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The role of risk measures in relating earthquake risks at building and portfolio levels

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Abstract
The potentially large spatial footprint of earthquake disasters and the increased concentration of population and values in dense urban areas call for an explicit consideration of seismic risk at a regional, building portfolio level. The relation between the building-level seismic risk and the portfolio-level seismic risk is helpful if one wants to meet a specific regional seismic risk tolerance level by specifying seismic risk targets for individual buildings. We examine four types of common risk measures and point to the importance of subadditivity, a risk measure mathematical property, for deriving a conservative upper-bound relation between the building-specific and the building portfolio seismic risks. Subadditive risk measures, such as the Expected Shortfall, allow estimates of conservative upper bounds of the portfolio-level seismic risk. In a case study, we show that nonsubadditive risk measures commonly used in earthquake engineering can lead to counter-intuitive and nonconservative perceptions of the regional seismic risk, especially when one extrapolates from individual buildings to the entire building portfolio. We also illustrate the advantages of subadditive risk measures for regional seismic risk assessment.

KEYWORDS
Expected Shortfall, performance-based design, regional earthquake risk analysis, risk measures, spatial aggregation, subadditivity

1 | INTRODUCTION

Earthquakes are one of the most devastating natural hazards threatening large regions and exposing inhabitants to high risk of casualties as well as direct and indirect financial losses. Using modern seismic design provisions to design structures reduces the seismic risk. Historically, the primary focus of seismic provisions is to prevent fatalities, caused by a complete or partial collapse of structures, by specifying design criteria for structural components and systems of individual structures. However, many stakeholders (i.e., code drafters, policy makers, governmental agencies, etc.) are not only interested in the entire range of seismic performance of individual structures (from immediate occupancy to collapse), but are also interested in the regional seismic performance of a portfolio of buildings. Indeed, various authors suggest that a discussion on satisfactory seismic performance should not focus only on the seismic performance of an individual building, that is, the building-specific seismic risk, but also on the seismic performance of building portfolios at a regional level.[1–3] In this context, knowing the relation between the building-specific and the spatially aggregated regional seismic risks is very beneficial.

Quantifying the building-specific seismic risk nowadays follows a well-established procedure rooted in the Performance-Based Earthquake Engineering (PBEE) framework. Introduced by the Pacific Earthquake Engineering Research Center (PEER) at the end of the 20th century, the first generation PEER–PBEE framework[4,5] formalized the concept of decision variables, in particular those associated with casualties and direct and indirect economic losses, and the probability-based method of computing the frequency of exceedance of a decision variable[6–9,10] for a building and its seismic hazard. The PEER–PBEE framework, summarized in a guideline issued by FEMA,[11] has been successfully applied in multiple studies to assess the seismic risk of both newly designed and existing buildings.[12–16]

In parallel with developing methods to estimate the building-specific seismic risk, tools for scenario-based risk estimation for building portfolios were used by the insurance sector and public agencies, that is, HAZUS by FEMA.[17] With increases in computational power, the progress in probabilistic seismic hazard analysis (PSHA), and the emergence of the PEER–PBEE framework, fully probabilistic regional risk assessments became possible. Such regional risk assessments are, for example, used for the pricing of natural catastrophe insurance products. Specifically, the PEER–PBEE framework has been extended in a simulation-based fashion to include spatial dependencies of the seismic damage and loss distributions.[18,19] This made it possible to estimate the exceedance frequencies for different decision variables, including building portfolio direct and indirect economic losses.[20–22] Such a bottom-up approach comes at the cost of additional assumptions (e.g., dependency models), which significantly influence the final regional risk estimates, and add to the calculation complexity.[23]

Despite the improvements in building-specific and regional seismic risk analysis capabilities, it is still of significant interest to relate the regional, portfolio-level, seismic risk to the seismic risks of the individual buildings in the portfolio. While building codes primarily aim to control the building-specific risk, their success in risk mitigation is often judged on a regional, portfolio-level. A recent report, published by the U.S. Federal Emergency Management Agency, calls for a change in codes and construction practices stating that *"… 20% to 40% of modern code-conforming buildings in an affected region would be unfit for occupancy following a large earthquake…"*.[24] A better understanding of the relation between building-specific and portfolio-level risk is key for policy-makers and code drafters interested in defining seismic risk targets for individual buildings that meet certain regional seismic risk tolerance levels.

Establishing a relation between building-specific and regional risk, however, is not trivial. The extent of areas where design ground-motions are exceeded can be quite large.[25] A few studies in the scientific literature revealed unexpected comparisons of earthquake-induced losses for individual buildings versus those at a portfolio level. For example, results presented in Crowley and Bommer[20] and DeBock and Liel[21] indicate that the risk of frequent portfolio losses is higher than an estimate obtained by computing frequent losses of individual buildings. Porter[3] compared the building-specific and the regional seismic risks for a scenario event on the Hayward fault[26] by examining the performance of a hypothetical portfolio of code-conforming buildings. The results are seemingly counter-intuitive: although the aforementioned buildings individually perform in line with the building code expectations (at the portfolio level, only a small fraction of the buildings, 0.8% of the inventory, collapses), approximately half of the buildings are likely to be significantly impaired in this scenario earthquake.

Relating marginal (i.e., building-specific) and aggregated (i.e., regional) risks is not unique to seismic risk assessment, and is certainly not new. In fact, it is central to many quantitative risk assessment tasks, including financial risk assessment. In particular, the relation between marginal (i.e., for an individual business unit) and aggregate (i.e., across all business units) financial risk gained much attention in the aftermath the 2008–2009 GFC, although the importance of this relation was already known in the academic literature.[27] The relation between marginal and aggregate risk lies at the heart of the post-GFC revisions of the regulatory provisions for prudent risk management of financial institutions.[28] As a consequence, the debate underlying the updated financial regulatory framework is also of great interest to seismic risk analysis.

The relation between marginal risk and aggregate risk depends, amongst other factors, on how risk is measured. Thus, the definition and the selection of risk measures—and their properties—become critical. Not surprisingly, the post-GFC debate focused on the proper selection of risk measures used in quantitative financial risk management. This study draws on that experience to fill the current knowledge gap on the relation between the building-specific and the regional seismic risks.

By analogy to the financial risk domain, the choice of risk measures and their properties is critical in defining the relation between building-specific and regional seismic risks. Therefore, in this paper, we first introduce the readers to risk measures and provide the necessary background on risk measure mathematical properties. We point to risk measure subadditivity as a desirable property of risk measures for regional seismic risk assessment. Specifically, we discuss two key advantages: First, subadditive risk measures allow for a decentralized computation of a conservative upper-bound of portfolio-level seismic risk from the risk measures evaluated for the individual elements of that portfolio. This upper bound can be quantified by analyzing the seismic risk separately for each portfolio element and neither requiring joint modeling of all elements nor specification of a dependency model amongst those elements. Second, the upper-bound relation can be inverted, allowing to set a regional (portfolio-level) seismic risk target that induces lower bounds of the seismic risk targets for individual buildings.

To illustrate the importance of risk measure subadditivity for regional seismic risk analyses, we present a case study in Section 3. First, we quantify the seismic risk in terms of direct earthquake-induced financial losses using different nonsubadditive and subadditive risk measures to examine the effects of varying individual building characteristics (e.g., strength) and varying building portfolio characteristics (e.g., size) on the building-specific and regional seismic risk. Motivated by the results of Porter,[3] we then study the seismic risk in terms of the number of buildings in a portfolio that simultaneously become impaired (nonusable) in a seismic event. Within this part of the case study, we also describe a way to set lower bounds for individual building risks that consistently limit the regional seismic risk. Crucially, we use the case study to show that nonsubadditive risk measures commonly used in earthquake engineering can lead to counter-intuitive and nonconservative perceptions of the regional seismic risk, especially when one extrapolates from individual buildings to the entire building portfolio.

2 | RISK MEASURES

A risk measure maps a random variable to a real number and, thus, condenses the set of (uncertain) outcomes of a risk analysis to a single value. Risk measures, their mathematical properties and potential pitfalls, have attracted research interest in the context of quantitative finance.[29] In the financial regulation context, risk measures are used to determine whether the risks taken by a financial institution are acceptable in terms of demonstrating that its capital reserves are adequate with a certain confidence level.

Value-at-Risk (VaR) and Expected Shortfall (ES) are two widely discussed risk measures in the quantitative finance academic literature. In simple terms, VaR is defined as the loss level that will not be exceeded with a certain confidence level during a certain period of time, hence VaR focuses on a single specific quantile level. Contrarily, Expected Shortfall is defined as the expected loss given that the loss is greater than the VaR at a certain confidence level. Thus, although VaR and ES are both a function of the time horizon and confidence level, they differ significantly in two aspects:

1. Quantile-based risk measures, such as VaR, aim to answer the question: "how bad can things get?" In contrast, ES investigates the question: "if things do get bad, what is the expected loss?"[30]
2. ES is subadditive, while VaR is (in general) not subadditive. Violation of subadditivity means that the risk of the aggregated portfolio (into which subportfolios have been merged) could be higher than the sum of the individual risks of the merged portfolios.[31]

In the context of regional seismic risk, the second point is crucial. Common risk metrics used in the PEER–PBEE framework share the same properties as VaR and are, in general, not subadditive. This may cause counterintuitive individual building versus building portfolio seismic risk comparisons.

The GFC was characterized by the synchronous default of multiple residential loans in the loan books of banking institutions.[32] The GFC demonstrated that financial institutions were systematically underestimating the credit risk of loan portfolios by assuming that if portfolios of residential home loans with different credit ratings are merged, the resulting default risk of the merged loan portfolio would be lower than the highest individual loan portfolio risk, due to diversification. However, managing credit risk by imposing an acceptable VaR exposure limit to individual loan portfolios of a bank does not necessarily mean that the VaR for the merged portfolio is also below the same VaR exposure threshold and, thus, acceptable for the bank.[33] Acharya et al.[34] notes that financial regulations prior to the GFC were designed to limit each institution's risk in isolation, but were failing to measure adequately the aggregate systemic financial risk. Further, in a paper by Huang et al., issued by the US Federal Reserve

Board,[35] it is noted that a proper risk measure needs to allow straightforward relations between the risk of individual banks and the whole banking/financial system. For this purpose, Huang et al.[35] underline that consistent aggregation from an individual risk level to a portfolio level is crucial, emphasizing the importance of risk measure subadditivity.

The global banking regulatory landscape was fundamentally revised in the aftermath of the GFC, affecting drastically the way banks quantify financial risk and protect against future global financial crises.[28] One notable revision is the adoption of ES instead of VaR as the primary measure to quantify risk of individual banks.[36,37] As Danielsson et al.[27] already discussed prior to the GFC, VaR can underestimate the joint downside risk of the assets held by the bank if the bank's risk is calculated as a simple sum of the individual asset VaR values for cases where the risk distribution is heavy tailed and largely asymmetric.[38]

By analogy, relating building-specific and regional seismic risks is essential if one is to avoid serious underestimates of the aggregate financial consequences of an earthquake in an urban region. Furthermore, it is important to develop seismic design frameworks to control the seismic risk exposure of regional building portfolios in association with the seismic risk exposure targets for individual buildings. Selecting appropriate seismic risk measures is a key first step towards achieving these goals.

The seismic risk of residential buildings is described in terms of different quantities of interest (e.g., fatalities, repair costs, level of post-earthquake functionality). The majority of studies report either the expected value (i.e., the average annual loss) or the value associated with a certain exceedance level (i.e., the loss with a certain mean annual frequency of exceedance) of the quantity of interest. Only a few studies explore different risk measures such as ES. Yoshikawa and Goda[39] employed ES and VaR to quantify the benefit of seismic upgrading for entire building portfolios. Rockafellar and Royset[40] study different risk measures in the context of engineering design optimization of an individual structure. Broccardo et al.[41] presented mathematical definitions and properties of different individual and societal risk metrics in the context of induced seismicity. Bodenmann et al.[42] extended the work of Galanis et al.[43] and quantified the benefit of seismic upgrading using various risk measures by focusing on an individual building situated in different hazard environments. To the best of our knowledge, a comparison of risk measures for the purpose of relating building-specific and regional seismic risks is missing in the earthquake engineering literature.

2.1 | The subadditivity property of risk measures

Consider a portfolio of s buildings and a set of non-negative random variables X_i, $i = 1, \dots, s$, that describe the consequence of an earthquake on building i in the portfolio in terms of, for example, the number of injured building occupants, direct financial property loss, or an indicator whether the building exceeded a level of damage that prohibits its use as a shelter-in-place after an earthquake. The random variable $Z = \sum_i X_i$ describes the spatially aggregated regional consequence to the entire building portfolio (e.g., the total number of injured people, the total direct financial property loss, or the total number of impaired buildings).

A risk measure $\rho(\cdot)$ is a mapping assigning a real number to a random variable. Specifically, $\rho(Z)$ assigns a real number to the total consequence random variable, while $\rho(X_i)$ maps the random consequence associated with the individual building i to a real number. For a subadditive risk measure, the following inequality holds true:

$$\rho(Z) = \rho\left(\sum_{i=1}^{s} X_i\right) \leq \sum_{i=1}^{s} \rho(X_i)\,. \tag{1}$$

Therefore, given a subadditive $\rho(\cdot)$, the building portfolio risk measure, $\rho(Z)$, is always lower or equal than the sum of risk measures $\rho(X_i)$ evaluated separately for each individual building.

First, subadditivity of a risk measure allows for a conservative approximation of portfolio-level risk without assuming any specific dependence structure among the individual buildings of the portfolio. That is important because estimating dependency models for regional seismic risk analyses is nontrivial and requires large amounts of data. Thus, it is common practice to additionally compute bounds of regional risk metrics assuming perfect correlation (e.g., a linear correlation coefficient of one). However, such an approach does not represent the worst-possible dependency structure and, thus, is not conservative in general.[44]

Second, subadditivity of a risk measure enables decentralized risk management. Consider, for example, a local authority interested in limiting the seismic risk on a regional scale to a certain risk threshold $\bar{r}$. To achieve this target, individual building risk thresholds, $\bar{r}_i$, are defined first such that $\sum \bar{r}_i \leq \bar{r}$. Then, verification of building-specific risk $\rho(X_i) \leq \bar{r}_i \; \forall i \in [1, s]$ ensures that the regional building portfolio risk is limited to $\bar{r}$, that is,

$$\rho(Z) \leq \sum_{i=1}^{s} \rho(X_i) \leq \bar{r} \, . \tag{2}$$

However, if a risk measure $\rho(\cdot)$ is nonsubadditive, the regional building portfolio risk might be larger than $\bar{r}$ even though the buildings satisfy individual risk thresholds $\bar{r}_i$, that is, are designed in accordance with the current seismic design provisions. Therefore, using a nonsubadditive risk measure gives no guarantee that the regional, portfolio level, risk threshold is satisfied.

2.2 | Examples of risk measures

In what follows, four risk measures are mathematically defined. The terminology used in these definitions relates to monetary earthquake consequences, for example, direct financial loss. However, as illustrated in a later section of the present study, the concepts also apply to other types of consequences. An interested reader is referred to Jonkman et al.[45] and Rockafellar and Royset[40] for a comprehensive overview. Thus, for the upcoming definitions, L refers to the non-negative random loss variable of either an individual building or the building portfolio with a nondecreasing cumulative distribution function (CDF) $F(l)$ and complementary cumulative distribution function (CCDF) $G(l) = 1 - F(l)$.

Instead of a probability of exceedance, losses induced by earthquakes are often stated in terms of their mean annual frequency of exceedance $\lambda(l)$ or the mean recurrence interval $1/\lambda(l)$. This study evaluates risk measures based on the distribution of maximum losses caused by any single event in 1 year. The corresponding loss exceedance probability curve is defined as

$$G(l) = 1 - \exp\left(-\lambda(l)\right) , \tag{3}$$

and is also known as the annual probability that l is exceeded in at least one event.[46] Note that, in this context, exceedance probabilities for the considered time horizon of 1 year are very low and thus $G(l) \approx \lambda(l)$ holds. The employed methodology to compute $\lambda(l)$ and $G(l)$ is explained in Section 3. Instead of the maximum loss caused by any event in a specific time horizon, decision makers may be interested in the cumulative losses from all events in that time horizon. While this is not the primary focus of the present study, we present mathematical details of both loss distributions in Appendix C, which also includes a derivation of Equation (3).

Expected Loss:
The Expected Loss, EL, risk measure is defined as

$$\rho(L) \equiv \mathrm{EL}(L) = \int_{\mathbb{R}^+} G(l)\mathrm{d}l \, . \tag{4}$$

Expected Loss is a widely used risk measure in earthquake engineering where it is also known as the average annual loss: EL is often employed in cost-benefit analysis to find optimal retrofit strategies[47,48]; the Italian guidelines for seismic risk classification of structures included EL as one of the governing criteria[49]; and, recently, O'Reilly and Calvi[50] proposed an approach for conceptual seismic design based on expected loss. In general, EL as a risk measure is appropriate for risk-neutral decision makers. Conversely, it is inappropriate for risk-averse decision-makers, who give more weight to low probability-high severity events, that is, to the right tail of the loss distribution.[51] EL is an additive risk measure, because the expected value of a linear combination of random variables is equal to the linear combination of the expected values of these random variables, that is, $\mathrm{EL}(L_1 + L_2) = \mathrm{EL}(L_1) + \mathrm{EL}(L_2)$, thus it fulfills the subadditivity property as defined in Equation (1).

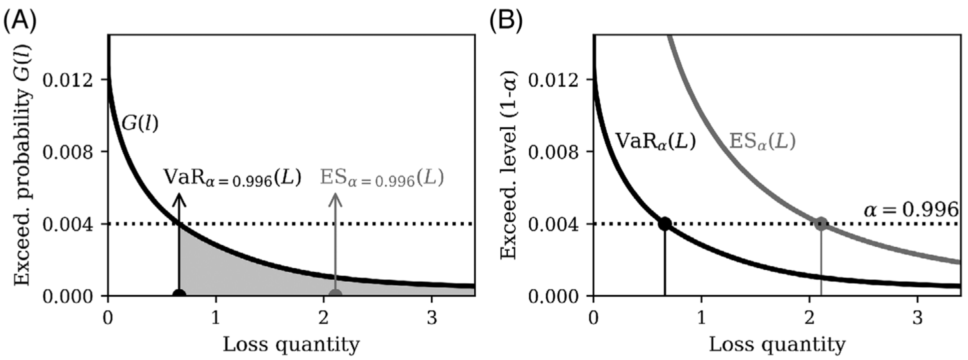

FIGURE 1 (A) Generic loss exceedance probability curve $G(l)$ and schematic illustration of risk measures Value-at-Risk (VaR_α) and Expected Shortfall (ES_α) at confidence level $\alpha = 0.996$. The shaded area shows the range of losses greater than VaR_α, which are taken into account by ES_α, and (B) risk measures VaR_α and ES_α as a function of exceedance level $1 - \alpha$.

Value-at-Risk:
Risk measure Value-at-Risk, VaR_α, at confidence level[1] $\alpha \in [0, 1]$ is defined as the smallest number l such that the probability that $L > l$ is no larger than $1 - \alpha$:

$$\rho(L) \equiv \mathrm{VaR}_\alpha(L) = \inf\{l \in \mathbb{R}^+ : G(l) \leq 1 - \alpha\}\,. \tag{5}$$

VaR_α, in probabilistic terms, refers simply to the α-quantile of the CDF $F(l)$, as indicated in Figure 1. Thus, it offers information only about the severity of losses occurring with exceedance probability higher than or equal to $1 - \alpha$ but neglects losses at smaller levels of exceedance. Furthermore, VaR is not subadditive in general, thus the sum of the building-specific risks could underestimate the spatially aggregated portfolio risk.[29] Note that because $G(l) \approx \lambda(l)$, VaR is closely related to the loss level l with a mean annual frequency of exceedance $\lambda(l) = 1 - \alpha$, for example, VaR at a confidence level α of 99.8% is approximately the same as the loss with a mean recurrence interval of 500 ($=1/(1-\alpha)$) years (the so-called 500 year loss). Expressing losses in terms of their mean recurrence interval or their exceedance frequency is very common within the earthquake engineering community. Because of their close resemblance to VaR, they also fail, in general, to satisfy the property of subadditivity, which is indicated by the results in Crowley and Bommer[20] and DeBock and Liel[21] discussed in the introduction of the present study.

Expected Shortfall:
Risk measure Expected Shortfall, ES_α, at confidence level $\alpha \in [0, 1]$ is the average over all Values at Risk at levels $u \geq \alpha$ and is defined as

$$\rho(L) \equiv \mathrm{ES}_\alpha(L) = \frac{1}{1-\alpha} \int_\alpha^1 \mathrm{VaR}_u(L)\mathrm{d}u\,. \tag{6}$$

If L is a continuous random variable, ES can also be expressed as the expected loss given that the loss exceeds VaR_α, $\mathrm{EL}(L \mid L \geq \mathrm{VaR}_\alpha(L))$ and, thus, by definition $\mathrm{ES}_\alpha \geq \mathrm{VaR}_\alpha$, which is shown in Figure 1. In the literature, this measure is also referred to as Conditional Value-at-Risk (CVaR), Tail Value-at-Risk (TVaR), or Conditional Tail Expectation (CTE), which are equivalent to the definition in Equation (6) for continuous random variables.[52] In contrast to VaR_α, Expected Shortfall takes into account severity of losses with probabilities of exceedance smaller than $1 - \alpha$, thus it is a "what-if" risk measure. Importantly, ES_α is a subadditive risk measure.[53]

Probable Maximum Loss (PML) is another risk measure term that is frequently used by the insurance and banking industry. In catastrophe modeling, PML commonly refers to the loss with a certain mean recurrence interval,[54] while a "PML assessment" in seismic due diligence for building property transactions may focus on losses conditional on a specified earthquake scenario.[55] For reference, we provide an overview of different risk measure terminologies in Appendix B.

[1] The term *confidence level* follows from the terminology and definition used in quantitative finance. We note that α in the present context corresponds to a *probability level* and should not be confused with the term *confidence interval* used in statistics.

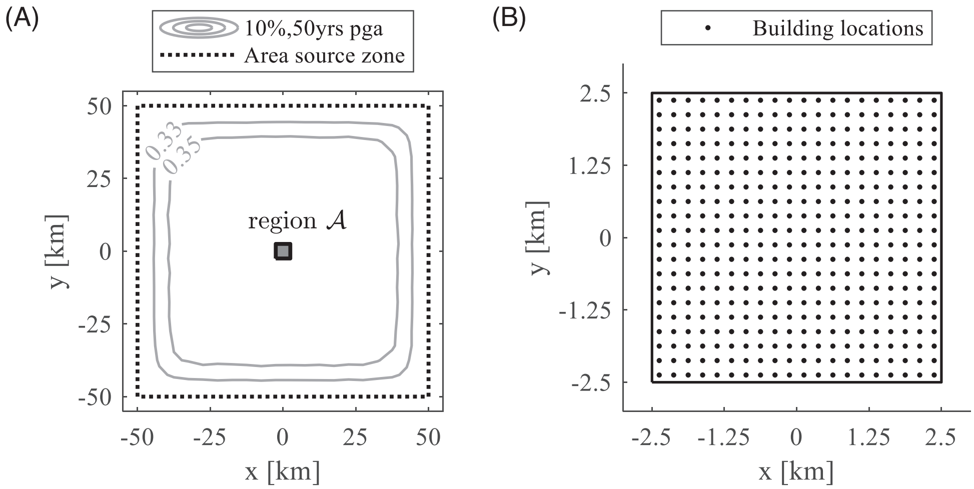

FIGURE 2 (A) Geometry of the virtual hazard environment with the area source zone (dotted line), contours of peak ground acceleration (pga) with a 10% probability of exceedance in 50 years (gray lines) and location of region $\mathcal{A}$. (B) Building locations of the portfolio within region $\mathcal{A}$. The grid-spacing between buildings is 250 m.

As pointed out above, only EL and ES satisfy the subadditivity property of a risk measure as defined in Equation (1), whereas subadditivity of VaR depends on the shape of the marginal (i.e., building-specific) loss distributions and the dependence structure between them. In the context of quantitative finance, nonsubadditive behavior of VaR has been observed in cases with: (1) very skewed marginal loss distributions, that are independent or partially dependent; (2) symmetric marginal loss distributions with highly asymmetric dependence structure; or (3) independent but heavy-tailed marginal loss distributions.[29]

3 | CASE STUDY

To understand the importance of subadditivity for earthquake engineering purposes, we present the principal results of a fictitious case study in the remainder of this paper, where we quantify the seismic risk on both the individual building and the portfolio levels. Section 3.1 presents the methodology and results in terms of direct financial losses, whereas Section 3.2 focuses on risk in terms of the total number of buildings that are unsafe to reoccupy after an earthquake event. The latter illustrates how the risk measures defined above can be applied to a nonfinancial setting and, especially, how subadditive risk measures can be used to relate individual building risk and spatially aggregated building portfolio risk.

We consider a building portfolio of 400 identical buildings, evenly spread in a 5×5 km region $\mathcal{A}$ that is centrally embedded in a virtual hazard environment characterized by a rectangular 100×100-km area source zone, as shown in Figure 2A. Earthquake magnitudes are assumed to follow a truncated Gutenberg–Richter distribution[56] defined in the [5,7] magnitude range with slope b equals 1.0. The mean annual frequency of exceeding the minimal magnitude anywhere within the source ν equals 0.5. The present study quantifies the seismic loss of a single building at site i for seismic event k conditional on the ground-motion intensity measure $im_{i,k}$ at that site. The probabilistic structure of $im_{i,k}$ is given by empirical ground motion models (GMMs), which are typically expressed as

$$\log im_{i,k} = \overline{\log im_{i,k}}(M, R, \theta) + \delta B_k + \delta W_{i,k} \, , \tag{7}$$

where $\overline{\log im_{i,k}}$ is the mean of the logarithms of $im_{i,k}$ as a function of magnitude M, source-to-site distance R and other parameters θ, while δB_k and $\delta W_{i,k}$ denote the between-event and within-event residual, respectively. These residuals are usually assumed to be independent random variables, normally distributed with zero means and standard deviations τ and ϕ, respectively. Single-site PSHA evaluates the mean annual frequency of exceeding a threshold im as

$$\lambda(im) = \nu \cdot \int_r \int_m G(im|m, r)|\mathrm{d}G(r|m)||\mathrm{d}G(m)| \, , \tag{8}$$

where $G(im|m,r)$ is derived based on Equation (7), $|\mathrm{d}G(m)|$ is the truncated Gutenberg–Richter distribution of earthquake magnitudes defined above, and $G(r|m) = P(R > r|m)$ describes the probability of exceeding a certain source-to-site distance r given a rupture of magnitude m. The present study employs the GMM of Akkar and Bommer,[57] which differs between three soil categories (rock, stiff, and soft soil) and three rupture mechanism (strike-slip, normal, and reverse). We assume stiff soil conditions within the entire hazard environment and treat all events as normal ruptures. The seismic hazard within an area of roughly 30 × 30km around the centroid of the area source zone is constant, for example, all buildings in region $\mathcal{A}$ have the same seismic hazard. This is indicated by the contours of the peak ground acceleration (pga) associated with a 10% probability of exceedance in 50 years in Figure 2A.

We characterize the 400 buildings in region $\mathcal{A}$ by their equivalent single-degree of freedom (ESDOF) systems using a linear-elastic/perfectly-plastic force-deformation response envelope. The buildings have a fundamental period of vibration T^* of 0.6 s and a base-shear yield strength coefficient C_y^* of 0.25 g. The latter is derived using $C_y^* = s_{ae}(T^*)/R$, where $s_{ae}(T^*)$ is the elastic, 5% damped pseudo-acceleration at T^* associated with a 10% probability of being exceeded in 50 years, and $R = 1.5$ is the strength reduction factor that accounts for the inelastic behavior and the over-strength of a building.

Intensity measures at multiple sites are necessary to estimate the seismic losses for a spatially distributed building portfolio. Specifically, $G(\mathbf{im}|m,r)$ is commonly modeled as a multivariate lognormal distribution, while the between-event residuals are fully correlated for a given event and correlation of within-event residuals depend on the distance between sites, here accounted for by the model of Esposito and Iervolino.[58]

3.1 | Direct financial property losses

This part of the case study first describes the methodology we employed to quantify direct financial property losses and estimate risk measures for an individual building and building portfolios. Consequently, we compute the seismic risk and its measures, and discuss how the performance of the considered risk measures is affected by key model parameters, such as the individual building seismic response properties, and the sizes of the considered region and the building portfolio. This part of the case study gives further insight into the potentially nonsubadditive behavior of risk measure VaR outlined above.

3.1.1 | Methodology

The methodology to quantify earthquake-induced direct financial losses is based on the PEER-PBEE framework, while its computer implementation is adopted from previous studies of the authors.[42,43] The most important steps to derive the risk measures for an individual building are illustrated first, followed by a description on how the building portfolio risk measures are evaluated. The total probability theorem is used to calculate the mean annual frequency of exceedance of a loss amount l for an individual building as

$$\lambda(l) = \sum_{dm} \int_{im} G(l|dm)|\mathrm{d}G(dm|im)||\mathrm{d}\lambda(im)| \,, \tag{9}$$

where $L = l$ is the seismically induced financial loss, $DM = dm$ is a damage measure, $|\mathrm{d}G(dm|im)| = P(DM = dm|im)$ is the so-called fragility function, $G(l|dm)$ is the cost function, a conditional CCDF expressing the probability that the financial loss is greater that l conditioned on a specific damage state, and $\lambda(im)$ is provided by a single-site PSHA as in Equation (8), where $IM = S_{ae}(T^*)$, the elastic spectral acceleration at the fundamental period of the building. Four damage grades dg_k, where $k \in \{1,2,3,4\}$, are used as a discrete damage measure, which classify the overall building damage state into grades ranging from slight to very heavy/complete damage. Damage grade thresholds are defined as a function of the ESDOF displacement ductility demand $\mu_{lim,k} \in \{0.7, 1.5, 0.5(1+\mu_{lim,4}), \mu_{lim,4}\}$ following Lagomarsino and Giovinazzi.[59] It is assumed that a building suffers from very heavy/complete damage if the displacement ductility demand exceeds a value of $\mu_{lim,4} = 4$. The fragility function $|\mathrm{d}G(dm|im)|$ is modeled as a lognormal distribution, whose parameters are derived using the SPO2IDA tool.[60] Finally, the loss $G(l|dm) = \mathrm{PBPV} \cdot P(DR > dr|dg_k)$ is computed as a product of the present building property value (PBPV) and a damage ratio $DR \in [0,1]$, where the probability distribution of the damage ratio conditional on a certain damage grade is described using a beta distribution with parameters stated in Dolce et al.[61]

Based on Equation (9), the risk measures $\mathrm{EL}(L_i)$, $\mathrm{VaR}_\alpha(L_i)$, and $\mathrm{ES}_\alpha(L_i)$ are evaluated with respect to the exceedance probability curve $G(l)$, computed by Equation (3), and their definitions (Equations 4–6).

The direct earthquake-induced loss quantification methodology described above is implemented in a Monte-Carlo simulation to estimate portfolio-level losses. For an event k, with a magnitude sampled from $|dG(m)|$ and a random location within the area source zone, a spatially correlated ground motion field describes $im_{i,k}$ at the site of buildings i, conditional on which realizations of damage measure $dg_{i,k}$ and subsequently loss $l_{i,k}$ are sampled. The portfolio loss for s buildings located in region $\mathcal{A}$ (Figure 2B) for event k is then estimated as $l_{\mathcal{A},k} = \sum_{i=1}^{s} l_{i,k}$. Denoting the number of simulated events by n, the mean annual frequency of exceeding a portfolio loss amount $l_{\mathcal{A},k} > l_{\mathcal{A}}$ is then approximated by

$$\lambda(l_{\mathcal{A}}) \approx \nu \cdot \frac{1}{n} \sum_{k=1}^{n} \mathbf{1}_{l_{\mathcal{A},k} > l_{\mathcal{A}}} , \tag{10}$$

where ν is the mean annual rate of exceedance of the minimal magnitude defined above and $\mathbf{1}$ is the indicator function taking a value of one if $l_{\mathcal{A},k} > l_{\mathcal{A}}$ and zero otherwise. Based on Equation (10), the risk measures $\rho \in \{\mathrm{VaR}_\alpha, \mathrm{ES}_\alpha, \mathrm{EL}\}$ for the buildings in a portfolio are evaluated as in the case of a single building discussed above.

The outlined procedure takes into account all earthquake events possibly produced by the assumed seismic hazard environment, which corresponds to a time-based analysis in the terminology of FEMA P58.[11] Conversely, in a scenario-based analysis, the consequences are modeled conditional on the occurrence of a specified seismic scenario. The present study performs such analyses to estimate conditional loss exceedance curves, based on which scenario-based EL, VaR and ES risk measures, SEL, SVaR, and SES, respectively, are evaluated.

3.1.2 | Results

Denote $\rho(L_{\mathcal{A}}) = \rho(L_1 + \cdots + L_s)$ as a building portfolio (regional) risk measure in terms of a monetary equivalent of direct earthquake-induced damage in region $\mathcal{A}$ (Figure 2), and $\rho^+(L_{\mathcal{A}}) = \rho(L_1) + \cdots + \rho(L_s)$ as its approximation via the sum of the individual (building-specific) risk measures. These quantities correspond to the left- and right-hand side of Equation (1), respectively. Whenever $\rho(L_{\mathcal{A}})$ is smaller or equal than $\rho^+(L_{\mathcal{A}})$, the risk measure is subadditive (under the assumed dependency structure among portfolio entities). To examine the behavior of different risk measures, a normalized subadditivity margin is defined as

$$\delta_\rho(L_{\mathcal{A}}) = \frac{\rho^+(L_{\mathcal{A}}) - \rho(L_{\mathcal{A}})}{\rho(L_{\mathcal{A}})} , \tag{11}$$

where positive values of δ_ρ indicate that an approximation via the sum of the building-specific risks measures provides a conservative estimate of the portfolio-level risk measure, and negative values imply nonsubadditivity for risk measure $\rho(\cdot)$ applied to a building portfolio in region $\mathcal{A}$. In other words, Equation 11 provides the percentage by which the portfolio risk measure $\rho(L_{\mathcal{A}})$ is under- or over-estimated when using $\rho^+(L_{\mathcal{A}})$. We highlight that the seismic hazard spatial dependency model[58] used to derive $\rho(L_{\mathcal{A}})$ is here considered as the "true" model only for illustration purposes. As discussed in Section 2.1, the correct hazard (and risk) dependency is generally more complex and often unknown in real risk assessment studies. For instance, Kang et al. argued that the response of similar buildings conditional on im exhibit positive dependence,[62] while most regional risk analyses, including ours, consider the response as conditionally independent. Consequently, the estimates of portfolio-level risk measures strongly depend on the assumed dependency structure. However, an approach based on upper bounds afforded by subadditive risk measures obviates this concern, as it depends only on the building-specific risks and is not affected by any dependency assumptions. It follows that the presented bounds ρ^+ are independent from the assumed spatial dependency model, while the subadditivity margins, δ_ρ, are.

Time-based analysis:
Table 1 states the computed ρ, ρ^+, and δ_ρ values for $\rho \in \{\mathrm{VaR}_\alpha, \mathrm{ES}_\alpha, \mathrm{EL}\}$ at exceedance levels $1-\alpha$ of 0.8 and 0.2% (confidence levels of 99.2 and 99.8%). Note that results are normalized with respect to the present portfolio property value PPPV, which is the sum of all PBPVs of the buildings in the portfolio. Because the latter is in this case study assumed constant for all buildings, the process corresponds simply to a normalization by the total number of buildings in the portfolio. As

TABLE 1 Building portfolio risk measured on the aggregated portfolio loss curve $\rho(L_{\mathcal{A}})$, its approximation via $\rho^{+}(L_{\mathcal{A}})$, and the normalized subadditivity margin δ_ρ for $\rho \in \{\mathrm{VaR}_\alpha, \mathrm{ES}_\alpha, \mathrm{EL}\}$ at two different confidence levels α.

		$\alpha = 0.992$		$\alpha = 0.998$	
	EL	**VaR**$_\alpha$	**ES**$_\alpha$	**VaR**$_\alpha$	**ES**$_\alpha$
$\rho(L_{\mathcal{A}})$ [PPPV]	0.1%	2.8%	9.7%	11.0%	23.2%
$\rho^{+}(L_{\mathcal{A}})$ [PPPV]	0.1%	1.6%	11.7%	13.5%	29.7%
δ_ρ	0%	−43%	21%	22%	28%

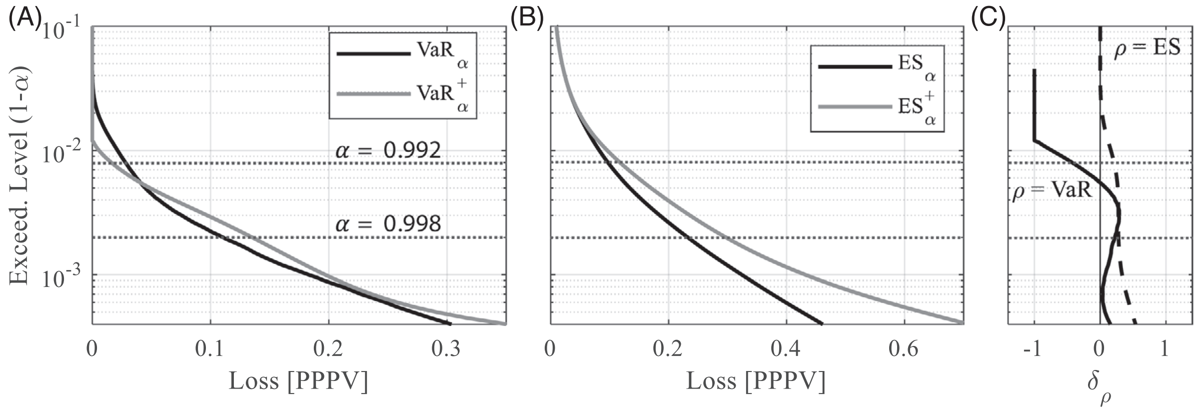

FIGURE 3 Direct financial losses for the building portfolio situated in region $\mathcal{A}$ quantified using (A) Value-at-Risk estimated on the aggregated portfolio loss curve VaR_α and its approximation via the sum of the building-specific risk measures VaR^{+}_α as a function of exceedance level $1-\alpha$ (B) ES_α and ES^{+}_α as a function of $1-\alpha$ and (C) normalized subadditivity margin δ_ρ for Value-at-Risk (solid line) and Expected Shortfall (dashed line). The dotted lines indicate two exceedance levels for which numerical values are provided in Table 1.

explained in Section 2.2, VaR_α is approximately the same as the loss with a mean recurrence interval of $1/(1-\alpha)$ years (here 125 and 500 years).

These results lead to several observations. First, risk measure VaR is nonsubadditive at the higher exceedance level of 0.8% and underestimates the portfolio risk by 43%, whereas it is subadditive for the lower exceedance level of 0.2%. Second, subadditivity of ES leads to positive δ_ρ at both confidence levels. Third, because the risk measure Expected Loss is additive, $\mathrm{EL} = \mathrm{EL}^{+}$ and $\delta_{\mathrm{EL}} = 0\%$.

Figures 3A and 3B plot VaR_α and VaR^{+}_α, and ES_α and ES^{+}_α, respectively, as a function of exceedance level $(1-\alpha)$. Figure 3C illustrates the normalized subadditivity margin for both measures. VaR_α is subadditive only for losses with exceedance levels lower than 0.5%, for example, the loss with an associated mean recurrence interval longer than 200 years, also called the 200-year loss, whereas the portfolio risk is underestimated for more frequent events. On the other hand, Figure 3B,C confirm subadditivity of ES_α over the whole range of the considered exceedance levels. It is worth noting that the normalized subadditivity margin for VaR_α varies from negative (nonconservative) to positive (conservative) as the exceedance level $(1-\alpha)$ decreases (i.e., the recurrence interval elongates), while this normalized margin increases monotonically for ES.

Scenario-based analysis:

A scenario is defined in terms of a certain earthquake magnitude m and an epicenter location $\mathbf{z} \in \mathbb{R}^2$. Specifically, a scenario is identified as the mode of the joint conditional probability density function $|dG(m, r|S_{ae}(T^*) > s_{ae})|$. Two scenarios are chosen based on hazard disaggregation for the site located at the center of the building portfolio (region $\mathcal{A}$). The threshold elastic spectral accelerations s_{ae} are set to a probability of exceedance of 50 and 2% in 50 years for the first and second scenario, respectively. The joint distribution is discretized using bins of 0.2 and 5 km for M and R, respectively. The identified scenarios are: scenario 1 (M5.5, 7.5 km) and scenario 2 (M6.7, 7.5 km). Whereas the magnitudes are different, close events with distances between 5 and 10 km contribute most to the exceedance of both s_{ae} thresholds. This is common for sites located in an area source that dominates the seismicity of the hazard environment.[63] For both scenarios, we then compute a conditional loss exceedance curve $G(l|m, r(\mathbf{z}))$ using the magnitudes described above and an epicenter located at $\mathbf{z} = [0, 7.5]$ km.

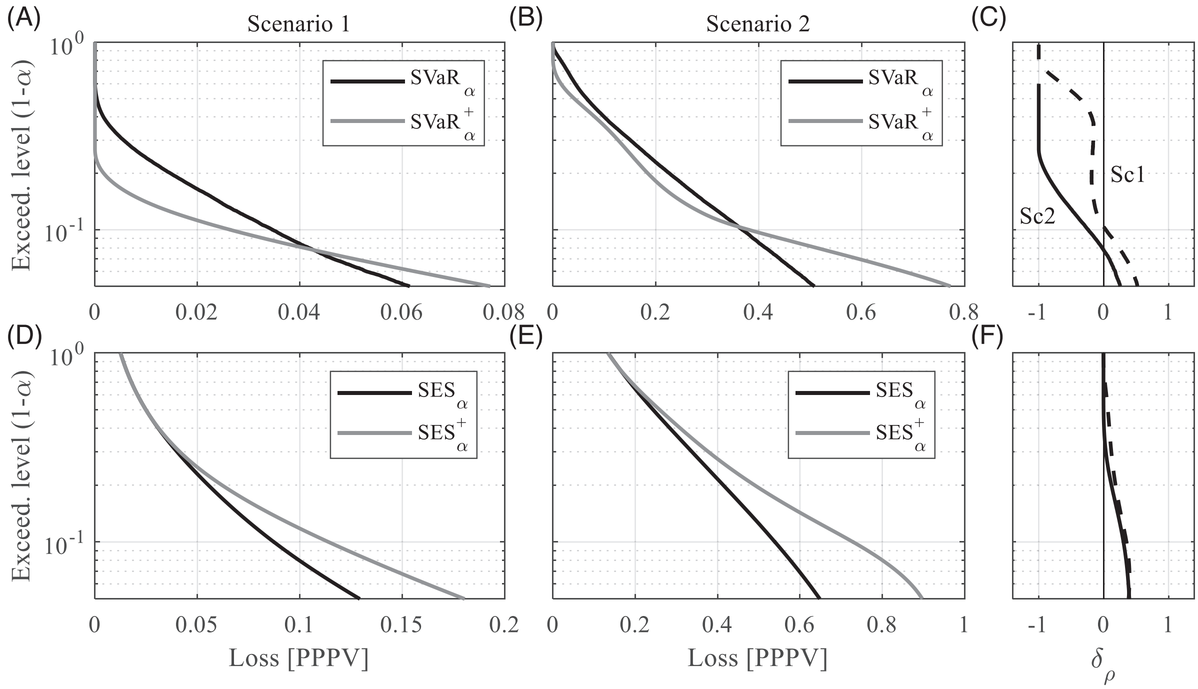

FIGURE 4 Direct financial losses for the building portfolio in region $\mathcal{A}$ conditional on two scenarios with magnitudes of 5.5 (Scenario 1) and 6.7 (Scenario 2). (A, B) SVaR_α and SVaR_α^+; (D, E) SES_α and SES_α^+. The normalized subadditivity margins for both scenarios are compared in (C) for Value-at-Risk and (F) for Expected Shortfall.

Figure 4A,B illustrates scenario risk measures SVaR_α and SVaR_α^+ for the two scenarios, whereas panel C compares the normalized subadditivity margins. In the lower magnitude scenario 1, VaR is nonsubadditive for exceedance levels higher than 7% (confidence level $\alpha = 93\%$), whereas for the higher magnitude scenario 2 in panel B, this exceedance threshold is higher, namely around 10%. Note that the median of the conditional loss distribution (e.g., $\mathrm{SVaR}_{0.5}$) is nonsubadditive in both scenarios. Panels D–F confirm subadditivity of scenario Expected Shortfall, namely SES_α is smaller or equal than SES_α^+ for all confidence levels. Interestingly, the δ_{SES} versus $(1-\alpha)$ curves in Figure 4F are similar in shape for both scenarios, compared to the strongly differing shapes of δ_{SVaR} versus $(1-\alpha)$ curves in Figure 4C. This suggests a higher "scenario-based sensitivity" for the SVaR.

Effects of building characteristics:
This section presents results on how varying building characteristics affect the performance of different risk measures. The base case (BC) building portfolio, discussed above, is characterized by the 0.25-g base-shear yield strength coefficient and the 0.6-s fundamental vibration of the building ESDOF systems. The low-strength (LS) and the high-strength (HS) building portfolios have a uniform ESDOF base-shear yield strength coefficient of 0.13 and 0.38 g, respectively. Notably, the yield displacements of the LS and HS building portfolio ESDOF systems are the same as those of the BC portfolio: thus, the fundamental periods of the ESDOF systems in the LS and HS building portfolios are 1.1 and 0.4 s, respectively. The building locations, the site class, and the virtual hazard environment are the same for the three building portfolios.

Figure 5 plots VaR_α and VaR_α^+ from a time-based analysis in panel A, whereas ES_α and ES_α^+ are plotted in panel C. The corresponding normalized subadditivity margins δ_ρ are shown in panels B and D, respectively. Recall that the normalized subadditivity margin provides information on the relative difference between the sum of risk measures evaluated on the building-specific loss distributions ρ^+ and the risk measure evaluated on the aggregated, portfolio loss distribution ρ. Therefore, one would expect ordering of the risk measures, for example, $\rho_{HS} < \rho_{BC} < \rho_{LS}$ and $\rho_{HS}^+ < \rho_{BC}^+ < \rho_{LS}^+$, but a similar normalized subadditivity margin, for example, $\delta_{\rho,HS} \approx \delta_{\rho,BC} \approx \delta_{\rho,LS}$. However, the lower building yield strength in the LS portfolio results in a vertical shift upwards of the δ_{VaR} versus $(1-\alpha)$ curves in Figure 5B, which means that the subadditive range of VaR_α in the LS case starts at exceedence levels of 0.8% instead of 0.5% as in the base case BC. The opposite behavior is observed for the HS case: increasing the building ESDOF yield strength leads to higher skewness (by a factor of 1.1 compared to the BC) and kurtosis (by a factor of 1.25 compared to the BC) of the building-specific loss

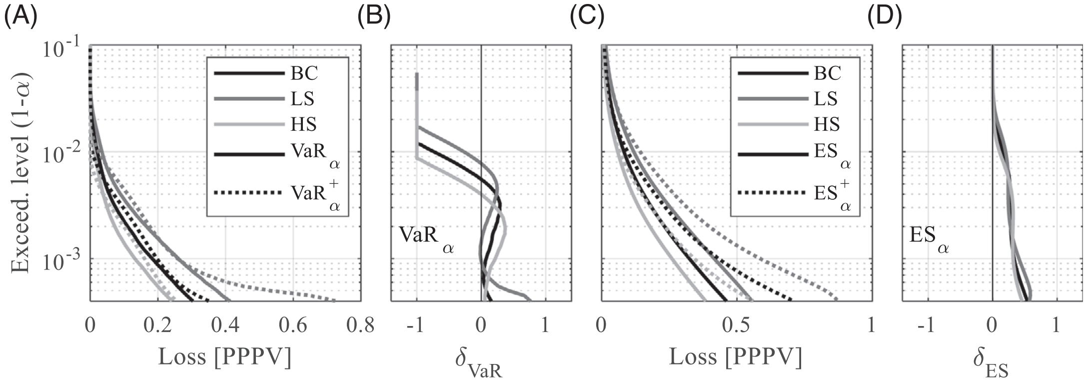

FIGURE 5 Direct financial losses for a portfolio situated in region $\mathcal{A}$ consisting of buildings with uniformly lower (LS) and higher (HS) base-shear yield strength coefficient compared to the base case (BC): (A) Value-at-Risk estimated on the aggregated portfolio loss curve VaR_α (solid lines) and its approximation via the sum of the building-specific risk measures VaR^+_α (dotted lines) as a function of exceedance level $1-\alpha$ (B) normalized subadditivity margin for VaR (C) ES_α and ES^+_α as a function of $1-\alpha$ and (D) normalized subadditivity margin for ES.

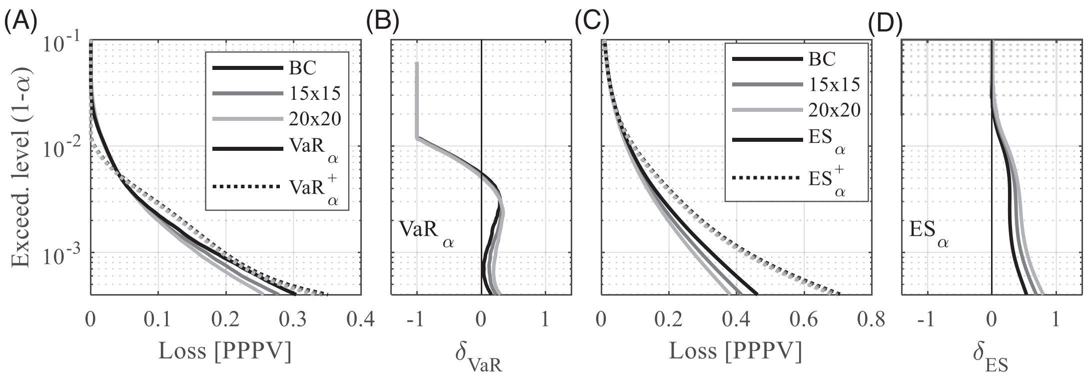

FIGURE 6 Direct financial losses for building portfolios of identical density situated in regions of dimensions 5×5 (BC), 15×15, and 20×20 km: (A) VaR_α (solid lines) and its approximation VaR^+_α (dotted lines) as a function of exceedance level $1-\alpha$ (B) normalized subadditivity margin for VaR (C) ES_α and ES^+_α as a function of $1-\alpha$ and (D) normalized subadditivity margin for ES.

distributions. Specifically, the probabilities of high-loss events are smaller and the probabilities of small- or no-loss events are higher compared to the LS and BS cases with weaker buildings. As stated in preceding sections, it is known from examples in the financial mathematics literature that the nonsubadditivity of quantile-based risk measures is affected by the skewness and kurtosis of the marginal (here the building-specific) distributions. Analogous to the scenario-based results shown in the preceding section, the normalized subadditivity margins for Expected Shortfall, shown in Figure 5D, are similar for all three building strength cases.

Effect of area size:

This set of studies considers varying the size of area (region $\mathcal{A}$ in Figure 2) over which a portfolio of buildings with identical density is distributed. Whereas the base case (BC) considers only 400 buildings distributed over 25 km^2, case 15×15 considers 3600 buildings distributed over 225 km^2, and case 20×20 considers 6400 buildings distributed over 400 km^2. The results are shown in Figure 6, following the pattern used in Figure 5.

Notice that the correlation length of the ground motion intensity spatial dependency model is kept constant at 19.3 km.[58] Therefore, for a given building portfolio density, the seismic risk measures are expected to be a function of the ratio between the correlation length of the hazard and a representative length of the "urban area" (e.g., its diagonal or diameter). The larger this ratio is, the larger is the dependency between the losses. Conversely, the smaller this ratio, the more statistically independent the losses become. Given this, for a larger area size and constant density, we expect more diversification within the portfolio, that is, the probability of an event affecting all buildings is smaller. Therefore, a risk measure ρ evaluated on the aggregated portfolio loss distribution, and normalized by PPPV (the solid lines in panels A and C of Figure 6) should be smaller for larger areas. On the other hand, the normalized value of the approximation

ρ^+ (the dotted lines in panels A and C of Figure 6) should be identical for the three cases because it does not include any diversification. Consequently, the relative difference between ρ and ρ^+, measured by the normalized subadditivity margin, should increase for larger constant-density areas. For Expected Shortfall, this is confirmed by the results in Figure 6D, whereas for VaR in Figure 6B, this only starts to become visible at exceedance levels below 0.3% when subadditivity sets in.

3.2 | Number of jointly impaired buildings

In addition to direct financial losses discussed above, the seismic performance of buildings is often categorized using discrete states describing the level of building's post-earthquake functionality. Re-occupancy relates to a state where the building can serve as a safe shelter. By analogy to Porter (2016), a building that is unsafe to reoccupy is called an impaired building. Then the annual rate of a single building being impaired is evaluated as

$$\lambda(impaired) = \int_{im} P(impaired|im)|\mathrm{d}\lambda(im)| \,, \tag{12}$$

where $\lambda(im)$ is provided by a single-site PSHA as in Equation (8) and $P(impaired|im)$ is the probability of a building being impaired conditional on a ground-motion intensity measure im. A common approach to estimate this probability is to define one or multiple damage-dependent criteria that, when exceeded, would render a building impaired, and then perform nonlinear dynamic response analyses using carefully selected ground-motion records to quantify the probability of building impairment. This approach is exemplified in Iervolino et al.,[16] where multiple researchers quantified the implicit seismic risk of modern and code-compliant Italian buildings with respect to building collapse and usability-preventing damage.

Whereas such studies help to address the differences in seismic risk of individual buildings, discussions on the acceptable level of individual building seismic risk should also include the regional seismic risk posed jointly by the buildings in the region. One option is to consider the regional seismic risk in terms of the number of jointly impaired buildings. Before presenting the results, we show how to use the risk measures discussed above to assess building impairment risk.

3.2.1 | Methodology

By analogy to Section 3.1, we start with defining the impairment risk for an individual building and then explain how portfolio impairment risk measures are evaluated. Consider a discrete random variable Y_i taking a value of 1 if building i is impaired in at least one earthquake event during time horizon t and 0 otherwise. Then the discrete probability masses for Y_i for different time horizons t are estimated as

$$P(Y_i = 1; t) = 1 - \exp\left(-\lambda(impaired_i)t\right) = 1 - P(Y_i = 0; t)\,, \tag{13}$$

where $\lambda(impaired_i)$ is the annual rate of building i being impaired, estimated via Equation (12). For the illustrative purpose in this case study, the event of a building being impaired is linked to an exceedance of the overall building damage grade dg_1. This means that a building is considered to be safe to reoccupy if it suffered no or only slight damage. Whenever the building is in damage grade 2 or higher, the building is considered as impaired. Note that this case study considers a fixed time horizon of 1 year, and the explicit notation of t is dropped in the following definitions and results.

This part of the case study focuses on risk measures Value-at-Risk and Expected Shortfall. However, because the building impairment state is a discrete (binary) variable, rather than a continuous monetized consequence, the terminology used for these risk measures is adapted to emphasize this difference, whereas their mathematical definitions in Equations (5) and (6) remain the same. Specifically, Value-at-Risk is called the α100% quantile, $q_\alpha(\cdot)$, and Expected Shortfall is referred to as the Tail Mean, $\mathrm{TM}_\alpha(\cdot)$. For the binary random variable Y_i the derivation of these two risk measures simplifies to

$$q_\alpha(Y_i) = \mathrm{VaR}_\alpha(Y_i) = \mathbf{1}_{\alpha > P(Y_i=0)}\,, \tag{14a}$$

$$\mathrm{TM}_\alpha(Y_i) = \mathrm{ES}_\alpha(Y_i) = \frac{P(Y_i = 1)}{1-\alpha}\mathbf{1}_{\alpha \le P(Y_i=0)} + \mathbf{1}_{\alpha > P(Y_i=0)}\,. \tag{14b}$$

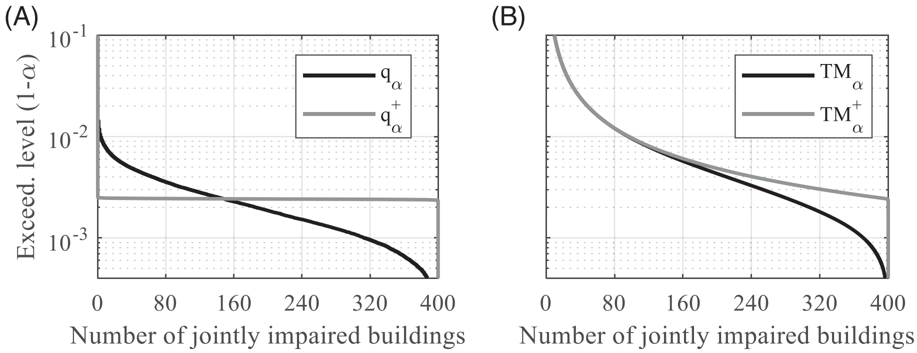

FIGURE 7 The number of jointly impaired buildings in the portfolio situated in region $\mathcal{A}$ as a function of exceedance level $(1-\alpha)$ quantified using (A) α100%-quantile based on the regional simulation q_α and its approximation via the sum of the building-specific risk measures q^+_α and (B) TM_α and TM^+_α.

From these definitions, it is obvious that both risk measures take a value of one for exceedance levels lower than $P(Y_i = 1)$. Clearly, these risk measures are of limited meaning for evaluating building-specific impairment risk. However, they are useful to approximate the regional building impairment risk, as illustrated in the following paragraph.

Consider the random variable $Y_\mathcal{A} = Y_1 + \dots + Y_s$, which counts the number of jointly impaired buildings within region $\mathcal{A}$ in at least one earthquake event in a 1 year time horizon, and has discrete support $z = 1, \dots, s$. To estimate the distribution of $Y_\mathcal{A}$, we use the results from the Monte-Carlo simulation described in the first part of this case study. Define $u_k = \sum_{i=1}^{s} \mathbf{1}_{dm_{i,k} > dg_1}$ as the number of impaired buildings in event k. Then, the CCDF of $Y_\mathcal{A}$ is evaluated as

$$P(Y_\mathcal{A} > z) = 1 - \exp\left(-\nu \cdot \frac{1}{n} \sum_{k=1}^{n} \mathbf{1}_{u_k > z}\right). \tag{15}$$

Given Equation (15), derivations of risk measures $q_\alpha(Y_\mathcal{A})$ and $\mathrm{TM}_\alpha(Y_\mathcal{A})$, stated by Equation (14) for individual buildings, follow from their definitions in Equations (5) and (6).

3.2.2 | Results

Motivated by the results for direct financial losses presented in Section 3.1, we compare the portfolio risk measures $\rho(Y_\mathcal{A})$ to their approximation via the sum of the building-specific risk measures $\rho^+(Y_\mathcal{A}) = \rho(Y_1) + \dots + \rho(Y_s)$. The results of a time-based analysis of building impairment are presented in the same way as the results for direct financial property losses presented in Figure 3 and Table 1. Figure 7 plots the number of jointly impaired buildings in region $\mathcal{A}$ as a function of the exceedance level $(1-\alpha)$ using q_α and q^+_α in panel A and TM_α and TM^+_α in panel B, respectively. Note that because the seismic hazard within the considered region is constant and because we assume identical buildings, $P(Y_i = 1) = 0.0025$ is the same for all buildings $i = 1, \dots, s$ in region $\mathcal{A}$. This leads to a $q^+_\alpha(Y_\mathcal{A}) = 0$ for $(1-\alpha) \geq 0.0025$ and a value of 400 which equals 100% of all buildings in region $\mathcal{A}$, for $(1-\alpha) < 0.0025$. The influence of this uniform individual building risk is examined later. The 99.5% quantile obtained from the regional simulation corresponds to the black curve in panel A at the exceedance level of $(1-\alpha)100\% = 0.5\%$ and equals $q_{0.995}(Y_\mathcal{A}) = 40$. This means that the maximum number of buildings jointly impaired by a single earthquake event occurring in 1 year exceeds 40 with a probability of less than 0.5%. This is roughly equivalent to the statement that the mean recurrence interval of an event where more than 10% (40/400) of the buildings in region $\mathcal{A}$ are jointly impaired is around 200 years ($\approx 1/(1-\alpha)$), which is considerably more frequent than the approximate mean recurrence interval of 400 years of an individual building being impaired. The nonsubadditivity of the q_α risk measure at this exceedance level is responsible for this counterintuitive result.

The results for the Tail Mean in panel Figure 7B confirm the subadditivity of this risk measure in the case of discrete random variables, which means that $\mathrm{TM}_\alpha(Y_\mathcal{A}) \leq \mathrm{TM}^+_\alpha(Y_\mathcal{A})$ for all exceedance levels. Focusing on the same exceedance level of 0.5% as above, $\mathrm{TM}_{0.995}$ equals 180 buildings or 45% of all buildings in the portfolio, which corresponds to the average of the 0.5% worst maximum numbers of jointly impaired buildings triggered by a single earthquake event in 1 year. The same measure approximated by the sum of the individual building risk measures is $\mathrm{TM}^+_{0.995} = 200$ buildings, as plotted

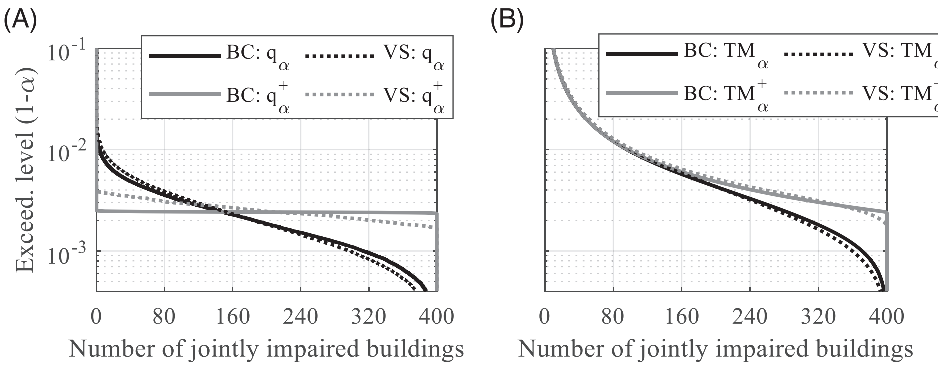

FIGURE 8 The number of jointly impaired buildings in a portfolio of varying, nonuniform yield strengths (VS) situated in region $\mathcal{A}$ as a function of exceedance level $(1-\alpha)$ quantified using (A) α100%-quantile based on the regional simulation q_α and its approximation via the sum of the building-specific risk measures q_α^+ and (B) TM_α and TM_α^+. Both are compared to the base case (BC) with uniform individual building risks illustrated in Figure 7.

by the gray curve in Figure 7B. The same figure indicates that the difference between TM_α and TM_α^+ increases as the exceedance level $(1-\alpha)$ gets closer to $P(Y_i = 1) = 0.0025$, whereas this difference is small for exceedance levels where TM_α^+, and thus also TM_α, are smaller than 80 buildings (roughly 20% of the portfolio).

To examine the effect of nonuniform building-specific risks, the yield base-shear strength coefficients of all building ESDOF systems from the base case BC in the first part of this case study are multiplied by a randomly chosen factor in the range [0.5,1.5], while the ESDOF systems yield displacement are kept constant and the same as in case BC. Figure 8 compares the outcomes of this analysis with varying yield strengths (VS), the dotted curves, to the outcomes of the base case (BC), the solid curves, already illustrated in Figure 7. The dotted gray curve in Figure 8A, illustrating q_α^+ versus $(1-\alpha)$, as well as the smoother transition zone for high values of TM_α^+ in Figure 8B reflect the effect of the nonuniform individual building risks. The black curves, obtained via the regional simulation, differ only very little from the gray curves obtained via the sum of the building-specific risks for both measures and specifically, the difference between TM_α^+ and TM_α remains small for exceedance levels where TM_α^+ is around 20% of all buildings in the portfolio.

A potential regional seismic performance objective could, thus, focus on specifying an exceedance level $(1-\alpha^*)$ such that the average of the $(1-\alpha^*)$100% worst maximum numbers of jointly impaired buildings triggered by a single event in 1 year, $\mathrm{TM}_{\alpha^*}(Y_\mathcal{A})$, is limited to x100% of all buildings in that region, here s. Then, because $\mathrm{TM}_{\alpha^*} \leq \mathrm{TM}_{\alpha^*}^+$, a limit for individual building risks Y_i can be derived as

$$\mathrm{TM}_{\alpha^*}^+(Y_\mathcal{A}) = s \cdot \frac{P(Y_i = 1)}{1-\alpha^*} \leq x \cdot s \Rightarrow \lambda(impaired_i) \leq -\ln\left(1 - x(1-\alpha^*)\right). \tag{16}$$

For example, consider a regional seismic performance objective specified in terms of limiting the 1% worst maximum numbers of jointly impaired buildings triggered by a single event in 1 year, $\mathrm{TM}_{0.99}$, to 20% of all buildings in that region. Then, based on Equation (16), the mean annual rate of a single building being impaired, $\lambda(impaired)$, should be limited to 0.002. The latter could then be compared to the implicit risk of individual, code-compliant buildings[16] to verify whether modifications of code provisions are necessary to attain the stated regional seismic performance objective.

3.3 | Summary

When focusing on the total portfolio-level direct financial property losses, the presented results indicate that risk measure Value-at-Risk is nonsubadditive for losses more frequent than the so-called 200-year loss. Furthermore, this threshold depends on the building-specific risks, indicating that for a portfolio of less vulnerable buildings, the range of nonsubadditive behavior can extend to less frequent losses because the building-specific loss distributions have higher skewness and a heavier tail. For scenario-based analyses, such quantile risk measures become subadditive only at low exceedance levels, for example, lower than 7 and 10% for the examined scenarios.

DeBock and Liel[21] suggested correction factors to approximate the spatially aggregated portfolio risk based on quantile-based measures evaluated for individual buildings. These factors are similar to the normalized subadditivity margin adopted in the present study. However, based on the presented sensitivity analysis, the generalization of such correction factors from one region (or building portfolio) to others could lead to nonconservative results. Instead, regional risk simulations are required to derive these region-specific correction factors for quantile-based risk measures. On the other hand, if subadditive risk measures, such as the Expected Shortfall, are used to quantify the building-specific risk, the spatially aggregated, portfolio-level risk can be conservatively approximated without the need for regional simulations.

Similar problems arise when focusing on risk in terms of the number of buildings that are impaired, rendered as unsafe to occupy, in the same event. Knowing the annual earthquake impairment rate for individual buildings in a portfolio does not allow for conclusions on the annual rate of an event where more than a certain percentage of the buildings become jointly impaired. Instead, establishing a relation between the former and the latter requires regional seismic risk simulations.[64] Contrarily, specifying a regional seismic risk target in terms of the subadditive risk measure Tail Mean, which is mathematically equivalent to Expected Shortfall, allows for a conservative definition of building-specific risk targets in terms of target annual earthquake impairment rates for individual buildings.

The presented case study employs a Gaussian dependency model for ground-motion im, which is arguably the most commonly used model in both practice and academia. In addition, we assumed that building-specific damage conditioned on im is independent. Yet, we note that the choice of adequate dependency models at building-specific and portfolio levels is still a topic of active research (e.g., Kang et al., 2022). For the case where such additional sources of "positive" dependencies are present, one might expect that the nonsubadditivity of quantile-based risk measures will exacerbate, while subadditive risk measures will continue to provide an upper bound on regional losses without requiring any specific assumptions on possible dependencies.

4 | CONCLUSIONS

The present study examines the use of different risk measures to estimate the seismic risks of individual buildings and building portfolios comprised of these individual buildings. The impetus for our work came from the intense discussions on appropriate risk measures for financial risk quantification taking place in the aftermath of the 2008–2009 GFC, as well as from the sometimes counterintuitive observations of the relations between the building-specific and spatially aggregated, regional seismic risks. The risk measure mathematical property of subadditivity is of particular importance in this setting, since it allows for: (1) the estimation of a conservative upper bound of regional risk when the building-specific risk measures are known but the dependency structure between them is not known, or when it is not feasible to conduct regional risk simulations with the same level of detail as used in individual building risk analyses; and (2) the definition of building-specific seismic risk targets that limit the regional seismic risk without relying on assumptions about dependency structure between building-specific risk targets. Subadditive risk measures enable consistent seismic risk governance for individual buildings and building portfolios.

The presented case study results indicate that subadditive risk measures, such as the Expected Shortfall, can be used to define the seismic performance of building portfolios on a spatially aggregated, regional level, as well as for disaggregating regional seismic performance goals to set the seismic performance goals for individual buildings. A subadditive risk measure guarantees that the approximation of regional seismic risk via the sum of building-specific risk measures (i.e., ES^+) is an upper bound to the risk measure evaluated on the aggregated portfolio loss distribution (i.e., ES) derived via regional seismic risk simulations. Our results also indicate that the relative difference between the approximate and the simulated seismic risk measures, as measured by the normalized subadditivity margin, is stable if individual building seismic characteristics change (i.e., higher or lower building base shear yield strengths are mandated by seismic design provisions), and that it adequately reflects changes in the building portfolio geometry (i.e., indicates more risk diversification if the same number of buildings are distributed over a larger area). Therefore, the subadditive Expected Shortfall risk measure and the normalized subadditivity margin can be utilized in regional seismic risk studies. Future work is planned to examine the behavior of the Expected Shortfall risk measure and the normalized subadditivity margin in regional seismic risk studies involving more complex seismic hazard environments and heterogeneous building portfolios.

AUTHOR CONTRIBUTIONS

Methodology: Lukas Bodenmann, Marco Broccardo, and Panagiotis Galanis. *Software*: Lukas Bodenmann. *Investigation*: Lukas Bodenmann. *Visualization*: Lukas Bodenmann. *Writing—original draft preparation*: Lukas Bodenmann. *Conceptualization*: Marco Broccardo and Panagiotis Galanis. *Writing—review & editing*: Marco Broccardo, Panagiotis Galanis, and Božidar Stojadinović. *Supervision*: Božidar Stojadinović. *Funding acquisition*: Božidar Stojadinović.

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CONFLICT OF INTEREST STATEMENT

The authors declare no conflicts of interest.

DATA AVAILABILITY STATEMENT

Data sharing is not applicable to this article as no new data were created in this study.

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APPENDIX A: GLOSSARY OF TECHNICAL ABBREVIATIONS

GFC	2008–2009 Global financial crisis
VaR	value-at-risk
ES	Expected Shortfall
EL	expected loss
CDF	cumulative distribution function
CCDF	complementary cumulative distribution function
PSHA	probabilistic seismic hazard analysis
GMM	ground-motion model
ESDOF	equivalent single-degree of freedom
PBPV	present building property value
PPPV	present portfolio property value

APPENDIX B: RISK MEASURES FOR EARTHQUAKE-INDUCED FINANCIAL LOSSES

Table B1 summarizes definitions and relations of different risk measure terminologies used for earthquake-induced financial losses. As explained in Section 2.2, this study considers a loss random variable L with CDF $F(l)$ and CCDF $G(l)$, where L is defined as the maximum loss caused by any single event in 1 year.

TABLE B1 Definitions and relations of different risk measure terminologies used for earthquake-induced financial losses.

Expected Loss (EL)	EL(L) is the expected value of the loss random variable with CDF $F(l)$.
Value-at-Risk (VaR)	For confidence level α, $\mathrm{VaR}_\alpha(L)$ corresponds to the smallest loss l such that the probability that $L > l$ is no larger than $1-\alpha$. In probabilistic terms, it refers to the α-quantile of the CDF $F(l)$.
Expected Shortfall (ES)	For confidence level α, $\mathrm{ES}_\alpha(L)$ is the average over all values-at-risk at levels $u \geq \alpha$. For continuous random variables, it corresponds to the expected loss given that the loss exceeds VaR_α.
Average Annual Loss (AAL)	Corresponds to the EL(L) for the herein used L.
Return Period Loss (RPL)	The x-year RPL corresponds to the loss associated with a given mean recurrence interval of x years. For the herein used L, and for large x, RPL is approximately the same as $\mathrm{VaR}_\alpha(L)$ with $\alpha = 1 - 1/x$.
Probable Maximum Loss (PML)	Corresponds either to the RPL (see above) or to the EL (see above) conditional on specified scenarios.

APPENDIX C: MAXIMUM AND CUMULATIVE LOSSES OVER TIME

This appendix provides further background material on the distributions of the maximum loss caused by a single event in a fixed time-horizon t and the cumulative losses due to all events in t, which we denote as $ML(t)$ and $CL(t)$, respectively. For this purpose, we introduce two additional random variables: $N(t)$ counts the number of randomly selected earthquake events that occur in t, and SL_n denotes the loss given the occurrence of the nth event. Earthquake occurrence over time is typically described by a homogeneous Poisson process with annual rate ν, thus $N(t)$ follows a Poisson distribution with parameter νt. Additionally, we assume that all SL_n are independent and identically distributed with CDF $F_{SL}(l)$ and CCDF $G_{SL}(l)$, and finally that $N(t)$ is statistically independent of all SL_n. These assumptions allow to describe the occurrence of events with $\{SL > l\}$ as a thinned Poisson process with reduced rate $\nu G_{SL}(l)$, and the number of such events in t, $N_{SL>l}(t)$, follows a Poisson distribution with parameter $\nu t G_{SL}(l)$. Note that the mean annual frequency of exceedance $\lambda(l)$, as computed in Section 3, is the same as the expected number of events with $\{SL > l\}$ in 1 year, $\lambda(l) = \mathbb{E}[N_{SL>l}(t = 1yr)]$, which results in following equation for the CCDF $G_{SL}(l)$

$$G_{SL}(l) = \lambda(l)/\nu \; . \tag{C1}$$

Note that in many cases, only events above a certain minimal magnitude are considered in which case the annual rate ν is the annual frequency of exceeding this minimal magnitude.

The maximum loss induced by a single event in t is defined as $ML(t) = \max_{1 \leq n \leq N(t)} SL_n$. The corresponding CCDF $G_{ML}(l;t) = P(ML(t) > l)$ can be derived as

$$\begin{aligned} G_{ML}(l;t) &= 1 - P(N(t)=0) - \sum_{n=1}^{\infty} P(\max(SL_1, SL_2, \ldots, SL_n) \leq l, N(t) = n) \\ &= 1 - \exp(-\nu t) - \sum_{n=1}^{\infty} (F_{SL}(l))^n \frac{(\nu t)^n}{n!} \exp(-\nu t) \\ &= 1 - \exp(-\nu t G_{SL}(l)) = 1 - \exp(-\lambda(l) t) \; . \end{aligned} \tag{C2}$$

Another derivation can be obtained via the fact that $G_{ML}(l;t)$ is the complement of the probability of having no event with $\{SL > l\}$ in time horizon t, for example, $G_{ML}(l;t) = 1 - P(N_{SL>l}(t) = 0)$. This explains why $G_{ML}(l;t)$ is also known as the probability that loss level l is exceeded by at least one event in time horizon t. As explained in Section 2.2, this study uses $G_{ML}(l; t = 1yr)$ as the loss exceedance probability curve to compute the risk measures from their definitions in Equations 4–6.

The cumulative loss induced by all events in t is defined as $CL(t) = \sum_{n=1}^{N(t)} SL_n$. The corresponding CCDF $G_{CL}(l;t) = P(CL(t) > l)$ is

$$\begin{aligned} G_{CL}(l;t) &= 1 - P(N(t) = 0) - \sum_{n=1}^{\infty} F_{SL}^{*n}(l) P(N(t) = n) \\ &= 1 - \exp(-\nu t) - \sum_{n=1}^{\infty} F_{SL}^{*n}(l) \frac{(\nu t)^n}{n!} \exp(-\nu t), \end{aligned} \tag{C3}$$

where F_{SL}^{*n} denotes an n-fold convolution of F_{SL}. Equation (C3) may be evaluated using recursive numerical algorithms, fast Fourier transforms, or Monte Carlo simulation.[65] Pandey and van der Weide[66] compared different algorithms in the context of earthquake losses. For the case of the expected cumulative losses, $\mathbb{E}[CL(t)]$, there exists an easier and widely applied solution

$$\mathbb{E}[CL(t)] = \mathbb{E}[N(t)] \cdot \mathbb{E}[SL] = \nu t \int_{\mathbb{R}^+} G_{SL}(l)\mathrm{d}l = t \int_{\mathbb{R}^+} \lambda(l)\mathrm{d}l. \tag{C4}$$

Thus, for a time horizon t of 1 year, the expected cumulative losses correspond to the area under the loss exceedance frequency curve $\lambda(l)$. While this study focuses on the distribution of maximum losses induced by a single event in 1 year, Bodenmann et al.[42] computed risk measures VaR and ES for both loss distributions using a time horizon of 50 years and a single building.