developed by the neoclassical tradition (Hahn, 1981). Sraffa also predates the discussion of the problem of the equilibrium value of money in a finite-horizon framework - a well-known statement of this problem is provided by Hahn (1965).

Finally, in his critical analysis of Keynes's Chapter 17 of the *General Theory*, Sraffa provides insights into the endogenous nature of the money supply with respect to economic activity: according to this approach, the monetary authority cannot target monetary aggregates through variations in the quantity of central bank reserves in the system (Moore, 1988).

Despite being absent in Keynes (1936), the endogenous money approach is a central tenet of the Post-Keynesian tradition[28] and it has been recently recognized as a faithful description of modern central bank practice (McLeay et al., 2014; Borio, 2019). In an endogenous money framework, several propositions derived from the '*quantity theory of money*' are obviated (Borio, 2019, p. 287).

Therefore, a reappraisal of Piero Sraffa's published and unpublished works enables a reconstruction of a coherent monetary theory, which is not less meaningful than Sraffa's works on value and distribution for economic theory.

The insights on money and banking developed by Sraffa during his career can help economists in the difficult - and still unsolved - task of integrating a

[28] Keynes's treatment of money in the General Theory differs from that in *A Treatise on Money*, where the banking system as a creator of deposits - and not only as an intermediary between savings and investments - is explicitly modeled. Besides the 'old' way of creating deposits by accepting cash or cheques by the public, Keynes describes a different procedure: "*But there is a second way in which a bank may create a claim against itself. It may itself purchase assets, i.e. add to its investments, and pay for them, in the first instance at least, by establishing a claim against itself. Or the bank may create a claim against itself in favour of a borrower, in return for his promise of subsequent reimbursement; i.e. it may make loans or advances.In both cases the bank creates the deposit; for only the bank itself can authorise the creation of a deposit in its books entitling the customer to draw cash or to transfer his claim to the order of someone else; and there is no difference between the two except in the nature of the inducement offered to the bank to create the deposit*" (Keynes, 1930, Ch. 2, p. 21)

fully-fledged theory of monetary fluctuations with a sound theory of value and distribution.

Chapter 3

Walrasian General Equilibrium and the Sraffian critique

3.1 Introduction

The subtitle of Piero Sraffa's famous *Production of Commodities by Means of Commodities* (Sraffa, 1960) is *Prelude to a Critique of Economic Theory*, reflecting the predominantly - albeit not uniquely - *critical* nature of the book.

The main object of Sraffa's criticism is "*the marginal theory of value and distribution*" (Sraffa, 1960, p. vii) - i.e., the neoclassical theory of value and distribution. According to this framework, it is possible to unequivocally determine the distribution of income associated with a Walrasian general equilibrium, given data about technology, agents' preferences and endowments.

According to Sraffa, economic theories do not have sufficient internal elements to determine income distribution: hence, observed economic values are the consequence of an exogenous distribution of real incomes (Bharadwaj, 1963) and income distribution is intrinsically indeterminate within economic models. Since the publication of (Sraffa, 1960), numerous authors have discussed the extent and the scope of Sraffa's critique.

The 1960s *Cambridge Capital Controversies* highlighted the destructive nature of Sraffa's critique for the 'aggregated' strands of neoclassical literature. Within this tradition, output and material inputs are aggregated, so that production relations can be described in terms of a composite 'commodity' produced as an output by means of labour and a composite 'commodity' entering the production process as an input (capital).

Given the aggregated nature of this class of models, relations between distributional variables - such as prices, interest rates and wages - and physical magnitudes - such as 'capital' and 'output' - can be unequivocally established

via monotonic functions. As a result, policy implications can be derived, such as the inverse relation between the value of capital and the interest rate.

During the 1960s debate, scholars in the Sraffian tradition demonstrated the existence of aggregation problems in these versions of neoclassical theory. The existence of such problems has been recognized by some neoclassical authors (Samuelson, 1966), whereas others have regarded them an empirically irrelevant *curiosum* (Sato, 1974) [1].

On the other hand, numerous neoclassical authors in the tradition of Arrow and Debreu (1954) and Debreu (1959) have argued that Sraffa's critique does not apply to the non-aggregated Walrasian version of neoclassical theory.

Furthermore, according to this strand of literature, Sraffian schemes can be derived as a *special case* of a Walrasian general equilibrium model by imposing suitable assumptions about the substitutability of the 'factors' of production, the nature of the prevailing technology and the absence of information about agents' preferences (Bliss, 1975; Burmeister, 1977; Hahn, 1982). Other authors, such as Mandler (1999b), acknowledge the theoretical and empirical relevance of Sraffian and classical *indeterminacy*, but frame it in the context of general equilibrium.

This paper studies the compatibility between the neoclassical and Sraffian approaches and aims to assess the claim that Sraffa's equations (henceforth *Sraffian schemes*) are a special case of a properly specified Walrasian general equilibrium model, building on the constructive and computable approach developed in Zambelli (2018b), Zambelli (2022a) and Zambelli (2022b). To do so, the paper takes inspiration from a thought experiment described in Hahn (1982) and discusses its implications.

The Chapter is organized as follows. Section 3.2 describes Frank Hahn's original thought experiment. Section 3.3 describes a neoclassical model used by a hypothetical 'neoclassical' economist on her arrival on an island where goods are produced by other goods and labour. Section 3.4 describes a generalized Sraffian model used by a hypothetical 'Sraffian' model on her arrival on the same island. Section 3.5 assesses the specialization claim for Sraffian schemes in light of specific characteristics of the two models: flexible or fixed production coefficients, uniform or non-uniform profit rates across industries, constant or variable relative prices over time, aggregation problems, and the constructive and computable nature of the two models. Section 3.6 concludes.

3.2 Hahn's 'island' experiment

Frank Hahn's 1982 paper *The Neo-Ricardians* is one of the most famous critical neoclassical contributions on Sraffa (1960). To dispute Sraffa's claim according to which the set of propositions in the book is designed to serve as the basis

[1] See Zambelli (2018a), Fisher (1969) and Felipe and Fisher (2003) for a comprehensive discussion of the debate on the neoclassical production function and for a empirically-based criticism of the foundations of the aggregate production function.

for a critique of the marginal theory of value and distribution, Hahn proposes a simple thought experiment:

> "*Imagine that Mr Sraffa and I, a representative of 'the marginal theory of value an distribution', arrive on an island. Mr Sraffa suggests that we should look at the very elaborate statistics of inputs and outputs for a given year which the islanders have collected and that we should each construct the input-output matrix. (A, a_0)*". (Hahn, 1982, p.358)

Assuming that the 'neoclassical' economist and the 'Sraffian' economist have the same mathematical skills and are allowed to observe the same economic data, Hahn shows that both economists can compute the Standard commodity and the price vectors for given choices of the uniform profit rate across industries. Hence, Hahn concludes that '*the only falsifiable entailment of the Sraffa equations is the postulate of a uniform rate*' (Hahn, 1982, p. 359).

In the subsequent sections of the paper, Hahn continues his example to show that:

a) the knowledge of technology alone is not sufficient to compute an equilibrium price vector both in Sraffian schemes and Walrasian general equilibrium (Hahn, 1982, pp. 360–362);

b) Sraffian schemes can be regarded as a special version of a more general neoclassical model - i.e., Walrasian general equilibrium (Hahn, 1982, pp. 362–367).

To prove statement a), Hahn devises a general equilibrium model in which Sraffian schemes arise out of the profit maximization problem of a set of firms under perfect competition. In this framework, the 'neoclassical' economist arriving on the island is provided information about all the alternative ways in which each good can be produced by means of goods and labour (Hahn, 1982, p. 359).

Hahn assumes constant returns to scale, no joint production, the uniformity between input and output prices of the same good, and the uniformity of the profit rate across sectors. Under this set of assumptions, Hahn considers a system in which firms maximize their unit profit functions when the difference between revenues and costs, where the latter include labour costs and the opportunity cost of investment, measured by the profit rate times the value of the means of production. In these profit functions, which turn out to be equivalent to Sraffian schemes, input coefficients (of goods and labour) are not taken as given, but are endogenously determined by profit-maximizing behavior (Hahn, 1982, p. 360).

This system is shown by Hahn to be equivalent to a Walrasian general equilibrium model where firms have access to a production function with constant returns to scale and maximize their profits. In such a model, the equilibrium

price vector, the endogenous input coefficients, and the wage rate can be derived from marginal products of the assumed production function (Hahn, 1982, p. 361).

The model, in line with Parts I and II of Sraffa (1960), exhibits one degree of indeterminacy, since the uniform profit rate is given. According to Hahn, such indeterminacy is deemed to be a common feature of a neoclassical and a Sraffian model, if the demand side of the economy (i.e., consumer preferences) is not modeled. Therefore, from the neoclassical point of view, the degree of freedom in Sraffa (1960) should not be interpreted as a failure of the marginal theory of value distribution, but rather as the result of an incomplete specification of the general equilibrium model in Sraffian schemes (Hahn, 1982, p. 362).

Building on these findings, Hahn aims to prove statement b) by setting out '*in search of the missing equation*' which determines the uniform rate of profit. To do so, he starts from a two-period intertemporal general equilibrium model in which profit-maximizing agents have access to a convex production set and endowments are given by past production decisions.

Then, Hahn introduces the definition of 'own rate of return' in the sense of Sraffa (1932a) and assumes that the sectoral profit rate in each industry is equal to the own rate of return of a commodity in terms of itself. Under this definition, the sectoral profit rate is equal to the ratio between the first-period price of a commodity and the second-period price of the same commodity.

In this setting, the system exhibits the same indeterminacy as the previously considered system, unless further equations are introduced. Hahn does so by introducing an excess demand function for second-period commodities so that the number of equations matches the number of unknowns (labour excess demand will be satisfied by Walras' Law) (Hahn, 1982, pp. 362-365).

At this point, Hahn discusses what he considers as the "*only falsifiable entailment of the Sraffa equations*" (Hahn, 1982, p. 359) - i.e., the postulate of a uniform rate of profits. Given the general equilibrium system discussed above, assuming that the rate of profits is equal across industries means imposing that all commodities have a uniform own rate of return and, hence, requiring that relative prices be constant between the two periods.

According to Hahn, this postulate proves that Sraffian schemes are a special case of a fully-fledged general equilibrium model:

> "*It will now be clear that Sraffa is considering a very special state [. . .] where the relative prices of 1976 wheat and barley are the same as those of 1977 wheat and barley. The neoclassical economist is quite happy with the more general situation*". (Hahn, 1982, p. 364)

Having reduced the number of unknowns, the 'special' Sraffian model with a uniform rate of profit is overdetermined. Therefore, according to Hahn, the system can only be solved if, given n goods, $n-1$ endowments are considered endogenous and become unknowns of the system.

This allows the neoclassical economist to "*generate all histories which are compatible with Mr Sraffa's demands. If Mr Sraffa lands on an island whose*

history does not belong to this set, he will be out of luck" (Hahn, 1982, p. 366). Hence, in this framework, solutions of Sraffian schemes with a uniform rate of profit will be dependent on available endowments and, thus, be a specialization of a general equilibrium model.

Hahn's argument has been criticized by Sraffian economists on several grounds[2]. According to a strand of literature, the true methodological differences between the two theories had not been fully grasped by Hahn (Petri, 2003; Garegnani, 2005). Other authors argue that the endogenization of endowments operated by Hahn in order to show the Sraffian 'special case' is a methodological error (Brancaccio, 2010). Furthermore, Pasinetti (2000) has argued that Hahn's criticism ignores results previously proven by Hahn himself on the problems with badly behaved production functions even for multi-sectoral neoclassical models (Hahn, 1966).

The following sections focus on a different aspect of the controversy between the two approaches. Given an economic system with the same observable data, a 'neoclassical' and a 'Sraffian' model will be compared with the aim of finding whether it is possible to claim that the latter is a special case of the former.

3.3 The neoclassical economist in the island

Let us suppose that a 'neoclassical' and a 'Sraffian' economist arrive on an island in which n goods are produced by means of goods and labour (no joint production is assumed).

Given a $n \times n$ matrix of endowments of the means of production $\mathbf{E} = [e_{ij}]$, which is the outcome of past production decisions, both economists can observe an input-output table where $\mathbf{A}$ is the $n \times n$ matrix of material inputs used up in the production process (with $\sum_{i=1}^{n} a_{ij} \leq \sum_{i=1}^{n} e_{ij} \ \forall \ j = 1, \ldots, n$) and $\mathbf{l}$ is the $n \times 1$ vector of labour inputs.

The island is populated by n_ω agents, who own shares of existing firms and/or sell their labour power to firms. For simplicity, we assume the existence of one firm per industry only.

Against this background, the neoclassical economist seeks to compute the exchange-values which ensure mutual consistency between the plans of producers and the plans of consumers, given the endowments of factors and ownership rights, consumers' preferences and the available technology. These exchange-values are consistent with Walrasian general equilibrium and, according to the neoclassical economist, can be used as a benchmark to evaluate whether agents in the island are producing and trading goods in or out of a general equilibrium.

To do so, the neoclassical economist seeks to define a production set - i.e., the set of n_z methods of production $\{\mathbf{A}_z\}_{z=1}^{n_z}$ including all the alternative ways in which the observed output could be produced by the n industries.

The information on the choice of available techniques for each firm is summarised in a set of n production functions $f_i(\mathbf{A}, l, \mathbf{\Theta})$ with $i = 1, \ldots, n$

[2]For an epistemological standpoint on the possibility to reduce Sraffian schemes to special cases of neoclassical theory, see Barrotta (1993).

and$\{\Theta_i\}_{i=1}^n$ being a set of parameters to be estimated:

$$b_i = f_i(\mathbf{a}_i, l_i, \Theta_i) \tag{3.1}$$

The neoclassical economist will, first and foremost, check the conditions under which firms maximize their profits.

Denoting by w the wage rate paid to workers (which is assumed to be uniform due to homogeneous labour), by $\mathbf{r}$ a $n \times 1$ vector of sectoral profit rates, and by $\mathbf{p}$ the $n \times 1$ the price vector, each firm i's profits can be defined as:

$$\pi_i = f_i(\mathbf{a}_i, l_i, \Theta_i)p_i - (1 + r_i)\mathbf{a}_i\mathbf{p} - wl_i \tag{3.2}$$

Hence, each firm i's problem is to find the prices, wage rate, profit rates and θ which maximise its pure profits under the constraints that prices, wages and profits are non-negative:

$$\begin{aligned} \max_{p_i, r_i, w, \theta} \quad & \pi_i(p_i, r_i, w, \Theta_i) && i = 1, \ldots, n \\ \text{s.t.} \quad & p_i \geq 0, && \forall\ i = 1, \ldots, n \\ & w \geq 0, \\ & r_i \geq 0, && \forall\ i = 1, \ldots, n \end{aligned} \tag{3.3}$$

If the production functions $f_i(\cdot)$ are continuously differentiable, first-order conditions for $\mathbf{a}_i^*$, l_i, p_i and w require that:

$$\frac{\partial \pi_i}{\partial a_{ij}} = \frac{\partial f_i(\cdot)}{\partial a_{ij}} p_i + f_i(\cdot)\frac{\partial p_i}{\partial a_{ij}} - (1 + r_i)p_j - (1 + r_i)a_{ij}\frac{\partial p_j}{\partial a_{ij}} = 0, \qquad i, j = 1, \ldots, n \tag{3.4}$$

$$\frac{\partial \pi_i}{\partial l_i} = \frac{\partial f_i(\cdot)}{\partial l_i} p_i + f_i(\cdot)\frac{\partial p_i}{\partial l_i} - w - l_i\frac{\partial w}{\partial l_i} = 0, \qquad i = 1, \ldots, n \tag{3.5}$$

As the length of the price vector does not matter, any $n \times 1$ vector η can be selected as a numeraire for the system:

$$\eta^T \mathbf{p} = 1 \tag{3.6}$$

If we assume that each production function is characterised by a parameter for each 'factor' of production including labor parameters - e.g., a Cobb-Douglas functional form - $\mathbf{\Theta}$ is a $n^2 + n$ matrix. Hence, the problem has $n^2 + 3n + 1$ unknowns.

In order to find a Walrasian price vector, it must be assumed that agents treat prices parametrically - i.e., that no agent can have an influence on equilibrium prices. Therefore, the following restrictions must apply:

$$\frac{\partial p_i}{\partial a_{ij}} = 0 \quad \forall\ i, j = 1, \ldots, n \tag{3.7}$$

$$\frac{\partial p_i}{\partial l_i} = 0 \quad \forall\ i = \ldots, n \tag{3.8}$$

Substituting (3.7) and (3.8) in (3.4) and (3.5), we get:

$$1 + r_i = \frac{\partial \pi_i}{\partial a_{ii}} \tag{3.9}$$

$$p_j = \frac{\partial \pi_i}{\partial a_{ij}} \frac{1}{1 + r_i} p_i \tag{3.10}$$

$$w = \frac{\partial p_i}{\partial l_i} p_i \tag{3.11}$$

Since the system formed by (3.6), (3.9), (3.10) and (3.11) is now composed of n^2+n+1 equations, there are $2n$ degrees of freedom, so that some specialization must be introduced in order to make the system determinate.

In order for neoclassical theory to fully explain income distribution, the neoclassical economist must be able to prove that rewards to each input into the production process sum to equality with total output. If constant returns to scale are assumed, the 'adding-up' problem (Hahn, 1982, p. 361) is solved if:

$$\sum_{j=1}^{n} \frac{\partial f_i(\cdot)}{\partial a_{ij}} a_{ij} + \frac{\partial f_i(\cdot)}{\partial l_i} l_i = b_i \tag{3.12}$$

which adds n equations to the system, thereby reducing the degree of indeterminacy of the system to n degrees of freedom.

At this point, the neoclassical economist would have completed her analysis of the 'supply' side of the economy, and she would turn to modeling the 'demand' side.

The net product (or surplus) of the island's economy which can be distributed to agents, following (Zambelli, 2018b) and Chapter 1 of this thesis, can be defined as:

$$\mathbf{s} = (\mathbf{B} - \mathbf{A})^T \mathbf{e} \tag{3.13}$$

where $\mathbf{e}$ is an $n \times 1$ summation vector whose elements are all equal to 1, T is the transpose operator; $\mathbf{s}$ is the $n \times 1$ surplus vector.

Given the net product of the economy $\mathbf{s}$ and the prevailing price vector $\mathbf{p}$ which solves (3.3), the income of each agent is based on the dividends of firms' profits (if they are shareholders) and the rewards for labor. Thus, the income of each individual agent can be captured by the following equation (Zambelli, 2022b):

$$\mathbf{H}\mathbf{s}^T\mathbf{p} = \begin{bmatrix} h_1^1 & \cdots & h_1^{n_\omega} \\ \vdots & \ddots & \vdots \\ h_n^1 & \cdots & h_n^{n_\omega} \\ h_w^1 & \cdots & h_w^{n_\omega} \end{bmatrix} \mathbf{s}^T\mathbf{p} \tag{3.14}$$

where $\mathbf{H}$ is a $(n+1) \times n_\omega$ matrix whose rows are associated with the sources of income - i.e., the n goods/industries and labour - and whose columns are associated with the each of the n_ω agents. The elements of $\mathbf{H}$ are shares of the value of the net product being distributed to each agent: hence, the sum of all elements of $\mathbf{H}$ must be equal to 1.

As a result, the income of agent ω is given by the following scalar:

$$y_\omega = \mathbf{e}_{(n+1)\times 1}^T \mathbf{h}^\omega \mathbf{s}^T\mathbf{p} \tag{3.15}$$

where $\mathbf{e}_{(n+1)\times 1}^T$ is a $(n+1) \times 1$ column summation vector and h_ω is the ω-th column of $\mathbf{H}$ corresponding to agent ω's income.

If the numeraire in (4.6) is taken to be the surplus vector $\mathbf{s}$ - i.e., $\mathbf{s}^T\mathbf{p} = 1$, the sum by columns of matrix $\mathbf{H}$ also yields the personal income per agent:

$$\mathbf{y} = \mathbf{H}^T \begin{bmatrix} \mathbf{p} \\ w \end{bmatrix}$$

The sum by rows of matrix $\mathbf{H}$ yields the vector of *functional distribution of income* $\mathbf{d}$ (Zambelli, 2018b, p. 803), which summarizes all the available information about the distribution of the net product to the different industries and to workers:

$$\mathbf{d}\mathbf{s}^T\mathbf{p} = \mathbf{H}\mathbf{e}_{n_\omega \times 1}\mathbf{s}^T\mathbf{p} \tag{3.16}$$

The above vector - whose elements are all between 0 and 1 and sum to 1 - is uniquely associated to a specific pattern of prices, profit rates and wage rates. When the surplus vector is considered as a numeraire, the left hand side of (3.16) can be simplified, but the unique relation between the distribution vector and the price vector still holds.

At this stage, the neoclassical economist takes her last step to determine a Walrasian general equilibrium price vector for the island's economy. In order to acquire their desired consumption goods, consumers exchange part of their endowments (i.e., incomes) with other agents.

Assuming no net investment in the island, we can assign as endowments the fraction of the total surplus (equal to total consumption) associated to agent's income, so that the value of agent's ω endowment is given by:

$$y_\omega = \sum_{i=1}^{n} y_{i\omega} \tag{3.17}$$

If we assume preferences of each agent ω to be defined by a utility function $U_\omega(\mathbf{c}_\omega)$, where $\mathbf{c}_\omega$ is agent ω's consumption vector $(n \times 1)$, the agent will face the following maximization problem:

$$\begin{aligned} \max_{\mathbf{c}_\omega} \quad & U_\omega(\mathbf{c}_\omega) \\ \text{s.t.} \quad & \mathbf{c}_\omega^T \mathbf{p} = y_\omega \end{aligned} \tag{3.18}$$

Under the usual neoclassical conditions on preferences, solving (3.18) requires the following first-order conditions to be equal to zero:

$$\frac{\partial \mathcal{L}}{\partial c_{i\omega}} = \frac{\partial U_\omega}{\partial c_{i\omega}} - \lambda \left(p_i + c_{i\omega} \frac{\partial p_i}{\partial c_{i\omega}} \right) = 0 \tag{3.19}$$

$$\frac{\partial \mathcal{L}}{\partial \lambda} = \mathbf{c}_\omega^T \mathbf{p} - y_\omega = 0 \tag{3.20}$$

where $\mathcal{L}$ is the Lagrangian obtained by (3.18), $c_{i\omega}$ is the consumption of good i by agent ω.

If we take $\mathbf{c}_\omega^*(\mathbf{p})$ as the consumption vector solving (4.18), the neoclassical economist will be able to define an excess demand function z_i for each good i by summing the individual excess demand functions over all consumers n_ω:

$$z_i(\mathbf{p}) = \sum_{\omega=1}^{n_\omega} c_{i\omega}^*(\mathbf{p}) - \sum_{\omega=1}^{n_\omega} y_{i\omega} = \sum_{\omega=1}^{n_\omega} c_{i\omega}^*(\mathbf{p}) - s_i(\mathbf{p}) = 0 \tag{3.21}$$

Finally, in order to properly define a Walrasian general equilibrium, all consumers must treat prices parametrically, so that none of their actions can have influence on prices:

$$\frac{\partial p_i}{\partial c_{i\omega}^*} = 0, \quad i = 1, \ldots, n \quad \omega = 1, \ldots, n_\omega \tag{3.22}$$

The system formed by (3.6), (3.9), (3.10), (3.11), (3.12) and (3.21) now includes $n^2 + 3n + 1$ independent equations and an equal number of unknowns. Hence, the neoclassical economist can derive the Walrasian general equilibrium prices, wages and profit rates which determine an equilibrium income distribution $\mathbf{d}$ and the choice of the parameters Θ yielding the most efficient technique given a production set and equilibrium prices.

This result can be used by the neoclassical economist as an benchmark in order to evaluate whether actual market outcomes are optimal. It must be noted that the model outlined in this section is not able to determine all distributive variables by knowing endowments and technology.

As in Hahn (1982), when the vector of profit rates $\mathbf{r}$ is not taken as given, the knowledge of preferences allows the neoclassical economist to model a demand side of the economy which, combined with the information of the supply side, is susceptible to determine the equilibrium income distribution.

3.4 The Sraffian economist in the island

When the Sraffian economist arrives on the island, she observes the output matrix $\mathbf{B}$, the endowments matrix $\mathbf{E}$ inherited from past production decisions, the

vector of labour inputs $\mathbf{L}$ and the set of available production methods $\{\mathbf{A}_z\}_{z=1}^{n_z}$.

If the Sraffian economist were to follow the method in Parts I and II of Sraffa (1960), he would be able to observe a unique given production method $\mathbf{A}$ (i.e., $n_z = 1$).

Subsequently, she could compute the set of exchange-values which would have to be observed for the system to reproduce itself - i.e., for all industries to have the sufficient purchasing power in order to repeat the production process at a given set of profit rates $\mathbf{r}$ with the same method of production and the same output[3].

Throughout his book, Sraffa makes the explicit assumption of a uniform profit rate. Many Sraffian scholars have interpreted this assumption as a consequence of the *gravitation* process of market prices around long-run normal prices in the *free competition* framework adopted by the classics (Garegnani, 1979).

In this framework, while the physical composition of capital continuously changes when capital is shifted across sectors by incumbent and entrants in search for higher profits, the rest position of the economy in the long run is associated with a uniform profit rate across sectors.

Other authors have interpreted the uniformity of profit rates not as an equilibrium condition, but as a physical property of the system and a logical corollary of the assumption of uniform wages given from outside the system (Sinha and Dupertuis, 2009; Sinha, 2021).

However, throughout this paper, no restriction will be imposed on the vector of profit rates. Although the assumption of uniform profit rates is useful for the critique of the Austrian theory of capital and the incorporated labour theory of value, the assumption is not required in order to analyze the self-replacing properties of the system.

If the Sraffian economist on the island does not restrict her analysis to the case of uniform profit rates, the system can still reproduce itself. As shown by Zambelli (2018b), there is a unique relation between functional income distribution and each possible configuration of prices, wage rates and profit rates; therefore, (Zambelli, 2018b) outlines a constructive methodology in order to associate each possible vector of distributional shares of income with a price-wage vector. This procedure allows to define a domain of self-replacing prices, wage rates and profit rates[4].

Moreover, different methods are available to the Sraffian economist if she

[3] "*The significance of the equations is simply this: that if a man fell from the moon on the earth, and noted the amount of things consumed in each factory and the amount produced by each factory during a year, he would deduce at which values the commodities must be sold, if the rate of interest must be uniform and the process of production repeated*". Sraffa (1927 or 1928), D3/12/7, quoted in Zambelli (2018b)

[4] For the purposes of our discussion, a constructive solution to a problem is defined as an explicit method of finding the solution with the implementation of an algorithm composed of a finite number of steps. As a consequence, a constructive existence proof is an algorithm whose implementation terminates with the construction of the proof of the object whose existence is postulated. See Velupillai (1989), Velupillai (2021) and Zambelli (2021) on the constructive nature of Sraffa's mathematics.

allows a choice of technique to take place, as in Part III of Sraffa (1960). An underlying production function with a given functional form could be assumed or estimated (as in Section 3.3), so that a *continuum* of techniques would be available; alternatively, a constructive methodology such as the *FVZ-algorithm* in Zambelli et al. (2017) can be used in order to determine the most efficient production method out of the set $\{\mathbf{A}_z\}_{z=1}^{n_z}$.

However, for the purposes of our experiment, the Sraffian economist aims at computing the prices, wage rates and profit rates such that each firm's revenues are equal to each firm's expenditures.

This conditions requires that the value of firms' sales in the market must match the value of firms' expenditures in each time period. Firms' outlays include the acquisition of goods required for production, the payment of wages for workers and the profits paid to shareholders. Therefore, this condition can be defined as a *bookkeeping equilibrium* which ensures market clearing.

Moreover, the absence of money and credit implies that no debt and credit relations are in place at the end of the period: all payments must have been settled within the end of the year[5].

As the incomes of individual agents are obtained from wages and profits, given a structure of ownership rights, the generation of a positive net product (or surplus) in the island can be used for consumption or as net investment in order to increase the scale of output in the next period.

If the system is to reproduce itself as in Parts I and II of Sraffa (1960), the whole surplus is used for consumption, whereas net investment is equal to zero. Therefore, the bookkeeping equilibrium condition requires that each agent's consumption exactly matches each agent's income.

Therefore, the Sraffian economist will consider a system of n industries which n commodities, assuming no joint production. For each industry, the bookkeeping equilibrium requires that the following condition holds for each industry:

$$b_i p_i = \mathbf{a}_i \mathbf{p} + r_i \mathbf{a}_i \mathbf{p} + w l_i \tag{3.23}$$

The n equations in (3.23) describe a *bookkeeping equilibrium*, in that the left hand side is the value of the revenues for each industry i ($b_i p_i$), whereas the right hand side is the value of expenditures incurred during the year. The latter is sum of the value of the means of production required to produce the output ($\mathbf{a}_i \mathbf{p}$), the value of goods allocated to the owners of the means of production as return on investment ($r_i \mathbf{a}_i \mathbf{p}$), and the value of goods allocated to workers ($w l_i$).

Similarities between (3.2) and (3.23) are apparent, although they can be interpreted in different ways. While the former is derived from the bookkeeping equilibrium condition, the latter is an objective function which each industry aims to maximize.

Furthermore, the quantity of output produced in (3.2) for each industry i is given by a production function, whereas output in (3.23) is at the observed

[5]The system would be able to reproduce if promises to pay are introduced in the model. See Venkatachalam and Zambelli (2021) and Venkatachalam and Zambelli (2022) for the study of Sraffian schemes with credit and debt relations

level b_i. This point has led many authors to argue that Sraffa (1960) "*simply takes the input/output table as given and unexplained*" (Hahn, 1982, p. 362). As it will be argued in Section 3.5, another interpretation is possible.

Once the bookkeeping equilibrium condition in (3.23) is established, the Sraffian economist will seek to compute the "*set of exchange-values which if adopted by the market restores the original distribution of the product and makes it possible for the process to be repeated*" (Sraffa, 1960, p. 4).

Hence, the Sraffian economist aims to compute the *self-replacing* prices which would enable all industries to repeat their production process and generate a surplus to be distributed among different types of consumers.

If production on the island were only able to provide subsistence - i.e, if no surplus was produced - the knowledge of the triple $(\mathbf{A}, \mathbf{l}, \mathbf{B})$ would be sufficient to determine the bookkeeping equilibrium prices.

In the subsistence condition, we would have $r_i = 0 \ \forall \ i$ and $w = 0$. If a numeraire is introduced by putting one of the n prices equal to unity, then the $n-1$ unknown prices would uniquely determine the $n-1$ remaining independent equations.

A positive surplus, however, should be distributed to workers and owners of means of production in each industry. According to one of the most fundamental tenets in Sraffa (1960), several allocations of the net product are consistent with the reproduction of the system, so that there is no available method to prioritize one particular allocation out of the set of all possible distributional patterns unless additional structures are imposed. The knowledge of the triple $(\mathbf{A}, \mathbf{l}, \mathbf{B})$ makes it possible to compute the quantity of the net product, but not its distribution among economic agents.

In reviewing Sraffa (1960), Bharadwaj (1963) argues that one of the most crucial insights of the book is that exchange-values (i.e., prices) cannot be determined independently from distribution, as there is a unique association between certain prices which enable the reproduction of the system and a given distribution of the net product. Therefore, to compute a domain of self-replacing prices, the Sraffian economist can work out all the possible distributional patterns for a given net product.

For each distributional pattern, a price vector ensuring equality between revenues and expenditures in each industry and for workers can be computed, with an associated vector of profit rates and an associated wage rate (which is assumed uniform for simplicity).

A constructive method to perform such a computation is provided in Zambelli (2018b). Given the triple $(\mathbf{A},\mathbf{l},\mathbf{B})$, the self-replacing condition requires that (3.23) holds.

Defining the surplus as in (3.13), a set of *functional distribution* vectors $\mathbf{d}$ can be constructed whose elements are shares of the surplus s being distributed to the n industries and workers. The vector $\mathbf{d}$ can be represented as follows:

$$\mathbf{d}\mathbf{S}^T\mathbf{p} = \begin{bmatrix} d_1\mathbf{S}^T\mathbf{p} \\ d_2\mathbf{S}^T\mathbf{p} \\ \vdots \\ d_n\mathbf{S}^T\mathbf{p} \\ d_w\mathbf{S}^T\mathbf{p} \end{bmatrix} = \begin{bmatrix} b_1p_1 - \mathbf{a}_1\mathbf{p} - l_1w \\ b_2p_2 - \mathbf{a}_2\mathbf{p} - l_2w \\ \vdots \\ b_np_n - \mathbf{a}_n\mathbf{p} - l_nw \\ \mathbf{e}^T\mathbf{l}w \end{bmatrix} = \begin{bmatrix} r_1\mathbf{a}_1\mathbf{p} \\ r_2\mathbf{a}_1\mathbf{p} \\ \vdots \\ r_n\mathbf{a}_1\mathbf{p} \\ \mathbf{e}^T\mathbf{l}w \end{bmatrix} \tag{3.24}$$

where $\mathbf{e}^T$ is a summation vector which makes it possible to sum the whole wage bill paid to workers in the $n+1$-th row of $\mathbf{d}$. As the elements of $\mathbf{d}$ are shares of $\mathbf{s}$, they must range between 0 and 1.

The cardinality of the set of feasible functional distribution vectors n_d is related to the desired precision of the computation and it is equal to the cardinality of the set of feasible prices, wages and profit rates. If the Sraffian economist defines all economic values in the domain of rational numbers, the definition of the finite number of vectors $\mathbf{d}$ is possible given a desired level of precision through which shares are defined.

Taking the surplus to be the numeraire of the system (so that $\mathbf{s}^T\mathbf{p} = 1$), (3.25) becomes simplified as follows (in a more compact matrix form):

$$\mathbf{d} = \begin{bmatrix} (\mathbf{B} - \mathbf{A}) & -\mathbf{l} \\ \mathbf{0}_{(1\times n)} & \mathbf{e}^T\mathbf{l} \end{bmatrix} \begin{bmatrix} \mathbf{p} \\ w \end{bmatrix} = \begin{bmatrix} \mathbf{R}\mathbf{A}\mathbf{p} \\ \mathbf{e}^T\mathbf{l}w \end{bmatrix} \tag{3.25}$$

The above equation displays a one-to-one correspondence between each vector $\mathbf{d}$ and each price-wage vector. Denoting by $\mathbf{p}_d$ and w_d the prices and wages associated with each $\mathbf{d}_k$ in $\{\mathbf{d}_k\}_{k=1}^{n_d}$, we have:

$$\left[\begin{array}{c} \mathbf{p}_d \\ \hdashline w_d \end{array}\right] = \left[\begin{array}{c:c} (\mathbf{B} - \mathbf{A}) & \mathbf{l} \\ \hdashline \mathbf{0}_{1\times n} & \mathbf{e}^T\mathbf{l} \end{array}\right]^{-1} \mathbf{d} \tag{3.26}$$

Moreover, given (3.23), (3.25) and (3.26), the vector of feasible profit rates is given by:

$$\mathbf{r}_d = \mathbf{d}_{n\times 1} \oslash (\mathbf{A}\mathbf{p}_d); \quad \mathbf{R}_d = \text{diag}(\mathbf{r}_d) \tag{3.27}$$

where $\oslash$ is the Hadamard division between each element of the first n rows of $\mathbf{d}$, representing the shares of the surplus distributed to each industry, and the value of the means of production, and $\mathbf{R}_d$ is a matrix whose elements are all zero except the diagonal, whose elements are those of the vector $\mathbf{r}_d$.

With equations (3.26) and (3.27), the Sraffian economist can uniquely determine all distributive values which ensure self-replacing given each distributional pattern.

The functional income distribution can also be related to the personal income distribution by defining the same matrix as in (3.14), where the net product is the numeraire:

$$\mathbf{H} = \begin{bmatrix} h_1^1 & \cdots & h_1^{n_\omega} \\ \vdots & \ddots & \vdots \\ h_n^1 & \cdots & h_n^{n_\omega} \\ h_w^1 & \cdots & h_w^{n_\omega} \end{bmatrix} \tag{3.28}$$

and where n_ω is the number of economic agents (consumers) in the island. In $\mathbf{H}$, the columns are associated to the shares of income distributed to each agent ω, and the rows are associated to the sources of income distributed to each agent. Therefore, the sum by rows of $\mathbf{H}$ yields the elements of the functional distribution vector $\mathbf{d}$ as in (3.16):

$$\mathbf{d} = \mathbf{H}\mathbf{e}_{n_\omega \times 1} \tag{3.29}$$

With the net product as numeraire, an income per agent matrix $\mathbf{Y}$ can be constructed:

$$\mathbf{Y} = \mathbf{H}^T \begin{bmatrix} \mathbf{p} \\ \mathbf{w} \end{bmatrix} \tag{3.30}$$

Under the self-replacing assumption, the net product is fully devoted to consumption. Therefore, if a social consumption matrix $\mathbf{C}$ is constructed, whose columns represent the goods consumed by each agent ω and whose rows represent the n goods available to agents, the sum by rows of C - i.e., the *aggregate* social consumption vector $\mathbf{c}_{n\times 1}$ - must be equal to the surplus vector

$$\mathbf{c}_{n\times 1} = \mathbf{s} = \mathbf{C}\mathbf{e}_{n_\omega \times 1} \tag{3.31}$$

Hence, despite demand has not been explicitly modeled yet by the Sraffian economist, the bookkeeping equilibrium conditions requires that the demand of consumption goods equals their supply if prices, wages and profit rates are consistent with self-replacing. Therefore, the bookkeeping equilibrium is also a market-clearing equilibrium: the excess demand function is equal to zero for all the n_d self-replacing price vectors.

Indeed, in self-replacing, agents face a budget constraint in which the value of their consumption bundle must be equal to the value of their income for all possible distributional patterns in n_d:

$$\mathbf{C}^T\mathbf{p}_d = \mathbf{H}^T \begin{bmatrix} \mathbf{p}_d \\ w_d \end{bmatrix} \tag{3.32}$$

Therefore, the Sraffian economist's final output will be the following system of equations:

$$\begin{aligned} \mathbf{B}\mathbf{p}_d &= (\mathbf{I} + \mathbf{R}_d)\mathbf{A}\mathbf{p}_d + \mathbf{l}w_d \\ \mathbf{s}^T\mathbf{p}_d &= 1 \end{aligned} \tag{3.33}$$

The above system of equations has $n+1$ equations but $2n+1$ unknowns - n prices p_i, n profit rates r_i and a wage rate w - thereby displaying indeterminacy.

Given (3.26), (3.27) and (3.33), the Sraffian economist can determine a general equilibrium condition with market clearing which is satisfied by a domain of self-replacing prices, wage rates and profit rates.

However, based on the observation of the inputs used up in the production process and the produced outputs and without imposing further structure, the Sraffian economist cannot determine the feasible economic solution out of all the possible n_d bookkeeping equilibria.

3.5 Special and general theories

At the end of the thought experiment, the neoclassical economist devises a method to determine the set of market-clearing price vectors which is also consistent with utility maximization by consumers and profit maximization by firms. This set of equilibrium price vectors - i.e., the Walrasian general equilibrium price vectors - ensures mutual consistency between the plans of all agents and determines the equilibrium income distribution.

On the other hand, the Sraffian economist develops a method to determine the domain of all market-clearing price vectors, associated with different allocations of the net product, but she is not able to prioritize a set of price vectors without imposing further structures. Economic theory, according to the Sraffian economist, does not have sufficient internal elements in order to predict the equilibrium distribution of income.

Hence, the Sraffian economist's model allows for a wide range of closures, provided that they entail one of the n_d price vectors belonging to the feasible domain. Based on this result, the Sraffian economist claims that she has found elements upon which a critique of the "*marginal theory of value and distribution*" (Sraffa, 1960, p. vii) can be attempted.

In this regard, neoclassical economists have argued that Sraffian schemes are not sufficient to undermine the foundations of the Walrasian general equilibrium model. Most authors admit that the Sraffian literature successfully highlighted problems in neoclassical 'parables' (Hahn, 1982, p. 370) in which capital goods and output are aggregated and in the Marshallian and Austrian theories of capital[6].

However, according to modern neoclassical theorists, Walrasian general equilibrium is immune from such a critique. In the absence of any notion of 'capital' as a quantity of heterogeneous goods which can be measured independently of prices, modern neoclassical theory cannot determine Walrasian general equilibrium prices with the knowledge of technology and endowments alone. The solution of the system requires agents' preferences (i.e., consumers' utility functions and firms' production functions), whose treatment is absent in the Sraffian tradition. When preferences are taken into account, Walrasian general equilibrium prices and distribution are fully determined.

[6] "*Even if some authors regarded these as parables, text books testify that they can be misused. Sraffa and his followers deserve the credit for making such misuse less likely in the future*". (Hahn, 1982, p. 370)

On the other hand, neoclassical authors have argued that Sraffian schemes should be interpreted as a specialization of the Walrasian general equilibrium model. The following subsections discuss several strands of neoclassical literature claiming that Sraffian schemes ought to be conceived as special cases of a more general neoclassical model.

3.5.1 Flexible production coefficients

Several authors claim that Sraffian schemes should be regarded as a case of linear activity model with fixed coefficients and a constant returns to scale technology, in the same tradition of Neumann (1945) and Leontief (1928).

In this regard, the neoclassical economist on the island might claim that (3.23) can be derived from (3.2) if the production set $\{\mathbf{A}\}_{z=1}^{n_z}$ has only one element and if (3.12) holds - i.e., if constant returns to scale are assumed (Hahn, 1982, p. 359-360). In the words of Hahn, Sraffa:

> "*takes the actual techniques of production as unexplained.* ***That being so, they may well be the outcome of the kind of choice which we have described****. Mr Sraffa landing on the island will never discover whether this is so or not. For he does not ask and of course he can calculate Standard ratios and commodities on the basis of* ***technological observation which are what they are because of a rational choice of technique****.*" (Hahn, 1982, p. 362; emphasis added)

Hahn's point can be modeled as follows. Under constant returns to scale, the firm's maximization problem can be rewritten in unit terms so that (3.2) becomes:

$$\pi_i^u(p_i, r_i, w) = p_i - (1 + r_i)\mathbf{a}_i^u\mathbf{p} - wl_i^u \tag{3.2'}$$

where π^u is the unit profit function, $\mathbf{a}_i^u$ is the vector of inputs per unit of output of commodity i, and l_i^u is the vector of labour inputs per unit of output of commodity i.

As the unit profit function grows linearly with the level of output, the maximization of pure profit per industry i shall occur when $\pi_i(p_i, r_i, w) = 0$; if $\pi_i(p_i, r_i, w) > 0$, firms will produce unbounded quantities and the sectoral rate of profit will be higher than r_i, whereas if $\pi_i(p_i, r_i, w) > 0$, firms will not produce.

It is straightforward to note that when the unit profit function is zero, multiplying the right hand side of (3.2') by the level of output b_i yields exactly (3.23), as $\mathbf{a}_i = b_i\mathbf{a}_i^u$ and $\mathbf{l}_i = b_i\mathbf{l}_i^u$.

The Sraffian economist might note that the opening sentence of Sraffa (1960) is specifically dedicated to the question of returns to scale:

> "*Anyone accustomed to think in terms of the equilibrium of demand and supply may be inclined, on reading these pages, to suppose that*

> ***the argument rests on a tacit assumption of constant returns in all industries***. *If such a supposition is found helpful,* ***there is no harm in the reader's adopting it as a temporary working hypothesis. In fact, however, no such assumption is made.*** *No changes in output at (any rate in Parts I and II) no changes in the proportions in which different means of production are used by an industry are considered so that no question arises as to the variation or constancy of returns.* ***The investigation is concerned exclusively with such properties of an economic system as do not depend on changes in the scale of production or in the proportion of 'factors'.***" (Sraffa, 1960, p. i; emphasis added)

Sraffa's point is that once the triple $(\mathbf{A}, \mathbf{l}, \mathbf{B})$ is observed, the set of self-replacing exchange-values does not depend from changes in output levels or the proportion of inputs. Hence, whatever is the production function generating the triple, the Sraffian economist will compute the same domain of self-replacing prices, profit rates and wage rates.

The Sraffian economist may concede that the constant returns to scale assumption is *sufficient* to derive a Sraffian scheme from a Walrasian general equilibrium framework. In fact, in the neoclassical economist's model, this assumption ensures determinacy by providing n linearly independent equations, and imposes that the assumed production function is able to ensure that input payments exhaust the total output.

However, the assumption is not *necessary* to have a Sraffian scheme; furthermore, the assumption is not even necessary to devise a Walrasian general equilibrium model.

If the Sraffian economists takes the prices, profit rates and wage rates $(\mathbf{p}_d, \mathbf{r}_d, w_d)$ which solve (3.33) for each distribution vector $\mathbf{d}$, she can estimate the matrix $\mathbf{\Theta}$ of parameters of each industry's production function using equations (3.9), (3.10) and (3.11) of the neoclassical model.

As it is straightforward to note, if we denote by $\mathbf{\Theta}_d$ the matrix of parameters of production functions of the i industries generating output $\mathbf{B}$ in the different distributional settings, we can expect that elements of $\mathbf{\Theta}_d$ are such that (3.12) holds only by a fluke.

For instance, Zambelli (2022a) estimates the parameters of a set of Cobb-Douglas production functions in a Sraffian scheme with no joint production in which the uniformity of profit rates across sectors is not assumed.

Using the numbers in p. 19 of Sraffa (1960) as observations of outputs and inputs, Zambelli (2022a) finds that none of the parameters of the industry-level production functions is such that the assumption constant returns to scale is verified (Zambelli, 2022a, p. 18).

Therefore, the Sraffian economist can argue that the assumption of constant returns to scale and fixed coefficients is the only possible specialization of a Walrasian general equilibrium model which is able to generate a linear activity model whose properties do not depend on preferences.

Although linear activity models bear similarities with Sraffian schemes, it cannot be claimed that Sraffian schemes are exclusively derived from a Walrasian general equilibrium model with linear technologies. The problem of the choice of technique, which is not treated in Parts I and II of Sraffa (1960), can be also examined through methodologies which do not require the assumption of specific functional forms or marginal products (Zambelli et al., 2017).

3.5.2 Non-uniform profit rates

Several neoclassical authors have argued that the assumption of a uniform profit rate in Sraffa (1960) is a crucial specialization of Sraffian schemes with respect to the Walrasian general equilibrium model. By taking $r_i = \overline{r} \ \forall \ i = 1, \ldots, n$, Sraffian prices and wages can be made independent of preferences and the rate of profits can be made independent of technology.

Frank Hahn argues that "*the only falsifiable entailment of the Sraffa equations is the postulate of a uniform rate of profit*" (Hahn, 1982, p. 359), and that all the other claims put forward in Sraffa (1960) are not relevant to neoclassical theory.

In order to demonstrate this point, Hahn devises a two-period intertemporal general equilibrium model with two commodities and labour (with constant returns to scale). A similar argument will be developed by using the neoclassical economist's model in Section 3.3.

This model can be reworked by noting that, within the assumed annual production cycle in year t, each industry i must have acquired inputs $\mathbf{a}_i$ in the previous production period $t-1$ at prices $\mathbf{p}_{t-1}$ in order to start production. After production, the output is taken to the market and sold at prices $\mathbf{p}_t$ in order to ensure that, at time t, the firm's shareholder are rewarded with a rate of profit $r_{i,t}$ and workers are rewarded with wage w_t (post-factum).

At time $t-1$, each shareholder should decide between lending his means of production to firm i in order to produce output $b_{i,t}$ or storing his means of production in order to sell them at time t[7]. For each shareholder to be indifferent between the two operations - i.e., investment or speculation - the price at which each industry i must sell its output is

$$p^e_{i,t} = \frac{p_{i,t-1}}{1 + r_{i,t}} \tag{3.34}$$

Isolating the rate of profit $r_{i,t}$ in (3.34) we get:

$$r_{i,t} = \frac{p_{i,t-1}}{p^e_{i,t}} - 1 \tag{3.35}$$

In (3.35), $(1 + r_{i,t})$ is the number of units of good i in period t that an agent can expect to obtain for one unit of good i in period $t-1$. In this regard, the rate of profit $r_{i,t}$ is the equivalent of the own rate of interest in Sraffa (1932a) or the 'wheat rate of interest on wheat' in Keynes (although, for the purposes

[7] It is a condition of *perfect arbitrage* (Solow, 1956, p. 80).

of Walrasian general equilibrium analysis, good i at two different time periods can be interpreted as two different goods altogether).

Assuming perfect foresight - i.e., agents have definite point expectations, allowing no uncertainty, of future variables, and agents' expectations are correct - and considering two different two goods j and k, their relative prices at time t are such that:

$$\frac{p_{j,t}}{p_{k,t}} = \frac{p_{j,t-1}}{p_{k,t-1}} \frac{1 + r_{k,t}}{1 + r_{j,t}} \tag{3.36}$$

If profit rates are assumed to be uniform, (3.36) implies that the relative prices of goods j and k are constant over time. This condition can take place in the context of a long-run equilibrium position, which is an interpretation of the assumption of a uniform rate of profit in Sraffa (1960) held by Neo-Ricardian economists, which Hahn defines as 'followers of Sraffa' (Hahn, 1982).

Assuming that profit rates are uniform across all industries i, time indices can be dropped and the neoclassical economist can then return to her original model in Section 3.3. However, if $r_i = \overline{r}$ for all i, the model loses $n-1$ unknowns and becomes overdetermined.

To make the system determinate, the neoclassical economist must increase the number of unknowns. Following Hahn (1982), the neoclassical economist may turn $n-1$ endowments into unknown variables to be determined jointly with the equilibrium price vector, the wage rate and the uniform profit rate.

Therefore, only one endowment is exogenously fixed in this case, with all other unknowns depending on it. If the endowments of goods on the island, inherited from previous production processes, are not compatible with the solution of the system, a positive price vector consistent with the assumption of a uniform profit rate and compatible with Walrasian general equilibrium might be impossible to find (Hahn, 1982, p. 366)[8].

The Sraffian economist may look at the problem in a different way. Using the model devised in Section 3.4, and assuming a given degree of precision, she spanned all the possible distribution patterns which are compatible with market clearing, and found all price vectors, wage rates and profit rates associated with each feasible - i.e., self-replacing - distribution vector.

Hence, in the Sraffian economist's model, uniform profit rates may occur in a subset of all feasible profit rates associated to specific distribution vectors. As the model is solely concerned with the conditions under which the system can reproduce itself, uniform profit rates will also occur as a special case in the Sraffian economist's model; moreover, uniform profit rates are associated with specific distributions of the shares of the net product, which need not be uniform across sectors (Zambelli, 2018b, p. 809–810).

To prioritize this specific subset of distribution vectors, the convergence of the economy towards this 'long-run' configuration should be proven[9].

[8] "*If Mr Sraffa lands on an island whose history does not belong to this set, he will be out of luck*" (Hahn, 1982, p. 366).

[9] Zambelli (2018b) shows that the assumption of a uniform profit rate is instrumental for

Therefore, the case of uniform profit rates is a specialization in both the models discussed in Sections 3.4 and 3.5. On the one hand, the neoclassical economist cannot claim that the uniformity of profit rates is the general proof that Sraffian schemes are generated by Walrasian general equilibrium models with constant returns to scale.

The Sraffian critique on the indeterminacy of economic models and the Sraffian insight on the nexus between prices and distribution do not rely on the assumption of a uniform rate of profit. The feasibility of a uniform profit rate *associated to positive Walrasian general equilibrium prices* depends on endowments *in the model* that the neoclassical economist devised in order to face the thought experiment on the island, where inputs used in up production process are observed, as well as available endowments.

On the other hand, based on the information provided in the Sraffian schemes alone, the Sraffian economist cannot regard the uniform profit rate as an equilibrium position to which the neoclassical model should converge in the long run. To prove the point, a specific convergence process based on the mobility of 'capital' across industries should be modeled.

3.5.3 Constant relative prices

Several authors have discussed the compatibility between neoclassical theory and Sraffian schemes in the context of intertemporal or sequential equilibrium models (Hahn, 1982; Mandler, 1999b, 2002; Garegnani, 2005) or in the context of overlapping generation models (Geanakoplos, 1989; Yoshihara and Kwak, 2022).

Most neoclassical models have considered some variations of the model in Section 3.3. Let us suppose that firms have purchased the required inputs at time $t-1$ at the prevailing prices at time $t-1$ and have thus begun the production process. At time t, the neoclassical economist observes the inputs used up in production and the output produced. She aims at computing the prices and wages at time t which make the plans of firms and consumers compatible, given technology, endowments and preferences.

However, the prevailing exchange-values at time t will not, in general, be independent of the exchange-values at time $t-1$. While the wage is paid post-factum at time t, material inputs (or 'factors') are paid at time $t-1$[10]. Hence, factor prices paid at time $t-1$ cannot be determined in the model until prices of the output produced by firms are also determined (Mandler, 1999a).

Sraffa in order to criticize the Marshallian and Austrian theory of capital. However, Sraffa's critique to the 'marginal theory of value and distribution' does not rely on the assumption of a uniform profit rate. For a Neo-Ricardian view of the long-run convergence to a uniform profit rate, see Garegnani (2002b), Garegnani (2002a) and Petri (2003). For a discussion on the difficulties of the convergence to uniform profit rates, see Steedman (1984), Nikaido (1978) and Nikaido (1983)

[10] According to the prevailing economic interpretation in neoclassical theory, firms would obtain 'factors' by borrowing them - or purchasing them - from households/consumers, which will benefit from income derived from profits or wages.

However, in order to transform agents' expectation in determinate equilibrium prices, all firms' expectation at time $t-1$ of output prices at time t must be correct and they must coincide due to the assumption of perfect foresight; alternatively, complete markets with simultaneous trading must be assumed, so that the prices of all goods at all dates can be determined simultaneously at time $t-1$ (which is equivalent to assuming a perfect foresight equilibrium).

The tradition of Walrasian general equilibrium stemming from Arrow and Debreu (1954) has heavily relied on such methods in order to make intertemporal equilibria determinate. Other authors have regarded these methods as an idealized conditions, thereby developing different models in which agents have different expectations of future prices in sequential trading (Hicks, 1939; Mandler, 1995; Grandmont, 1982), or where markets are not supposed to be complete (see, among others, Geanakoplos (1990)).

If the neoclassical economist on the island considers (3.34), substitutes it in (3.2) , and - since the wage is paid post-factum at time t - imposes $w_t = \frac{w_{t-1}}{1+r_i}$, she gets the following redefinition of the profit function:

$$\pi_{i,t} = f_i(\mathbf{a}_i, l_i, \theta_i)_i p^e_{i,t} - \mathbf{a}_i \mathbf{p}_{t-1} - w_t l_i \tag{3.2"}$$

From (3.2"), assuming constant returns to scale and perfect foresight ($p^e_{i,t} = p_{i,t}$), the neoclassical economist can derive the same model described in Section 3.3. The model will have n^2+3n+1 independent equations and an equal number of unknowns, with $\mathbf{p}_{t-1}$ replacing $\mathbf{p}$ and $\mathbf{p}_t$ replacing $\mathbf{r}$[11].

In this setting, it could be argued that the model derived from (3.2") generates Sraffian schemes as a special 'long-run' case in which, due to the assumption of a uniform profit rate, present and future prices are constrained to be proportional, so that relative prices are bound to be in the same proportion for all commodities over time:

$$\frac{p_{j,t}}{p_{j,t-1}} = \frac{p_{k,t}}{p_{k,t-1}} \quad \forall\ j,k\ \in i = 1,\dots,n \tag{3.37}$$

Such a condition might also apply to neoclassical models of balanced growth, in which endowments grow at a uniform rate across sectors in each period.

The Sraffian economist's model in Section 3.4, however, aims at computing prices consistent with the reproduction of the system in the current production cycle given the observed quantities of inputs and outputs. If no further structures are imposed on the model, no convergence process towards a long-run position can be explained by the model itself.

Furthermore, the model in Section 3.4 can be extended so as to relax the self-replacing assumption, without making the system inherently determinate. For instance, Venkatachalam and Zambelli (2021) and Venkatachalam and Zambelli (2022) introduce credit and debt instruments in Sraffian schemes.

In this context, revenues and expenditures will not be equal for some industries: a part of the output will be sold to agents which do not have the necessary

[11] The formulation in (3.2") differs from the one adopted in Mandler (1999b), which includes different prices for the same commodity when used as an input or as a produced output.

purchasing power (in real terms), so that IOUs are issued to be paid back in the future. Although this is not a self-replacing state, the system is still indeterminate, as multiple distribution vectors are compatible with the repayment of debt and credit over time. In this class of models, non-neutrality of money - which is not observed in an Arrow-Debreu general equilibrium model with complete markets - is observed and treated.

Moreover, a non-zero net investment may be introduced in order to study the dynamics of the system over time and allow for the investigation of the conditions leading to growth or contraction of the economy. In this case, a matrix of investment $\mathbf{Inv}_t$ can be introduced so that (3.31) should be modified as follows:

$$\mathbf{s}_t = \mathbf{c}_t + \tilde{\mathbf{i}}_t \tag{3.31'}$$

where $\mathbf{c}_t$ is the aggregate social consumption vector and $\tilde{\mathbf{i}}_t = \mathbf{Inv}_t \mathbf{e}_{n_\omega \times 1} - \mathbf{E}_t \mathbf{e}_{n_\omega \times 1}$ is the aggregate net investment vector, defined as the excess of the gross investment vector over the vector of social endowments.

Therefore, even in the absence of the self-replacing assumption and in the absence of constant relative prices over time, Sraffian schemes remain indeterminate without the imposition of further structures.

3.5.4 Aggregation problems, constructivity and computational difficulties

In Section 3.3, the neoclassical economist's model is closed by the condition in (3.21) stating that the excess demand of each of the n goods must be equal to zero. Each aggregate excess demand z_i is defined by the sum of the individual consumption of each good i demanded by each agent ω which maximizes (3.18) for all agents in n_ω.

To effectively compute the price vector in (3.21), the neoclassical economist needs an algorithm which is able to halt when the aggregate excess demand equals to zero in a finite number of steps.

Constructing the aggregate excess demands in (3.21) is possible if all the utility functions U_ω are given. At this point, unless some external data generating process is known or assumed, the neoclassical economist should be able to estimate the parameters associated to each utility functions when the price vector is Walrasian. However, this would add at least n_ω equations to the system, thereby generating further degrees of indeterminacy: the parameter space of individual utility functions should be such that a positive price vector is calculated in (3.21).

Using a more general specification, the island's economy could be simply specified in terms of an aggregate excess demand function; the function could be thought of as being derived from the richer specification in Section 3.3.

While several economists have attempted to provide a more general solution to the problem through this method, the *Sonnenschein-Mantel-Debreu* (henceforth *SMD*) theorem has led such endeavors up a blind alley due to a serious

aggregation problem (Sonnenschein, 1973; Mantel, 1974; Debreu, 1974). This result shows that for any arbitrary function $\lambda(\mathbf{p})$ which satisfies continuity, homogeneity of degree zero and Walras' law ($\mathbf{p}\lambda(\mathbf{p}) = 0$), there exists an economy of n_ω consumers whose aggregate excess demand function coincides with $\lambda(\mathbf{p})$ and a general equilibrium exists.

As a consequence, the *SMD* theorem implies that (3.21) is satisfied for any function satisfying the aforementioned conditions, regardless of the individual utility maximization process faced by the n_ω consumers in (3.18). In the words of Mas-Colell et al. (1995), *SMD* is really an "*anything goes*" theorem.

A corollary of *SMD* is that the *tatonnement* process which Walrasian economics relies on to prove the convergence to an equilibrium does not always find a unique and stable equilibrium (Ackerman, 2002).

Moreover, *SMD* implies that, without the assumption of stronger restrictions, the neoclassical model in Section 3.3 would not be able to produce falsifiable predictions about aggregate market variables. Any observed data could be related to a function with the properties of $\lambda(\mathbf{p})$, unless further structure is imposed, and any function with the same properties could serve as an excess demand function.

It could be argued that *SMD* does not impair the capacity of Walrasian general equilibrium to produce testable predictions, but motivates the need for equilibrium refinements. If this is the case, however, Sraffian schemes cannot be regarded as a specialization of Walrasian general equilibrium due to the absence of assumptions on agents' preferences, as these assumptions cannot be inferred from the properties of excess demand functions and are, by definition, specifically related to the refinement which is adopted.

On top of that, *constructivity* and *computability* represent even more pressing problems for the neoclassical model. Even if the equilibrium concept described by the neoclassical economist could be refined, the mathematical instruments used in the model may lead to multiple equilibria (i.e., a form of indeterminacy) or to the inability of effectively computing the equilibrium price vector.

In modern general equilibrium theory (Arrow et al., 1971), fixed-point theorems in the versions of Brouwer or Kakutani are the most widely accepted mathematical tool in proving the existence of a Walrasian equilibrium.

According to this theorem, for any continuous function $f(\cdot)$ mapping a compact convex subset of the Euclidean space to itself there is a point x_0 such that $f(x_0) = x_0$; the version in Kakutani (1941) generalizes the theorem to set-valued functions.

Although Brouwer's fixed point theorem has been used in a wide range of economic applications, the author of such a discovery came to reject its validity due to its *non-constructive* content (Brouwer, 1952). The crucial point in Brouwer's original argument relied on a proof by contradiction regarding the possibility to find a function mapping a compact convex subset of the Euclidean space to itself without a fixed point. In his subsequent argument, Brouwer re-stated the fixed-point theorem without relying on the deductive proof.

However, to successfully compute a Walrasian price vector, the neoclassical economist should be able to make the excess demand function effectively

computable (Velupillai and Zambelli, 2013).

The problem was highlighted by the famous economist Kenneth Arrow in 1986:

> "*[T]he claim the excess demands are not computable is a much profounder question for economics that the claim that equilibria are not computable. The former challenges economic theory itself; if we assume that human beings have calculating capacities not exceeding those of Turing machines, then* ***the non-computability of optimal demands is a serious challenge to the theory that individuals choose demands optimally***." (Letter to Alan Lewis, emphasis added, cited in Velupillai (2014), p. 38)

In order to prove the possibility of computing excess demands, authors such as Scarf (1982) build on the *Uzawa equivalence theorem*, which states that any algorithm ensuring the computation of equilibria of arbitrary economies specified in terms of aggregate excess demand functions also ensures the computation of fixed points of arbitrary mappings of the simplex into itself (Uzawa, 1962).

However, several authors in the tradition of constructive mathematics have analyzed the existence of such an algorithm, concluding that excess demands are ultimately not computable. Within this tradition, the definition of *Turing Machine* as a digital computer with unlimited memory is a well-accepted mathematical formalization of a finite algorithm.

Under the *Church-Turing thesis*, according to which every recursive function can be effectively computed if and only if it has a Turing Machine representation, it is possible to check whether Walrasian general equilibrium price vectors and excess demands can be computed by a Turing Machine.

Applying this concept to the Walrasian general equilibrium framework, it could be argued that an aggregate excess demand function is 'Turing-computable' if there is an algorithm such that an equilibrium price vector can be computed, given an arbitrary degree of precision, and if there is an algorithm explaining how many steps of computation the first algorithm must execute before it is guaranteed that an equilibrium has been reached (Richter and Wong, 1999).

The procedure developed by Scarf (1973) is a natural candidate for such an algorithm, and it is based upon the equivalence between the existence of a Walrasian general equilibrium and the existence of a Brouwerian fixed point in Uzawa (1962).

The crucial point of the theorem is to devise a continuous excess demand function which satisfies Walras' Law and homogeneity from an arbitrary continuous function mapping the unit simplex onto itself, such that the equilibrium price vector, implied by the aggregate excess demand function, is also a fixed point for the arbitrary function from which it is derived. Such an algorithm, defined as a constructive or recursion-theoretic procedure, does not exist (Velupillai, 2009, p. 1407–1409).

Hence, the absence of such an algorithm implies that there is no finite list of instructions which permits a Turing machine to decide effectively, for each

possible candidate (arbitrary) price vector, whether or not it is an element of the subset of the price vectors for which the aggregate excess demand is zero and halt when such a price vector is found. The Uzawa equivalence theorem is then *undecidable* and recursively unsolvable[12] (Jones, 1974, p. 725), as the underlying decision procedure would never halt.

The absence of implementable, algorithmic, solutions is certain when the economic magnitudes in Section 3.3 model are defined in the domain of real numbers, given the uncountable nature of irrational numbers. However, even when the economic magnitudes are defined in the domain of rational numbers, the absence of an algorithmic procedure is still a problem facing the neoclassical economist who wishes to find the price vector for which the aggregate excess demand is equal to zero.

Furthermore, the procedure devised by Scarf (1973) is able to *approximate* a fixed point but not to *effectively compute* it (Richter and Wong, 1999, p. 10–12), thereby making it impossible for the neoclassical economist to effectively compute the price vector in (3.21).

Some neoclassical economists, such as Smale (1976), have recognized the difficulties in the fixed-point approach and in using real numbers:

> " *We return to the subject of equilibrium theory. The existence theory of the static approach is deeply rooted to the use of the mathematics of fixed point theory.* ***Thus one step in the liberation from the static point of view would be to use a mathematics of a different kind****. Furthermore, proofs of fixed point theorems traditionally use difficult ideas of algebraic topology, and this has obscured the economic phenomena underlying the existence of equilibria. Also the economic equilibrium problem presents itself most directly and with the most tradition not as a fixed point problem, but as an equation, supply equals demand. Mathematical economists have translated the problem of solving this equation into a fixed point problem.* ***I think it is fair to say that for the main existence problems in the theory of economic equilibrium, one can now bypass the fixed point approach and attack the equations directly to give existence of solutions****, with a simpler kind of mathematics and even mathematics with dynamic and algorithmic overtones*". (Smale, 1976, p. 290; emphasis added)

Solving the excess demand problem in the way suggested by Smale (1976) would require that the system of equations in Section 3.3 is solved for non-negative valued, rational-number variables - i.e., the kind of numerical domain which can be *homomorphic* to observed economic systems. Such an environment would require the characterization of excess demand functions as parametric Diophantine equations[13].

[12] For a different proof of the undecidability of the Uzawa equivalence theorem, see Tanaka (2009).

[13] See Velupillai (2009) for a discussion of the problems related to the general solvability of excess demands in the form of Diophantine equations.

Therefore, characterizing the island's economy as a class of aggregate excess demand functions and the Walrasian general equilibrium price vector as the fixed point of such functions may leave the neoclassical economist unsatisfied in her search for a determinate solution to the problem of distribution.

A constructive and computable solution to the neoclassical economist's problem has been provided by Zambelli (2022b). Building on generalized Sraffian schemes as exemplified in Section 3.4, Zambelli (2022b) introduces a finite number of consumers whose preferences are mapped by Cobb-Douglas utility functions with randomly generated parameters.

Having defined an arbitrary degree of precision as the difference between aggregate demand for each good i and the available surplus for each good i, and a set of possible combinations of firms' densities across sectors, Zambelli (2022b) computes an aggregate excess demand function for each virtual economy associated to each vector of firms' density. For each virtual economy, a set of possible functional distribution vectors and a related set of personal income distribution vectors are computed.

Out of 1711 different virtual economies, a Walrasian equilibrium is found in 480 economies (Zambelli, 2022b). In 373 of the virtual economies in which Walrasian price vectors are found, there are multiple equilibria. The total number of Walrasian equilibrium points for the whole set of economies is 1782 out of a 301 million total market-clearing points (obtained by multiplying 176,219 possible functional income distributions by 1711 possible distribution of firms across sectors).

Hence, a constructive definition of the problem can be found if Sraffian schemes are regarded as budget constraints for the island's economy and if the full set of all possible market-clearing price vectors is computed, given an arbitrarily defined degree of precision.

In this context, the Sraffian economist's model provides a *general* set of bookkeeping market-clearing equilibria. If this set includes one or more equilibria whose associated price vector is such that the aggregate excess demand constructed out of the specified n_ω utility functions is equal to the available surplus, then at least one Walrasian equilibria can be computed in the island's economy.

However, in this procedure, the Walrasian case can be prioritized with respect to other cases only via a suitable *specialization* of the general model in Section 3.4, with the imposition of further structures on the model.

As the existence of a Walrasian price vector is not guaranteed in general, several elements of the set of self-replacing distribution vector will not be consistent with the maximization of the utility functions of consumers. Furthermore, several price vectors that are consistent with utility maximization will not be consistent with market-clearing.

3.6 Conclusions

This Chapter has investigated the conditions under which Sraffian schemes can be derived as a special case from a Walrasian general equilibrium model.

By devising a thought experiment in the spirit of Hahn (1982) and applying the constructive methodology developed by Zambelli (2018b), the chapter has shown that when a neoclassical and a Sraffian economist land on the same island and try to compute prices, profit rates and wage rates according to their different equilibrium concepts, no general conclusion can be drawn as to whether one of the two models is a special case of the other.

Common arguments about the nature of Sraffian schemes as a special case of Walrasian general equilibrium have been disputed. Building on the interpretation of Sraffa in Bharadwaj (1963) and Zambelli (2018b), it has been shown that arguments based on the flexibility of production coefficients, the uniformity of profit rates across sectors, intertemporal specification of Sraffian schemes, and topological existence theorems of Walrasian general equilibrium are not sufficient in order to prove that Sraffian schemes arise out a specialization of neoclassical theory.

Moreover, it has been shown that when Sraffian schemes are set in a fully-fledged neoclassical model, including a neoclassical choice of technique (Zambelli, 2022a) and a set of consumer preferences mapped by utility functions (Zambelli, 2022b), income distribution is still indeterminate.

As Walrasian excess demand functions cannot be effectivized, the neoclassical economist on the island would find herself unable to compute the Walrasian price vector and prioritize the associated distributional pattern over the domain of self-replacing distributional patterns and market-clearing price vectors defined and computed by the Sraffian economist.

On the other hand, the constructive method underlying Sraffian schemes can provide an effective procedure in order to check whether one or more market-clearing price vectors is also Walrasian, given a production set, a set of consumer preferences and an arbitrarily defined degree of precision.

However, as shown by the numerical example in Zambelli (2022b), only a subset of price vectors which happen to be Walrasian in the domain of market-clearing price vectors. Furthermore, the model in Section 3.3 shows that flexible production coefficients and factor substitutability does not reduce the degree of indeterminacy, as in Zambelli (2022a). To prove that the Walrasian case must be prioritized, further specializations of the general *indeterminate* model are required.

Hence, Sraffa's '*prelude to a critique of economic theory*' and - more specifically - his critique of the 'marginal theory of value and distribution' cannot be regarded as a special case of Walrasian general equilibrium in the context of computable and constructive methodologies. When topological existence theorems, devised in the context of formalist models, are not sufficient to demonstrate the existence of a general equilibrium, the indeterminacy of income distribution is a feature of both Sraffian and neoclassical economic models.

Chapter 4

Output gap, participation and minimum income: a proposal for Italy

4.1 Background and motivation

A decade after the 2008–09 Great crisis[1], economic conditions in the Euro area are improving, albeit the recovery is feeble, and the labor market has not been catching up at a stable pace. According to Eurostat, Euro Area (EA19) aggregate unemployment stood at 8.3% in June 2018, after having reached a peak of 12.1% in 2013[2].

Despite the apparent improvement, wage growth remains subdued and measures of underemployment which go beyond the narrow definition of unemployment do not point to a strong recovery of labor market conditions. Moreover, a high dispersion of unemployment rates still persists among countries, with Greece, Italy and Spain showing worse labor market conditions compared to Central and Northern European countries.

To this regard, Figures 1 and 2 show broader estimates of labor underutilization in the Euro area and in Italy, respectively, including part-time workers, people seeking work but not immediately available, and people available to work but not seeking.

While all indicators of labor underutilization sharply decline from 2014 onwards, in 2017 the broader measure of labor market slack hit 18% in the Euro

[1] This Chapter reprints a paper published in the *Journal of Post Keynesian Economics*, 2021, written jointly with Walter Paternesi Meloni and Pasquale Tridico. For the published version see Bracci et al. (2021).

[2] On January 2020, Eurostat indicated a positive but moderate improvement concerning Euro area (EA19) unemployment rate (7.4%). Importantly, the reader should take into account that the present work has been conceived in 2018 and developed before the outbreak of the Covid-19 emergency.

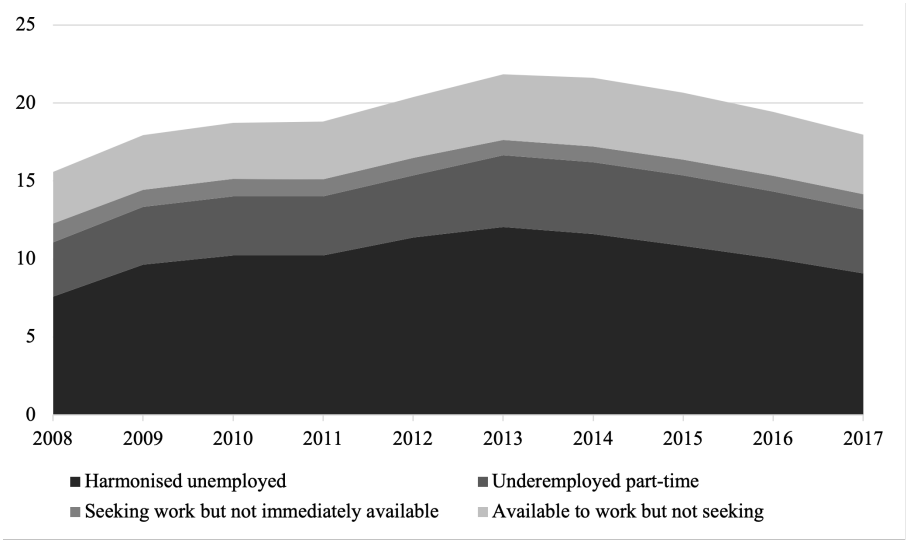

Figure 4.1: Broader estimates of labor underutilization in the Euro area. Source: elaborations on OECD and Eurostat. All components are expressed as a percentage of the active population. The latest observations are for the yearly average of 2017.

area and 26.1% in Italy (expressed as a percentage of the active population). Moreover, while the bulk of labor underutilization in the Euro area is given by the official unemployment rate, discouraged workers – i.e. people that are available to work but are not seeking it – hit 11.6% in Italy, while the same magnitude stood at 3.8% in the Euro area in 2017. Instead, in the United States, the broader measure of unemployment (the U-6 index, provided by the U.S. Bureau of Labor Statistics) including discouraged people and other marginally attached workers stood at 7.5% in July 2018, far below its peak of 2009 (17.1%).

Hence, the persistent labor market slack in the Euro area indicates the possibility of expanding employment through discretionary fiscal policies without incurring in significant inflationary pressures. This implies that potential GDP of Euro area economies may be strongly underestimated, particularly in Mediterranean countries.

Along these lines, the former European Central Bank (henceforth, ECB) President Mario Draghi has stated that "*estimates of the size of the output gap have to be made with caution* [...] *if substantially more workers can be drawn into the labor force, it would be possible for the labor market to strengthen further without generating wage pressures*" (Draghi, 2018).

In a recent speech, also the current ECB President Christine Lagarde has admitted the presence of a 'hidden slack': in her words, "*studies that assume that the output gap has been much larger have, in general, outperformed those*

that use traditional estimates" (Lagarde, 2020) [3].

Exploiting this additional non-inflationary fiscal space might be extremely beneficial not only for the Euro area as a whole, but in particular also for countries that experience high degrees of labor underutilization combined with prolonged economic stagnation such as Italy.

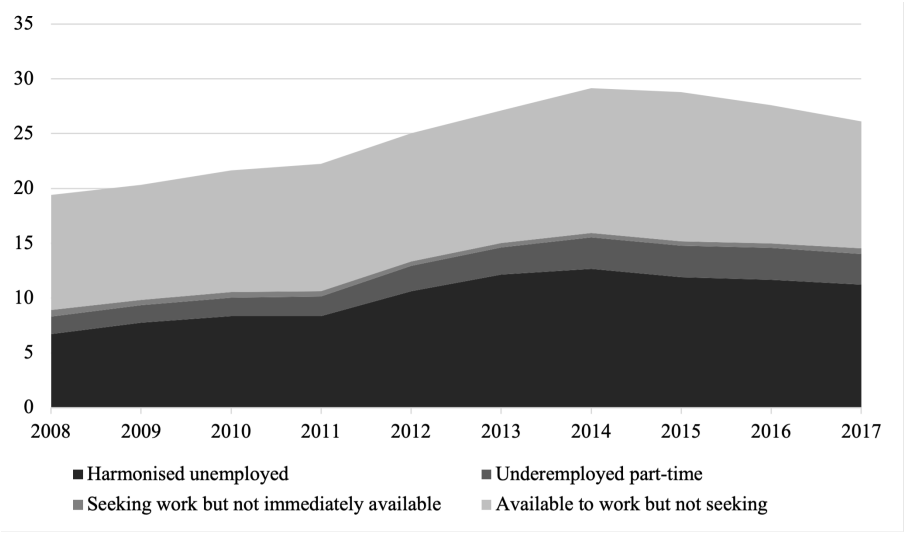

Figure 4.2: Broader estimates of labor underutilization in Italy. Source: elaborations on OECD and Eurostat. All components are expressed as a percentage of the active population. The latest observations are for the yearly average of 2017.

Due to the fundamental relevance of the potential output in determining the fiscal spaces for government, it is crucial to understand, at least in general terms, how it is estimated. In fact, the debate on potential GDP has recently shifted from the theoretical definition to the methods for its estimation.

On the one hand, a substantial consensus has emerged, and it has been endorsed by European institutions, in considering potential GDP as the maximum level of income that can be achieved by fully utilizing available resources (capital and labor) and that is compatible with stable inflation (Okun, 1962; Gordon and Clark, 1984).

On the other hand, the estimates of structural unemployment, i.e. the unemployment rate consistent with non-accelerating inflation (the NAIRU or NAWRU)[4], proposed by the European Commission (henceforth, EC) for the es-

[3] See on this Jarociński and Lenza (2018)

[4] It is worth pointing out that in the EU fiscal framework reference is made to the NAWRU (non-accelerating wage rate of unemployment) instead of the NAIRU when assessing the 'natural' or 'structural' rate of unemployment. While the latter refers to the traditional price inflation features in the Phillips curve, the former refers to wage inflation. Despite this difference, they can be interpreted in the same way: when the actual unemployment rate is

timation of potential GDP, have turned out to be highly pro-cyclical and hence methodologically debatable (Coibion et al., 2017).

In this regard, two discussion papers by the EC deserve to be mentioned: the former proposes an alternative methodology to the NAWRU-based one to reduce pro-cyclicality issues (Lendvai et al., 2015); the latter refers to a method which mixes information about the business cycle and labor market conditions (Hristov et al., 2017). The criticism concerning pro-cyclicality, along with the documented revisions made to output gap estimates from 2001 onwards (Darvas, 2013) and the discrepancies among estimations made by different institutions (Darvas, 2019), stimulated a lively campaign against 'nonsense' output gaps, which culminated in a number of empirical works casting doubt on the plausibility of the official estimations of output gaps in the Euro area periphery[5]. Having considered the role of output gap in designing fiscal consolidation strategies in the European Union (henceforth, EU), the controversy has extended from the academic debates to the political sphere.

With respect to the Italian case, which is the subject of this Chapter, the former Minister of Economy and Finance, Pier Carlo Padoan, explicitly considered the existing EU fiscal rules to penalize Italy[6]. Padoan held that alternative methodologies for the estimation of potential GDP would generate additional budgetary flexibility, as Italy would be running a balanced budget in structural terms since 2015.

According to Padoan, in recent years Italy's structural deficits estimated according to the current EU rules are based on underestimated output gaps. Consequently, the official calculation of fiscal space is overly restrictive, while alternative measures would allow for more fiscal space[7].

We fully recognize the relevance and policy implications of this debate, to which our work relates and aims at contributing. Nevertheless, our Chapter will not focus on technical aspects and methodologies concerning the estimation of potential output, nor will propose alternative strategies for the computation of 'unobservable' variables (e.g. the NAIRU).

Specifically, we aim at suggesting an expansionary policy measure consisting in deficit-financing a conditional minimum income (CMI) for inactive workers that transit into the labor force. The measure would contribute to generate a greater fiscal stance as increasing participation will increase potential output, and therefore the size of output gap. Our policy suggestion, which is motivated by the high degree of underemployment featuring the Italian economy, combines a minimum income with an incentive to rejoin the labor market.

---

below this 'equilibrium' rate, inflationary pressures would take place. Throughout the article, we shall use the terms NAIRU and NAWRU interchangeably.

[5] See for example the note by Adam Tooze published on 30th April 2019 by Social Europe (website: www.socialeurope.eu/output-gap-nonsense). The reader may also refer to the works by Robin Brooks and Greg Basile published by the Institute of International Finance (website: www.iif.com).

[6] The interview was released at *Le Figaro*. See AGI.

[7] In addition to Padoan, also Cacciotti et al. (2013), Cottarelli et al. (2014), Fioramanti (2014), Frale and De Nardis (2017), and MEF (Italian Ministry of Finance) (2017b) posed similar questions to the output gap estimations for Italy.

In doing so, it is also consistent with the 14th principle of the European Pillar of Social Rights: "*Everyone lacking sufficient resources has the right to adequate minimum income benefits ensuring a life in dignity at all stages of life, and effective access to enabling goods and services. For those who can work, minimum income benefits should be combined with incentives to (re)integrate into the labour market*".

The rest of the Chapter goes as follows. Section 4.2 illustrates the main characteristics and rules of the EU fiscal framework. In Section 4.3 we provide some elements about the Italian context. In Section 4.4 we outline our proposal and we simulate the results of the introduction of the CMI. Section 4.5 discusses the main criticisms of our proposal. Section 4.6 summarizes and concludes.

4.2 The output gap and the fiscal framework of the Euro area

To understand the functioning of our policy proposal, let us start by introducing some elements describing the fiscal framework in the Euro area. Currently, it is based on the revised Stability and Growth Pact (SGP), which includes two policy instruments.

The first one is the 'preventive arm'. In order to ensure sustainable fiscal policies over the economic cycle, this instrument aims at setting medium-term objectives (MTO) for the structural deficit of each Euro area member. The second one is the 'corrective arm', which imposes corrective actions to member countries whose national budget exceeds the Maastricht Treaty reference value of 3% in nominal terms or whose public debt exceeds the reference value of 60%. Member countries are expected to reach their MTOs by adjusting their structural budget at a rate of 0.5% of GDP as a benchmark if they are not compliant with their MTOs.

Potential GDP assumes a crucial role in the determination of budgetary flexibility. Specifically, the latter is a positive function of the output gap, that is the difference between actual and potential GDP (expressed as a percentage of potential GDP). Basically, the higher is potential GDP, the greater the output gap is for a given actual GDP. The output gap (OG) is calculated as follows[8]:

$$OG = \left[\frac{Y}{Y_{Pot}} - 1\right] \times 100 \tag{4.1}$$

where Y is the actual GDP and Y_{Pot} is the potential GDP computed through an estimation procedure - see (European Commission, 2017). Different methods can be used to estimate potential GDP[9] Particularly, the methodology of the

[8]Following Keynesian insights, fiscal policy is supposed to be countercyclical, hence a greater fiscal stance is allowed when the economy stands well below its potential.

[9]Putting it simply, the methods of computation of potential GDP can be clustered in two families: those ones based on the use of filtering techniques (implying the smoothing of actual GDP data), and those ones based on the production function approach (grounded on the estimation of potential levels of the factors of production). For a broader overview of these

EC for estimating potential GDP of a country is based on a standard Cobb-Douglas production function with constant returns to scale (Havik et al., 2014), which encompasses the measure of the total factor productivity (TFP) through the conventional method of the Solow residual, the net capital stock (K) and the potential labor force (L), as in equation (2):

$$Y_{Pot} = TFP \times (L_{Pot})^{\alpha} \times K^{1-\alpha} \tag{4.2}$$

where α is the estimated labor output elasticity[10], $(1-\alpha)$ is the estimated capital output elasticity, and L_{Pot} is the potential labor force. The latter is computed as in equation (4.3):

$$L_{Pot} = PART_{rate} \times POP_{15-74} \times HOURS \times (1 - NAWRU) \tag{4.3}$$

where $PART_{rate}$ is the participation rate (the ratio between the labor force and the working age population) smoothed with a Hodrick-Prescott filter, POP_{15-74} is the working age population (from 15 to 74 years in the EU framework), $HOURS$ is the trend component of the sum of hours worked in a given year, and $NAWRU$ is the non-accelerating wage rate of unemployment.

Intuitively, equation (3) suggests that the potential labor force is positively influenced by the participation rate and negatively relates to the NAWRU. Hence, as more working age individuals transit from inactivity to employment or unemployment, the potential labor force increases, ceteris paribus, thereby boosting potential GDP.

On the other hand, increases in the 'structural' component of unemployment – captured by the NAWRU – decrease the potential labor force and, consequently, lower potential GDP. The output gap is crucial for the calculation of the structural balance (SB), which in each year is obtained as in equation (4.4) by subtracting the output gap multiplied by an empirically estimated semi-elasticity to the nominal deficit, minus any politically negotiated 'one-off' (that is, exceptional expenditures for natural disasters or banking crises):

$$SB = BB - \beta \times OG - OFF \tag{4.4}$$

where BB is the nominal budget balance-to-GDP ratio, β is the semi-elasticity of the structural balance with respect to the output gap (estimated

methods, see Cerra and Saxena (2000) and Congressional Budget Office (2004). Recently, some challenging attempts to introduce innovative techniques — or interesting modifications to the existing ones — for the computation of potential GDP have been proposed by Fontanari et al. (2020), Charles et al. (2018) and Li and Mendieta-Muñoz (2020).

[10]Equation (2) is homogeneous of degree 1. Since constant returns to scale and perfect competition are assumed, this functional form does not allow for the presence of any 'residual' in the distribution of product. Moreover, being in this framework marginal products equal to their remuneration (in physical terms), the parameter a should reflect the labor share on GDP. In the EC model for the computation of potential GDP, it is fixed for all EU countries at 0.65 - see Fioramanti (2014).

for Italy at 0.55)[11], and OFF is the ratio of *una tantum* expenditures to GDP.

In addition to these technical aspects, the adoption of the EU fiscal framework by member states follows a detailed timeline, which is known as the European Semester: essentially, this represents a roadmap to be followed, accompanied by an exchange between the EC and each member state, to comply with the Stability and Growth Pact [12].

As we shall see in the rest of the Chapter, understanding the complete technicalities of the fiscal framework of the Euro area, as well as the whole process through which governments of member countries are supposed to comply with, is crucial for the reading of our policy suggestion, in terms of viability, effectiveness and implications.

4.3 A bird's-eye view of Italy

4.3.1 GDP stagnation and the labor market

Before turning to our proposal, some features and specificities of the country should be reported to frame the socio-economic context[13]. In 2019, Italy was the only large economy in the Eurozone which had not recovered yet from the 2008–09 Great crisis: although returned to positive (but very modest) growth rates, total and per capita GDP were still below their pre-crisis levels (see Figure 3).

However, the slowdown of the economy seems to have started well before the 2007 financial crash: Italy's economic performances started deteriorating in the second half of the 1990s.

Many commentators looked at this prolonged slowdown as mainly depending on supply-side factors. Accordingly, the causes of stagnation have been often sought in the slowdown of productivity, which in turn would have depended on poor ICT investments, low company dimensions and labor/capital misallocation phenomena Bugamelli et al. (2018). Moreover, low quality of education, an unfriendly business environment and high public debt would have negatively contributed to low economic performances Romei (2018).

This view, however, has been challenged on several grounds. In particular, it seems to overlook that fiscal austerity measures implemented after the 2010–11 European sovereign debt crisis did not favor economic recovery, contrary to what

[11] The semi-elasticity captures the reactivity of the budget balance (in % of GDP) to a change in the output gap. Its value stems from a weighted average of the individual elasticities of each government revenue and spending item. For technical issues, see Mourre et al. (2014) and Price et al. (2014). Recently, Mourre et al. (2019) have offered an update and further analyses on the estimation of this parameter.

[12] The EC yearly analyses in detail EU member states' economic and structural reforms programmes and provides them with a set of recommendations for the subsequent 12–18 months. During this time, each member state has to align their budgetary and economic policies with the objectives and rules agreed at EU level, within the previously mentioned targets. For a more detailed picture, see European Commission (2017).

[13] For a comprehensive overview of the Italian context, see Cesaratto and Zezza (2018) and Storm (2019).

advocated by the 'expansionary austerity' supporters (Alesina and Ardagna, 1998, 2010) - see Botta and Tori (2018) for a critique.

Moving from the recent literature documenting the negative effects of austerity policies on output and employment (Guajardo et al., 2014; Jordà and Taylor, 2016) and from the idea that aggregate demand growth - and hence expansionary fiscal policies - may have effects which go beyond the short run, as recently documented by Girardi et al. (2020), Italy's prolonged stagnation may not be viewed as independent from fiscal consolidation and structural reforms (mainly concerning a continuous process of labor market flexibilization).

From this perspective, the recent policy agenda, after the sovereign debt crisis, is likely to have hindered demand both directly, i.e. with a long series of primary fiscal surpluses, and indirectly, by reducing the labor income share and consequently to further dampen internal demand, in line with findings according to which Italy is a wage-led economy (Onaran and Obst, 2016).

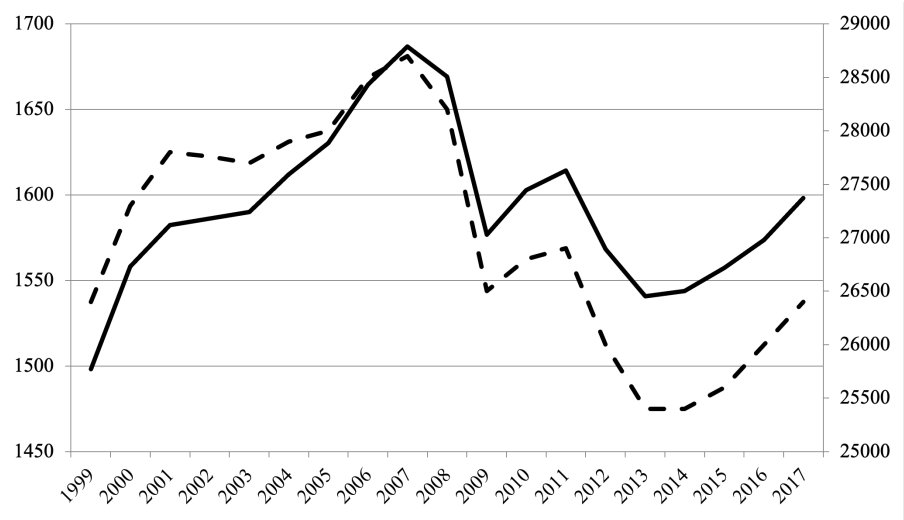

Figure 4.3: Italy's real GDP dynamics. GDP, billions of Euro at constant 2010 prices, bold line, left scale. Per capita GDP, Euro at constant 2010 prices, dashed line, right scale. Source: elaborations on AMECO.

Endorsing a Keynesian perspective, some of the causes of persistent labor market slack in Italy can be traceable in the worrying GDP stagnation documented in Figure 4.3. Remarkably, weak labor market performances are witnessed by a number of macroeconomic indicators, such as low employment rates, low activity rates and high unemployment rates. According to Eurostat, in 2017 they amounted, respectively, to 58%, 62.3% and 11.3%, far beyond the EA19 average (64.2%, 66.4% and 9.1%, respectively). The comparison of participation rates with other countries or aggregates indicates that Italy's participation rate was 8.1 percentage points lower than the EU average and 6.6 p.p. lower than the OECD average (Figure 4.4).

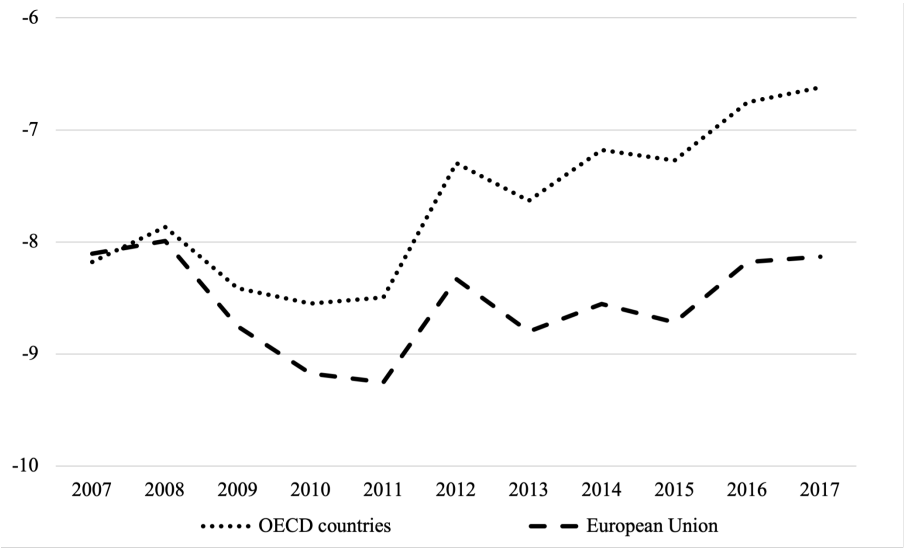

Figure 4.4: Gaps in participation. The figure reports the dynamic pattern of the difference between Italy's participation rate and the average participation rate of two aggregates. Source: elaborations on OECD Stats, Labour Force Statistics, labour force participation rate (15-64).

Combined with high unemployment and low employment rates, far below-average participation rates contribute to shed light on the issue of labor under-utilization which concerns the Italian economy.

According to ISTAT (National Statistics Institute) (2019), labor underutilization in Italy is, on the one hand, related to cyclical factors, namely to the stagnation following the 'double dip' recession; on the other hand, it presents structural components, linked to the size, the specialization and the territorial placement of the productive system, as well as to skills mismatch.

Overall, in 2017 Italy showed about 6 million unused work forces in absolute terms (2.9 million unemployed and more than 3 million discouraged individuals). If we artificially included potential labor forces (i.e. inactive people which are interested in working, while discouraged) within the active population, the inactivity gap between Italy and EA19 would pass from 4.1 to 3.4 percent.

Moreover, underemployment testifies the existence of labor market slack in Italy: in 2017, around 1 million individuals (4.4% of total employment) declared to work less weekly hours compared to their availability. For the same year, ISTAT (National Statistics Institute) (2019) stated that the phenomenon of over-education concerned 5.57 million employed, which represent 24.2% of the total employed and 35% of graduates employed.

The lack of adequate job opportunities may lead to the decision to migrate abroad, a growing phenomenon in recent years (from 40.000 in 2008, to almost 115.000 people in 2017), especially among the youngest people and those with

a high level of education.

While an OECD (2019) work attributed both high unemployment and low participation to skills and territorial mismatch, Italy's low participation rates may also be the outcome of poor job opportunities and underemployment. In this regard, Zezza (2017), by using an approach similar to that of the U-6 index adopted in in the US, estimated an underemployment of approximately 30% in 2017. Combined with some explorative evidence suggesting the existence of a long-run relationship between unemployment and participation (Nemore et al., 2021), which in particular holds for women (Ozerkek et al., 2013), this element supports the existence of a 'discouraged worker' effect.

From this perspective, high and persistent levels of unemployment and underemployment would further confirm that labor market slack is the consequence of low GDP performances, which in turn would depend on stagnating demand.

In addition to the existing literature grounded on the Keynesian tradition – among which, Cesaratto and Zezza (2018), Paternesi Meloni and Stirati (2018) and Storm (2019), with this latter using the evocative expression "*suffocation of aggregate demand*" to describe one of the main traits of the recent Italian economic history – such lack in demand can be testified by the size of Italy's output gap in the last decade.

A comparison across years and with different countries can be useful. On the one hand, the size of Italy's output gap has been higher in recent times then during the previous decades: while in the 1980s and the 1990s the average output gap was roughly -0.5, it stood at 1.7 in the first half of 2000s (average 2000-2007), and then collapsed at 1.85 in the post-crisis period (average 2008-2017).

On the other side, after 2011–12 Italy's output gap was systematically lower than that of other 'big' economies: as shown in Figure 5, a sizeable gap held before 2014, and – despite a recovery in 2017 – the difference with the European average was still about 1 p.p. of potential GDP.

From our perspective, such poor GDP performance, which translated in a worrisome slack in the labor market, should be faced with important policy measures. Recently, a specific initiative has been undertaken by the Italian government to support unemployed, working poor and inactive workers, while creating incentives to actively seek new job opportunities.

We refer to the so-called '*Reddito di Cittadinanza*' (henceforth, RDC), a program of conditional income support started in the first half of 2019: specifically, the program supports household incomes which are below the national poverty line. It pays an income and an housing support up to €780 per month (as an integration of other social transfers) for a single individual to eligible beneficiaries, while requiring the active participation of beneficiaries to job-seeking efforts of public and private job centers.

According to the National Security Institute (INPS), which manages the applications and payments, about 960 thousand households were involved in the program on September 2019[14]. Similar to our proposal, the RDC aims

[14] After the outbreak of the pandemic emergency, the number of beneficiaries dramatically

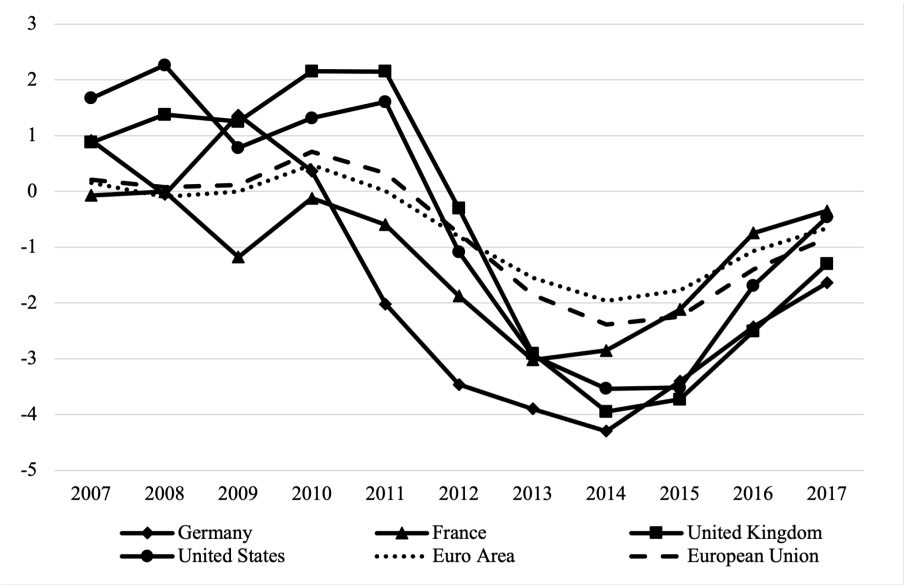

Figure 4.5: Gaps in output gap. The figure reports the dynamic pattern of the difference between Italy's output gap and the output gap of different countries or aggregates. Source: elaborations on AMECO.

at reactivating discouraged people while providing them an income support conditional to active job seeking.

However, our proposal directly targets discouraged individuals (which are outside the labor force by definition) and provides support to their individual income, whereas the RDC targets household incomes which are below the poverty line, regardless of their occupational status.

By targeting a broader spectrum of beneficiaries (and not exclusively inactive people), one can argue that the impact of RDC on Italy's participation rate may be weakened. However, according to an estimate by the National Statistics Institute (ISTAT), the measure is expected to increase the active labor force by 470.000 units in the second quarter of 2020, confirming the tendency of this kind of policies to increase participation[15]. Similar to what we expect from our proposal, the labor supply shock stemming from RDC has been estimated by the Italian Ministry of Finance (MEF) to widen the output gap in the first five years, due to an increase of the labor gap in absolute value (as a result of the upward revision of the participation rate) which is partly offset by a reduction of the TFP gap.

increased up to 3.1 million people, that is 1.3 million households (September 2020). Source: INPS.

[15] See MEF (Italian Ministry of Finance) (2019) (p. 33) for further details.

4.3.2 Actual vs. structural unemployment

According to EU institutions, Italy's high unemployment rates should be to a large extent considered as a structural phenomenon: for 2018, the estimated NAWRU was 9.9%, while actual unemployment rate settled at 10.6% (source: AMECO).

Broadly speaking, this means that discretionary fiscal policies would have been able to reduce Italy unemployment by only 0.7 percent points (i.e. the size of the unemployment gap), while more generous fiscal policies would only accelerate the pace of inflation. Consistently, the main objective of policy makers should be lowering the structural unemployment through liberalization processes (in both the goods and the labor market) and by targeting the education system.

In this framework, unemployment benefits and active labor market policies may help, at least for short periods, to cope with unemployment, while no role is recognized for sustained expansionary fiscal policies, which are almost exclusively considered able to upsurge inflation.

Nevertheless, as indicated by Lang et al. (2020), the notion of a non-accelerating inflation rate of unemployment is "*a well-established but controversial feature of modern macroeconomics*" (p. 19). The notion of NAIRU is grounded on two assumptions that are not shared by Keynesian and post-Keynesian economics, namely the neutrality of money and the irrelevance (at least in the long run) of aggregate demand for the determination of output and employment.

Inevitably, this allows us to consider the notion of NAIRU as crucially related to the view that persistent unemployment is principally caused by excessive labor market rigidities (due to trade union power, strict unemployment protection and other labor market institutions), whereas aggregate demand has no lasting effect on unemployment in case hysteresis is not at work[16].

Despite its controversial theoretical foundations - see among others Stirati (2016), Stockhammer (2008) and Stockhammer (2011), the NAIRU is among the milestones of the current policy-making toolkit. Then, the only existing debates concern its empirical estimation. Particularly for Italy, this is immediately testified by remarkable discrepancies in NAIRU estimations – and then in potential output and output gap – stemming from different international institutions (Fioramanti et al., 2015; Frale and De Nardis, 2017).

This depends on different estimation strategies of 'unobservable' variables. In this regard, the commonly-agreed methodology for the computation of the NAWRU resorts to the standard unobserved component framework to estimate time-varying conceptual variables. Then, by assuming that its dynamics is generated by a stochastic linear process, the unemployment rate is decomposed into the NAWRU plus a gap.

In doing so, the Kalman filter procedure is used: intuitively, while the trend (or 'structural') component is usually interpreted as a random walk without

[16] For a critique on the concept of NAIRU, see also Stirati (2001), Sawyer (2002) and Storm and Naastepad (2007); Girardi et al. (2020) provide some empirical support against the independency of potential GDP of aggregate demand.

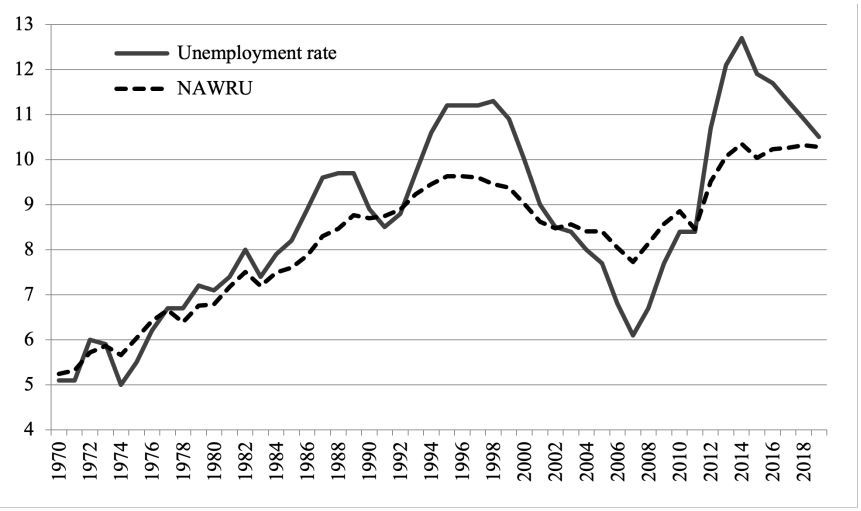

Figure 4.6: Italy's actual unemployment rate and NAWRU. Source: AMECO (data extracted on March 2018).

drift, the cyclical one is estimated by referring to specific economic relationships. Particularly, a negative correlation between unemployment rate and inflation rate (in the spirit of the Phillips curve) is assumed to hold – for the complete procedure, see Havik et al. (2014).

Notwithstanding very sophisticated statistical procedures, the estimates of the NAIRU have been criticized by several scholars. Although the European Commission frequently refines and updates NAWRU estimation procedures, some scholars argued that structural unemployment still largely traces actual unemployment (Ball, 2014; Palumbo, 2015; Gechert et al., 2016; Blanchard, 2018; Jump and Stockhammer, 2019), to the point that also the European Commission (Hristov et al., 2017) has recently recognized that NAWRU estimates which follow this scheme tend to be excessively pro-cyclical[17].

Hence, the reliability of potential output estimates, and consequently the size of structural deficits, are called into question. All in all, this brief overview indicates that both the notion of NAIRU/NAWRU and its empirical estimations should be considered as highly questionable: due to both analytical and empirical issues, we cannot exclude that past levels of unemployment (and then of economic activity) would influence NAIRU estimations, and then those ones of potential GDP and output gap.

This seems to be especially true for Italy, where actual unemployment and the NAWRU systematically tend to comove, as it is evident from Figure 6.

[17] A certain degree of pro-cyclicality has been explained through the presence of labor market rigidities (Rusticelli et al., 2015). In parallel, the 'hysteresis hypothesis' has been advanced to describe labor market 'anomalies' in European Countries (Blanchard and Summers, 1986; Havik et al., 2014).

Such path-dependency casts doubt on the very substance of the concept of NAIRU - see Blanchard (2018) - and leaves an open question: in case structural unemployment would effectively depend on actual unemployment, significant expansionary measures would be able to lower both actual unemployment and the NAIRU[18]. This element will be further discussed in Section 4.5 in connection with our policy proposal.

4.4 The proposal: simulation results and policy implications

As previously discussed, a consensus emerges, particularly among Post-Keynesian scholars, on the criticisms regarding both the notion and the estimations of the NAIRU. We share this view, as well as the policy implications of the use of this approach.

However, the main contribution of this Chapter does not concern technical issues on the estimation of potential output, nor we are going to propose alternative strategies for the computation of non-inflationary unemployment. Specifically, in this work we present a measure consisting in deficit-funding a CMI of €780 per month for 1 million discouraged individuals that accept to restart seeking work, thereby re-entering the labor force by registering at the job center[19].

As a result, the participation rate would increase, boosting ceteris paribus the potential labor force as stated in equation (4.3), which would increase Italy's potential output – as indicated in equation (4.2) – and consequently the output gap and the allowed fiscal stance. In doing so, however, we have to deal with the thorny issue of the path-dependency of the NAIRU, which should be taken into account also for the evaluation of our measure.

Let us start with some data concerning the potential pool of beneficiaries, an element which is crucial for the reliability of our proposal. In Italy, the active labor force is estimated by ISTAT in line with common Eurostat guidelines: according to ISTAT's definition of unemployment, an individual is classified as unemployed if a) she has not worked at least one hour in the week of inquiry, b) she has actively sought work in the last 4 weeks, and c) she is available to start a new job within two weeks.

Monitoring the impact of the CMI program on the active labor force is possible, as ISTAT's statistical inquiry of active labor force is performed weekly. Moreover, activities such as a daily consultation of online job offers or job-seeking activities at job centers are valid causes to change an individual's status

[18] To this regard, Girardi et al. (2020) have recently estimated the effects of demand impulses on key macroeconomic outcomes, finding that autonomous demand shocks fosters GDP, lowers unemployment and upsurges other factors which may stimulate potential growth (such as productivity, participation, and capital stock).

[19] As in the scheme of the RDC, the sum represents the minimum monthly income needed to reach, on average among regions, the reference 'absolute poverty' threshold in Italy, cf. (ISTAT (National Statistics Institute), 2018).

from inactive to unemployed. According to the Eurostat Labor Force Survey, which is grounded on ISTAT surveys, in 2017 there were 3.13 million discouraged individuals in Italy, which contributed to a very low employment rate.

According to the definition provided by ISTAT, those people are explicitly outside the labor force, as they are defined as promptly available to work (in just two weeks), but they have actually 'given up' job searching activities because of the perceived low probability of finding vacancies[20]. Meanwhile, the official unemployment rate stood at 10.9% in February 2018, with roughly 2.9 million people actively seeking work in absolute terms. By definition, the aforementioned 3.13 million discouraged individuals would be available to work but have not been seeking it for the latest four weeks: hence, if these individuals joined the labor market, Italy's participation rate can be revised upwards, thereby improving the economic potential of Italy.

In order to increase the participation rate along these lines, our proposal entails the payment of a minimum income – different from existing unemployment benefits – as an incentive for discouraged people to subscribe to employment centers and attend re-training courses[21].

Of course, an appropriate selection system has to be modeled, as it should effectively target people who are outside the labor force. Accordingly, in managing the applications priority should be given to individuals who join the labor force for the first time, or at least after a long period of inactivity. This can be assessed by monitoring how long ago was the precedent job position of the candidate. The same strategy could also prevent gaming phenomena[22]. We now proceed by estimating the impact of the entrance of discouraged individuals into the labor force on Italy's potential GDP as an exercise of comparative statistics for 2016. Our calculation is based on EC's data, according to which potential hours worked amounted to 1,725.1 per worker, the working age population totaled 45,276,000 individuals, the participation rate stood at roughly 60% and the net capital stock was estimated at €5,162.1 billion (source: AMECO). Accordingly, and considering a 10.1% NAWRU as estimated by the EC, in 2016 potential GDP in Italy totaled €1,603 billion. As actual GDP (at constant 2010) prices totaled €1,573 billion, the resulting official output gap amounted to -1.9, thereby allowing a cyclical correction of about €16 billion to the nominal budget balance.

We then calculate the effects of the measure by computing the cyclical correction to the nominal budget, i.e. the term OG multiplied by β in equation (4), under different hypotheses. Out of 3.344 million discouraged individuals

[20] For the third quarter of 2018, the official statistics on Italy indicate a vacancy rate of 1.1% in industry and services sector, which is approximately equal to 270 thousand vacancies.

[21] Our policy proposal would not imply a reduction in unemployment benefits since people involved would be outside the labor market, and hence they are not be getting any unemployment benefits.

[22] One can argue that a low-paid worker can be encouraged to quit the job in case the CMI is higher than the salary. While the monitoring of the precedent job can formally mitigate this issue, the introduction of a minimum wage level at least equal to the CMI would discourage these behaviors.

Scenarios (2016 data)	*Newcomers in the labor force (million)*	*Cost of the measure (billion €)*	*Participation rate (15-74)*	*Potential GDP (billion €)*	*Output gap*	*Cyclical correction (billion €)*	*Δ Cyclical correction (billion €)*
Actual	-	-	60.34%	1,602.900	-1.9	16.138	-
Policy 1	1	9.360	62.54%	1,640.799	-4.1	35.748	19.611
Policy 2	2	18.720	64.75%	1,678.233	-6.3	54.248	38.111
Policy 3	3	28.080	66.96%	1,715.222	-8.3	71.736	55.598

Table 4.1: Comparative statics model for 2016 (different policy scenarios).

[23], in our policy scenarios for 2016 we consider, respectively, 1, 2 and 3 million newcomers in the labor force, or in other words potentially involved in the CMI measure (Table 4.1)[24].

In case 1 million individuals entered the labor force, the participation rate would increase from 60.3% to 62.5%[25]. Assuming constant TFP and NAWRU, potential GDP would hit €1,641 billion, thereby bringing the output gap to -4.1. The cyclical correction to the structural budget balance would therefore hit €35 billions, increasing the available fiscal space by €19.611 billion. Larger additional fiscal stances would be achieved in case the policy would involve 2 or 3 million inactive workers.

The emerging additional fiscal space could be devoted to discretionary fiscal measures: in our case, these resources would be used precisely in order to deficit-financing the CMI and to potentiate the job centers (more details on this are provided for 2017). Obviously, this would negatively impact the structural deficit since it is reasonable to suppose that the relative increase in actual deficit will be greater than the relative increase in potential output (about 2.3%), but the measure would also produce real effects[26].

For these reasons, we set forth a more sophisticated empirical exercise for the subsequent year. Improvements involve both technicality and policy impact evaluation. In detail, we simulate the reintroduction of 1 million discouraged workers into the labor force in 2017 by implementing the estimation procedure adopted by the Output Gap Working Group (OGWG), a technical body set up to provide advice to the ECOFIN and the EC, in the 2017 Autumn Forecasts. Specifically, we make use of RATS routines and the GAP package[27].

[23] Source: ISTAT, potential labor force. Even if the most common data by ISTAT concern the age group 15–64, we used the 15–74 statistics to be consistent with EC procedures.

[24] In case the scope of measure would not be enough to involve all discouraged individuals, a selection system would be necessary to identify the beneficiaries. For instance, this mechanism may take into consideration the economic condition of the candidates, by given priority to the poorest ones.

[25] The simulation, which results are reported in Table 4.1, starts from 27,317.6 million workers in the total labor force.

[26] To be fair, also the actual unemployment rate would increase, but this would not significantly impact the real conditions of involved people since they would pass from inactive to unemployed, with no changes on their personal situation. The expansionary fiscal policy is however supposed to stimulate the economy and then to foster employment opportunities, as we will show in the simulation proposed for 2017.

[27] Estimates of potential GDP have been computed via the RATS routines made available by the OGWG at the CIRCABC public repository. We have followed the European Commission's methodology which entails the estimation of TFP—in its trend and cycle components—and

Scenarios (2017 data)	*Newcomers in the labor force (million €)*	*Cost (billion €)*	*Participation rate (15-74)*	*Potential GDP (billion €)*	*Output gap*	*Cyclical correction (billion €)*	*Δ Cyclical correction (billion €)*
Actual	-	-	62.27%	1,605.43	-0.594	5.247	-
Policy	1	9.360	64.69%	1,628.28	-1.989	17.565	12.318

Table 4.2: Simulation model for 2017 (single policy scenario).

This improvement is needed in order to cope with potential criticisms of the comparative statistics exercise[28]. For 2017, the EC's estimates entail a 10.3% NAWRU, a €1,605.43 billion potential GDP and a €1,595.9 billion actual GDP (at constant 2010 prices). The relative output gap stood at -0.59, associated with a €5.25 billion cyclical correction to the nominal budget balance.

According to our simulation (reported in Table 4.2), if 1 million discouraged individuals (out of 3.13 million, which count as additional potential workforce according to ISTAT) had joined the labor force, the participation rate would have increased from 62.3% to 64.8%, potential GDP would have increased by €23 billion, and the resulting output gap would have been equal to -1.98; as a result, the labor force shock would have produced an additional fiscal space of €12.318 billion[29].

Assuming a monthly payment of €780 per individuals, the additional nominal budget deficit would make it possible to finance a CMI for the supposed 1 million newcomers in the labor force (which would cost €9.360 billion) and to support the training activities of employment centers.

As already mentioned, the implementation of our proposal implies a higher structural deficit. The latter amounted to approximately 2.09% potential GDP in 2017 (roughly €33.5 billion at constant 2010 prices). Within the existing framework, Italy would have had to undertake fiscal consolidation aimed at improving the structural balance by 1.6 percent points within the subsequent three years[30].

In case all the additional fiscal space generated by the measure would be utilized (this is not the case of our simulation, as we shall see below), structural deficit would hit 2.76% (see Table 4.3). This value is below the 3% Maastricht threshold, the exceeding of which would activate the 'corrective arm'. Nevertheless, even within the action of the 'preventive arm', the higher structural deficit nested in our proposal would raise Italy's medium-term consolidation effort (see Section 4.2).

However, attention should be paid to the real effects of the measure. While we are aware that supporting the employability of individuals alone is not suffi-

NAWRU through the GAP package, that is also publicly available at the CIRCABC repository.

[28] Basically, the procedure we follow in the simulation allow us to consider as 'endogenous' all the variables in our empirics, then taking into account to feasible side-effects of the increasing participation rate on TFP and the NAWRU. See Section 4.5 for a discussion.

[29] Discrepancies in participation rates with the previous exercise would depend on the fact that the simulation for 2017 considers only the civilian labor force.

[30] According to the existing fiscal framework, Italy still has to cope with a prospective structural deficit of 0.5% GDP, that is the medium-term budgetary objective.

cient to create jobs, the stimulus provided by the minimum income to aggregate demand can be successful in fostering output, and consequently employment.

In this regard, we evaluated the real impact of our measure on economic activity. In doing so, however, the true stimulus might be lower than the additional fiscal stance since some of the beneficiaries of the CMI could lose other social transfers.

Accordingly, we cautiously suppose that only €9.198 out of €12.318 billion of the additional fiscal space translate in the budget expansion. Specifically, the €2.958 billion exceeding the cost of the measure will be, in our scheme, devoted to the enhancement of job centers.

While it is reasonable to assume that this effort will completely translate into a fiscal expansion, we suppose that this will occur for only €6.240 out of the €9.360 billion (two thirds of the sum of the payments of the CMI). By assuming a 1.1 fiscal multiplier, as in the Italian Treasury Econometric Model (ITEM) employed by the Italian Finance Ministry[31], the measure would boost real GDP by €10.118 billions, which ceteris paribus would imply a 0.633% higher real GDP. By assuming an employment elasticity of 0.5, in accordance with ECB (European Central Bank) (2016), we also estimate an effect of about 72 thousands additional employees[32].

Table 4.3 presents additional evidence concerning public finance indicators, computed outside the simulation procedure and starting from the above-mentioned fiscal expansion of €9.198 billion (in real terms). Together with the effects on output, from the actual to the policy scenario we calculate an increase in the nominal budget balance (in % of actual GDP) and in the structural deficit, with this latter passing from 2.09 to 2.62 (in % of potential GDP).

While impacting on deficit, it is interesting to note that this measure would not negatively affect the debt-to-GDP ratio, which in the policy scenario exhibits a slight reduction due to the positive effects on output. The real effect, however, will reduce the size of the ex-post output gap, which would settle at -1.37 once considered the real impact of the fiscal expansion (an element which will be discussed in Section 4.5).

Of course, restoring workers' employability is not sufficient to ensure labor absorption: active labor market policies and training have effects on employment as long as new vacancies are created - see Tridico (2019), ch. 4 - while active labor market policies are effective only when vacancies and job seekers grow at the same rate (i.e. when the economy lies on the Beveridge curve).

This seems not to be the case of Italy, as discussed in Section 4.3, where high unemployment should be faced through more ambitious expansionary policies. In this regard, the idea that fiscal stimuli should be regarded as the best policy

[31] Being our stimulus a combination of subsides for the beneficiaries and public effort in reinforcing the job centers, we refer to the fiscal multiplier of total expenditure calculated by MEF (Italian Ministry of Finance) (2017a).

[32] In the simulation reported in Table 4.2, for 2017 we start from 25.7 million workers involved in the civilian labor force. The simulation has been carried out by adding one million persons to the labor force and the unemployment. This increases the unemployment rate up to 14.62%, whose value settles at 14.32% if we consider the expansionary effect of the measure. See Table 4.3 for details.

Table 4.3: Actual and policy macroeconomic scenarios for 2017. Source: AMECO (data extracted on March 2018).

	Actual	Policy	*Notes*
GDP (billion, constant 2010 prices)	1,595.89	1,606.01	(a)
Potential GDP (billion)	1,605.43	1,628.28	(b)
Participation rate (15-74)	62.27	64.69	(c)
Unemployment rate	11.30	14.33	(d)
Labor Force	25,720,493	26,720,493	(e)
Output gap	-0.594	-1.368	(f)
Cyclical correction	-0.327	-0.752	(g)
Nominal budget balance-to-GDP ratio	-2.10	-2.62	(h)
Gov' deficit (billion, current prices)	36.017	45.902	(i)
Gov' deficit (billion, constant 2010 prices)	33.514	42.712	(j)
GDP (billion, current prices)	1,715.10	1,725.97	(k)
Structural deficit (% of potential GDP)	2.09	2.62	(l)
Gov' consolidated gross debt (billion, current prices)	2,266.4	2,276.3	(m)
Debt-to-GDP ratio	132.14	131.88	(n)

(a) AMECO data for the actual scenario. For policy scenario, we consider a fiscal stimulus of €9.198 billion (in real terms), to be multiplied for 1.1 and hence added to the actual GDP.

(b) AMECO data for the actual scenario. For policy scenario,

(c) AMECO data for the actual scenario. For policy scenario, considering 1 million newcomers in the labor force.

(d) AMECO data for the actual scenario. For policy scenario, considering 1 million newcomers in the labor force and the employment effect of the measure. We estimate such effect by applying the employment elasticity of 0.5 to an actual employment of 24.616 million persons (source: AMECO). Specifically, we applied the 0.5 coefficient to the GDP growth of 0.634%, resulting in 0.317% employment growth.

(e) Data refer to the civilian labor force provided by AMECO data for the actual scenario. For policy scenario, considering 1 million newcomers in the labor force.

(f) AMECO data for the actual scenario. For the policy scenario, we calculate the ex-post output gap by considering also the real effect of the measures, and hence by dividing the GDP of the policy scenario for the estimated potential GDP. Note that the output gap estimated in Table 2 (the ex-ante output gap) does not consider the real effects of the measure, and therefore it would be equal to the ratio of actual GDP and the policy potential GDP (in per cent of the policy potential GDP).

(g) Output gap times semi-elasticity to the nominal deficit ($\beta = 0.55$).

(h) AMECO data for the actual scenario. For policy scenario, Gov' deficit (current prices) divided by GDP (current prices).

(i) For actual scenario, GDP (current prices) times deficit-to-GDP ratio. For policy scenario, actual Gov' deficit (constant 2010 prices) plus the fiscal stimulus of €9.198 billion.

(j) For actual scenario, Gov' deficit (current prices) divided by GDP deflator. For the policy scenario, actual Gov' deficit (constant 2010 prices) plus the fiscal stimulus of €9.198 billion.

(k) AMECO data for the actual scenario. For policy scenario, GDP (constant 2010 prices) multiplied by GDP deflator.

(l) Gov' deficit (constant 2010 prices) divided by potential GDP.

(m) AMECO data for the actual scenario. For policy scenario, actual Gov' consolidated gross debt plus the fiscal stimulus of €9.198 billion, with this latter multiplied by the GDP deflator.

(n) Gov' consolidated gross debt divided by GDP (current prices).

GDP deflator for 2017 (base year 2010) is 1.075.

answer to the actual scenario of hysteresis in unemployment and prolonged stagnation is currently considered also from authors belonging to the New Keynesian tradition (Krugman, 2012; Summers, 2015; Summers and Stansbury, 2019).

If we admit the idea that the decrease of structural unemployment can be achieved through the reduction of actual unemployment (Orlandi et al., 2012), this can occur through demand-side policies, as the empirical evidence has recently questioned the relationship between labor market flexibility and employment (Tridico, 2013; Kleinknecht et al., 2016; Brancaccio et al., 2020).

4.5 Criticism and the case of Germany

4.5.1 Main points of criticism

The present proposal can be criticized along three lines. The first strand of criticism argues that this measure is nothing more than an 'accounting trick' that the EC will never approve[33].

Secondly, it has been claimed that whereas such measure would have real effects, the increase in actual GDP would reduce the output gap (as we actually find for 2017) and then reduce the additional fiscal stance.

Thirdly, one may argue that the increase in the unemployment rate caused by the transition of discouraged individuals into unemployment would automatically translate into an increase in the NAWRU, thereby neutralizing much of the effect of the participation rate on the potential labor force. In what follows, we deal with these potential shortcomings.

With respect to the first critique, admitting that the proposal is uniquely related to accounting issues concerning the pool of discouraged and inactive people can contribute to legitimize it from a procedural point of view, since we are measuring the effects on potential GDP by following the calculation procedures of the EC.

Specifically, if the measure effectively increases participation, the computation of the output gap will be grounded on the 'new' active labor force stemming from actual surveys, and hence from the potential labor force a higher contribution to potential GDP will hold. Of course, this would crucially depend on the initial possibility of the government to deficit-financing the CMI. We are conscious of the political implications of this proposal, but they are not object of our discussion here.

Concerning the second critique, one can argue that in case minimum income payments translate immediately into additional consumption, actual GDP will also increase. Consequently, the output gap would not be so large, and the additional fiscal space would not emerge (Garnero, 2018).

Nevertheless, as we indicate in our simulation for 2017, where we consider also the effects of the measure on output and employment, the ex-post output

[33]See Claudio Paudice's interview to Daniel Gros reported in the HuffPost Italia. The interview refers to the work by Tridico and Paternesi Meloni (2018), where the general features of this proposal have been introduced.

gap is larger (-1.37) than the actual (-0.59), implying a discrepancy between actual and potential GDP of €22.2 billion (if we did not consider the real effect on GDP, it would amount at €32.4 billions). This value is far beyond the amount of the stimulus.

Therefore, a sufficiently larger (to implement the measure) output gap would hold even if we consider also the real effect of the CMI. Indeed, a further element has to be taken into consideration: a delay between the increase in potential GDP and the expected increase in actual GDP is likely to be in line with the official procedures of the European Semester, since the EC preventively admits (in year $t-1$) a certain output gap with respect to GDP estimation for year t. Subsequently, the EC updates both estimations of potential GDP and the compliance with the medium-run goal criteria both during year t and ex-post in year $t+1$ (see European Commission (2017), pp. 24–26).

For what concerns the third critique, issues are even more debatable from both a theoretical and a methodological standpoint. Ideally, if including discouraged individuals into the labor force would mechanically induce an increase of the NAIRU (being the activated people translated in the unemployment and at the same time considered as structural unemployed), this could reveal possible weaknesses regarding its estimations, as discussed in Section 4.2.

However, it should be noted that the importance of broader estimates of labor underutilization for the measurement of the economic potential of countries has been stressed by European Commission (2017) and ECB (European Central Bank) (2017): both of them underlined the fact that the presence of 'hidden' unemployment in the form of inactivity impairs a correct assessment of labor market conditions and negatively affects the impact of monetary policy on economic activity.

Moreover, while it is reasonable that an increase in actual unemployment has an impact on the NAWRU – in line with the hysteresis argument advocated by several leading economists (Blanchard, 2018; Cottarelli et al., 2014) – the transformation of cyclical into structural unemployment happens over time, thereby leaving room for an initial increase in potential GDP[34].

In fact, the output gap estimation method officially adopted at EU level allows for a 'gradual' effect of the unemployment rate on the NAWRU. In this regard, the statement by Draghi (2018) reported in Section 4.1 seems to support this interpretation. Moreover, for the case of Italy it is straightforward to note how changes in the NAWRU regularly track changes in actual unemployment

[34]As a matter of fact, the NAIRU theory has become the prevalent explanation of unemployment in the European framework, and it is often used to advocate policies aiming at reducing welfare provision (particularly unemployment benefits) to decrease actual unemployment rates (Stockhammer, 2008). However, this view has been criticized by authors who endorse a different perspective on the inflation-unemployment nexus (Sawyer, 2002; Arestis and Sawyer, 2005; Stirati and Meloni, 2018).According to this alternative approach, output and employment essentially depend on aggregate demand also in the long run, while income distribution can be affected by the bargaining power of parties. See Stirati (2001) and Serrano (2019) for an interpretation of inflation and hysteresis according to these lines of interpretation, as well as Kurz and Salvadori (1995) and Petri (2004) for advanced expositions of the analytical background.

(as reported in Figure 6).

If causality ran from actual unemployment to the NAWRU, expansionary policies would decrease actual and consequently structural unemployment (Stirati, 2016). Therefore, an additional stimulus to aggregate demand triggered by the introduction of the minimum income can promote labor absorption, thereby contributing to the reduction of both the 'additional' unemployment generated by the entrance of discouraged individuals into the labor force.

4.5.2 Labor supply shocks and the case of Germany

However, other factors such as the effect of an influx of discouraged individuals and long-term unemployed on aggregate labor productivity, TFP and the NAWRU can eventually reduce the impulse provided by the labor supply shock to the labor contribution to potential output, thereby reducing the related fiscal space in structural terms. The use of the commonly-agreed methodology and technicalities, as we did in the simulation for 2017, should in principle exclude this potential bias.

However, we find of interest to introduce a paramount reference case. In order to understand the potential impact of increasing participation on potential output, let us refer to the request presented by the German Finance Ministry to the German desk of the Directorate-General for Economic and Financial Affairs (DG ECFIN) of the EC in 2016. Two notes presented for consideration of the Output Gaps Working Group report the results of the request by the German Ministry (DG ECFIN, 2016b,a).

Specifically, the German Ministry requested the DG ECFIN to take migration effects into account within the 2016 Autumn economic forecast. Based on immigration scenarios provided by the German Council of Economic Experts and the German desk of ECFIN, the two notes compared the 'mechanical' method for assessing the impact of asylum seekers on potential output derived by the official production function methodology with a more refined approach which distinguishes between the participation rate of the working age natives and the working age migrants.

Instead of computing a smooth trend over the global labor force, the two works apply a Hodrick-Prescott filter to the native-born labor force, whereas estimates of participation rates for migrant labor force are unfiltered as they already refer to trend rates. The comparison between the participation rates computed via the standard mechanical approach and the refined one shows negligible differences between the two methods in estimating potential output after the migration shock. In 2015 and 2016, potential output growth would deviate from the standard mechanical approach, by not more than 0.2%, whereas in the medium term the impact becomes smaller than 0.1%.

The small entity of the potential growth rate bias is mostly due to the assumption that the whole migrant labor force is low-skilled, consistent with the German Council of Economic Experts (2016), Deutsche Bundesbank (2017) and Deutsche Bundesbank (2016). Regardless of the potential growth bias, the standard mechanical approach implemented in the Spring 2016 economic

forecast for Germany takes the migration shock into account: as Figure 7 shows, Germany held a constant cyclically adjusted budget balance equal to 1.1% of potential GDP between 2015 and 2016, while the German output gap stood at -0.5% in 2014 and 2015, and at -0.2% in 2016, finally turning to zero in 2017.

The forecasts in DG ECFIN (2016b) and DG ECFIN (2016a) are based upon the assumptions presented in German Council of Economic Experts (2016) and the endogenously computed employment and participation rates within the EC's QUEST model (Ratto et al., 2009). In German Council of Economic Experts (2016), four different scenarios for future inflows of migrants and refugees are compared. In the baseline scenario, 80% of the 1 million refugees arrived in Germany in 2015 seek asylum, while 72% of asylum seekers are of working age. Hence, 0.576 million migrants in 2015 are added to the working age population.

The participation rate of migrants is assumed to be a function of the number of years they have already been in Germany, and it increases from 40% in 2015 to 62% in 2020. Moreover, using a cohort approach, it is assumed that each year in which a migrant person stays in Germany, the associated probability to be unemployed decreases: the unemployment rate of migrants is thus set to decline from 80% in 2015 to 32% in 2020. In the baseline scenario, the Germany's government is set to spend €800 per month in benefits for asylum applicants, €550 per month for 75% of the recognized refugees, and a lump sum measure of €2,000 per migrant person for qualification and training measures including language courses in the first year after recognition.

Different scenarios are then computed with respect to the inflows of migrants, the speed of recognition of asylum seekers and the pace of integration in the labor market. Each scenario is connected with a different level of fiscal outlays and produces different results in terms of fiscal sustainability: the quicker is integration in the labor market, the less becomes the fiscal effort required to pay benefits for asylum seekers and recognized refugees. Based on the information provided by DG ECFIN (2016b) and DG ECFIN (2016a), it is possible to understand that the goal of the German Finance Ministry in requesting an adjustment for the migration shock to the DG ECFIN has been to favor a recognition of the related fiscal effort as a response to a structural change in Germany's labor market.

Several other sources have interpreted the migration shock as a structural change in the dynamics of labor supply. Deutsche Bundesbank (2017) revised its estimate of potential output upwards after the influx of refugees, owing to the impact of the migration shock on the potential labor force and its indirect effects on productivity growth and investment (including housing). Deutsche Bundesbank (2017) estimated potential output to grow at 1.3% per year and the Germany's output gap to improve over the medium term, whereas the actual output gap stayed negative until 2017, allowing the cyclically adjusted budget balance to be constant over the 2015–2016 period.

Similarly, Deutsche Bundesbank (2016) estimated the short-term and medium-term projection of Germany's potential output to be influenced by different immigration scenarios: with higher immigration, potential growth increases by 1.3% (compared to 1.2% in case of lower immigration) in the

2016–2020 period, and by 1.0% in the 2021–2025 period (compared to 0.7% in case of lower immigration). The contribution of hours worked to potential growth is estimated to be 0.3% in the 2016–2020 period (compared to 0.1% with lower immigration), and -0.1% in the 2021–2025 period (compared to -0.4% with lower immigration).

All in all, evidence from Germany may suggest that a labor supply shock of inactive workers, such as the reintroduction of 1 million discouraged individuals into the labor, can contribute to boost Italy's potential GDP. Hence, the fiscal expenditures for a minimum income conditional to the reactivation of long-term unemployed and discouraged people out of the labor force through active labor market policies might be at least partially subtracted from the computation of the cyclically adjusted budget balance and the structural balance, which is the relevant magnitude for the evaluation of compliance with EU budgetary rules.

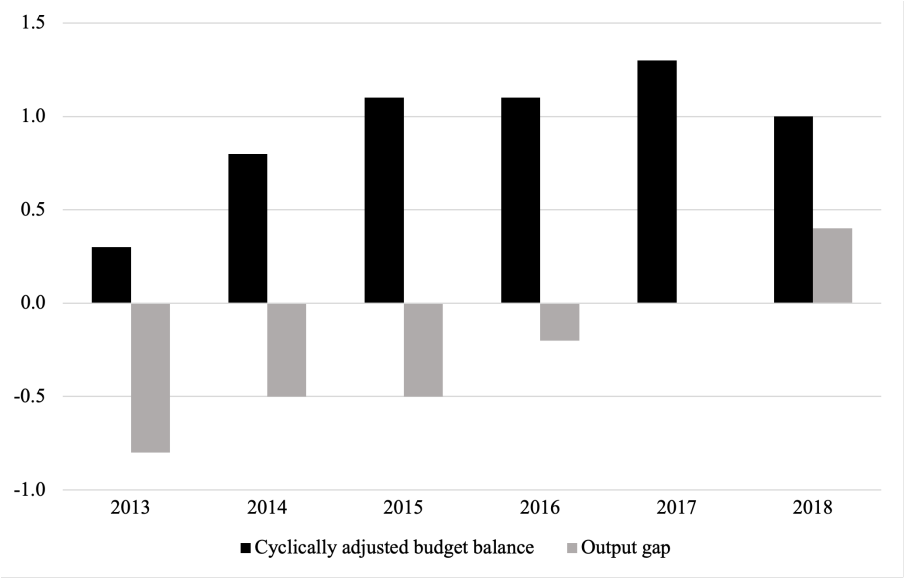

Figure 4.7: Germany's cyclically adjusted budget balance and output gap. Note: both measures are expressed in % of potential GDP. Source: AMECO (data extracted on March 2018).

A further caveat has to be discussed. It has been argued that the influx of discouraged people into the labor force can hardly be translated into an increase of the potential labor force, and this would happen since these workers would be typically low-skilled and untrained (Monacelli, 2018).

Consequently, the NAWRU would increase and average TFP would decrease, thereby negatively offsetting the increase in participation. If this was the case, there would be no additional fiscal space available, to the point that the overall impact of the measure on the output gap could also be negative in a very extreme case.

Remarkably, this argument assumes that workers 'activated' by the CMI would almost totally translate in additional structural unemployment. While the procedure followed for 2017 via RATS routines and the GAP package makes any possible side-effect endogenous, we leave a thorough estimation of the response of the NAWRU and TFP to a labor supply shock for future research. Nevertheless, a comparison with what has recently happened in Germany provides a useful comparative indicator.

Germany experienced a huge labor supply shock consisting in an influx of migrants which in 2015 involved about 900,000 asylum seekers (OECD, 2017). Such inflow has effectively contributed to increase the potential labor force: Deutsche Bundesbank (2016) forecasts an increase of approximately 2 million workers from 2017 to 2025. The age structure of immigrants is extrapolated on the basis of the pattern observed in previous years, in which young immigrants predominated; hence, the inflow of immigrants is expected to offset the decline in participation rates of native-born workers due to ageing.

Moreover, the dynamics of the NAWRU in Germany have not been significantly influenced by the increase in unemployment due to the inflow of migrants: the NAWRU stood at 5.3% in 2013 and decreased to 4.1% in 2016, while it is estimated at 3.6% in 2018 (source: AMECO). According to Romei et al. (2017), about 20% of refugees from crisis-afflicted countries managed to find employment as of June 2017, with more than 50% of refugees and asylum seekers still on integration courses.

Most of the jobs created for refugees are low-skilled, with 57% of employers reporting that they had little difficulties in integrating refugees, 29% reporting no difficulties and 13% reporting considerable difficulties mostly due to language issues.

Deutsche Bundesbank (2016) also found that according to a generalized estimate of the Phillips curve, upward wage pressures resulting from the expansion of domestic demand have exceeded the wage-dampening impact of immigration. However, most of the dampening effect is not to be attributed to immigration pushing down the demand wages of domestic workers, but it is due to the migrant employment composition which generally favors low-skilled jobs with below-average wages.

An extrapolation of these outcomes to the case of Italy is not straightforward. According to Monacelli (2018), a decrease in average wages in Italy as a result of the influx of discouraged individuals in the labor force might dampen labor force participation as many workers that are currently 'discouraged' can be expected to be employed in informal labor markets, especially in the Southern regions of the country.

Thus, there would be an incentive for these individuals to keep working in informal markets and at the same time earn the minimum income, thereby decreasing the demanded wage in informal markets and increasing the demand for labor in these markets. This would dampen the potential labor force. In order to prevent this outcome, fighting illegal work and well-designed training programs to be managed by employment centers are key: if the latter are successful, the positive labor supply shock is not likely to be translated in an

increase in structural unemployment.

Concerning technical matters, the production function approach for the estimation of potential output implies that in the short run the inflow of discouraged workers increases the potential labor force and the unemployment gap widens, as the hike in the unemployment rate is interpreted as cyclical. If unemployment does not decrease over time – i.e. if there is a slow integration of previously inactive workers in the labor market – the Kalman filter approach begins interpreting the increase in unemployment as a trend component, thereby increasing the NAWRU (cf. Section 4.2). The speed of labor absorption is thus crucial in promoting an increase in the productive capacity of the economy, and thus prevent an excessive increase in the structural unemployment.

4.6 Conclusions

Focusing on the case of Italy, this Chapter presents a proposal for promoting a transition of discouraged individuals into the labor force by means of a minimum income conditional to active job seeking. Increasing participation would boost potential output and, other things being equal, widen the stance for fiscal policy.

A simulation of the impact of such a measure on Italy's structural balance has been performed using the production function approach implemented by the EC. For 2016, the simulation consists of a comparative statics exercise, whereas for 2017 a labor supply shock (1 million additional workers in the active population) has been introduced in the procedure used by the OGWG.

In 2016, the upward revision in the participation rate caused by the labor supply shock would increase the available fiscal space by about €19 billions, whereas in 2017 an equivalent labor supply shock would increase fiscal space by about €12 billion. This additional fiscal stance would have been used to deficit-financing a conditional minimum income for inactive workers that transit into the labor force. This measure could be implemented due to the greater fiscal stance generated by the increase in output gap, stemming from the increasing participation.

Finally, by assuming the fiscal multiplier estimated by the Italian Ministry of Finance, we forecast the impact of the introduction of this measure on national income, deficit and debt ratios.

Further research should assess the impact of a labor supply shock to Italy's potential labor force on aggregate wage dynamics, the likelihood of sectoral labor shortages and the dynamics of both NAIRU and TFP. These factors provide a crucial indication as to how quickly an increase in the official unemployment rate would be reabsorbed over time, and whether the reactivation of discouraged people would have a considerable impact on the medium-term dynamics of potential output.

Notwithstanding the feasible real impact of this measure, we are conscious that a minimum income to inactive people should be a complementary measure to the implementation of more ambitious and inclusive employment policies. The CMI would, however, represent a welfare measure designed to fight ab-

solute poverty and to encourage the reintegration of inactive people into the workforce. Obviously, the greater employability stemming from re-training is not sufficient itself to solve the problem of unemployment and underemployment. Nevertheless, the stimulus to aggregate demand arising from the deficit spending may contribute to sustain economic recovery and then employment growth.

However, labor market slack in Italy remains a relevant issue. High and persistent unemployment, particularly among young people, has to be faced through substantial demand stimulus, whose size would be remarkably greater than the measure here proposed. Of course, we are aware that significant expansionary fiscal policies, which we consider extremely relevant to sustain aggregate demand (especially after economic downturns, or during periods of stagnation), are not feasible in the current institutional context of the Euro area, featured by fiscal strictness. Nevertheless, and if politically accepted, our proposal can still represent an attempt to foster GDP and employment, as well as to mitigate poverty and income inequality.

Bibliography

Ackerman, F. (2002). Still dead after all these years: interpreting the failure of general equilibrium theory. *Journal of economic methodology*, 9(2):119–139.

Alesina, A. and Ardagna, S. (1998). Tales of fiscal adjustment. *Economic policy*, 13(27):488–545.

Alesina, A. and Ardagna, S. (2010). Large changes in fiscal policy: taxes versus spending. *Tax policy and the economy*, 24(1):35–68.

Arestis, P. and Sawyer, M. (2005). Aggregate demand, conflict and capacity in the inflationary process. *Cambridge Journal of Economics*, 29(6):959–974.

Arrow, K. J. and Debreu, G. (1954). Existence of an equilibrium for a competitive economy. *Econometrica: Journal of the Econometric Society*, pages 265–290.

Arrow, K. J., Hahn, F., et al. (1971). *General competitive analysis*. Holden-Day.

Ball, L. (2014). Long-term damage from the great recession in OECD countries. *European Journal of Economics and Economic Policies: Intervention*, 11(2):149–160.

Ball, L. and Romer, D. (1990). Real rigidities and the non-neutrality of money. *The Review of Economic Studies*, 57(2):183–203.

Barens, I. and Caspari, V. (1997). Own-rates of interest and their relevance for the existence of underemployment equilibrium positions. In Harcourt, G. and Riach, P., editors, *A 'Second Edition' of the General Theory, volume 1*, pages 283–303. Routledge.

Barro, R. J. (1978). Unanticipated money, output, and the price level in the united states. *Journal of political economy*, 86(4):549–580.

Barrotta, P. (1993). A special case? An epistemological analysis of hahn's critique of Sraffian economics. *History of Economic Ideas*, pages 141–170.

Bharadwaj, K. (1963). Value through exogenous distribution. *The Economic Weekly*, pages 1450–1454.

Bharadwaj, K. (1970). On the Maximum Number of Switches between two Production Systems. *Swiss Journal of Economics and Statistics (SJES)*, 106(IV):409–429.

Bindseil, U. (2014). *Monetary policy operations and the financial system*. Oxford University Press.

Blanchard, O. (2018). Should we reject the natural rate hypothesis? *Journal of Economic Perspectives*, 32(1):97–120.

Blanchard, O. J. and Summers, L. H. (1986). Hysteresis and the European unemployment problem. *NBER macroeconomics annual*, 1:15–78.

Bliss, C. (1975). *Capital Theory and the Distribution of Income*. Elsevier.

Böhm-Bawerk, E. (1923). *The positive theory of capital*, volume 2. GE Stechert & Company, reprint.

Borio, C. (2019). On money, debt, trust, and central banking. *Cato J.*, 39:267–302.

Botta, A. and Tori, D. (2018). The theoretical and empirical fragilities of the expansionary austerity theory. *Journal of Post Keynesian Economics*, 41(3):364–398.

Bracci, G., Paternesi Meloni, W., and Tridico, P. (2021). Output gap, participation and minimum income: a proposal for Italy. *Journal of Post Keynesian Economics*, 44(4):643–676.

Brancaccio, E. (2010). On the impossibility of reducing the surplus approach to a neoclassical 'special case': A criticism of Hahn in a Solowian context. *Review of Political Economy*, 22(3):405–418.

Brancaccio, E., De Cristofaro, F., and Giammetti, R. (2020). A meta-analysis on labour market deregulations and employment performance: no consensus around the IMF-OECD consensus. *Review of Political Economy*, 32(1):1–21.

Brouwer, L. E. J. (1952). An intuitionist correction of the fixed-point theorem on the sphere. *Proceedings of the Royal Society of London. Series A. Mathematical and Physical Sciences*, 213(1112):1–2.

Bugamelli, M., Lotti, F., Amici, M., Ciapanna, E., Colonna, F., D'Amuri, F., Giacomelli, S., Linarello, A., Manaresi, F., Palumbo, G., et al. (2018). Productivity growth in Italy: a tale of a slow-motion change. (422).

Burmeister, E. (1968). On a theorem of Sraffa. *Economica*, 35(137):83–87.

Burmeister, E. (1977). The irrelevance of Sraffa's analysis without constant returns to scale. *Journal of Economic Literature*, pages 68–70.

Cacciotti, M., Frale, C., and Teobaldo, S. (2013). A new methodology for a quarterly measure of the output gap. Technical Report 6, Government of the Italian Republic (Italy), Ministry of Economy and Finance, Department of the Treasury Working Paper.

Cerra, V. and Saxena, S. (2000). Alternative methods of estimating potential output and the output gap: an application to Sweden. *IMF Working Paper*.

Cesaratto, S. and Zezza, G. (2018). What went wrong with Italy, and what the country should now fight for in europe. Technical report, FMM Working Paper No. 37, Hans-Böckler-Stiftung, Macroeconomic Policy Institute (IMK), Forum for Macroeconomics and Macroeconomic Policies (FFM).

Charles, S., Dallery, T., and Marie, J. (2018). Réflexions et suggestions autour du concept d'output gap. *Revue Interventions économiques. Papers in Political Economy*, (60).

Chiodi, G. (2010). The means of subsistence and the notion of 'viability' in Sraffa's surplus approach. In *Computable, Constructive & Behavioural Economic Dynamics*, pages 344–356. Routledge.

Ciccarone, G. (1998). Prices and distribution in a Sraffian credit economy. *Review of Political Economy*, 10(4):399–413.

Clower, R. (1967). A reconsideration of the microfoundations of monetary theory. *Economic Inquiry*, 6(1):1–8.

Coibion, O., Gorodnichenko, Y., and Ulate, M. (2017). The cyclical sensitivity in estimates of potential output. Technical report, National Bureau of Economic Research.

Congressional Budget Office (2004). A summary of alternative methods for estimating potential GDP. Technical report.

Costantini, O. (2015). The cyclically adjusted budget: history and exegesis of a fateful estimate. *Institute for New Economic Thinking Working Paper series*, (24).

Costantini, O. (2018). Invented in america: Birth and evolution of the cyclically adjusted budget rule, 1933–61. *History of Political Economy*, 50(1):83–117.

Cottarelli, C., Giammusso, F., and Porello, C. (2014). Politica di bilancio ostaggio della stima del Pil potenziale. Technical report, Lavoce.info.

Darvas, Z. (2013). Mind the Gap! and the way structural budget balances are calculated. Technical report, Bruegel.org.

Darvas, Z. (2019). Uncertainty over output gap and structural-balance estimates remains elevated. Technical report, Bruegel.org.

De Cecco, M. (1993). Piero sraffa's 'monetary inflation in italy during and after the war': an introduction. *Cambridge journal of economics*, 17(1):1–5.

Debreu, G. (1959). *Theory of value: An axiomatic analysis of economic equilibrium*. Yale University Press.

Debreu, G. (1974). Excess demand functions. *Journal of mathematical economics*, 1(1):15–21.

Deleplace, G. (2014). The essentiality of money in the Sraffa Papers. In *Towards a New Understanding of Sraffa*, pages 139–166. Springer.

Deutsche Bundesbank (2016). Demographic change, immigration and the potential output of the German economy.

Deutsche Bundesbank (2017). The current influx of refugees—projected impact on the labour market and public finances.

DG ECFIN (2016a). Adding migration effects to potential output estimation explicitly — the example of Germany (2015–2020) — update. Technical report, Note for the attention of the Output Gap Working Group.

DG ECFIN (2016b). Adding migration effects to potential output estimation explicitly—the example of Germany (2015–2020). Technical report, Note for the attention of the Output Gap Working Group.

Donzelli, F. (2004). Ascesa e declino della nozione di "saggio d'interesse proprio" in Sraffa. *mimeo*.

Donzelli, F. (2006). Chapter 17 of Keynes' General Theory: The Mystery Unveiled. Technical report, Universitá degli Studi di Milano Working Paper.

Draghi, M. (2018). Monetary Policy in the Euro Area. Technical report, Speech at the Conference Organized by the Institute for Monetary and Financial Stability, Frankfurt, March 14.

Dvoskin, A. and Feldman, G. D. (2021). On the role of finance in the Sraffian system. *Review of Political Economy*, 33(2):261–277.

ECB (European Central Bank) (2016). The employment-GDP relationship since the crisis. Technical report, ECB Economic Bulletin.

ECB (European Central Bank) (2017). Economic Bulletin, issue 3/2017 – Boxes. Assessing labour market slack. Technical report.

Eggertsson, G. B., Mehrotra, N. R., and Robbins, J. A. (2019). A model of secular stagnation: Theory and quantitative evaluation. *American Economic Journal: Macroeconomics*, 11(1):1–48.

European Commission (2017). Vade mecum on the Stability and Growth Pact. Technical Report 52.

European Commission (2019). Vade mecum on the Stability and Growth Pact. Technical Report 101.

European Commission (2020). Communication from the Commission: Economic governance review. Technical report.

Felipe, J. and Fisher, F. M. (2003). Aggregation in production functions: what applied economists should know. *Metroeconomica*, 54(2-3):208–262.

Ferguson, C. (1969). *The Neoclassical theory of production and distribution.* Cambridge University Press,.

Fioramanti, M. (2014). Potential output, output gap and fiscal stance: is the EC estimation of the NAWRU too sensitive to be reliable? Technical report, MPRA Paper No. 73762.

Fioramanti, M., Padrini, F., and Pollastri, C. (2015). La stima del pil potenziale e dell'output gap: analisi di alcune criticità. *Ufficio Parlamentare di Bilancio, Nota di lavoro*, 1.

Fisher, F. M. (1969). The existence of aggregate production functions. *Econometrica: Journal of the Econometric Society*, pages 553–577.

Fisher, I. (1896). Appreciation and Interest. *Publications of the American Economic Association*, 11(4):1–98.

Fontanari, C., Palumbo, A., and Salvatori, C. (2020). Potential output in theory and practice: a revision and update of Okun's original method. *Structural Change and Economic Dynamics*, 54:247–266.

Frale, C. and De Nardis, S. (2017). Quando il gap si fa incerto: stime alternative del potenziale e dell'output gap nell'economia italiana. Technical report.

Franke, R. (1988). Integrating the financing of production and a rate of interest into production price models. *Cambridge Journal of Economics*, 12(2):257–272.

Friedman, M. (1956). The quantity theory of money: a restatement. *Studies in the Quantity Theory of Money*, 5:3–31.

Garegnani, P. (1979). Notes on consumption, investment and effective demand: a reply to joan robinson. *Cambridge Journal of Economics*, 3(2):181–187.

Garegnani, P. (2002a). On some supposed obstacles to the tendency of market prices towards natural prices. In *Equilibrium and Economic Theory*, pages 149–180. Routledge.

Garegnani, P. (2002b). Sraffa's price equations: a stationary economy or 'normal positions'? In Boehm, S., Gehrke, C., Kurz, H., and Sturn, R., editors, *Is There Progress in Economics?* Edward Elgar, Cheltenham and Northampton.

Garegnani, P. (2005). Capital and intertemporal equilibria: a reply to Mandler. *Metroeconomica*, 56(4):411–437.

Garnero, A. (2018). Ma il reddito di cittadinanza non si finanzia da solo. *Lavoce.info*.

Geanakoplos, J. (1989). Overlapping generations model of general equilibrium. In *General Equilibrium*, pages 205–233. Springer.

Geanakoplos, J. (1990). An introduction to general equilibrium with incomplete asset markets. *Journal of Mathematical Economics*, 19(1):1–38.

Gechert, S., Rietzler, K., and Tober, S. (2016). The european commission's new NAIRU: Does it deliver? *Applied Economics Letters*, 23(1):6–10.

German Council of Economic Experts (2016). Chapter 1 — Economic Policy — Focus on future viability. In *Annual Economic Report*.

Girardi, D., Paternesi Meloni, W., and Stirati, A. (2020). Reverse hysteresis? Persistent effects of autonomous demand expansions. *Cambridge Journal of Economics*, 44(4):835–869.

Gordon, R. J. (2012). Is US economic growth over? faltering innovation confronts the six headwinds. Technical report, National Bureau of Economic Research.

Gordon, R. J. and Clark, P. K. (1984). Unemployment and potential output in the 1980s. *Brookings Papers on Economic Activity*, 1984(2):537–568.

Grandmont, J.-M. (1982). Temporary general equilibrium theory. *Handbook of mathematical economics*, 2:879–922.

Grieve, R. H. (2015). Keynes and Sraffa on own-rates: a present-day misunderstanding. *Contributions to Political Economy*, 34(1):1–16.

Guajardo, J., Leigh, D., and Pescatori, A. (2014). Expansionary austerity? International evidence. *Journal of the European Economic Association*, 12(4):949–968.

Hahn, F. (1965). On some problems of proving the existence of equilibrium in a monetary economy. In Hahn, F. and Brechling, F. P. R., editors, *The Theory of Interest Rates*, pages 31–58. London: Macmillan.

Hahn, F. (1981). *Money and inflation*. The MIT Press.

Hahn, F. (1982). The Neo-Ricardians. *Cambridge Journal of Economics*, 6(4):353–374.

Hahn, F. H. (1966). Equilibrium dynamics with heterogeneous capital goods. *The Quarterly Journal of Economics*, pages 633–646.

Harcourt, W. J. and Sardoni, C. (1993). Piero sraffa:'monetary inflation in italy during and after the war'. *Cambridge Journal of Economics*, 17(1):7–26.

Havik, K., Mc Morrow, K., Orlandi, F., Planas, C., Raciborski, R., Röger, W., Rossi, A., Thum-Thysen, A., Vandermeulen, V., et al. (2014). The production function methodology for calculating potential growth rates & output gaps. Technical report, Directorate General Economic and Financial Affairs (DG ECFIN).

Hayek, F. (1931). *Prices and Production*. George Routledge and Sons, Ltd, London.

Hayek, F. A. (1932). Money and Capital: a reply. *The Economic Journal*, 42(166):237–249.

Hicks, J. (1973). *Capital and time: a neo-Austrian theory*. Clarendon Press.

Hicks, J. R. (1939). *Value and Capital*. Oxford: Clarendon Press.

Holston, K., Laubach, T., and Williams, J. C. (2017). Measuring the natural rate of interest: International trends and determinants. *Journal of International Economics*, 108:S59–S75.

Hristov, A., Planas, C., Roeger, W., Rossi, A., et al. (2017). NAWRU estimation using structural labour market indicators. Technical report, European Commission.

Hume, D. (1752). *Political Discourses*. R. Fleming, For A. Kincaid and A. Donaldson.

ISTAT (National Statistics Institute) (2018). Il benessere equo e sostenibile in Italia. Technical report.

ISTAT (National Statistics Institute) (2019). Il mercato del lavoro 2018: verso una lettura integrata. Technical report.

Jarociński, M. and Lenza, M. (2018). An inflation-predicting measure of the output gap in the euro area. *Journal of Money, Credit and Banking*, 50(6):1189–1224.

Jevons, W. S. (1876). *Money and the Mechanism of Exchange*. D. Appleton.

Jones, J. P. (1974). Recursive undecidability – an exposition. *The American Mathematical Monthly*, 81(7):724–738.

Jordà, Ò. and Taylor, A. M. (2016). The time for austerity: estimating the average treatment effect of fiscal policy. *The Economic Journal*, 126(590):219–255.

Jump, R. C. and Stockhammer, E. (2019). Reconsidering the natural rate hypothesis. Technical report, FMM Working Paper.

Kakutani, S. (1941). A generalization of Brouwer's fixed point theorem. *Duke mathematical journal*, 8(3):457–459.

Kaldor, N. (1960). *Essays on Economic Stability and Growth*. London: Duckworth.

Kalecki, M. (1937). The principle of increasing risk. *Economica*, 4(16):440–447.

Keynes, J. M. (1930). *A Treatise on Money: Pure theory of money, Vol. I.* Macmillan, London.

Keynes, J. M. (1936). *The General Theory of Money, Interest and Employment*. London: Macmillan.

Keynes, J. M. (1937). The theory of the rate of interest. In Gayer, A. D., editor, *The Lessons of Monetary Experience: Essays in Honor of Irving Fisher*, volume 1973. London, Allen and Unwin.

Kleinknecht, A., Kwee, Z., and Budyanto, L. (2016). Rigidities through flexibility: flexible labour and the rise of management bureaucracies. *Cambridge Journal of Economics*, 40(4):1137–1147.

Krugman, P. (2012). Monetary vs. fiscal policy, revisited. *New York Times*.

Kurz, H. and Salvadori, N. (1995). *Theory of Production. A Long-Period Analysis*. Cambridge University Press.

Kurz, H. D. (2000). The Hayek-Keynes-Sraffa controversy. *Critical essays on Piero Sraffa's legacy in economics*, page 257.

Kurz, H. D. (2014). Keynes, Sraffa, and the latter's "secret skepticism". In Kurz, H. D. and Salvadori, N., editors, *Revisiting Classical Economics*, pages 122–142. Routledge.

Kurz, H. D. and Salvadori, N. (2021). White elephants and other non-basic commodities: Piero Sraffa and Krishna Bharadwaj on the role and significance of the distinction between basics and non-basics. *The Indian Economic Journal*.

Lachmann, L. M. (1986). Austrian economics under fire: the Hayek-Sraffa duel. In Grassl, W. and Smith, B., editors, *Austrian Economics: Historical and Philosophical Background*, pages 225–243.

Lagarde, C. (2020). The monetary policy strategy review: Some preliminary considerations. Technical report, Speech at "ECB and Its Watchers XXI".

Lane, P. R. (2019). Determinants of the real interest rate. Technical report, Remarks at the National Treasury Management Agency of Dublin.

Lang, D., Setterfield, M., and Shikaki, I. (2020). Is there scientific progress in macroeconomics? The case of the NAIRU. *European Journal of Economics and Economic Policies: Intervention*, 17(1):19–38.

Lendvai, J., Salto, M., and Thysen, A. T. (2015). Structural unemployment vs. NAWRU: implications for the assessment of the cyclical position and the fiscal stance. *Economic papers*, (552):1–28.

Leontief, W. W. (1928). *Die wirtschaft als Kreislauf*. Laupp.

Lerner, A. P. (1952). The essential properties of interest and money. *The Quarterly Journal of Economics*, pages 172–193.

Levhari, D. and Samuelson, P. A. (1966). The nonswitching theorem is false. *The Quarterly Journal of Economics*, 80(4):518–519.

Li, M. and Mendieta-Muñoz, I. (2020). Are long-run output growth rates falling? *Metroeconomica*, 71(1):204–234.

Lucas, R. E. (1972). Expectations and the neutrality of money. *Journal of Economic Theory*, 4:103–124.

Lutz, F. A. and Hague, D. C. E. (1961). *The theory of capital*. Palgrave Macmillan.

Magill, M. and Quinzii, M. (2002). *Theory of Incomplete Markets*. MIT Press.

Mandler, M. (1995). Sequential indeterminacy in production economies. *Journal of Economic Theory*, 66(2):406–436.

Mandler, M. (1999a). *Dilemmas in economic theory: Persisting foundational problems of microeconomics*. Oxford University Press.

Mandler, M. (1999b). Sraffian indeterminacy in general equilibrium. *The Review of Economic Studies*, 66(3):693–711.

Mandler, M. (2002). Classical and neoclassical indeterminacy in one-shot versus ongoing equilibria. *Metroeconomica*, 53(3):203–222.

Mantel, R. R. (1974). On the characterization of aggregate excess demand. *Journal of economic theory*, 7(3):348–353.

Mas-Colell, A. (1989). Capital theory paradoxes: anything goes. In *Joan Robinson and modern economic theory*, pages 505–520. Springer.

Mas-Colell, A., Whinston, M. D., Green, J. R., et al. (1995). *Microeconomic theory*. New York: Oxford University Press.

McCombie, J. S. L. (2001). What does the aggregate production function show? Further thoughts on Solow's "second thoughts on growth theory". *Journal of Post Keynesian Economics*, 23(4):589–615.

McLeay, M., Radia, A., Thomas, R., et al. (2014). Money creation in the modern economy. *Bank of England Quarterly Bulletin*, 54(1):14–27.

MEF (Italian Ministry of Finance) (2017a). Nota di aggiornamento al Documento di Economia e Finanza. Technical report.

MEF (Italian Ministry of Finance) (2017b). Relevant factors influencing debt developments in italy. Technical report.

MEF (Italian Ministry of Finance) (2019). Documento di Economia e Finanza 2019.

Monacelli, T. (2018). Reddito di cittadinanza o gioco delle tre carte? *Lavoce.info*.

Moore, B. J. (1988). *Horizontalists and verticalists: the macroeconomics of credit money*. Cambridge University Press.

Mourre, G., Astarita, C., Princen, S., et al. (2014). Adjusting the budget balance for the business cycle: the EU methodology. Technical report, European Commission.

Mourre, G., Poissonnier, A., Lausegger, M., et al. (2019). The semi-elasticities underlying the cyclically-adjusted budget balance: an update and further analysis. Technical report, European Commission.

Muth, J. F. (1961). Rational expectations and the theory of price movements. *Econometrica: Journal of the Econometric Society*, pages 315–335.

Naldi, N. (2015). Sraffa and Keynes on the concept of commodity rates of interest. *Contributions to Political Economy*, 34(1):17–30.

Nemore, F., Caferra, R., and Morone, A. (2021). Unemployment and labor force participation in Italy. *International Journal of Manpower*, 42(8):1440–1449.

Neumann, J. v. (1945). A model of general economic equilibrium. *The Review of Economic Studies*, 13(1):1–9.

Newman, P. (1962). Production of Commodities by Means of Commodities. *Schweizerische Zeitschrift für Volkswirtschaft und Statistik*, 98:58–75.

Nikaido, H. (1978). Do profit rates equalize by movement of money capital in Marx's scheme of reproduction? *Working Paper no. 7812, Modelling Research Group, Department of Economics*.

Nikaido, H. (1983). Marx on competition. *Zeitschrift für nationalökonomie*, 43(4):337–362.

OECD (2017). Who bears the cost of integrating refugees? Technical report, OECD Migration Policy Debates n. 13.

OECD (2019). Recent trends in the Italian labour market. In *Strengthening Active Labour Market Policies in Italy*. OECD.

Okun, A. M. (1962). Potential GNP: its measurement and significance. In *Proceedings of the Business and Economic Statistics Section of the American Statistical Association*, pages 89–104. Alexandria, VA: American Statistical Association.

Onaran, O. and Obst, T. (2016). Wage-led growth in the EU15 member-states: the effects of income distribution on growth, investment, trade balance and inflation. *Cambridge Journal of Economics*, 40(6):1517–1551.

Orlandi, F. et al. (2012). Structural unemployment and its determinants in the EU countries. Technical report, Directorate General Economic and Financial Affairs (DG ECFIN), European Commission.

Ozerkek, Y. et al. (2013). Unemployment and labor force participation: a panel cointegration analysis for european countries. *Applied Econometrics and International Development*, 13(1):67–76.

Palumbo, A. (2015). Studying growth in the modern classical approach: Theoretical and empirical implications for the analysis of potential output. *Review of Political Economy*, 27(3):282–307.

Panico, C. (1985). Market forces and the relation between the rates of interest and profits. *Contributions to Political Economy*, 4(1):37–60.

Panico, C. (1988). *Interest and Profit in the Theories of Value and Distribution*. Springer.

Pasinetti, L. L. (1966). Paradoxes in capital theory. A symposium: Changes in the rate of profit and switches of techniques. *The Quarterly Journal of Economics*, 80(4):503–517.

Pasinetti, L. L. (2000). Critique of the neoclassical theory of growth and distribution. *PSL Quarterly Review*, 53(215).

Paternesi Meloni, W. and Stirati, A. (2018). Austerità in Italia: i sacrifici alimentano il debito. *Economia e Politica*.

Patinkin, D. and Steiger, O. (1989). In search of the "Veil of Money" and the "Neutrality of Money": A Note on the Origin of Terms. *The Scandinavian Journal of Economics*, 91(1):131–146.

Petri, F. (2003). Introduction by Fabio Petri. In Hahn, F. and Petri, F., editors, *General equilibrium: problems and prospects*. Routledge.

Petri, F. (2004). *General equilibrium, capital and macroeconomics: A key to recent controversies in equilibrium theory*. Edward Elgar Publishing.

Pivetti, M. (1991). *An essay on money and distribution*. Springer.

Price, R., Dang, T.-T., and Guillemette, Y. (2014). New tax and expenditure elasticity estimates for EU budget surveillance. Technical report, OECD Economics Department Working Papers, 1174/2014.

Quandt, R. E. (1961). Review (of PCC). *The Journal of Political Economy*, 69(5):500.

Rachel, Ł. and Summers, L. H. (2019). On secular stagnation in the industrialized world. Technical report, National Bureau of Economic Research.

Radner, R. (1968). Competitive equilibrium under uncertainty. *Econometrica: Journal of the Econometric Society*, pages 31–58.

Ratto, M., Roeger, W., and in't Veld, J. (2009). QUEST III: An estimated open-economy DSGE model of the euro area with fiscal and monetary policy. *Economic Modelling*, 26(1):222–233.

Richter, M. K. and Wong, K.-C. (1999). Non-computability of competitive equilibrium. *Economic Theory*, 14(1):1–27.

Romei, V. (2018). Why Italy's economy is stagnating. *Financial Times*.

Romei, V., Ehrenberg-Shannon, B., Maier-Borst, H., and Chazan, G. (2017). How well have Germany's refugees integrated? *Financial Times*.

Rusticelli, E. et al. (2015). Rescuing the phillips curve: Making use of long-term unemployment in the measurement of the NAIRU. *OECD Journal: Economic Studies*, 2014(1):109–127.

Samuelson, P. A. (1962). Parable and realism in capital theory: the surrogate production function. *The Review of Economic Studies*, 29(3):193–206.

Samuelson, P. A. (1966). A summing up. *The Quarterly Journal of Economics*, 80(4):568–583.

Sato, K. (1974). The neoclassical postulate and the technology frontier in capital theory. *The Quarterly Journal of Economics*, 88(3):353–384.

Sawyer, M. (2002). The NAIRU, aggregate demand and investment. *Metroeconomica*, 53(1):66–94.

Scarf, H. (1973). *The computation of economic equilibria (in collaboration with Terje Hansen)*. Number 24. Yale University Press.

Scarf, H. E. (1982). The computation of equilibrium prices: an exposition. *Handbook of mathematical economics*, 2:1007–1061.

Serrano, F. (2019). Mind the gaps: the conflict augmented Phillips curve and the Sraffian supermultiplier. *TD IE-UFRJ*.

Shaikh, A. (1974). Laws of production and laws of algebra: the humbug production function. *The review of economics and statistics*, pages 115–120.

Sidrauski, M. (1967). Rational choice and patterns of growth in a monetary economy. *The American Economic Review*, 57(2):534–544.

Simon, H. A. (1979). On parsimonious explanations of production relations. *The Scandinavian Journal of Economics*, pages 459–474.

Simon, H. A. and Levy, F. K. (1963). A note on the cobb-douglas function. *The Review of Economic Studies*, 30(2):93–94.

Sinha, A. (2021). Interpreting the nature of Sraffa's equations: A critique of Garegnani's interpretation. In *A Reflection on Sraffa's Revolution in Economic Theory*, pages 1–48. Springer.

Sinha, A. and Dupertuis, M.-S. (2009). Sraffa's system: Equal rate of profits and the notion of center of gravitation. *Journal of Economic Behavior & Organization*, 71(2):495–501.

Smale, S. (1976). Dynamics in general equilibrium theory. *The American Economic Review*, 66(2):288–294.

Solow, R. M. (1956). A contribution to the theory of economic growth. *The quarterly journal of economics*, 70(1):65–94.

Sonnenschein, H. (1973). Do Walras' identity and continuity characterize the class of community excess demand functions? *Journal of economic theory*, 6(4):345–354.

Sraffa, P. (1920). *L'inflazione monetaria in Italia durante e dopo la guerra*. Premiata Scuola Tip. Salesiana.

Sraffa, P. (1922a). The bank crisis in Italy. *The Economic Journal*, 32(126):178–197.

Sraffa, P. (1922b). Italian banking today. *The Manchester Guardian Commercial*, pages 675–676.

Sraffa, P. (1925). Sulle relazioni fra costo e quantità prodotta. *Annali di economia*, 2(1):277–328.

Sraffa, P. (1932a). Dr. Hayek on money and capital. *The Economic Journal*, 42(165):42–53.

Sraffa, P. (1932b). Money and Capital: A rejoinder. *The Economic Journal*, 42(166):249–251.

Sraffa, P. (1960). *Production of Commodities: Prelude to a Critique of Economic Theory*. Cambridge University Press.

Steedman, I. (1984). Natural prices, differential profit rates and the classical competitive process. *The Manchester School*, 52(2):123–140.

Stirati, A. (2001). Inflation, unemployment and hysteresis: an alternative view. *Review of Political Economy*, 13(4):427–451.

Stirati, A. (2016). Blanchard, the NAIRU, and economic policy in the Eurozone. *Institute for New Economic Thinking Blog Post, 31 March*.

Stirati, A. and Meloni, W. P. (2018). A short story of the Phillips curve: from Phillips to Friedman...and back? *Review of Keynesian Economics*, 6(4):493–516.

Stockhammer, E. (2008). Is the NAIRU theory a monetarist, new Keynesian, post Keynesian or a Marxist theory? *Metroeconomica*, 59(3):479–510.

Stockhammer, E. (2011). Wage norms, capital accumulation, and unemployment: a post-Keynesian view. *Oxford Review of Economic Policy*, 27(2):295–311.

Storm, S. (2019). Lost in deflation: Why Italy's woes are a warning to the whole Eurozone. *International Journal of Political Economy*, 48(3):195–237.

Storm, S. and Naastepad, C. (2007). It is high time to ditch the NAIRU. *Journal of Post Keynesian Economics*, 29(4):531–554.

Summers, L. H. (2015). Demand side secular stagnation. *American economic review*, 105(5):60–65.

Summers, L. H. and Stansbury, A. (2019). Whither central banking. *Project syndicate*, 23.

Tanaka, Y. (2009). Undecidability of Uzawa equivalence theorem and Cantor's diagonal argument. *Applied Mathematics E-Notes*, 9:1–9.

Tridico, P. (2013). The impact of the economic crisis on EU labour markets: A comparative perspective. *International Labour Review*, 152(2):175–190.

Tridico, P. (2019). *Economia del lavoro: analisi macroeconomica, evidenze empiriche e politiche del lavoro*. Mondadori Università.

Tridico, P. and Paternesi Meloni, W. (2018). Pil potenziale e tasso di partecipazione: proposta per finanziare il reddito minimo. *Economia e Politica*.

Uzawa, H. (1962). Walras' existence theorem and Brouwer's fixed point theorem. *The Economic Studies Quarterly*, 13(1):59–62.

Varri, P. (1979). Basic and non-basic commodities in Mr. Sraffa's price system. *Metroeconomica*, 31(1):55–72.

Velupillai, K. V. (1989). The existence of the standard system: Sraffa's constructive proof. *Political Economy*, 5(1):3–12.

Velupillai, K. V. (2000). *Computable economics: the Arne Ryde memorial lectures*. Oxford University Press on Demand.

Velupillai, K. V. (2006). Algorithmic foundations of computable general equilibrium theory. *Applied Mathematics and Computation*, 179(1):360–369.

Velupillai, K. V. (2007). Taming the incomputable, reconstructing the nonconstructive and deciding the undecidable in mathematical economics. *University of Trento*.

Velupillai, K. V. (2009). Uncomputability and undecidability in economic theory. *Applied Mathematics and Computation*, 215(4):1404–1416.

Velupillai, K. V. (2014). Constructive and computable Hahn–Banach theorems for the (second) fundamental theorem of welfare economics. *Journal of Mathematical Economics*, 54:36–39.

Velupillai, K. V. (2016). Seven kinds of computable and constructive infelicities in economics. *New Mathematics and Natural Computation*, 12(03):219–239.

Velupillai, K. V. (2021). Definitions, assumptions, propositions and proofs in Sraffa's PCMC. In Sinha, A., editor, *A Reflection on Sraffa's Revolution in Economic Theory*. Springer.

Velupillai, K. V. and Zambelli, S. (2013). Computability and algorithmic complexity in economics. In *A Computable Universe: Understanding and Exploring Nature as Computation*, pages 303–331. World Scientific.

Venkatachalam, R. and Zambelli, S. (2021). Sraffa, Money and Distribution. In Sinha, A., editor, *A Reflection on Sraffa's Revolution in Economic Theory*. Springer.

Venkatachalam, R. and Zambelli, S. (2022). Self-replacing prices with credit and debt. *Structural Change and Economic Dynamics*, 60:451–466.

Von Mises, L. (1912). *Theorie des Geldes und der Umlaufsmittel*. Munich: Duncker und Humblot.

Wicksell, K. (1898). The influence of the rate of interest on commodity prices. *Economic Journal*, 17:213–20.

Wicksell, K. (1901). Forelasningar i Nationalekonomi. *Vol. I, Lund: Berlingska Boktrycheriet*.

Wicksell, K. (1906). Lectures on Political Economy. Volume 2: Money and Credit.

Woodford, M. (2003). *Interest and Prices: Foundations of a Theory of Monetary Policy*. Princeton University.

Yoshihara, N. and Kwak, S. H. (2022). Sraffian indeterminacy of steady-state equilibria in the Walrasian general equilibrium framework. *Discussion Paper Series 734, Institute of Economic Research, Hitotsubashi University*.

Zaghini, E. (1967). On non-basic commodities. *Schweizerische Zeitschrift für Volkswirtschaft und Statistik*, 103:257–266.

Zambelli, S. (2004). The 40% neoclassical aggregate theory of production. *Cambridge Journal of Economics*, 28(1):99–120.

Zambelli, S. (2018a). The aggregate production function is NOT neoclassical. *Cambridge Journal of Economics*, 42(2):383–426.

Zambelli, S. (2018b). Production of Commodities by Means of Commodities and Non-uniform Rates of Profits. *Metroeconomica*, 69(4):791–819.

Zambelli, S. (2021). A comment on 'Definitions, assumptions, propositions and proofs in Sraffa's PCMC'. *A Reflection on Sraffa's Revolution in Economic Theory*.

Zambelli, S. (2022a). Sraffa's Production of Commodities by Means of Commodities with flexible coefficients production functions. *mimeo*.

Zambelli, S. (2022b). When supply does not meet demand. In Venkatachalam, R., editor, *Artificial Intelligence, Learning and Computation in Economics and Finance*. Springer.

Zambelli, S., Fredholm, T., and Venkatachalam, R. (2017). Robust measurement of national technological progress. *Structural Change and Economic Dynamics*, 42:38–55.

Zezza, G. (2017). Dealing with unemployment: The Italian case. Technical report, Paper presented at ASTRIL conference, Rome, December 14-15.