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Arbitrating the S_8 discrepancy with growth rate measurements from redshift-space distortions

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ABSTRACT

Within the Lambda cold dark matter (ΛCDM) model, measurements from recent cosmic microwave background (CMB) and weak lensing (WL) surveys have uncovered a $\sim 3\sigma$ disagreement in the inferred value of the parameter $S_8 \equiv \sigma_8\sqrt{\Omega_{\rm m}/0.3}$, quantifying the amplitude of late-time matter fluctuations. Before questioning whether the S_8 discrepancy calls for new physics, it is important to assess the view of measurements other than CMB and WL ones on the discrepancy. Here, we examine the role of measurements of the growth rate $f(z)$ in arbitrating the S_8 discrepancy, considering measurements of $f\sigma_8(z)$ from redshift-space distortions (RSDs). Our baseline analysis combines RSD measurements with geometrical measurements from baryon acoustic oscillations (BAO) and Type Ia Supernovae (SNeIa), given the key role of the latter in constraining $\Omega_{\rm m}$. From this combination and within the ΛCDM model, we find $S_8 = 0.762^{+0.030}_{-0.025}$, and quantify the agreement between RSD + BAO + SNeIa and *Planck* to be at the 2.2σ level: the mild disagreement is therefore compatible with a statistical fluctuation. We discuss combinations of RSD measurements with other data sets, including the E_G statistic. This combination increases the discrepancy with *Planck*, but we deem it significantly less robust. Our earlier results are stable against an extension where we allow the dark energy equation of state w to vary. We conclude that, from the point of view of combined growth rate and geometrical measurements, there are hints, but no strong evidence yet, for the *Planck* ΛCDM cosmology overpredicting the amplitude of matter fluctuations at redshifts $z \lesssim 1$. From this perspective, it might therefore still be premature to claim the need for new physics from the S_8 discrepancy.

Key words: cosmological parameters – dark energy – dark matter.

1 INTRODUCTION

The concordance Lambda cold dark matter (ΛCDM) model provides a wonderful fit to a wide variety of observations (see, for instance, Riess et al. 1998; Perlmutter et al. 1999; Aghanim et al. 2020; Aiola et al. 2020; eBOSS Collaboration 2021). However, the increase in precision and sensitivity of recent surveys, brought about by remarkable experimental developments, has uncovered intriguing discrepancies among parameters inferred from independent measurements.

One of these discrepancies is the well-known H_0 tension, referring to discrepancies between various late- and early-time independent measurements of the Hubble constant H_0 (see e.g. Freedman et al. 2019; Riess et al. 2019; Aghanim et al. 2020; Aiola et al. 2020; Wong et al. 2020). Whether the H_0 tension calls for new physics, and what this new physics might be, are the subject of an ongoing and rapidly evolving research direction.[1] A milder yet not less enduring discrepancy is also present between cosmic microwave background (CMB) and low-redshift probes of the amplitude of matter fluctuations, affecting σ_8 (the present-day linear theory amplitude of matter fluctuations averaged in spheres of radius $8\,h^{-1}$Mpc) and the matter density parameter $\Omega_{\rm m}$: this discrepancy is best captured by the parameter $S_8 \equiv \sigma_8\sqrt{\Omega_{\rm m}/0.3}$, which reflects the main degeneracy direction of weak lensing (WL) measurements.

Within the context of the ΛCDM model, CMB anisotropy measurements from *Planck* and ACT + WMAP indicate S_8 values of 0.834 ± 0.016 (Aghanim et al. 2020) and 0.840 ± 0.030 (Aiola et al. 2020), respectively. On the other hand, the value of S_8 inferred by a host of WL and galaxy clustering measurements is typically lower than the CMB-inferred values, ranging between 0.703 and 0.782: examples of surveys reporting lower values of S_8 include CFHTLenS (Joudaki et al. 2017a), KiDS-450 (Joudaki et al. 2017b), KiDS-450 + 2dFLenS (Joudaki et al. 2018), KiDS + VIKING-450 (KV450; Hildebrandt et al. 2020), DES-Y1 (Troxel et al. 2018), KV450 + BOSS (Tröster et al. 2020), KV450 + DES-Y1 (Asgari

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[1]See, for instance, Bernal, Verde & Riess (2016), Mörtsell & Dhawan (2018), Poulin et al. (2019), Kreisch, Cyr-Racine & Doré (2020), Vagnozzi (2020), Visinelli, Vagnozzi & Danielsson (2019), Sakstein & Trodden (2020), Hill et al. (2020), Ballesteros, Notari & Rompineve (2020), Braglia et al. (2020), Efstathiou (2020, 2021), Das & Ghosh (2020), Choudhury, Hannestad & Tram (2021), Brinckmann, Hyeok Chang & LoVerde (2020), De Felice, Mukohyama & Pookkillath (2021) for discussions concerning the H_0 tension and possible solutions, and Verde, Treu & Riess (2019), Di Valentino et al. (2021a, 2021b) for recent reviews.

et al. 2020; Joudaki et al. 2020), a reanalysis of the BOSS galaxy power spectrum (Ivanov, Simonović & Zaldarriaga 2020), KiDS-1000 (Asgari et al. 2021), and KiDS-1000 + BOSS + 2dFLenS (Heymans et al. 2021). *Planck* Sunyaev–Zeldovich cluster counts also infer a rather low value of $S_8 = 0.774 \pm 0.034$ (Ade et al. 2016). To balance the discussion, it is also worth remarking that KiDS-450 + GAMA (van Uitert et al. 2018) and HSC SSP (Hamana et al. 2020) indicate higher values of S_8, of $0.800^{+0.029}_{-0.027}$ and $0.804^{+0.032}_{-0.029}$, respectively.

While the status of the S_8 discrepancy is perhaps somewhat less clear than that of the H_0 tension, it is beyond question that there overall is some disagreement between high- and low-redshift probes of the amplitude of matter fluctuations (see, for instance, Di Valentino et al. 2020c, for a concise review of the problem). It is thus worthwhile to investigate whether new physics might solve or at least alleviate the S_8 discrepancy, a possibility that has been investigated in several works. Models that have been contemplated in this sense include for example active and sterile neutrinos (Battye & Moss 2014; MacCrann et al. 2015; Feng, Zhang & Zhang 2017; Vagnozzi et al. 2017; Mccarthy et al. 2018), ultra-light axions (Hlozek et al. 2015), decaying dark matter (DM; Enqvist et al. 2015; Chudaykin, Gorbunov & Tkachev 2018; Di Valentino et al. 2018; Abellán et al. 2020; Chen et al. 2021; Pandey, Karwal & Das 2020; Xiao et al. 2020; Abellán, Murgia & Poulin 2021), extended or exotic DM and/or dark energy (DE) models and interactions (Kunz, Nesseris & Sawicki 2015; Kumar & Nunes 2016; Pourtsidou & Tram 2016; Gariazzo et al. 2017; Benetti, Graef & Alcaniz 2018; Buen-Abad et al. 2018; Kumar, Nunes & Yadav 2018, 2020a, b; Poulin et al. 2018; Archidiacono et al. 2019; Di Valentino et al. 2019b, 2020b; Lambiase et al. 2019; Vagnozzi et al. 2019; Chamings et al. 2020; Dutta et al. 2020; Heimersheim et al. 2020; Jiménez et al. 2020; Choi, Yanagida & Yokozaki 2021) including unified dark sector models (Camera, Martinelli & Bertacca 2019), modified gravity models (Dossett et al. 2015; De Felice & Mukohyama 2017; Nesseris, Pantazis & Perivolaropoulos 2017; Kazantzidis & Perivolaropoulos 2018, 2019; Barros et al. 2020; De Felice, Nakamura & Tsujikawa 2020; Skara & Perivolaropoulos 2020; Zumalacarregui 2020; Marra & Perivolaropoulos 2021), and more generally extended parameter spaces (Di Valentino & Bridle 2018; Di Valentino, Melchiorri & Silk 2020), among the others. It is also worth noting that most of the models invoked to address the S_8 discrepancy do so at the expense of worsening the H_0 tension, and vice versa (see e.g. Vagnozzi et al. 2018; Poulin et al. 2018; Kumar et al. 2019a; Hill et al. 2020; Alestas & Perivolaropoulos 2021), highlighting the importance of a conjoined analysis of the two tensions (Di Valentino et al. 2020b; Di Valentino, Linder & Melchiorri 2020a).

The possibility that the S_8 discrepancy might be at least partially due to systematics cannot be completely excluded, as discussed for instance in Efstathiou & Lemos (2018) in the context of the KiDS-450 measurements. In this sense, it is important to look at the S_8 discrepancy through different eyes, i.e. through data sets other than CMB and WL measurements, which might be able to arbitrate the discrepancy or at least point us towards the ingredients needed to resolve it. To draw a parallel with the H_0 tension, the inverse distance ladder take on the tension has been instrumental towards narrowing down plausible solutions (Bernal et al. 2016; Aylor et al. 2019; Lemos et al. 2019; Schöneberg, Lesgourgues & Hooper 2019; Knox & Millea 2020). Broadly speaking, the question we are then interested in is: '*Is there strong evidence from data other than weak lensing measurements for the Planck ΛCDM cosmology overpredicting the amplitude of matter fluctuations at $z \lesssim 1$?*' In other words, we want to compare the CMB and WL inferences of S_8 against other techniques that can also measure the amplitude of the spectrum of matter fluctuations. Anticipating the answer to the previous question, we will find that there are indeed hints from combined growth and geometrical measurements, but no strong evidence.

We shall address this question making use of measurements of the growth rate of matter density perturbations $f(z)$, as inferred from the peculiar velocities arising from redshift-space distortion (RSD) measurements (Kaiser 1987), which typically constrain the combination $f\sigma_8(z)$. We will combine RSD measurements with two additional classes of probes: *(a)* geometrical probes of distances and expansion rates such as baryon acoustic oscillations (BAO), uncalibrated Supernovae Type Ia (SNeIa), and cosmic chronometer (CC) measurements; *(b)* the E_G statistic (Zhang et al. 2007), which measures a combination of gravitational lensing, galaxy clustering, and RSDs, probing a combination of the two metric potentials, and which is insensitive to galaxy bias and σ_8 in the linear regime. We will assess the status of the S_8 discrepancy in light of the RSD + BAO + SNeIa(+CC) and RSD + E_G data set combinations, both within the concordance ΛCDM model and within the 1-parameter wCDM extension where the DE equation of state (EoS) w is allowed to vary, to check whether the discrepancy can be alleviated within this extension. We note that recent related analyses have also been conducted in e.g. Nesseris et al. (2017), Efstathiou & Lemos (2018), Kazantzidis & Perivolaropoulos (2018), Quelle & Maroto (2020), Skara & Perivolaropoulos (2020), Li et al. (2021), Benisty (2021), and Garcia-Quintero, Ishak & Ning (2020).

The rest of this paper is then structured as follows. In Section 2, we present the data sets and statistical methodology used in our analysis. Our results are discussed in Section 3, with Section 3.1 reporting the results within the wCDM model. We draw concluding remarks in Section 4. We invite the busy reader to skip to Fig. 1, Table 1, and especially Fig. 2, where they will find the main results of this paper conveniently summarized.

2 DATA SETS AND METHODOLOGY

In the following, we first discuss the data sets we make use of. We then discuss our analysis methods, in particular our choice of cosmological parameters and tension metric used to assess the concordance or discordance between the adopted data sets and the *Planck* CMB measurements, within the context of the cosmological models being considered.

2.1 $f\sigma_8$ measurements

As discussed in the introduction, the key data set we will use to try and arbitrate the S_8 discrepancy, independently of CMB and WL measurements, are RSD measurements. Recall that RSDs are a velocity-induced mapping from real- to redshift-space due to line-of-sight peculiar motions of objects, which introduce anisotropies in their clustering patterns (Kaiser 1987). This effect depends on the growth of structure, making RSD probes sensitive to the combination $f\sigma_8$, with f the logarithmic derivative of the linear growth rate $D(a)$ with respect to the scale factor a:

$$f(a) \equiv \frac{\mathrm{d}\ln D(a)}{\mathrm{d}\ln a}\,. \tag{1}$$

On sub-horizon scales and in the linear regime, the evolution equation for $f(a)$ is given by

$$\frac{\mathrm{d}f(a)}{\mathrm{d}\ln a} + f^2 + \left(2 + \frac{1}{2}\frac{\mathrm{d}\ln H(a)^2}{\mathrm{d}\ln a}\right) f - \frac{3}{2}\Omega_{\mathrm{m}}(a) = 0\,, \tag{2}$$

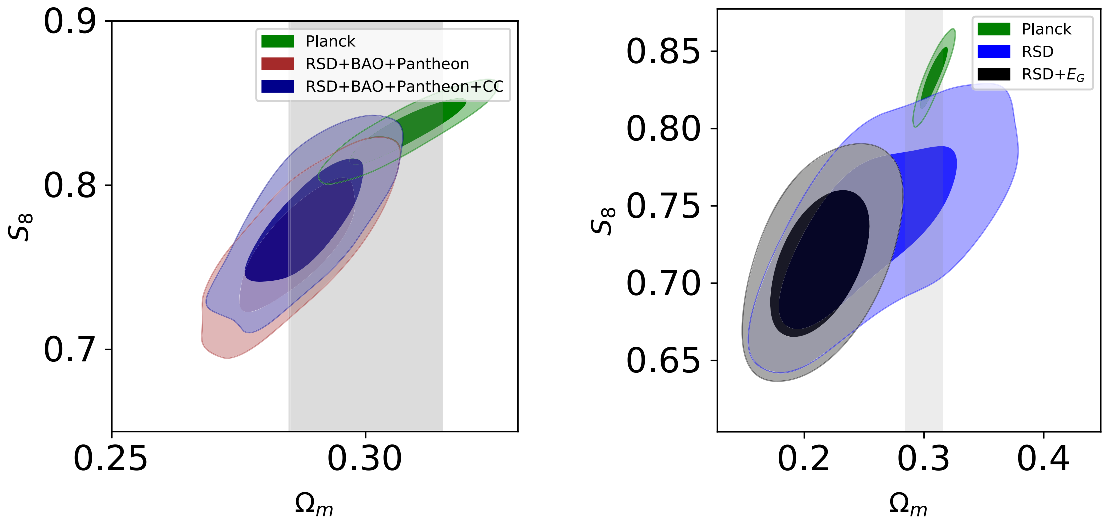

Figure 1. *Left-hand panel*: 2D joint posterior distributions in the S_8–Ω_m plane, with the corresponding 68 per cent C.L. and 95 per cent C.L. contours, obtained from the following data sets/data set combinations within the ΛCDM model: *Planck* (green contours), *RSD + BAO + Pantheon* (magenta contours), and *RSD + BAO + Pantheon + CC* (dark blue contours). *Right-hand panel*: as for the left-hand panel, but considering the *Planck* (green contours), *RSD* (light blue contours), and *RSD + E_G* (black contours) data set combinations, respectively. In both the left-hand and right-hand panels, the vertical grey bands denote the 68 per cent C.L. interval on $\Omega_m = 0.298 \pm 0.015$ obtained from *BAO + Pantheon*. The level of agreement or tension between *Planck* and data set combinations considered is quantified in the two rightmost columns of Table 1. The *RSD + BAO + Pantheon* and *RSD + BAO + Pantheon + CC* data set combinations are the most robust ones, and should be considered as baseline data set combinations.

Table 1. 68 per cent C.L. intervals on the matter density parameter Ω_m, the present-day linear theory amplitude of matter fluctuations averaged in spheres of radius $8\,h^{-1}$Mpc σ_8, and $S_8 \equiv \sigma_8\sqrt{\Omega_m/0.3}$, inferred from the data sets/data set combinations given in the leftmost column, within the ΛCDM model. The two rightmost columns quantify the level of agreement or tension between *Planck* and the data sets in question, using either the 1D T_{S_8} tension metric given by equation (10), or the more robust quadratic tension metric estimator given by equation (11). We encourage the use of the latter as reference value for the amount of tension. For the *RSD + BAO + Pantheon* and *RSD + BAO + Pantheon + CC* data set combinations, the level of agreement with *Planck* is at the $\simeq 2\sigma$ level: at this level the mild disagreement, if any, is still consistent with a possible statistical fluctuation.

Data set	Ω_m	σ_8	S_8	Tension (equation 10)	Tension (equation 11)
RSD + BAO + Pantheon	0.286 ± 0.008	$0.781^{+0.021}_{-0.019}$	$0.762^{+0.030}_{-0.025}$	2.1σ	2.2σ
RSD + BAO + Pantheon + CC	0.288 ± 0.008	$0.793^{+0.018}_{-0.020}$	$0.777^{+0.026}_{-0.027}$	1.8σ	2.1σ
RSD + E_G	$0.200^{+0.020}_{-0.030}$	$0.870^{+0.039}_{-0.050}$	0.710 ± 0.029	3.7σ	5.3σ
RSD	$0.254^{+0.038}_{-0.058}$	$0.804^{+0.048}_{-0.071}$	$0.739^{+0.036}_{-0.040}$	2.3σ	3.1σ
BAO + Pantheon	0.298 ± 0.015	–	–	–	–

where $\Omega_m(a) \equiv \Omega_{m,0}a^{-3}H_0^2/H(a)^2$, with $\Omega_{m,0} \equiv \Omega_m$ the matter density parameter today, and $H(a)$ is the Hubble rate as a function of scale factor. Within the ΛCDM model, and assuming gravity is described by General Relativity (GR), $f(a)$ scales to good approximation as $f(a) \propto \Omega_m(a)^{0.55}$ (Lahav et al. 1991).

Let us now consider the matter overdensity field δ_m. On sub-horizon scales, and assuming that DE does not cluster, the growth equation that governs the evolution of δ_m is given by

$$\delta_m''(a)+\left(\frac{3}{a}+\frac{H'(a)}{H(a)}\right)\delta_m'(a)-\frac{3}{2}\frac{\Omega_m(a)}{a^2}\delta_m(a)=0, \tag{3}$$

with the prime denoting a derivative with respect to the scale factor a. It is worth noting that equation (3) admits a closed-form solution in terms of the Gaussian hypergeometric function ${}_2F_1$:

$$\delta_m(a) = a\,{}_2F_1\left[\frac{1}{3},1;\frac{11}{6};a^3\left(1-\frac{1}{\Omega_m}\right)\right]. \tag{4}$$

Redshift surveys can constrain the quantity $f(a)\sigma_8(a) \equiv f\sigma_8(a)$ [or equivalently $f\sigma_8(z)$], which is given by

$$f\sigma_8(a) = a\frac{\delta_m'(a)}{\delta_m(a_0)}\sigma_{8,0}\,, \tag{5}$$

with f given by equation (1), and $\sigma_8(a)$ given by

$$\sigma_8(a) = \frac{\delta_m(a)}{\delta_m(1)}\sqrt{\int_0^\infty \mathrm{d}k\,\frac{k^2P(k)W_R^2(k)}{2\pi^2}}\,. \tag{6}$$

where $W_R(k) = 3[\sin(kR)/kR - \cos(kR)]/(kR)^2$ is the Fourier transform of the top-hat window function, with R the appropriate scale over which the RMS normalization of matter fluctuations is being computed.

Several measurements of $f\sigma_8(a)$ from a variety of different surveys, each making different assumptions (in particular assumptions on the reference value of Ω_m) and subject to different systematics, exist in the literature. Before using any one of them, it is imperative to assess their internal consistency. Such an analysis was recently performed in the context of a Bayesian model comparison framework in Sagredo, Nesseris & Sapone (2018), which was able to identify potential

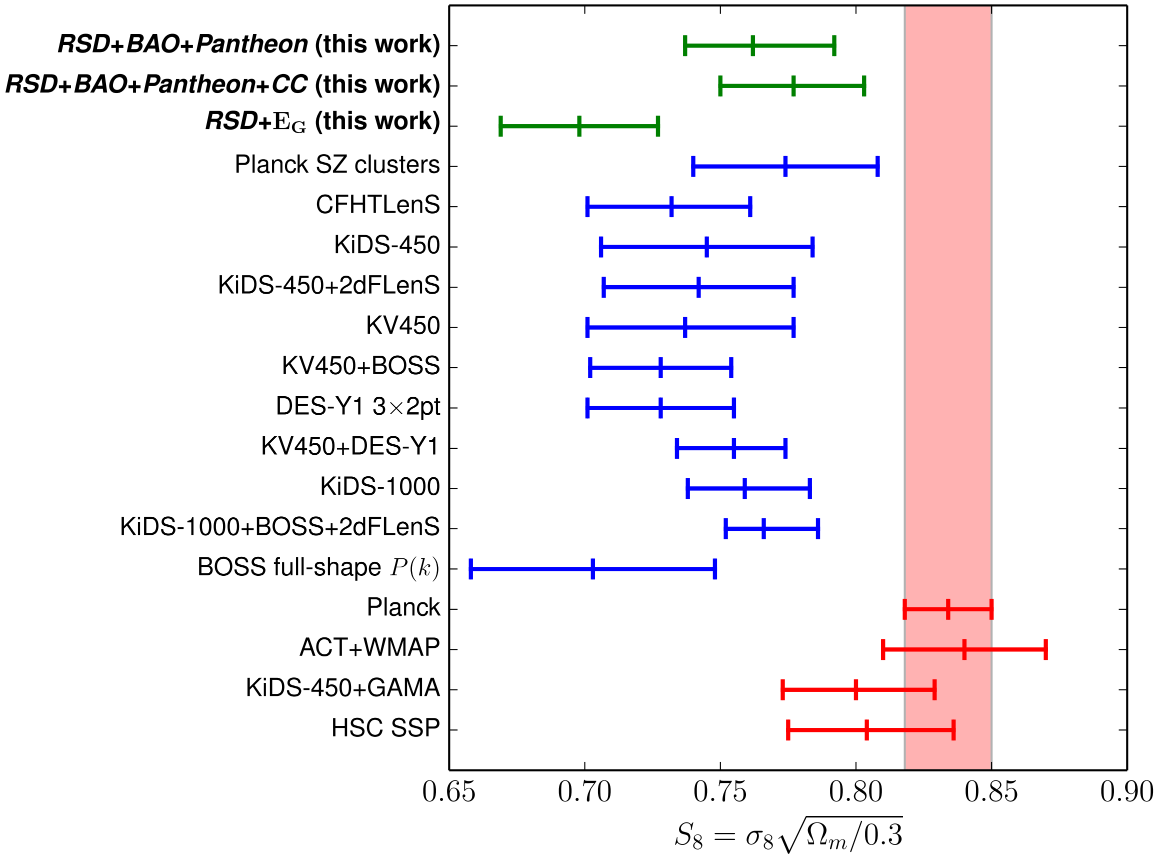

Figure 2. Whisker plot displaying 68 per cent C.L. intervals on $S_8 \equiv \sigma_8\sqrt{\Omega_{\rm m}/0.3}$, as inferred from a wide variety of measurements within the ΛCDM model. The colour coding is such that green bars indicate our new results, blue bars indicate probes that infer an overall lower value of S_8 (mostly WL surveys), and red bars indicate probes that infer an overall higher value of S_8. The red band denotes the 68 per cent C.L. interval on $S_8 = 0.834 \pm 0.016$ determined by *Planck* CMB measurements. From top to bottom, the reported measurements and surveys are: $S_8 = 0.762^{+0.030}_{-0.025}$ from *RSD* + *BAO* + *Pantheon* (our work); $S_8 = 0.777^{+0.026}_{-0.027}$ from *RSD* + *BAO* + *Pantheon* + *CC* (our work); $S_8 = 0.710 \pm 0.029$ from *RSD* + E_G (our work); $S_8 = 0.774 \pm 0.034$ from *Planck* Sunyaev-Zeldovich cluster counts (Ade et al. 2016); $S_8 = 0.732^{+0.029}_{-0.031}$ from CFHTLenS (Joudaki et al. 2017a); $S_8 = 0.745 \pm 0.039$ from KiDS-450 (Joudaki et al. 2017b); $S_8 = 0.742 \pm 0.035$ from KiDS-450 + 2dFLenS (Joudaki et al. 2018); $S_8 = 0.737^{+0.040}_{-0.036}$ from KV450 (Hildebrandt et al. 2020); $S_8 = 0.728 \pm 0.026$ from KV450 + BOSS (Tröster et al. 2020); $S_8 = 0.782 \pm 0.027$ from the DES-Y1 3 × 2pt analysis (Troxel et al. 2018); $S_8 = 0.755^{+0.019}_{-0.021}$ from KV450 + DES-Y1 (Asgari et al. 2020; Joudaki et al. 2020); $S_8 = 0.759^{+0.024}_{-0.021}$ from KiDS-1000 (Asgari et al. 2021); $S_8 = 0.766^{+0.020}_{-0.014}$ from KiDS-1000 + BOSS + 2dFLenS (Heymans et al. 2021); $S_8 = 0.703 \pm 0.045$ from a reanalysis of the BOSS galaxy power spectrum (Ivanov et al. 2020); $S_8 = 0.834 \pm 0.016$ from *Planck* (Aghanim et al. 2020); $S_8 = 0.834 \pm 0.016$ from ACT + WMAP (Aiola et al. 2020); $S_8 = 0.800^{+0.029}_{-0.027}$ from KiDS-450 + GAMA (van Uitert et al. 2018); and $S_8 = 0.804^{+0.032}_{-0.029}$ from HSC SSP (Hamana et al. 2020).

outliers as well as subsets of data affected by systematics or new physics. It is worth noting that, within a ΛCDM + GR framework, RSD measurements of $f\sigma_8$ essentially measure the combination $\sigma_8\Omega_{\rm m}^{0.55}$, which up to a known constant is closely related to S_8.

In this work, we shall make use of the RSD measurements of $f\sigma_8(z)$ provided in table I of Sagredo et al. (2018), consisting of 22 measurements of $f\sigma_8(z)$ in the redshift range $0.02 < z < 1.944$ obtained from the following surveys: 2dFGRS (Song & Percival 2009), 2MASS (Davis et al. 2011), SDSS-II LRGs (Samushia, Percival & Raccanelli 2012), First Amendment SNeIa + IRAS (Turnbull et al. 2012; Hudson & Turnbull 2013), WiggleZ (Blake et al. 2012), GAMA (Blake et al. 2013), BOSS DR11 LOWZ (Sanchez et al. 2014), BOSS DR12 CMASS (Chuang et al. 2016), SDSS DR7 MGS (Howlett et al. 2015) and SDSS DR7 (Feix, Nusser & Branchini 2015), FastSound (Okumura et al. 2016), Supercal SNeIa + 6dFGS (Huterer et al. 2017), VIPERS PDR-2 (Pezzotta et al. 2017), and eBOSS DR14 quasars (Zhao et al. 2019). We refer to these measurements as *RSD*, and further note that these are commonly referred to as the 'Gold 2018' sample in the literature.

We note that in principle many more measurements of $f\sigma_8$ other than the adopted ones are available (see e.g. table II of Nesseris et al. 2017). However, as noted in Nesseris et al. (2017), Sagredo et al. (2018), within this enlarged set, not all the measurements are independent, and hence should not be used at the same time without a proper modelling of the cross-covariance. The extensive analyses of Nesseris et al. (2017) and Sagredo et al. (2018) have allowed for the overlap between these measurements to be minimized, while in turn ensuring that their independence is maximized. Our analysis properly accounts for the covariance between measurements at different redshifts originating from the same analysis (as e.g. in the case of the WiggleZ and eBOSS measurements). Finally, we note that our analysis properly accounts for the so-called growth correction, first discussed in Nesseris et al. (2017), which corrects for the different assumptions of each survey concerning the fiducial values of $\Omega_{\rm m}$ and σ_8.

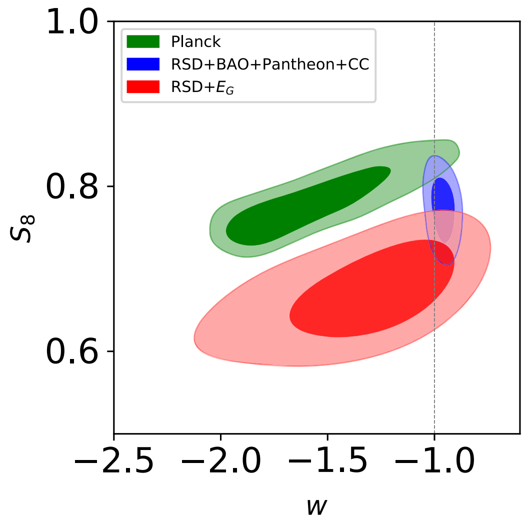

Figure 3. 2D joint posterior distributions in the S_8-w plane, with the corresponding 68 per cent C.L. and 95 per cent C.L. contours, obtained from the following data sets/data set combinations within the wCDM model: *Planck* (green contours), *RSD + BAO + Pantheon + CC* (blue contours), and *RSD* + E_G (red contours). Using the more robust quadratic tension metric estimator given by equation (11), we infer that *Planck* and *RSD + BAO + Pantheon + CC* are in agreement at the 2.2σ level, while we infer that *Planck* and *RSD* + E_G are in tension at the 3.5σ level.

2.2 E_G measurements

The E_G statistic was first proposed in Zhang et al. (2007) as a means of testing deviations from GR, while avoiding potential degeneracies with galaxy bias and σ_8. E_G measures a combination of gravitational lensing, galaxy clustering, and RSDs, probing a combination of the two metric potentials, and is insensitive to galaxy bias and σ_8 in the linear regime. These measurements will be of interest to us given their dependence on the growth factor f. E_G is defined as the expectation value of the estimator $\hat{E}_G$, originally defined as (Zhang et al. 2007)

$$\hat{E}_G = \frac{aC_{\kappa g}(\ell\,,\Delta\ell)}{3H_0^2\sum_\alpha j_\alpha(\ell\,,\Delta\ell)P_{vg}^\alpha}\,, \qquad (7)$$

where for a given multipole ℓ and bin of size $\Delta\ell$, and wavenumbers labelled by k_α, $C_{\kappa g}$ is the lensing convergence-galaxy overdensity cross-correlation, P_{vg} is the galaxy velocity-overdensity cross-spectrum, and j_α is an appropriate weighting function that transforms P_{vg} to an angular power spectrum. The expectation value of equation (7), and hence E_G, is given by (Zhang et al. 2007)

$$E_G = \left[\frac{a\nabla^2(\Psi+\Phi)}{3H_0^2 f\delta_{\rm m}}\right]\,, \qquad (8)$$

where Ψ and Φ are the two Newtonian potentials, which appear in the perturbed FLRW metric in conformal Newtonian gauge, and are equal to each other in GR and in the absence of anisotropic stress. For other works examining important theoretical or observational aspects of E_G as a means of testing fundamental physics, we refer the reader for instance to Reyes et al. (2010), Amendola et al. (2013b), Pullen, Alam & Ho (2015), Blake et al. (2016), Leonard, Ferreira & Heymans (2015), Pullen et al. (2016), Alam et al. (2017a), de la Torre et al. (2017), Amon et al. (2018), Singh et al. (2019), Blake et al. (2020), and Zhang et al. (2021).

Assuming that on the largest scales gravity is correctly described by GR, equation (8) reduces to (Zhang et al. 2007; Amendola et al. 2013b; Leonard et al. 2015)

$$E_G(z) = \frac{\Omega_{\rm m}}{f(z)}\,, \qquad (9)$$

which is clearly independent of σ_8 and linear bias. Moreover, note that E_G is expected to be scale-independent not only within GR, but more generally within any theory of gravity captured by a scale-dependent effective Newtonian constant, with a scale-independent relationship between Φ and Ψ. Note, however, that E_G is strictly speaking scale-independent only at linear level. On smaller scales, non-linearities associated with galaxy clustering, galaxy biasing, and WL, make E_G slightly scale-dependent (see e.g. Leonard et al. 2015, for further discussions). From equation (9), we see that within ΛCDM + GR, $E_G \propto \Omega_{\rm m}^{-0.45}$. It is also clear that combining RSD measurements of $f\sigma_8(z)$ with E_G measurements can enormously help in disentangling $f(z)$ and $\sigma_8(z)$ (Skara & Perivolaropoulos 2020). This allows for better constraints on σ_8, which in turn can help arbitrate the S_8 discrepancy.

In this work, we make use of the E_G measurements compiled in table 7 of Pinho, Casas & Amendola (2018), which we collectively refer to as E_G. This consists of nine measurements of $E_G(z)$ in the range $0.09 < z < 0.48$. Of these nine points, four have been obtained from a joint analysis of RCSLenS and CFHTLenS imaging and WiggleZ and BOSS spectroscopy (Blake et al. 2016); two from a joint analysis of CFHTLenS imaging and VIPERS spectroscopy (de la Torre et al. 2017); and three from a joint analysis of KiDS-450 imaging and 2dFLenS, BOSS, and GAMA spectroscopy (Amon et al. 2018). These E_G measurements probe scales in the range $3\,h^{-1}$ Mpc $< R < 60\,h^{-1}$ Mpc and well in the linear regime. We treat the nine E_G measurements as being statistically uncorrelated, thus approximating the likelihood as being a multivariate Gaussian in E_G with diagonal covariance matrix: note that Blake et al. (2016), de la Torre et al. (2017), and Amon et al. (2018) suggest that the covariance between E_G measurements at two different redshift bins from the same analysis may be neglected, whereas to the best of our knowledge the covariance between measurements from different analyses has not been estimated in the literature.

2.3 Other measurements

In addition to $f\sigma_8$ (RSD) and E_G measurements, we consider three additional geometrical measurements of distances and expansion rates, based on the use of standard rulers, standard candles, and standard clocks:

(i) BAO distance and expansion rate measurements from the 6dFGS (Beutler et al. 2011), SDSS-DR7 MGS (Ross et al. 2015), and BOSS DR12 (Alam et al. 2017b) galaxy surveys, as well as from eBOSS DR14 Lyman-α (Lyα) absorption (de Sainte Agathe et al. 2019) and Lyα-quasars cross-correlation (Blomqvist et al. 2019). These consist of isotropic BAO measurements of $D_V(z)/r_d$ (with $D_V(z)$ and r_d the spherically averaged volume distance, and sound horizon at baryon drag, respectively) for 6dFGS and MGS, and anisotropic BAO measurements of $D_M(z)/r_d$ and $D_H(z)/r_d$ [with $D_M(z)$ the comoving angular diameter distance and $D_H(z) = c/H(z)$ the Hubble distance] for BOSS DR12, eBOSS DR14 Lyα, and eBOSS DR14 Lyα-quasars cross-correlation. At the time of writing, the covariance matrix for the legacy eBOSS BAO measurements (eBOSS Collaboration 2021) was not publicly available, which is the reason why we instead opted for these older measurements. At any rate, we expect that adopting these newer measurements should not qualitatively affect our results.

(ii) Type Ia Supernovae (SNeIa) distance moduli measurements from the *Pantheon* sample, consisting of 1048 SNeIa in the range $0.01 < z < 2.3$ (Scolnic et al. 2018). These measurements constrain the uncalibrated luminosity distance $H_0 d_L(z)$, or in other words the slope of the late-time expansion rate (which in turn constrains Ω_m). We refer to this data set as *Pantheon*.

(iii) CC measurements of $H(z)$. These consist of measurements of $H(z)$ from the differential age evolution of massive, early-time, passively evolving galaxies, which act as standard clocks (Jimenez & Loeb 2002). We make use of 31 CC measurements of $H(z)$ in the range $0.07 < z < 1.965$, compiled in Jimenez et al. (2003), Simon, Verde & Jimenez (2005), Stern et al. (2010), Moresco et al. (2012, 2016), Zhang et al. (2014), Moresco (2015), and Ratsimbazafy et al. (2017). We refer to this data set as *CC*.

We consider three different data set combinations, all of which involve the *RSD* data set: *RSD* + *BAO* + *Pantheon*, *RSD* + *BAO* + *Pantheon* + *CC*, and *RSD* + E_G. Of the three, we consider the *RSD* + *BAO* + *Pantheon* one to be the most robust one, and treat it as our baseline data set combination. In particular, combining *BAO* and *Pantheon* measurements produces tight constraints on Ω_m which, once combined with the *RSD* measurements, improves the constraints on σ_8. We note that, as we are assuming the validity of the ΛCDM model at high redshifts, we can compute the sound horizon r_s given a big bang Nucleosynthesis (BBN) prior on ω_b (discussed in the paragraph below). Hence, the *BAO* + *Pantheon* combination corresponds to an *inverse distance ladder* anchored to the early-Universe determination of r_s. In addition, we will occasionally also report the constraints we obtain from the *RSD* data set alone.

Model-wise, we consider a standard ΛCDM + GR model, spanned by the following four parameters: the Hubble constant H_0 or equivalently the reduced Hubble constant $h \equiv H_0/(100\,\mathrm{km\,s^{-1}\,Mpc^{-1}})$, the physical baryon density $\omega_b \equiv \Omega_b h^2$, the physical cold dark matter density $\omega_c \equiv \Omega_c h^2$, and σ_8. The matter density parameter today Ω_m is treated as a derived parameter, whose value is given by $\Omega_m = (\omega_b + \omega_c)/h^2$. Another important derived parameter is $S_8 \equiv \sigma_8\sqrt{\Omega_m/0.3}$. To constrain the physical baryon density, we adopt a Gaussian prior on ω_b from BBN: $100\omega_b = 2.233 \pm 0.036$ (Mossa et al. 2020). In what follows, the use of the BBN prior on ω_b will be implicitly assumed. With the exception of ω_b, for which we adopt a Gaussian prior as discussed above, we adopt flat priors on all other cosmological parameters. At a later stage, we consider a 1-parameter extension of the previous model, where the DE EoS w is allowed to vary. We refer to this extended model as wCDM.[2]

We note, however, that our results might actually be seen applying more generally than just to ΛCDM. In fact, the evolution equation for δ_m, equation (3), which is our main equation as far as the interpretation of RSD measurements goes, really only assumes *(a)* the validity of GR, and that *(b)* DE does not cluster. The amount of matter is then constrained by the *BAO* + *Pantheon* data set combination. While we will keep referring to the ΛCDM model in the remainder of our paper, the reader should keep in mind that the associated results are in fact more general than that.

We use Markov chain Monte Carlo (MCMC) methods to sample the posterior distributions of the parameters considered. To generate our MCMC chains, we make use of the cosmological MCMC sampler `MontePython` (Blas, Lesgourgues & Tram 2011; Audren et al. 2013; Brinckmann & Lesgourgues 2019), while theoretical predictions for the cosmological observables are computed through `class` (Blas et al. 2011; Lesgourgues 2011).. We monitor the convergence of the generated MCMC chains via the Gelman–Rubin parameter $R-1$ (Gelman & Rubin 1992), and require $R-1 < 0.001$ for the chains to be considered converged.

[2]Although in principle interesting, we do not consider an extended cosmology involving spatial curvature Ω_K since it is known that, despite the apparent indication for a closed Universe from *Planck* primary CMB measurements (Di Valentino, Melchiorri & Silk 2019a; Handley 2021), Ω_K is too well constrained close to spatial flatness by combining *Planck* data with other data sets which break the geometrical degeneracy (Ryan, Doshi & Ratra 2018; Park & Ratra 2019; Efstathiou & Gratton 2020; Vagnozzi et al. 2020; Cao, Ryan & Ratra 2021; Chudaykin, Dolgikh & Ivanov 2021; Vagnozzi, Loeb & Moresco 2021). In addition, including Ω_K as a free parameter results in most cases in the S_8 discrepancy being considerably worsened (see for instance, Di Valentino, Melchiorri & Silk 2021c).

2.4 Tension metrics

Once we have obtained constraints on the above cosmological parameters, and in particular the derived parameters Ω_m and S_8, our next goal is to quantify the level of concordance or discordance (if any) between the data set combinations we have considered and the *Planck* measurements. Consider two data sets i and j for which the inferred values of S_8 are $S_{8,i} \pm \sigma_{S_{8,i}}$ and $S_{8,j} \pm \sigma_{S_{8,j}}$, respectively. Then, if one focuses solely on S_8, a naïve 1D tension metric, which we refer to as T_{S_8}, can be constructed by the following:

$$T_{S_8} \equiv \frac{S_{8,i} - S_{8,j}}{\sqrt{\sigma^2_{S_{8,i}} + \sigma^2_{S_{8,j}}}}\,, \tag{10}$$

where the value of T_{S_8} can directly be interpreted as level of tension in equivalent Gaussian σs. We note that T_{S_8} was already used in a similar context by Hildebrandt et al. (2017) and Joudaki et al. (2017b). While this tension metric is a good starting point, it can underestimate the level of tension due to its only focusing on one particular direction of parameter space. A more robust tension metric should instead take the whole parameter space into consideration, accounting for correlations between parameters.

To construct a more robust tension metric, we make use of the quadratic estimator proposed in Addison et al. (2016), which robustly assesses whether the differences between correlated parameters inferred from two different data sets are consistent with zero. Considering once more two independent data sets i and j, we can assess the level of concordance or discordance between the two by considering the vector of differences of mean parameter values, treating it as being distributed according to a multivariate Gaussian distribution with zero mean and covariance given by the sum of the covariance matrices of the individual data sets. In practice, we construct the following test statistic:

$$\chi^2 = (\mathbf{x}_i - \mathbf{x}_j)^T(\mathcal{C}_i + \mathcal{C}_j)^{-1}(\mathbf{x}_i - \mathbf{x}_j)\,, \tag{11}$$

where $\mathbf{x}_i$ and $\mathbf{x}_j$ are the vectors containing the mean values for the cosmological parameters inferred from data sets i and j, respectively, and similarly $\mathcal{C}_i$ and $\mathcal{C}_j$ are the covariance matrices for these data sets. It can easily be seen that equation (11) essentially corresponds to a generalized Mahalanobis distance between $\mathbf{x}_i$ and $\mathbf{x}_j$.

The significance of a given value of the test statistic χ^2 is then converted to an equivalent Gaussian σ level. We compute the test statistic in equation (11) over the whole 4D parameter space (5D when we also vary the DE EoS w), to fully account for correlations between the parameters. For each data set, we estimate the parameter mean vector and covariance matrix directly from our MCMC chains. In closing, we also note that the same quadratic tension estimator was recently used by the ACT collaboration in Aiola et al. (2020) to quote the level of concordance with the *Planck* measurements. For a selection of other tension metrics discussed in the recent literature, we refer the reader to e.g. Karpenka, Feroz & Hobson (2015), MacCrann et al. (2015), Lin & Ishak (2017a, b), Adhikari & Huterer (2019), Raveri & Hu (2019), Nicola, Amara & Refregier (2019), Handley & Lemos (2019a, b), Garcia-Quintero et al. (2019), Lemos et al. (2020), and Raveri, Zacharegkas & Hu (2020).

3 RESULTS

We first work within the context of the ΛCDM model. In a first instance, we consider our baseline data set combination: *RSD* + *BAO* + *Pantheon*. From this data set combination, we infer 68 per cent confidence level (C.L.) constraints of $\Omega_m = 0.286 \pm 0.008$, $\sigma_8 = 0.7808^{+0.021}_{-0.019}$, and $S_8 = 0.762^{+0.030}_{-0.025}$. We note that *BAO* + *Pantheon* produce tight constraints on $\Omega_m = 0.298 \pm 0.015$. Using the 1D T_{S_8} tension metric given by equation (10), the value of S_8 is found to be in 2.1σ agreement with the *Planck* determination, for which $S_8 = 0.834 \pm 0.016$. Adopting instead the more robust quadratic tension metric estimator given by equation (11), we find that the concordance between *RSD* + *BAO* + *Pantheon* and *Planck* decreases. However, we find that the two data sets are still in agreement at the 2.2σ level. The agreement between the two data sets is admittedly not perfect: there is clearly a mild disagreement between the two, with *RSD* + *BAO* + *Pantheon* preferring lower values of S_8. However, we believe any reference to tensions is certainly premature, since a $\simeq 2\sigma$ agreement/disagreement could still be compatible with a statistical fluctuation.

Including the *CC* data set does not qualitatively alter the previous conclusions. In this case, we find $\Omega_m = 0.288 \pm 0.008$, $\sigma_8 = 0.7929^{+0.018}_{-0.020}$ and $S_8 = 0.777^{+0.026}_{-0.027}$. Again, using the 1D T_{S_8} and quadratic tension metrics, we find that *RSD* + *BAO* + *Pantheon* + *CC* and *Planck* are in agreement at the 1.8σ and 2.1σ level, respectively. While again there is clearly a mild disagreement with *Planck*, with *RSD* + *BAO* + *Pantheon* + *CC* preferring lower values of S_8, this disagreement is at a level that could be compatible with a statistical fluctuation.

Overall, the main message of the first part of our results therefore is: combining a wide range of RSD measurements of $f\sigma_8(z)$ with an inverse distance ladder constructed out of BAO and Hubble flow SNeIa and anchored to the high sound horizon value predicted within ΛCDM, while returning a slightly lower value of S_8, gives *no strong evidence for the Planck ΛCDM cosmology overpredicting the amplitude of matter fluctuations at* $z \lesssim 1$. In this sense, the *RSD* + *BAO* + *Pantheon*(+ *CC*) data set combination would suggest that it might be premature to invoke new physics to address the S_8 discrepancy, in qualitative agreement with the earlier results of Efstathiou & Lemos (2018).

We now consider the *RSD* + E_G data set combination, which we find leads to rather unexpected results. In particular, we recover extremely low values for $\Omega_m = 0.200^{+0.020}_{-0.030}$ and $S_8 = 0.698 \pm 0.029$, respectively. The extremely low value of Ω_m is in strong tension with any independent probe of Ω_m, e.g. BAO (eBOSS Collaboration 2021) and SNeIa (Scolnic et al. 2018), including probes which by themselves already tend to favour low values of Ω_m, such as cluster counts (Ade et al. 2016; Sakr et al. 2018; see also Zubeldia & Challinor 2019 for revised constraints). The recovered extremely low value of S_8 is also in mild tension with WL measurements, which by themselves already prefer a lower value of S_8 as discussed in Section 1. Using the 1D T_{S_8} and quadratic tension metrics, we find that *RSD* + E_G and *Planck* are in tension at the 4.2σ and 5.3σ levels, respectively. In this case, it is very clear that focusing only on S_8 underestimates the level of the tension. It is worth noting that, following Blake et al. (2016), de la Torre et al. (2017), Amon et al. (2018), and Pinho et al. (2018) we have treated the E_G measurements as being independent, i.e. neglecting the covariance between them. To the best of our knowledge, the covariance between all the available E_G measurements has yet to be robustly quantified in the literature. We can generically expect that including the covariance between these measurements, if any, might reduce the significance of the tension, if only by virtue of enlarged error bars.

We note that these results are in qualitative agreement with those of Skara & Perivolaropoulos (2020), who also found that a similar data set combination exacerbated the S_8 discrepancy at a similar level. We also note that the inferred low value of S_8 is in qualitative agreement with the value inferred from the reanalysis of the BOSS full-shape power spectrum of Ivanov et al. (2020), which finds $S_8 = 0.703 \pm 0.045$. If taken at face value, these results could indicate a weakening of gravity at low redshifts, as suggested in Skara & Perivolaropoulos (2020), where a model in which the lensing and growth effective Newton's constants G_L and $G_{\rm eff}$ weaken was studied in this context. Similar hints were found in related works, including Nesseris et al. (2017), Kazantzidis & Perivolaropoulos (2018, 2019), Perivolaropoulos & Kazantzidis (2019).

It is also worth noting that similar hints in E_G data were found in Pullen et al. (2016), where combining *Planck* 2015 CMB lensing maps and the galaxy velocity field reconstructed from the BOSS DR11 CMASS sample, an E_G measurement of $E_G(z = 0.57) = 0.243 \pm 0.060 \pm 0.013$ was found, discrepant at the 2.6σ level from the GR expectation of $E_G(z = 0.57) = 0.402 \pm 0.012$ given the *Planck* and BOSS measurements. Possible systematic errors were studied in detail and found to be subdominant compared to the statistical error, and in any case unable to restore agreement with GR (see fig. 11 of Pullen et al. 2016). It is worth noting that the later related work of Alam et al. (2017a) finds no evidence for these deviations.

We note that possible tensions between E_G measurements and *Planck* might be related to the '*lensing is low*' (LIL) problem. This amounts to the observation that galaxy clustering measurements, together with standard galaxy–halo connection models, predict a galaxy–galaxy lensing signal which is higher by $\simeq$ 20–40 per cent compared to observations (Leauthaud et al. 2017). Possible explanations for the LIL problem range from an incomplete/incorrect galaxy–halo connection model, baryonic physics, additional systematics, or new physics (see e.g. Lange et al. 2019; Yuan, Eisenstein & Leauthaud 2020; Zu 2020; Lange et al. 2021; Yuan et al. 2021). However, none of the proposed scenarios have been fully able to address the problem. There is also some debate as to how much do uncertainties on photometric redshifts impact or bias the inferred S_8, and therefore the discrepancy with CMB measurements (see e.g. Joudaki et al. 2017a; Efstathiou & Lemos 2018). We also note that measurements of the cross-correlation between CMB lensing and galaxy overdensities have systematically been reporting evidence of a deficit of power on large scales (see e.g. Liu & Hill 2015; Kuntz 2015; Giannantonio et al. 2016; Pullen et al. 2016; Giusarma et al. 2018). While this lack of power might be related to the LIL problem, it might also be at least partially due to contamination from the thermal Sunyaev–Zel'dovich effect (see e.g. Darwish et al. 2020).

In view of these possible problems with E_G measurements, we caution the reader against over-interpreting the results obtained from the *RSD* + E_G data set combination, and to consider our *RSD* + *BAO* + *Pantheon*(+ *CC*) results as being the baseline ones. At the same time, it is worth noting that E_G and WL measurements are closely related – in fact, all our E_G measurements were obtained from analyses which made use of WL data (from RSCLenS, CFHTLenS, and KiDS-450). In this sense, the *RSD* + E_G combination does not allow us to assess the status of the S_8 discrepancy in a way which is completely independent of WL surveys. On the other hand, this can be achieved by the *RSD* + *BAO* + *Pantheon*(+ *CC*) data set combination(s), which is one of the reasons why we invite the reader to consider the results obtained from the latter as being our baseline ones.

Given these tensions, we also do not combine the *RSD* + E_G and *BAO* + *Pantheon*(+ *CC*) data sets. In closing we finally note that, while most independent analyses infer values of $\Omega_{\rm m}$ in the ballpark of $\simeq 0.3$, a few analyses do infer rather low values of $\Omega_{\rm m}$: these include a combination of DES cluster counts and WL inferring $\Omega_{\rm m} = 0.179^{+0.031}_{-0.038}$ (Abbott et al. 2020), as well as the KiDS-450+2dFLenS and KiDS-450+2dFLenS + GAMA analyses, which infer $\Omega_{\rm m} = 0.23^{+0.038}_{-0.038}$ (Joudaki et al. 2018) and $\Omega_{\rm m} = 0.25^{+0.03}_{-0.03}$ (Amon et al. 2018), respectively, all in extremely strong tension with *Planck*. However, these studies themselves appear to suggest that the cause of these low values of $\Omega_{\rm m}$ can be tracked back, at least partially, to systematics. These systematics are argued to most likely concern the modelling of the WL signal rather than the cluster counts one, although adopting a higher richness threshold in the selection of clusters appears to reduce the tension with other probes (Abbott et al. 2020).

Finally, in order to investigate whether the tension between *RSD* + E_G and *Planck* is entirely or mostly due to the E_G data set and the possible problems discussed previously, we also consider the *RSD* data set alone. In this case, we still find rather low values of $\Omega_{\rm m} = 0.227^{+0.068}_{-0.033}$ and $S_8 = 0.734^{+0.036}_{-0.040}$. Using the 1D T_{S_8} and quadratic tension metrics, we find that *RSD* and *Planck* are in tension at the 2.8σ and 3.1σ levels, respectively. These results are in qualitative agreement with earlier works in Nesseris et al. (2017), Kazantzidis & Perivolaropoulos (2018, 2019), and Perivolaropoulos & Kazantzidis (2019), which also identified tensions between RSD and *Planck* measurements at the 2.5–3σ level which, if taken at face value, point towards a lack of gravitational power in structures on intermediate and small cosmological scales, which could indicate a time-dependent (weakening) gravitational constant.

Our results are summarized in Fig. 1, where we show the joint S_8–Ω_m constraints obtained from the different data set combinations considered, in the whisker plot of Fig. 2, where we compare our inferred values of S_8 to those inferred from a number of independent surveys (mentioned earlier in Section 1), and in Table 1, where we summarize our constraints and level of concordance/discordance between the data set combinations considered and *Planck*. In particular, from Fig. 2 we see that the values of S_8 we infer from our *RSD* + *BAO* + *Pantheon* and *RSD* + *BAO* + *Pantheon* + *CC* data set combinations, while in $\simeq 2\sigma$ agreement with *Planck*, are in better agreement with the value inferred from various WL surveys.

3.1 wCDM

Earlier we found the *RSD* + *BAO* + *Pantheon*(+ *CC*) data set combination to be in $\simeq 2\sigma$ agreement with *Planck*. While this level of agreement does not call for new physics, it is worth noting that the value of S_8 we inferred is none the less lower than that of *Planck* and moves in the direction of the value inferred from WL measurements. In this sense, we believe it is still worth exploring whether extended models may improve the agreement between these two data sets. With this in mind, we repeat the previous analysis for the wCDM model, where the DE EoS w is allowed to vary.

For the *RSD* + *BAO* + *Pantheon* + *CC* data set combination, we infer 68 per cent C.L. constraints of $w = -0.96 \pm 0.04$, $\Omega_{\rm m} = 0.293^{+0.008}_{-0.009}$, $\sigma_8 = 0.781 \pm 0.021$, and $S_8 = 0.775^{+0.027}_{-0.030}$. In particular, we find the inferred value of w to be in excellent agreement with the cosmological constant value $w = -1$. Moreover, using the quadratic estimator of equation (11), we find *RSD* + *BAO* + *Pantheon* + *CC* and *Planck* to be in agreement at 2.2σ within the wCDM model. Therefore, the extension allowing for w to vary has essentially left the level of concordance/discordance between these two probes unchanged compared to the value within the ΛCDM model discussed earlier. Dropping the *CC* data set leads to essentially identical results.

If we instead consider the *RSD* + E_G data set combination, we infer 68 per cent C.L. constraints of $w = -1.31^{+0.33}_{-0.15}$, $\Omega_{\rm m} = 0.209^{+0.017}_{-0.027}$, $\sigma_8 = 0.809^{+0.066}_{-0.047}$, and $S_8 = 0.670^{+0.037}_{-0.036}$ in line with the earlier results within ΛCDM supporting a lower matter density. While the inferred value of w is consistent with $w = -1$ within better than 1σ, we notice a curious trend towards phantom values $w < -1$. This is directly related to the preference for lower values of $\Omega_{\rm m}$, given the positive correlation between w and $\Omega_{\rm m}$. Using the quadratic estimator of equation (11), we find *RSD* + E_G and *Planck* to be in 3.5σ tension within the wCDM model. While this figure is significantly lower than the 5.3σ obtained earlier within ΛCDM, mostly by virtue of the larger error bars, the amount of tension between the two probes remains significant.

As for our earlier results, given the possible issues with the E_G measurements, we urge the reader to take our *RSD* + *BAO* + *Pantheon*(+ *CC*) results as baseline. With this in mind, the main result of this section is that our previous results obtained within the ΛCDM model, and in particular the inferred level of concordance between *Planck* and a combination of RSD and inverse distance ladder measurements, is stable against a minimal parameter space extension where the DE EoS w is allowed to vary. Freeing up w does not improve the level of agreement between these two probes.

4 CONCLUSIONS

The ΛCDM model is, without question, an extremely successful one. Despite its many successes there are persisting hints, in the form of cosmological tensions, that this model might be about to break down. However, before claiming the definitive failure of an otherwise extremely successful, albeit phenomenological model, it is important to check whether these hints persist when viewed from a different perspective.

In this spirit, we have re-assessed the S_8 discrepancy between CMB and WL probes of the amplitude of matter fluctuations. We have examined this tension from the point of view of RSD measurements of the growth rate, and more precisely of $f\sigma_8(z)$. A robust assessment of the RSDs take on the S_8 discrepancy cannot afford to leave out geometrical data in the form of BAO and high-z SNeIa measurements, given the importance of these measurements in constraining $\Omega_{\rm m}$.

The cosmological constraints we infer from our baseline combination of RSD, BAO, and *Pantheon* SNeIa data (eventually including CC measurements), and in particular the inferred value of S_8, are somewhat intermediate between the WL and *Planck* CMB results (although tending towards the former), as is visually shown by the two uppermost bars in Fig. 2. Using the tension metric defined in equation (11), we find the *RSD* + *BAO* + *Pantheon* combination to be in agreement with *Planck* at the 2.2σ level. From this perspective, the hints for a S_8 discrepancy from growth rate data, if any, could be ascribable to a statistical fluctuation. These results, though obtained adopting a more up-to-date set of RSD measurements, agree qualitatively with the earlier results of Efstathiou & Lemos (2018).

We have also combined RSD measurements with measurements of the E_G statistic, which measures a combination of gravitational lensing, galaxy clustering, and RSDs (Zhang et al. 2007). We have found the *RSD* + E_G combination to be in 5.3σ tension with *Planck* (see the third bar from the top of Fig. 2), ultimately due to the extremely low inferred value of $\Omega_{\rm m}$. We caution the reader against over-interpreting the results arising from the *RSD* + E_G combination, and to take the results coming from the *RSD* + *BAO* + *Pantheon* (+ *CC*) data set combination being our baseline ones, for reasons discussed in more depth in Section 3. Finally, we have examined the stability of our results against a minimal parameter space extension where we free the DE EoS w, and have found our results to be qualitatively unchanged.

Our initial goal was to answer the question: '*Is there evidence from data other than weak lensing measurements for the Planck* Λ*CDM cosmology overpredicting the amplitude of matter fluctuations at* $z \lesssim 1$?' From the perspective of growth rate measurements, the answer is that there are hints at the $\approx 2\sigma$ level, but no definitive evidence of a tension: in this sense, we believe it might still be too early to claim evidence for new physics in light of the S_8 discrepancy. We also note that new physics models constructed to alleviate the S_8 discrepancy should not do so at the expense of worsening the H_0 tension (and vice versa). It is noteworthy that many proposed models fail in doing so (see e.g. the discussion in Alestas & Perivolaropoulos 2021), suggesting that if the S_8 discrepancy does indeed call for new physics, a joint solution to the S_8 and H_0 tensions will likely involve a rather non-trivial physical scenario. Future more precise measurements from the CMB (Abazajian et al. 2016; Ade et al. 2019; Lee et al. 2019), growth rate (DESI Collaboration 2016; Ivezić et al. 2019; Weltman et al. 2020), and WL (Amendola et al. 2013a; DESI Collaboration 2016; Ivezić et al. 2019) sides will certainly shed more light on the issue, and will either confirm or disprove whether new physics is needed in this context.

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DATA AVAILABILITY

The data underlying this article will be shared on request to the corresponding author.

REFERENCES

Abazajian K. N. et al., 2016, preprint (arXiv:1610.02743)
Abbott T. M. C. et al., 2020, Phys. Rev. D, 102, 023509
Abellán G. F., Murgia R., Poulin V., Lavalle J., 2020, preprint (arXiv:2008.09615)
Abellán G. F., Murgia R., Poulin V., 2021, preprint (arXiv:2102.12498)
Addison G. E., Huang Y., Watts D. J., Bennett C. L., Halpern M., Hinshaw G., Weiland J. L., 2016, ApJ, 818, 132
Ade P. A. R. et al., 2016, A&A, 594, A24
Ade P. et al., 2019, J. Cosmol. Astropart. Phys., 02, 056
Adhikari S., Huterer D., 2019, J. Cosmol. Astropart. Phys., 1901, 036
Aghanim N. et al., 2020, A&A, 641, A6
Aiola S. et al., 2020, J. Cosmol. Astropart. Phys., 12, 047
Alam S., Miyatake H., More S., Ho S., Mandelbaum R., 2017a, MNRAS, 465, 4853
Alam S. et al., 2017b, MNRAS, 470, 2617
Alestas G., Perivolaropoulos L., 2021, MNRAS, 504, 3956
Amendola L. et al., 2013a, Living Rev. Relativ., 16, 6
Amendola L., Kunz M., Motta M., Saltas I. D., Sawicki I., 2013b, Phys. Rev. D, 87, 023501
Amon A. et al., 2018, MNRAS, 479, 3422
Archidiacono M., Hooper D. C., Murgia R., Bohr S., Lesgourgues J., Viel M., 2019, J. Cosmol. Astropart. Phys., 10, 055
Asgari M. et al., 2020, A&A, 634, A127
Asgari M. et al., 2021, A&A, 645, A104
Audren B., Lesgourgues J., Benabed K., Prunet S., 2013, J. Cosmol. Astropart. Phys., 02, 001
Aylor K., Joy M., Knox L., Millea M., Raghunathan S., Wu W. L. K., 2019, ApJ, 874, 4
Ballesteros G., Notari A., Rompineve F., 2020, J. Cosmol. Astropart. Phys., 11, 024
Barros B. J., Barreiro T., Koivisto T., Nunes N. J., 2020, Phys. Dark Universe, 30, 100616
Battye R. A., Moss A., 2014, Phys. Rev. Lett., 112, 051303
Benetti M., Graef L. L., Alcaniz J. S., 2018, J. Cosmol. Astropart. Phys., 07, 066
Benisty D., 2021, Phys. Dark Universe, 31, 100766
Bernal J. L., Verde L., Riess A. G., 2016, J. Cosmol. Astropart. Phys., 10, 019
Beutler F. et al., 2011, MNRAS, 416, 3017
Blake C. et al., 2012, MNRAS, 425, 405
Blake C. et al., 2013, MNRAS, 436, 3089
Blake C. et al., 2016, MNRAS, 456, 2806
Blake C. et al., 2020, A&A, 642, A158
Blas D., Lesgourgues J., Tram T., 2011, J. Cosmol. Astropart. Phys., 07, 034
Blomqvist M. et al., 2019, A&A, 629, A86
Braglia M., Ballardini M., Emond W. T., Finelli F., Gumrukcuoglu A. E., Koyama K., Paoletti D., 2020, Phys. Rev. D, 102, 023529
Brinckmann T., Hyeok Chang J., LoVerde M., 2020, preprint (arXiv:2012.11830)
Brinckmann T., Lesgourgues J., 2019, Phys. Dark Universe, 24, 100260
Buen-Abad M. A., Schmaltz M., Lesgourgues J., Brinckmann T., 2018, J. Cosmol. Astropart. Phys., 01, 008
Camera S., Martinelli M., Bertacca D., 2019, Phys. Dark Universe, 23, 100247
Cao S., Ryan J., Ratra B., 2021, MNRAS, 504, 300
Chamings F. N., Avgoustidis A., Copeland E. J., Green A. M., Pourtsidou A., 2020, Phys. Rev. D, 101, 043531
Chen A. et al., 2021, Phys. Rev. D, 103, 123528
Choi G., Yanagida T. T., Yokozaki N., 2021, J. High Energy Phys., 01, 127
Choudhury S. R., Hannestad S., Tram T., 2021, JACP, 2021, 84
Chuang C.-H. et al., 2016, MNRAS, 461, 3781
Chudaykin A., Dolgikh K., Ivanov M. M., 2021, Phys. Rev. D, 103, 023507
Chudaykin A., Gorbunov D., Tkachev I., 2018, Phys. Rev. D, 97, 083508
Darwish O. et al., 2020, MNRAS, 500, 2250
Das A., Ghosh S., 2020, preprint (arXiv:2011.12315)
Davis M., Nusser A., Masters K., Springob C., Huchra J. P., Lemson G., 2011, MNRAS, 413, 2906
De Felice A., Mukohyama S., 2017, Phys. Rev. Lett., 118, 091104
De Felice A., Mukohyama S., Pookkillath M. C., 2021, Phys. Lett. B, 816, 136201
De Felice A., Nakamura S., Tsujikawa S., 2020, Phys. Rev. D, 102 063531
de la Torre S. et al., 2017, A&A, 608, A44
de Sainte Agathe V. et al., 2019, A&A, 629, A85
DESI Collaboration, 2016, preprint (arXiv:1611.00036)

Di Valentino E. et al., 2021a, Astropart. Phys., 131, 102605
Di Valentino E. et al., 2021b, preprint (arXiv:2103.01183)
Di Valentino E. et al., 2021c, Astropart. Phys., 131, 102604
Di Valentino E., Bøehm C., Hivon E., Bouchet F. R., 2018, Phys. Rev. D, 97, 043513
Di Valentino E., Bridle S., 2018, Symmetry, 10, 585
Di Valentino E., Linder E. V., Melchiorri A., 2020a, Phys. Dark Universe, 30, 100733
Di Valentino E., Melchiorri A., Mena O., Vagnozzi S., 2020b, Phys. Dark Universe, 30, 100666
Di Valentino E., Melchiorri A., Silk J., 2019a, Nat. Astron., 4, 196
Di Valentino E., Melchiorri A., Mena O., Vagnozzi S., 2019b, Phys. Rev. D, 101, 063502
Di Valentino E., Melchiorri A., Silk J., 2020, J. Cosmol. Astropart. Phys., 01, 013
Di Valentino E., Melchiorri A., Silk J., 2021, ApJ, 908, L9
Dossett J. N., Ishak M., Parkinson D., Davis T., 2015, Phys. Rev. D, 92, 023003
Dutta K., Ruchika Roy A., Sen A. A., Sheikh-Jabbari M. M., 2020, Gen. Relativ. Gravit., 52, 15
eBOSS Collaboration, 2021, Phys. Rev. D, 103, 083533
Efstathiou G., 2020, preprint (arXiv:2007.10716)
Efstathiou G., 2021, MNRAS, 505, 3866
Efstathiou G., Gratton S., 2020, MNRAS, 496, L91
Efstathiou G., Lemos P., 2018, MNRAS, 476, 151
Enqvist K., Nadathur S., Sekiguchi T., Takahashi T., 2015, J. Cosmol. Astropart. Phys., 09, 067
Feix M., Nusser A., Branchini E., 2015, Phys. Rev. Lett., 115, 011301
Feng L., Zhang J.-F., Zhang X., 2017, Eur. Phys. J. C, 77, 418
Freedman W. L. et al., 2019, ApJ, 882, 34
Garcia-Quintero C., Ishak M., Fox L., Lin W., 2019, Phys. Rev., D100, 123538
Garcia-Quintero C., Ishak M., Ning O., 2020, J. Cosmol. Astropart. Phys., 12, 018
Gariazzo S., Escudero M., Diamanti R., Mena O., 2017, Phys. Rev. D, 96, 043501
Gelman A., Rubin D. B., 1992, Stat. Sci., 7, 457
Giannantonio T. et al., 2016, MNRAS, 456, 3213
Giusarma E., Vagnozzi S., Ho S., Ferraro S., Freese K., Kamen-Rubio R., Luk K.-B., 2018, Phys. Rev. D, 98, 123526
Hamana T. et al., 2020, PASJ, 72, 16
Handley W., 2021, Phys. Rev. D, 103, L041301
Handley W., Lemos P., 2019a, Phys. Rev., D100, 023512
Handley W., Lemos P., 2019b, Phys. Rev., D100, 043504
Heimersheim S., Schöneberg N., Hooper D. C., Lesgourgues J., 2020, J. Cosmol. Astropart. Phys., 12, 016
Heymans C. et al., 2021, A&A, 646, A140
Hildebrandt H. et al., 2017, MNRAS, 465, 1454
Hildebrandt H. et al., 2020, A&A, 633, A69
Hill J. C., McDonough E., Toomey M. W., Alexander S., 2020, Phys. Rev. D, 102, 043507
Hlozek R., Grin D., Marsh D. J. E., Ferreira P. G., 2015, Phys. Rev. D, 91, 103512
Howlett C., Ross A., Samushia L., Percival W., Manera M., 2015, MNRAS, 449, 848
Hudson M. J., Turnbull S. J., 2013, ApJ, 751, L30
Huterer D., Shafer D., Scolnic D., Schmidt F., 2017, J. Cosmol. Astropart. Phys., 05, 015
Ivanov M. M., Simonović M., Zaldarriaga M., 2020, J. Cosmol. Astropart. Phys., 05, 042
Ivezić v. et al., 2019, ApJ, 873, 111
Jiménez J. B., Bettoni D., Figueruelo D., Teppa Pannia F. A., 2020, J. Cosmol. Astropart. Phys., 08, 020
Jimenez R., Loeb A., 2002, ApJ, 573, 37
Jimenez R., Verde L., Treu T., Stern D., 2003, ApJ, 593, 622
Joudaki S. et al., 2017a, MNRAS, 465, 2033
Joudaki S. et al., 2017b, MNRAS, 471, 1259
Joudaki S. et al., 2018, MNRAS, 474, 4894
Joudaki S. et al., 2020, A&A, 638, L1
Kaiser N., 1987, MNRAS, 227, 1
Karpenka N. V., Feroz F., Hobson M. P., 2015, MNRAS, 449, 2405
Kazantzidis L., Perivolaropoulos L., 2018, Phys. Rev. D, 97, 103503
Kazantzidis L., Perivolaropoulos L., 2019, preprint (arXiv:1907.03176)
Knox L., Millea M., 2020, Phys. Rev. D, 101, 043533
Kreisch C. D., Cyr-Racine F.-Y., Doré O., 2020, Phys. Rev. D, 101, 123505
Kumar S., Nunes R. C., 2016, Phys. Rev. D, 94, 123511
Kumar S., Nunes R. C., Yadav S. K., 2018, Phys. Rev. D, 98, 043521
Kumar S., Nunes R. C., Yadav S. K., 2019a, Eur. Phys. J. C, 79, 576
Kumar S., Nunes R. C., Yadav S. K., 2019b, MNRAS, 490, 1406
Kuntz A., 2015, A&A, 584, A53
Kunz M., Nesseris S., Sawicki I., 2015, Phys. Rev. D, 92, 063006
Lahav O., Lilje P. B., Primack J. R., Rees M. J., 1991, MNRAS, 251, 128
Lambiase G., Mohanty S., Narang A., Parashari P., 2019, Eur. Phys. J. C, 79, 141
Lange J. U., Leauthaud A., Singh S., Guo H., Zhou R., Smith T. L., Cyr-Racine F.-Y., 2021, MNRAS, 502, 2074
Lange J. U., Yang X., Guo H., Luo W., van den Bosch F. C., 2019, MNRAS, 488, 5771
Leauthaud A. et al., 2017, MNRAS, 467, 3024
Lee A. et al., 2019, Bull. Am. Astron. Soc., 51, 147
Lemos P., Köhlinger F., Handley W., Joachimi B., Whiteway L., Lahav O., 2020, MNRAS, 496, 4647
Lemos P., Lee E., Efstathiou G., Gratton S., 2019, MNRAS, 483, 4803
Leonard C. D., Ferreira P. G., Heymans C., 2015, J. Cosmol. Astropart. Phys., 12, 051
Lesgourgues J., 2011, preprint (arXiv:1104.2932)
Li E.-K., Du M., Zhou Z.-H., Zhang H., Xu L., 2021, MNRAS, 501, 4452
Lin W., Ishak M., 2017a, Phys. Rev., D96, 023532
Lin W., Ishak M., 2017b, Phys. Rev., D96, 083532
Liu J., Hill J. C., 2015, Phys. Rev. D, 92, 063517
MacCrann N., Zuntz J., Bridle S., Jain B., Becker M. R., 2015, MNRAS, 451, 2877
Marra V., Perivolaropoulos L., 2021, preprint (arXiv:2102.06012)
Mccarthy I. G., Bird S., Schaye J., Harnois-Deraps J., Font A. S., Van Waerbeke L., 2018, MNRAS, 476, 2999
Moresco M. et al., 2016, J. Cosmol. Astropart. Phys., 05, 014
Moresco M., 2015, MNRAS, 450, L16
Moresco M., Verde L., Pozzetti L., Jimenez R., Cimatti A., 2012, J. Cosmol. Astropart. Phys., 07, 053
Mörtsell E., Dhawan S., 2018, J. Cosmol. Astropart. Phys., 09, 025
Mossa V. et al., 2020, Nature, 587, 210
Nesseris S., Pantazis G., Perivolaropoulos L., 2017, Phys. Rev. D, 96, 023542
Nicola A., Amara A., Refregier A., 2019, J. Cosmol. Astropart. Phys., 1901, 011
Okumura T. et al., 2016, Publ. Astron. Soc. Japan, 68, 38
Pandey K. L., Karwal T., Das S., 2020, J. Cosmol. Astropart. Phys., 07, 026
Park C.-G., Ratra B., 2019, Astrophys. Space Sci., 364, 134
Perivolaropoulos L., Kazantzidis L., 2019, Int. J. Mod. Phys. D, 28, 1942001
Perlmutter S. et al., 1999, ApJ, 517, 565
Pezzotta A. et al., 2017, A&A, 604, A33
Pinho A. M., Casas S., Amendola L., 2018, J. Cosmol. Astropart. Phys., 11, 027
Poulin V., Boddy K. K., Bird S., Kamionkowski M., 2018, Phys. Rev. D, 97, 123504
Poulin V., Smith T. L., Karwal T., Kamionkowski M., 2019, Phys. Rev. Lett., 122, 221301
Pourtsidou A., Tram T., 2016, Phys. Rev. D, 94, 043518
Pullen A. R., Alam S., He S., Ho S., 2016, MNRAS, 460, 4098
Pullen A. R., Alam S., Ho S., 2015, MNRAS, 449, 4326
Quelle A., Maroto A. L., 2020, Eur. Phys. J. C, 80, 369
Ratsimbazafy A., Loubser S., Crawford S., Cress C., Bassett B., Nichol R., Väisänen P., 2017, MNRAS, 467, 3239
Raveri M., Hu W., 2019, Phys. Rev., D99, 043506
Raveri M., Zacharegkas G., Hu W., 2020, Phys. Rev. D, 101, 103527
Reyes R., Mandelbaum R., Seljak U., Baldauf T., Gunn J. E., Lombriser L., Smith R. E., 2010, Nature, 464, 256
Riess A. G. et al., 1998, AJ, 116, 1009

Riess A. G., Casertano S., Yuan W., Macri L. M., Scolnic D., 2019, AJ, 876, 85
Ross A. J., Samushia L., Howlett C., Percival W. J., Burden A., Manera M., 2015, MNRAS, 449, 835
Ryan J., Doshi S., Ratra B., 2018, MNRAS, 480, 759
Sagredo B., Nesseris S., Sapone D., 2018, Phys. Rev. D, 98, 083543
Sakr Z., Ilić S., Blanchard A., Bittar J., Farah W., 2018, A&A, 620, A78
Sakstein J., Trodden M., 2020, Phys. Rev. Lett., 124, 161301
Samushia L., Percival W. J., Raccanelli A., 2012, MNRAS, 420, 2102
Sanchez A. G. et al., 2014, MNRAS, 440, 2692
Schöneberg N., Lesgourgues J., Hooper D. C., 2019, J. Cosmol. Astropart. Phys., 10, 029
Scolnic D. M. et al., 2018, ApJ, 859, 101
Simon J., Verde L., Jimenez R., 2005, Phys. Rev. D, 71, 123001
Singh S., Alam S., Mandelbaum R., Seljak U., Rodriguez-Torres S., Ho S., 2019, MNRAS, 482, 785
Skara F., Perivolaropoulos L., 2020, Phys. Rev. D, 101, 063521
Song Y.-S., Percival W. J., 2009, J. Cosmol. Astropart. Phys., 10, 004
Stern D., Jimenez R., Verde L., Kamionkowski M., Stanford S., 2010, J. Cosmol. Astropart. Phys., 02, 008
Tröster T. et al., 2020, A&A, 633, L10
Troxel M. A. et al., 2018, Phys. Rev. D, 98, 043528
Turnbull S. J., Hudson M. J., Feldman H. A., Hicken M., Kirshner R. P., Watkins R., 2012, MNRAS, 420, 447
Vagnozzi S., 2020, Phys. Rev. D, 102, 023518
Vagnozzi S., Dhawan S., Gerbino M., Freese K., Goobar A., Mena O., 2018, Phys. Rev. D, 98, 083501
Vagnozzi S., Di Valentino E., Gariazzo S., Melchiorri A., Mena O., Silk J., 2020, preprint (arXiv:2010.02230)
Vagnozzi S., Giusarma E., Mena O., Freese K., Gerbino M., Ho S., Lattanzi M., 2017, Phys. Rev. D, 96, 123503
Vagnozzi S., Loeb A., Moresco M., 2021, ApJ, 908, 84
Vagnozzi S., Visinelli L., Mena O., Mota D. F., 2019, MNRAS, 493, 1139
van Uitert E. et al., 2018, MNRAS, 476, 4662
Verde L., Treu T., Riess A. G., 2019, Nat. Astron., 3, 891
Visinelli L., Vagnozzi S., Danielsson U., 2019, Symmetry, 11, 1035
Weltman A. et al., 2020, Publ. Astron. Soc. Aust., 37, e002
Wong K. C. et al., 2020, MNRAS, 498, 1420
Xiao L., Zhang L., An R., Feng C., Wang B., 2020, J. Cosmol. Astropart. Phys., 01, 045
Yuan S., Eisenstein D. J., Leauthaud A., 2020, MNRAS, 493, 5551
Yuan S., Hadzhiyska B., Bose S., Eisenstein D. J., Guo H., 2021, MNRAS, 502, 3582
Zhang C., Zhang H., Yuan S., Zhang T.-J., Sun Y.-C., 2014, Res. Astron. Astrophys., 14, 1221
Zhang P., Liguori M., Bean R., Dodelson S., 2007, Phys. Rev. Lett., 99, 141302
Zhang Y. et al., 2021, MNRAS, 501, 1013
Zhao G.-B. et al., 2019, MNRAS, 482, 3497
Zu Y., 2020, preprint (arXiv:2010.01143)
Zubeldia I. n., Challinor A., 2019, MNRAS, 489, 401
Zumalacarregui M., 2020, Phys. Rev. D, 102, 023523

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